Probing quantum entanglement with GPD

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with Michael Fucilla, 2509.05267
with Jakob Schoenleber, 2511.04537
with Michael Fucilla, Bowen Xiao, 2604.11697
with Victor Martinez-Fernandez, in progress

Quantum entanglement at the LHC

SCIAM $1 for 90 days

SEPTEMBER 25, 2024 | 4 MIN READ

'Spooky Action at a Distance' Observed in Quarks for the Fir Time

Physicists report the first observations of quar entanglement in top and anti-top quarks, the known fundamental particles and their antim counterparts, inside the Large Hadron Collide

BY DAN GARISTO & NATURE

Article

Observation of quantum entanglement with top quarks at the ATLAS detector

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The ATLAS Collaboration

Entanglement is a key feature of quantum mechanics, with applications in fields such as metrology, cryptography, quantum information and quantum computation. It has been observed in a wide variety of systems and length scales, ranging from the microscopic to the macroscopic. However, entanglement highest accessible energy scales. Here we report the

Voir en français

LHC experiments at CERN observe quantum entanglement at the highest energy yet

The results open up a new perspective on the complex world of quantum physics

SEPTEMBER, 2024

Observation of quantum entanglement in top quark pair produc proton–proton collision

The CMS Collaboration

CERN, Geneva, Switzerland

E-mail: cms-publication-committee-chair@cern.ch

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The ATLAS detector, part of the Large Hadron Collider, sits 100 meters below ground, where it measures various properties of high-energy particles. Maximilien Brice/CERN

Quantum Observables for Collider Physics 2026

Apr 20 – 24, 2026
CERN
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Participant List

101 participants

Last Name	First Name	Affiliation
Afik	Yoav	University of Chicago
Arisi	Angelo	Purdue University
Bae	Joonwoo	Korea Advanced Institute of Science and Technology
Barr	Alan	University of Oxford

Quantum entanglement at the EIC?

No mention in the NAS report, yellow report.

Still, entanglement is ubiquitous in QCD.

entanglement entropy (Kovner, Lublinsky, Kharzeev, Levin, Kutak, Hentschinski, Dumitru,...) ,

spin-orbit entanglement (2404.04208, Bhattacharya, Boussarie, YH; 2410.16082, YH, Montgomery)

Spin-spin entanglement at the EIC

2506.12889, Qi, Guo, Xiao
2509.05267, Fucilla, YH
2510.23773, Cheng, Han, Trifinopoulos
2511.04537, YH, Schoenleber
2604.11697, Fucilla, YH, Xiao

Preliminaries

Classical bit: 0 or 1

Qubit: Superposition of 0 and 1 Any two-level quantum system:
electron, quark, photon, Schrodinger's cat...

$$|\psi\rangle = a|\mathrm{dead}\rangle + b|\mathrm{alive}\rangle \qquad |a|^2 + |b|^2 = 1$$

A two-qubit system can be entangled

$|\psi\rangle = |0\rangle|1\rangle$ not entangled (separable)

`pure state'

$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle - |1\rangle|0\rangle)$ `maximally' entangled (Bell state)

Density matrix

At colliders, qubit pairs are produced millons of times with different spin states
→ mixed states described by the density matrix

$$\rho = |01\rangle\langle 01|$$ pure, not entangled

$$\rho = \frac{1}{2}|00\rangle\langle 00| + \frac{1}{2}|11\rangle\langle 11|$$ mixed, not entangled

$$\rho = \frac{1}{2}(|01\rangle\langle 01| - |01\rangle\langle 10| - |10\rangle\langle 01| + |10\rangle\langle 10|) = |\Psi^-\rangle\langle\Psi^-|$$ pure, entangled

Peres-Horodecki criterion

In general, it is difficult to judge whether a given density matrix represents an entangled state or not.

Fortunately, for two-qubit states, there is a quick recipe.

$$\rho_{AB} = \frac{1}{4}\left(1 \otimes 1 + b_1^i(\sigma^i \otimes 1) + b_2^j(1 \otimes \sigma^j) + C_{ij}(\sigma^i \otimes \sigma^j)\right)$$

polarization correlation

The system is separable (not entangled) iff the partial transpose ρ^{T_2} is nonnegative definite.

Bell-nonlocality

Clauser-Horne-Shimony-Holt (CHSH) inequality

$$\left| n_1^i C_{ij} (n_2^j + n_4^j) + n_3^i C_{ij} (n_2^j - n_4^j) \right| \leq 2$$

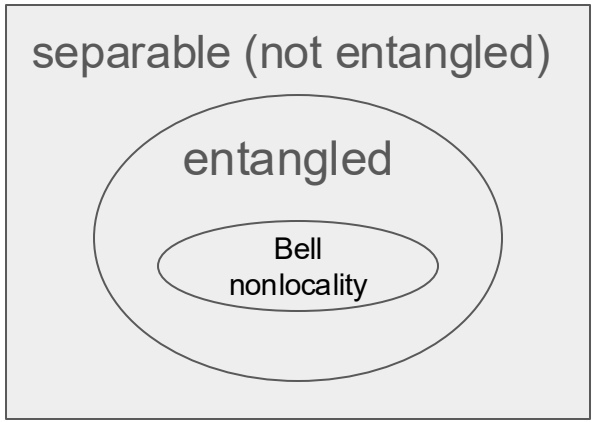
Quantum mechanics can violate this inequality → 2022 Nobel prize

If the largest two eigenvalues of $C^T C$ satisfy

$\mu_1 + \mu_2 > 1$ → `Bell nonlocality'

Horodecki, Horodecki, Horodecki (1995)

In general, Bell nonlocality requires deeper quantum correlation than entanglement



Magic

Entanglement is crucial for quantum computers, but not sufficient
Quantum circuits consisting only of `stabilizer states' (including Bell states) have no quantum advantage

Universal quantum computer requires certain nonclassical resources called `magic'

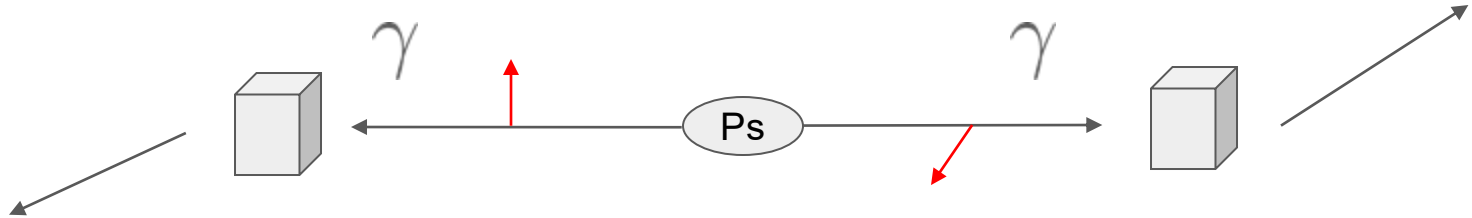
One way to quantify magic: 2nd Renyi entropy

$$M_2 = -\ln\left(\frac{1+B_n^4+\bar{B}_n^4+C_{nn}^4+C_{rr}^4+C_{kk}^4+C_{kr}^4+C_{rk}^4}{1+B_n^2+\bar{B}_n^2+C_{nn}^2+C_{rr}^2+C_{kk}^2+C_{kr}^2+C_{rk}^2}\right)$$

Can colliders produce pairs with large magic? White, White 2406.07321

Theoretical upper bound $M_2^{\max} = \ln\frac{16}{7} \approx 0.827$ Liu, Low, Yin 2502.17550

Entanglement in positronium decay



Two photons linearly polarized at right angles.

$$\frac{1}{\sqrt{2}}(|x\rangle|y\rangle - |y\rangle|x\rangle)$$

Maximally entangled Bell state

The Angular Correlation of Scattered Annihilation Radiation*

C. S. Wu and I. Shaknov
Pupin Physics Laboratories, Columbia University, New York, New York
November 21, 1949

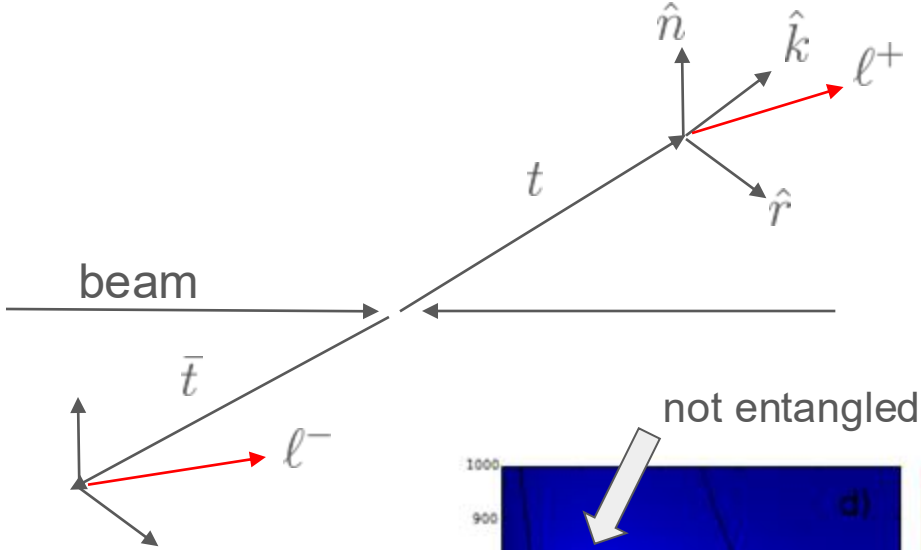
As early as 1946, J. A. Wheeler[1] proposed an experiment to verify a prediction of pair theory, that the two quanta emitted in the annihilation of a positron-electron pair, with zero relative angular momentum, are polarized at right angles to each other. This suggestion involves coincidence measurements of the scattering of both the annihilation photons at various azimuths. The detailed theoretical investigations were reported

Entanglement in $t\bar{t}$ production at the LHC

Two production mechanisms

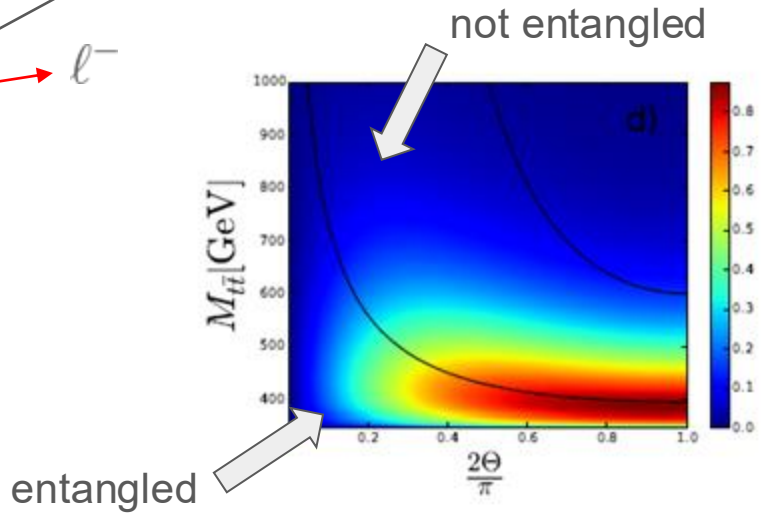
$gg \to t\bar{t}$ $q\bar{q} \to t\bar{t}$

$$\frac{d\sigma}{d\cos\varphi} \propto 1 - (\alpha_+)^2 \frac{\mathrm{Tr}[C]}{3} \cos\varphi$$



φ : angle between ℓ^+ and ℓ^- measured in respective rest frames

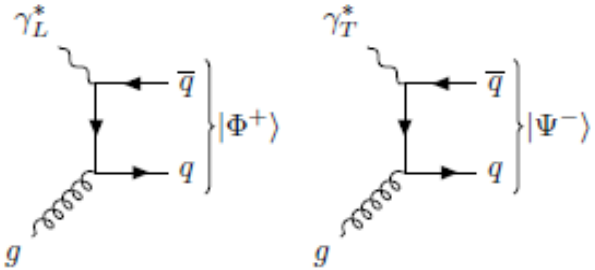
The pair is entangled if $D = \frac{1}{3}\mathrm{Tr}[C] < -\frac{1}{3}$

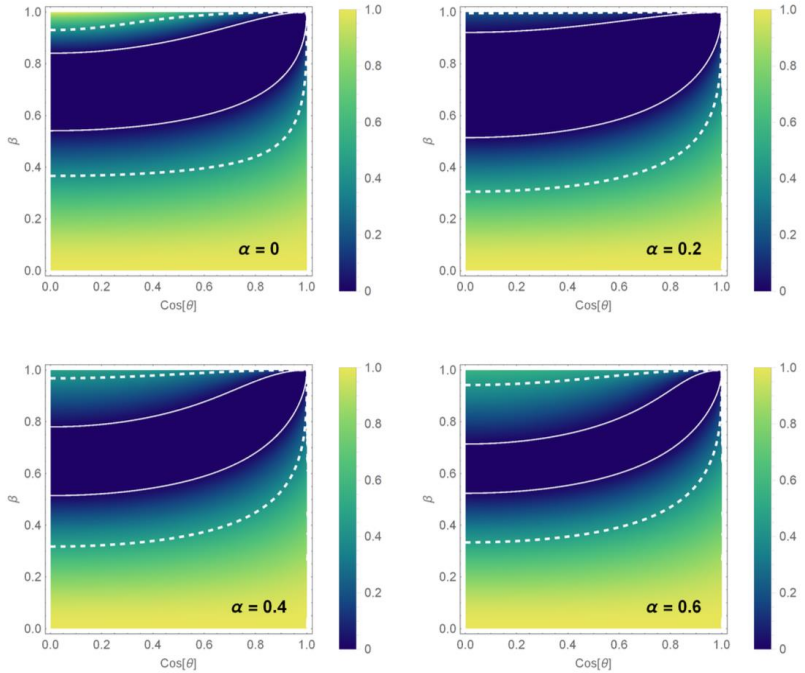


Inclusive quark-antiquark production in DIS

Spin density matrix in one-gluon exchange

Qi, Guo, Xiao, 2506.12889





Longitudinal photon:
Always maximally entangled and maximally Bell-nonlocal

Transverse photon:

Bell nonlocality $\subset$ entanglement

Can we combine the study of entanglement with the usual topics of EIC (TMD, GPD,...)?

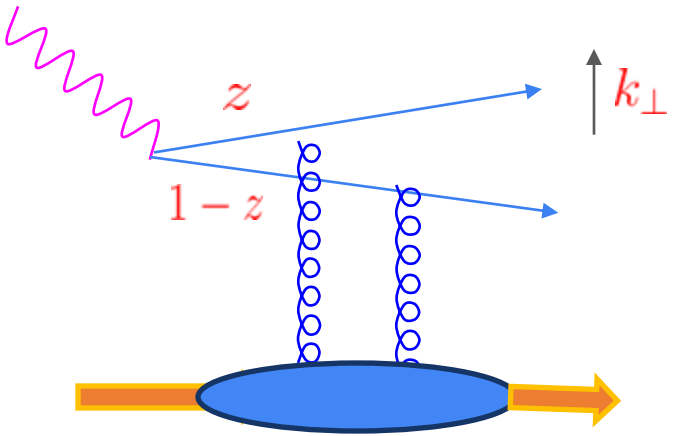
Exclusive dijet production

Quark-antiquark pair production via color singlet (Pomeron) exchange

Regge limit --> kt factorization, (G)TMD
Nikolaev, Zakharov (1994) Bartels, Lotter, Wusthoff (1996)

Scattering amplitude purely imaginary
BFKL Pomeron exchange

Medium energy --> collinear factorization, GPD
Braun, Ivanov (2005)



Spin density matrix in the Regge limit

Fucilla, YH, 2509.05267

Longitudinal case: the same as before
→ maximal entanglement/Bell nonlocality

$$\widetilde{C}^T_{nn} = -\frac{4\alpha_{em}e_q^2}{N_c} z\bar{z}k_\perp^2 (T_1+T_2)^2,$$

Transverse case:

$$\begin{pmatrix} C_{nn} & 0 & 0 \\ 0 & C_{rr} & C_{rk} \\ 0 & C_{kr} & C_{kk} \end{pmatrix}$$

$$\begin{aligned}\widetilde{C}^T_{rr} &= \frac{\alpha_{em}e_q^2}{2z\bar{z}N_c}\Big[2T_1(T_1+T_2)k_\perp m\sqrt{1-\beta^2}\sin\theta(\beta+(1-2z)\cos\theta)+T_1^2m^2\left(1-\beta^2\right)\sin^2\theta \\ &\quad -(T_1+T_2)^2k_\perp^2\Big(\left(1-\beta^2\right)\sin^2\theta+2z(1-z)\left(\left(2-\beta^2\right)\cos^2\theta-1\right)\Big)\Big],\end{aligned}$$

$$\begin{aligned}\widetilde{C}^T_{kk} &= \frac{\alpha_{em}e_q^2}{2z\bar{z}N_c}\Big[2T_1(T_1+T_2)k_\perp m\sqrt{1-\beta^2}\sin\theta(\beta-(1-2z)\cos\theta)+T_1^2m^2\left(\cos^2\theta-\beta^2\right) \\ &\quad +(T_1+T_2)^2k_\perp^2\Big(\beta^2-2z(1-z)+\cos^2\theta\big(2z(1-z)\left(2-\beta^2\right)-1\big)\Big)\Big],\end{aligned}$$

$$\begin{aligned}\widetilde{C}^T_{rk} &= \frac{\alpha_{em}e_q^2}{2z\bar{z}N_c}\Big[2T_1(T_1+T_2)k_\perp m\left(z-\beta^2+\cos^2\theta\left(1-\left(2-\beta^2\right)z\right)\right) \\ &\quad +\sqrt{1-\beta^2}\sin\theta\left(T_1^2m^2(\cos\theta-\beta)+(T_1+T_2)^2k_\perp^2\left(\beta-(1-2z)^2\cos\theta\right)\right)\Big],\end{aligned}$$

$$\begin{aligned}\widetilde{C}^T_{kr} &= \frac{\alpha_{em}e_q^2}{2z\bar{z}N_c}\Big[2T_1(T_1+T_2)k_\perp m\left(z-\left(1-\beta^2\right)\sin^2\theta-\left(2-\beta^2\right)z\cos^2\theta\right) \\ &\quad +\sqrt{1-\beta^2}\sin\theta\left(T_1^2m^2(\beta+\cos\theta)-(T_1+T_2)^2k_\perp^2\left(\beta+(1-2z)^2\cos\theta\right)\right)\Big].\end{aligned}$$

Density matrix depends on the TMD gluon distribution of the target.

Spin density matrix in the Regge limit

Fucilla, YH, 2509.05267

Longitudinal case: the same as before
→ maximal entanglement/Bell nonlocality

$$\widetilde{C}^T_{nn} = -\frac{4\alpha_{em}e_q^2}{N_c} z\bar{z}k_\perp^2 (T_1+T_2)^2,$$

Tran

$$C_{nn}^2 + C_{rk}^2 + C_{kr}^2 + C_{kk}^2 - C_{nn}^2 = 1$$

$$C_{nn} = -C_{rr}C_{kk} + C_{kr}C_{rk}$$

$\sin^2\theta$

$-\beta^2)$

$$\begin{pmatrix} & & \\ & & \\ 0 & C_{kr} & C_{kk} \end{pmatrix}$$

$$\widetilde{C}^T_{rk} = \frac{\alpha_{em}e_q^2}{2z\bar{z}N_c}\Big[2T_1(T_1+T_2)k_\perp m\left(z-\beta^2+\cos^2\theta\left(1-\left(2-\beta^2\right)z\right)\right)$$
$$+\sqrt{1-\beta^2}\sin\theta\left(T_1^2m^2(\cos\theta-\beta)+(T_1+T_2)^2k_\perp^2\left(\beta-(1-2z)^2\cos\theta\right)\right)\Big],$$

Density matrix depends on the TMD gluon distribution of the target.

$$\widetilde{C}^T_{kr} = \frac{\alpha_{em}e_q^2}{2z\bar{z}N_c}\Big[2T_1(T_1+T_2)k_\perp m\left(z-\left(1-\beta^2\right)\sin^2\theta-\left(2-\beta^2\right)z\cos^2\theta\right)$$
$$+\sqrt{1-\beta^2}\sin\theta\left(T_1^2m^2(\beta+\cos\theta)-(T_1+T_2)^2k_\perp^2\left(\beta+(1-2z)^2\cos\theta\right)\right)\Big].$$

Entanglement = Bell nonlocality in the Regge limit

Fucilla, YH, 2509.05267

Entanglement (Peres-Horodecki)

$$\max(\Delta_1, \Delta_2) > 0$$

$$\Delta_1 = \sqrt{(C^T_{rr} - C^T_{kk})^2 + (C^T_{rk} + C^T_{kr})^2} - 1 + C^T_{nn},$$

$$\Delta_2 = \sqrt{(C^T_{rr} + C^T_{kk})^2 + (C^T_{rk} - C^T_{kr})^2} - 1 - C^T_{nn}.$$

Bell nonlocality (Horodecki^3)

$$1 < \mu_1 + \mu_2 \leq 2$$

Largest two of C^2_{nn}

$$\frac{C^2_{rr} + C^2_{kr} + C^2_{kk} + C^2_{rk} \pm \sqrt{\left(C^2_{rr} + C^2_{kr} - C^2_{kk} - C^2_{rk}\right)^2 + 4\left(C_{rr}C_{rk} + C_{kr}C_{kk}\right)^2}}{2}$$

Entanglement = Bell nonlocality in the Regge limit

Fucilla, YH, 2509.05267

Entanglement (Peres-Horodecki)

$$\max(\Delta_1, \Delta_2) > 0$$

$$\Rightarrow C_{nn} \neq 0$$

$$\Delta_1 = \sqrt{(C^T_{rr} - C^T_{kk})^2 + (C^T_{rk} + C^T_{kr})^2} - 1 + C^T_{nn},$$

$$\Delta_2 = \sqrt{(C^T_{rr} + C^T_{kk})^2 + (C^T_{rk} - C^T_{kr})^2} - 1 - C^T_{nn}.$$

Bell nonlocality (Horodecki^3)

$$1 < \mu_1 + \mu_2 \leq 2$$

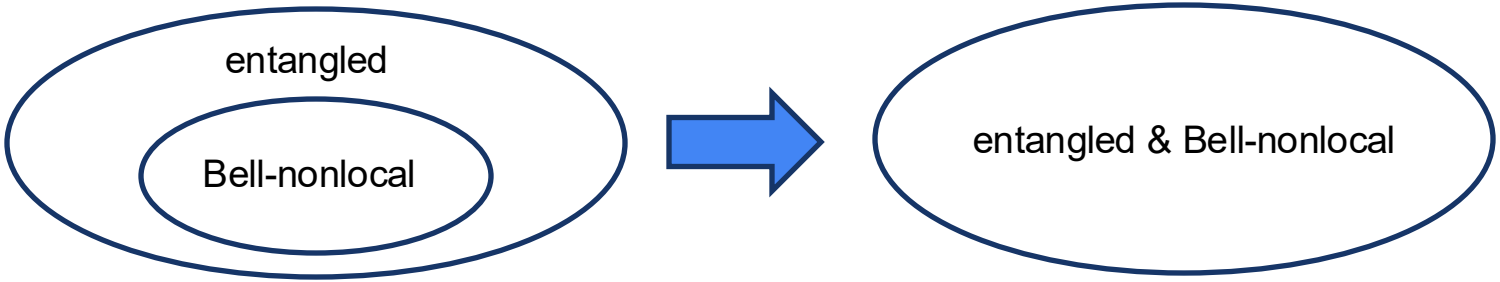
$$\Rightarrow C_{nn} \neq 0$$

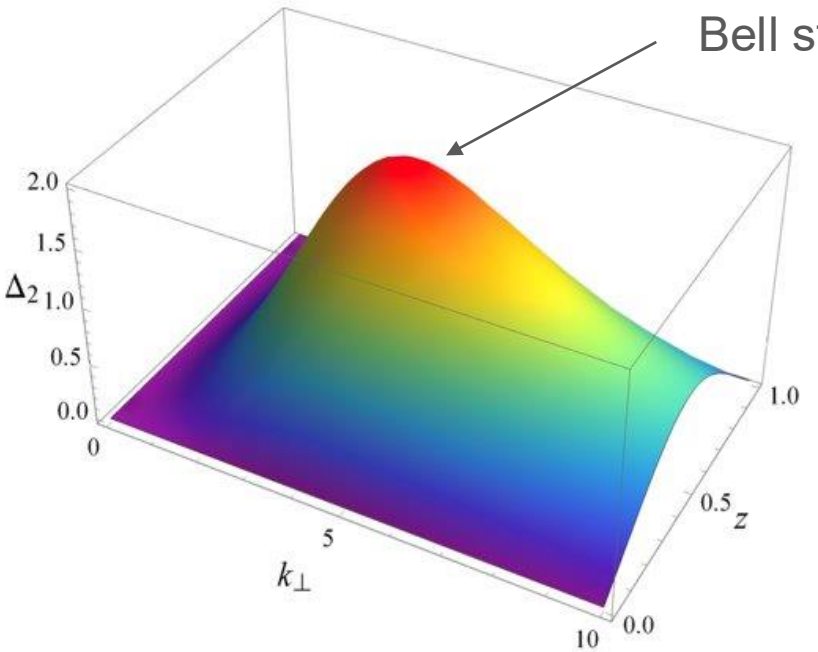
Largest two of C^2_{nn}

$$\frac{C^2_{rr} + C^2_{kr} + C^2_{kk} + C^2_{rk} \pm \sqrt{\left(C^2_{rr} + C^2_{kr} - C^2_{kk} - C^2_{rk}\right)^2 + 4\left(C_{rr}C_{rk} + C_{kr}C_{kk}\right)^2}}{2}$$

The two conditions become identical.

Entanglement and Bell nonlocality necessary and sufficient!





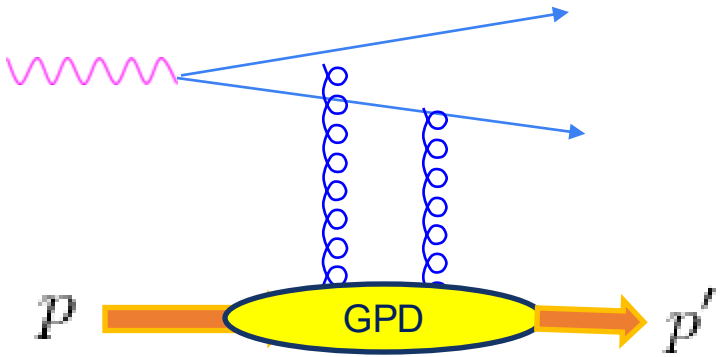
$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_q^n|-\rangle_{\bar{q}}^n - i|-\rangle_q^n|+\rangle_{\bar{q}}^n\right)$$

Everywhere entangled,
Everywhere Bell-nonlocal

Maximal entanglement/Bell nonlocality
at one point in phase space

Spin density matrix in collinear factorization + GPD

YH, Schoenleber, 2511.04537



$$I_L^g = \int_{-1}^{1} dx F_g(x,\xi)\left(\frac{4\xi(1-\beta)(x^2+\xi^2)}{(x^2-\xi^2+i\epsilon)^2} - \frac{2\xi(1-2\beta)}{x^2-\xi^2+i\epsilon}\right)$$

GPD

$$C_{nn}^T = \frac{(|X|^2-|Y|^2)k_\perp^2}{k_\perp^2(|X|^2+|Y|^2)+2m^2|I_L^g|^2}$$

$$C_{rr}^T = \frac{k_\perp^2}{k_\perp^2(|X|^2+|Y|^2)+2m^2|I_L^g|^2}\left[-\frac{k_\perp^2-(1-2z)^2m^2}{k_\perp^2+(1-2z)^2m^2}|X|^2+\frac{k_\perp^2-m^2}{k_\perp^2+m^2}|Y|^2 + \frac{8z\bar{z}m^4|I_L^g|^2}{(k_\perp^2+(1-2z)^2m^2)(k_\perp^2+m^2)} + 4m^2\left(-\frac{(1-2z)\mathrm{Re}[XI_L^{g*}]}{k_\perp^2+(1-2z)^2m^2}+\frac{\mathrm{Re}[YI_L^{g*}]}{k_\perp^2+m^2}\right)\right],$$

$$C_{kk}^T = \frac{1}{k_\perp^2(|X|^2+|Y|^2)+2m^2|I_L^g|^2}\left[k_\perp^2\frac{k_\perp^2-(1-2z)^2m^2}{k_\perp^2+(1-2z)^2m^2}|X|^2+k_\perp^2\frac{k_\perp^2-m^2}{k_\perp^2+m^2}|Y|^2 - \frac{2m^2(k_\perp^4-(1-2z)^2m^4)|I_L^g|^2}{(k_\perp^2+(1-2z)^2m^2)(k_\perp^2+m^2)} + 4m^2k_\perp^2\left(\frac{(1-2z)\mathrm{Re}[XI_L^{g*}]}{k_\perp^2+(1-2z)^2m^2}+\frac{\mathrm{Re}[YI_L^{g*}]}{k_\perp^2+m^2}\right)\right],$$

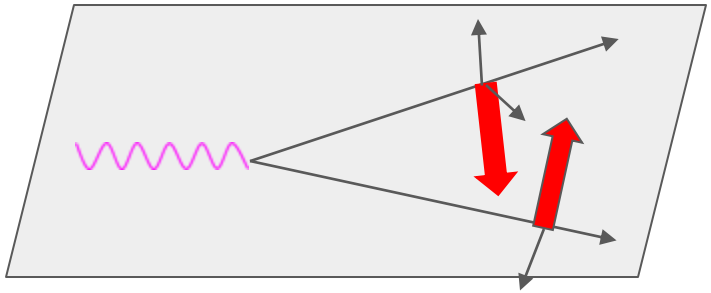
$$C_{rk}^T = \frac{2k_\perp m}{k_\perp^2(|X|^2+|Y|^2)+2m^2|I_L^g|^2}\left[\frac{(1-2z)k_\perp^2|X|^2}{k_\perp^2+(1-2z)^2m^2}+\frac{k_\perp^2|Y|^2}{k_\perp^2+m^2} - \frac{2(1-z)m^2(k_\perp^2+(1-2z)m^2)|I_L^g|^2}{(k_\perp^2+m^2)(k_\perp^2+(1-2z)^2m^2)} - \frac{k_\perp^2-(1-2z)^2m^2}{k_\perp^2+(1-2z)^2m^2}\mathrm{Re}[XI_L^{g*}]-\frac{k_\perp^2-m^2}{k_\perp^2+m^2}\mathrm{Re}[YI_L^{g*}]\right],$$

$$C_{kr}^T = \frac{2k_\perp m}{k_\perp^2(|X|^2+|Y|^2)+2m^2|I_L^g|^2}\left[\frac{(1-2z)k_\perp^2|X|^2}{k_\perp^2+(1-2z)^2m^2}-\frac{k_\perp^2|Y|^2}{k_\perp^2+m^2} + \frac{2zm^2(k_\perp^2-(1-2z)m^2)|I_L^g|^2}{(k_\perp^2+m^2)(k_\perp^2+(1-2z)^2m^2)} - \frac{k_\perp^2-(1-2z)^2m^2}{k_\perp^2+(1-2z)^2m^2}\mathrm{Re}[XI_L^{g*}]+\frac{k_\perp^2-m^2}{k_\perp^2+m^2}\mathrm{Re}[YI_L^{g*}]\right].$$

Spontaneous polarization

Massive quark and antiquark transversely (normal to the production plane) polarized even in unpolarized collisions.

--> single spin asymmetry



Interference between real and imaginary parts of GPD amplitudes.

$$B_n^T = \frac{-2mk_\perp \mathrm{Im}[(X+Y)I_L^{g*}]}{k_\perp^2(|X|^2+|Y|^2)+2m^2|I_L^g|^2}, \qquad \bar{B}_n^T = \frac{-2mk_\perp \mathrm{Im}[(X-Y)I_L^{g*}]}{k_\perp^2(|X|^2+|Y|^2)+2m^2|I_L^g|^2}.$$

Vanishes in the Regge limit (no real part in the amplitude)

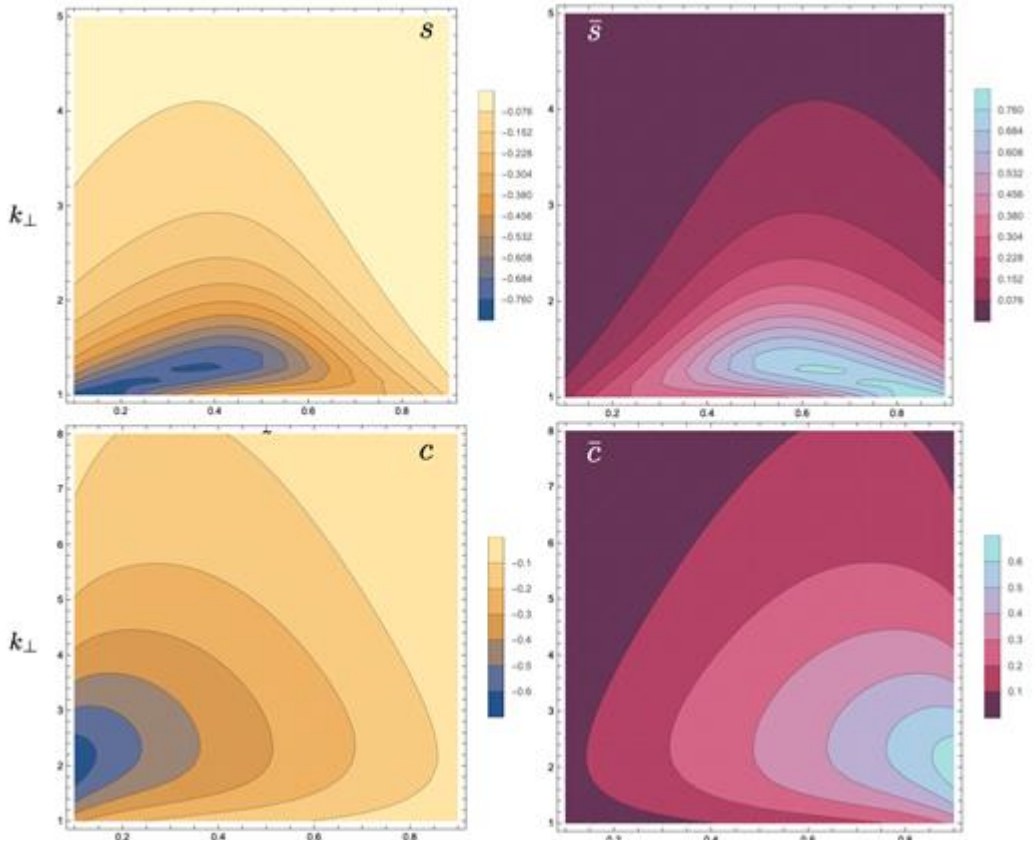
Giant transverse polarization

YH, Schoenleber, 2511.04537

Imaginary part of GPD amplitudes not suppressed by α_s

Up to 80% polarization from leading twist pQCD!

Realized in low-medium CM energies at EIC.

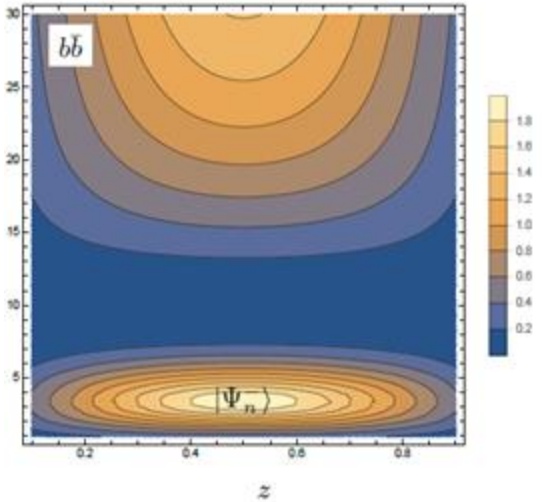


Bell nonlocality $\subset$ entanglement away from the Regge limit

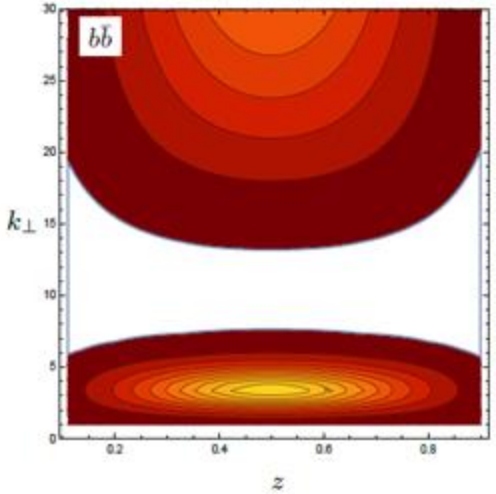
$$(C_{rr}^T)^2 + (C_{rk}^T)^2 + (C_{kr}^T)^2 + (C_{kk}^T)^2 - (C_{nn}^T)^2 - 1$$
$$= 8k_\perp^2 m^2 \frac{(\mathrm{Re}[X I_L^{g*}])^2 + (\mathrm{Re}[Y I_L^{g*}])^2 - (|X|^2 + |Y|^2)|I_L^g|^2}{((|X|^2 + |Y|^2)k_\perp^2 + 2|I_L^g|^2 m^2)^2}$$

2511.04537

Ultraperipheral collisions (UPC) at LHC, W=500GeV



Everywhere entangled



Bell nonlocality not everywhere seen

Entanglement and Bell-nonlocality in DIS

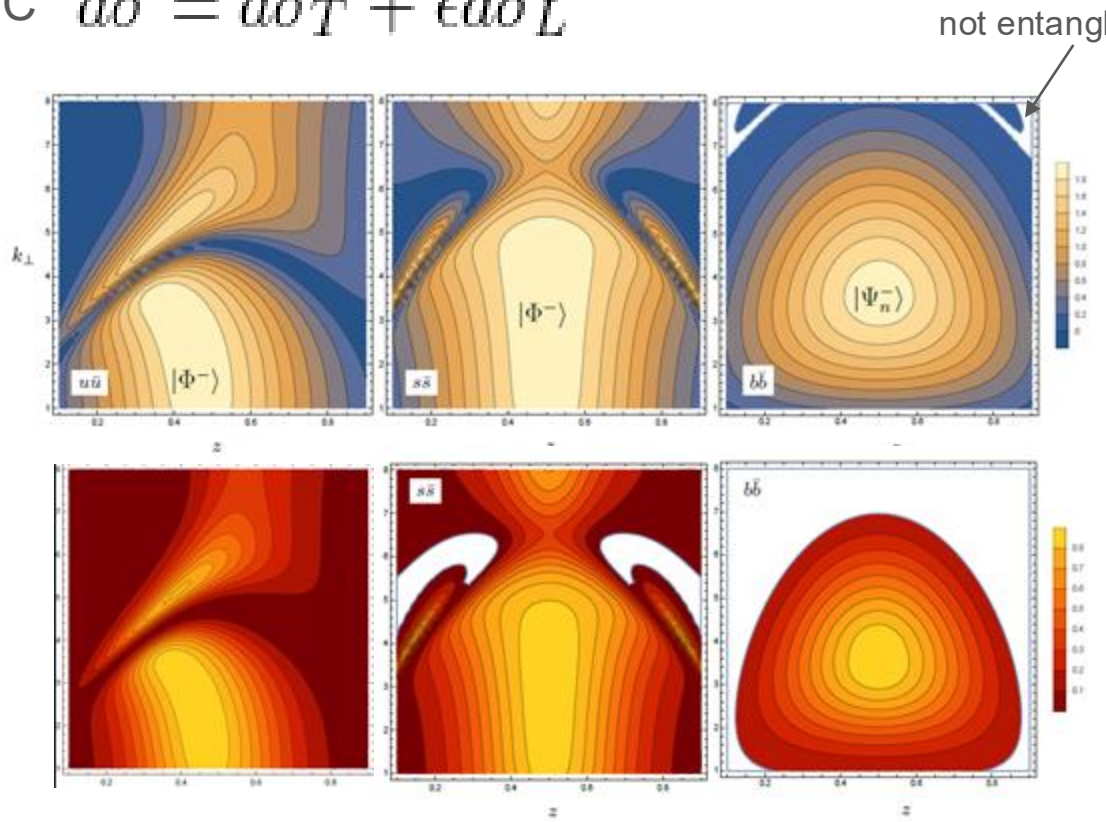
2511.04537

L/T separation challenging at EIC $d\sigma = d\sigma_T + \epsilon d\sigma_L$

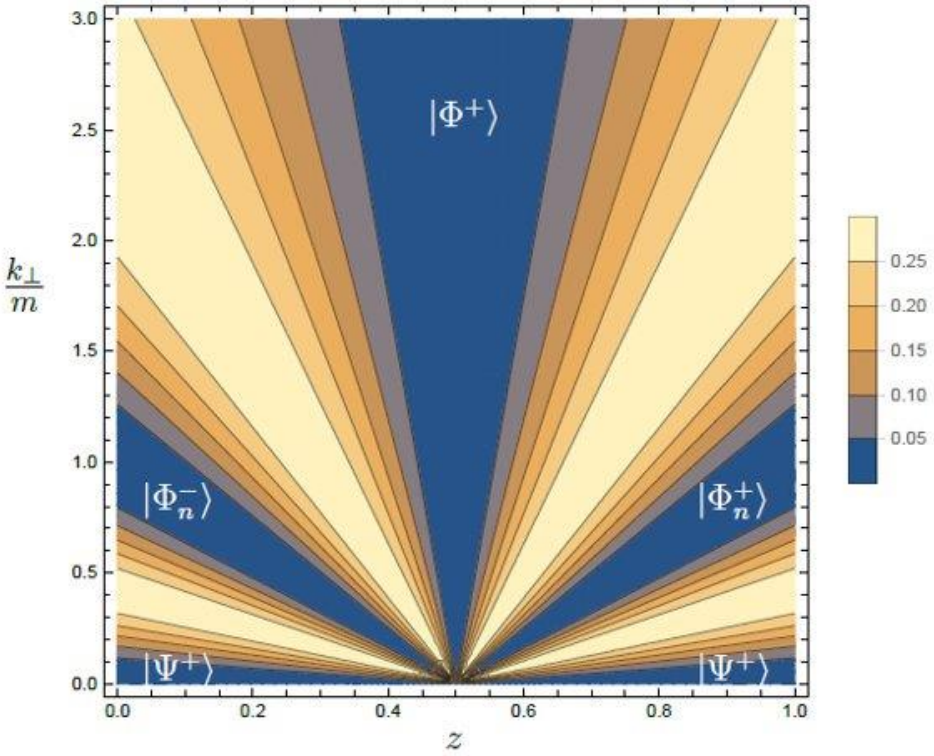
$Q = 10$ GeV, $W = 30$ GeV

Pairs almost everywhere entangled

Bell nonlocality not everywhere seen for massive quarks



Magic (longitudinal photon)



Maximally entangled everywhere.

Magic vanishes only for Bell states.

$$M_2^{L,\max} = \ln \frac{4}{3} \approx 0.288$$

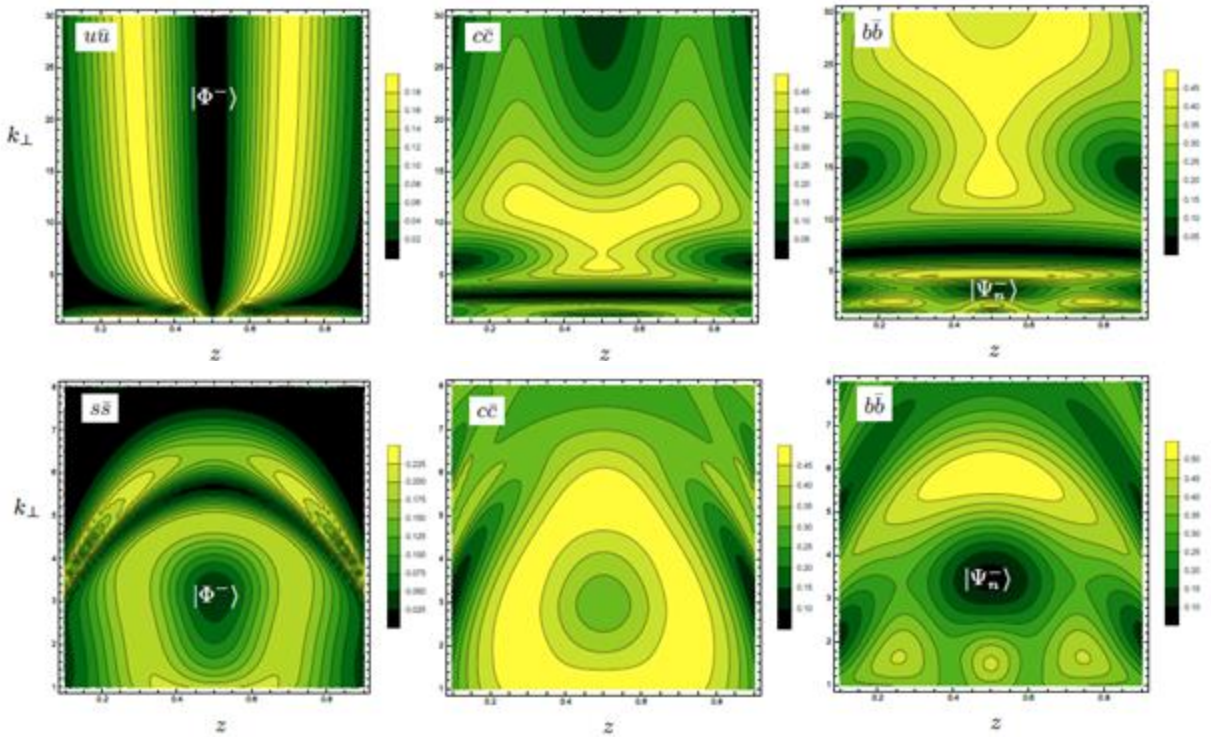
Magic (transverse+longitudinal)

2511.04537

UPC (LHC)
W=500GeV

DIS (EIC)

$W = 30$ GeV, $Q = 10$ GeV



The highest value we found ~0.58
slightly less than $M_2^{\max}|_{B=B=0} = \ln\frac{9}{5} \approx 0.588$

Observable

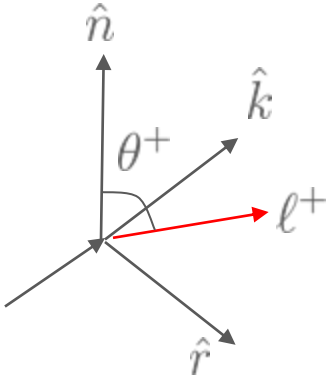
Can the quark-level correlation survive?

$$b \xrightarrow{7.0\%} \Lambda_b \xrightarrow{11\%} X_c^+ + \mu^- + \bar{\nu}_\mu,$$
$$c \xrightarrow{6.4\%} \Lambda_c^+ \xrightarrow{3.5\%} \Lambda + \mu^+ + \nu_\mu,$$
$$s \xrightarrow{2.8\%} \Lambda \xrightarrow{64\%} p + \pi^-,$$

Fragmentation into heavy baryon and self-analyzing weak decay

$$C_{nn}^T = \frac{9}{\alpha_+ \alpha_-} \langle \cos\theta_+ \cos\theta_- \rangle$$

Prediction $\langle \cos\theta_+ \cos\theta_- \rangle \geq 0$



In practice, a number of issues:
spin transfer in hadronization, low branching ratio, decoherence, etc.

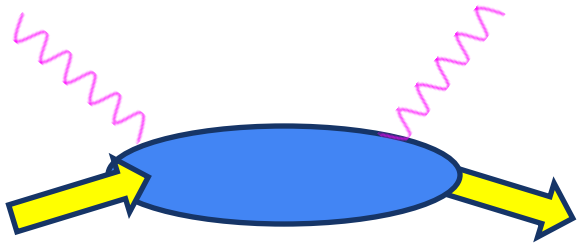
Ways to reconstruct spin correlation without resorting to weak decays? Cheng, Yan (2025)

Entanglement in real Compton scattering off nucleon

YH, Martinez-Fernandez, in preparation

No-go theorem: no entanglement in unpolarized Compton when all amplitudes are real

--> polarized Compton scattering



Low-energy (<140 MeV)

Charge, magnetic moment, electric/magnetic polarizabilities,

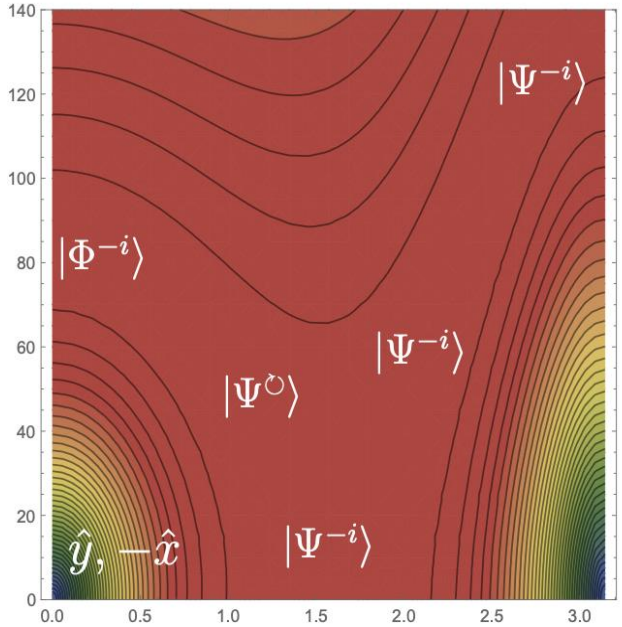
Medium energy (<1 GeV)

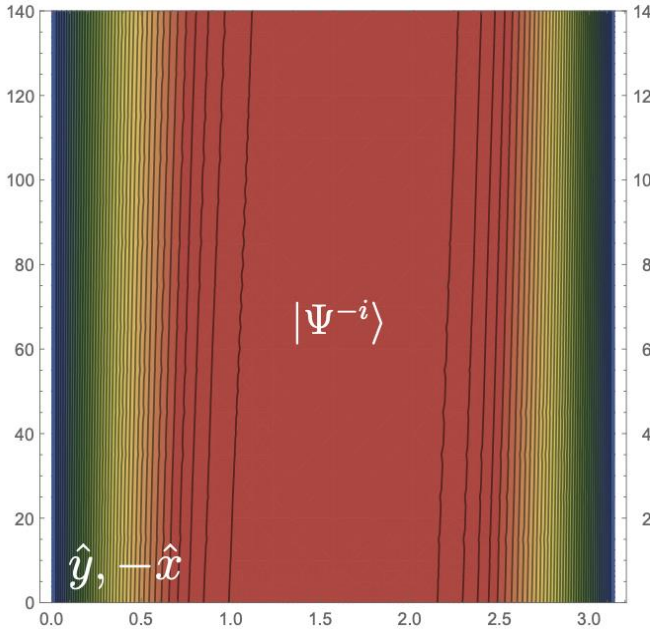
Dispersion relation, chiral perturbation theory

High energy (>>1 GeV)

QCD factorization for wide-angle Compton scattering

Low energy, neutron target





with electric and magnetic polarizabilities

without polarizabilities

Can final photon and nucleon polarizations be measured?
Bylund, Defurne, Guichon. (2022)

Wide-angle Compton scattering

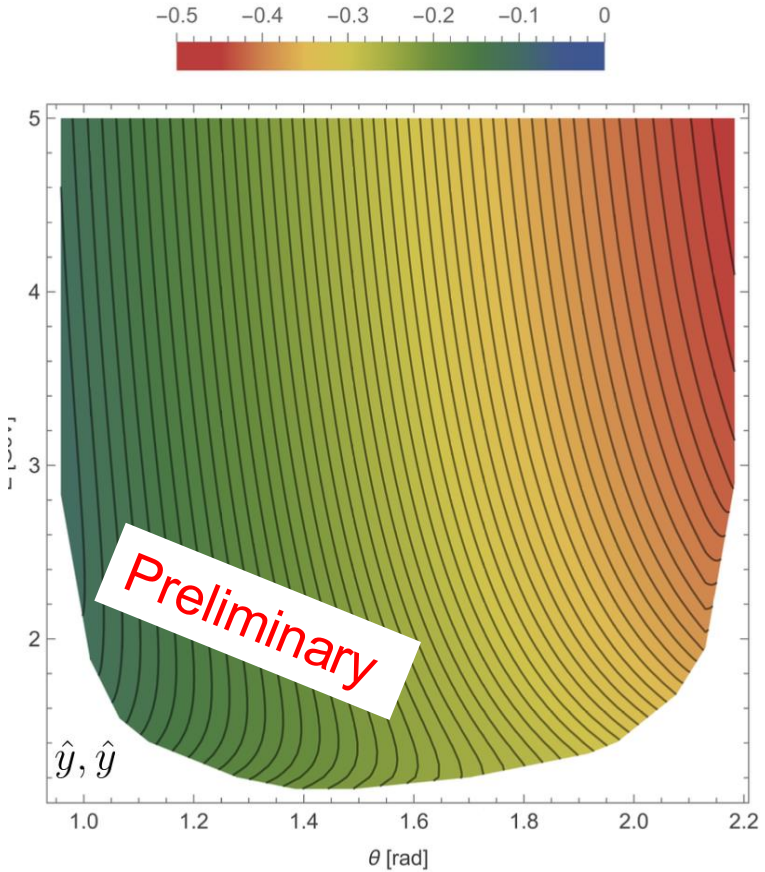
QCD factorization in terms of GPD when

$$s \sim -t \sim -u$$

Radyushkin (1998)
Diehl, Feldmann, Jakob, Kroll (1999)
Huang, Kroll, Morii (2002)

Negative moment of GPD involved

$$\int_{-1}^{1} \frac{dx}{x} H^q(x, \eta = 0, t)$$



Conclusions

Quantum entanglement at the EIC:
novel topic beyond the Yellow report.

A new class of observables sensitive to GPD and TMD,
one that connects/appeals to broader physics communities.