# Reconstructing the Full Kinematic Dependence of GPDs from Pseudo-Distributions

**K. Orginos**
William & Mary
W&M

**H. Dutrieux**, R.G. Edwards, **J. Karpie**, C. Mezrag,
C. Monahan, K. Orginos, A. Radyushkin,
D. Richards, E. Romero, S. Zafeiropoulos

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C. Egerer

## Motivation: Why GPDs?

### 3D Tomography

GPDs encode the transverse spatial distribution of partons at fixed longitudinal momentum $x$ — yielding a 3D picture of the nucleon (Burkardt 2000).

### Spin & Mechanical Properties

Ji sum rule: orbital angular momentum of quarks from GPD moments. Energy-momentum tensor $\Rightarrow$ pressure and shear forces inside the proton (Ji 1996).

### Experiments + Lattice

DVCS at JLab, COMPASS, future EIC probes GPDs. Lattice QCD offers complementary first-principles access — bypassing deconvolution issues.

GPDs **generalise** both PDFs (forward limit, $t \to 0$) and elastic form factors ($\int dx$)

## GPD Kinematics: $H(x,\xi,t)$ and $E(x,\xi,t)$

### Three kinematic variables

- $x$ longitudinal momentum fraction
- $\xi$ skewness: $\xi = -q^+/2P^+$
- $t = \Delta^2$ squared momentum transfer

### Limits

- $H(x,0,0) = q(x)$ — reduces to PDF
- $\int_{-1}^{1} dx\, H(x,\xi,t) = F_1(t)$

### Key properties

- **Polynomiality** (Lorentz invariance):

$$\int_{-1}^{1} dx\, x^n H(x,\xi,t) = \sum_{k=0}^{n} A_{n,k}(t)\, \xi^k$$

- **Positivity**:

$$\left| H - \frac{\xi^2}{1-\xi^2} E \right|^2 \leq \frac{q(x_+)\, q(x_-)}{1-\xi^2}, \quad x_\pm = \frac{x \pm \xi}{1 \pm \xi}$$

- ERBL region $|x| < \xi$: off-forward
- DGLAP region $|x| > \xi$: parton-like

**This work:** first lattice extraction of $H^{u-d}$ and $E^{u-d}$ as functions of *all three variables simultaneously*

# From Lattice QCD to GPDs: Pseudo-Distributions

**Compute 3-pt functions**

Matrix elements with space-like Wilson line $z$. Pairs $(p_f, p_i)$ with varying momenta and transfers $\Delta$.

**Reduced matrix element**

Ratio cancels UV divergences $\Rightarrow$ RG-invariant $\mathcal{M}(\nu, \bar{\nu}, t, z^2)$ with well-defined continuum limit.

**Ioffe-time distributions**

Lorentz-invariant objects converging to GPDs as $z^2 \to 0$. Variables: $\nu = P{\cdot}z$, $\bar{\nu} = -(q{\cdot}z)/2$.

**Perturbative matching**

One-loop kernel $\to \overline{\text{MS}}$ at $\mu_0 \sim 1/\sqrt{-z^2}$, then RGE evolution to $\mu = 2$ GeV.

Power corrections $\mathcal{O}(z^2\Lambda^2_{\text{QCD}})$ and $\mathcal{O}(z^2 t)$ limit usable $z$ range • Key advantage: no deconvolution from cross-sections

## Step 1: Three-Point Functions on the Lattice

GPDs parametrize light-cone ($z^+ = 0$, $\mathbf{z}_\perp = \mathbf{0}$) off-forward matrix elements:

$$\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \left\langle N(p_f)\right| \bar\psi\left(-\tfrac{z}{2}\right) \gamma^+ \hat W\left(-\tfrac{z}{2}, \tfrac{z}{2}\right) \psi\left(\tfrac{z}{2}\right) \left|N(p_i)\right\rangle \Big|_{z^+=0}$$

$$= \frac{1}{P^+}\, \bar u(p_f) \left[\gamma^+ H^q(x,\xi,t) + \frac{i\sigma^{+\nu} q_\nu}{2m} E^q(x,\xi,t)\right] u(p_i) \qquad (1)$$

with $P \equiv \frac{1}{2}(p_i + p_f)$, $q \equiv p_f - p_i$, $t \equiv q^2$, $\xi \equiv -q^+/2P^+$

**On the Euclidean lattice** only space-like ($z^2 < 0$) separations are accessible. The three-point function yields the bare matrix element:

$$M^\mu(p_f, p_i, z) = \left\langle N(p_f)\right| \bar\psi\left(-\tfrac{z}{2}\right) \gamma^\mu \hat W\left(-\tfrac{z}{2}, \tfrac{z}{2}\right) \psi\left(\tfrac{z}{2}\right) \left|N(p_i)\right\rangle, \qquad z^2 < 0 \qquad (2)$$

Eqs. (1) and (2) share the same Lorentz-invariant form factors — space-like computations relate to light-cone GPDs up to matching controlling collinear divergences and UV-divergence renormalization

## Step 2: UV Renormalization and Lorentz Structure

UV divergences of $M^\mu$ depend only on $|z|$ and cancel in the ratio:

$$\mathcal{M}^\mu(p_f, p_i, z) = \frac{M^\mu(p_f, p_i, z)}{M^0(0, 0, z)} \tag{3}$$

With Ioffe time $\nu = P\cdot z,\ \bar{v} = -(q\cdot z)/2,\ \xi = \bar{v}/\nu,\ \mathcal{M}^\mu$ admits an 8-amplitude Lorentz decomposition:

$$\begin{aligned}\mathcal{M}^\mu = {} & \langle\langle \gamma^\mu \rangle\rangle \mathcal{A}_1 + z^\mu \langle\langle \mathbb{1} \rangle\rangle \mathcal{A}_2 + i\langle\langle \sigma^{\mu z} \rangle\rangle \mathcal{A}_3 + \tfrac{i}{2m}\langle\langle \sigma^{\mu q} \rangle\rangle \mathcal{A}_4 + \tfrac{q^\mu}{2m}\langle\langle \mathbb{1} \rangle\rangle \mathcal{A}_5 \\ & + \tfrac{i}{2m}\langle\langle \sigma^{zq} \rangle\rangle \left[P^\mu \mathcal{A}_6 + q^\mu \mathcal{A}_7 + z^\mu \mathcal{A}_8\right]\end{aligned} \tag{4}$$

where $\langle\langle \Gamma \rangle\rangle \equiv \bar{u}(p_f)\,\Gamma\,u(p_i)$ and each $\mathcal{A}_k = \mathcal{A}_k(\nu, \bar{v}, t, z^2)$ is Lorentz-invariant. **As $z^2 \to 0$,** the amplitudes connect to GPDs via the generalized Ioffe-time distributions (GITD):

$$\begin{pmatrix} \mathcal{H}^q \\ \mathcal{E}^q \end{pmatrix}(\nu, \bar{v}, t) = \int_{-1}^{1} dx\, e^{-ix\nu} \begin{pmatrix} H^q \\ E^q \end{pmatrix}(x, \xi, t) \tag{5}$$

$$\mathcal{H}^q(\nu, \bar{v}, t, z^2) = \mathcal{A}_1 - \xi\,\mathcal{A}_5 \tag{6}$$

$$\mathcal{E}^q(\nu, \bar{v}, t, z^2) = \mathcal{A}_4 + \nu\,\mathcal{A}_6 - 2\bar{v}\,\mathcal{A}_7 + \xi\,\mathcal{A}_5 \tag{7}$$

## Perturbative Matching to $\overline{\mathrm{MS}}$

**Step 1:** Apply one-loop matching kernel at scale $\mu_0^2 \sim 1/|z^2|$:

$$\bar{\mathcal{F}} = \mathcal{F} + \frac{\alpha_s(\mu_0^2) C_F}{2\pi} \left[ L + \ln \tfrac{e^{2\gamma_E+1}}{4}\, B \right] \otimes \mathcal{F} + \mathcal{O}(z^2 \Lambda_{\mathrm{QCD}}^2) + \mathcal{O}(z^2 t) \tag{8}$$

where $\bar{\alpha} \equiv 1 - \alpha$ and

$$L \otimes \mathcal{F} = \int_0^1 d\alpha \left[ 4 \left( \frac{\ln \bar{\alpha}}{\bar{\alpha}} \right)_+ \cos(\bar{\alpha}\bar{\nu}) - 2 \frac{\sin(\bar{\alpha}\bar{\nu})}{\bar{\nu}} + \delta(\bar{\alpha}) \right] \mathcal{F}(\alpha\nu, \alpha\bar{\nu}, t) \tag{9}$$

**Step 2:** Evolve to target scale $\mu$ via the RGE:

$$\frac{d}{d \ln \mu^2} \bar{\mathcal{F}}(\nu, \bar{\nu}, t, \mu^2) = \frac{\alpha_s(\mu^2) C_F}{2\pi} B \otimes \bar{\mathcal{F}} \tag{10}$$

where

$$B \otimes \mathcal{F} = \int_0^1 d\alpha \left[ \left( \frac{2\alpha}{\bar{\alpha}} \right)_+ \cos(\bar{\alpha}\bar{\nu}) + \frac{\sin(\bar{\alpha}\bar{\nu})}{\bar{\nu}} - \frac{\delta(\bar{\alpha})}{2} \right] \mathcal{F}(\alpha\nu, \alpha\bar{\nu}, t) \tag{11}$$

Plus-prescription: $\int_0^1 d\alpha\; G(\alpha)_+ f(\alpha x) = \int_0^1 d\alpha\; G(\alpha) \big[ f(\alpha x) - f(x) \big]$

# Double Distributions: the Key Novelty

## What are Double Distributions?

Amplitudes $\mathcal{A}(\nu, \bar{\nu}, t)$ admit a 2D Fourier representation:

$$\mathcal{A}(\nu, \bar{\nu}, t) = \iint_{\Omega} d\beta\, d\alpha\; e^{-i\beta\nu - i\alpha\bar{\nu}}\; a(\beta, \alpha, t)$$

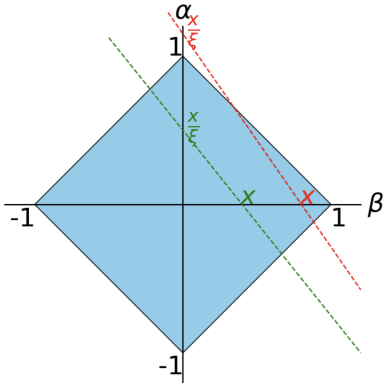
Support $\Omega$: rhombus $|\beta| + |\alpha| \leq 1$.

GPDs are **Radon transforms** of DDs:

$$H(x, \xi, t) = \iint_{\Omega} d\beta\, d\alpha\; \delta(x - \beta - \alpha\xi)\; h(\beta, \alpha, t)$$

- Polynomiality automatic — built-in by construction
- One DD gives GPDs at **all** values of $\xi$
- DDs at least as fundamental as GPDs

## DD domain $\Omega$ & Radon lines




Green: $0 < x < \xi$ (ERBL) Red: $\xi < x < 1$ (DGLAP)

## Parity Constraints on Double Distributions

Since $H^q$ and $E^q$ are $\xi$-even and real, the DDs $h^q$ and $e^q$ are even in $\alpha$. Separating the $\beta$-parity:

$$h^{q(-)}(\beta, \alpha, t) = h^q(\beta, \alpha, t) + h^q(-\beta, \alpha, t) \tag{12}$$

$$h^{q(+)}(\beta, \alpha, t) = h^q(\beta, \alpha, t) - h^q(-\beta, \alpha, t) \tag{13}$$

the integrals over the full rhombus $\Omega$ reduce to the upper triangle $\Omega^+ = \{(\beta, \alpha) \in [0, 1]^2 \,|\, \beta + \alpha \leq 1\}$:

$$\mathrm{Re}\, \mathcal{A}_1(\nu, \bar{\nu}, t, z^2) = 2 \int_{\Omega^+} \mathrm{d}\beta\, \mathrm{d}\alpha\ \cos(\beta\nu) \cos(\alpha\bar{\nu})\, h^{q(-)}(\beta, \alpha, t, z^2) \tag{14}$$

$$\mathrm{Im}\, \mathcal{A}_1(\nu, \bar{\nu}, t, z^2) = -2 \int_{\Omega^+} \mathrm{d}\beta\, \mathrm{d}\alpha\ \sin(\beta\nu) \cos(\alpha\bar{\nu})\, h^{q(+)}(\beta, \alpha, t, z^2) \tag{15}$$

**Re** and **Im** parts of the lattice amplitude decouple — they constrain independently the $\beta$-**even** ($h^{q(-)}$, linked to the $x$-even GPD) and $\beta$-**odd** ($h^{q(+)}$, linked to the $x$-odd GPD) components of $h^q$. Identical structure holds for $e^q$ via the combination $\mathcal{A}_4 + \nu\mathcal{A}_6 - 2\bar{\nu}\mathcal{A}_7$.

# Lattice Setup

**Ensemble:** a094m358

| | |
|---|---|
| Lattice spacing | $a = 0.094$ fm |
| Pion mass | $m_\pi = 358$ MeV |
| Volume | $32^3 \times 64$ |
| Configurations | 1490 ($4\times$ prev.) |
| Max momentum | $\lvert p \rvert$ up to 2.7 GeV |
| Max Ioffe time | $\nu$ up to $\sim 7$ |
| Kinematics | 249 pairs $(p_f, p_i)$ |

**Key improvements vs. HadStruc:2024rix**

- $4\times$ more statistics (1490 vs. 348 configs)
- 63 new pairs at momenta up to 2.7 GeV (vs. 1.4 GeV)
- Doubled Ioffe-time range: $\nu_{\max} \sim 7$ (vs. $\sim 3.5$)
- Denser skewness coverage
- Full 3D $(x, \xi, t)$ reconstruction vs. fixed $\xi$
- DD representation: polynomiality enforced by construction

JLab/W&M/LANL/MIT/Marseille — 2+1 flavour Wilson clover fermions — distillation framework

## Energy Dispersion Relation from 2-Point Functions

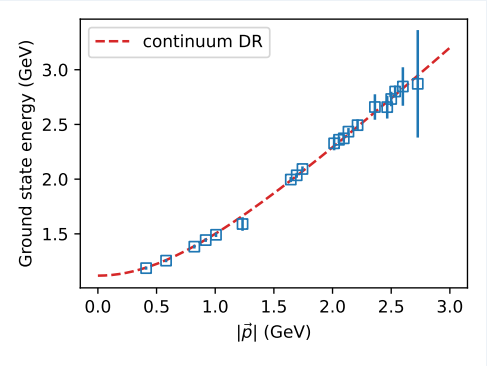
The 2-point function spectral decomposition:

$$C_{2\mathrm{pt}}(\vec{p}, T) = \sum_n \frac{|\mathcal{Z}_n(\vec{p})|^2}{2E_n(\vec{p})} e^{-E_n(\vec{p})T} \quad (16)$$

Ground-state energies $E_0(\vec{p})$ extracted by multi-state fits. Consistency with the continuum dispersion relation

$$E_0^2(\vec{p}) = m_N^2 + |\vec{p}|^2$$

validates the lattice setup and the momentum reach.




extracted $E_0(\vec{p})$ vs. $|\vec{p}|$

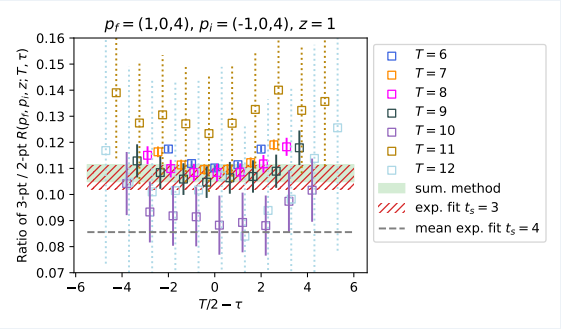
# Ratio Method: Extracting the Matrix Element

3-point spectral decomposition:

$$C_{3\text{pt}}^{[\gamma^\mu]} = \sum_{n',n} \frac{\mathcal{Z}_{n'}\mathcal{Z}_n^\dagger}{4E_{n'}E_n} \langle n'|\mathring{\mathcal{O}}^\mu|n\rangle \times e^{-E_{n'}(T-\tau)}\, e^{-E_n\tau} \qquad (17)$$

Ratio isolating the ground-state signal:

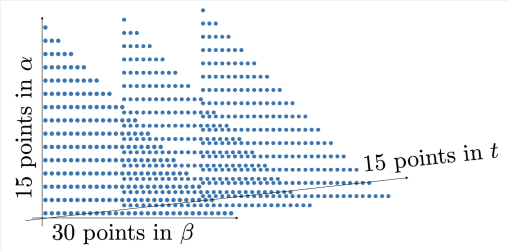
$$R^\mu \equiv \frac{C_{3\text{pt}}^\mu}{C_{2\text{pt}}(\vec{p}_f;T)} \sqrt{\frac{C_{2\text{pt}}(\vec{p}_i;T-\tau)\, C_{2\text{pt}}(\vec{p}_f;\tau)\, C_{2\text{pt}}(\vec{p}_f;T)}{C_{2\text{pt}}(\vec{p}_f;T-\tau)\, C_{2\text{pt}}(\vec{p}_i;\tau)\, C_{2\text{pt}}(\vec{p}_i;T)}} \xrightarrow{\tau,T\to\infty} \frac{M^\mu}{\sqrt{4E_0(\vec{p}_f)E_0(\vec{p}_i)}} \cdot \frac{1}{\sqrt{2}} \qquad (18)$$




$R^0$ plateau for representative $(\vec{p}_f, \vec{p}_i)$ pairs

# The Inverse Problem & Gaussian Process Regularization

**Challenge:** reconstruct $a(\beta, \alpha, t)$ from finite noisy Fourier measurements $M(\nu_k, \bar{\nu}_k, t_k) \Rightarrow$ severely under-constrained




Discretize $f^q$ on a $30 \times 15 \times 15$ mesh in $(\beta, \alpha, t)$:

$$\mathcal{M}_k = (B_{ki}) \cdot (f_i^q)$$

# mesh vertices $i \gg$ # data points $k$

$\Rightarrow$ Bayesian regularisation via GPR

# GPR: Bayesian Setup

**Bayes' theorem**

$$P(f_i^q \mid M, I) = \frac{P(M \mid f_i^q)\ P(f_i^q \mid I)}{P(M \mid I)}$$

**Prior $\mathcal{N}(g_i, K)$**

$$P(f_i^q \mid I) = \frac{e^{-\frac{1}{2}(f^q - g)^T K^{-1} (f^q - g)}}{\sqrt{\det(2\pi K)}}$$

$K$: covariance kernel (physics-motivated)
$g_i = 0$: prior mean

**Likelihood**

$$P(M \mid f_i^q) = \frac{e^{-\frac{1}{2}(M - Bf^q)^T \Sigma^{-1} (M - Bf^q)}}{\sqrt{\det(2\pi\Sigma)}}$$

Correlated $\chi^2$ of $Bf^q$ with data
$\Sigma$: lattice data covariance

## GPR: Analytic Posterior

Prior & likelihood Gaussian + $B$ linear $\Rightarrow$ posterior is Gaussian with **analytic** mean & covariance — no nonlinear minimisation required

**Posterior mean**

$$\langle f_i^q \rangle = g_i + KB^T[\Sigma + BKB^T]^{-1}(M_k - B \cdot g)$$

**Posterior covariance**

$$H = K - KB^T[\Sigma + BKB^T]^{-1}BK$$

# GPR: The Prior Kernel

## Covariance kernel $K_{ij}$

$$K_{ij} = \sigma^2 (\beta_i \beta_j)^{-\gamma} \left[(1-\beta_i)(1-\beta_j)\right]^{\delta} \left[(1-\alpha_i-\beta_i)(1-\alpha_j-\beta_j)\right]^{\eta} \times \exp\left(-\frac{(\ln \beta_i - \ln \beta_j)^2}{2l_\beta^2}\right) \exp\left(-\frac{(\beta_i\beta_j)^{-\gamma}(\alpha_i-\alpha_j)^2}{2l_\alpha^2}\right) \exp\left(-\frac{(t_i-t_j)^2}{2l_t^2}\right)$$

### Power-law factors: boundary behavior

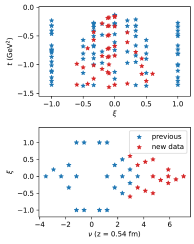
- $\sigma^2$: overall prior variance
- $(\beta_i\beta_j)^{-\gamma}$: allows small-$\beta$ singularity
- $[(1-\beta_i)(1-\beta_j)]^{\delta}$: large-$\beta$ vanishing
- $[\cdots]^{\eta}$: cancellation on edges of $\Omega^+$

### RBF factors: smoothness

- $l_\beta$ on *log* scale: flexibility at small $\beta$
- $l_\alpha$: smoothness in $\alpha$
- $l_t$: smoothness in $t$

7 hyperparameters $(\sigma, \gamma, \delta, \eta, l_\beta, l_\alpha, l_t)$ varied over 100 samples $\times$ 10 posterior draws
$\Rightarrow$ model uncertainty estimate

# Kinematic Coverage




Blue: 186 pairs

(HadStruc:2024rix) Red: 63 new pairs

## Coverage highlights

**249** momentum pairs $(p_f, p_i)$

$\nu$ **up to 7** max Ioffe time at $z = 6a$

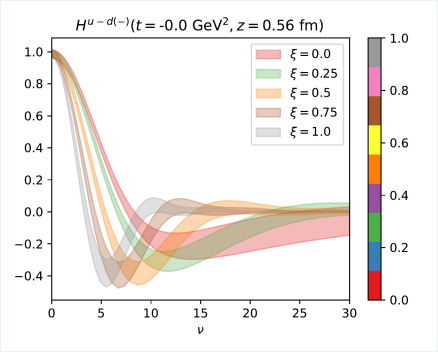
$\xi \in [0, 0.5]$ skewness range

$t \in [0, -1.4\,\mathbf{GeV}^2]$ momentum transfer range

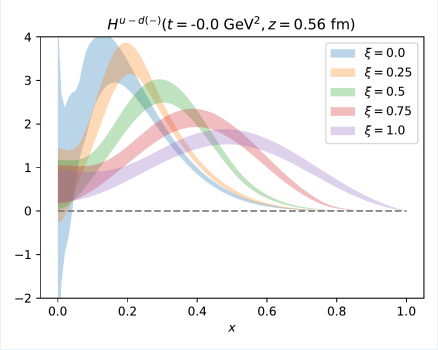
$z = 1a \ldots 6a$ Wilson line lengths

# GPR Baseline Reconstruction

**GITD:** $\mathrm{Re}\,\mathcal{H}^{u-d}$ **vs** $\nu$**,** $z = 6a = 0.56$ **fm**



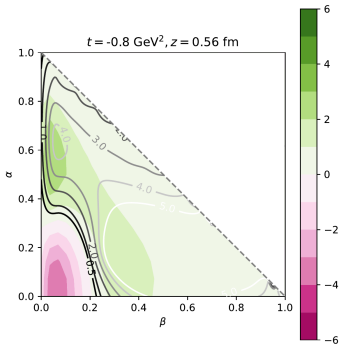

**GPD:** $H^{u-d}(x, \xi, t)$**,** $z = 6a$




- $\mathcal{M}(\nu{=}0) = 1$ exactly: $\int dx\, H$ independent of $\xi$ (polynomiality)
- Reconstruction faithful at intermediate $x$; prior-driven at $x \to 0, 1$
- $x$-even (real part) and $x$-odd (imaginary part) both extracted
- $E^{u-d}$ also reconstructed; correlated $\chi^2 / N_{\mathrm{data}} \sim 1.7$ ($H$), $\sim 1.8$ ($E$)

## Double Distribution Reconstruction

$h^{u-d}(\beta, \alpha)$ **at** $t = -0.8$ **GeV**$^2$




colour: central values  contours: $n\sigma$ from zero

### What the DD shows us

- Complex structure at small $\beta, \alpha$: GPR allows profiles inaccessible to simple parametric forms
- Negative central values at small $\beta, \alpha$ compatible with zero within uncertainty — well-behaved
- Uncertainty grows as $\beta \to 0$ (prior-driven, low Fourier sensitivity)
- **First** such DD extracted directly from lattice QCD

GPR mesh: $30 \times 15 \times 15$ in $(\beta, \alpha, t) \Rightarrow$ 3390 d.o.f.

# Model Dependence Assessment

Vary 7 kernel hyperparameters over physical ranges $\rightarrow$ 100 random sets $\rightarrow$ 10 posterior samples each $=$ 1000 replicas

| HP | Range | Controls |
|---|---|---|
| $\sigma$ | $1\times$–$2\times$ | overall variance |
| $\gamma$ | 0.1–0.5 | small-$\beta$ singularity |
| $\delta$ | 1–3 | large-$\beta$ suppression |
| $\eta$ | 0–2 | edge cancellation |
| $l_\beta$ | 0.3–1.0 | $\beta$ smoothness |
| $l_\alpha$ | 0.1–0.4 | $\alpha$ smoothness |
| $l_t$ | 0.2–0.5 | $t$ smoothness |

## Key findings

- 50–55% of HP volume excluded (unphysical large-$\nu$ extrapolation)
- $z = 0.28$ fm: baseline too optimistic — significant spread under variation
- $z = 0.56$ fm: much better agreement — larger Ioffe-time range reduces model bias
- Imaginary part at $\xi = 0$: less HP-sensitive
- Half replicas use RDDA prior mean — negligible difference in posterior

## Perturbative Matching & Evolution

$$\bar{\mathcal{F}}(\nu, \bar{\nu}, \mu_0^2) = \mathcal{F}(\nu, \bar{\nu}, z^2) + \frac{\alpha_s C_F}{2\pi}\left[L + \ln\frac{e^{2\gamma_E+1}}{4}B\right] \otimes \mathcal{F} + \mathcal{O}(z^2\Lambda_{\rm QCD}^2)$$

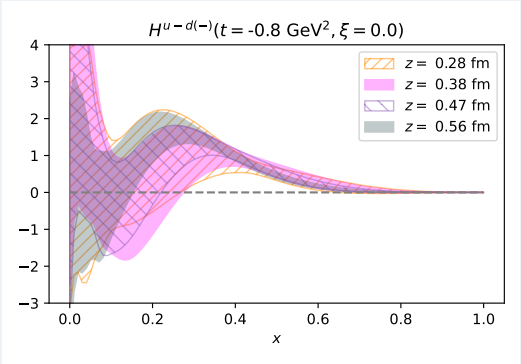
**Step 2 — RG evolution to $\mu = 2$ GeV:**

$$\frac{d}{d\ln\mu^2}\bar{\mathcal{F}}(\nu, \bar{\nu}, t, \mu^2) = \frac{\alpha_s(\mu^2)C_F}{2\pi}\, B \otimes \bar{\mathcal{F}}$$

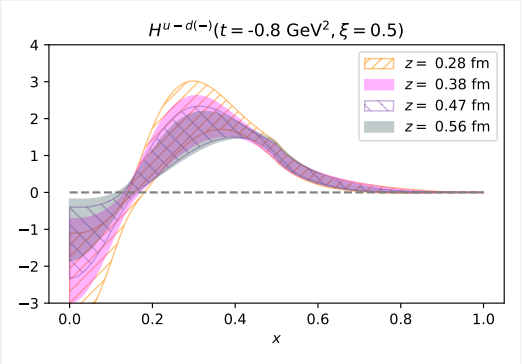
- Matching partially cancels evolution — matched GPD stays close to input (standard pseudo-dist. behaviour)
- Evolution benchmarked vs. APFEL++ to $\sim 10^{-3}$ in $x$-space
- Scale variation $\mu_0 \times \sqrt{2}$: negligible vs. statistical and model uncertainties
- $z = 3a, 4a, 5a, 6a$ results consistent after matching

$\alpha_s(2\,\text{GeV}) = 0.28$
$\Lambda_{\rm QCD} = 165\,\text{MeV}$
200 Ioffe-time points, $\nu \in [0, 600]$
RK4 solver for evolution ODE

# Perturbative Matching: $H^{u-d}_{\text{even}}$ at $\mu = 2$ GeV



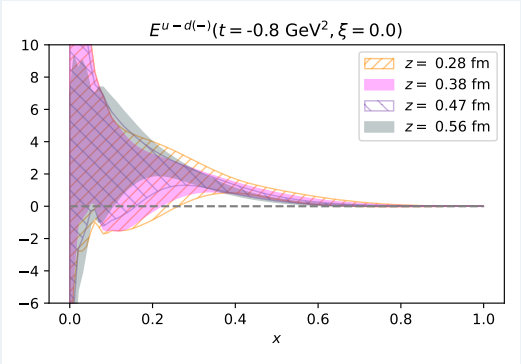

$\xi = 0, \quad t = -0.8\,\text{GeV}^2$



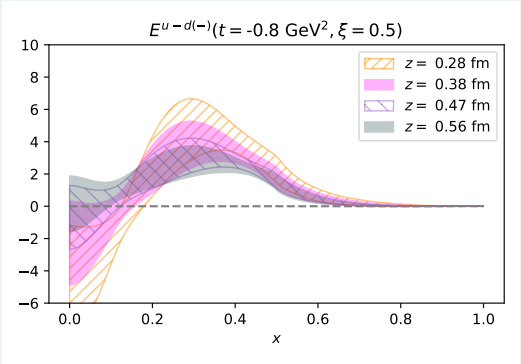

$\xi = 0.5, \quad t = -0.8\,\text{GeV}^2$

Matched results at $z = 3a$ and $z = 6a$ agree well — no strong evidence of higher-twist contamination at the $z$ values used. Scale variation uncertainty negligible vs. statistical and model dependence.

# Perturbative Matching: $E^{u-d}_{\text{even}}$ at $\mu = 2$ GeV




$\xi = 0, \quad t = -0.8\,\text{GeV}^2$

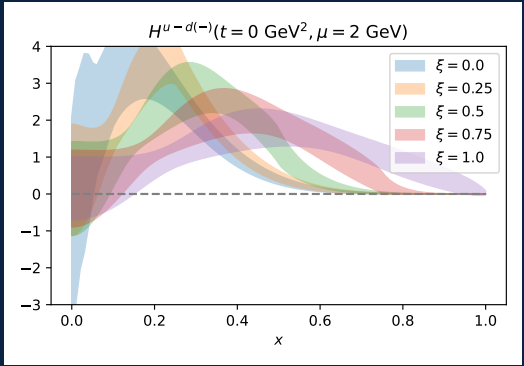



$\xi = 0.5, \quad t = -0.8\,\text{GeV}^2$

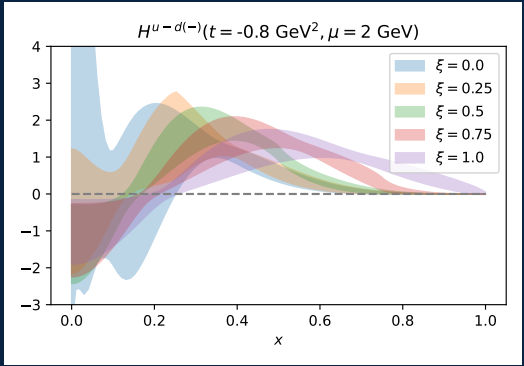
$E^{u-d}_{\text{even}}$ is noisier but shows consistent matched results across $z$ values. Non-zero skewness ($\xi = 0.5$) reconstruction benefits from the extended kinematic coverage of this work.

# Final Results: $H^{u-d}_{\text{even}}$ at $\mu = 2$ GeV

$z = 5a = 0.47$ fm, $t_s = 3$, hyperparameter variation for statistical uncertainty. Systematic: quadrature sum of $\Delta z$ and $\Delta t_s$ variations. Data on Zenodo: `doi:10.5281/zenodo.19695472`



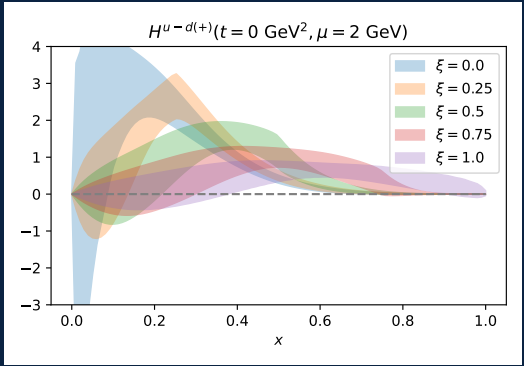

$t = 0$



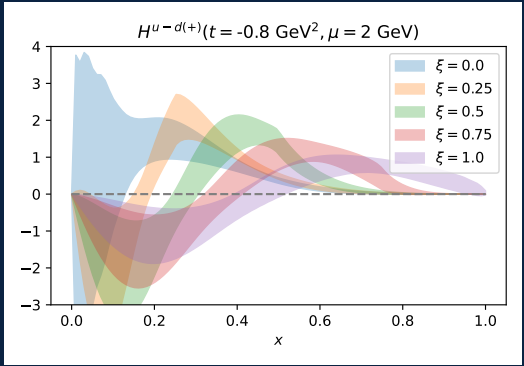

$t = -0.8\ \text{GeV}^2$

# Final Results: $H^{u-d}_{\text{odd}}$ at $\mu = 2$ GeV

$z = 5a = 0.47$ fm, $t_s = 3$, hyperparameter variation for statistical uncertainty. Systematic: quadrature sum of $\Delta z$ and $\Delta t_s$ variations. Data on Zenodo: `doi:10.5281/zenodo.19695472`



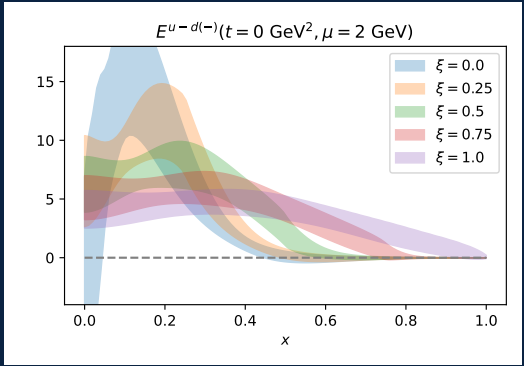

$t = 0$



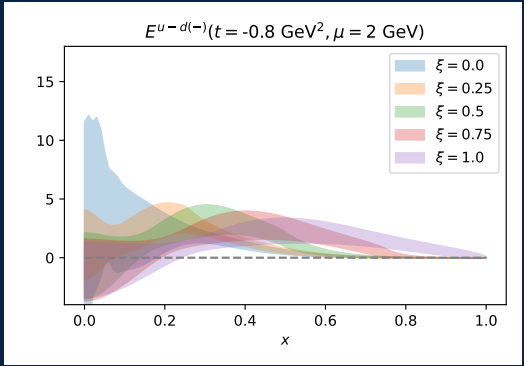

$t = -0.8\ \text{GeV}^2$

# Final Results: $E^{u-d}_{\text{even}}$ at $\mu = 2$ GeV

$z = 5a = 0.47$ fm, $t_s = 3$, hyperparameter variation for statistical uncertainty. Systematic: quadrature sum of $\Delta z$ and $\Delta t_s$ variations. Data on Zenodo: `doi:10.5281/zenodo.19695472`



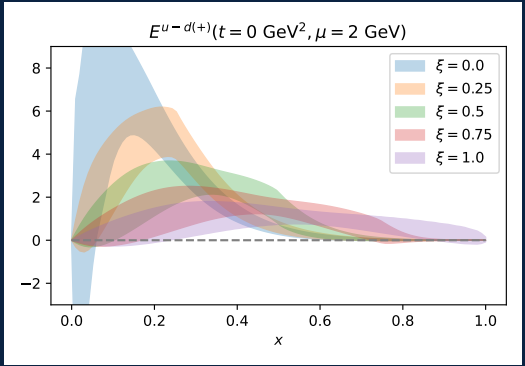

$t = 0$



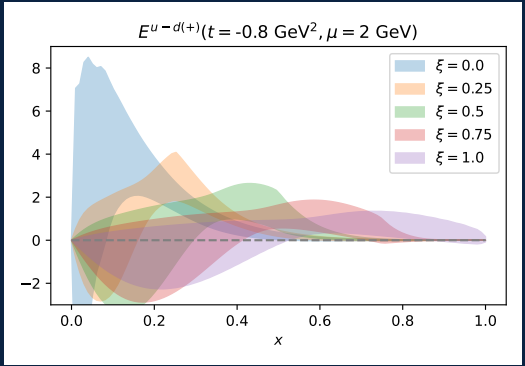

$t = -0.8\,\text{GeV}^2$

# Final Results: $E^{u-d}_{\text{odd}}$ at $\mu = 2$ GeV

$z = 5a = 0.47$ fm, $t_s = 3$, hyperparameter variation for statistical uncertainty. Systematic: quadrature sum of $\Delta z$ and $\Delta t_s$ variations. Data on Zenodo: `doi:10.5281/zenodo.19695472`




$t = 0$




$t = -0.8\ \text{GeV}^2$

# Phenomenological Comparison & Positivity Bounds

## GK model comparison at $t = 0$

- GK model uses RDDA: single parameter $N$ controls $\xi$-dependence
- Fair agreement at intermediate $x$
- $\langle x \rangle_{u-d} = 0.21(1)$ (ours) vs. GK 0.17 vs. NNPDF40NLO $0.150(5)$ — expected at $m_\pi = 358$ MeV
- Steeper $t$-dependence in GK (physical $m_\pi$)

## Positivity bound

$$\left| H - \frac{\xi^2}{1-\xi^2} E \right|^2 \leq \frac{q(x_+)\, q(x_-)}{1-\xi^2}$$

$$x_\pm = \frac{x \pm \xi}{1 \pm \xi}$$

✓ **Positivity satisfied!**

**First** lattice extraction satisfying both **polynomiality** (by construction via DDs) and **positivity** (tested a posteriori) — a longstanding goal of GPD phenomenology

# Generalized Form Factors from GPD Moments

$$\int_{-1}^{1} dx\, x^{n-1} H(x,\xi,t) = \sum_{\substack{k=0 \\ k \text{ even}}}^{n-1} A_{n,k}(t)\,\xi^k + \frac{1+(-1)^n}{2}\,\xi^n C_n(t)$$

Value at t = 0

| GPD $H^{u-d}$ | | | | GPD $E^{u-d}$ | | | |
|---|---|---|---|---|---|---|---|
| $A_{1,0}$ **0.943** ±15 | | | | $B_{1,0}$ **3.17** ±6 | | | |
| $A_{2,0}$ **0.215** ±10 | | | | $B_{2,0}$ **0.38** ±2 | | | |
| $A_{3,0}$ **0.070** ±4 | $A_{3,2}$ **0.39** ±18 | | | $B_{3,0}$ **0.12** ±2 | $B_{3,2}$ **0.6** ±2 | | |
| $A_{4,0}$ **0.025** ±2 | $A_{4,2}$ **0.06** ±3 | | | $B_{4,0}$ **0.035** ±6 | $B_{4,2}$ **0.08** ±4 | | |
| $A_{5,0}$ **0.010** ±4 | $A_{5,2}$ **0.04** ±2 | $A_{5,4}$ **0.14** ±5 | | $B_{5,0}$ **0.014** ±11 | $B_{5,2}$ **0.08** ±6 | $B_{5,4}$ **0.2** ±4 | |
| $A_{6,0}$ **0.0057** ±14 | $A_{6,2}$ **0.019** ±8 | $A_{6,4}$ **0.04** ±3 | | $B_{6,0}$ **0.007** ±3 | $B_{6,2}$ **0.02** ±1 | $B_{6,4}$ **0.05** ±3 | |

- GPR moments vs. previous polynomial fit
- $n = 4, 5, 6$ GFFs accessible from extended $\nu$ range
- Higher $k$: larger magnitude, steeper $t$-dependence — invalidates factorized Ansatz
- EFFs: $z$-dependent shift from higher-twist or discretization

# Context: Competing Lattice GPD Extractions

| Group | Framework | Method | Polynomiality | GPDs | $m_\pi$ | $p_{\max}$ |
|---|---|---|---|---|---|---|
| **This work (2025)** | Pseudo-dist. | DD via GPR | Yes (built-in) | $H, E$ all $(x, \xi, t)$ | 358 MeV | 2.7 GeV |
| Lin et al. (2023) | LaMET ($\chi$QCD) | Param. fit, fixed $\xi$ | Not explicit | $H^{u-d}$ | $\sim$450 MeV | $\sim$2 GeV |
| Bhattacharya et al. | LaMET (varies) | Param. fit, fixed $\xi$ | Not explicit | $H$ (various) | varies | $\sim$2 GeV |
| Holligan et al. 2025 | Pseudo-dist. | Moment extraction | Partial | GFFs | 358 MeV | 1.4 GeV |
| Chu et al. 2025 | LaMET (asym. frames) | Lorentz-inv. amplitudes | Not explicit | $H, E$ nonzero $\xi$ | $\sim$260 MeV | $\sim$2 GeV |

**Key differentiators of this work:**

- **DD extraction $\Rightarrow$ polynomiality built in**
- **Full 3D reconstruction in $(x, \xi, t)$ simultaneously**
- **Hyperparameter variation quantifies model uncertainty**

# Context: Related and Precursor Work

## Bayesian Inference with GPs for PDFs

Candido et al., Eur. Phys. J. C **84**, 716 (2024)

Bayesian GP framework for PDF determination from lattice Ioffe-time data; demonstrates applicability of nonparametric methods to parton physics.

## Nonparametric Reconstruction from Fourier Data

Dutrieux et al., Phys. Rev. D **111**, 034515 (2025)

Systematic study of the 1D inverse problem: kernel design, mesh resolution, hyperparameter sensitivity. Direct methodological precursor to the 3D extension carried out here.

## Towards Unpolarized GPDs from Pseudo-Distributions

HadStruc (Dutrieux et al.), JHEP **08** (2024) 162

First lattice extraction of skewness-dependent GPD moments using the $(\nu, \bar{\nu})$ framework. Established the kinematics, ratio method, and analysis pipeline extended in this work.

## GPs for Inferring Parton Distributions

Medrano et al., arXiv:2510.21041 (2025)

GPR for 1D PDF reconstruction from lattice Ioffe-time data. Introduces kernel design, mesh, and Bayesian posterior framework adopted in this work.

**This work:** first generalization of GP methodology from 1D (PDF in $\nu$) to 3D $(\beta, \alpha, t)$ — enabling full kinematic reconstruction of GPDs

## Systematic Uncertainties & Limitations

| Systematic | Status | Comment |
|---|---|---|
| **Statistical** | Included | Full jack-knife; 4× more configs |
| **Excited states** | Partial | $t_s = 3$ vs $t_s = 4$; added in quadrature |
| **Model dependence** | Partial | 100 HP samples × 10 posterior draws |
| **Perturbative matching** | Heuristic | Scale variation $\mu_0 \times \sqrt{2}$; NLO only |
| **Discretization** | Not assessed | Single ensemble; no continuum limit |
| **Finite volume** | Not assessed | $m_\pi L = 5.4$; unquantified |
| **Higher twist** | Not assessed | $\mathcal{O}(z^2\Lambda^2_{\mathrm{QCD}})$ and $\mathcal{O}(z^2 t)$; visible in EFFs |
| **Unphysical $m_\pi$** | Not corrected | $m_\pi = 358$ MeV vs 140 MeV |

## Conclusions

- **First** lattice extraction of double distributions from lattice data — full $(x, \xi, t)$ GPD dependence with polynomiality *enforced by construction*
- Multidimensional GP regression: flexible non-parametric regularisation with partial model uncertainty quantification
- $H^{u-d}$ and $E^{u-d}$ extracted at $\mu = 2$ GeV; fair agreement with Goloskokov-Kroll model at $t = 0$
- **Positivity bounds satisfied** a posteriori — first such demonstration from first-principles lattice QCD
- GFFs up to $n = 6$ extracted; skewness-dependent moments $A_{n,k}$ with $k > 0$ now accessible
- Dominant unquantified systematics: discretisation errors, higher-twist, unphysical pion mass

**Outlook: physical $m_\pi$ · continuum limit · gluon GPDs · isoscalar GPDs**