Evolution of pion-nucleon transition distribution amplitudes

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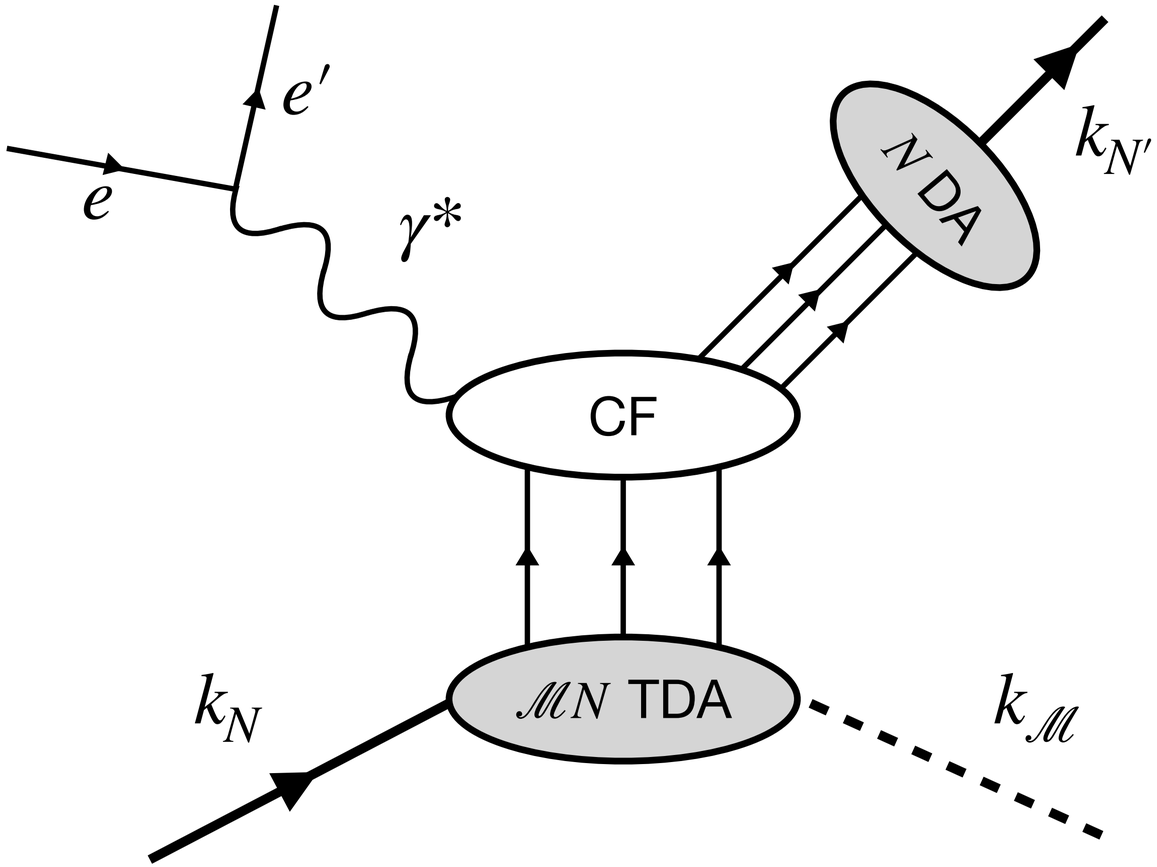
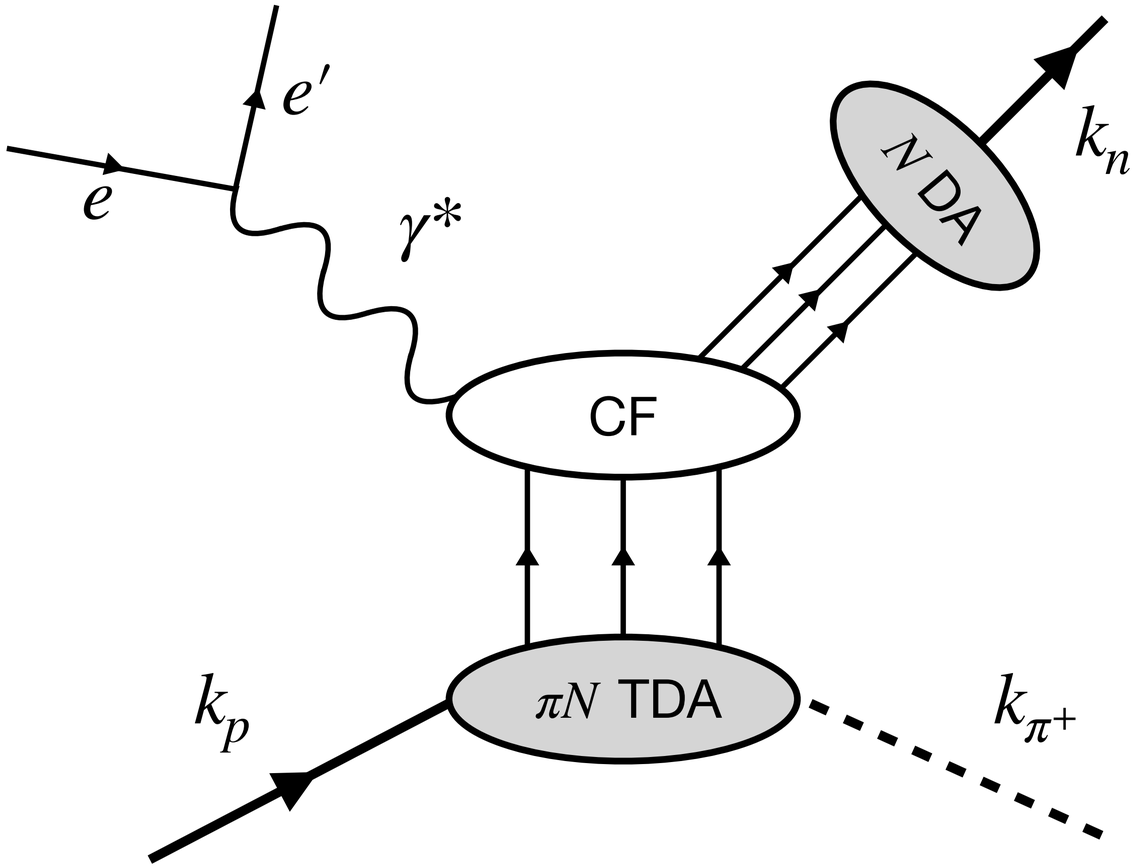
Exclusive meson electroproduction



$$eN \to e'N'\mathscr{M}$$

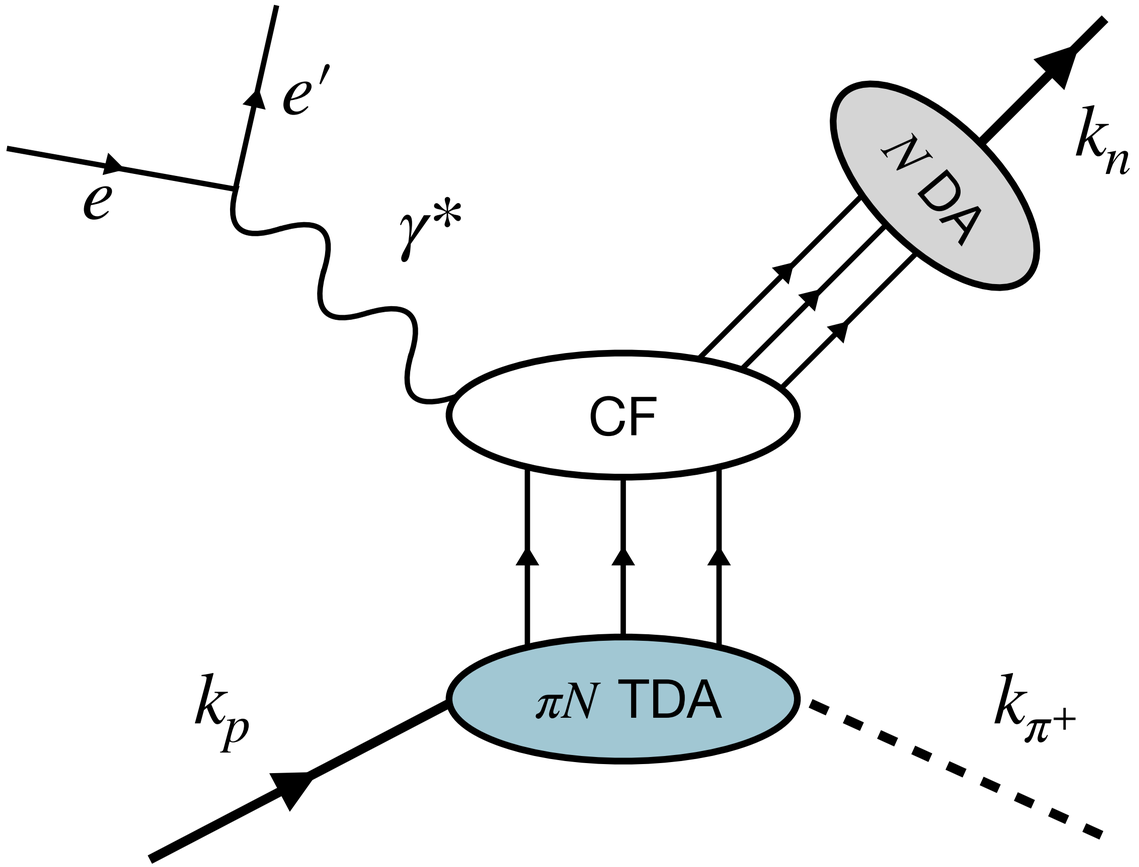
Exclusive meson electroproduction



$$ep \to e'n\pi^+$$

measured at JLab [*Phys. Lett. B* ***780****, 340-345, (2018)*]

Exclusive meson electroproduction



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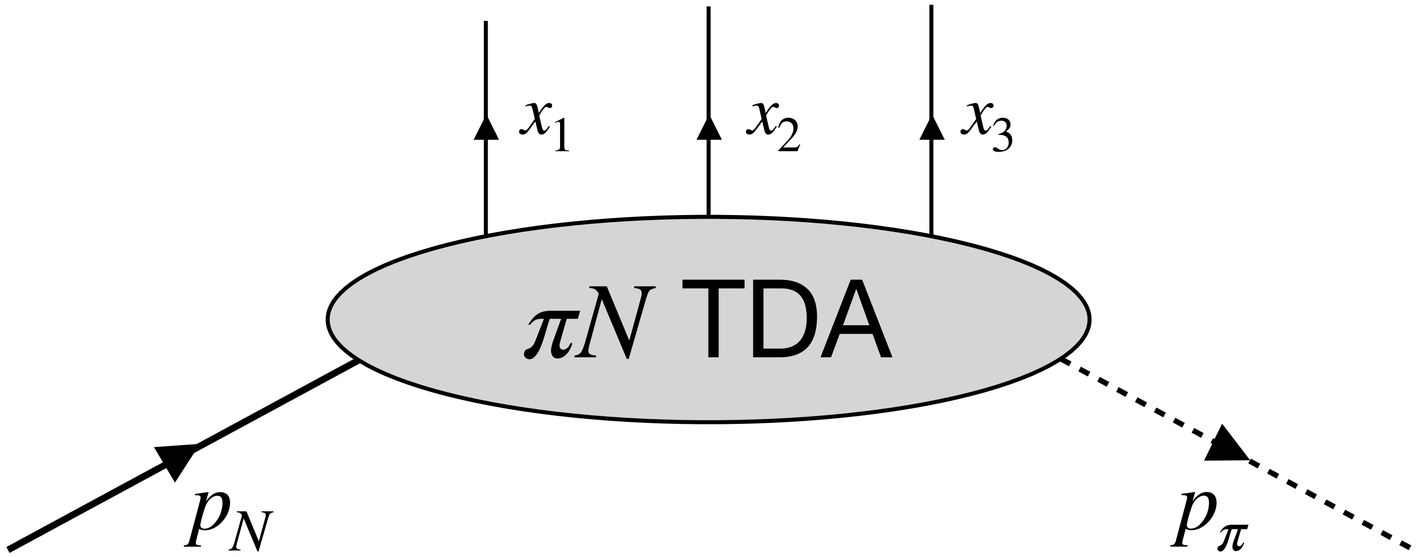
Pion-nucleon TDAs

$$\langle \pi \,|\, u_{\lambda_1}(z_1) u_{\lambda_2}(z_2) d_{\lambda_3}(z_3) \,|\, N, \Lambda \rangle \sim \int [dx] e^{iP^+ \sum_j x_j z_j} f(x_1, x_2, x_3)$$



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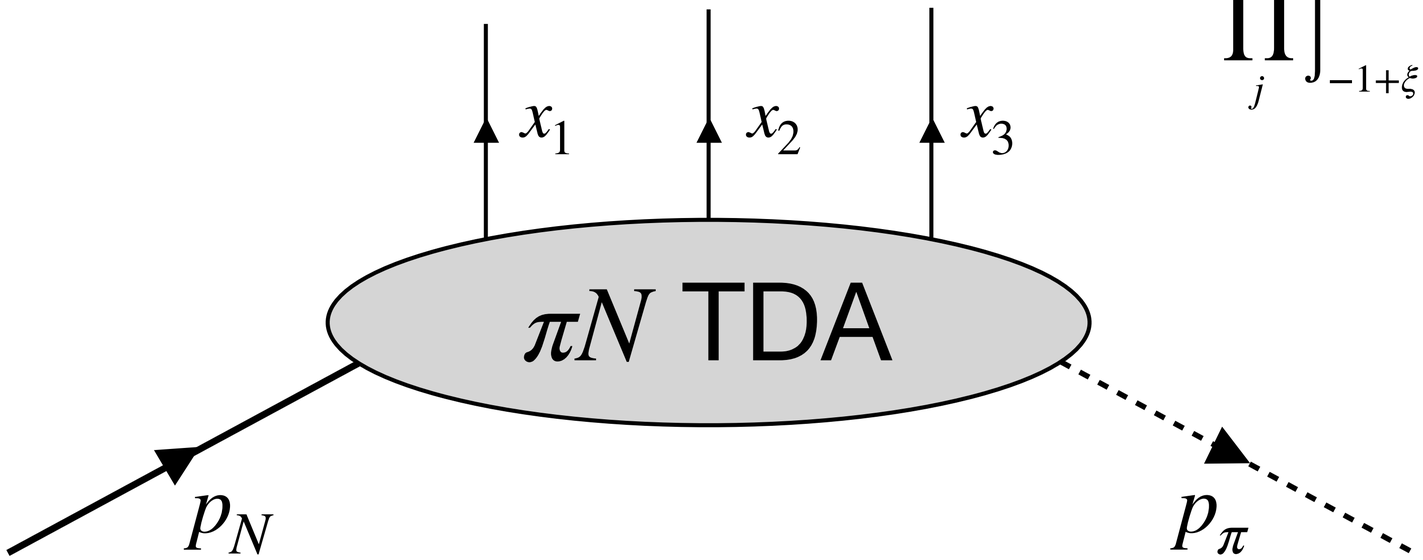
momentum conservation:

$$\prod_j \int_{-1+\xi}^{+1+\xi} dx_j \, \delta(x_1 + x_2 + x_3 - 2\xi)$$

$$\xi = \frac{p_N^+ - p_\pi^+}{p_N^+ + p_\pi^+}$$



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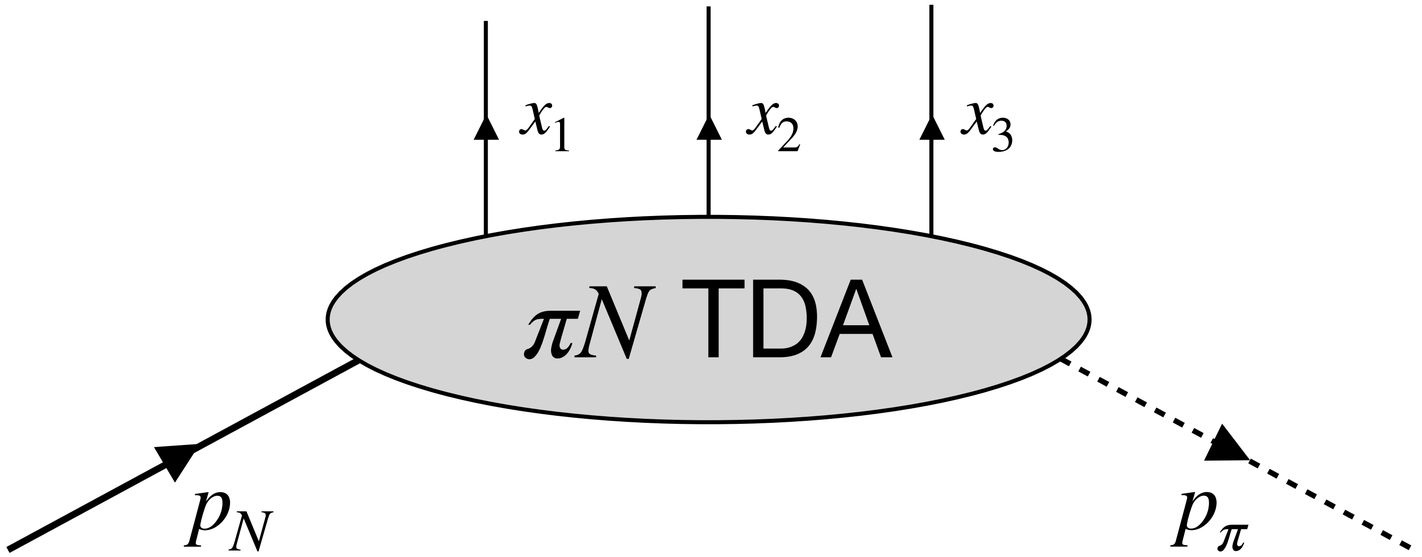
Helicity amplitudes

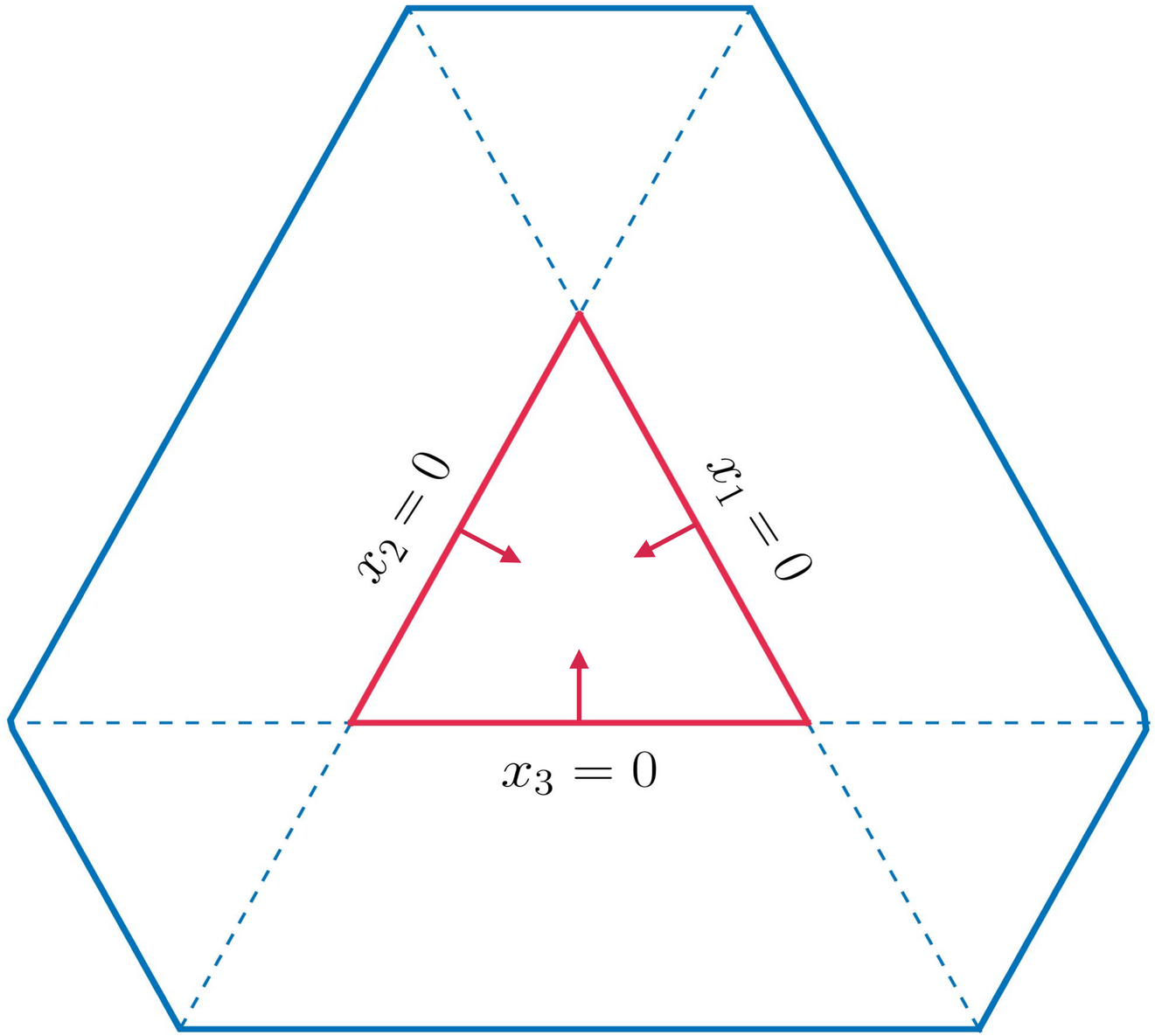
$$T^{\Lambda}_{\lambda_1 \lambda_2 \lambda_3}$$



TDA support

$$w = x_3 - \xi$$
$$w \in [-1, +1]$$

$$v = \frac{x_1 - x_2}{2}$$
$$v \in [-d, +d]$$
$$d = 1 - \left| \frac{\xi + w}{2} \right|$$



TDA support

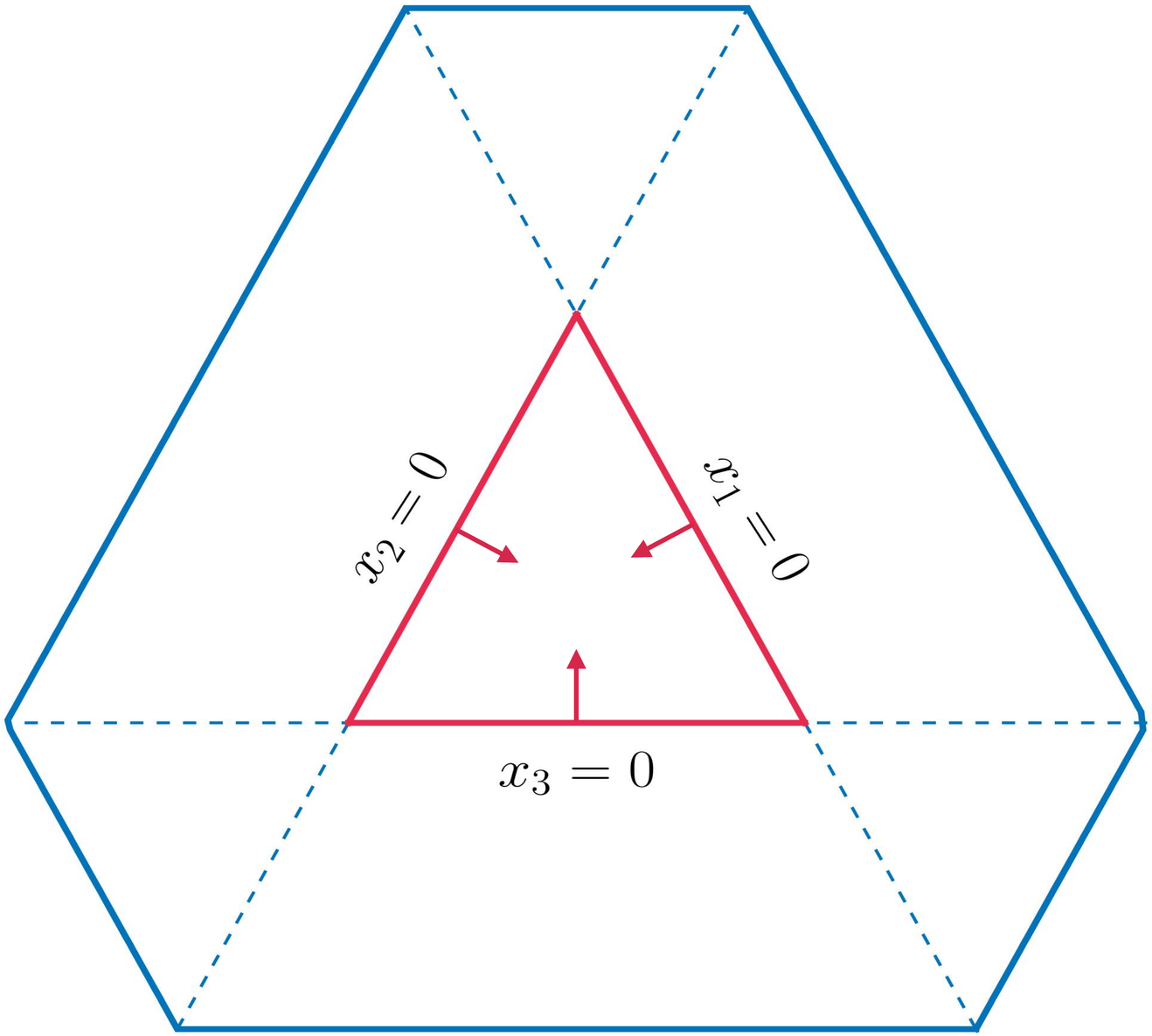
$$w = x_3 - \xi$$

$$w \in [-1, +1]$$

$$v = \frac{x_1 - x_2}{2} \rightarrow t = \frac{v}{d}$$

$$v \in [-d, +d]$$

$$J(w) = 1 - \left| \frac{\xi + w}{2} \right|$$



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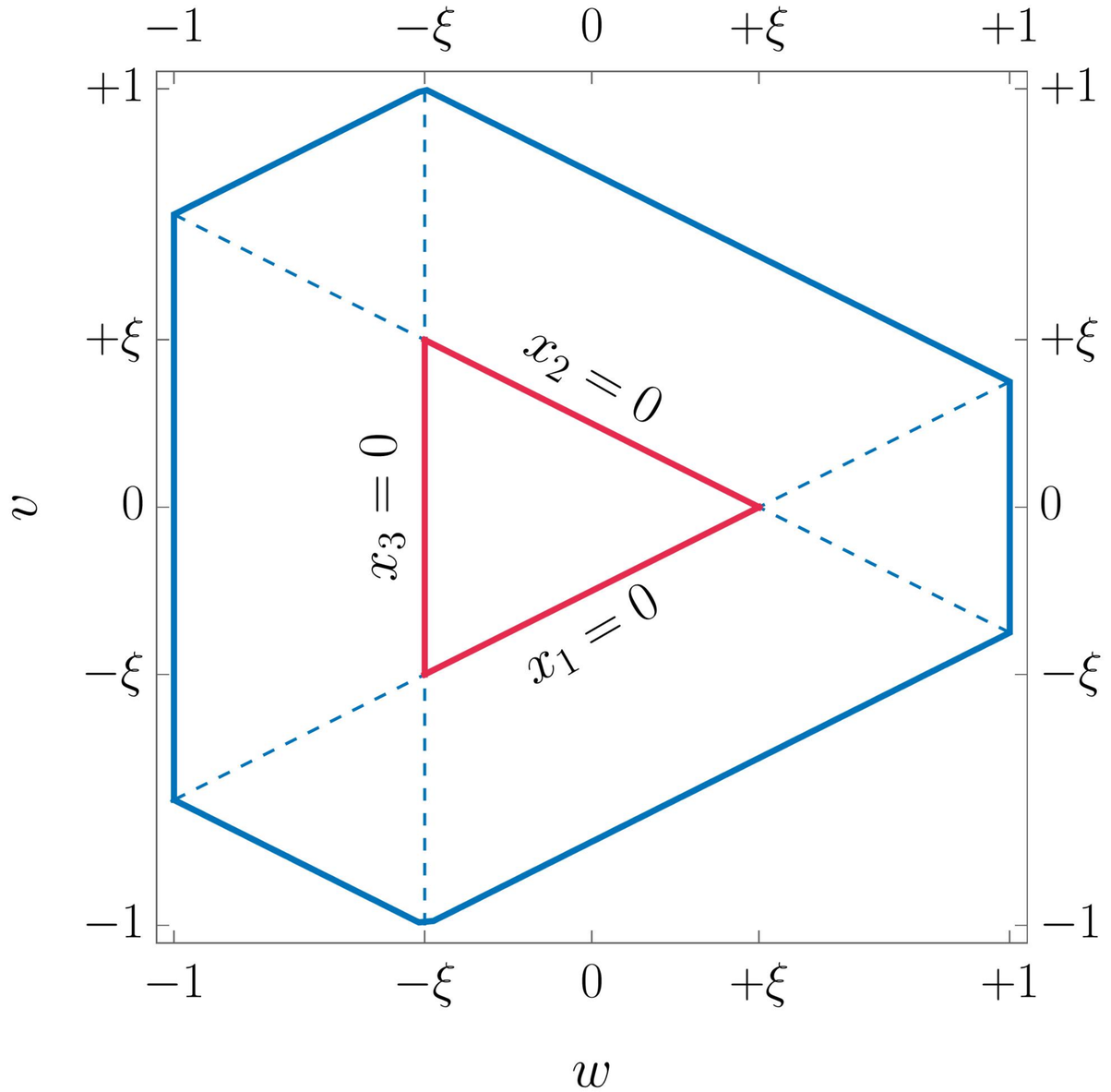
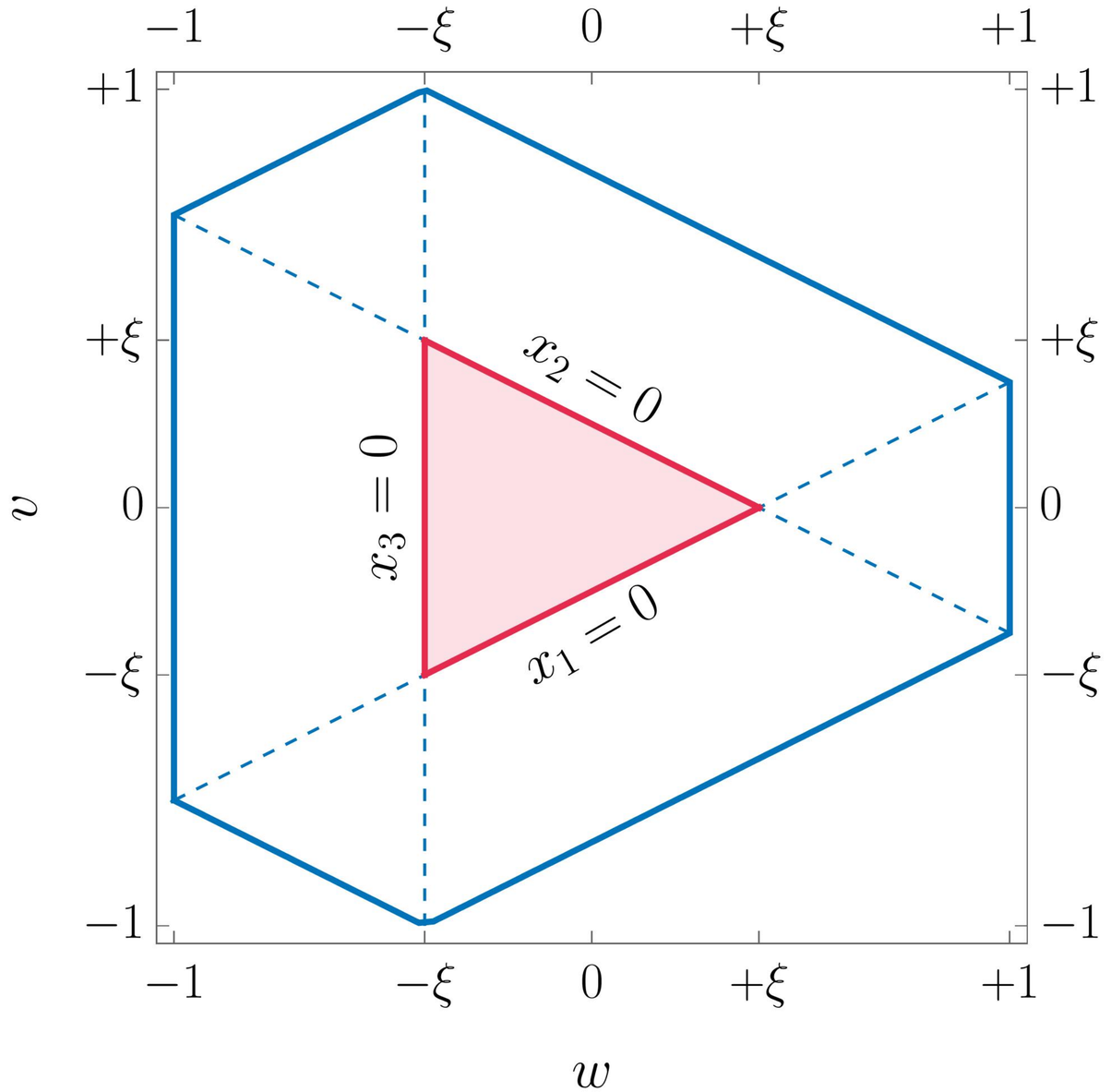
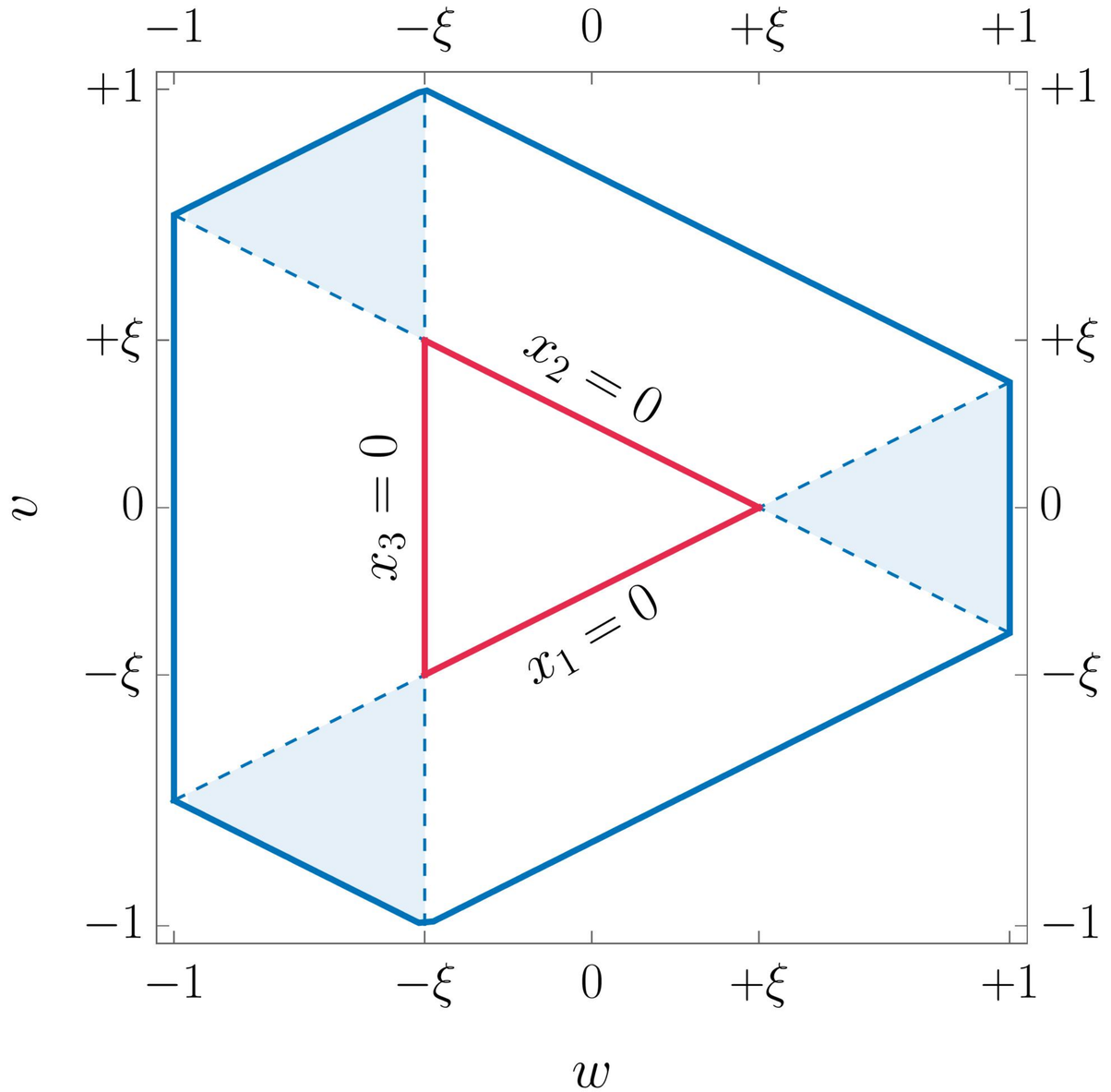


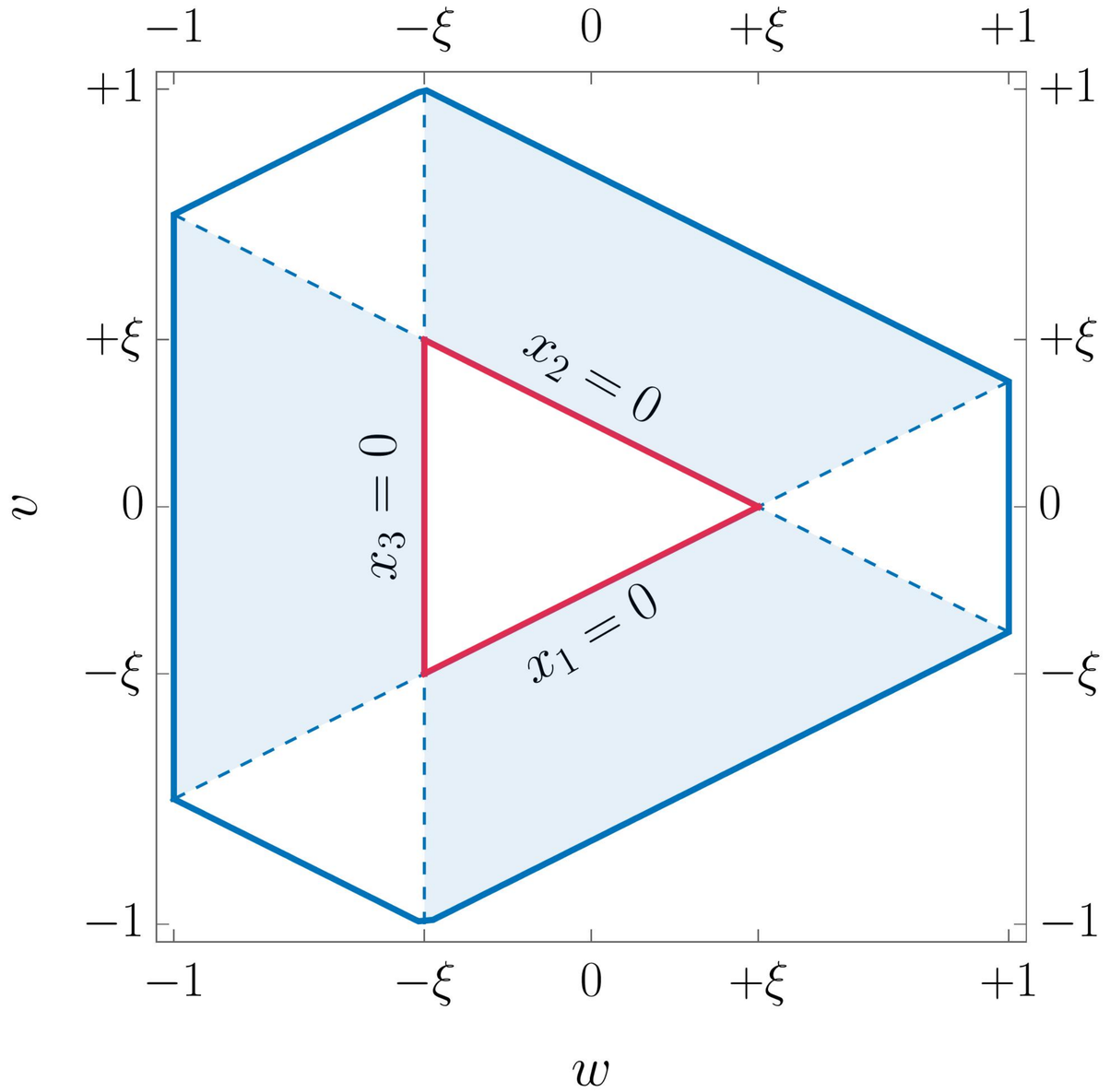
ERBL region



DGLAP type I region



DGLAP type II region



Polynomial expansion

$$T^{\Lambda}_{\lambda_1\lambda_2\lambda_3}(w,t;\mu) = \sum_{i,j=0}^{n} c_{ij}(\mu)\, P_i(t)\Pi_j(w)$$

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$h_n(y), q_n(y)$

orthonormal polynomials

$$\begin{cases} P_i(t) = h_i(t)\sqrt{\mu(t)} & \rightarrow \int_{-1}^{+1} \mu(t) h_i(t) h_k(t) = \delta_{ik} \\ \Pi_j(w) = q_j(w)\sqrt{\dfrac{\eta(w)}{J(w)}} & \rightarrow \int_{-1}^{+1} \eta(w) q_j(w) q_k(w) = \delta_{jk} \end{cases}$$

Polynomial expansion

$$T^{\Lambda}_{\lambda_1\lambda_2\lambda_3}(w,t;\mu) = \sum_{i,j=0}^{n} c_{ij}(\mu)\boxed{P_i(t)\Pi_j(w)}$$

Gegenbauer polynomials

$$h_n(y), q_n(y) \propto C_n^{(3/2)}(y)$$

$$\begin{cases} P_i(t) = h_i(t)\sqrt{\mu(t)} & \rightarrow \int_{-1}^{+1} \mu(t)h_i(t)h_k(t) = \delta_{ik} \\ \Pi_j(w) = q_j(w)\sqrt{\frac{\eta(w)}{J(w)}} & \rightarrow \int_{-1}^{+1} \eta(w)q_j(w)q_k(w) = \delta_{jk} \end{cases}$$

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Polynomial expansion

$$\int_{-1}^{+1} dw \int_{-1}^{+1} dt\, J(w)\, P_i(t)\Pi_j(w)\, T^{\Lambda}_{\lambda_1\lambda_2\lambda_3}(w,t;\mu)$$

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Polynomial expansion

TDA model at initial scale

$$\int_{-1}^{+1} dw \int_{-1}^{+1} dt\, J(w)\, P_i(t)\Pi_j(w)\, T^{\Lambda}_{\lambda_1\lambda_2\lambda_3}(w,t;\mu)$$

$$T^{\Lambda}_{\lambda_1\lambda_2\lambda_3}(w,t;\mu) = \sum_{i,j=0}^{n} c_{ij}(\mu) P_i(t)\Pi_j(w)$$

Evolution equations

$$\frac{dT^{\Lambda}_{\lambda_1\lambda_2\lambda_3}(\mathbf{x};\mu)}{d\ln\mu} = -\frac{\alpha_S(\mu)}{2\pi}\left[\left(1+\frac{1}{N_c}\right)\mathbf{H} + \frac{3}{2}C_F\right]T^{\Lambda}_{\lambda_1\lambda_2\lambda_3}(\mathbf{x};\mu)$$

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(two-particles kernels) $\quad \hat{\mathscr{H}}^{12} + \hat{\mathscr{H}}^{13} + \hat{\mathscr{H}}^{23} - h_{\lambda_1\lambda_2\lambda_3}$

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(two-particles kernels) $\hat{\mathscr{H}}^{12}+\hat{\mathscr{H}}^{13}+\hat{\mathscr{H}}^{23}-h_{\lambda_1\lambda_2\lambda_3}$

$$\lambda_i=\lambda_j\neq\lambda_k \;\rightarrow\; h_{\lambda_i\lambda_j\lambda_k}=\mathscr{H}^{+}_{ik}+\mathscr{H}^{+}_{jk}$$

$$\lambda_i=\lambda_j=\lambda_k \;\rightarrow\; h_{\lambda_i\lambda_j\lambda_k}=0$$

Kernel evaluation

$$(\hat{\mathscr{H}}^{12} \otimes T)(x_1, x_2; \mu) = \int \mathscr{D}x' \, \Phi^{12}(x_1, x_2, x_1', x_2') T(x_1', x_2'; \mu)$$

Kernel evaluation

$$\int_{-1+\xi}^{+1+\xi} dx_1' \int_{-1+\xi}^{+1+\xi} dx_2' \, \delta(x_1 + x_2 - x_1' - x_2')$$

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notation: $b_{ij}(x_1, x_2) \doteq P_i\big(t(x_1, x_2)\big)\Pi_j\big(w(x_1, x_2)\big)$

Kernel evaluation

1. $$\int dx_1 dx_2 \, b_{ij}(x_1, x_2) \frac{dT(x_1, x_2; \mu)}{d \ln \mu}$$

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2. $$\int dx_1 dx_2\, b_{ij}(x_1, x_2)(\hat{\mathscr{H}}^{12} \otimes T)(x_1, x_2; \mu)$$ T expansion

$$= \sum_{kl} c_{kl}(\mu) \int dx_1 dx_2\, b_{ij}(x_1, x_2)(\hat{\mathscr{H}}^{12} \otimes b_{kl})(x_1, x_2)$$

Kernel evaluation

$$\frac{dc_{ij}(\mu)}{d\ln\mu} = -\frac{\alpha_S(\mu)}{2\pi}\sum_{kl} c_{kl}(\mu)\ \mathcal{I}^{12}_{ijkl}$$

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rank-4 tensor

$$\int dx_1 dx_2\, b_{ij}(x_1,x_2)\int \mathscr{D}x'\,\Phi^{12}(x_1,x_2,x_1',x_2')\,b_{kl}(x_1',x_2')$$

3D numerical integrals

Evolved TDA

$$\frac{dc_{ij}(\mu)}{d\ln\mu} = -\frac{\alpha_S(\mu)}{2\pi}\sum_{kl} c_{kl}(\mu)\left[\left(1+\frac{1}{N_c}\right)\mathbf{I}_{ijkl} + \frac{3}{2}C_F\,\mathbf{1}\right]$$

$$\downarrow$$

$$\mathcal{I}^{12}_{ijkl} + \mathcal{I}^{13}_{ijkl} + \mathcal{I}^{23}_{ijkl} - \mathcal{I}^{\lambda_1\lambda_2\lambda_3}_{ijkl}$$

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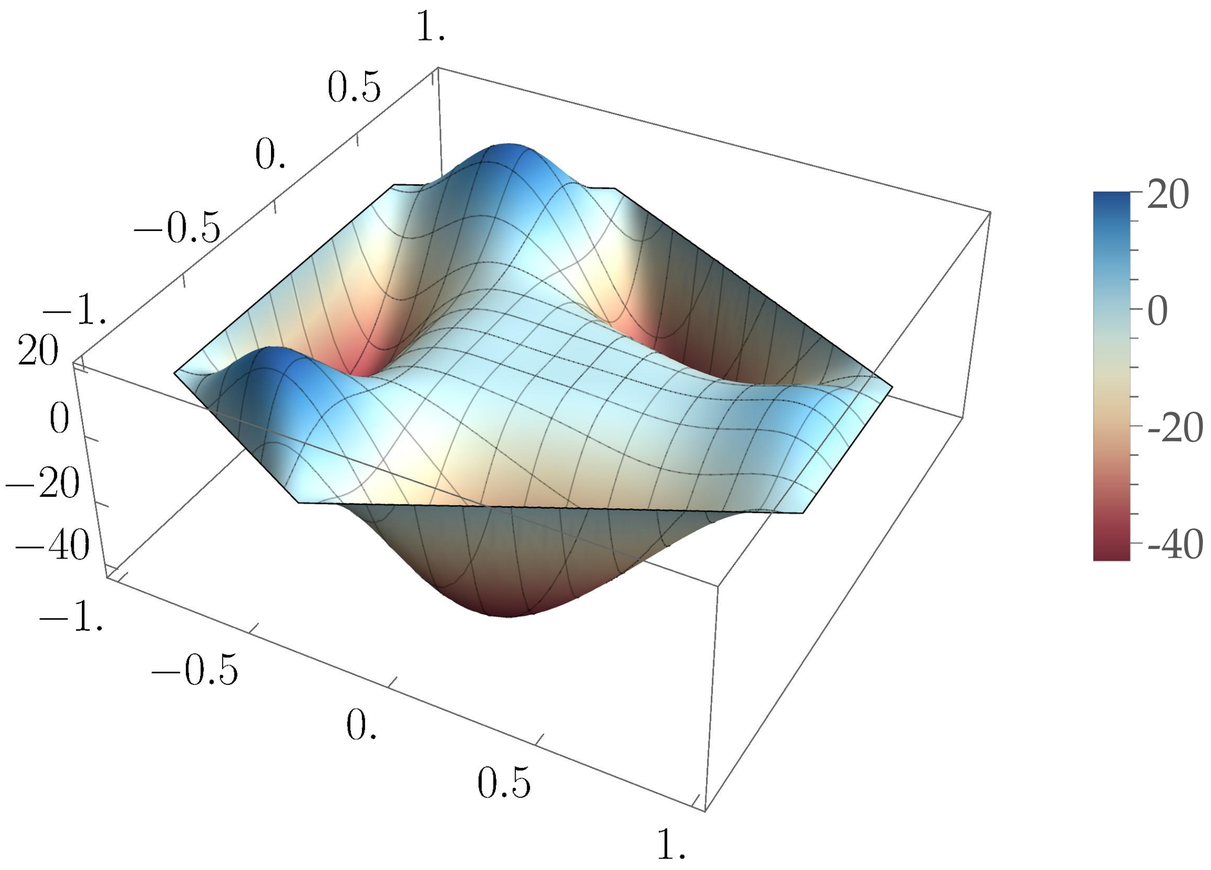
evaluate $c_{ij}(\mu_I)\ \forall i,j \Rightarrow$ solve for $c_{ij}(\mu_F)$

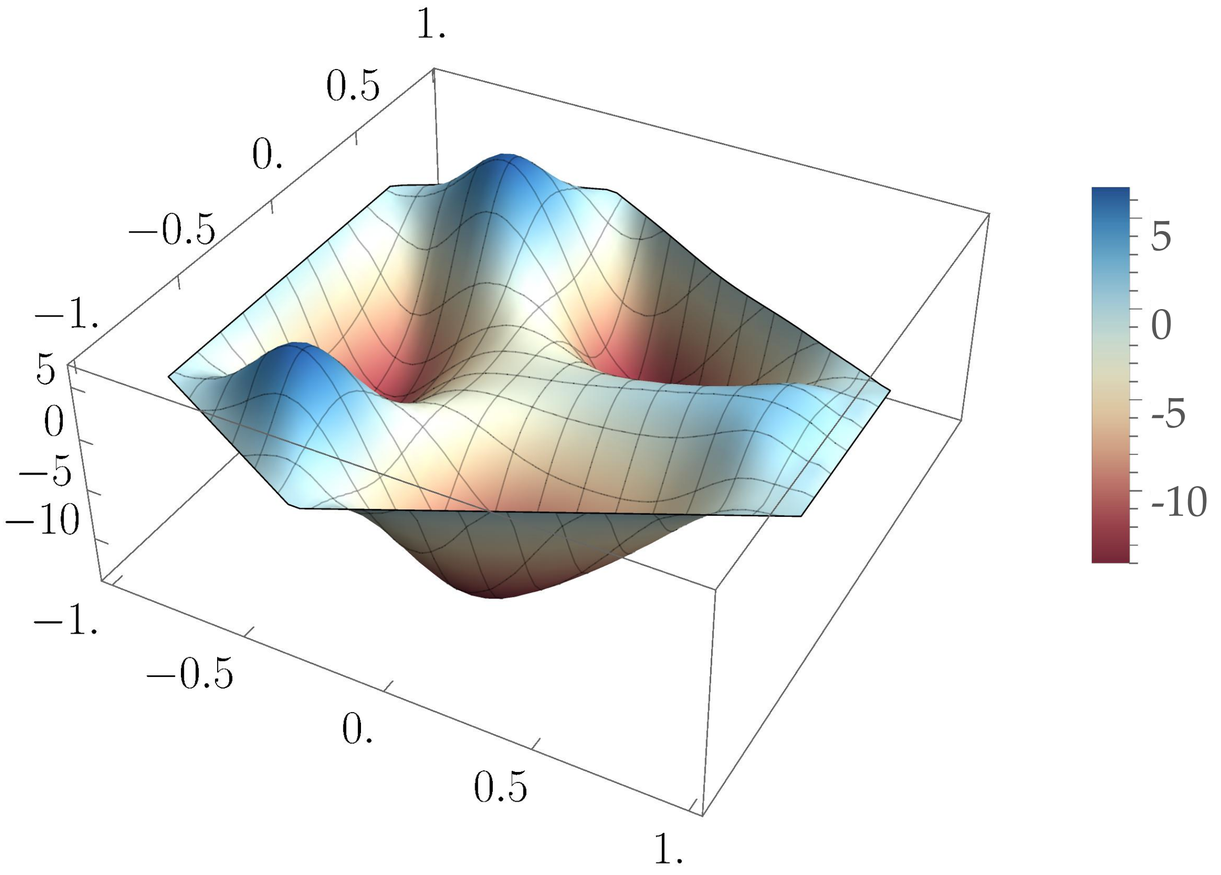
$$T^{\Lambda}_{\lambda_1\lambda_2\lambda_3}(w,t;\mu_F) = \sum_{i,j=0}^{n} c_{ij}(\mu_F)\,P_i(t)\Pi_j(w)$$

Results

$T_{\uparrow\uparrow\downarrow}(w, v; \mu = 1GeV)$



$T_{\uparrow\uparrow\downarrow}(w, v; \mu = 10GeV)$



initial model built from *Phys. Rept.* ***940***, *1-121, (2021)*

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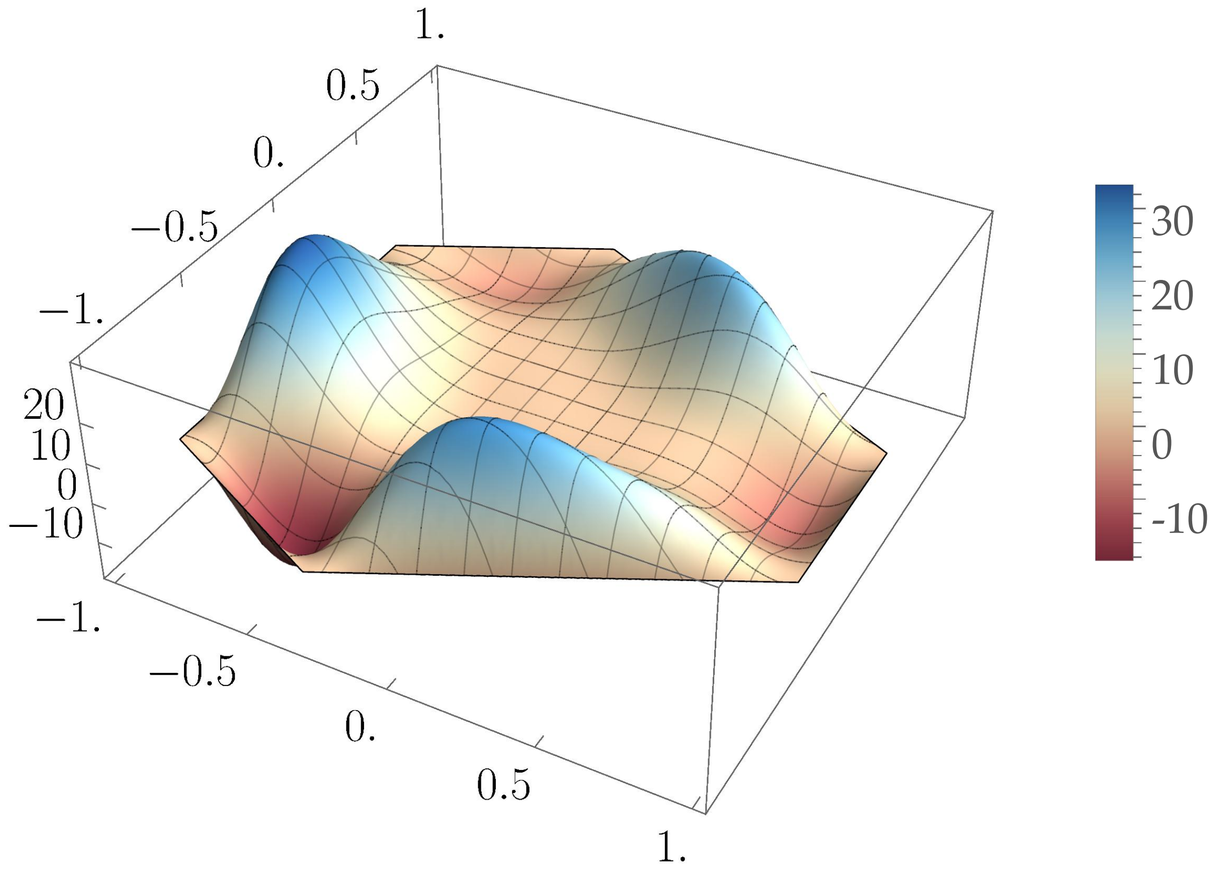

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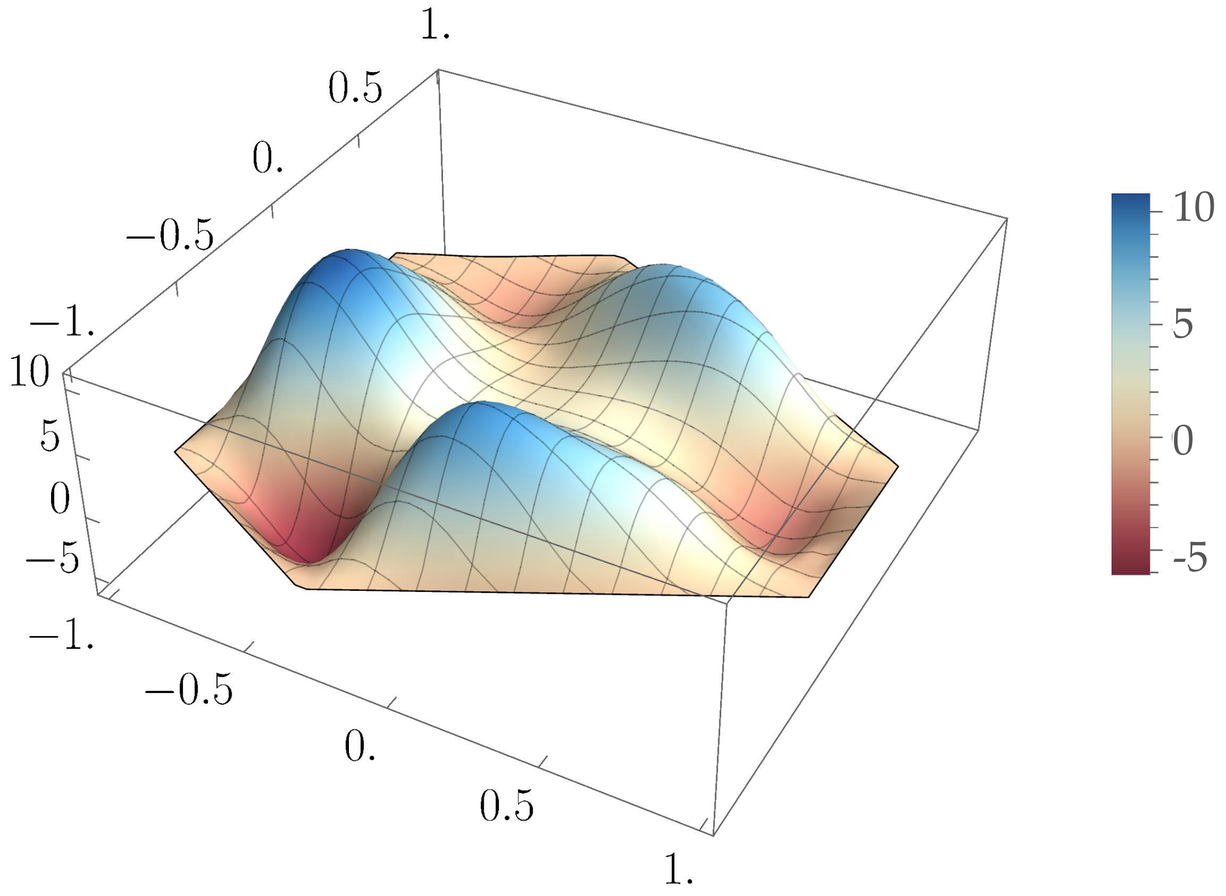

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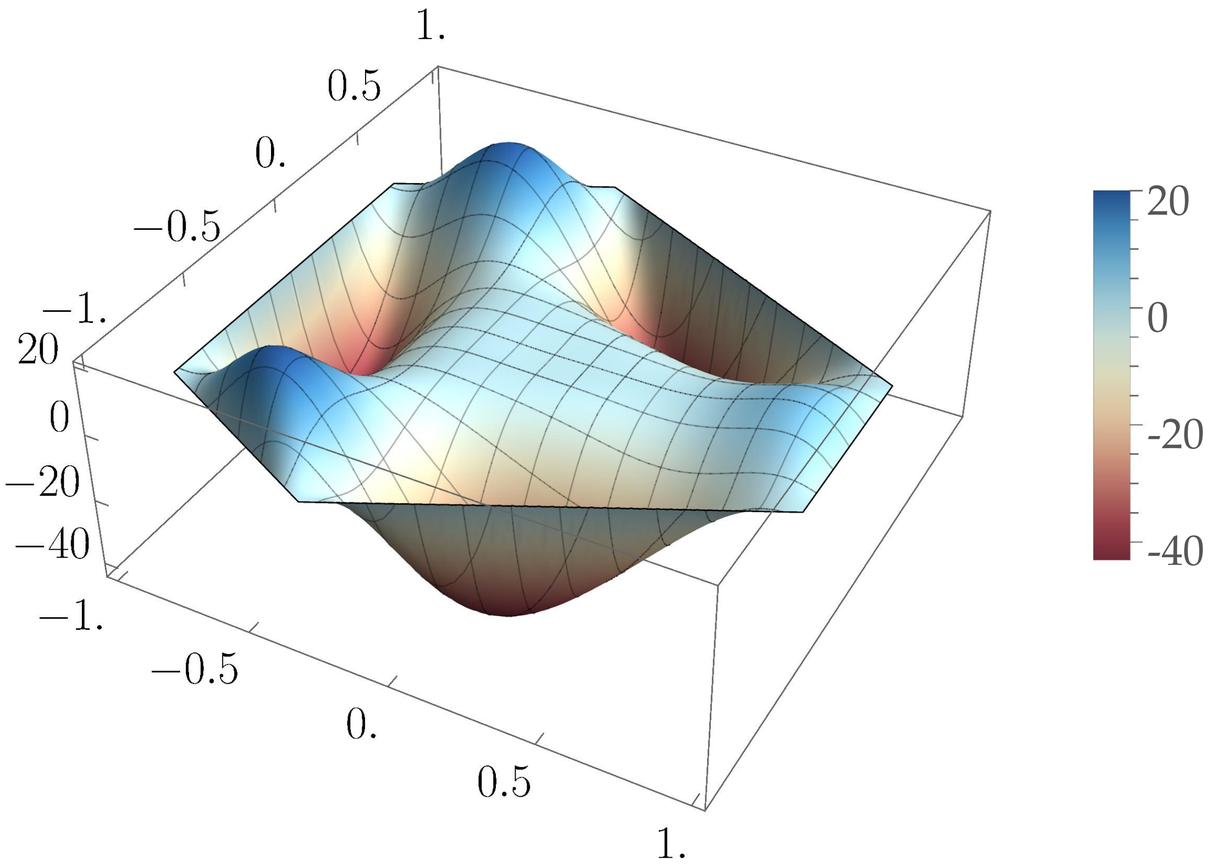
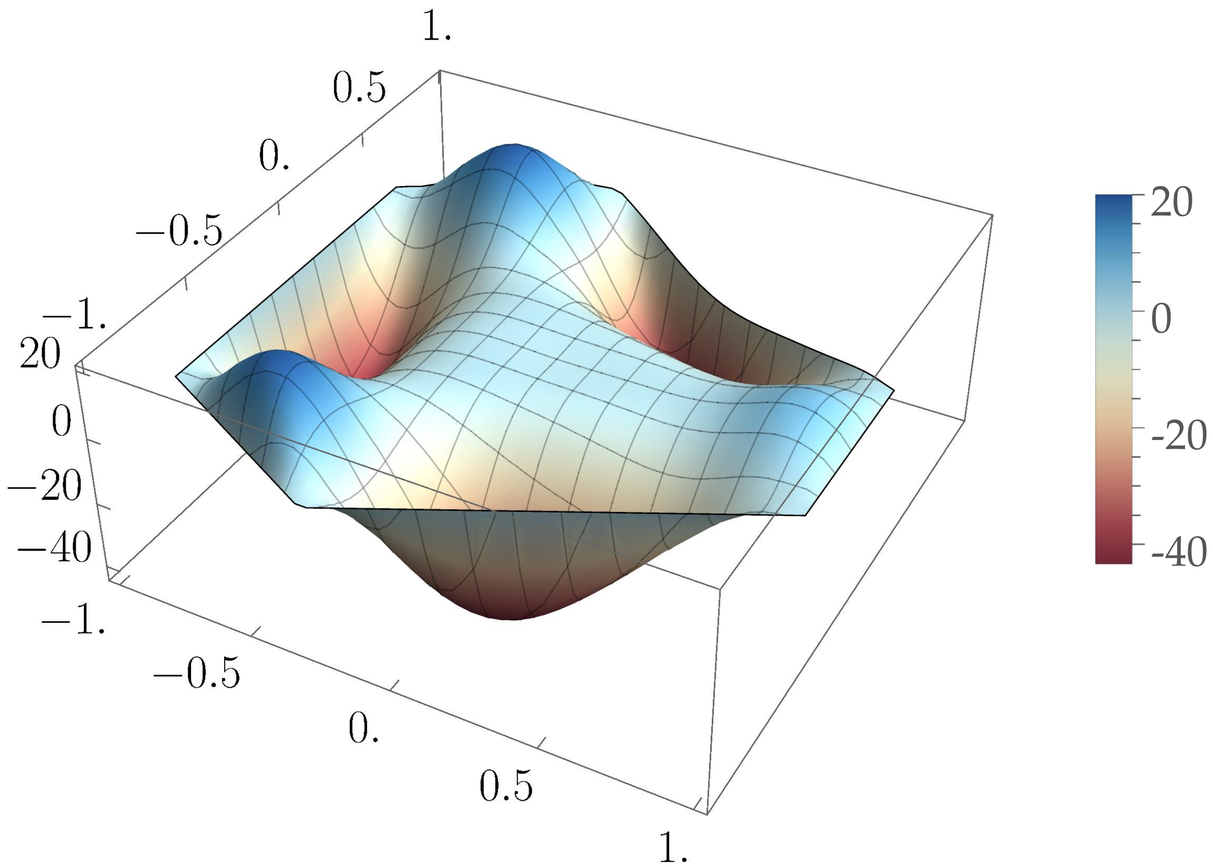
- TDA expansion on polynomial base → **avoid interpolation**
- Evolution "**transferred**" on expansion coefficients → model complexity has low impact on the solution
- Kernel computation is **model independent**, but **lenghty** → needs to be improved

Initial distribution $T_{\uparrow\uparrow\downarrow}$

model



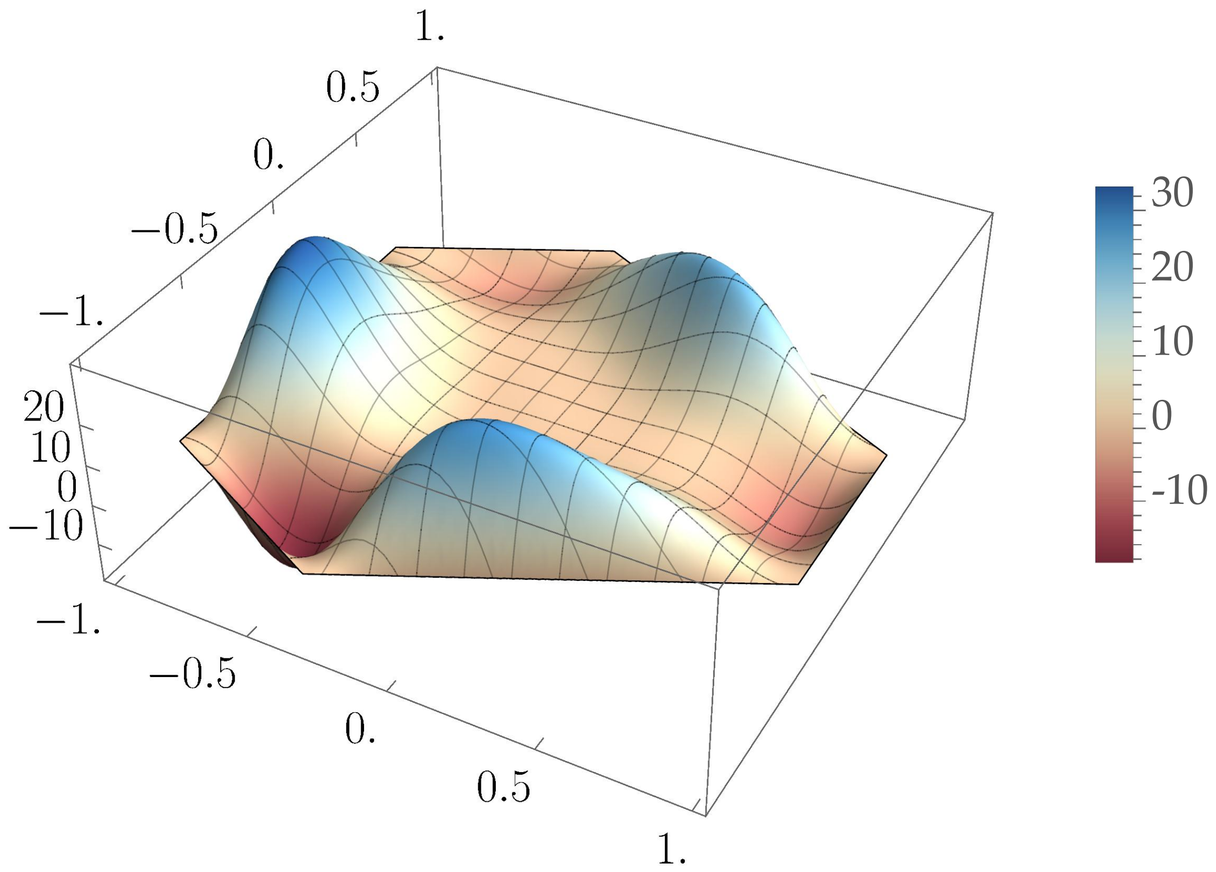
polynomial expansion



maximum difference ~ 3%

Initial distribution $T_{\downarrow\uparrow\uparrow}$

model



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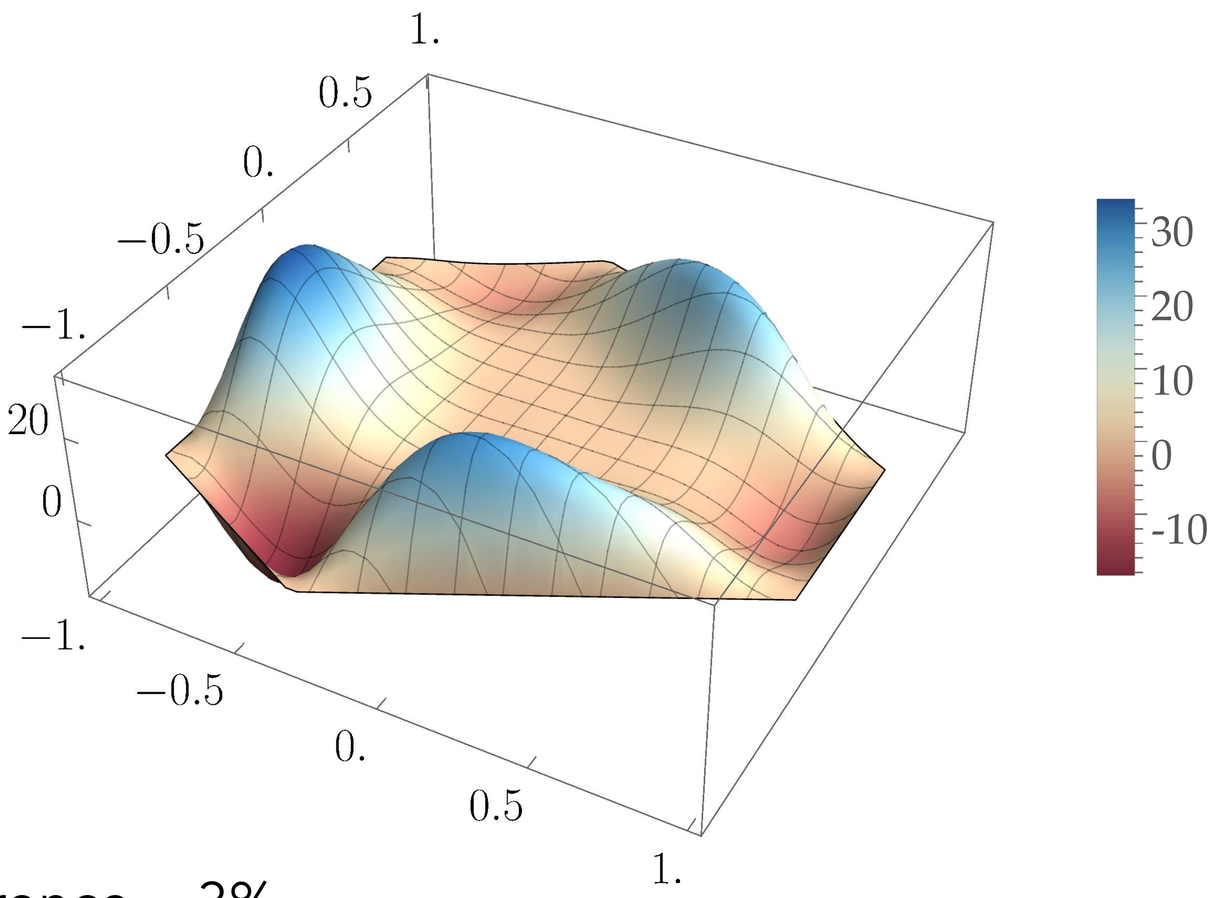

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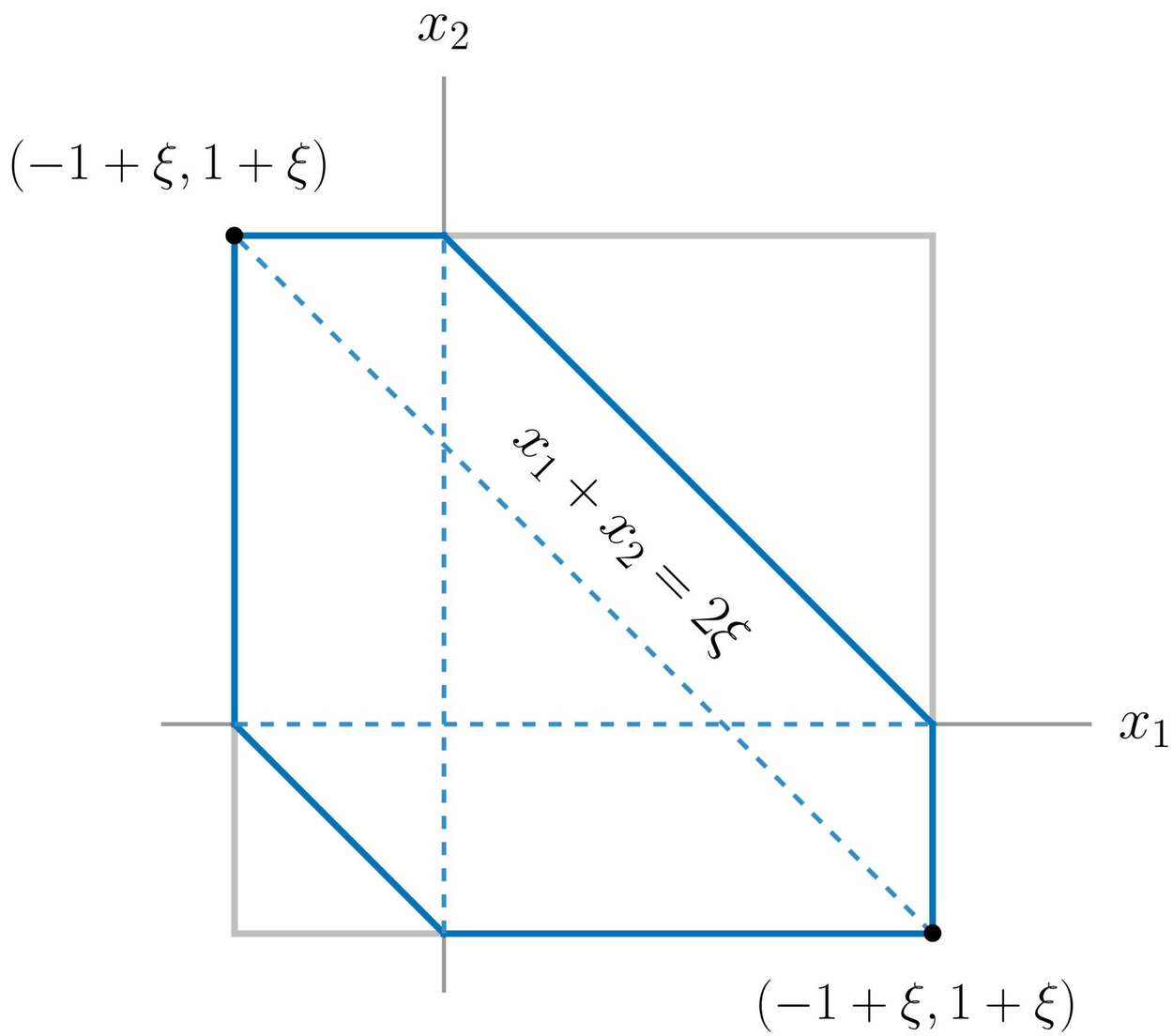
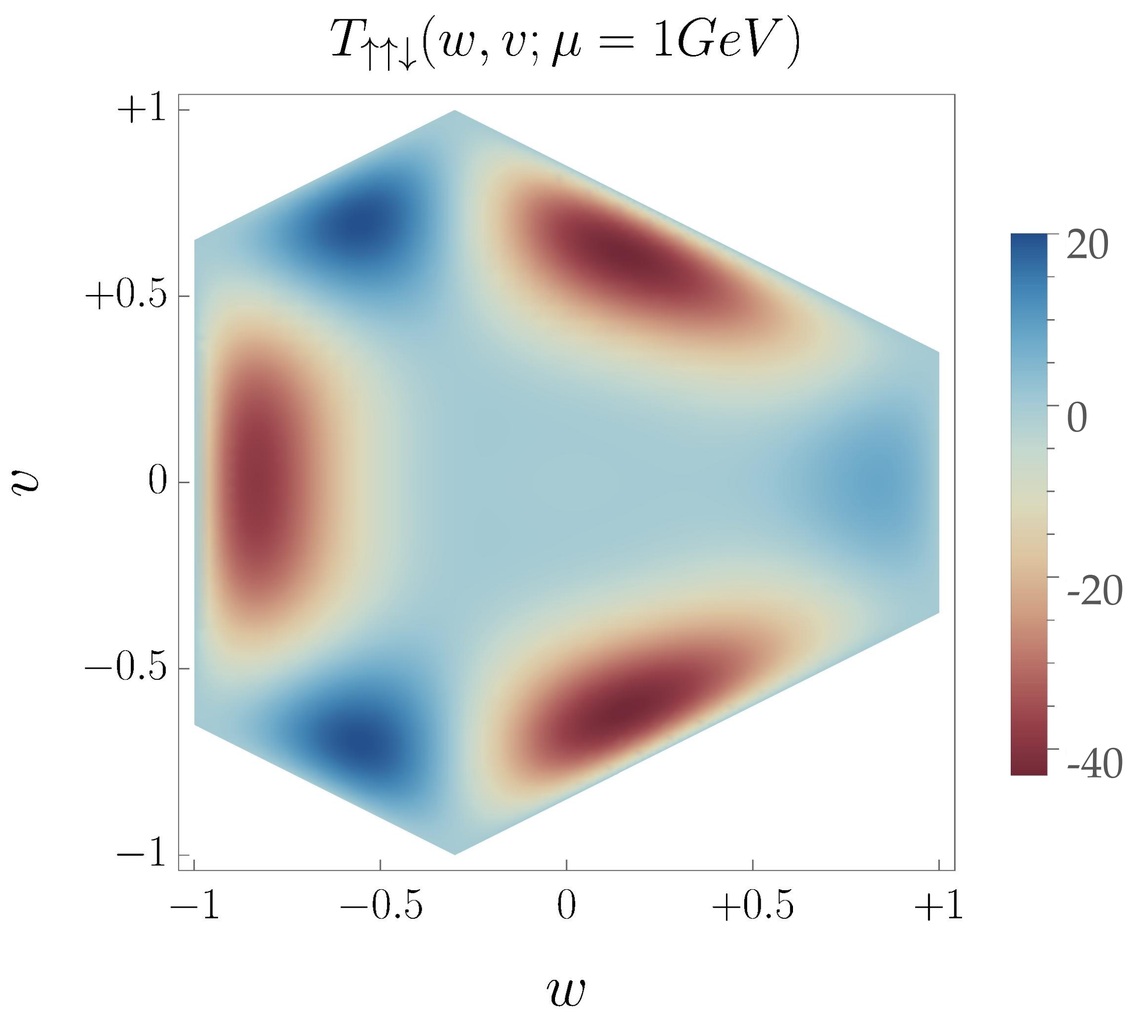
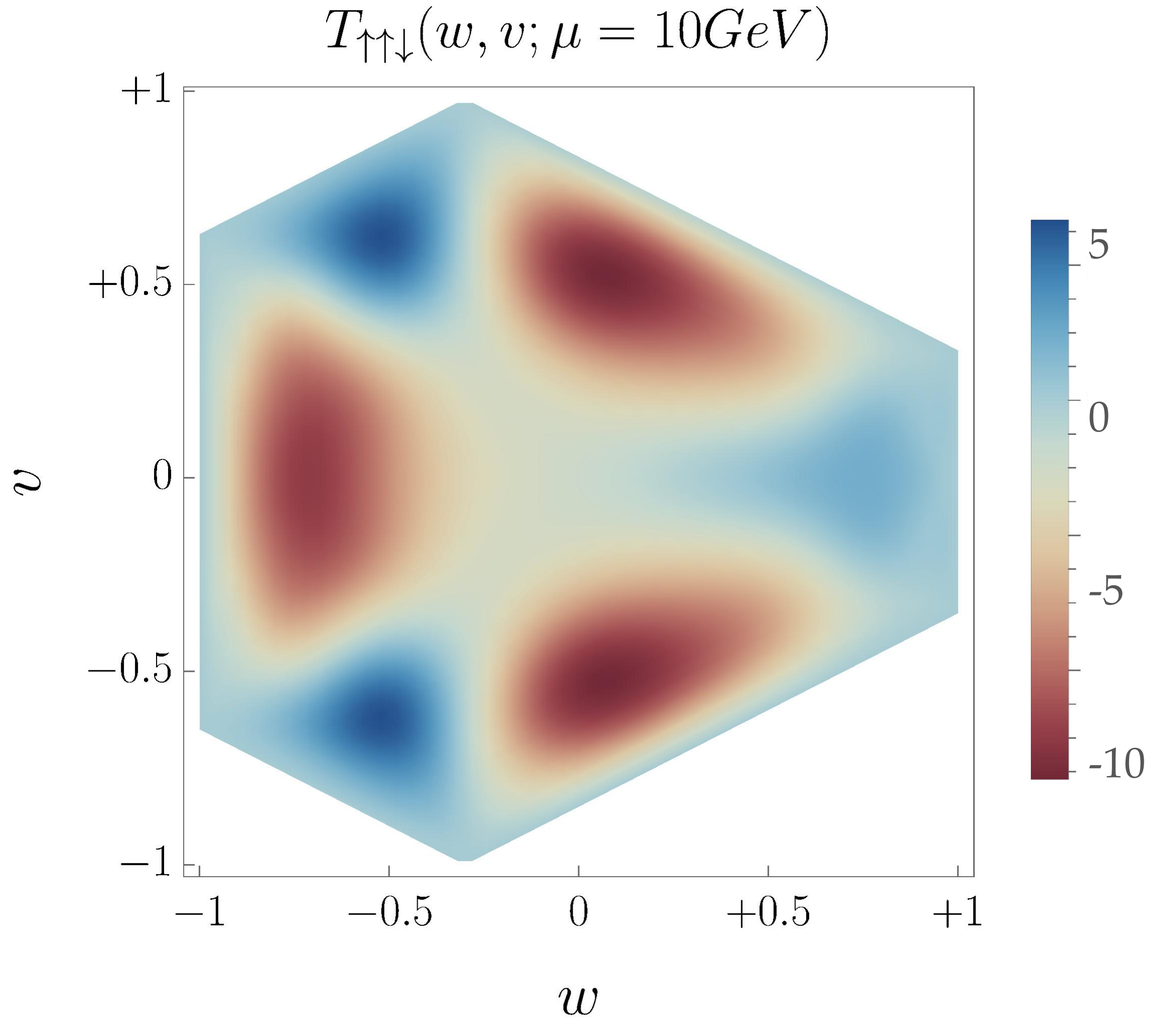
Projection on the base

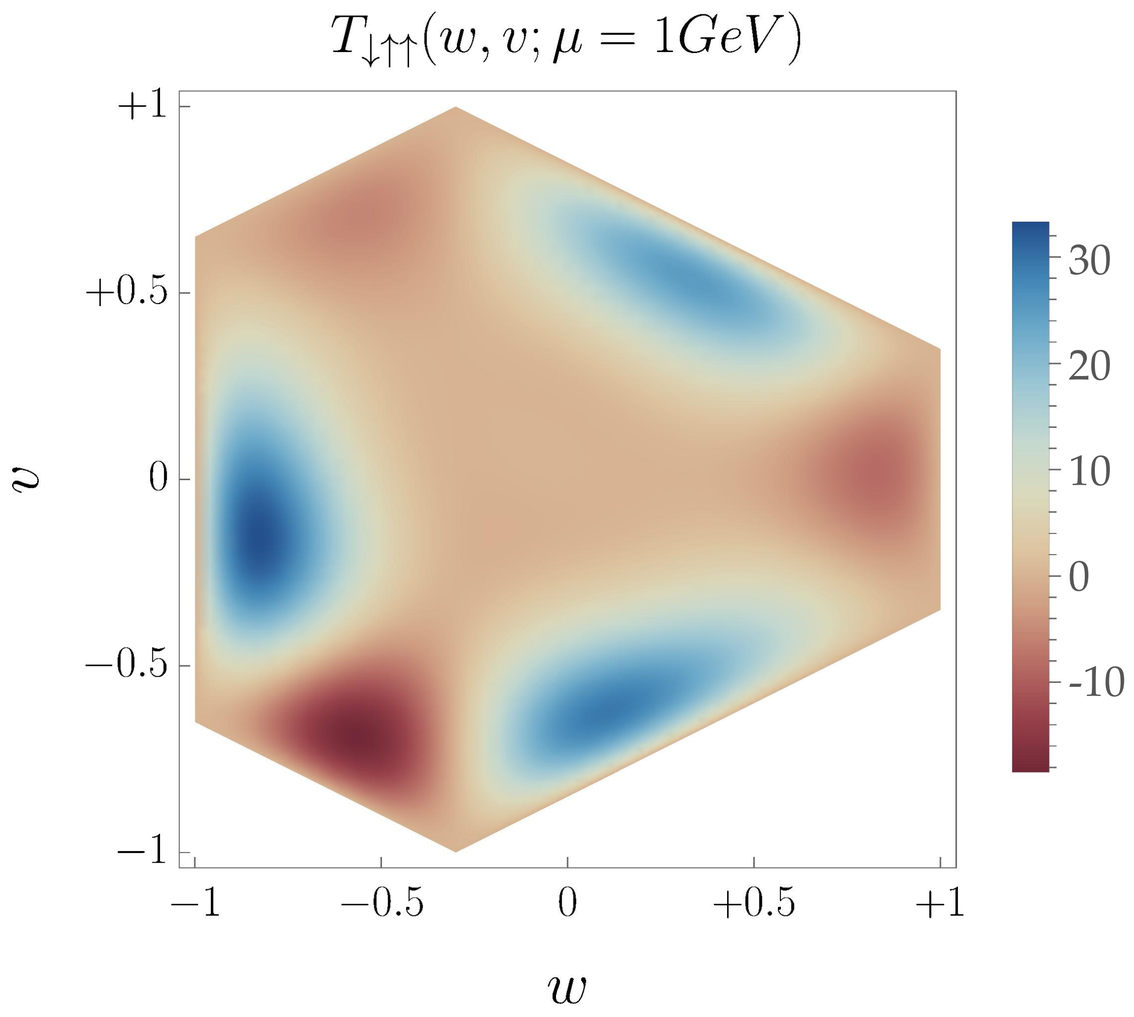


Results


$T_{\uparrow\uparrow\downarrow}(w, v; \mu = 1GeV)$
+1
+0.5
0
−0.5
−1
v
−1
−0.5
0
+0.5
+1
w
20
0
-20
-40


$T_{\uparrow\uparrow\downarrow}(w, v; \mu = 10GeV)$
+1
+0.5
0
−0.5
−1
v
−1
−0.5
0
+0.5
+1
w
5
0
-5
-10

Results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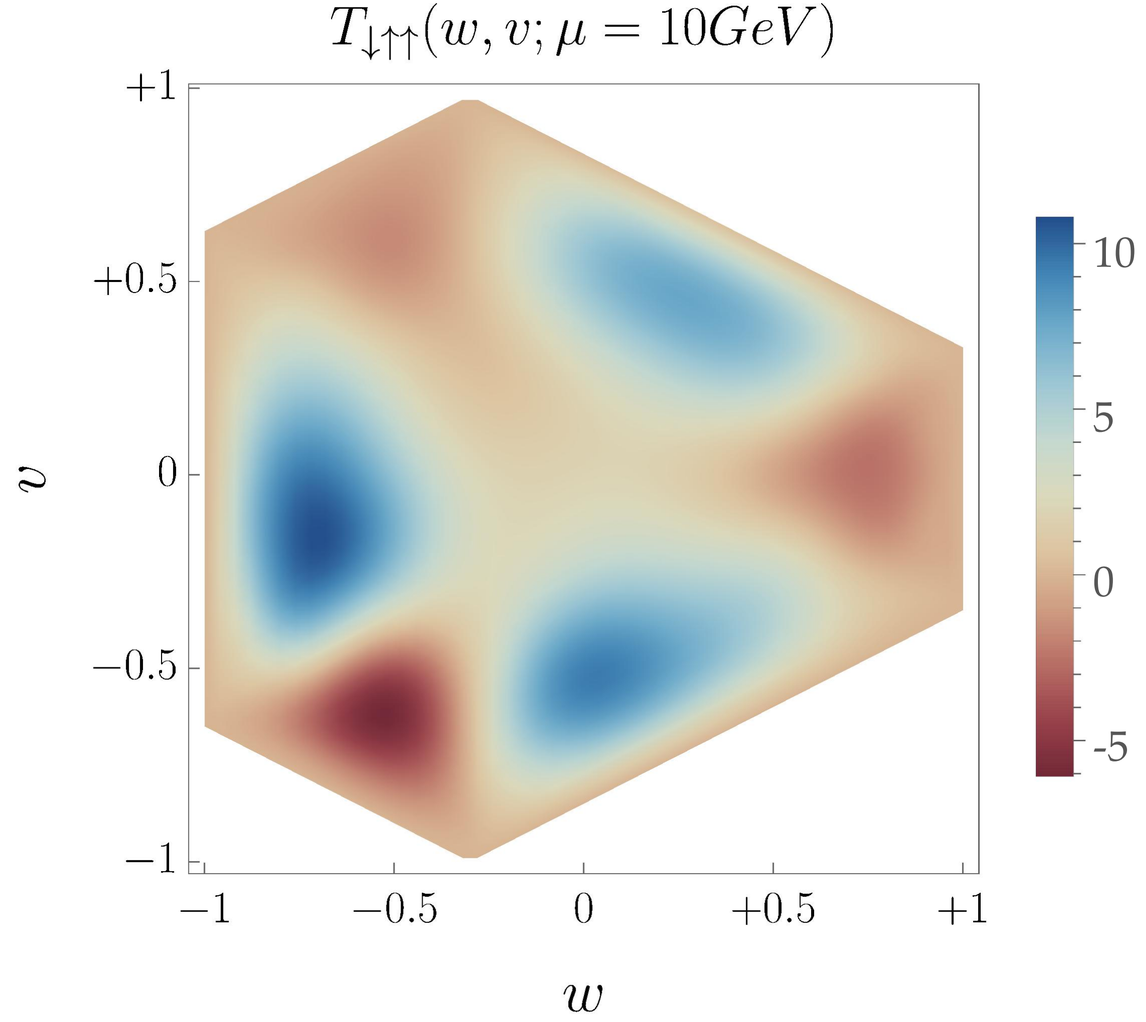
+1
+0.5
0
−0.5
−1
v
−1
−0.5
0
+0.5
+1
w
30
20
10
0
-10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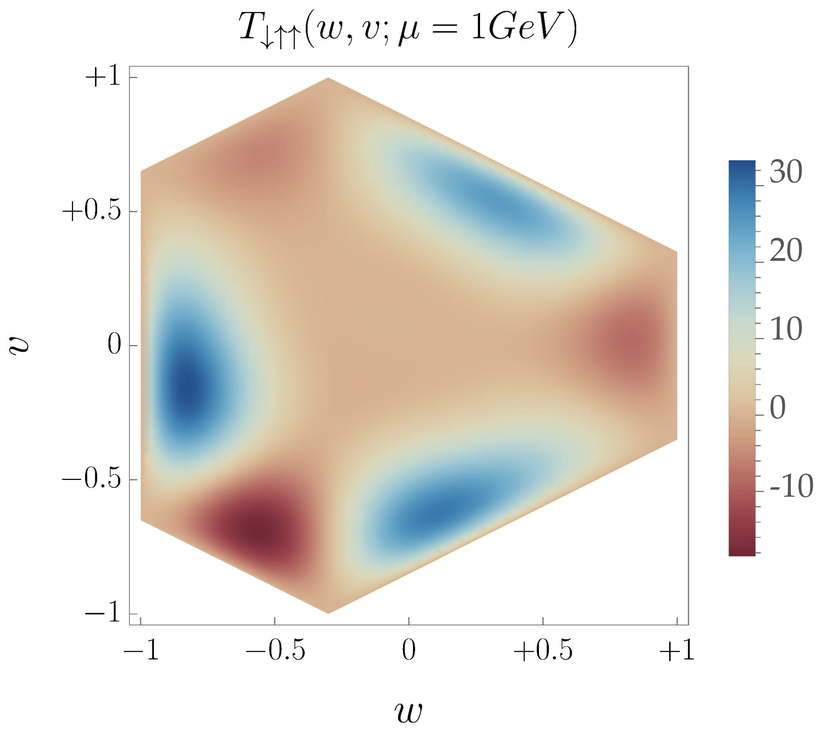
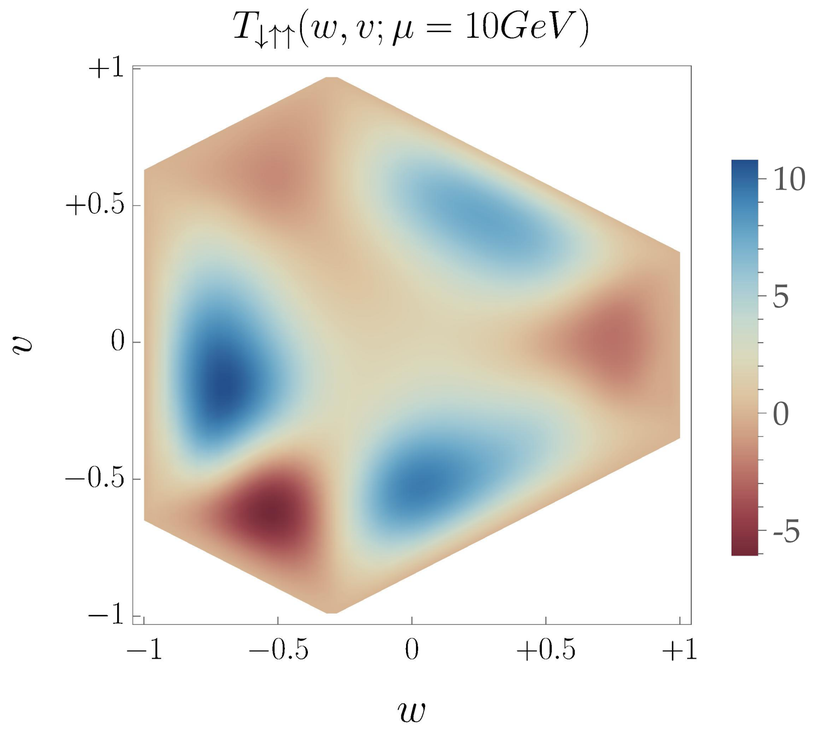
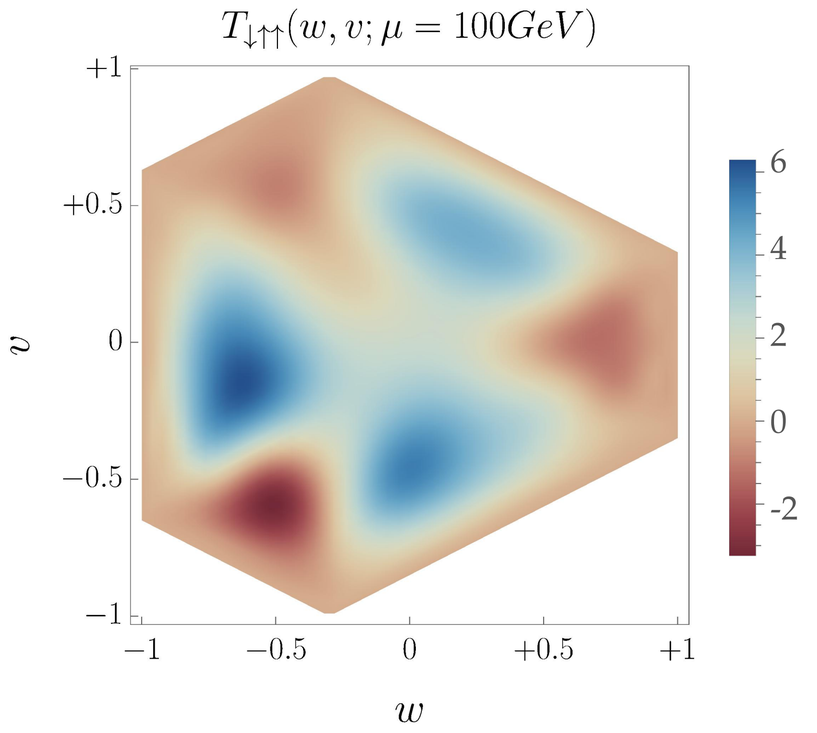
-5

Results


$T_{\downarrow\uparrow\uparrow}(w, v; \mu = 1GeV)$
+1
+0.5
0
−0.5
−1
v
−1
−0.5
0
+0.5
+1
w
30
20
10
0
-10


$T_{\downarrow\uparrow\uparrow}(w, v; \mu = 10GeV)$
+1
+0.5
0
−0.5
−1
v
−1
−0.5
0
+0.5
+1
w
10
5
0
-5


$T_{\downarrow\uparrow\uparrow}(w, v; \mu = 100GeV)$
+1
+0.5
0
−0.5
−1
v
−1
−0.5
0
+0.5
+1
w
6
4
2
0
-2