北京大学

Lattice Effective Field Theory study of nuclear beta decays

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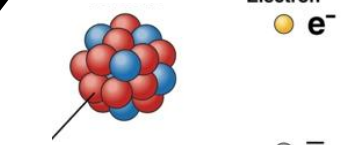
Outline

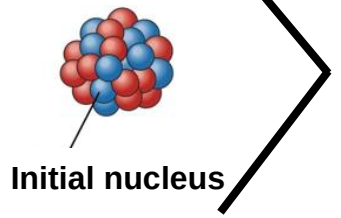
- Introduction
  - Nuclear β-decays
  - Nuclear Lattice Effective Field Theory
- Applications
  - Simple system benchmark: ^{3}H- and ^{6}He-decays
  - Revealing the anomalous long lifetime of ^{14}C
- Summary and outlooks

Low-energy description of nuclear β decay

- Nuclear β decay is triggered by the effective weak Hamiltonian with current-current interactions。

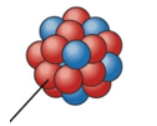
$$\langle \text{final nucleus}, e^-, \bar{\nu} | H_W = -\frac{G_F}{\sqrt{2}} V_{ud} \int d^3\boldsymbol{x}\, \underbrace{j_\mu^{(-)}(\boldsymbol{x})}_{\text{lepton current}} \underbrace{\mathcal{J}^{(+),\mu}(\boldsymbol{x})}_{\text{nuclear current}} + \text{h.c.} | \text{Initial nucleus} \rangle$$

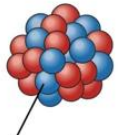




- What matters for nuclear physicsts is the matrix element of the nuclear weak current。

$$\langle \text{final nucleus} | \mathcal{J}^{(+),\mu} | \text{Initial nucleus} \rangle$$



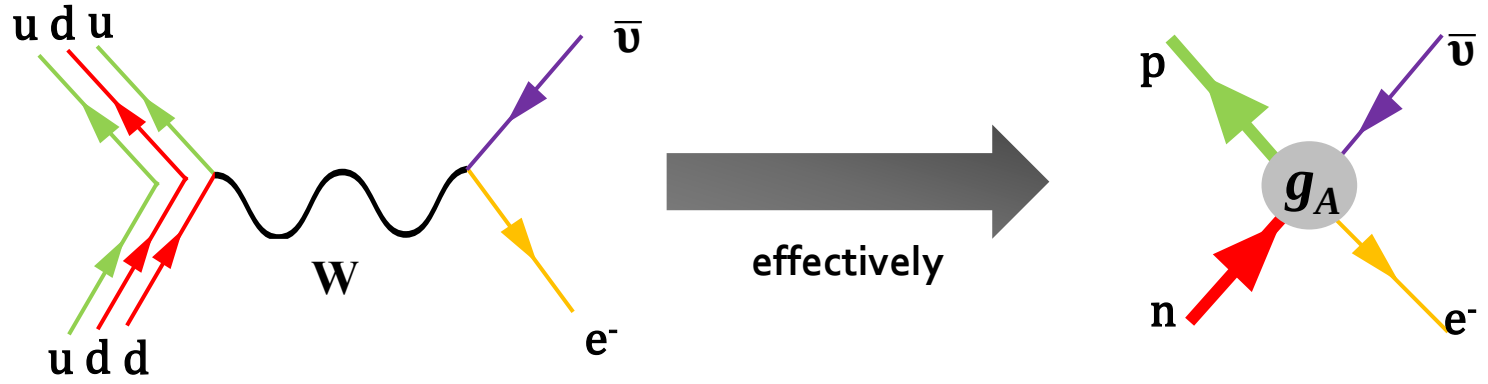


- Therefore, the study of nuclear β decay requires two sides of information.

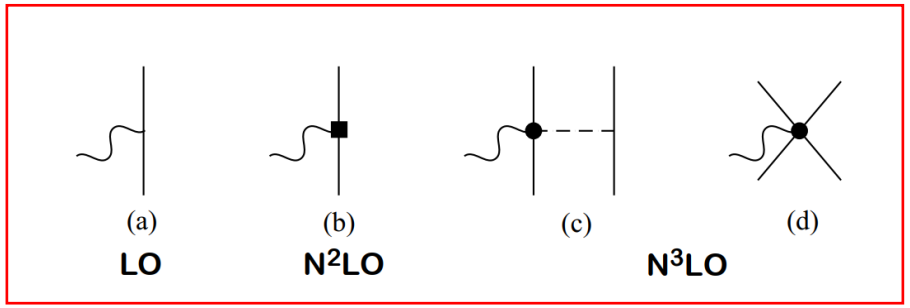
  - ► Expression and realization of the nuclear weak current.
  - ► Wave functions of the initial and final nuclei.

Nuclear weak current

- At leading-order, the nuclear weak current is given by a structure-independent one-body operator, describing the transition from a single neutron into a single proton



- Higher-order corrections emerge due to relativistic effects and many-body correlations, which can be systematically organized in the framework of chiral EFT.



- Pioneered by Park *et al.*

  T. S. Park et al, *Phys. Rept.* 233 (1993)

- Developed by the JLab-Pisa group and Bochum-Bonn group .

  A. Baroni et al, *Phys. Rev. C* 93, 015501 (2016)

  H. Krebs et al, Annals *Phys.* 378 (2017)

Nuclear Lattice Effective Field Theory

- Nuclear Lattice Effective Field Theory (NLEFT) is an *ab initio* nuclear many-body method

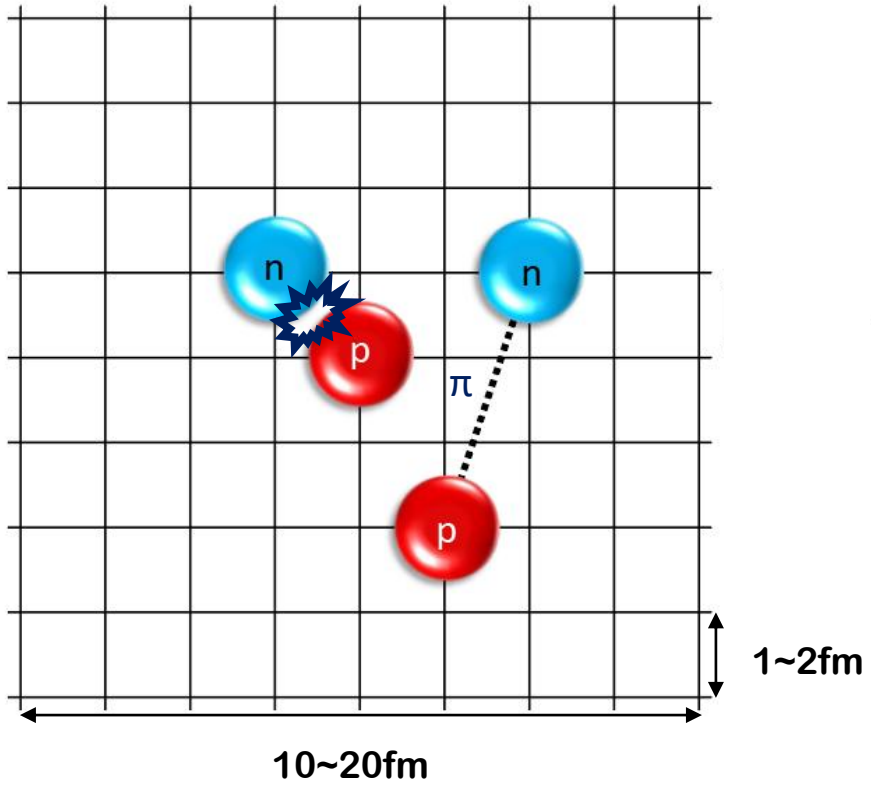
Dean Lee, Prog. *Part. Nucl. Phys.* 63, 117 (2009)

Lähde, Meißner, "Nuclear Lattice Effective Field Theory", Springer (2019)

NLEFT = Chiral EFT + Lattice Methd

Also see talks by Hang

- Protons and neutrons are point-like particles moving on a discrete, finite-volume cubic lattice
- Nuclear forces are discretized and regularized by the finite lattice spacing $a \approx$ 1-2 fm
- Observables are calculated through the stochastic lattice quantum Monte Carlo (MC) technique
- Computational cost depends polynomially on the mass number (applied to heavy nuclei successfully)



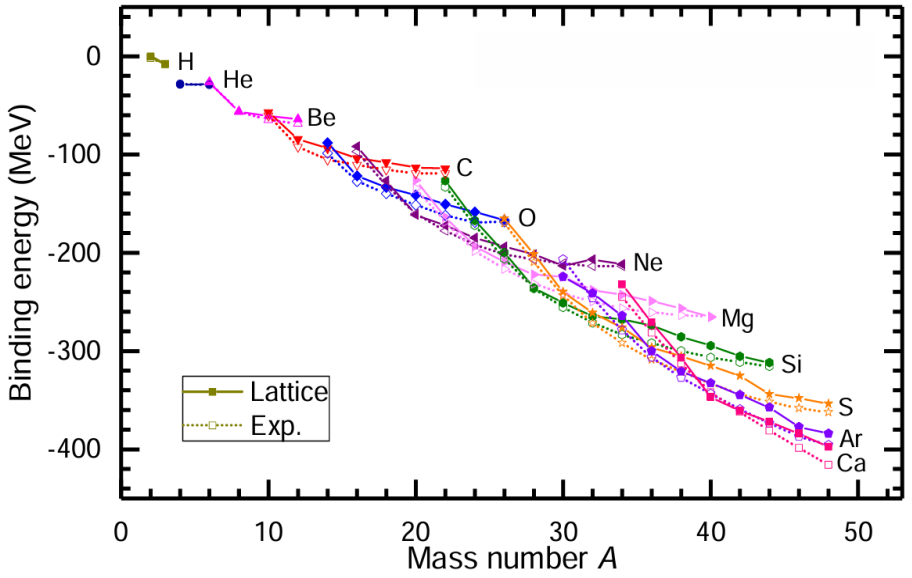
Z.-W. Niu and B.-N. Lu, *Phys. Rev. Lett.* 135, 222504(2025)

F. Hildenbrand et al, , *Phys. Rev. Lett.* 136, 062501(2026)

Lattice nuclear forces

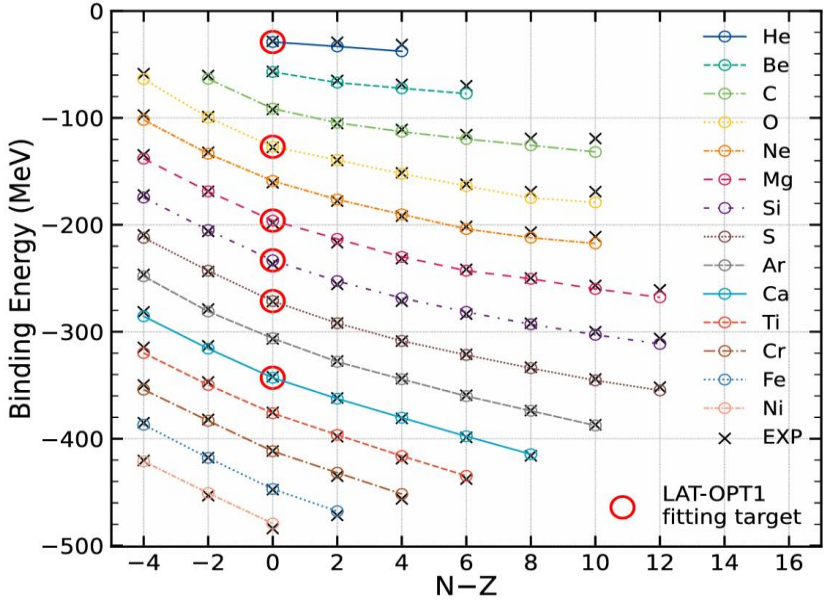
- There is an important class of interactions which are simple but incorporate the essential elements of realistic nuclear forces.

$$H_{\mathrm{SU(4)}} = K + \frac{C_2}{2!}\sum_n :\tilde{\rho}^2(n): + \frac{C_2}{3!}\sum_n :\tilde{\rho}^3(n):$$



B.-N. Lu et al, *Phys. Lett. B*, 797, 134864(2019)

$$H_{\mathrm{LAT\text{-}OPT1}} = H_{\mathrm{SU(4)}} + V_{\mathrm{spin-orbit}}$$



Z.-W. Niu and B.-N. Lu, *Phys. Rev. Lett.* 135, 222504(2025)

- Due to exact/approximate Wigner SU(4) symmetry, these interactions can be treated nonperturbatively in MC simulations.

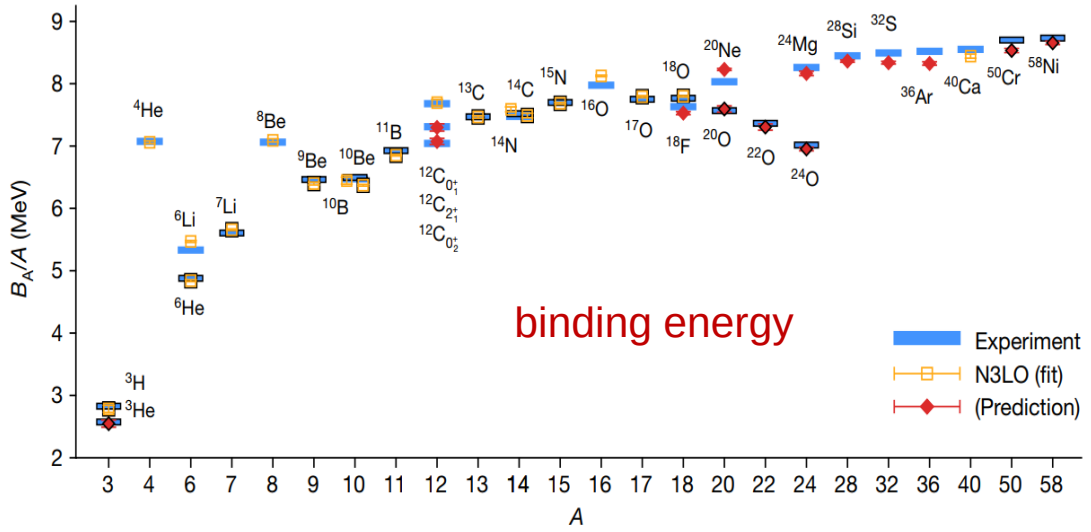
Lattice nuclear forces

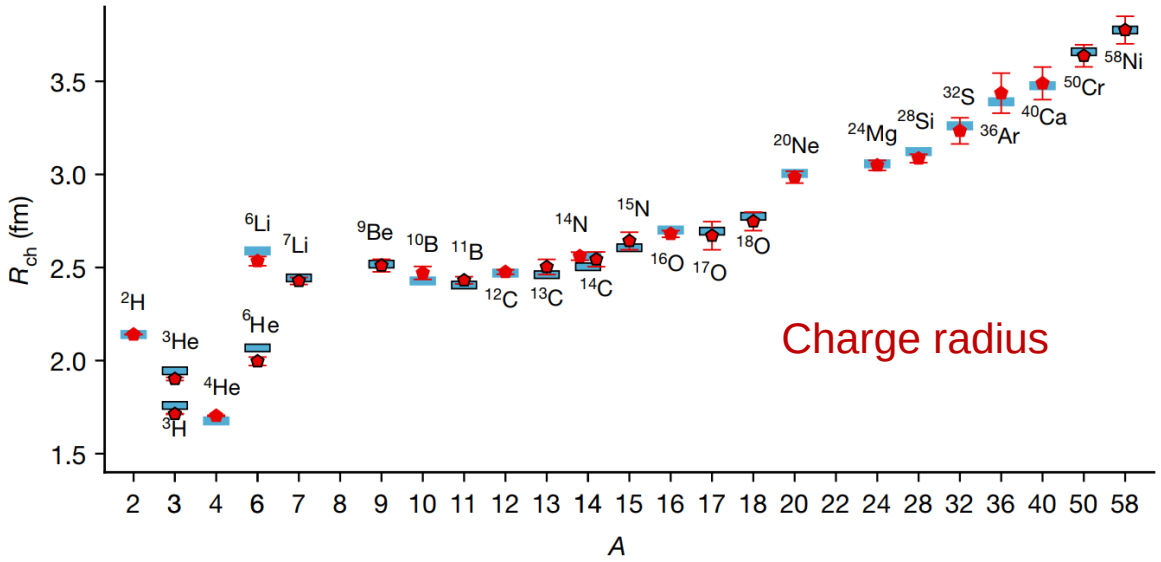
- To precisely describe nuclear structures, properties and transitions, high-fidelity lattice interactions have been constructed from chiral EFT.

high-fidelity N^{3}LO lattice chiral Hamiltonian (based on wavefunction matching method)

S. Elhatisari et al, *Nature* 630 (2024)

$$H_{\chi}^{\mathrm{wfm}}$$





- Due to severe sign problems, the high-fidelity lattice interactions are solved perturbatively in Monte Carlo simulations.

Lattice simulation

- Imaginary-time projection method is employed to project out the ground state.

$$|\Psi_0\rangle \propto \lim_{\tau\to\infty} e^{-H\tau/2}|\Psi_T\rangle$$

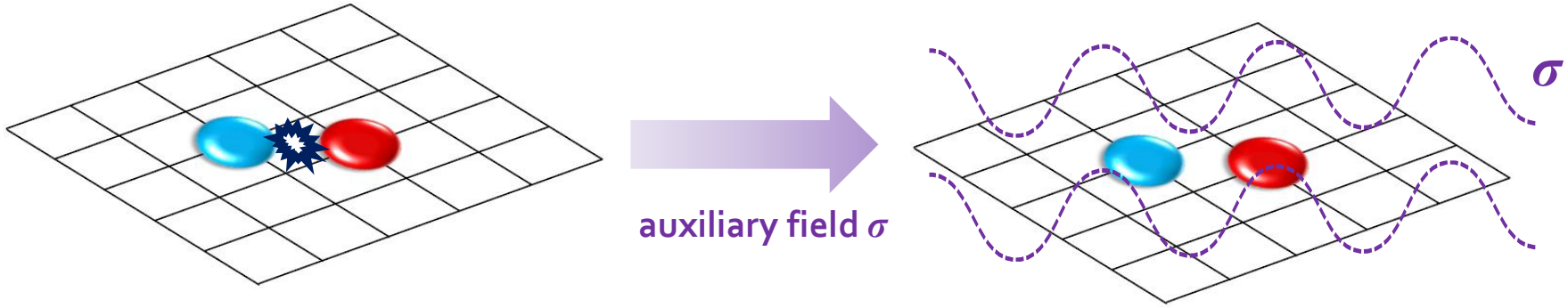
ground state of H — initial trial state

- Observables are extracted from ratios between nuclear correlation functions.

$$\langle O\rangle = \lim_{\tau\to\infty} \frac{\langle\Psi_T|e^{-H\tau/2}Oe^{-H\tau/2}|\Psi_T\rangle}{\langle\Psi_T|e^{-H\tau}|\Psi_T\rangle}$$

- Auxiliary field transformation is applied, turning the problem into the evaluation of integrals about the auxiliary field σ .

$$\lim_{\tau\to\infty} \frac{\langle\Psi_T|e^{-H\tau/2}Oe^{-H\tau/2}|\Psi_T\rangle}{\langle\Psi_T|e^{-H\tau}|\Psi_T\rangle} \xrightarrow{\text{auxiliary field } \sigma} \frac{\int \mathcal{D}\sigma\ e^{-\sigma^2/2} f_O[\sigma]}{\int \mathcal{D}\sigma\ e^{-\sigma^2/2} f_I[\sigma]}$$



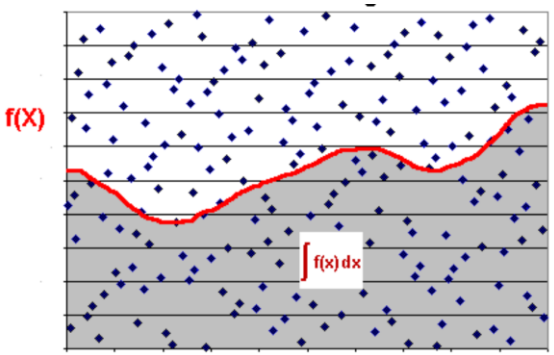
Monte Carlo sign problem

- Stochastic sampling method is employed to evaluate the high-dimensional integral.

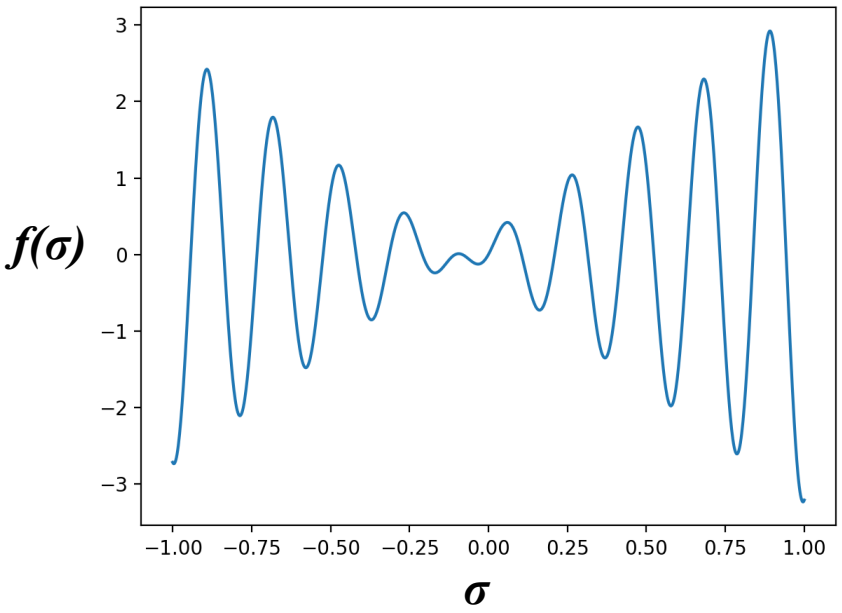
  N. Metropolis et al, *J. Chem. Phys.* 21, 06(1953).

  S. Duane, *Nucl. Phys. B* 257, 652(1985).

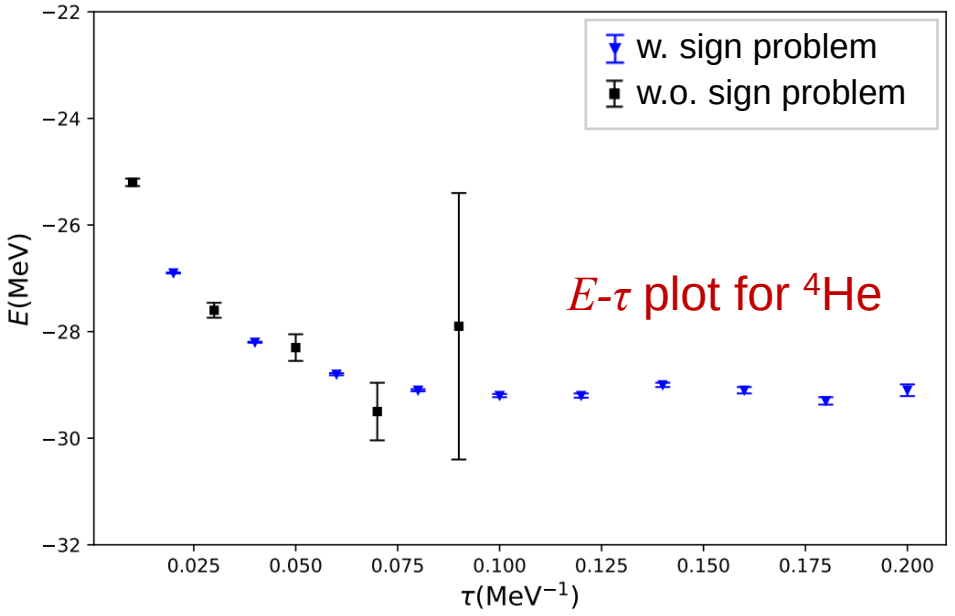
$$\frac{\int \mathcal{D}\sigma\ e^{-\sigma^2/2} f_O[\sigma]}{\int \mathcal{D}\sigma\ e^{-\sigma^2/2} f_I[\sigma]}$$



- Non-positive and highly-oscillating integrands would greatly reduce the efficiency of stochastic sampling, known as the MC sign problem.



- High-fidelity interactions suffer from strong sign problems, making their calculation extremely difficult.



Perturbative alleviation of sign problems

- The solution to the sign problem is a common challenge for all quantum Monte Carlo simulations.

Troyer. M and Wiese, U.-J., *Phys. Rev. Lett.* 94, 170201(2005)

- To solve the high-fidelity Hamiltonian $\boldsymbol{H_\chi}$, non-degenerate perturbation theory is employed in NLEFT calculations.

$$|\Psi_\chi\rangle = |\Psi_S^{(0)}\rangle + \sum_{n\neq 0} |\Psi_S^{(n)}\rangle \frac{\langle\Psi_S^{(n)}|(H_\chi - H_S)|\Psi_S^{(0)}\rangle}{E_S^{(0)} - E_S^{(n)}} + \cdots$$

$|\Psi_S^{(0)}\rangle$

Ground state of H_S with mild sign problems

B.-N. Lu et al, *Phys. Rev. Lett.*128, 242501(2022)

$$\sum_{n\neq 0} \frac{|\Psi_S^{(n)}\rangle\langle\Psi_S^{(n)}|}{E_S^{(0)} - E_S^{(n)}}$$

Excited-state contributions of H_S, extracted from correlation functions of H_S

- Successfully applied to nuclear property and structure studies.

S. Elhatisari et al, *Nature* 630 (2024)
S.-H. Shen et al, *Phys. Rev. Lett.* 134, 162503(2025)
Z.-X. Ren et al, *Phys. Rev. Lett.* 135, 152502(2025)

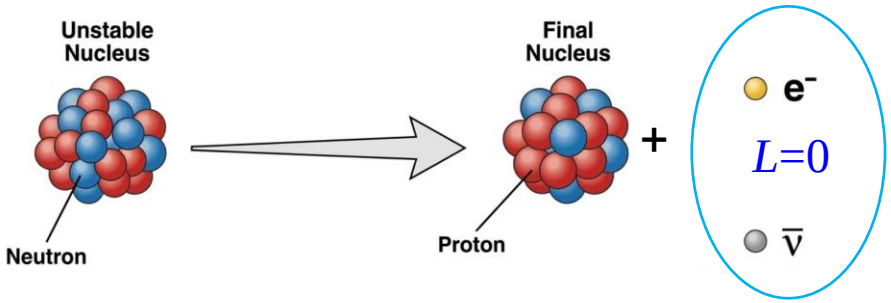
Benchmarking simple systems: ^{3}H- and ^{6}He-decays

T. W., X. Feng and B.-N. Lu, *Phys. Rev. C* 112, 025502 (2025)

Choice of object

- β-decay has never been studied through NLEFT before, so simple systems should first be benchmarked with.

  ► Choice of decay types: allowed β-decay.



➥ Fermi type $M_F = \langle f||\mathcal{J}^0||i\rangle$

➥ Gamow-Teller type $M_{\mathrm{GT}} = \langle f||\boldsymbol{\mathcal{J}}||i\rangle$

  ► Choice of nuclei: ^{3}H and ^{6}He.

  ➥ Both belonging to allowed β-decay type.

  ➥ Simple structure.

  ➥ Small mass number.

  ➥ Extensively studied in literature.

Technical development

- Two major technical issues challenge the NLEFT simulation of nuclear β-decays.

  ► Multichannel difficulty: new correlation functions & new sampling methods.

$$M_{\mathrm{GT}} = \lim_{\tau\to\infty} \frac{\langle \Psi_T^f | e^{-H\tau/2} \mathcal{J} e^{-H\tau/2} | \Psi_T^i \rangle}{\sqrt{\langle \Psi_i^f | e^{-H\tau} | \Psi_T^i \rangle \langle \Psi_i^f | e^{-H\tau} | \Psi_T^f \rangle}}$$

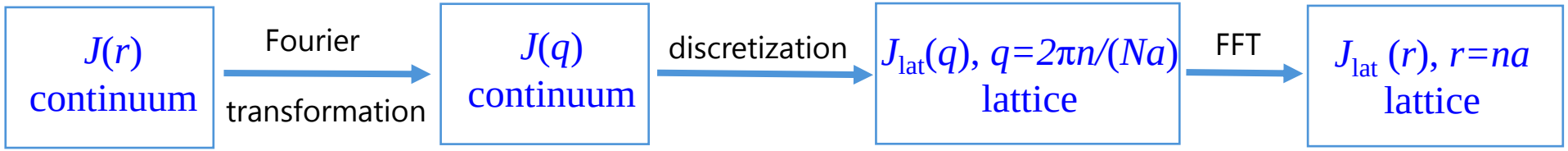
auxiliary field σ

$$\frac{\int \mathcal{D}\sigma\; e^{-\sigma^2/2} f_{\mathcal{J}}^{(i\to f)}[\sigma]}{\sqrt{\left(\int \mathcal{D}\sigma\; e^{-\sigma^2/2} f_I^{(i)}[\sigma]\right)\left(\int \mathcal{D}\bar{\sigma}\; e^{-\bar{\sigma}^2/2} f_I^{(j)}[\bar{\sigma}]\right)}}$$

Initial/final trial states constructed as shell-model ground states

Configurations of σ generated from a carefully designed PDF

► Realization of higher-order currents: direct lattice discretization.

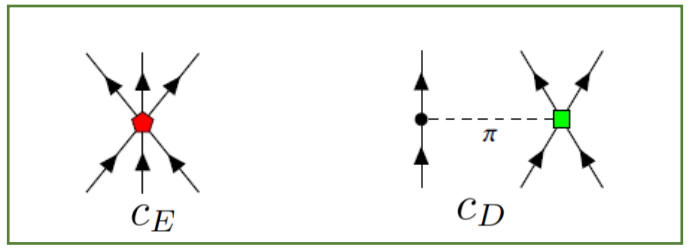


^{3}H results

- We use ^{3}H β decay to fix 3N forces, as is widely done in literature.

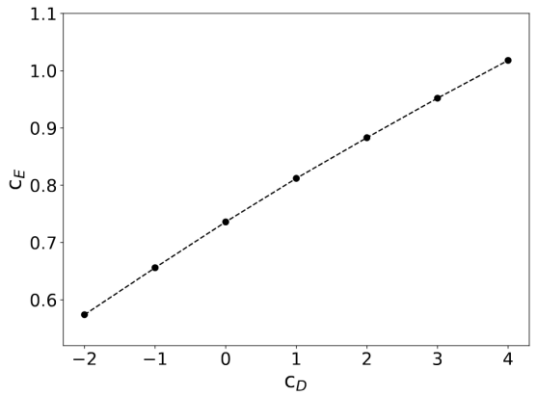
$$E(^3\mathrm{H}) = -8.482\mathrm{MeV}$$

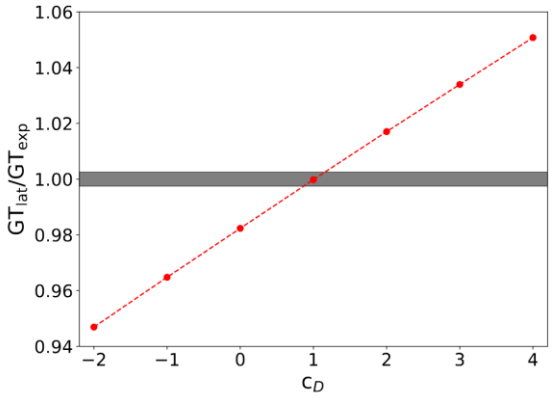
$$\mathrm{GT}(^3\mathrm{H} \to ^3\mathrm{He}) = 2.3297(58)$$



$$c_D = 1.01(12)$$

$$c_E = 0.81(1)$$





- We work out order-by-order contributions to the GT matrix element, finding weak corrections due to higher-order currents

	LO	N^{2}LO	N^{3}LO(OPE)	N^{3}LO(CT)
M_{GT}	2.355(17)	-0.010(2)	-0.070(1)	0.039(3)

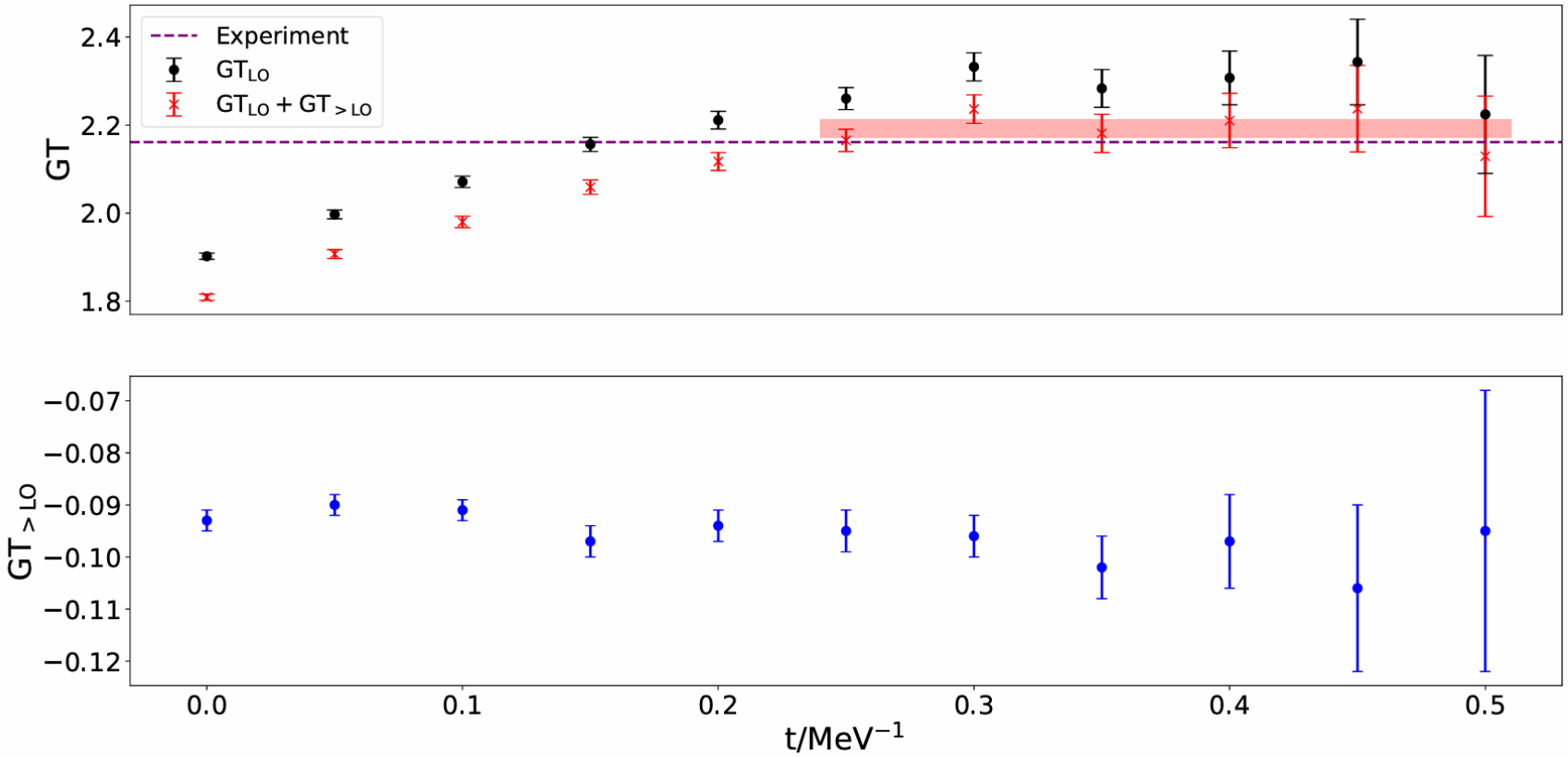
Negative corrections compared to continuum-based calculations

P. Gysbers et al, *Nature Phys.* 15, 428 (2019)

A. Baroni et al, *Phys. Rev. Lett.* 98, 044003 (2018)

……

^{6}He results



	LO	N^{2}LO	N^{3}LO(OPE)	N^{3}LO(CT)	Total-LO	Total	Expt. [96]
M_{GT}	2.289(19)	-0.024(2)	-0.104(1)	0.032(1)	-0.096(2)	2.192(20)	2.161(4)

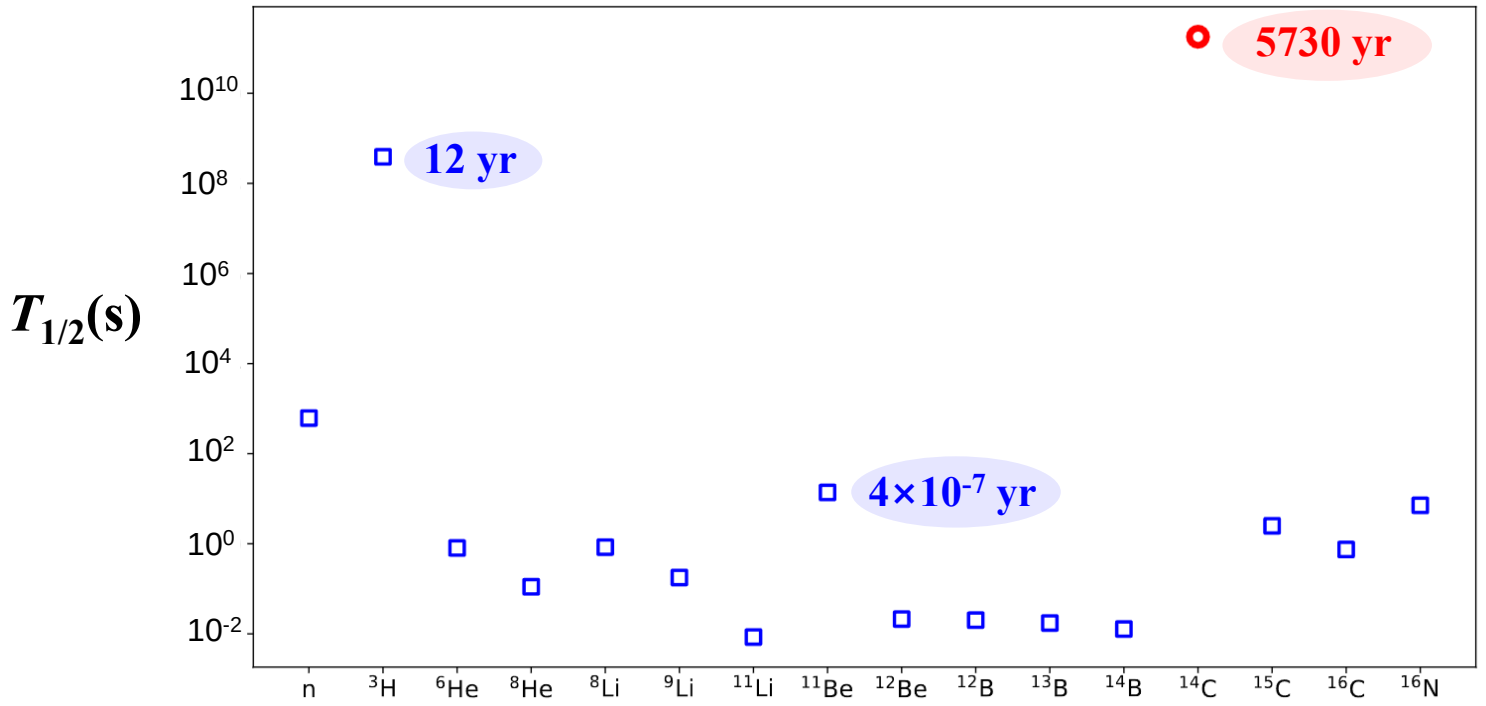
It is time for more challenging systems!

Revealing the anomalous long lifetime of ^{14}C

T. W., S. Elhatisari, X. Feng, B.-N. Lu and U.-G. Meissner, in preparation

The lifetime puzzle of ^{14}C

- The 5730-year half-life of ^{14}C, the physical basis of radiocarbon dating, is anomalously long.



most hindered allowed β-decay process

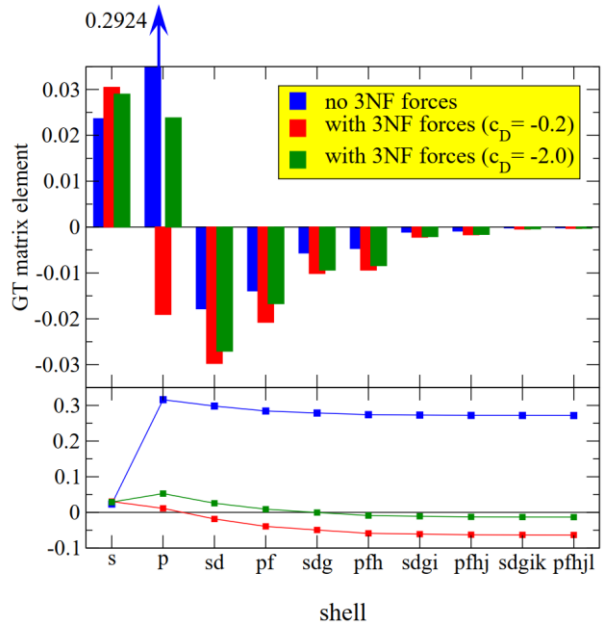
- Although of allowed type, the Gamow-Teller matrix element of ^{14}C β-decay is unexpectedly close to 0.

$$M_{GT} \approx 0.002 << 1$$

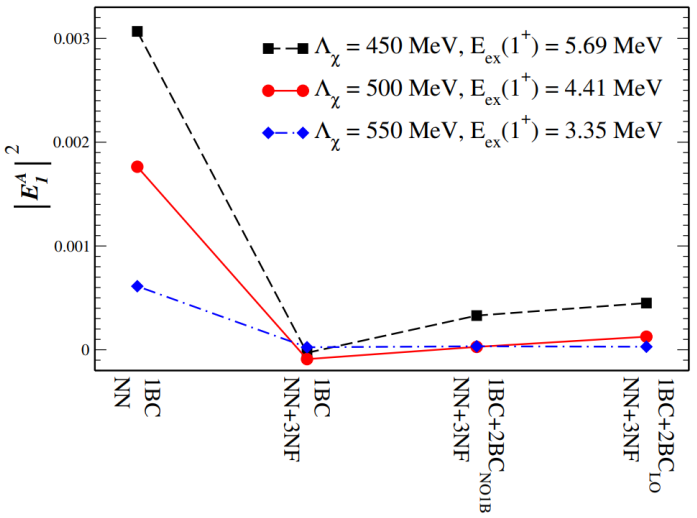
Chou et al, *Phys. Rev. C* 47 (1993)

Previous *ab initio* studies

- The β-decay of 14C has been studied in the framework of *ab initio* no-core shell model and coupled cluster methods, which reveals the sensitivity of M_{GT} to 3N forces.



Maris, P. et al, *Phys. Rev. Lett.* 106, 202502 (2011)



Ekström, A. et al, *Phys. Rev. Lett.* 113,262504 (2014)

The shell model is supposed to give some simplicity from the complexity of the nuclear many body system. The results of this paper (ab initio calculation) are far from simple.

——Igal Talmi

I. Talmi, *Physics* 4 (2022)

- However, the underlying mechanism is not clearly stated.

Shell model explanation

- Nuclear shell model treats ^{14}C and ^{14}N as an ^{16}O-core plus two holes of *p*-shell.



- The angular momentum of ^{14}C and ^{14}N is determined by the coupling of the two holes. I. Talmi, *Physics* 4, 2022

^{14}C(J=0) $|^{14}\mathrm{C}\rangle_{\mathrm{SM}} = x|^1S_0\rangle + y|^3P_0\rangle$

^{14}N(J=1) $|^{14}\mathrm{N}\rangle_{\mathrm{SM}} = \alpha|^3S_1\rangle + \beta|^3D_1\rangle + \gamma|^1P_1\rangle$

- Due to the special structure of the GT transition operator, the process is forbidden once ^{14}N is occupied by D-wave.

$$O_{\mathrm{GT}}^{1\mathrm{N}} = -g_A \sum_{n=1}^{A} \boldsymbol{\sigma}_n \tau_n^+$$

no orbital dependence

$$\langle L_f | O_{\mathrm{GT}}^{1\mathrm{N}} | L_i \rangle = 0 \quad (L_i \neq L_f)$$

selection rule of *L*

- The D-wave dominance of ^{14}N wave function has been verified experimentally, but the mechanism is unclear.

Negret, A. et al, *Phys. Rev. Lett.* 97,062502(2006)

Optimization of lattice interactions

- Our calculation of ^{14}C β decay employs lattice chiral interactions of different orders.

H_S **LO chiral Hamiltonian** H_χ **N^{3}LO chiral Hamiltonian**

- Since the sensitivity of ^{14}C β decay to 3N forces is revealed by previous works, two strategies are used to fit the 3N LECs.

H_χ $\left\{ \begin{array}{l} H_\chi^{\mathrm{3N_{wfm}}} \\ H_\chi^{\mathrm{3N_{opt}}} \end{array} \right.$

Strategy Ⅰ: fitted to energies of selected nuclei S. Elhatisari et al, *Nature* 630 (2024)

Strategy Ⅱ (optimized): additionally fitted to the magnetic moment μ and the excitation energy ΔE of ^{14}N(1$^+$)

- μ and ΔE of ^{14}N work as key constraint for the spin-orbital and tensor components of 3N forces.

$$V_{\mathrm{3N}} = V_{\mathrm{3N}}^{\mathrm{C}} + V_{\mathrm{3N}}^{\mathrm{SO}} + V_{\mathrm{3N}}^{\mathrm{T}} + \cdots$$

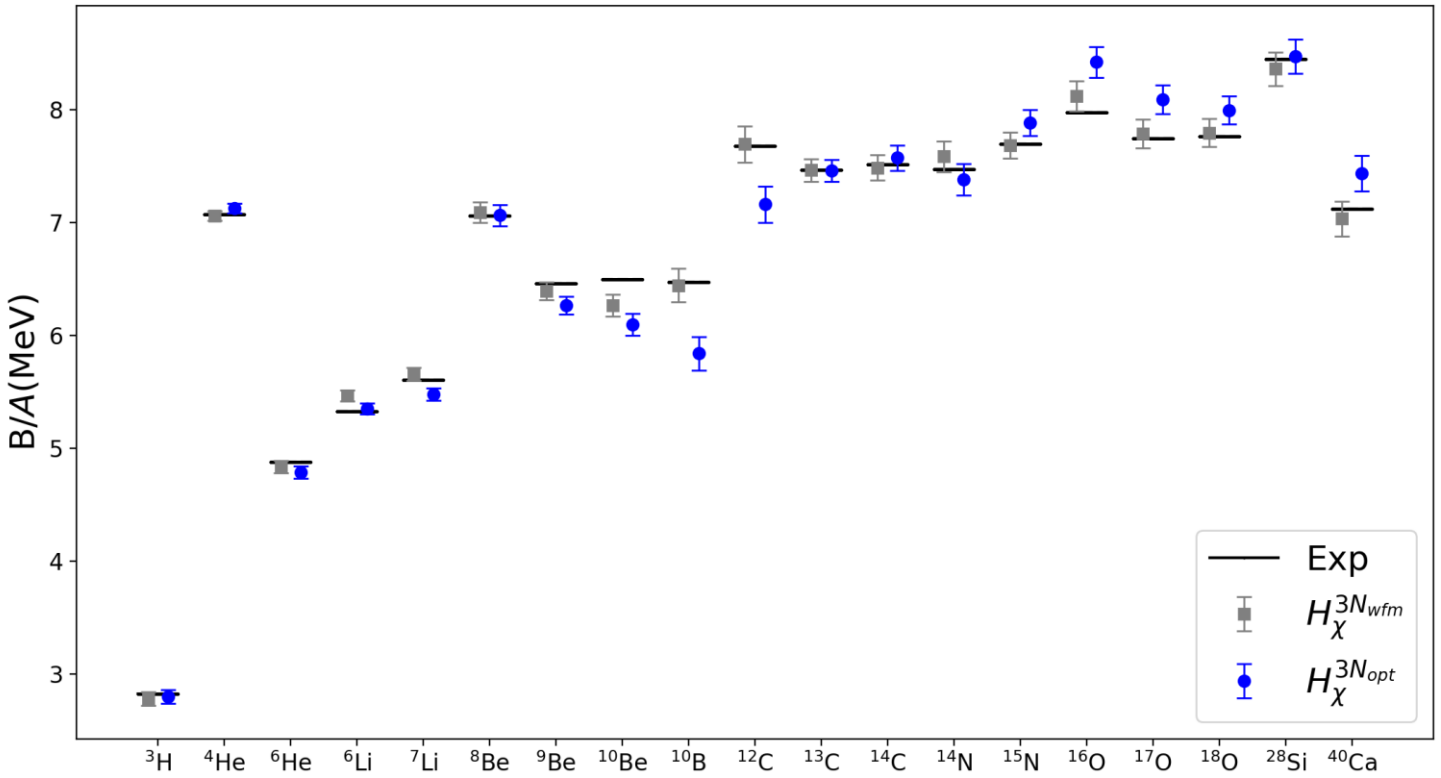
Constraint by μ and ΔE

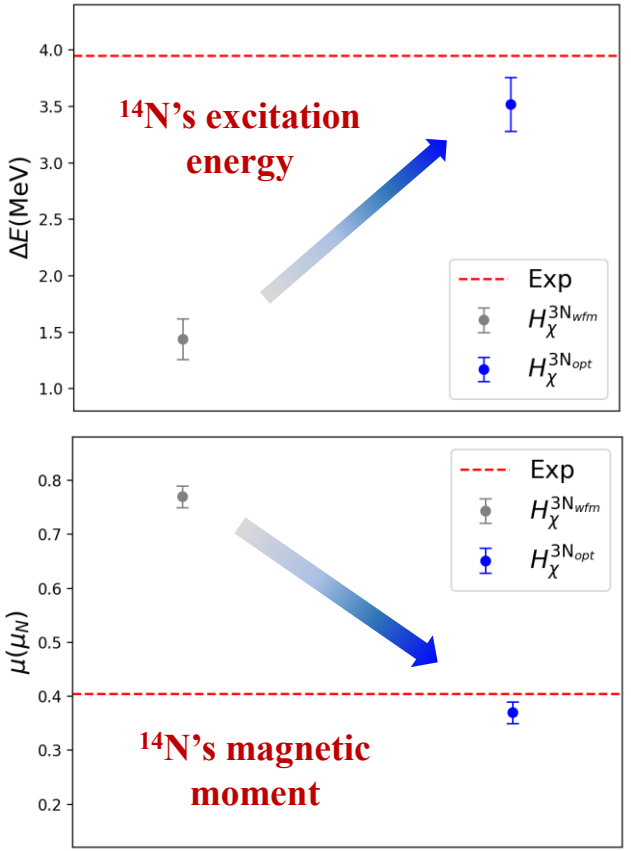
$V_{\mathrm{3N}}^{\mathrm{C}}$ central component

$V_{\mathrm{3N}}^{\mathrm{SO}}$ $V_{\mathrm{3N}}^{\mathrm{T}}$ Spin-orbital and tensor components

Nuclear property results

- The two N^{3}LO Hamiltonian both give acceptable results of nuclear energies, consistent with experimental values within 2~3σ for most nuclei.





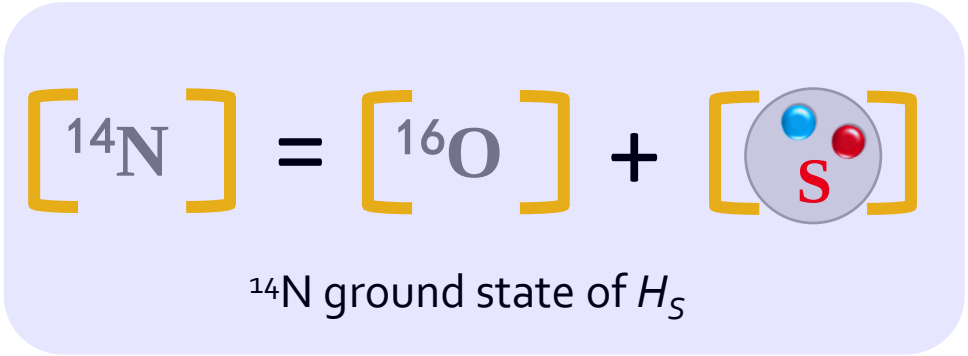
- On the other hand, the old Hamiltonian gives bad predictions on μ and ΔE.

Perturbative difficulty

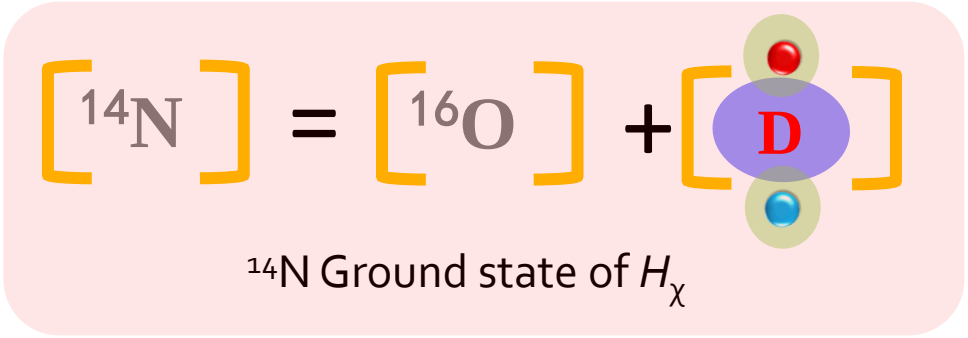
- We find the matrix element of H_S much larger than the experimental value!

$$M_{GT}(\mathrm{LO}) \approx 2.4 \qquad M_{GT}(\mathrm{exp}) \approx 0.002 \qquad M_{GT}(\mathrm{LO}) << M_{GT}(\mathrm{exp})$$

- The LO Hamiltonian misses essential correlations, whose ground state of ^{14}N is found to be dominated by S-wave.



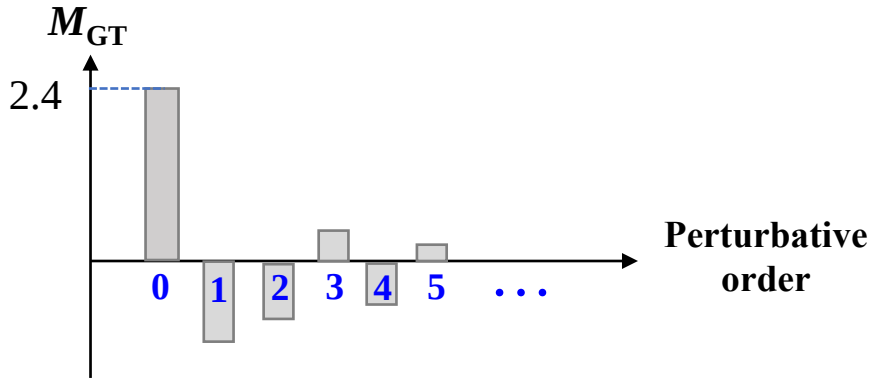
^{14}N ground state of H_S



^{14}N Ground state of H_χ

- Solving H_χ and expanding M_{GT} around the 'bad' wave function would greatly hamper the convergence of perturbative method.

Maybe even be divergent!



Improved perturbative method

- We try to systematically optimize the valence structure of the LO wave function, following the idea of degenerate perturbation theory.

  ▶ **Step 1:** construct optimized LO wave function using low-lying eigenstates of $\boldsymbol{H_S}$

$|\Psi_0\rangle,\ |\Psi_1\rangle,\ |\Psi_2\rangle,\ \cdots$

low-lying eigenstate of H_S



$$|\Phi_0\rangle = c_0|\Psi_0\rangle + c_1|\Psi_1\rangle + c_2|\Psi_2\rangle + \cdots$$

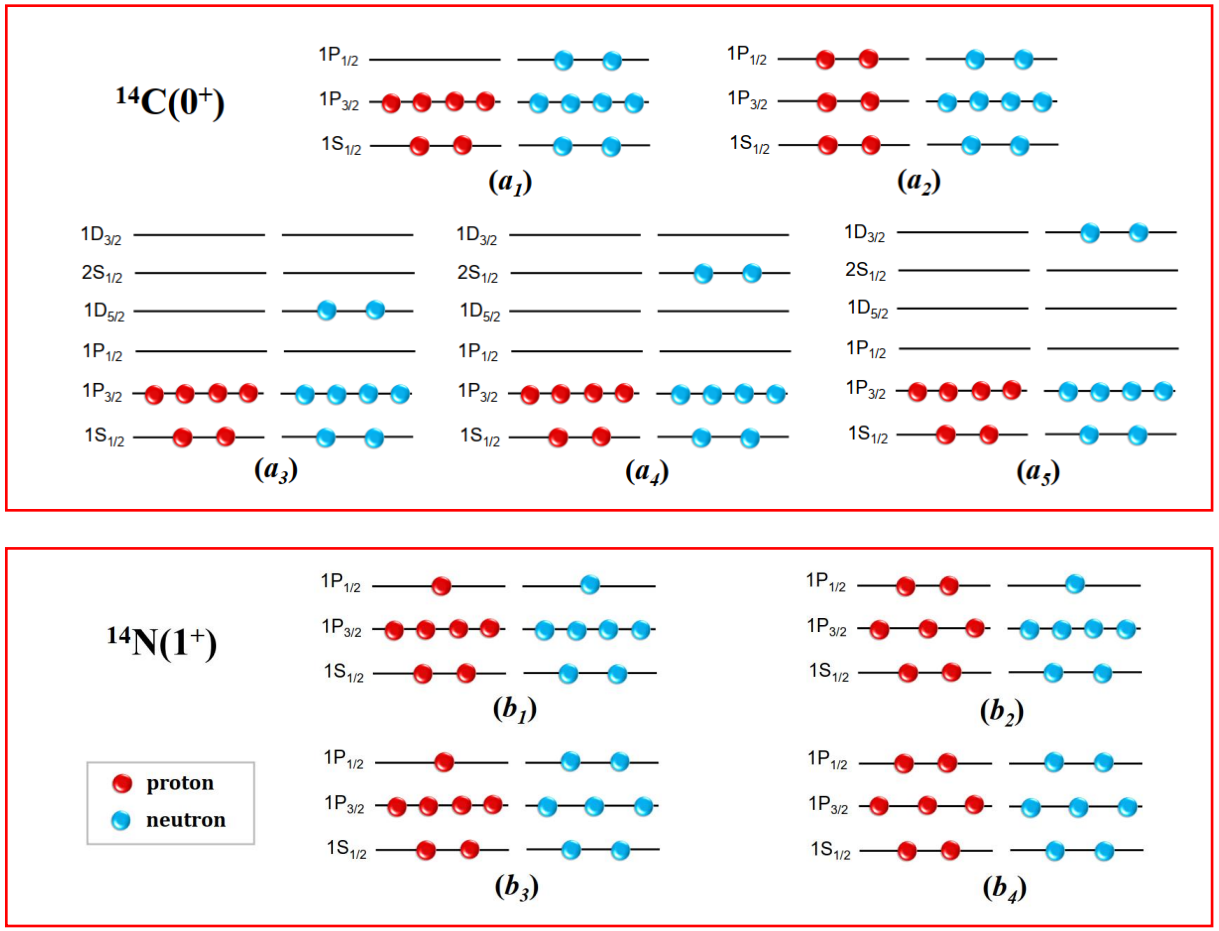
ground state of H_χ

  ▶ **Step 2:** determine the unknown coefficients $\{c_0, c_1, c_2, \ldots\}$ by calculating the variational minimum with respect to H_χ

$$\{c_0, c_1, c_2, \ldots\}_{\text{opt}} = \min_{\{\boldsymbol{c}\}} \frac{\langle\Phi_0(\boldsymbol{c})|H_\chi|\Phi_0(\boldsymbol{c})\rangle}{\langle\Phi_0(\boldsymbol{c})|\Phi_0(\boldsymbol{c})\rangle}$$

Numerical challenges

- This is an even more challenging multi-channel calculation compared to the previous ^{3}H and ^{6}He calculation.



Trial states for ^{14}C and ^{14}N

Number of correlation functions needed ×100

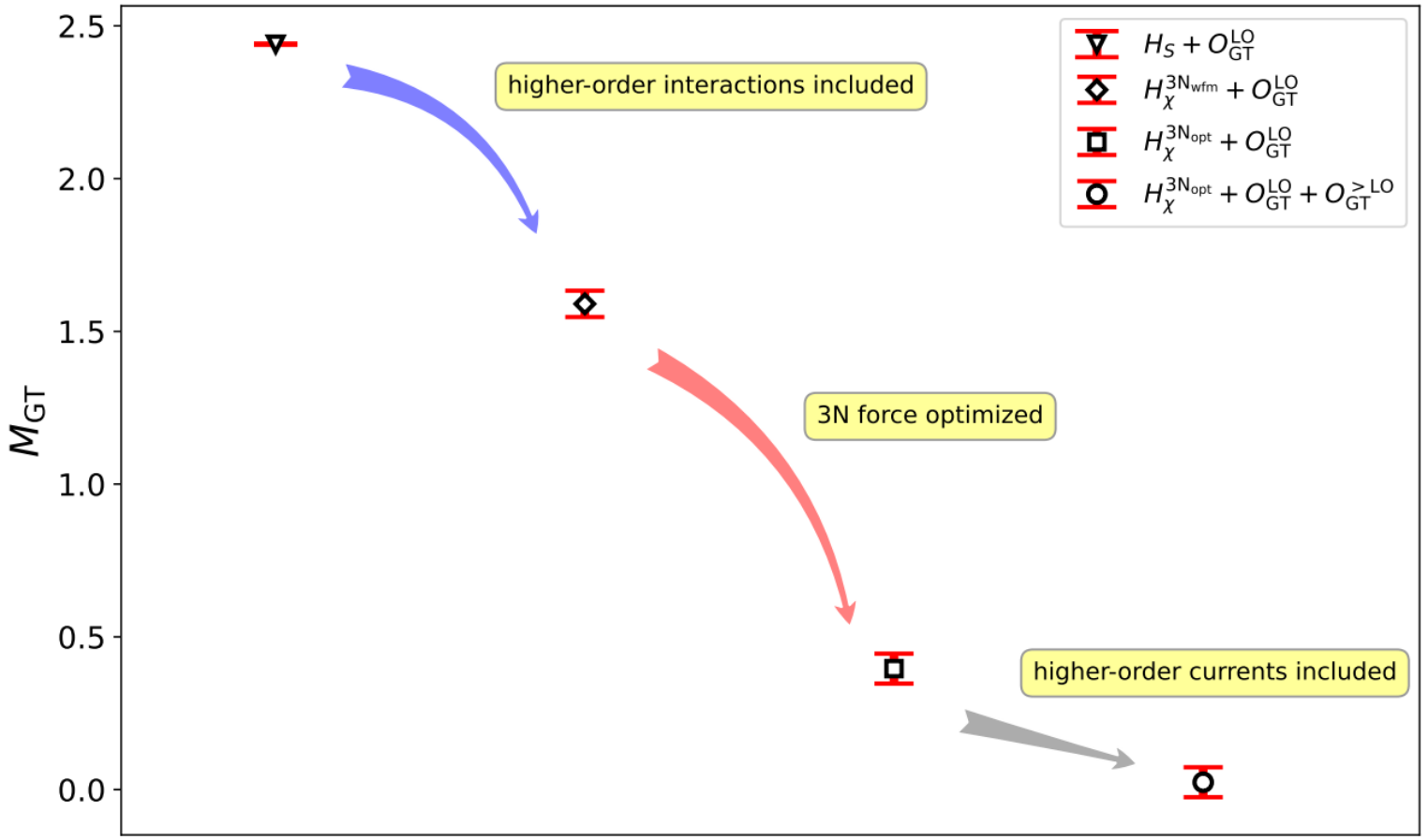
After a lot of hard works on

- ▶ **Sampling methods**
- ▶ **code structure**

Computation accelerated by 50~60 times!

How the interaction influences M_{GT}

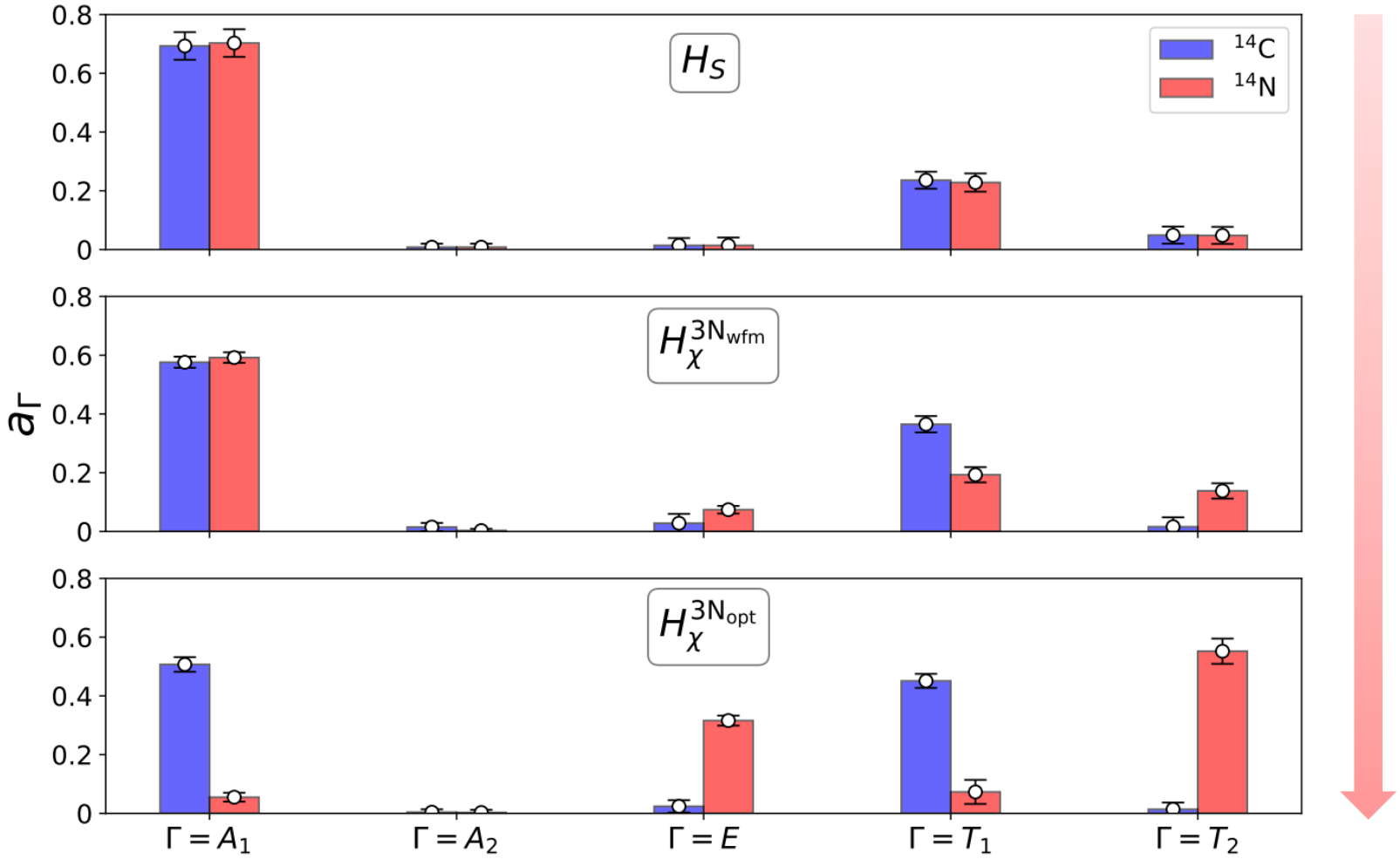
- With the systematic optimization of interactions, the matrix element gets closer to the experiment.
- 3N-force optimization is crucial for the quenching of he matrix element.
- Corrections of high-order currents are smaller, but essential for explaining the experiment.



How the interaction shapes nuclear structure

- We project the orbital wave functions of the initial and final nuclei on different irreducible representations of the cubic group, from which we gain the composition of different orbital angular momenta L。

^{14}C's wave function always occupied by ***L*=0/1**



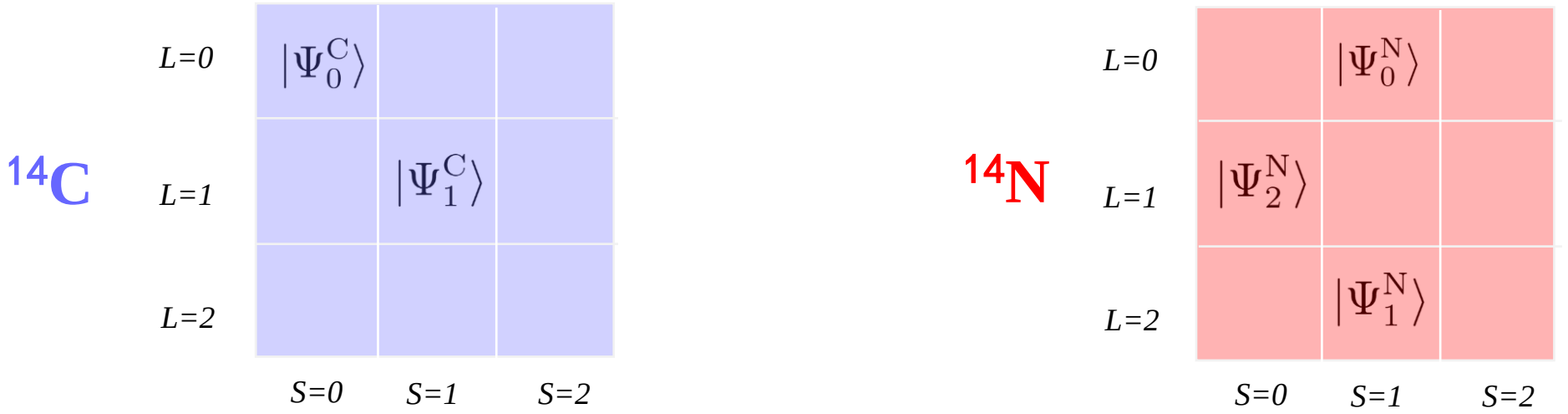
^{14}N's wave function gradually dominated by ***L*=2**

consistent with the shell model!

Duality between NLEFT and shell model

- As higher-order interactions, why do 3N forces influence the structure of ^{14}N so much?
- By projecting out different spin (S) and orbital angular momentum (L) components, we find that S and L are almost good quantum numbers for the low-lying eigenstates of $\boldsymbol{H_S}$

$$|\Phi_0\rangle = c_0|\Psi_0\rangle + c_1|\Psi_1\rangle + c_2|\Psi_2\rangle + \cdots$$



- Clearly, there is a strong duality between the shell model ansatz and the LEFT eigenstates

SM	$\|^{14}\mathrm{C}\rangle_{\rm SM} = x\|^1S_0\rangle + y\|^3P_0\rangle$	$\|^{14}\mathrm{N}\rangle_{\rm SM} = \alpha\|^3S_1\rangle + \beta\|^3D_1\rangle + \gamma\|^1P_1\rangle$
NLEFT	$\|^{14}\mathrm{C}\rangle_{\rm LAT} = \tilde{x}\|\Psi_0^{\rm C}\rangle + \tilde{y}\|\Psi_1^{\rm C}\rangle$	$\|^{14}\mathrm{N}\rangle_{\rm LAT} = \tilde{\alpha}\|\Psi_0^{\rm N}\rangle + \tilde{\beta}\|\Psi_1^{\rm N}\rangle + \tilde{\gamma}\|\Psi_2^{\rm N}\rangle$

Mechanism for ^{14}N's structure shaped by 3N forces

- Using the above duality, we determine the S-, P- and D-wave components of ^{14}N ground state.

$$\left(\overline{H_\chi}\right)_{k,k'} = \langle \Phi_k^{\mathrm{N}} | H_\chi | \Phi_{k'}^{\mathrm{N}} \rangle - \delta_{k,k'} \langle \Phi_0^{\mathrm{N}} | H_\chi | \Phi_0^{\mathrm{N}} \rangle$$

- If only 2N interactions are considered, the ground state of ^{14}N is dominated by S-wave.

$$\overline{H_{\chi,2\mathrm{N}}} = \begin{bmatrix} 0 & -1.32 & -4.46 \\ -1.32 & 1.98 & -4.44 \\ -4.46 & -4.44 & 16.31 \end{bmatrix} \begin{matrix} S \\ D \\ P \end{matrix} \quad (\mathrm{MeV})$$

$$S \qquad D \qquad P$$

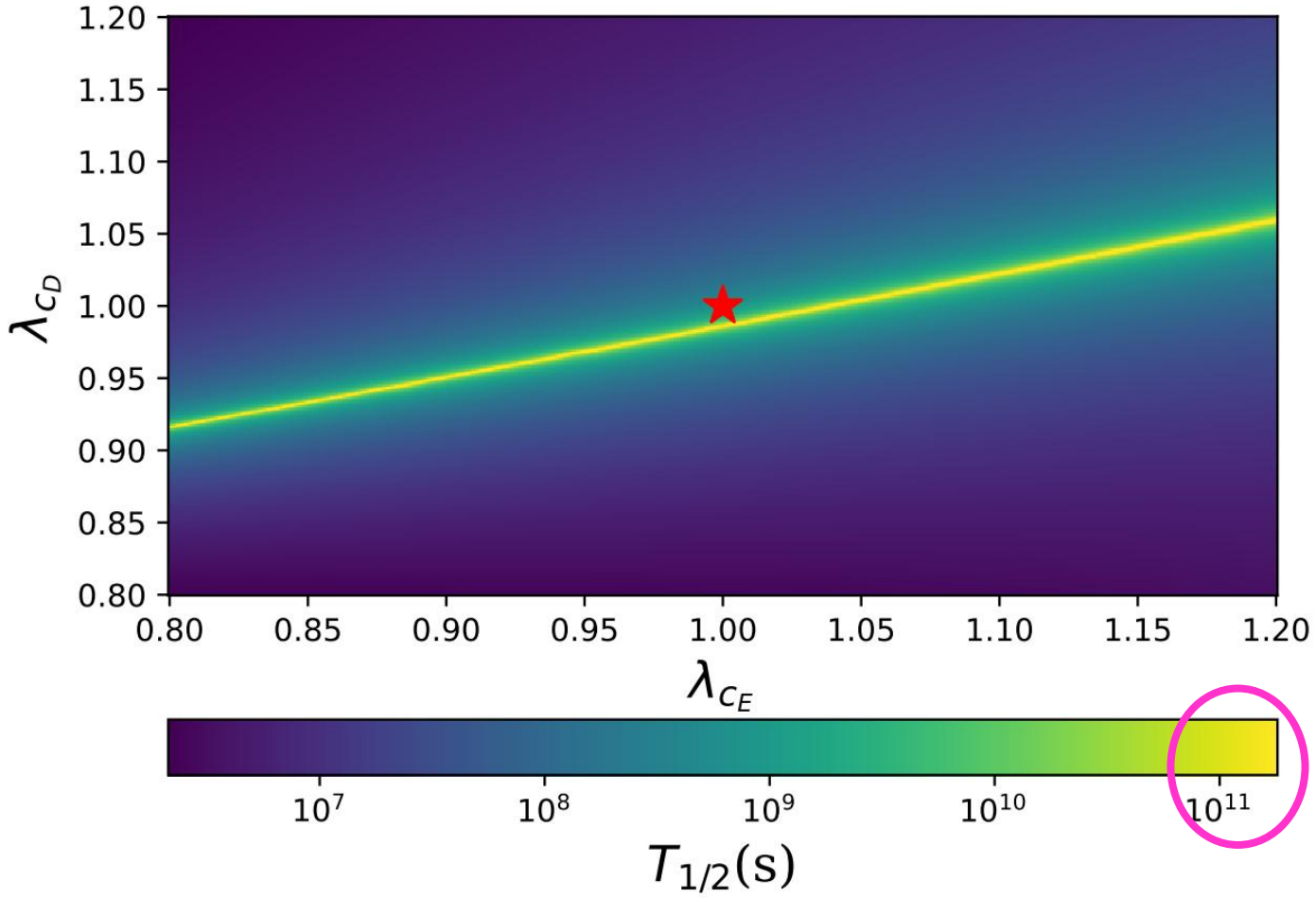
- For 3N's contribution, the D-wave term is lower than S-wave, which is further enhanced by 3N optimization, leading to the D-wave dominance.

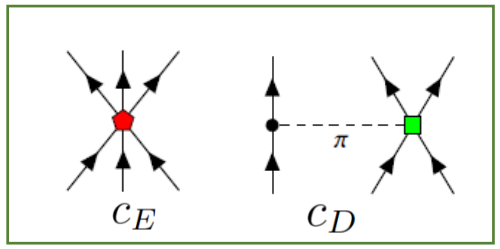
$$\overline{H_{\chi,3\mathrm{N}}}: \begin{bmatrix} 0 & 1.61 & 0.09 \\ 1.61 & -1.68 & 0.14 \\ 0.09 & 0.14 & 3.54 \end{bmatrix} \xrightarrow{\text{3N optimization}} \begin{bmatrix} 0 & 2.62 & 0.08 \\ 2.62 & -4.58 & 0.14 \\ 0.08 & 0.14 & 1.89 \end{bmatrix} \quad (\mathrm{MeV})$$

- **It is fine-tuning of 3N forces on the energy levels of different shell configurations that critically shapes the inner structure of ^{14}N and quenches M_{GT}**

Sensitivity of ^{14}C's lifetime to 3N forces

- To show how sensitive ^{14}C's lifetime is to the variation of 3N forces, we plot it out explicitly.





$$V_{3\mathrm{N}}^{(c_D)} \rightarrow \lambda_{c_D} V_{3\mathrm{N}}^{(c_D)}$$

$$V_{3\mathrm{N}}^{(c_E)} \rightarrow \lambda_{c_E} V_{3\mathrm{N}}^{(c_E)}$$

$T_{1/2}$（exp）=1.8×10^{11}s

Outlook

- Extend NLEFT studies of β-decay to heavier nuclei

  ➥ Provide ab initio input for astrophysical community

- Generalization to other rare/exotic weak transition processes

  ➥ Neutrinoless double β decays?

- Nuclear structure aspects

  ➥ Configuration mixing, many-body correlations, single-particle and collective motions…

Many thanks to my supervisor and collaborators:
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Thank you for your attention

Monte Carlo sampling

- Compared to nuclear property calculation, β-decay is a genuine two-channel process, making its Monte Carlo sampling more difficult. .

$$\langle O \rangle = \lim_{\tau\to\infty} \frac{\langle \Psi_T | e^{-H\tau/2} O e^{-H\tau/2} | \Psi_T \rangle}{\langle \Psi_T | e^{-H\tau} | \Psi_T \rangle}$$

$$\frac{\int \mathcal{D}\sigma\ e^{-\sigma^2/2} f_O[\sigma]}{\int \mathcal{D}\sigma\ e^{-\sigma^2/2} f_I[\sigma]}$$

$$P[\sigma] \propto e^{-\sigma^2/2} f_I[\sigma]$$

$$M_{\mathrm{GT}} = \lim_{\tau\to\infty} \frac{\langle \Psi_T^f | e^{-H\tau/2} \mathcal{J} e^{-H\tau/2} | \Psi_T^i \rangle}{\sqrt{\langle \Psi_i^f | e^{-H\tau} | \Psi_T^i \rangle \langle \Psi_i^f | e^{-H\tau} | \Psi_T^f \rangle}}$$

$$\frac{\int \mathcal{D}\sigma\ e^{-\sigma^2/2} f_{\mathcal{J}}^{(i\to f)}[\sigma]}{\sqrt{\left(\int \mathcal{D}\sigma\ e^{-\sigma^2/2} f_I^{(i)}[\sigma]\right)\left(\int \mathcal{D}\bar{\sigma}\ e^{-\bar{\sigma}^2/2} f_I^{(j)}[\bar{\sigma}]\right)}}$$

$$P[\sigma] \propto e^{-\sigma^2/2}(f_I^{(i)}[\sigma] + f_I^{(f)}[\sigma])$$

L	0	1	2	3	4
Γ	A_1	T_1	$E \oplus T_2$	$A_2 \oplus T_1 \oplus T_2$	$A_1 \oplus E \oplus T_1 \oplus T_2$

Table 1. Decompositions of orbital angular momentum $L \leq 4$ into irreps Γ of the octahedral group O.