QUARK-GLUON TOMOGRAPHY COLLABORATION

Temple University

Argonne NATIONAL LABORATORY

PennState Berks

Monte Carlo extraction of GPDs from DVCS data

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Introduction

- The goal:
  - Perform a global analysis of Compton Form Factors (CFFs) and Generalized Parton Distributions (GPDs) from available experimental data using machine learning
- Developed a fitting framework that is backward differentiable to enable use of stochastic gradient descent optimization
- Conducted preliminary fits of a simple parametric model to currently available Deeply Virtual Compton Scattering (DVCS) data

GPDs

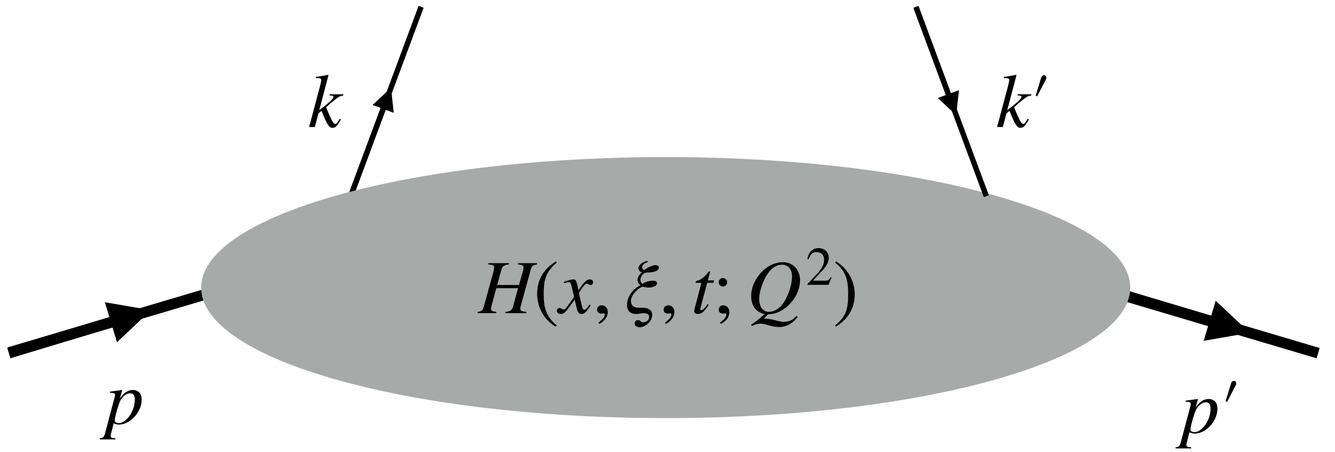
- Functions of x, ξ, and t:

$$x = \frac{k^+ + k'^+}{p^+ + p'^+} \quad \xi = \frac{p'^+ - p^+}{p^+ + p'^+} \quad t = (p' - p)^2$$

- Also depend on energy scale (Q^2):
  - Evolution equations:

$$\frac{\mathrm{d}H^a(x,\xi,t)}{\mathrm{d}\ln Q^2} = \int \mathrm{d}x P^a(x,\xi) H^a(x,\xi,t;Q_0^2)$$

- For each parton flavor, four leading twist GPDs:
  - H, E, $\tilde{H}$, $\tilde{E}$



GPD Properties

- Symmetric in ξ
- Polynomiality:

$$\int_{-1}^{1} \mathrm{d}x\, x^s H^a(x,\xi,t;\mu^2) = \sum_{i=0\ (\mathrm{even})}^{s} (2\xi)^i\, A^a_{s+1,i}(t,\mu^2) + \mathrm{mod}(s,2)\,(2\xi)^{s+1}\, C^a_{s+1}(t,\mu^2),$$

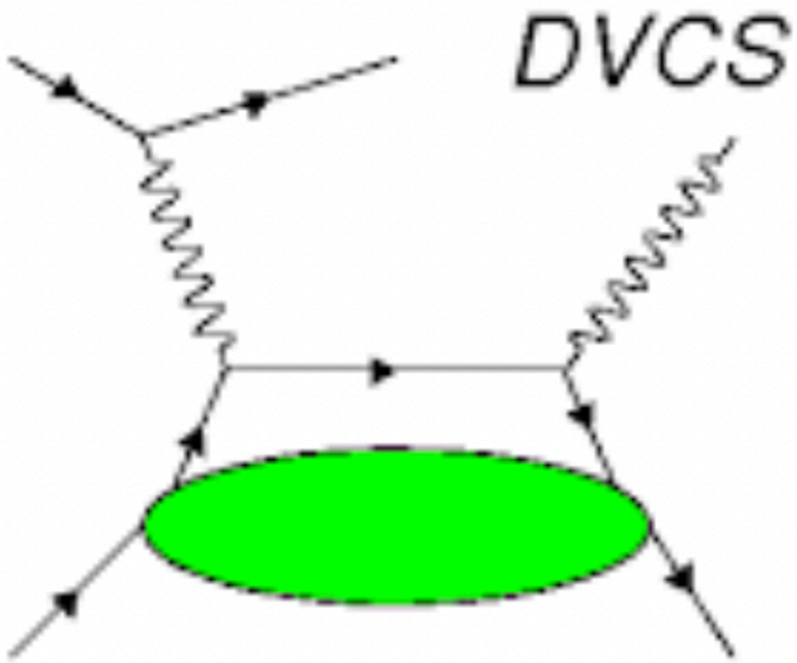
- Forward Limit:
  - When $\xi, t \to 0$, H and $\tilde{H}$ map to Parton Distribution Functions (PDFs):

$$H^q(x,0,0) = q(x)\,\Theta(x) - \bar{q}(-x)\,\Theta(-x),$$

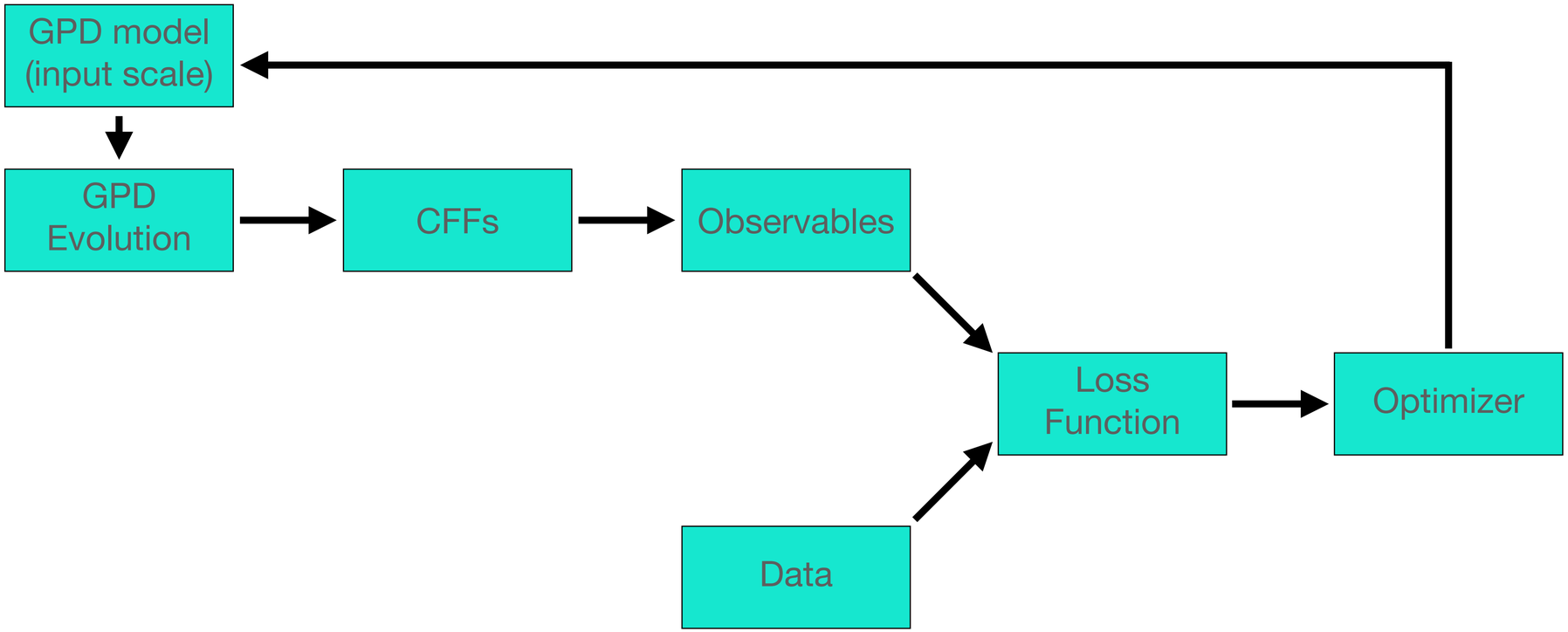
Observables

- Deeply virtual Compton Scattering (DVCS)
  - Compton Form Factors (CFFs):

$$\mathcal{H}(\xi, t, Q^2) = \int_{-1}^{1} dx \sum_a C^a(x, \xi, Q^2, \mu^2)\, H^a(x, \xi, t; \mu^2)$$



The fitting framework



- All pieces are backward differentiable to facilitate machine learning

The fitting framework

- GPD evolution:
  - NLL evolution using tiktaalik (Freese, et. al. Computer Physics Communications 311 (2025) 109552, https://github.com/quantom-collab/tiktaalik)
- CFF calculation:
  - NLO hard coefficients from tiktaalik (currently available in dev branch)
- Optimizer: Pytorch Adam optimizer (Stochastic Gradient Descent)
- Loss: Chi Squared

$$\chi^2(\boldsymbol{a}) = \sum_{i,e}\left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}}\right)^2 + \sum_k (r_e^k)^2 + \left(\frac{1 - N_e}{\delta N_e}\right)^2$$

The fitting framework

- JAM methodology:
  - Monte Carlo fit:
    - Conduct multiple fits (called replicas):
      - For each replica:
        - Starting parameters are randomly sampled
        - Data values sampled from Gaussian distribution
    - Calculate average and standard deviation of all replicas

Parametric Model

- GPD model:
  - Utilize double distributions to guarantee polynomiality

$$H^f(x,\xi,t;\mu_0^2) = \int_\Omega d\beta d\alpha\, \delta(x-\beta-\xi\alpha)\left[H^f_{DD}(\beta,\alpha,t;\mu_0^2) + \xi\delta(\beta)D^f(\alpha,t;\mu_0^2)\right]$$

$$E^f(x,\xi,t;\mu_0^2) = \int_\Omega d\beta d\alpha\, \delta(x-\beta-\xi\alpha)\left[E^f_{DD}(\beta,\alpha,t;\mu_0^2) - \xi\delta(\beta)D^f(\alpha,t;\mu_0^2)\right]$$

$$\tilde{H}^f(x,\xi,t;\mu_0^2) = \int_\Omega d\beta d\alpha\, \delta(x-\beta-\xi\alpha)\left[\tilde{H}^f_{DD}(\beta,\alpha,t;\mu_0^2)\right]$$

$$\tilde{E}^f(x,\xi,t;\mu_0^2) = \int_\Omega d\beta d\alpha\, \delta(x-\beta-\xi\alpha)\left[\tilde{E}^f_{DD}(\beta,\alpha,t;\mu_0^2)\right]$$

  - Parameterize the double distributions and the D-term

Parametric Model

- Double distributions at the input scale consist of two parts:

$$H^f_{DD}(\beta, \alpha, t; \mu_0^2) = f(\beta) h^f(\alpha, \beta, t)$$

  - Forward limit $f(\beta)$:

    - For H and $\tilde{H}$, fix using recent JAM unpolarized and polarized Parton Distribution Functions (PDFs) from C. Cocuzza

    - For E and $\tilde{E}$, must fit forward limit parameters:

$$f(\beta) = N |\beta|^{-a} (1 - |\beta|)^b$$

Parametric Model

- Double distributions at the input scale consist of two parts:

$$H^f_{DD}(\beta, \alpha, t; \mu_0^2) = f(\beta) h^f(\alpha, \beta, t)$$

  - Profile function $h(\alpha, \beta, t)$:

    - Use Golskokov-Kroll model (Kroll, Moutarde, Sabatie, Eur. Phys. J. C (2013) 73:2278)

$$h(\alpha, \beta, t) = \frac{\Gamma(2n+2)}{2^{2n+1}\Gamma^2(n+1)} \frac{\left[(1-|\beta|)^2 - \alpha^2\right]^n}{(1-|\beta|)^{2n+1}} |\beta|^{-\alpha_p t} e^{\beta_p t}$$

Parametric Model

- D-term:
  - Use first term of the Gegenbauer polynomial parameterization (Goeke et. al. Prog. Part. Nucl. Phys. 47, 401 (2001))

$$D^{q+}(x/\xi,t) = (1-(x/\xi)^2) d_1^{q+} C_1^{3/2}(x/\xi) \left(1 - \frac{t}{\lambda^{q+}}\right)^{-\alpha_D^{q+}}$$

$$D^{g}(x/\xi,t) = (1-(x/\xi)^2) d_1^{g} \left(1 - \frac{t}{\lambda^{g}}\right)^{-\alpha_D^{g}},$$

Parametric Model

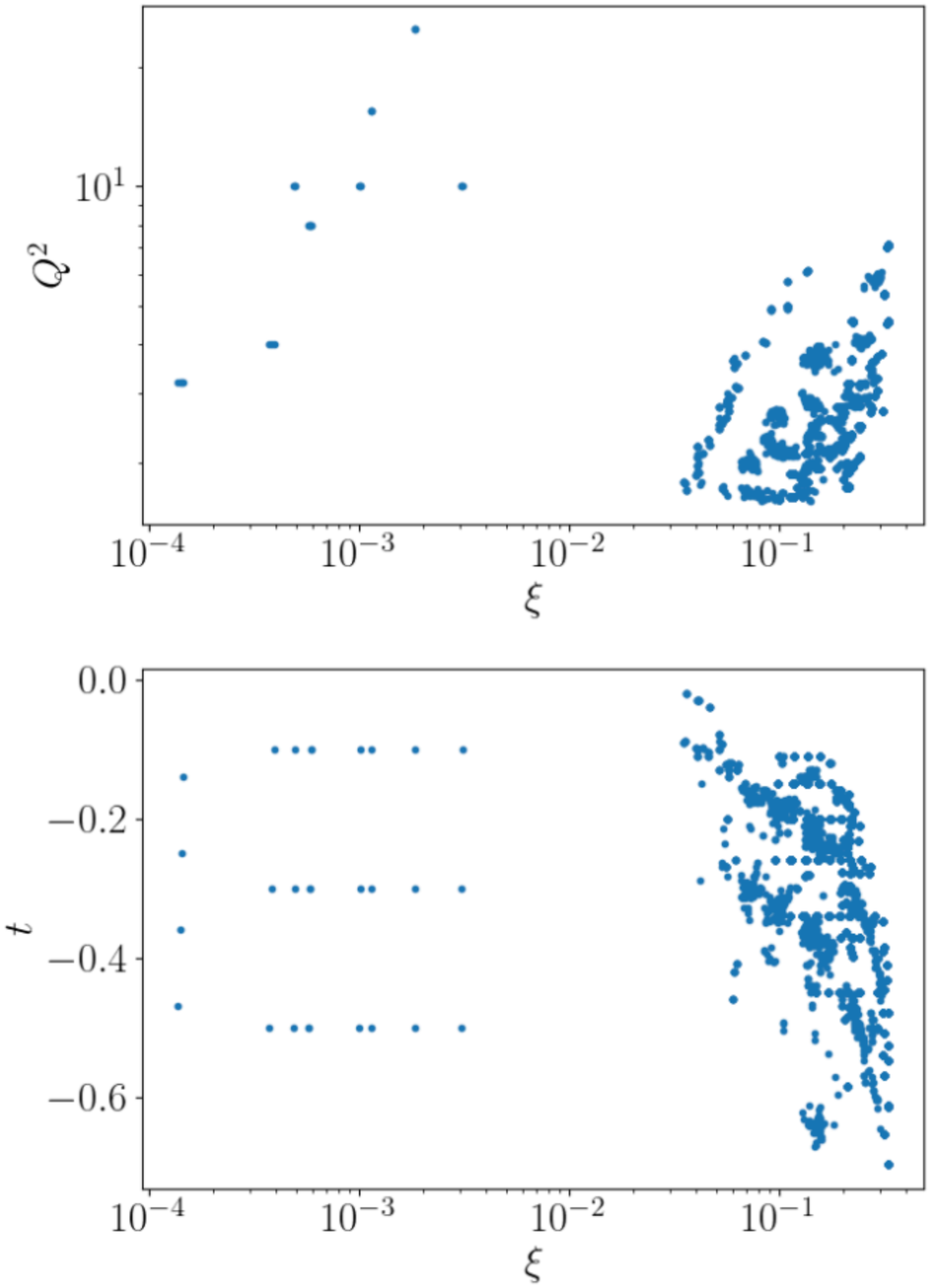
- Since quark GPDs only contribute to DVCS in a charge weighted sum over flavors, we parameterize the quark singlet combinations

$$F^{q+} = F^{q} + F^{\bar{q}}$$

- Assume s+ GPDs are zero at input scale.
- Total of 62 fitted model parameters

Data

- Fitting available DVCS data from:
  - HERA
  - HERMES
  - Hall A (proton and neutron target)
  - CLAS (proton and neutron target)
  - COMPASS
- Kinematic filters
  - $Q^2 > 1.6129\ GeV^2$
  - $t/Q^2 < 0.2$
  - $-t < 0.7\ GeV^2$
- Total of 7577 data points



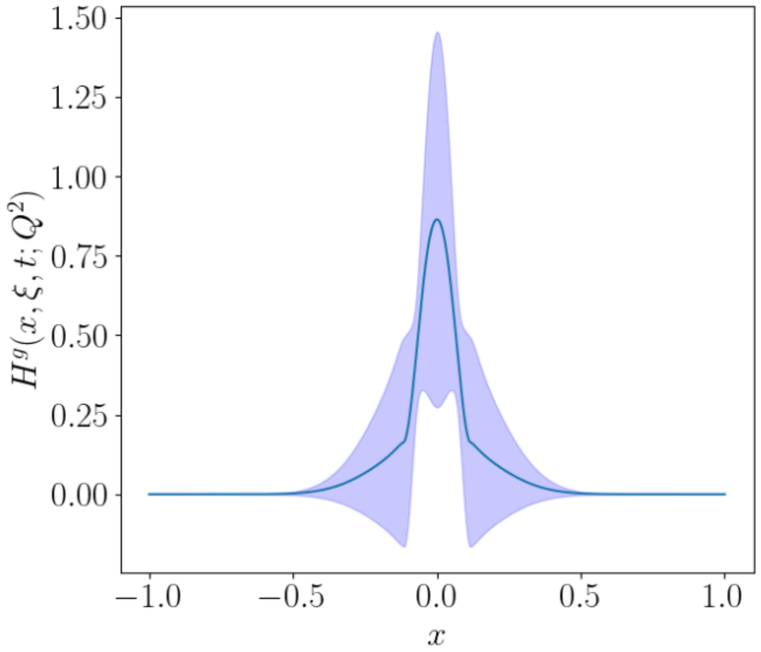
Preliminary Fit Results

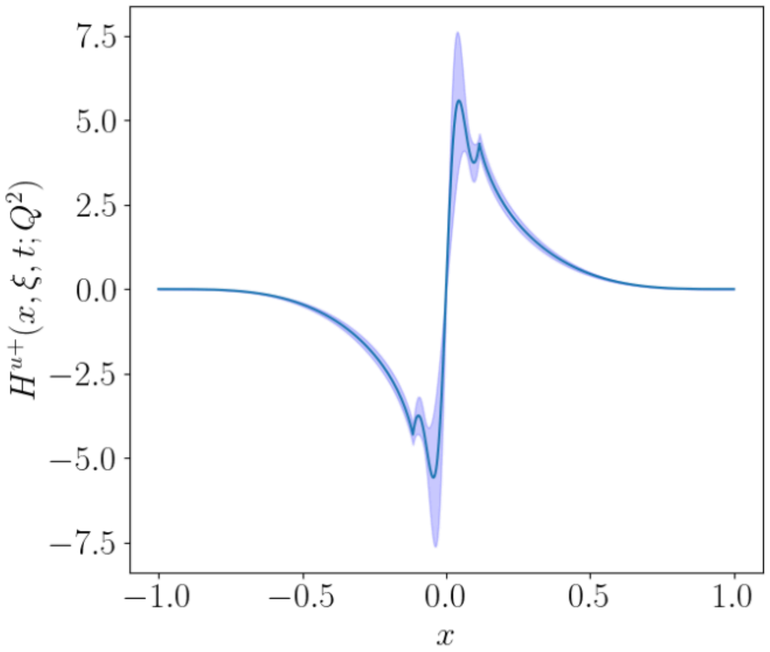
- Results of 10 replicas:
  - Chi Squared per number of points:
    - Overall: 1.078

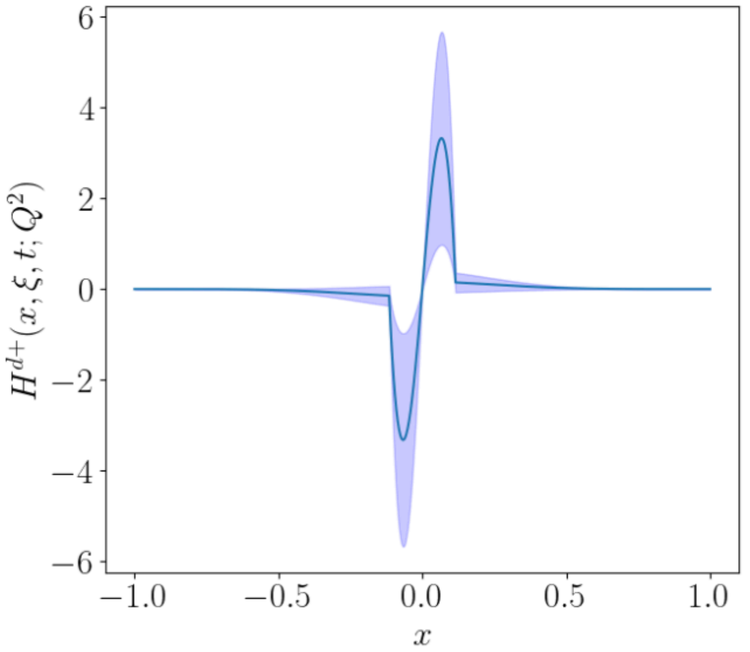
Results (H GPD)

Preliminary

$$\xi = 0.115, t = -0.127, Q^2 = 1.6\ \mathrm{GeV}^2$$





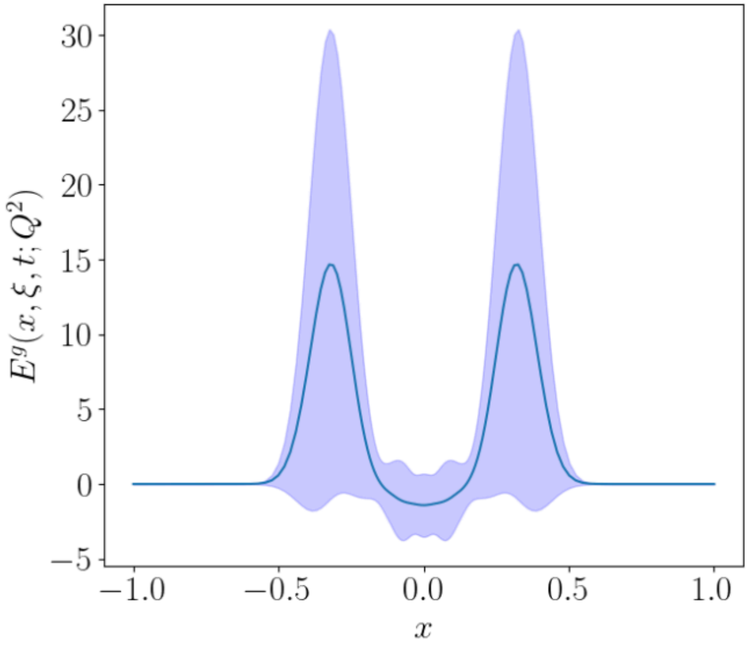


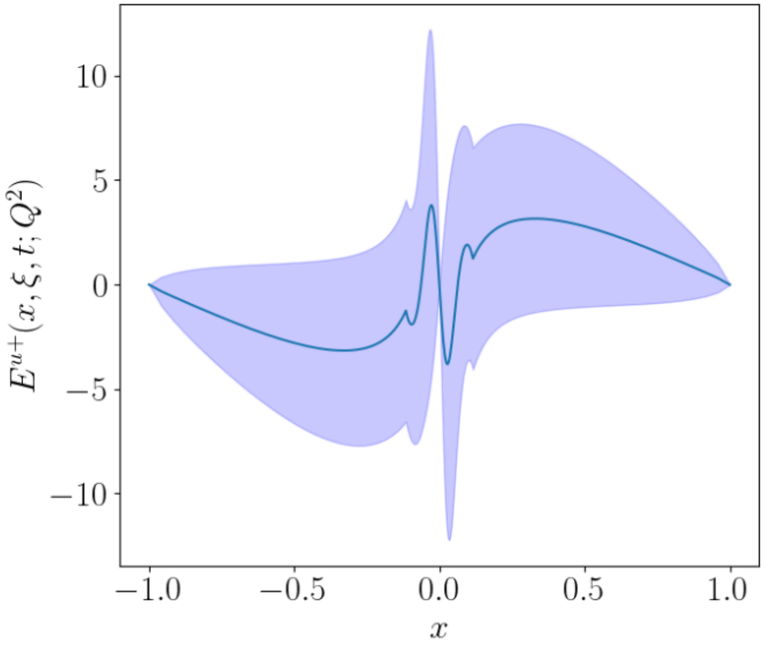
Preliminary

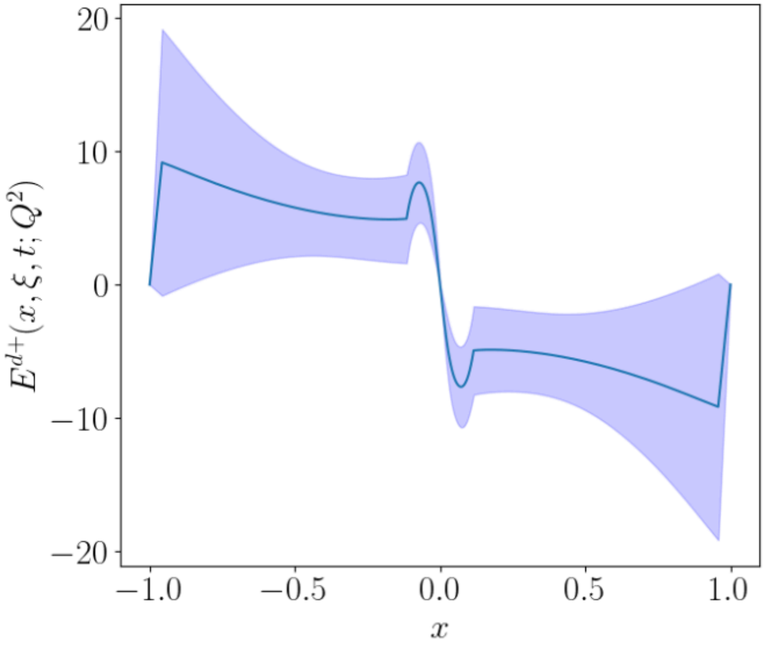
Results (E GPD)

Preliminary

$$\xi = 0.115, t = -0.127, Q^2 = 1.6\ \mathrm{GeV}^2$$





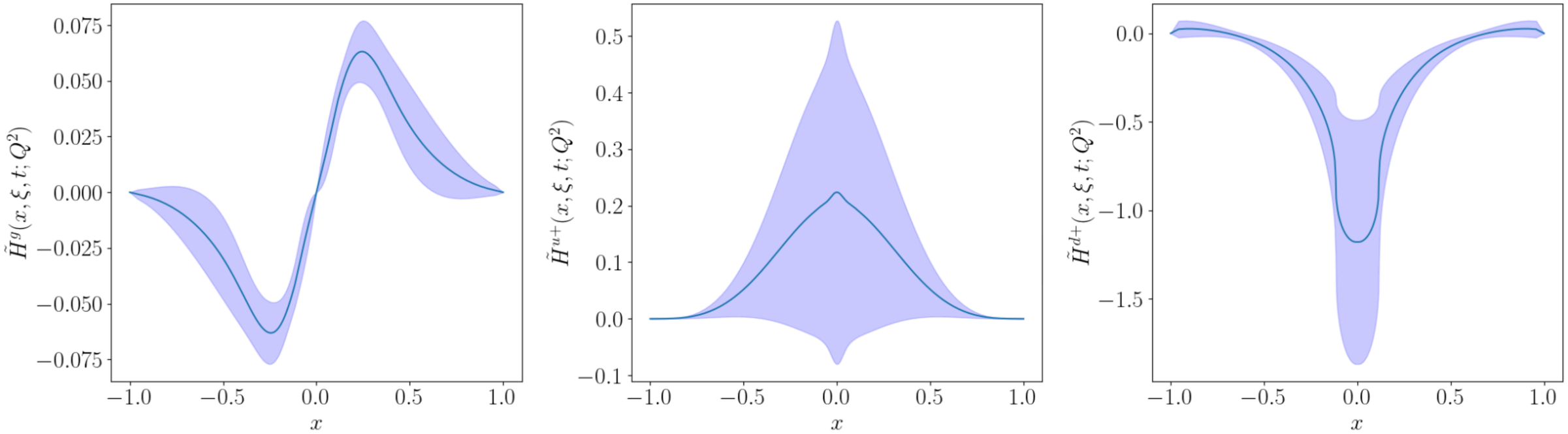


Preliminary

Results ($\tilde{H}$ GPD)

Preliminary

$$\xi = 0.115, t = -0.127, Q^2 = 1.6\ \mathrm{GeV}^2$$

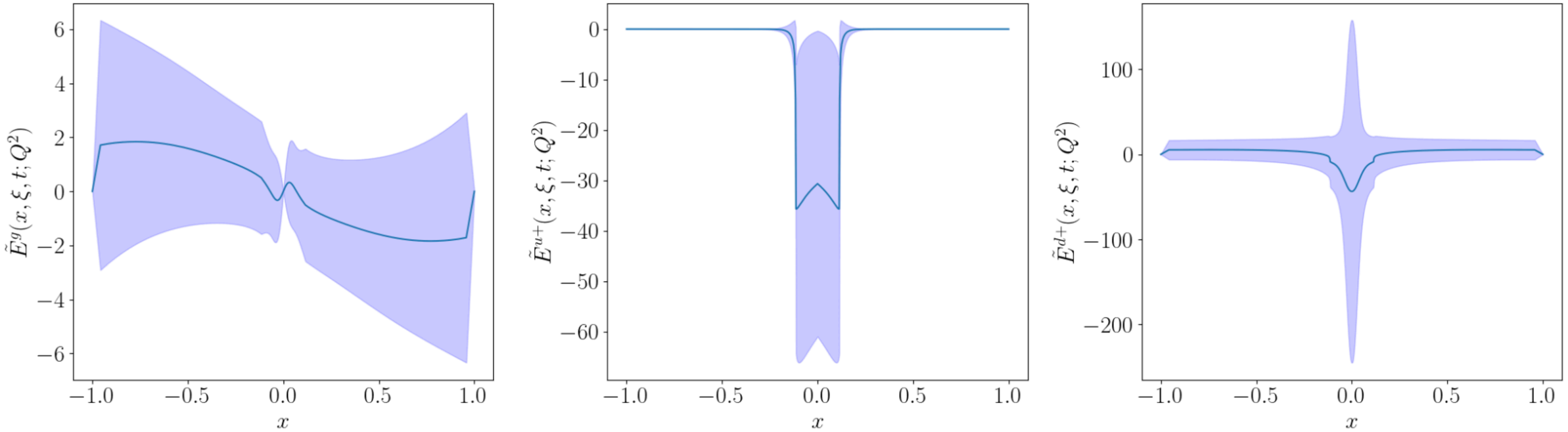


Preliminary

Results ($\tilde{E}$ GPD)

Preliminary

$$\xi = 0.115, t = -0.127, Q^2 = 1.6\ \mathrm{GeV}^2$$



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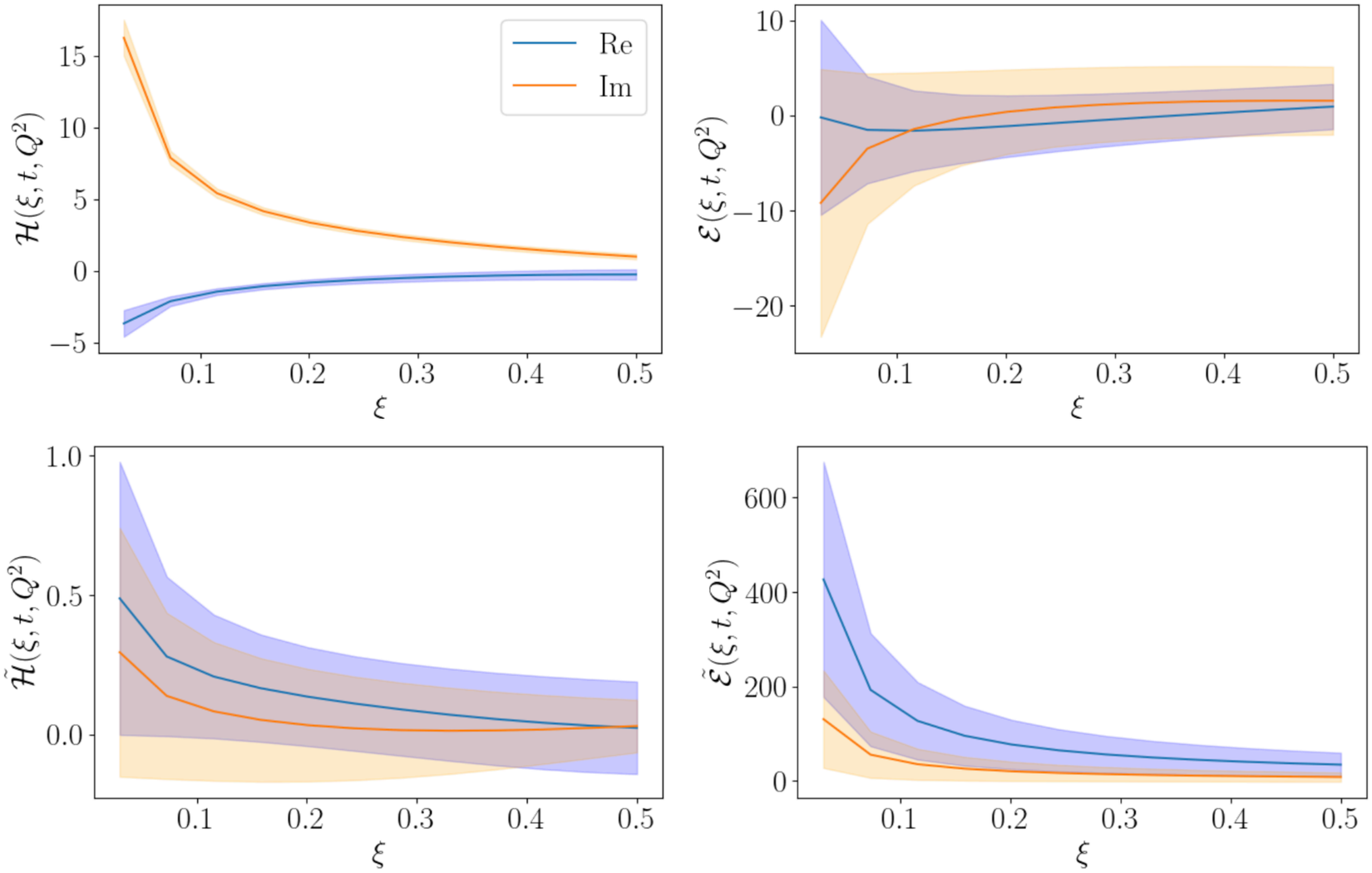
Results (CFFs)

Preliminary

$$t = -0.127, Q^2 = 1.6\ \mathrm{GeV}^2$$



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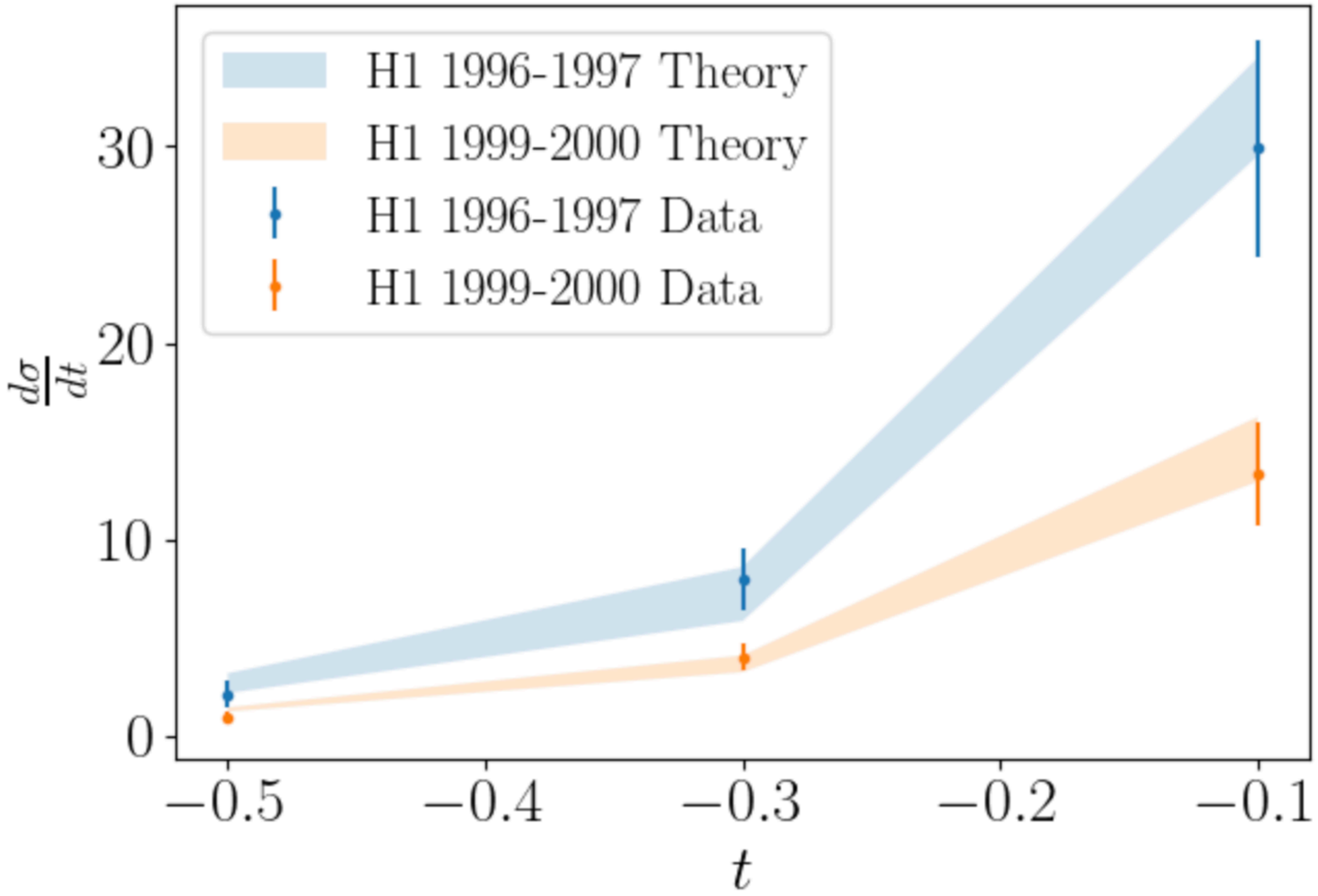
Results (Comparison with H1 2005, Eur.Phys.J.C 44 (2005) 1-11)



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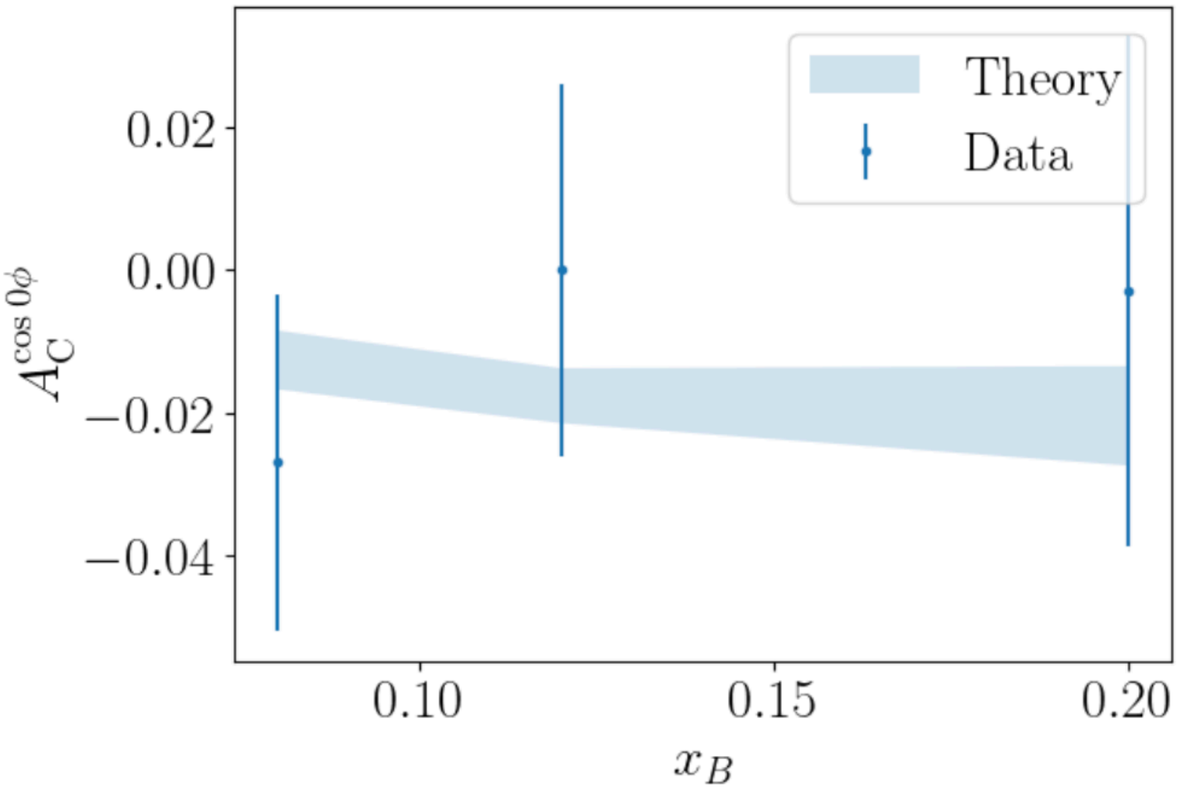
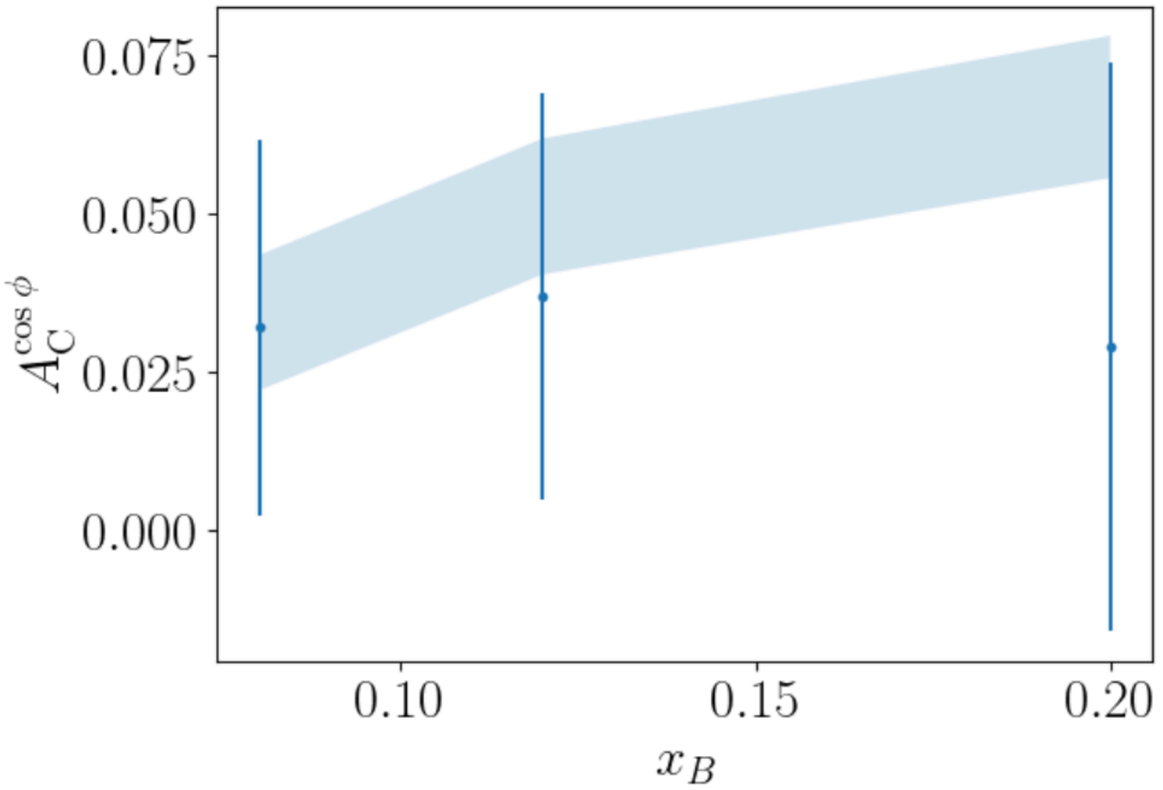
Results (Comparison with HERMES 2008, JHEP 06 (2008) 066)





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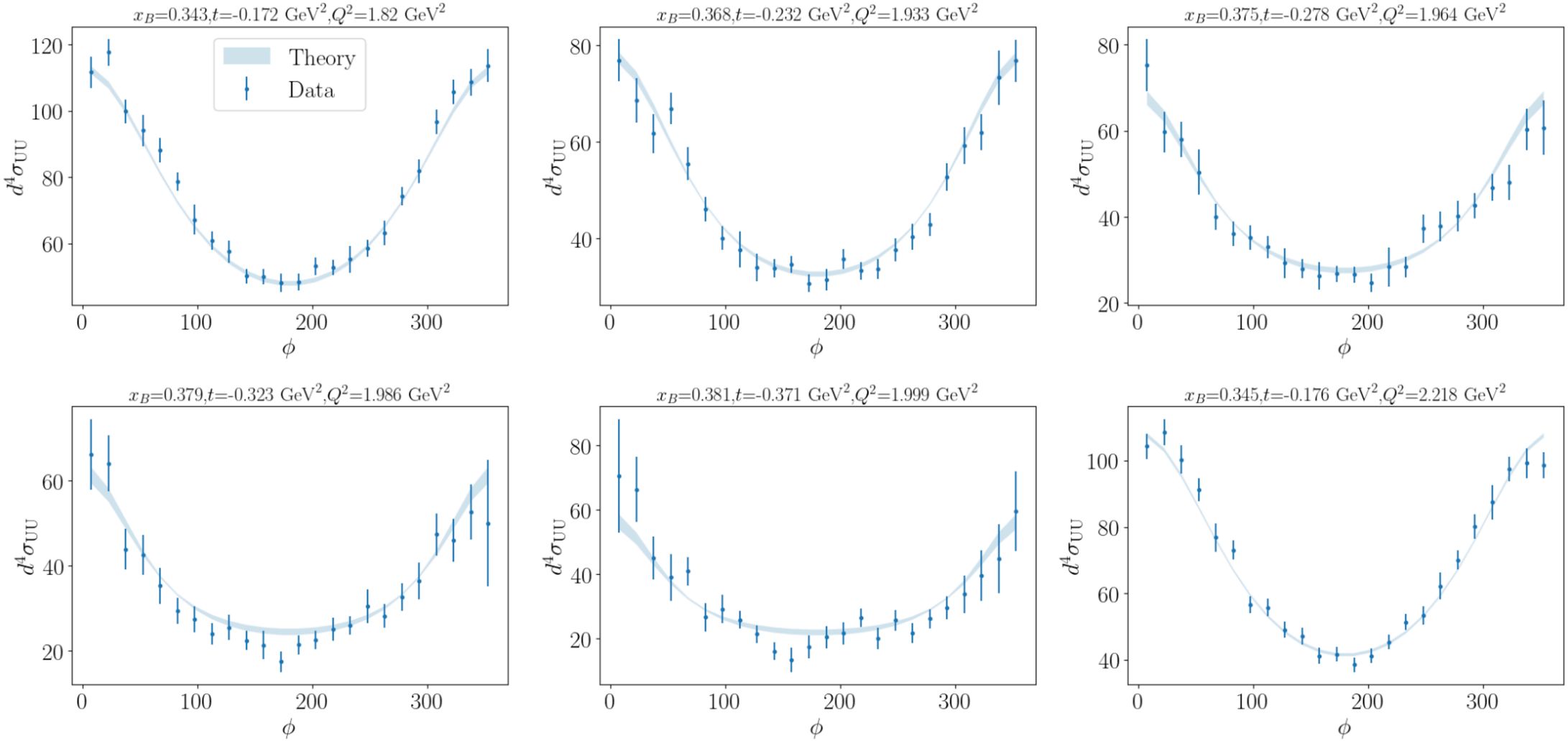
Results (Comparison with Hall A 2015, Phys.Rev.C 92 (2015) 5, 055202)



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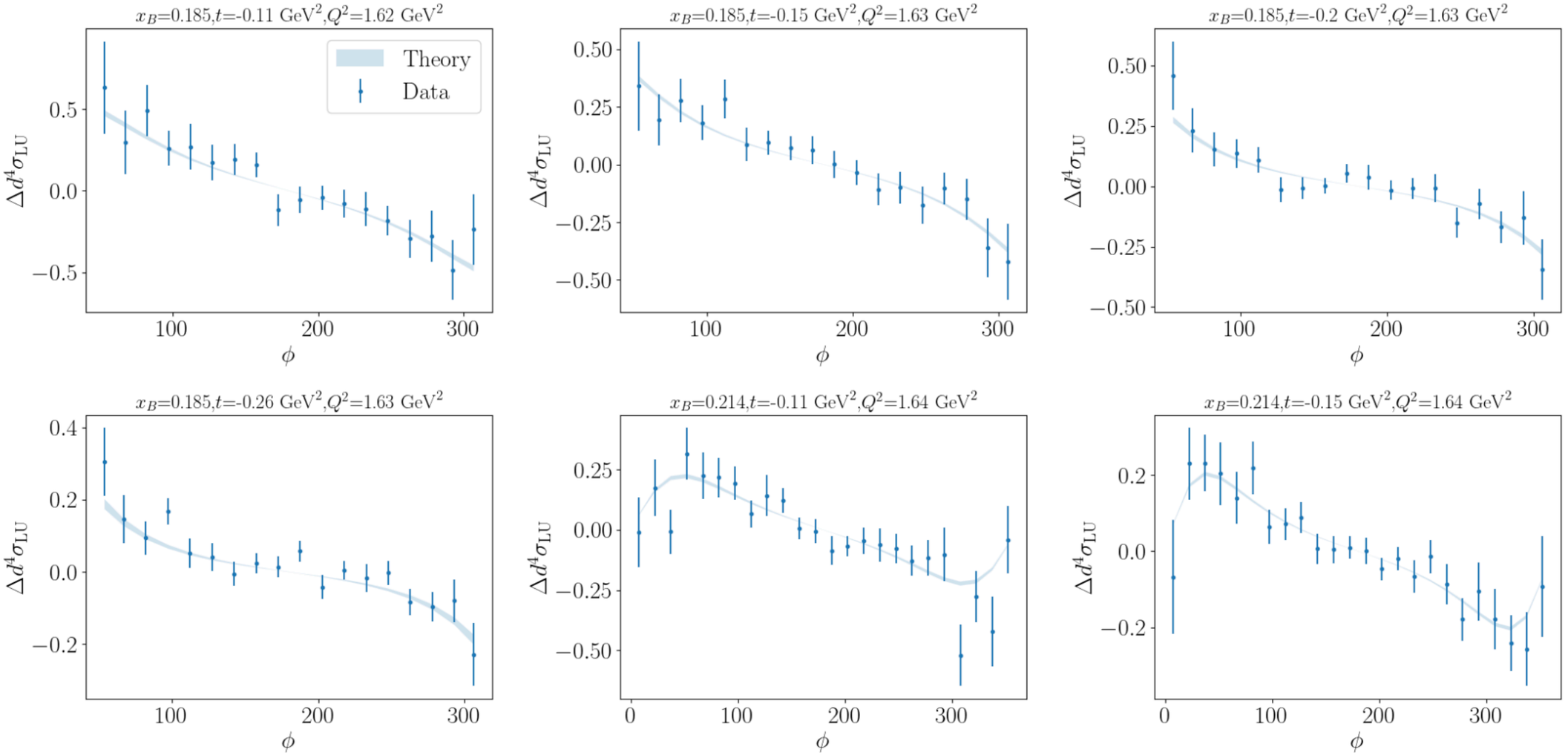
Results (Comparison with CLAS 2015, Phys.Rev.Lett. 115 (2015) 21, 212003)



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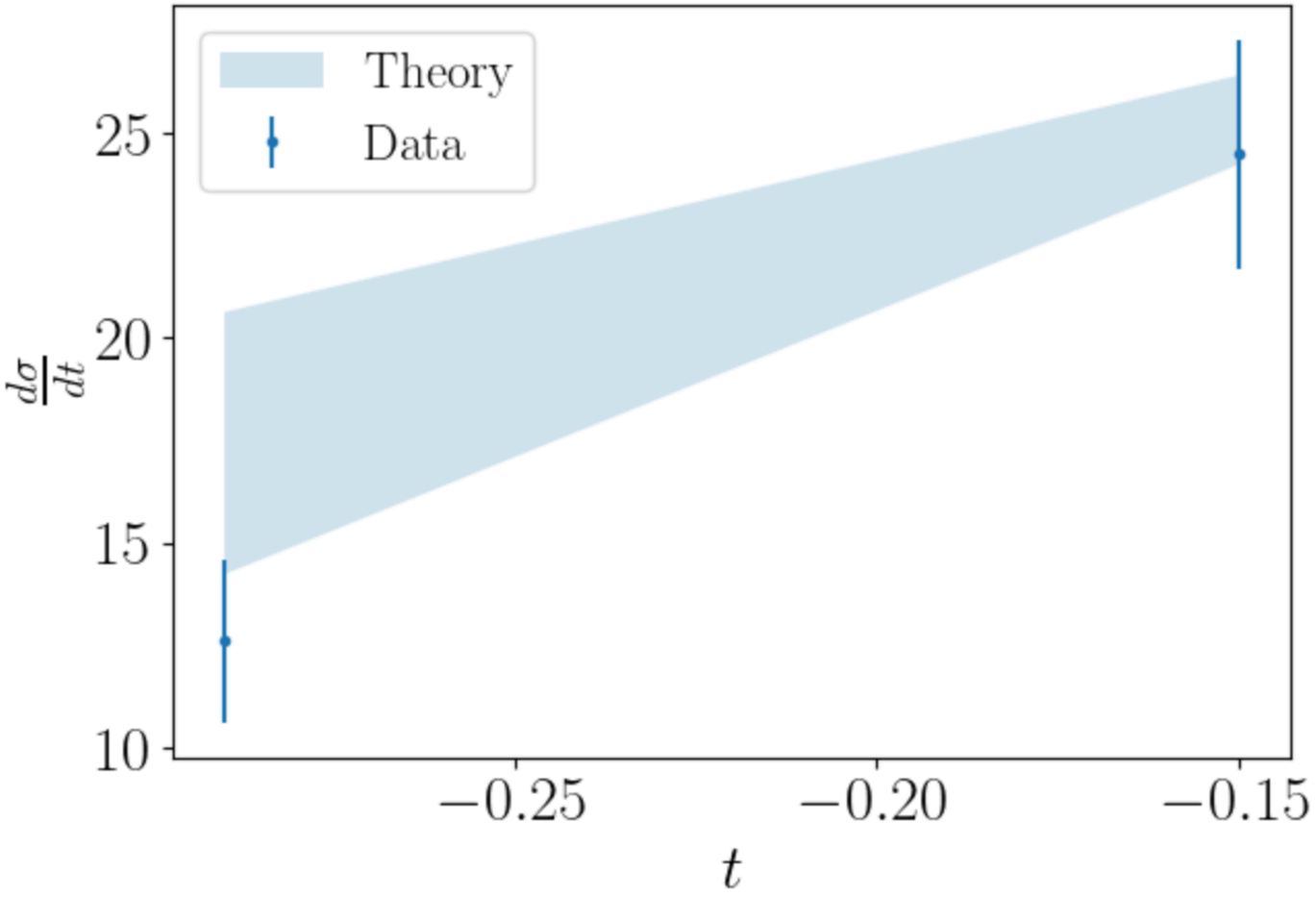
Results (Comparison with COMPASS 2019, Phys.Lett.B 793 (2019) 188-194)



Preliminary

Summary and Next Steps

- Summary:
  - Developed ML ready framework for Bayesian Monte Carlo extraction of GPDs from data
  - Preliminary fitting of available DVCS data using a simple parameterization achieved a very good chi square per number of points (1.078). Generating more replicas to get complete results.