GPD factorization at large t

Jakob Schönleber

Technical University Munich

Based on [Y. Hatta, JS, 2025]

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- GPD $H_q(x,\xi,t)$ depends on the momentum scale t. If $Q \equiv \sqrt{-t} \gg \Lambda \equiv \Lambda_{\rm QCD}$ we might expect to have factorization (schematically)

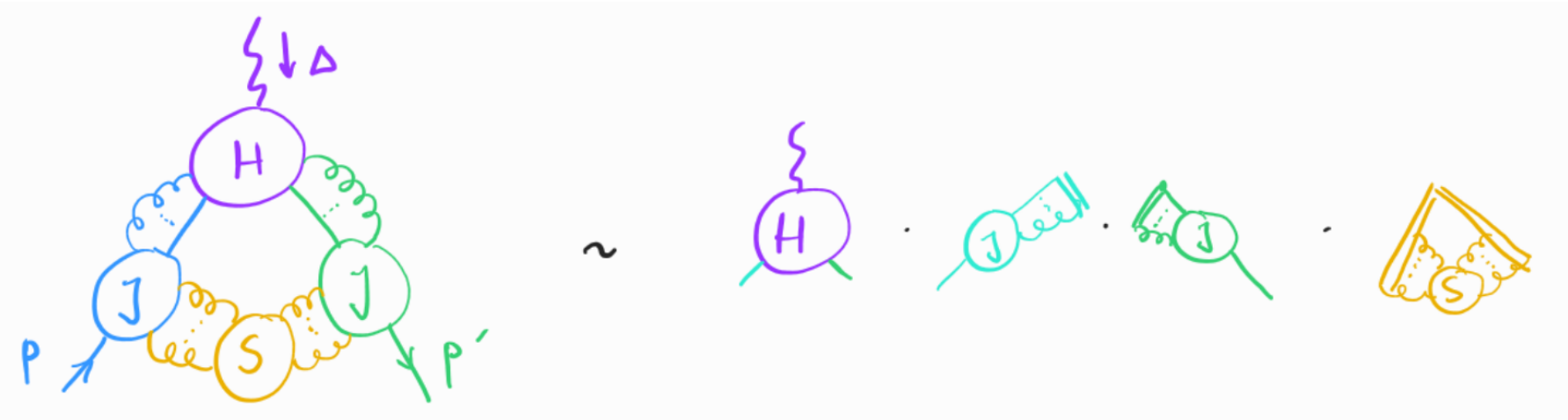
$$H_q(t) \stackrel{?}{=} K(t,\mu) \otimes f(\mu)(1+\mathcal{O}(\Lambda/Q))$$

- Form factors, E&M F_1 and GFF $A_q, \ldots$ are moments of the GPD, so a similar factorization should hold
- For $F_1 = \int dx\, H$ this is an old problem going back to the $70's$
- Form factors involve an incoming and outgoing hadron states

$$F_1(Q^2) = \langle p' | \, \bar{\psi} \gamma^\mu \psi \, | p \rangle \,, \qquad t = -Q^2 = (p'-p)^2$$

- If Q^2 is large the hadron has to "turn", which means that all large momentum partons have to participate in the hard interaction and all remaining partons must be "soft"
- Focus here on the **nucleon**

- In the unphysical case of massless and off-shell external quark states we have the factorization of the Sudakov form factor $F = HJJ^*S + \mathcal{O}(1/Q)$.

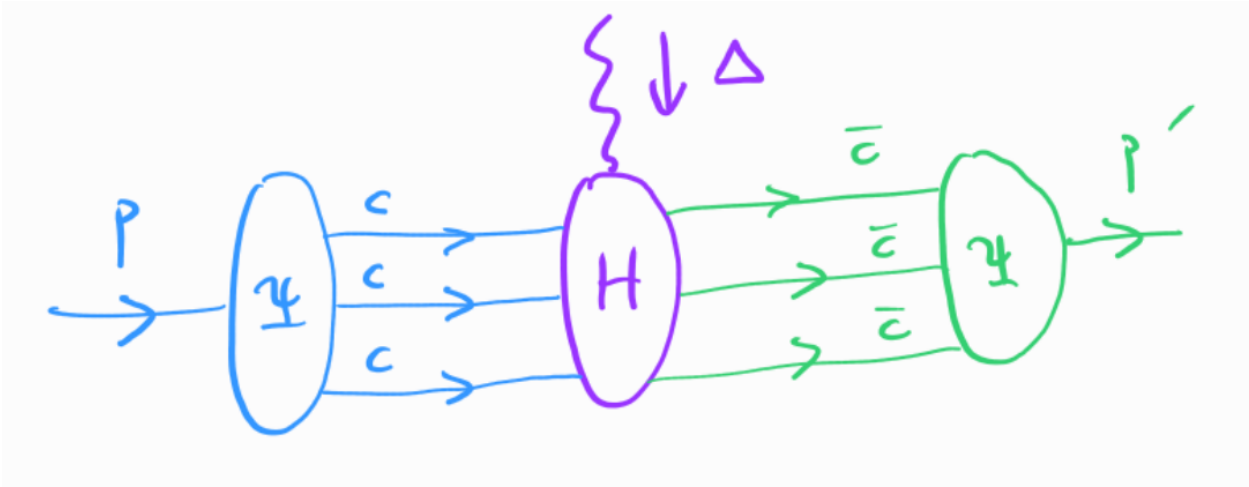


- External quarks states (in particular the functions $J = \langle 0 | \, q \, | p \rangle$) have no meaning beyond fixed order perturbation theory
- Does not easily generalize to the nucleon

- Earliest discussions with pQCD [Efremov, Radyushkin, 1976], [Brodsky, Lepage, 1980]: Claim that leading contribution comes from 3 valence (collinear) quark Fock space component:

$$F(Q^2) = h_q \otimes \Psi \otimes \Psi^* \, (1 + \mathcal{O}(1/Q)) \sim \alpha_s(Q)^2/Q^4$$

"Hard scattering contribution"



- Momentum modes

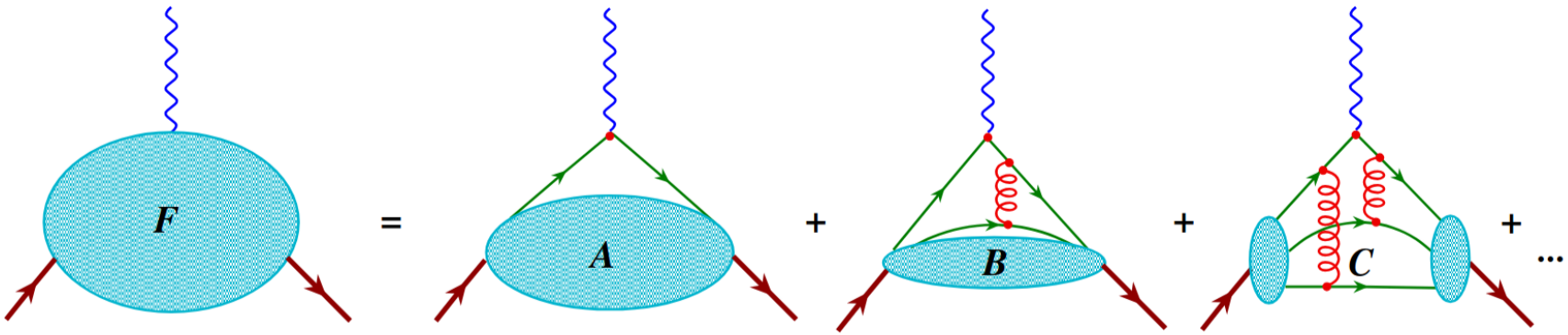
collinear "c" $(\overset{+}{Q}, \Lambda^2/Q, \overset{\perp}{\Lambda})\,,$ anti-collinear "$\bar{c}$" $(\Lambda^2/Q, \overset{-}{Q}, \overset{\perp}{\Lambda})$

This is **not** the standard GPD frame where both in- and outgoing proton are collinear in the same direction

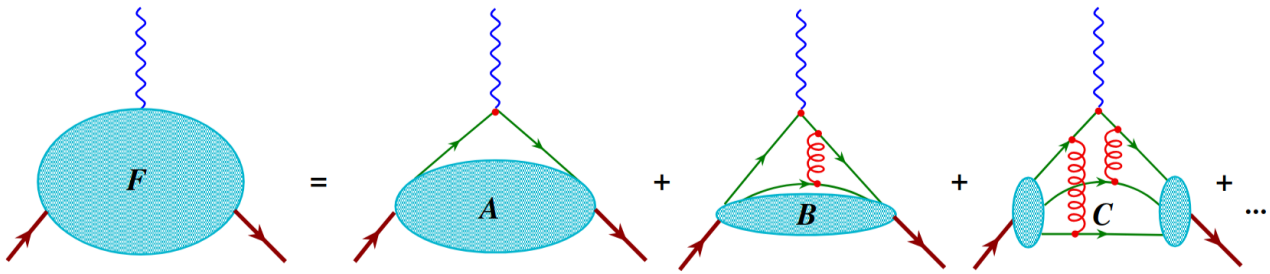
- $\Psi \simeq \langle 0| \, u \, u \, d \, |p\rangle$ are baryon distribution amplitudes

- Problems with the hard scattering contribution:
  - It does not describe the data well at moderately (experimentally accessible) values $-t \lesssim 10\,\text{GeV}^2$ [Isgur, Llewellyn Smith, 1989], [Radyushkin, 1998]
  - There are soft divergences starting at two-loop α_s^4/Q^4 order which break "collinear factorization" (more on that later)
- It was realized that other contributions are also important (or even more important). Summary from [Braun, Lenz, Wittmann, 2006]:

$$F_1(Q^2) \simeq \mathcal{A}(Q^2) + \frac{\alpha_s}{Q^2}\mathcal{B}(Q^2) + \frac{\alpha_s^2}{Q^4}\mathcal{C}(Q^2) + \ldots$$



$$F_1(Q^2) \simeq \mathcal{A}(Q^2) + \frac{\alpha_s}{Q^2}\mathcal{B}(Q^2) + \frac{\alpha_s^2}{Q^4}\mathcal{C}(Q^2) + \ldots$$



- From [Braun, Lenz, Wittmann, 2006]: "*Various estimates suggest that* $\mathcal{A}(Q^2) \lesssim 1/Q^6$ *and* $\mathcal{B}(Q^2) \lesssim 1/Q^4$ *[and* $\mathcal{C}(Q^2) \sim Q^0$*]* "
- It was suggested (from phenomenology) in [Isgur, Llewellyn Smith, 1989], [Radyushkin, 1998] that at experimentally accessible values $-t \lesssim 10\,\text{GeV}^2$ the form factor is dominated by the $\mathcal{A}$ contribution.
- In fact, $\mathcal{A}$ was already mentioned in pre QCD times, going back to Feynman. It is called "**Feynman contribution**" or "soft mechanism"

- Since soft spectator quarks are soft

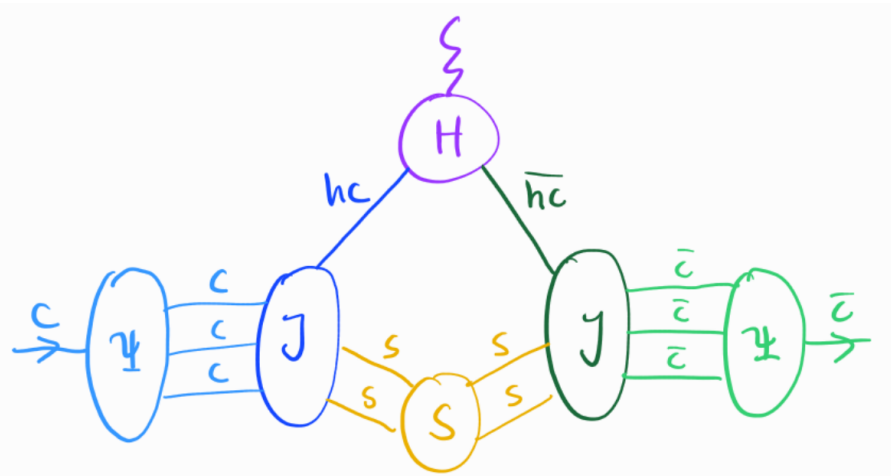
$$s \sim (\Lambda, \Lambda, \Lambda)$$

active partons are not c and $\bar{c}$ but hard-collinear hc and hard-anti-collinear $\overline{hc}$

$$hc \sim (Q, \Lambda, \sqrt{Q\Lambda} \text{ or } \Lambda), \qquad \overline{hc} \sim (\Lambda, Q, \sqrt{Q\Lambda} \text{ or } \Lambda)$$

- This introduces a new "intermediate" scale ΛQ, which is virtuality of the active quarks
- Corresponding to the hierarchy $Q \gg \sqrt{\Lambda Q} \gg \Lambda$ we have the further factorization [Kivel, Vanderhaegen, 2010]

$$\mathcal{A}(Q^2) = C \otimes J \otimes J^* \otimes S \otimes \Psi \otimes \Psi^* (1 + \mathcal{O}(\sqrt{\Lambda/Q})) \sim \frac{\alpha_s(\sqrt{\Lambda Q})^4}{Q^4}$$



- In practice we have to distinguish kinematical regions
  - **asymptotic region**

$$10 - 25\,\text{GeV}^2 \ll Q^2 \to \infty$$

    The $Q\Lambda \gg 1\,\text{GeV}^2$ is a perturbative scale. Less relevant for phenomenology.
  - **pre-asymptotic region**

$$1\,\text{GeV}^2 \ll Q^2 \lesssim 10 - 25\,\text{GeV}^2$$

    Then $Q\Lambda \lesssim 1\,\text{GeV}^2$
  - **non-perturbative region**

$$Q^2 \lesssim 1\,\text{GeV}^2$$

    No large-t factorization.
- We are interested in the pre-asymptotic region. Since $\alpha_s(\sqrt{Q\Lambda})$ is not small, $\mathcal{A}$ should not be further factorized beyond the simple Sudakov, i.e. integrate out only the modes with virtuality between Q^2 and ΛQ
- *We assume, however, that power estimates still hold*

- In **Soft-collinear effective theory** (SCET) we have

$$\mathcal{A} = C(Q^2, \mu) \times \underbrace{\langle p' | \, \bar{\chi}_{\overline{hc}} \gamma_\perp^\mu \chi_{hc} \, | p \rangle \, (\Lambda Q, \mu)}_{\equiv f_1^q(\Lambda Q, \mu)},$$

  where χ_{hc} is the hard-collinear SCET quark field with a hard-collinear Wilson lines attached for gauge invariance

- $f_1^q = f_1^q(t, \mu)$ is a non-perturbative function. It still has an intrinsic t dependence!

- C is the matching coefficient of the Sudakov form factor (known to three loops)

$$C(Q^2, \mu) = 1 + \frac{\alpha_s C_F}{4\pi} \left(-\ln^2 \frac{Q^2}{\mu^2} + 3 \ln \frac{Q^2}{\mu^2} - 8 + \frac{\pi^2}{6} \right) + \mathcal{O}(\alpha_s^2)$$

- Now consider GPDs. The hard scattering contribution was considered in [Hoodbhoy, Ji, Yuan, 2003]. For GFFs [Tong, Ma, Yuan, 2022]
- [Hatta, JS, 2025]: Feynman contribution to GPDs/GFFs
- Going to GPD case essentially means that we replace the local E&M current with a non-local current separated by light-like distance $z^2 = 0$

$$\bar{\psi}(0)\gamma^\mu\psi(0) \longrightarrow \bar{\psi}(-z/2)\gamma^\mu[-z/2, z/2]\psi(z/2)$$

- By definition, at large t, z must not be along the c or $\bar{c}$ directions of the in- and outgoing protons. This means that the IR structure can not depend on the length of z, so that only the hard matching kernel gets modified. In momentum space:

$$C(t, \mu) \to K(x, \xi, t, \mu)$$

- One-loop is easy to calculate

$$K_{qq}(x,\xi,t) = \delta(1-x) + \frac{\alpha_s C_F}{2\pi}\theta(x-\xi)\left[\frac{\delta(1-x)}{2}\left\{-\ln^2\frac{\mu_F^2}{-t} - 3\ln\frac{\mu_F^2}{-t} + \frac{\pi^2}{6} - \ln^2\frac{1+\xi}{1-\xi}\right\}\right.$$
$$+ \ln\frac{\mu_{\rm UV}^2}{-t}\left\{\frac{1+x^2-2\xi^2}{1-\xi^2}\left[\frac{1}{1-x}\right]_+^{\xi} + \delta(1-x)\left(\frac{3}{2} - \ln\frac{1+\xi}{1-\xi}\right)\right\}$$
$$+ \frac{(1+x^2-2\xi^2)\ln(1-\xi^2) - (1-x)^2}{1-\xi^2}\left[\frac{1}{1-x}\right]_+^{\xi} - 2\frac{1+x^2-2\xi^2}{1-\xi^2}\left[\frac{\ln(1-x)}{1-x}\right]_+^{\xi}\Bigg]$$
$$+ \frac{\alpha_s C_F}{2\pi}\theta(|\xi|-x)\left\{\ln\frac{\mu_{\rm UV}^2}{-t}\frac{x+\xi}{1+\xi}\left(\frac{1}{2\xi} + \frac{1}{1-x}\right) - \frac{x+\xi}{2\xi(1+\xi)}\right.$$
$$+ \frac{1}{2\xi(1-x)(1-\xi^2)}\left((1-x)(x+\xi)(1+\xi)\ln\left(\frac{4\xi^2}{\xi^2-x^2}\right) + 2\xi x^2\ln\frac{2\xi(1+\xi)}{(x+\xi)(1-x)}\right.$$
$$\left.\left. + 2\xi^3\ln\left(\frac{\xi+x}{\xi-x}\right) + 2\xi\ln\left(\frac{(1+\xi)(\xi-x)}{2\xi(1-x)}\right) + 4\xi^3\ln\left(\frac{1-x}{1+\xi}\right)\right)\right\} + \mathcal{O}(\alpha_s^2).$$

- K_{qq} experiences Sudakov suppression

$$K_{qq}(Q^2, \mu_F = \sqrt{\Lambda Q}) = K_{qq}(Q^2, \mu_F = Q) \quad \underbrace{U(Q, \sqrt{Q\Lambda})}_{= \exp\left(-\frac{\alpha_s C_F}{8\pi}\ln^2\frac{Q^2}{\Lambda^2} + \ldots\right)} ,$$

$\Rightarrow$ Feynman contribution is subleading at asymptotically large $Q^2 \to \infty$ relative to $H \otimes \Psi \otimes \Psi^*$. In pre-asymptotic region, U is not very small, so no substantial Sudakov suppression

- Factorization is multiplicative

$$H_q^{\text{Feyn}}(x,\xi,t) = \underbrace{K_{qq}(x,\xi,t,\mu)}_{\delta(1-x)+\mathcal{O}(\alpha_s(Q))}\ f_1^q(t,\mu) + \mathcal{O}(1/Q)$$

- t dependence does not fully factorize, **but x dependence does!**
- The same f_1^q appears in the E&M form factor (After all $F_1 = \int dx\, H_q$)
- Any ratio of moments (of the Feynman contribution) are perturbatively calculable (up to power corrections). In particular

$$\frac{A_q^{\text{Feyn}}(Q^2,\mu_{\text{UV}})}{F_{1q}^{\text{Feyn}}(Q^2)} = 1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{8}{3}\ln\frac{Q^2}{\mu_{\text{UV}}^2} - \frac{52}{9}\right) + \mathcal{O}(\alpha_s^2) + \underbrace{\mathcal{O}(1/Q)}_{\text{more next slide}}$$

$$\overset{Q^2\approx 4\text{GeV}^2}{\approx} 0.8 \quad \text{to} \quad 0.9$$

- Rough large Q^2 extrapolations from data and lattice exist [Dutrieux et al., 2026]

$$0.54 \lesssim \frac{A_q(Q^2)}{F_{1q}(Q^2)} \lesssim 0.96$$

Next-to-leading corrections

- Biggest power correction: "soft kinematic correction"

$$\langle P_2|O^q(z_2,z_1)|P_1\rangle = e^{-iz_1\bar{w}\cdot P_1+iz_2 w\cdot P_2}\langle P_2|O^q(0,0)|P_1\rangle(1+\mathcal{O}(1/Q))$$

- Physical origin: The active parton momentum fraction is not exactly unity

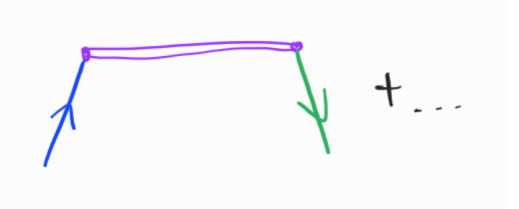
$$x = 1 - \underbrace{y}_{\sim\Lambda/Q}$$

- They can be taken heuristically taken into account using the overlap representation of the GPD in terms of light-cone wave-functions, see [Hatta, JS, 2025] $\rightarrow$ However, one then looses the universality

Next-to-leading corrections

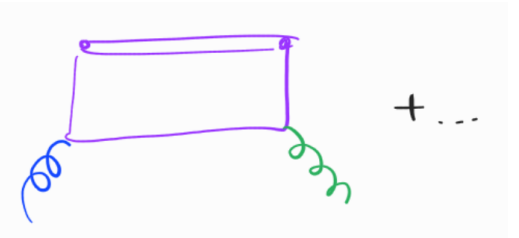
- SCET gives a systematic way of getting power estimates from various contributes. This gives

$$K_{qq}(x,\xi,t)\underbrace{\langle p'|\,\bar{\chi}_{\overline{hc}}\gamma_\perp^\mu\chi_{hc}\,|p\rangle}_{\equiv f_1^q}\sim\boxed{\alpha_s^0/Q^4}$$



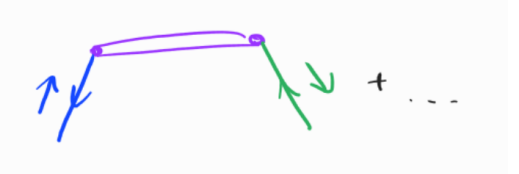
- Quark-in-gluon Feynman contribution to quark GPD

$$K_{qg}(x,\xi,t)\underbrace{\langle p'|\,F_{\overline{hc}}^{+\mu}F_{hc\mu}^{+}\,|p\rangle}_{\equiv f_1^g}\sim\boxed{\alpha_s^1/Q^5}$$



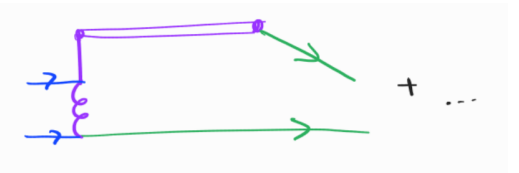
- Quark-in-antiquark Feynman contribution to quark GPD

$$K_{q\bar{q}}(x,\xi,t)\underbrace{\langle p'|\,\bar{\chi}_{hc}\gamma_\perp^\mu\chi_{\overline{hc}}\,|p\rangle}_{\equiv f_1^{\bar{q}}}\sim\boxed{\alpha_s^0/Q^6}$$



- Two active quarks and one soft spectator

$$K_{qqqq}(x,\xi,t)\,\langle p'|\,\bar{\chi}_{\overline{hc}}\bar{\chi}_{\bar{c}}\chi_{hc}\chi_c\,|p\rangle\sim\boxed{\alpha_s^1/Q^5}$$



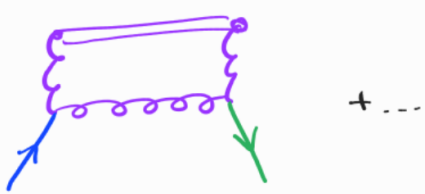
Gluon GPD

- Treatment should be analogous for gluon GPD, but the power estimates are different

$$H_g = H_g^{\text{Feyn}} + \underbrace{h_g \otimes \Psi \otimes \Psi^*}_{\sim \alpha_s^2/Q^4}$$

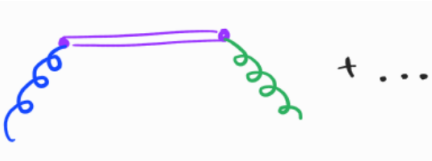
- SCET gives a systematic way of getting power estimates from various contributes. This gives

$$K_{gq}(x,\xi,t) \underbrace{\langle p' | \bar{\chi}_{\overline{hc}} \gamma_\perp^\mu \chi_{hc} | p \rangle}_{\equiv f_1^q} \sim \boxed{\alpha_s^1/Q^4}$$



- Gluon-in-gluon Feynman contribution to gluon GPD

$$K_{gg}(x,\xi,t) \underbrace{\langle p' | F_{\overline{hc}}^{+\mu} F_{hc\mu}^{+} | p \rangle}_{\equiv f_1^g} \sim \boxed{\alpha_s^0/Q^5}$$



- f_1^g contribution should be roughly of the same size as f_1^q contribution to H_g^{Feyn}

What about Mesons?

- Straightforward to perform the same analysis for mesons. Get

$$H_{q/M} = \underbrace{K_{qq} f_1^{q/M}}_{\text{Feynman contr.}} + \underbrace{h'_q \otimes \phi_M \otimes \phi_M^*}_{\text{hard scattering contr.}} + \dots ,$$

  where ϕ_M is the meson DA and $H_{q/M}$ is quark-in-meson GPD

- Important difference:

$$K_{qq} f_1^{q/M} \sim \frac{\alpha_s^0}{Q^3} \qquad \text{and} \qquad h'_q \otimes \phi_M \otimes \phi_M^* \sim \frac{\alpha_s^1}{Q^2}$$

  - Feynman contribution is power suppressed by $1/Q$ compared to hard scattering, but hard scattering contribution has additional α_s^1, so they a roughly of the same size
  - No soft divergences due to different power law behaviour

To summarize

- We have identified the following contributions ($\alpha_s \equiv \alpha_s(Q)$):
  - Nucleon:

$$H_q(x,\xi,t) \sim \underbrace{\frac{\alpha_s^2}{Q^4}}_{\text{hard}} + \underbrace{U}_{\text{Sudakov}} \Bigg(\underbrace{\frac{1}{Q^4}}_{\text{soft } qq} + \underbrace{\frac{1}{Q^5}}_{\text{soft } qq \text{ kin.pc.}} + \underbrace{\frac{\alpha_s}{Q^5}}_{\text{soft } qg} + \ldots \Bigg) + \ldots$$

$$H_g(x,\xi,t) \sim \underbrace{\frac{\alpha_s^2}{Q^4}}_{\text{hard}} + \underbrace{U}_{\text{Sudakov}} \Bigg(\underbrace{\frac{1}{Q^5}}_{\text{soft } gg} + \underbrace{\frac{\alpha_s}{Q^4}}_{\text{soft } gq} + \ldots \Bigg) + \ldots$$

  - Meson:

$$H_q(x,\xi,t) \sim \underbrace{\frac{\alpha_s}{Q^2}}_{\text{hard}} + \underbrace{U}_{\text{Sudakov}} \Bigg(\underbrace{\frac{1}{Q^3}}_{\text{soft } qq} + \ldots \Bigg) + \ldots$$

- In pre-asymptotic regime: soft/Feynman $>$ hard/ERBL for nucleon and soft/Feynman $\approx$ hard/ERBL for meson.

Final comments and conclusions

- Have fully characterized the full leading power factorization of the GPD at leading power in large-t limit, this includes the moments, such as E&M form factor F_1 and A GFFs
- Found that while t dependence does not fully factorize, the x (and ξ) dependence actually does $H_q^{\text{Feyn}}(x,\xi,t) \approx K_{qq}(x,\xi,t) f_1^q(t)$. This approximation may be decent for a sizable range (optimistic) of $4\,\text{GeV}^2 \lesssim Q^2 \lesssim 25\,\text{GeV}^2$
- At leading power the only non-perturbative input f_1^q can be extracted from $F_1^q(t) \approx C(t) f_1^q(t)$ data.
- One can make a connection to the traditional framework involving light-cone wave functions [Hatta, JS, 2025] (no time to go into this in this talk). This might be a further avenue to study $f_1^q(t)$.
- Unfortunately, for the gluon GPD another contribution involving another non-perturbative function $f_1^g(t)$ appears which should be comparable to the contribution involving f_1^q, so the ratios of H_g to F_1^q is not fully perturbative in practice.