Gluonic gravitational form factors and threshold J/ψ Photoproduction

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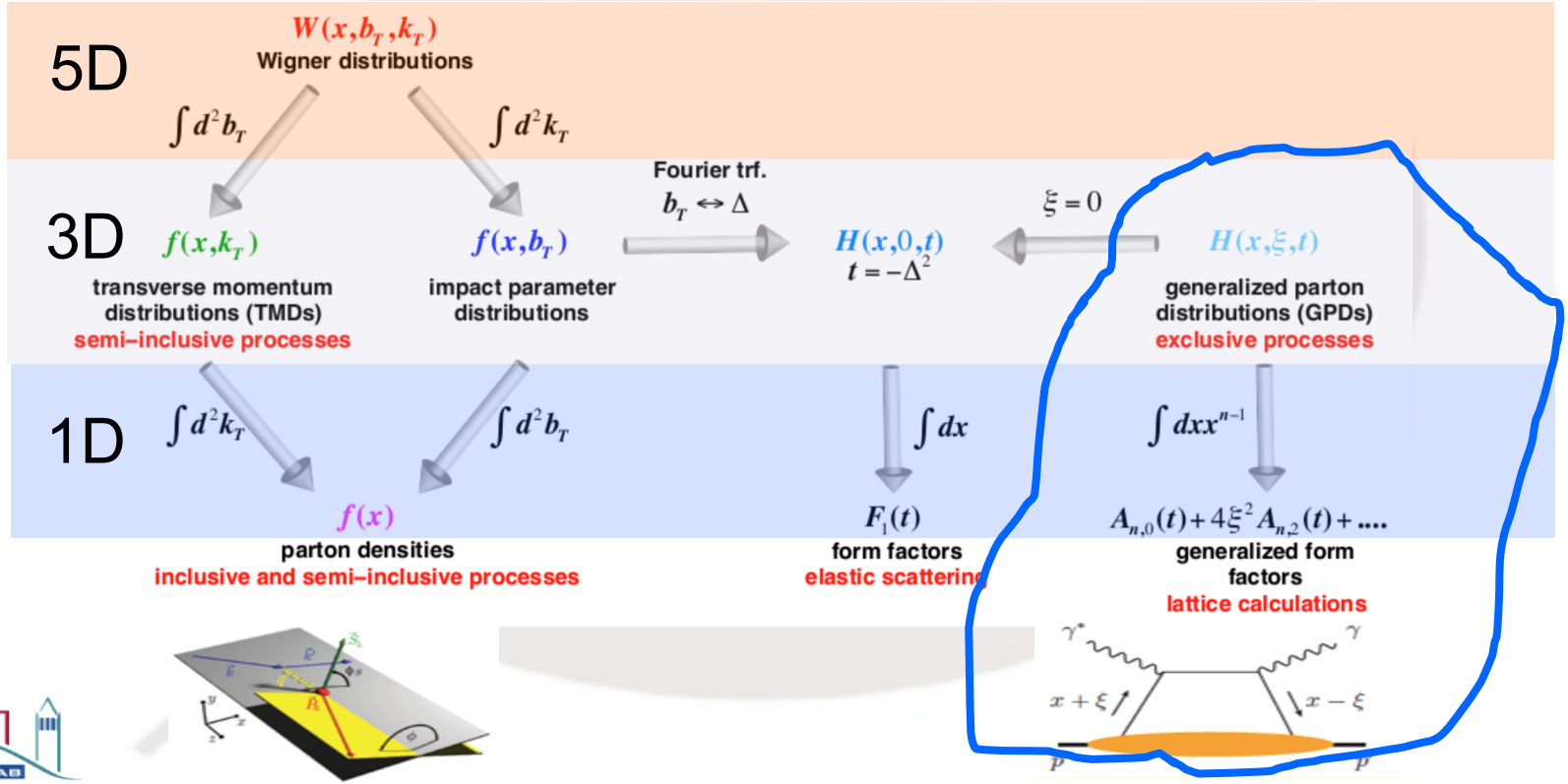
Gravitational Form Factors: important aspect of tomography

- **Wigner distributions (Belitsky, Ji, Yuan)**



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Energy-Momentum Tensor and GFFs

- Quark and gluon contributions to EMT

$$T_q^{\mu\nu} = \frac{1}{4}\bar{\psi}_f \left(-i\overleftarrow{D}^\mu\gamma^\nu - i\overleftarrow{D}^\nu\gamma^\mu + i\overrightarrow{D}^\mu\gamma^\nu + i\overrightarrow{D}^\nu\gamma^\mu\right)\psi_f$$

$$T_g^{\mu\nu} = F^{a,\mu\lambda}F^{a,\nu}_{\lambda} + \frac{1}{4}g^{\mu\nu}F^{a,\sigma\rho}F^a_{\sigma\rho}\,,$$

$$T^\mu_\mu = \frac{\beta(g_s)}{2g_s}F^{a,\mu\nu}F^a_{\mu\nu}$$

- Gravitational Form Factors (EMT Form Factors)

$$\langle P',s'|T_a^{\mu\nu}(0)|P,s\rangle = \bar{u}_s(P')\left[A_a(t)\gamma^{(\mu}\bar{P}^{\nu)} + B_a(t)\frac{i\bar{P}^{(\mu}\sigma^{\nu)\rho}\Delta_\rho}{2M}\right.$$

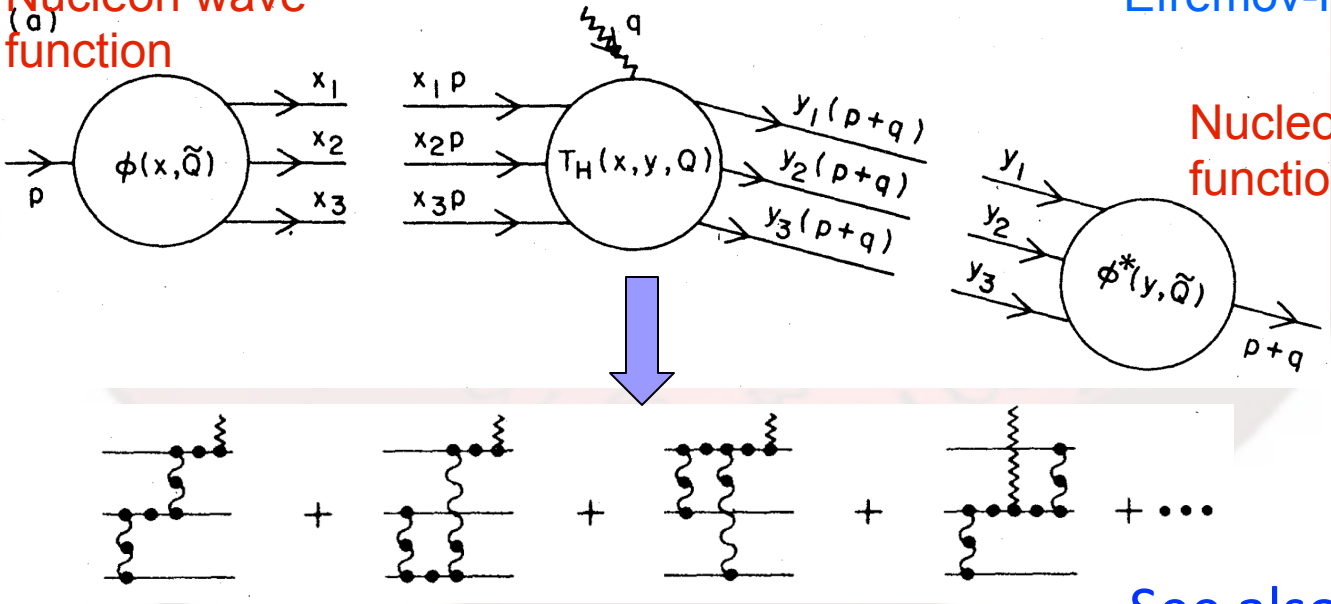
$$\left. + C_a(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \overline{C}_a(t)Mg^{\mu\nu}\right]u_s(P)\,,$$

Kobzarev-Okun 1962; Pagels 1966; Ji 1997

Perturbatively, one can compute the form factors at large momentum transfer

Lepage-Brodsky 1980
Efremov-Radyushkin 1980

Nucleon wave function



Nucleon wave function

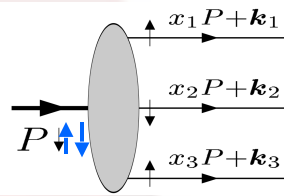
See also talk by Jacob

Light-cone Wave Functions

Lepage-Brodsky 1980

- They are building blocks for the hadron structure

$$|P\rangle = \sum_{n,\lambda_i} \int \overline{\Pi}_i \frac{dx_i d^2k_{\perp i}}{\sqrt{x_i} 16\pi^3} \phi_n(x_i, k_{\perp i}, \lambda_i) |n : x_i, k_{\perp i}, \lambda_i\rangle$$



- Fock state of n-partons: momentum fractions, transverse momenta, helicities
- Can be used to compute the form factors, GPDs, and hard exclusive scattering amplitudes

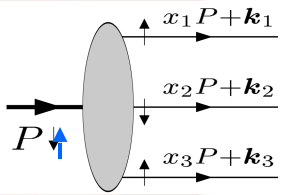
Nucleon's 3-quarks WF

Ji, Ma, Yuan, 2002
Boffi, Pasquini, 2004

- According to the general structure, six independent light-cone wave functions for three quarks component:

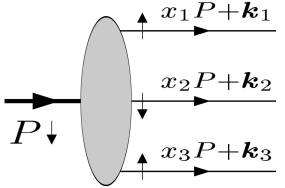
$$|P\uparrow\rangle_{1/2} = \int d[1]d[2]d[3]\left(\tilde{\psi}^{(1)}(1,2,3)\right) \times \frac{\epsilon^{abc}}{\sqrt{6}} u^\dagger_{a\uparrow}(1)\left(u^\dagger_{b\downarrow}(2)d^\dagger_{c\uparrow}(3) - d^\dagger_{b\downarrow}(2)u^\dagger_{c\uparrow}(3)\right)|0\rangle$$

$L_z=0$



$$|P\downarrow\rangle_{1/2} = \int d[1]d[2]d[3]\left((k_1^x - ik_1^y)\tilde{\psi}^{(3)}(1,2,3) + (k_2^x - ik_2^y)\tilde{\psi}^{(4)}(1,2,3)\right) \times \frac{\epsilon^{abc}}{\sqrt{6}}\left(u^\dagger_{a\downarrow}(1)u^\dagger_{b\uparrow}(2)d^\dagger_{c\uparrow}(3) - d^\dagger_{a\downarrow}(1)u^\dagger_{b\uparrow}(2)u^\dagger_{c\uparrow}(3)\right)|0\rangle \ ,$$

$|L_z|=1$



Distribution amplitudes

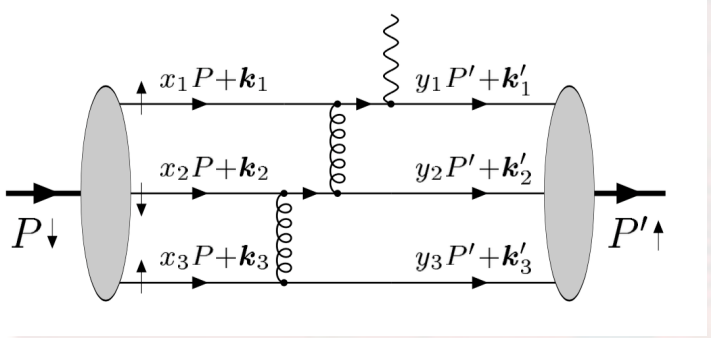
- Integrate out the transverse momentum
  - Twist-three (leading-twist)

$$\Phi_3(y_i) = -2\sqrt{6}\int \frac{d^2\vec{k}'_{1\perp}d^2\vec{k}'_{2\perp}d^2\vec{k}'_{3\perp}}{(2\pi)^6}\delta^{(2)}(\vec{k}'_{1\perp}+\vec{k}'_{2\perp}+\vec{k}'_{3\perp})\tilde{\psi}^{(1)}(1,2,3)$$

  - Twist-four (Braun-Fries-Mahnke-Stein 2000)

$$\begin{aligned}\Psi_4(x_1,x_2,x_3) &= -\frac{2\sqrt{6}}{x_2 M}\int \frac{d^2\vec{k}_{1\perp}d^2\vec{k}_{2\perp}d^2\vec{k}_{3\perp}}{(2\pi)^6}\delta^{(2)}(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp}) \\ &\quad \times\vec{k}_{2\perp}\cdot\left[\vec{k}_{1\perp}\tilde{\psi}^{(3)}(1,2,3)+\vec{k}_{2\perp}\tilde{\psi}^{(4)}(1,2,3)\right] . \\ \Phi_4(x_2,x_1,x_3) &= -\frac{2\sqrt{6}}{x_3 M}\int \frac{d^2\vec{k}_{1\perp}d^2\vec{k}_{2\perp}d^2\vec{k}_{3\perp}}{(2\pi)^6}\delta^{(2)}(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp}) \\ &\quad \times\vec{k}_{3\perp}\cdot\left[\vec{k}_{1\perp}\tilde{\psi}^{(3)}(1,2,3)+\vec{k}_{2\perp}\tilde{\psi}^{(4)}(1,2,3)\right] .\end{aligned}$$

Form factor calculations



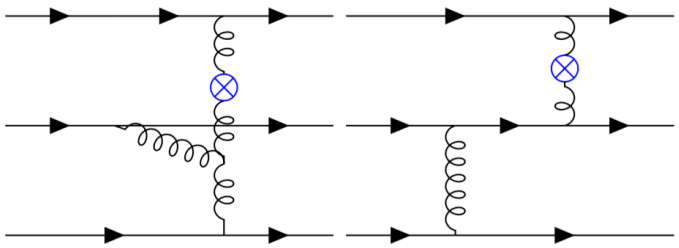
Compute the partonic scattering amplitudes, convert to hadron's
Leading-twist: direct integration of k_t, higher-twist: need k_t-expansion

- Two gluon exchanges are needed to generate large momentum transfer
- Helicity-non-flip has power behavior, F_1~1/t^2
- Helicity-flip amplitude has power behavior, F_2~1/t^3

Brodsky-Lepage 1981 for F_1
Belitsky-Ji-Yuan 2002 for F_2

Gravitational FFs

$$\langle P', s'|T_a^{\mu\nu}(0)|P, s\rangle = \bar{u}_s(P')\Big[A_a(t)\gamma^{(\mu}\bar{P}^{\nu)} + B_a(t)\frac{i\bar{P}^{(\mu}\sigma^{\nu)\rho}\Delta_\rho}{2\Lambda} + C_a(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{\Lambda} + \bar{C}_a(t)\Lambda g^{\mu\nu}\Big]u_s(P),$$



- No contribution from three-gluon vertex diagram
- A_g~1/t^2
- B_g, C_g scale as 1/t^3, $\bar{C}_g$ scales as 1/t^2

$$\mathcal{A} = \frac{4\pi^2\alpha_s^2 C_B^2}{3t^2}\Big(I_{13} + I_{12} + I_{31} + I_{32}\Big), \qquad I_{ij} = \frac{x_i + y_i}{\bar{x}_i\bar{y}_i x_i x_j y_i y_j}$$

Tong-Ma-Yuan, 2103.12047
2203.13493

Gravitational form factors for Pions: no much difference

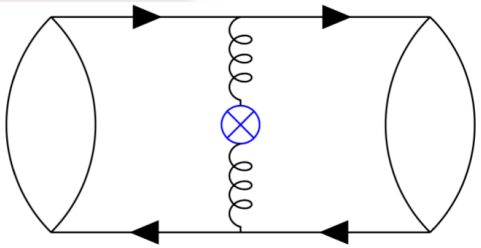
- Pion case

$$\langle P'|T_g^{\mu\nu}|P\rangle = 2\bar{P}^\mu\bar{P}^\nu A_g^\pi(t) + \frac{1}{2}(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)C_g^\pi(t) + 2m^2 g^{\mu\nu}\overline{C}_g^\pi(t)$$

$$A_g^\pi(t) = C_g^\pi(t) = \frac{4m^2}{t}\overline{C}_g^\pi(t)$$

$$= \frac{4\pi\alpha_s C_F}{-t}\int dx_1 dy_1 \phi^*(y_1)\phi(x_1)\left(\frac{1}{x_1\bar{x}_1} + \frac{1}{y_1\bar{y}_1}\right)$$

  - A_g=C_g!!
  - Cbar cancels between quarks and gluons
    - Quark part from GPD quark at large-t (Hoodbhoy-Ji-Yuan 2003)



Tong-Ma-Yuan, 2103.12047;
earlier work: Tanaka, PRD 2018
latest work: Coriano et al., 2026

Implication: scalar factor factors (F^2 term)

Talks by Keh-Fei and Jerry

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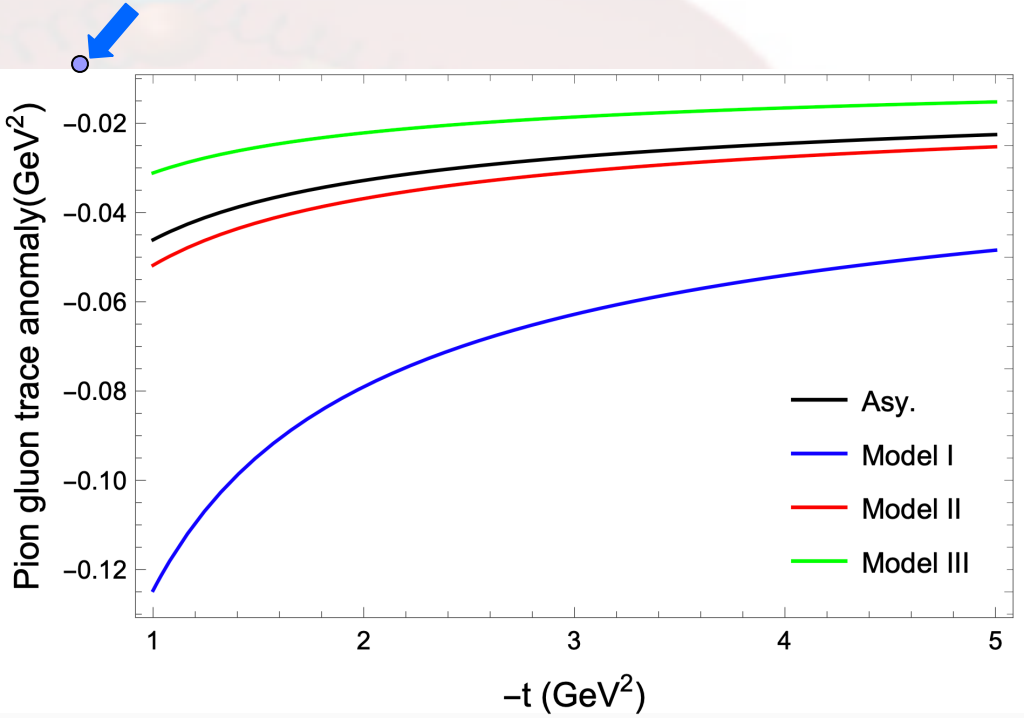
Scalar form factor of Pion

In the perturbative region:

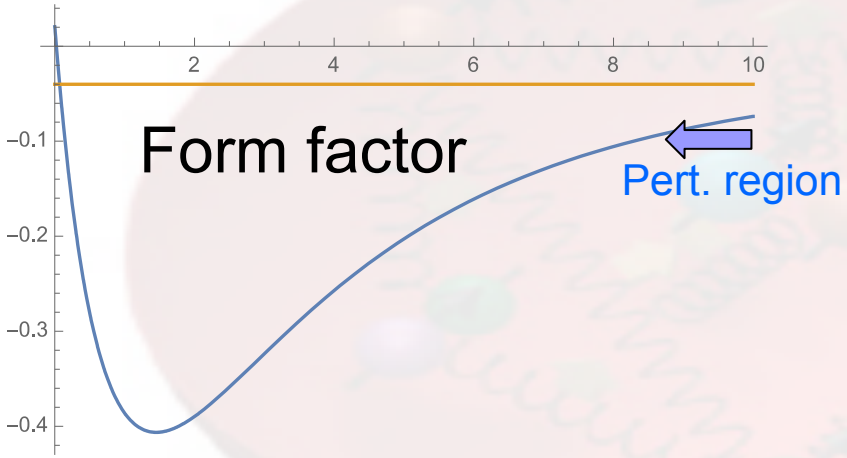
$$\langle P'|\frac{\beta(g)}{2g}F^2|P\rangle = -3\alpha_s^2(-t)C_F\left(11-\frac{2n_f}{3}\right)f_\pi^2 \times\left[1+\sum_{n=1}^{\infty}a_{2n}(\mu_0^2)L_{2n}^{\gamma_{2n}/b}(-t)\right]^2,$$

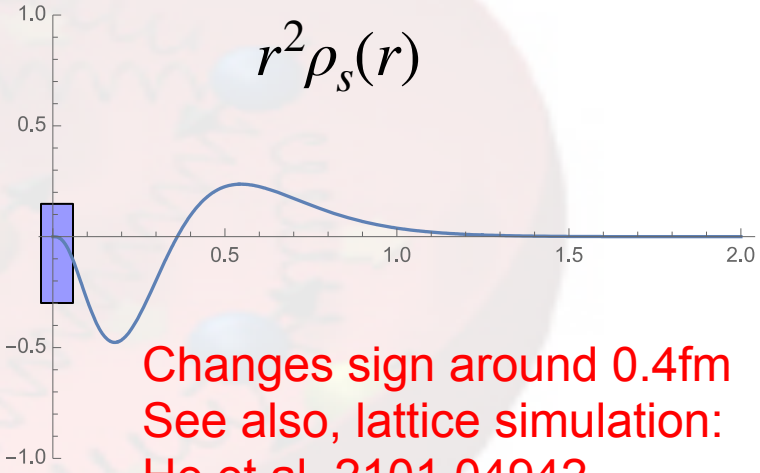
at t=0, it leads to Pion mass



See also: X. Gao et al, 2206.04084

Parameterization at low-t from lattice calculations:





Changes sign around 0.4fm
See also, lattice simulation:
He et al, 2101.04942

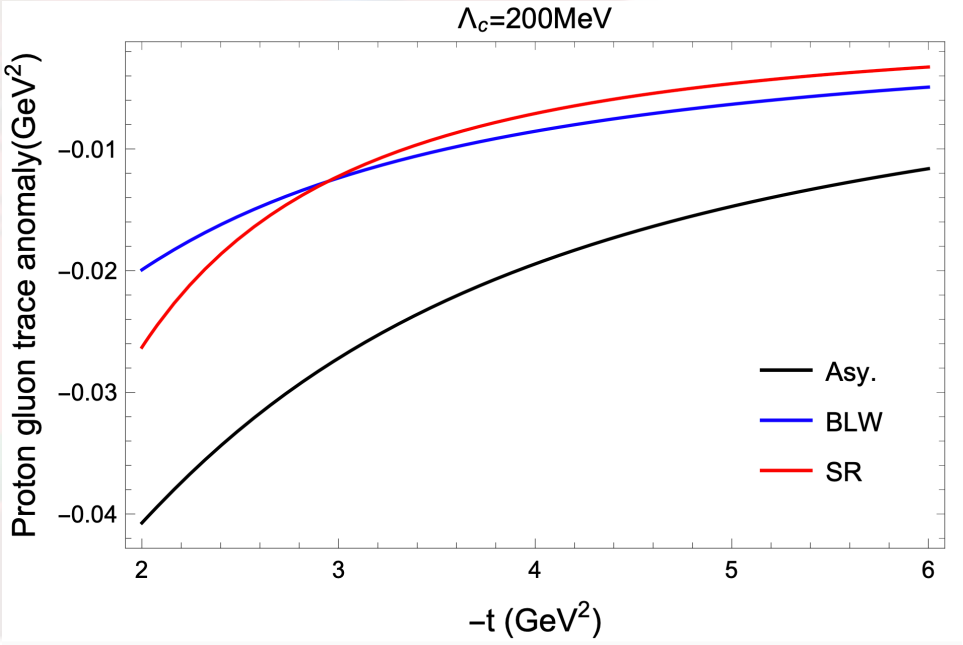
$$\rho_{s,m}(r) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} G_{s,m}(-\vec{\Delta}^2)$$

Scalar form factor of Nucleon

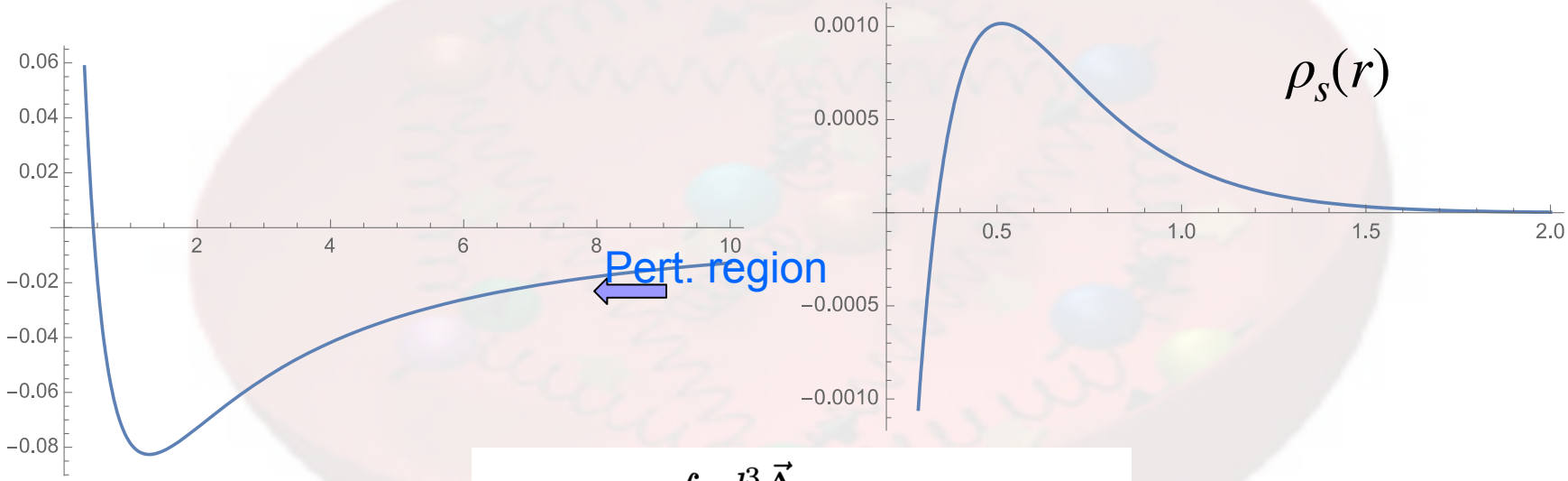
In the perturbative region:

$$G_p(t) = \int [dx][dy]\{x_3\Phi_4(x_1,x_2,x_3)\mathcal{G}_\Phi(\{x\},\{y\}) + x_1\Psi_4(x_2,x_1,x_3)\mathcal{G}_\Psi(\{x\},\{y\})\}\Phi_3(y_1,y_2,y_3)$$

$$\mathcal{H}_\Psi = \frac{C_B^2 M_p}{6t^2}(4\pi\alpha_s)^2\Big[x_3\left((y_1-y_3)\,\bar{x}_1 + x_1 y_2\right)T_1 - T_3 + \bar{x}_3\left(y_3\bar{x}_3 + x_3\bar{y}_3\right)\tilde{T}_1 + x_3\left(y_2\bar{x}_2 + x_2\bar{y}_2\right)(\tilde{T}_2 - T_2) + x_3\left(y_1-\bar{y}_1\right)(T_4+T_5) + \left(y_3\bar{x}_3 + x_3\bar{y}_3\right)(\tilde{T}_4+\tilde{T}_5)\Big]$$



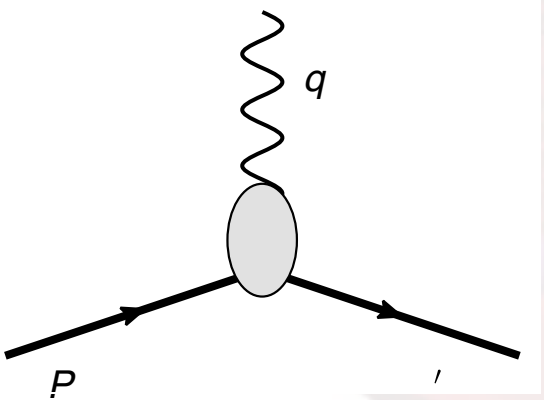
Parameterization at low-t from lattice calculations



$$\rho_{s,m}(r) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} G_{s,m}(-\vec{\Delta}^2)$$

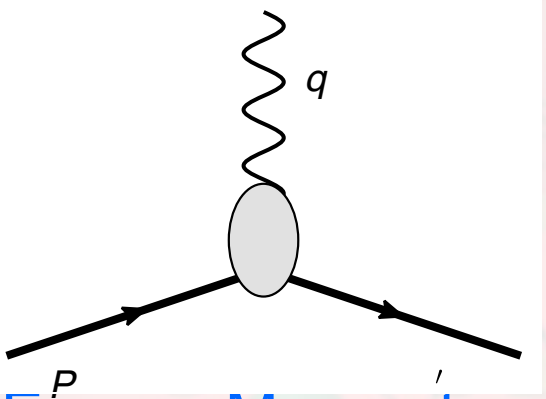
How do we measure the GFFs if we have strong gravity ☺

Photon



EM current

Graviton



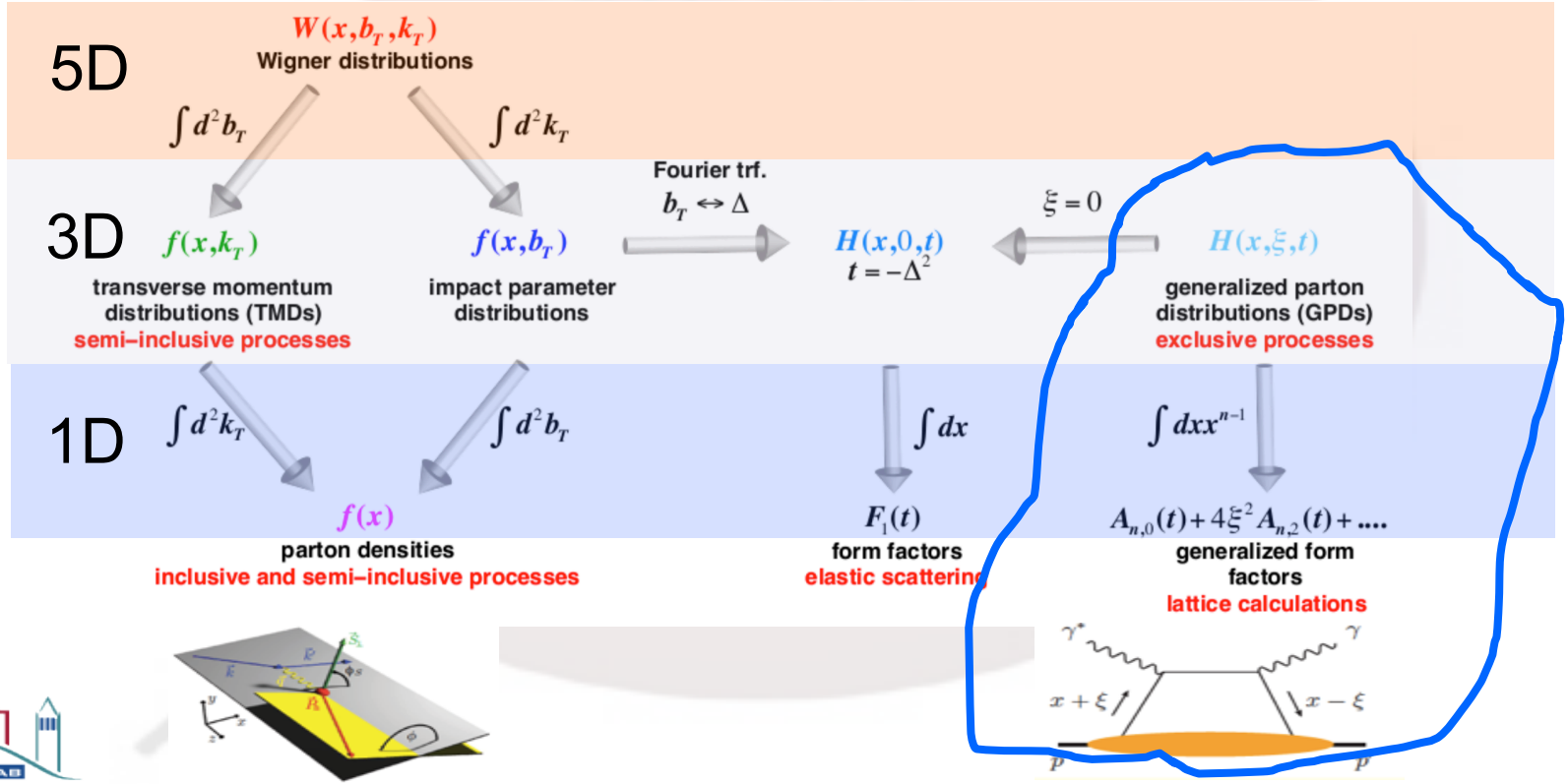
Energy-Momentum current

$$\langle P', s'|T_a^{\mu\nu}(0)|P, s\rangle = \bar{u}_s(P')\Big[A_a(t)\gamma^{(\mu}\bar{P}^{\nu)} + B_a(t)\frac{i\bar{P}^{(\mu}\sigma^{\nu)\rho}\Delta_\rho}{2\Lambda} + C_a(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{\Lambda} + \bar{C}_a(t)\Lambda g^{\mu\nu}\Big]u_s(P),$$

Kobzarev-Okun 1962; Pagels 1966; Ji 1997

How to measure the GFFs: through DPDs

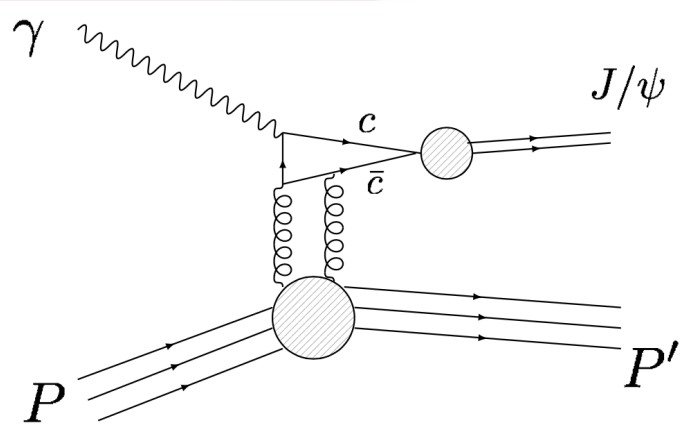
- **Wigner distributions (Belitsky, Ji, Yuan)**



Kharzeev's Proposal

Kharzeev, 1996, 2021
Kharzeev et al., 1999

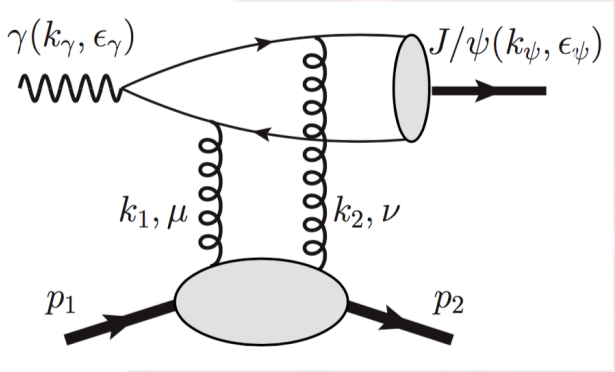
- Vector meson dominance: photon transition to heavy quarkonium
- The amplitude proportional to heavy quarkonium-nucleon interaction
  - Applying OPE, lowest order gluonic operator contributes
    - Peskin 1979; Luke-Manohar-Savage 1992; Kharzeev 1996

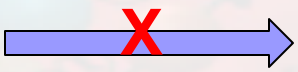


$$\frac{d\sigma_{\gamma P\to\psi P}}{dt} = \frac{1}{64\pi s}\frac{1}{|\mathbf{p}_{\gamma cm}|^2}|\mathcal{M}_{\gamma P\to\psi P}(t)|^2$$

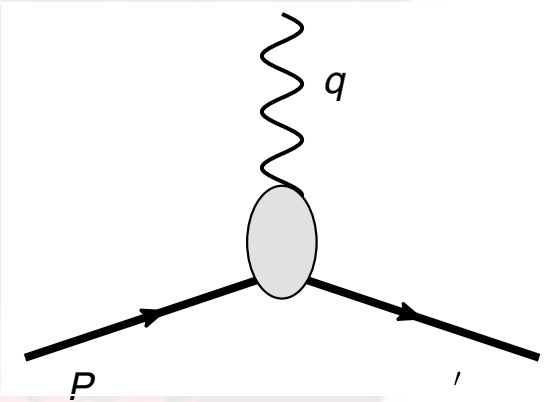
$$G(t) = \frac{M}{\left(1-\frac{t}{m_s^2}\right)^2}$$

Rutherford scattering with heavy quarkonium



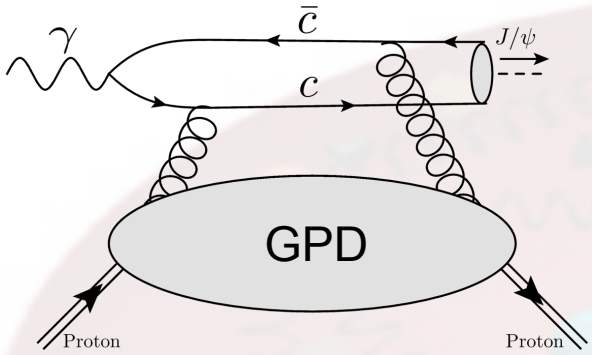


Graviton



- The QCD dynamics involved in the on-shell photon (massless) transition to a massive heavy quarkonium does not allow a simple interpretation

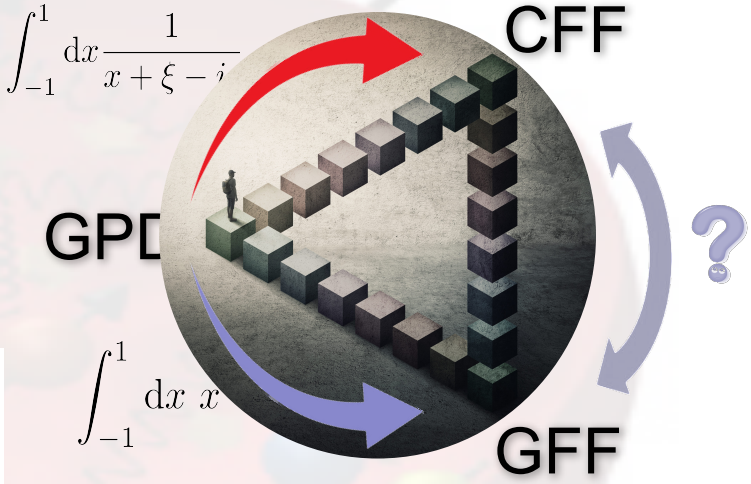
Near-threshold J/ψ photo-production to probe GPDs



$$G(t,\xi) = \frac{1}{2\xi}\int_{-1}^{1} \mathrm{d}x \mathcal{A}(x,\xi) F_g(x,\xi,t)$$

$$\mathcal{A}(x,\xi) \equiv \frac{1}{x+\xi-i0} - \frac{1}{x-\xi+i0}$$

- We can then extract the GFFs from the gCFFs, utilizing the large skewness kinematics in the near-threshold region in the heavy quark limit.



Y. Guo et. al. Phys. Rev. D 103 9, 096010

Earlier references in GPD formalism: Hoodbhoy 1996; Koempel-Kroll-Metz-Zhou 2012 and references therein

Taylor expansion of the hard coefficient

$$G(t,\xi) = \sum_{n=0}^{\infty} \frac{1}{\xi^{2n+2}} \int_{-1}^{1} dx x^{2n} F_g(x,\xi,t)$$

- Only the leading term corresponds to the gluonic gravitational form factors

$$G(t,\xi) = \frac{1}{\xi^2(\bar{P}^+)^2} \langle P'| \frac{1}{2} \sum_{a,i} F^{a,+i}(0) F^{a,+}{}_i(0) |P\rangle$$

$$= \frac{1}{2\xi^2(\bar{P}^+)^2} \left\langle P' \left| T_g^{++} \right| P \right\rangle ,$$

Boussarie-Hatta 2020;
Hatta-Strikman 2021;
Guo-Ji-Liu 2021;

Systematics of large-ξ expansion

$$G(t,\xi) = \frac{1}{2\xi}\int_{-1}^{1} \mathrm{d}x \mathcal{A}(x,\xi) F_g(x,\xi,t)$$

$$\mathcal{A}(x,\xi) \equiv \frac{1}{x+\xi-i0} - \frac{1}{x-\xi+i0}$$

- Model

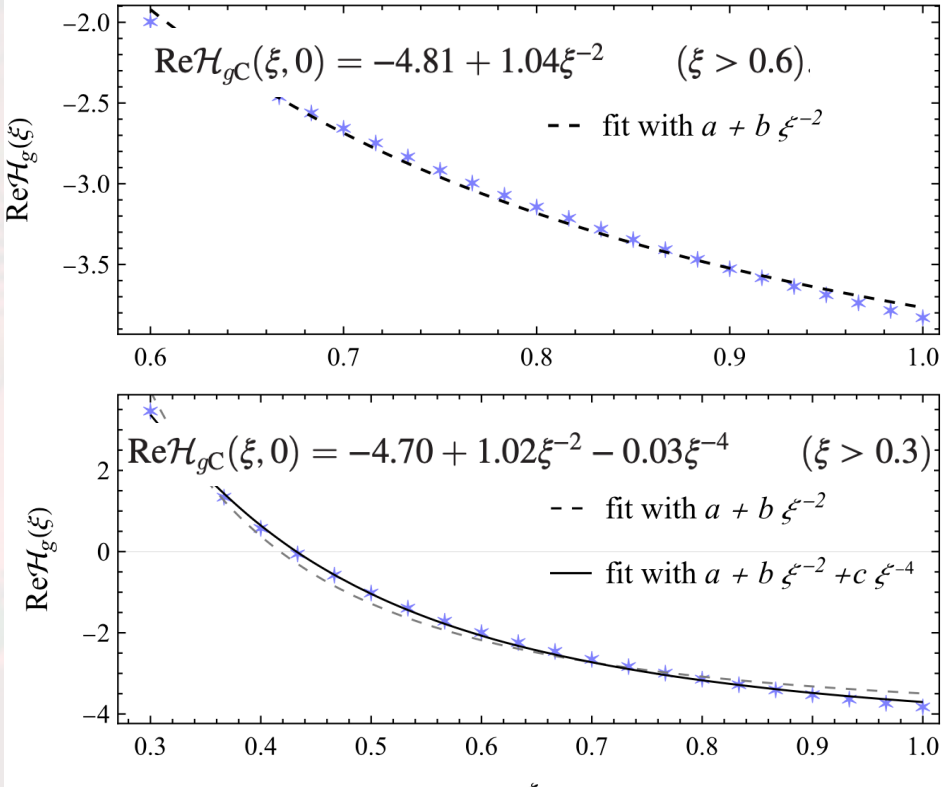
Radyushkin 1999

$$H_g(x,\xi,t) = H_g^{DD}(x,\xi,t) + |\xi|\theta(|\xi|-|x|)D_g(x,\xi,t),$$

$$E_g(x,\xi,t) = E_g^{DD}(x,\xi,t) - |\xi|\theta(|\xi|-|x|)D_g(x,\xi,t),$$

$$H_g^{DD}(x,\xi,t) = \int_{-1}^{1}\mathrm{d}\beta\int_{-1+|\beta|}^{1-|\beta|}\mathrm{d}\alpha\delta(x-\beta-\alpha\xi) \times \pi_g(\alpha,|\beta|)|\beta| f_g(|\beta|,t),$$

$$\pi_g(\alpha,|\beta|) = \frac{15}{16}\frac{((1-|\beta|)^2-\alpha^2)^2}{(1-|\beta|)^5}$$



Guo-Ji-Yuan, PRD109.014014 (2024)

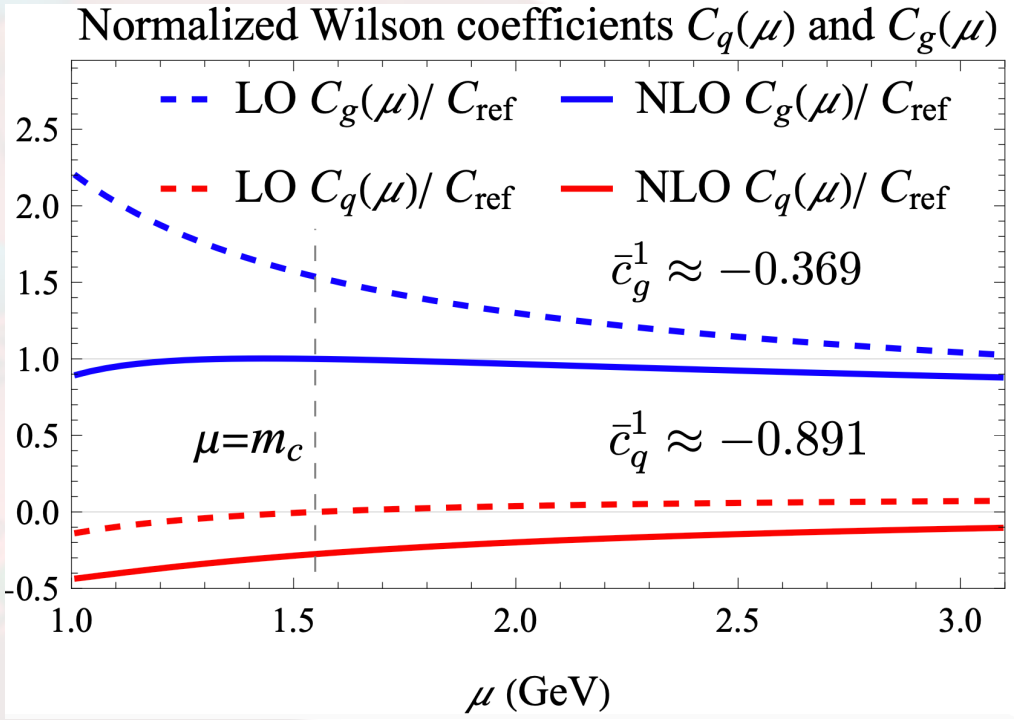
Next-leading order

Guo-Yuan-Zhao, 2501.10532

$$\mathrm{Re}\mathcal{H}_g(\xi,t) = \sum_{n=0}^{\infty} \frac{1}{\xi^{2n+2}} \bar{C}_g^{(2n+1)} \bar{H}_g^{(2n+1)}(\xi,t)$$

$$\bar{C}_g^{(1)} = \frac{5}{4} + \alpha_S \left[\bar{c}_g^1 - \frac{55}{16\pi} \log\left(\frac{m_c^2}{\mu_F^2}\right) \right]$$

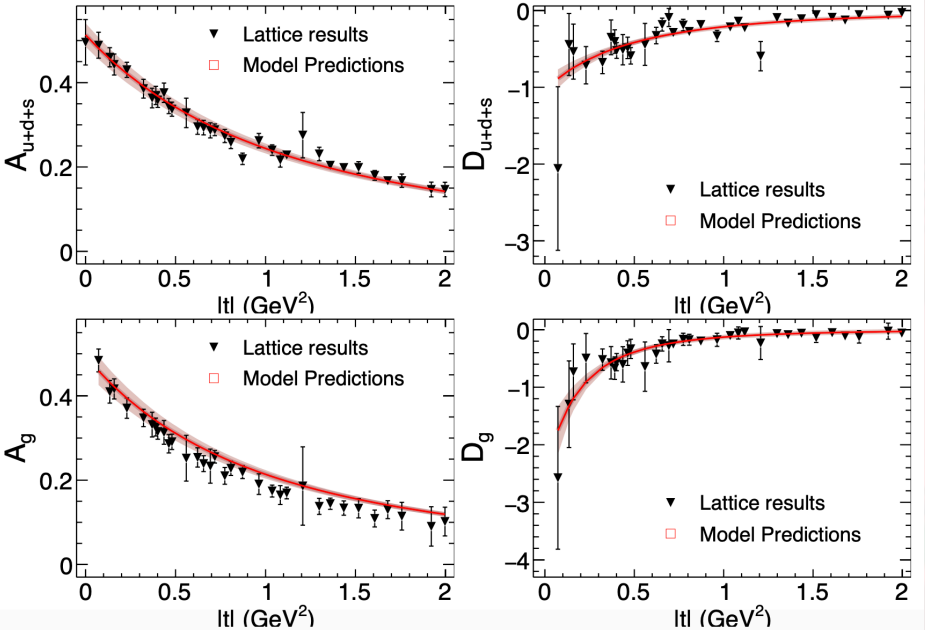
$$\bar{C}_q^{(1)} = 0 + \alpha_S \left[\bar{c}_q^1 + \frac{10}{9\pi} \log\left(\frac{m_c^2}{\mu_F^2}\right) \right] +$$

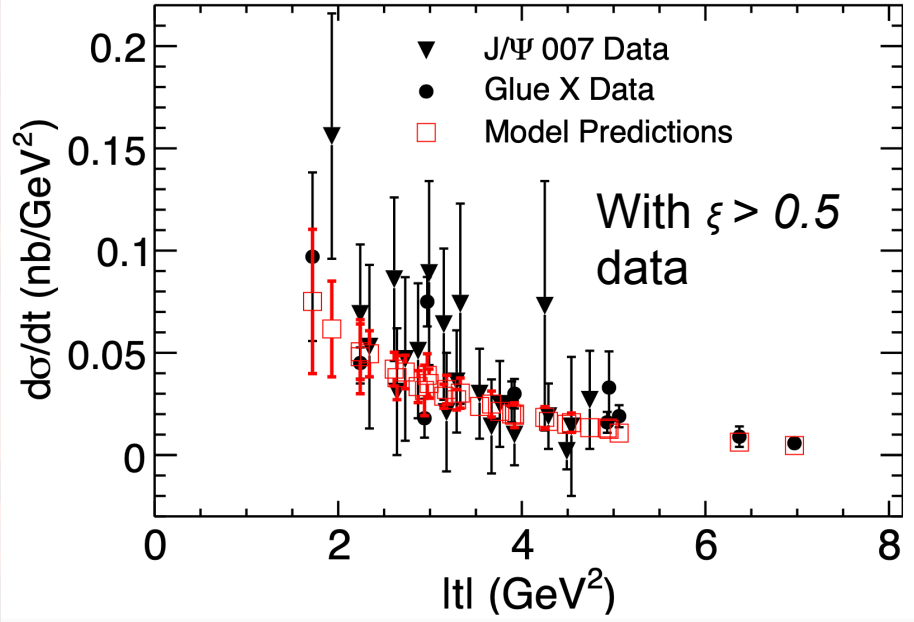


References for NLO GPD calculations:
Ivanov-Schafer-Szymanowski-Krasnikov, hep-ph/0401131;
Chen-Qiao, 1903.00171.
Flett-Gracey-Jones-Teubner, 2105.07657

Combined fit to the exp. and lattice

Bayesian Inferring Nucleon's Gravitation Form Factors via Near-threshold J/ψ Photoproduction, Guo-Yuan-Zhao, 2501.1053





Lattice: Pefkou et. al. Phys. Rev. D 105, 054509 (2022), Phys. Rev. Lett. 132, 251904 (2024)

Conclusion and Looking forward

- Gravitational form factors provide intuitive picture on the fundamental properties of hadrons
  - Mass distribution, mechanic properties
- With high precision data from current and future experiments at Jlab and EIC, we need to improve the theory precision, for GFFs and GPDs in general
  - Carry out higher order perturbative QCD corrections (NLO+NNLO+..) and demonstrate the convergence
  - Implement unbiased analysis: extending to wider kinematics; including more processes in the fit (DVCS, other mesons); AI/ML etc.