Finite-t and Target Mass Corrections for the Short-Distance Expansion of Quasi (Pseudo) GPDs

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Generalized Parton Distributions (GPDs)

- GPDs provide a comprehensive 3D image of the nucleon, correlating the longitudinal momentum fraction with the transverse spatial distribution of partons.

$$\langle p'|\bar{q}(z_1 n)\not{n}[z_1 n, z_2 n]q(z_2 n)|p\rangle$$
$$= \int_{-1}^{1} dx\, e^{-i(Pn)[z_1(\xi-x)+z_2(x+\xi)]}\left\{\bar{u}(p')\not{n}u(p)M(x,\xi,t) - \frac{2(Pn)}{2m_N}\bar{u}(p')u(p)E(x,\xi,t)\right\}$$

where $M = H + E$, $\xi = -\frac{(\Delta n)}{2(Pn)}$, $P_\mu = \frac{1}{2}(p+p')_\mu$, $\Delta_\mu = (p'-p)_\mu$, $t = \Delta^2$, $n^2 = 0$.

- GPDs $H(x,\xi,t)$, $E(x,\xi,t)$, $\widetilde{H}(x,\xi,t)$, $\widetilde{E}(x,\xi,t)$ enter hard exclusive processes, such as deeply virtual Compton scattering (DVCS).
- **Lattice QCD approach:** Compute nonlocal equal-time correlators $\langle p'|\bar{q}(z)\Gamma q(0)|p\rangle$ with large P_z.

Motivation and Goal of this work

Problem

- Lattice calculations suffer from power corrections $\sim m_N^2/P_z^2$ and t/P_z^2 – so-called **kinematic corrections**.
- Kinematic power corrections restore the broken translation invariance and can be large, limiting extraction of GPDs at moderate $|t|$.

Our goal

Calculate systematically **kinematic** twist-2, twist-3 and twist-4 contributions to the short-distance expansion of $\langle p'|\bar{q}(zv)\Gamma q(0)|p\rangle$ up to $\mathcal{O}(v^2/P_v^2)$.

- We provide explicit expressions for the power corrections for all GPDs and to all orders in the short-distance expansion.
- Numerical estimates for a realistic GPD model.

Ref.: *Braun and Manashov, JHEP 01 (2012) 085; Braun, Manashov, Pirnay, Phys. Rev. D86 (2012) 014003*

Gegenbauer Moments of the GPDs

- Gegenbauer moments of the GPDs are given by reduced matrix elements of local conformal operators. For the vector operators

$$O_N^{(\not{n})}(y) = (-\partial_+)^N \bar{q}(y) C_N^{3/2}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) \not{n} q(y)$$

- The matrix element is

$$(-i)^N \langle p'|O_N^{(\not{n})}|p\rangle = \bar{u}(p')\not{n}u(p)\, M_N^{(n)}(t) - \frac{Pn}{m_N}\bar{u}(p')u(p)\, E_N^{(n)}(t)$$

with

$$M_N^{(n)}(t) = \sum_{m,\text{even}}^{N} M_{Nk}(t)(\Delta n)^m (Pn)^{N-m}\,, \quad E_N^{(n)}(t) = \sum_{m,\text{even}}^{N+1} E_{Nm}(t)(\Delta n)^m (Pn)^{N-m}$$

and similar for the axial vectors.

Quasi/Pseudo-GPDs on the Lattice

- Lattice calculates:

$$Q^{(\gamma_\mu)}(z_1 v, z_2 v) = \bar{q}(z_1 v)\gamma_\mu [z_1 v, z_2 v] q(z_2 v), \quad v^2 < 0$$

Matching to light-cone GPDs (factorization):

$$\langle p'|Q^{(\Gamma)}(z_1 v, z_2 v)|p\rangle = C_\Gamma(z_1 v, z_2 v; \mu_F^2) \otimes \langle p'|Q^{(\not{n})}(z_1 n, z_2 n)|p\rangle^{\mu_F^2} + \mathcal{O}\Big(\frac{1}{(Pv)}\Big)$$

We consider **asymmetric frame** $z_1 = z$, $z_2 = 0$ (favored on lattice) and introduce the lattice asymmetry parameter: $\eta = -\frac{(\Delta v)}{2(Pv)}$.

- The parameterization of the off-forward nucleon matrix element

$$\begin{aligned}\langle p'|Q^{(\gamma_\mu)}|p\rangle^V = {} & \bar{u}(p')\not{v} u(p) \left\{ \frac{v^\mu}{v^2}\mathcal{M}^{(v)} + \left(\frac{P^\mu}{(Pv)} - \frac{v^\mu}{v^2}\right)\mathcal{M}^{(P)} + \frac{\Delta_\perp^\mu}{(Pv)}\mathcal{M}^{(\Delta)} \right\} \\ & + \bar{u}(p')\left(\gamma_\mu - \frac{P_\mu \not{v}}{(Pv)}\right) u(p)\mathcal{M}^{(\gamma)} \\ & - \frac{(Pv)}{m_N}\bar{u}(p')u(p)\left\{ \frac{v^\mu}{v^2}\mathcal{E}^{(v)} + \left(\frac{P^\mu}{(Pv)} - \frac{v^\mu}{v^2}\right)\mathcal{E}^{(P)} + \frac{\Delta_\perp^\mu}{(Pv)}\mathcal{E}^{(\Delta)} \right\}\end{aligned}$$

- Our discussion focuses on $\mathcal{M}^{(P)}$ as example.

Separation of Kinematic vs. Dynamical Corrections

- The divergence of leading-twist conformal operators vanishes in a free theory; it only emerges due to gauge interactions, making it an intrinsically multi-particle phenomenon.
- The QCD equations of motion (EOM) transform these divergences into complex sums of **quark-antiquark-gluon ($\bar{q}gq$) operators**. Consequently, standard extraction methods using on-shell quark matrix elements fail.
- It is difficult to isolate the target **kinematic contributions** from the background of **dynamical power corrections** (which are driven by genuine quark-gluon correlations).
- The separation of kinematic and dynamical terms must be strictly robust against factorization scale changes.
- The method of **[Braun and Manashov, 2012]** incorporates the kinematic corrections, which avoids having to solve multi-operator renormalization group equations directly.
- Kinematic power corrections come from two distinct sources: **Nachtmann-type** and **the higher-twist operators**.

$$\mathcal{M}^{(P)} = \mathcal{M}_{t2}^{(P)} + \mathcal{M}_{t3}^{(P)} + \mathcal{M}_{t4}^{(P)} + \dots$$

$\mathcal{M}^{(P)}$ as Example

- Twist-2 contributions:

$$(\mathcal{M}^{(P)})_{t2} = \sum_{N=0}^{\infty} 2(2N+3) \sum_{k=0}^{\infty} \frac{(izP_v)^{N+k}}{k!} \frac{\Gamma(N+k+1)}{\Gamma(2N+k+4)} (\mathcal{M}^{(P)}_{Nk})_{t2}$$

$$(\mathcal{M}^{(P)}_{Nk})_{t2} = (N+k+1) \sum_{\substack{m=0 \\ \text{even}}}^{N} (-2\eta)^{m+k} M_{Nm} - \frac{v^2}{4P_v^2} \frac{N+k-1}{N+k+1} \sum_{\substack{m=-2 \\ \text{even}}}^{N-2} (-2\eta)^{m+k} \mathbb{M}^{(4)}_{Nkm}$$

$$\mathbb{M}^{(4)}_{Nkm} = (m+k+1)(m+k+2)tM_{N,m+2} + (N-m)(N-m-1)\Big(m_N^2 - \frac{t}{4}\Big) M_{Nm}$$

This is an example of the **Nachtmann-type contribution**.

- Twist-3 contributions:

$$(\mathcal{M}^{(P)})_{t3} = \sum_{N=0}^{\infty} \frac{(2N+3)}{(N+1)(N+2)} \sum_{k=0}^{\infty} \frac{(izP_v)^{N+k+1}}{k!} \frac{\Gamma(N+k+2)}{\Gamma(2N+k+4)} (\mathcal{M}^{(P)}_{Nk})_{t3}$$

$$(\mathcal{M}^{(P)}_{Nk})_{t3} = \frac{1}{N+k+2} \frac{v^2}{P_v^2} \sum_{\substack{m=-2 \\ \text{even}}}^{N-2} (-2\eta)^{m+k+1} \Big\{ \mathbb{M}^{(4)}_{Nkm} - (N+k)(m+k+2)tM_{N,m+2} \Big\}$$

Twist-4 Contributions

$$(\mathcal{M}^{(P)})_{t4} = \frac{1}{2}\sum_{N=0}^{\infty}\frac{2N+3}{(N+1)^2(N+2)^2}\sum_{k=0}^{\infty}\frac{(izP_v)^{N+k+1}}{k!}\frac{\Gamma(N+k+2)}{\Gamma(2N+k+4)}\left[(\mathcal{M}^{(P),A}_{Nk})_{t4} + T_{Nk}(\mathcal{M}^{(P),B}_{Nk})_{t4}\right]$$

$$(\mathcal{M}^{(P),A}_{Nk})_{t4} = 0\,, \qquad (\mathcal{M}^{(P),B}_{Nk})_{t4} = \frac{v^2}{P_v^2}\sum_{\substack{m=0,\\ \text{even}}}^{N-2}(N+k)(-2\eta)^{m+k+1}(\partial\mathbb{M})_{Nm}$$

$$(\partial\mathbb{M})_{Nm} = (m+2)(2N-m+1)tM_{Nm+2} - (N-m)(N-m-1)\left(m_N^2 - \frac{t}{4}\right)M_{Nm}$$

- T_{Nk} are very complicated (but rational numbers).
- For $\eta = 0$, twist-4 contributions vanish – simplifies analysis.

Short Distance Expansion

Evaluating the full multi-twist framework for the primary amplitude $\mathcal{M}^{(P)}$, we obtain the exact full-twist expressions for the higher moments. The result is expressed as a sum over conformal moments:

$$\mathcal{M}^{(P)} = \sum_{N=0}^{\infty} \frac{\Gamma(3/2)[iz(Pv)]^N}{4^N \Gamma(N+3/2)} \mathcal{M}_N^{(P)}$$

with $\mathcal{M}_N^{(P)}$ given by combinations of Gegenbauer moments $M_{Nm}(t)$.

$N = 2$ moment $(\eta = 0)$

$$\mathcal{M}_2^{(P)} = M_{20}(t) \left[1 - \frac{1}{18} \frac{v^2 P^2}{P_v^2} - 3 \frac{v^2 t}{P_v^2} \frac{M_{00}(t)}{M_{20}(t)} - \frac{1}{18} \frac{v^2 t}{P_v^2} \frac{M_{22}(t)}{M_{20}(t)} \right]$$

$N = 3$ moment

$$\mathcal{M}_3^{(P)} = M_{30}(t) \left[1 - \frac{3}{16} \frac{v^2 P^2}{P_v^2} - \frac{25}{6} \frac{v^2 t}{P_v^2} \frac{M_{10}(t)}{M_{30}(t)} - \frac{1}{16} \frac{v^2 t}{P_v^2} \frac{M_{32}(t)}{M_{30}(t)} \right]$$

The GPD Model

To estimate the physical size of these corrections, we use a valence quark double-distribution model (GK12):

$$M^q(x,\xi,t) = \int\int_{|\beta|+|\alpha|\leq 1} d\alpha d\beta \delta(x-\beta-\xi\alpha) f^q(\beta,\alpha,t)$$

with a Regge-like t-dependence:

$$f^q(\beta,\alpha,t) = q(\beta,t)h(\beta,\alpha), \quad q(\beta,t) = \theta(\beta)\beta^{-0.5-\alpha' t}(1-\beta)^3 e^{Bt}$$

We set the Regge slope $\alpha' = 0.9$ GeV^{-2} and profile function $h(\beta,\alpha) = \frac{3}{4}\frac{(1-|\beta|)^2-\alpha^2}{(1-|\beta|)^3}$.

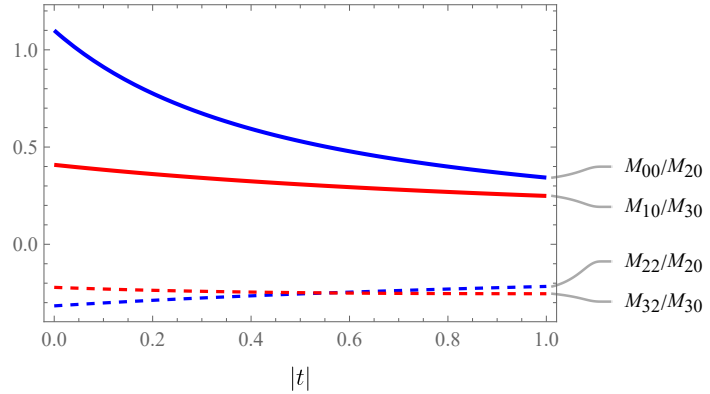
The Gegenbauer moments of the GPD can be expressed directly in terms of integrals of the DD:

$$M_{00}(t) = \iint\limits_{|\beta|+|\alpha|\leq 1} d\alpha d\beta\, f^q(\beta,\alpha,t)\,, \qquad M_{10}(t) = 6\iint\limits_{|\beta|+|\alpha|\leq 1} d\alpha d\beta\, \beta\, f^q(\beta,\alpha,t)$$

Ref.: *Radyushkin, Phys. Rev. D 56 (1997) 5524; Kroll, Moutarde and Sabatie, Eur. Phys. J. C 73 (2013) 2278*

Form Factor Ratios in Power Corrections

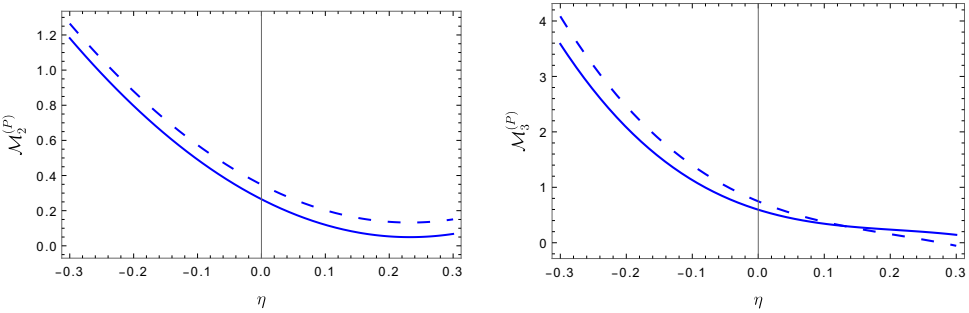
For $\eta = 0$, the correction relies on specific ratios like M_{00}/M_{20} and M_{10}/M_{30}.



These ratios decrease with momentum transfer, which mitigates the huge numerical prefactors seen in the analytical equations.

Kinematic Corrections

Let's fix $|t| = 1$ GeV^2 and $|P_v|/|v| = 2$ GeV. We plot the moments as a function of the lattice asymmetry parameter η.



- At the symmetric point ($\eta = 0$), the total kinematic correction modifies the $N = 2$ moment by roughly $\mathbf{24}\%$.
- For the $N = 3$ moment, the modification reaches $\mathbf{20}\%$.

Discussions

Key observations

1. Kinematic corrections are **significant** ($\sim 20-25\%$) for a realistic lattice QCD setup: For $|t| = 1$ GeV^2 and $|P_v|/|v| = 2$ GeV and for a realistic GPD model.
2. They are dominated by the twist-three contributions.
3. Since the moment ratios decrease with momentum transfer, the kinematic corrections increase slowly with $|t|$.
4. At $\eta = 0$, twist-4 corrections vanish, as the contribution of the twist-four operator $\partial^\mu \mathcal{O}_{\{\mu\alpha\}}$ includes the factor η.

Implications for lattice QCD

To achieve a higher precision in GPD moments, these kinematic corrections **must be taken into account**. They restore translation invariance broken by truncating the twist expansion.

Dynamical (genuine) twist-4 corrections from quark-gluon correlations are separate and expected to be smaller at large $|t|$.

Summary and Conclusions

What has been achieved

- Systematic calculation of kinematic power corrections $\sim m_N^2/P_v^2$ and t/P_v^2 for quasi/pseudo-GPDs up to twist-4.
- Explicit expressions for Gegenbauer moments $\mathcal{M}_N^{(P)}$, $\mathcal{E}_N^{(P)}$, ...
- Numerical estimates show corrections are $\sim 20-25\%$ at $|t| = 1\,\text{GeV}^2$, $|P_v|/|v| = 2\,\text{GeV}$.
- The operator-level results are valid for arbitrary quark positions.
- Results are ready for implementation in lattice analyses.

Thank you for your attention!

Back up

$$T_{Nk} = \frac{1}{N+k}\left\{\frac{1}{2}N(N+3) + \frac{\Gamma(N+2)}{\Gamma(N+k+2)}\frac{S(N,k)-\bar{S}(N,k)}{\gamma_N}\right\}.$$

where

$$S(N,k) = \frac{1}{2}(N+2)\sum_{m=0}^{N-1}\left\{\frac{(-1)^{N-m+1}\Gamma(N+2)}{\Gamma(m+2)\Gamma(N-m+2)} + 1 - \frac{(N+3)(N-m)}{(N+1)(N+2)}\right\}$$
$$\times \frac{\Gamma(m+k+2)}{\Gamma(m+1)}\frac{1}{N-m+1}{}_3F_2\left(\begin{matrix}-k, N-m+1, N-m+1\\ -m-k-1, N-m+2\end{matrix};1\right),$$

$$\bar{S}(N,k) = (N+2)\sum_{m=0}^{N-1}\left\{\frac{(-1)^{N-m+1}\Gamma(N+1)}{\Gamma(m+1)\Gamma(N-m+3)} + \frac{1}{2}\right\}\left\{\frac{(m+1)}{(N-m)}\frac{\Gamma(N+k+2)}{\Gamma(N+2)}\right.$$
$$\left. - \frac{\Gamma(m+k+2)}{\Gamma(m+1)}\frac{1}{N-m+1}{}_3F_2\left(\begin{matrix}-k, N-m+1, N-m+1\\ -m-k-1, N-m+2\end{matrix};1\right)\right\}.$$