Borromean nuclei: structure and reactions

Nuclear theory for astrophysics: Bridging bound state and continuum physics

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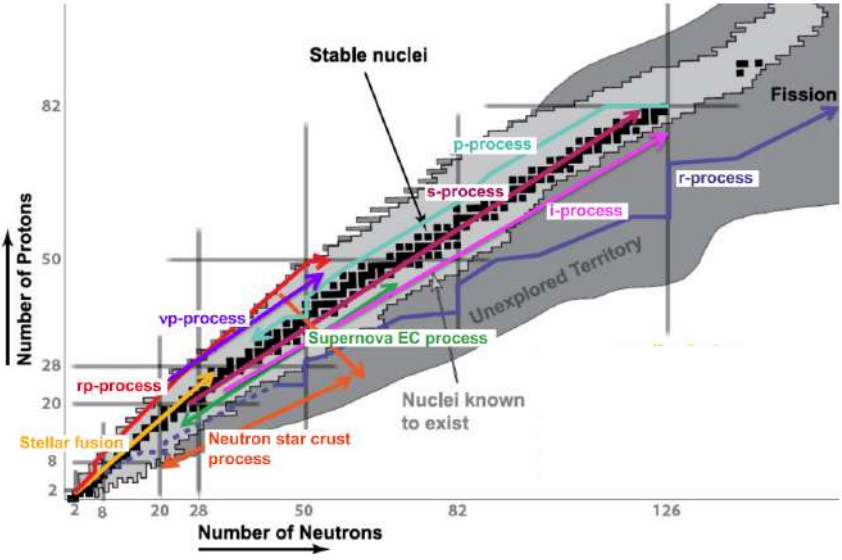
Facility for Rare Isotope Beams
Michigan State University

20/05/2026

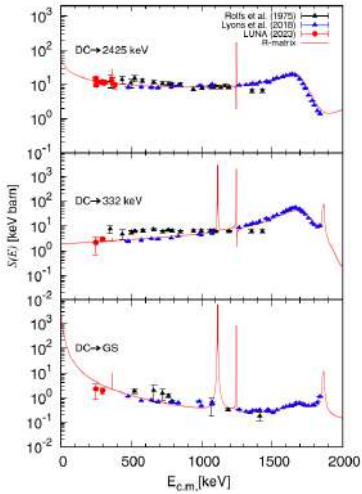




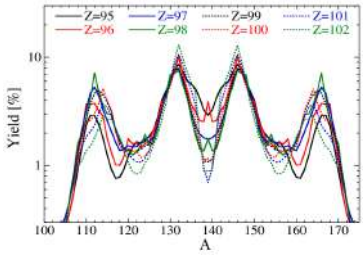


H. Schatz, J. Phys. G. 2022

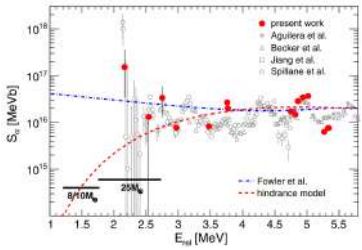
Reaction rates are the goal



Masha et al., Phys. Rev. C 2023

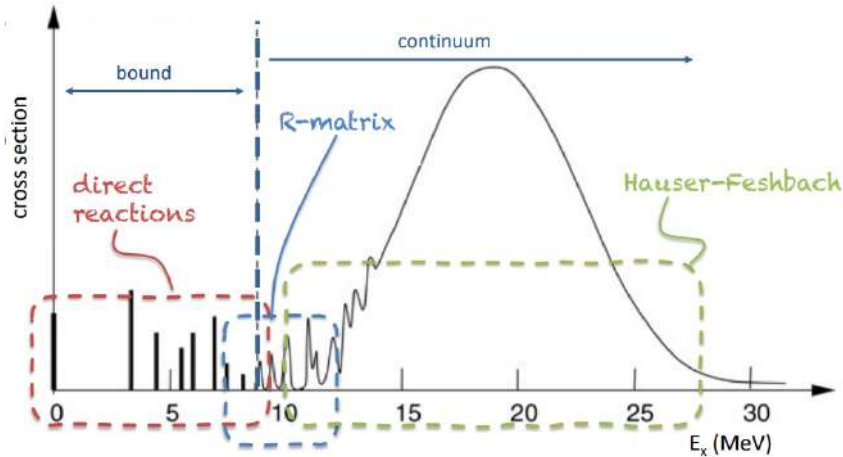


Goriely et al., Phys. Rev. Lett. 2013



Fruet et al., Phys. Rev. Lett. 2020

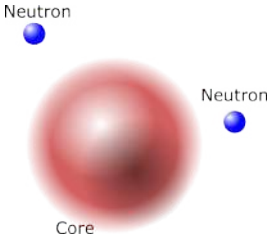
No model can predict everything

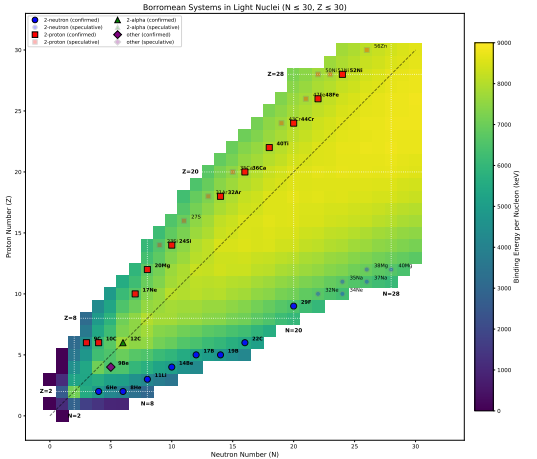


Slide from G. Potel Aguilar

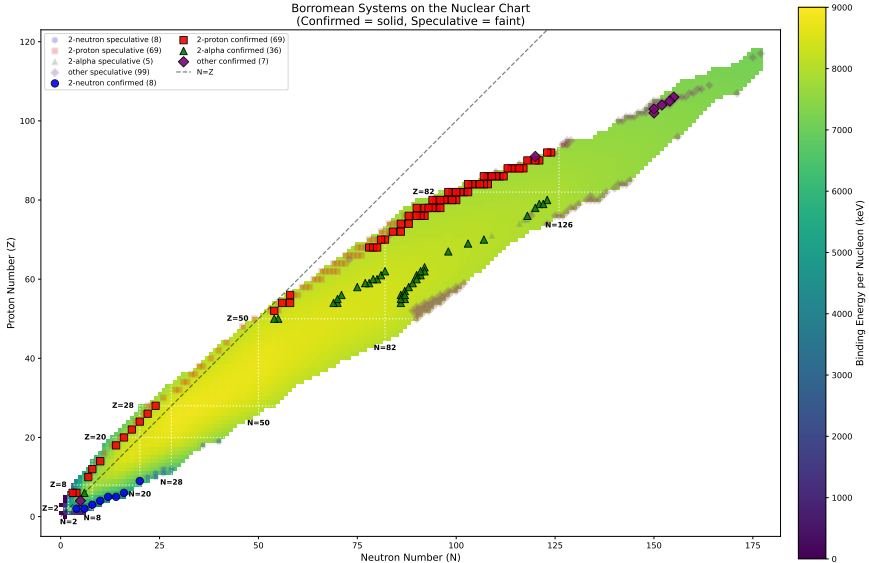
Clusterisation and few-body effects emerge at the extremes of binding

Extreme case: Borromean neutron halos e.g. ^{11}Li, ^{16}Be, ^{22}C, predicted to continue along neutron dripline.





How many Borromean nuclei are there?



Overview

1. Framework for calculating reaction rates involving Borromean nuclei
   - Two-neutron halo, different theoretical and experimental challenges
   - Description of calculable R-matrix approach
   - Case study of ^{22}C with Bayesian uncertainty quantification
2. Aerial survey of heavy Borromean nuclei
   - Molecular nuclei formed from halos
   - Challenges in describing cluster reactions in heavy nuclei

Why do we want to know about Borromean systems?



Reasons for interest in Borromean systems:

- Presence across nuclear chart including along r-process
- Universal properties of loosely-bound systems
- Importance of the continuum to calculate reaction rates
- Includes some systems of great astrophysical significance e.g. ^{12}C

Definition(s) of Borromean:
(A) The nucleus can be divided into three clusters c_n such that

1. There is a bound state of all three clusters
2. There is no bound state of any pair of clusters

(B) The continuum with the lowest energy threshold for breakup is a three-body continuum.

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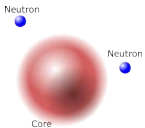
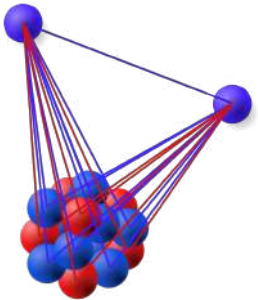
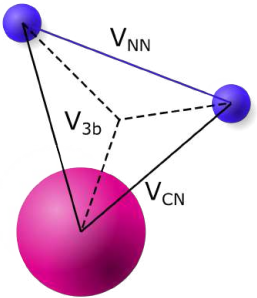


Two neutron halos:

- Insight into behaviour of neutron-rich systems
- Support for upcoming experimental campaigns (e.g. ^{40}Mg)
- Possibility of extension to increasing complexity (different clusters)

Few-body methods are ideal for clusterised systems





Few-body representation: effective interactions

V_{CN} is the mean-field of the core

How do theorists describe 2-neutron halos?

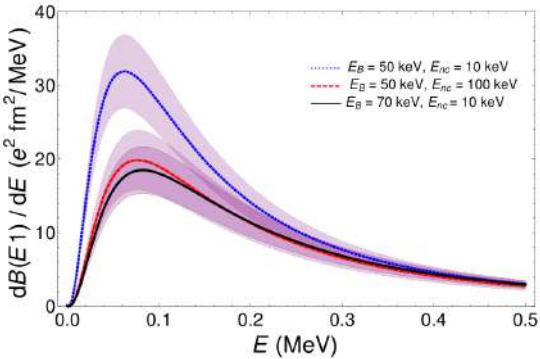


Figure from Acharya et al. 2016

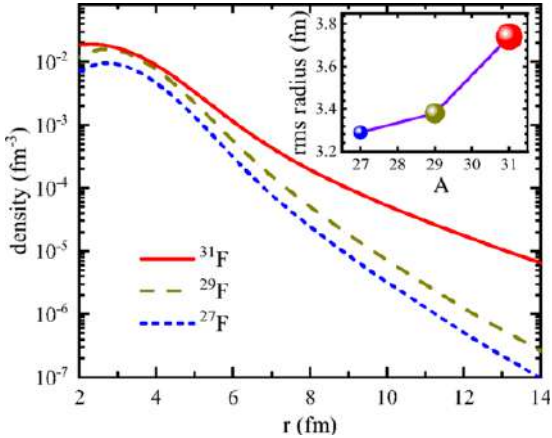


Figure from Michel et al. 2020

How have 2-neutron halos been studied?

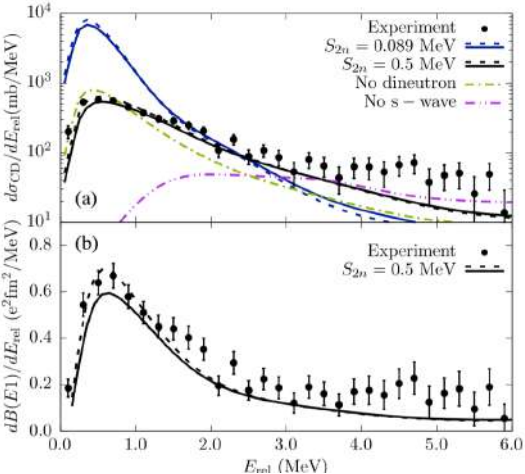
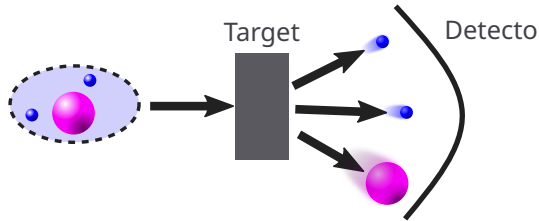


Figure from Cook et al. 2020



Experimental challenges

- Yields of neutron-rich isotopes limited.
- Low measurement efficiency due to neutron detection.
- Model-dependent extraction of dipole strength from cross-section.

Reaction observables can be calculated once we know about states

General expression for an observable describing anything* we care about

$$A = \langle \Psi_{scat} | \hat{A} | \Psi_{bound} \rangle$$

3-body model

*anything includes dipole strength, interaction with a target for breakup, knockout,...

What do we want from a three-body model?

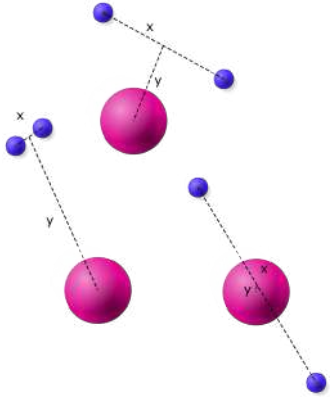
Wishlist:

- Bound **and** scattering states
- Non-resonant continuum
- Ability to extract detailed information about partial waves
- Scalable to larger, more complicated systems
- Fast, parallel code

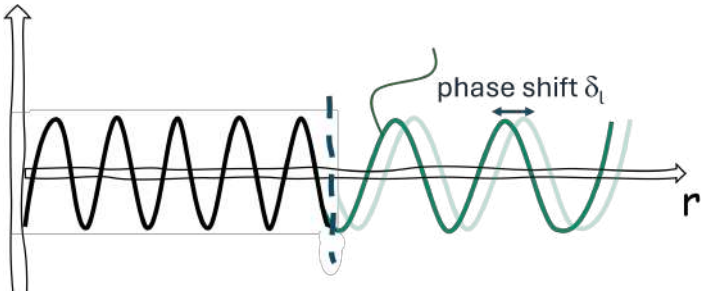
Three-body models have been developed before:

- Faddeev calculations for structure and quasi-bound states (*Thompson*, *Nunes*, *Danilin and Lazauskas*, *Carbonell*)
- 3-body CDCC using Gaussian basis and transformed harmonic oscillator wavefunctions (*Rodriguez-Gallardo*, *Casal*, *Arias*,...)
- Gamow-coupled channel approach using complex momenta (*Wang*, *Nazarewicz*, *Michel*, ...)
- Halo EFT approach based on low-energy constants (*Göbel*, *Hammer*, *Phillips*,...)

The three-body bound and scattering solutions are computed using the R-matrix and hyperspherical harmonics



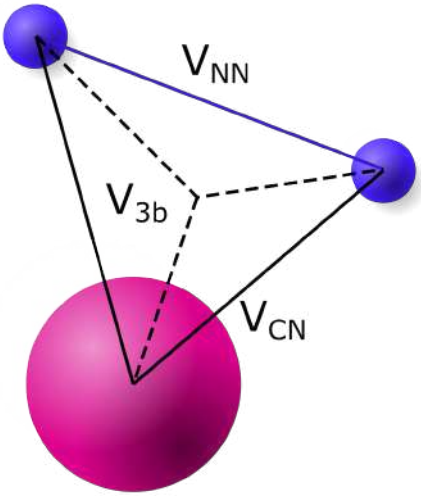
Channel radius in terms of $\rho = \sqrt{x^2 + y^2}$



Consistent treatment of the bound and scattering states → crucial for Borromean systems

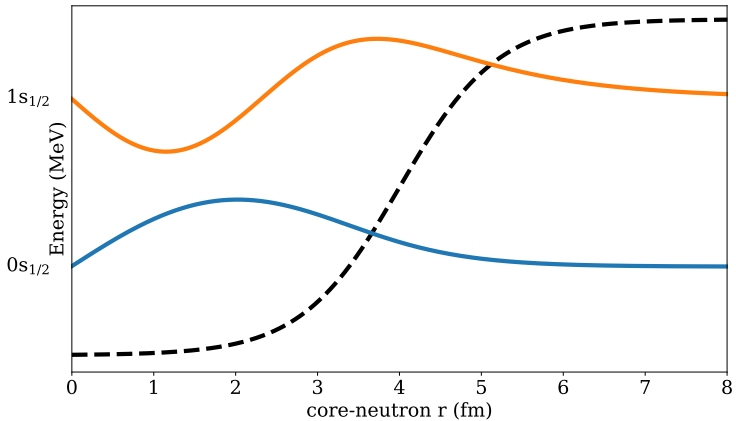
Descouvemont & Baye, Rep. Prog. Phys. 2010

What are the ingredients of our three-body Hamiltonian?



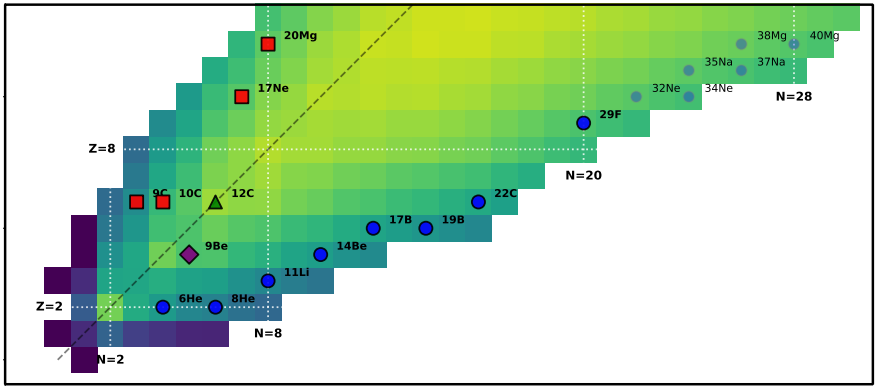
- Inert core (plan to extend to core deformation)
- Neutron-neutron potential: simple Minnesota interaction
- **Core-neutron potential:** this is where we can include structure information

How is structure information encoded in the interaction?



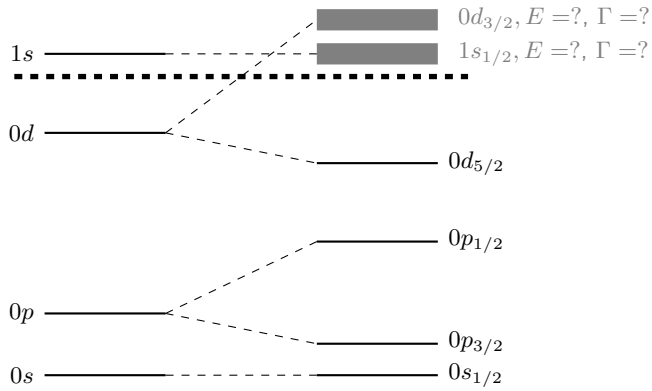
Core-neutron potentials contains deep bound states that are occupied by core nucleons, which will have to be removed.

^{22}C is a good example of what we can do



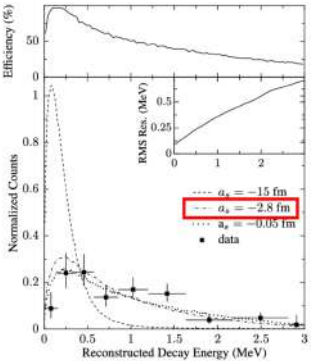
^{21}C: what structure information do we use?

- s-wave virtual state?
- d-wave resonance?
- spin-orbit splitting?

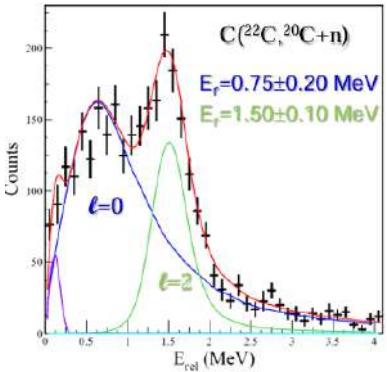


Shell model levels for n+^{20}C shown

Knockout and decay reactions tell us the structure of ^{21}C



Mosby et al. 2013: Measures ^{21}C scattering length with large uncertainty



Leblond et al. 2016: Measures energy of a d-wave resonance in ^{21}C.

Tension!

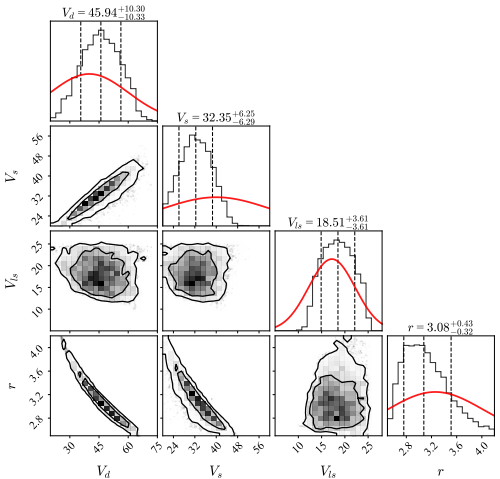
Conflicting measurements and interpretations $\rightarrow$ ^{22}C information can help resolve

We vary the core-neutron interaction parameters to reproduce the error in ^{21}C (^{20}C+n) data

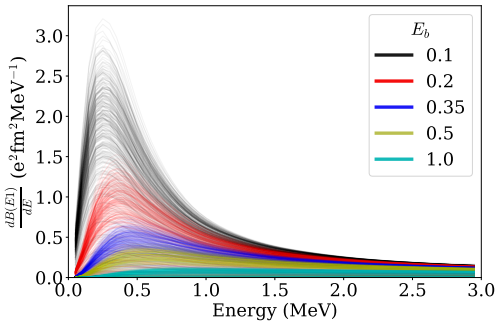
Bayesian study of two-body (core-neutron) interaction parameters

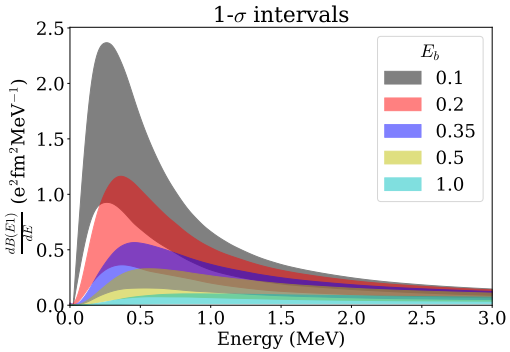
Interactions are Woods-Saxon plus spin-orbit in each partial wave.

- V_d (Woods-Saxon depth)
- V_s (Woods-Saxon depth)
- V_{ls} (Strength of surface spin-orbit)
- r (Woods-Saxon radius)
- The threshold energy value (S_{2n}) is poorly known: 0.42±0.94 MeV (AME2003) or -0.14±0.46 MeV (Gaudefroy et al. 2012): we test five values between 0.1 and 1.0 MeV.



By sampling, we can construct credible bands for the dipole strength function

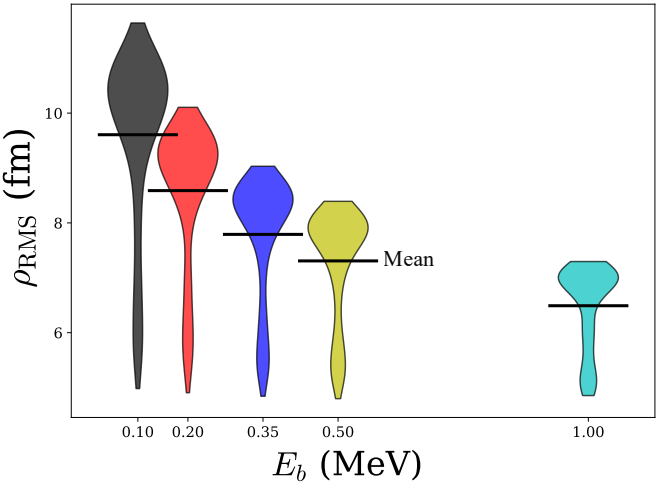




B(E1) uncertainty

The uncertainty on the dipole strengths still seems to be quite large, combining error from bound states and scattering states.

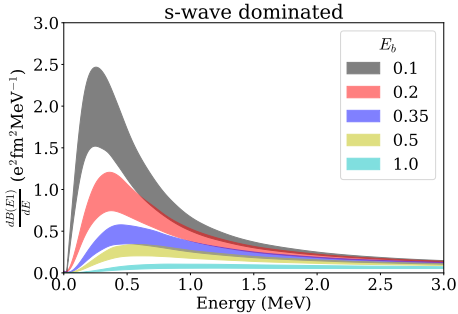
The binding energy influences the compactness of states



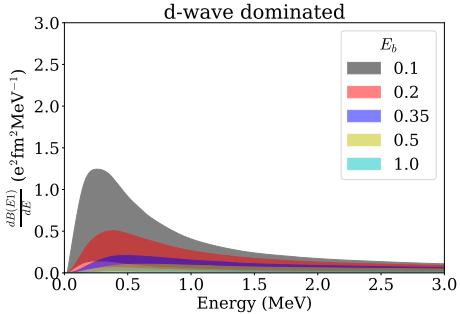
- Violin plot: 300 samples at each binding energy
- The trend of decreasing means is normal
- The bimodal structure is strange

Changing the three-body force adjusts the binding energy, scattering states stay the same.

If we separate out the shell effects, we can see behaviour more clearly

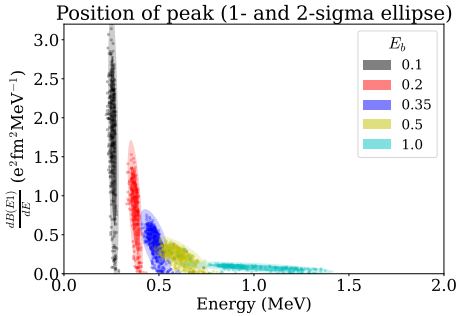


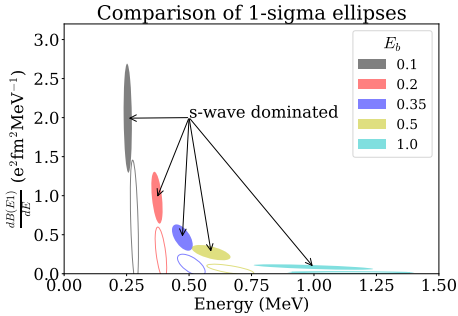
Strong sign of universality in dependence on binding energy



Note: low statistics make these 1-σ intervals unclear.

How much could we say with data?





Measurement of the peak of the $B(E1)$ distribution allows simultaneous information about the three-body binding energy and the two-body structure information.

Now we need some data to compare to

Bayesian exploration of potential parameters suggests that B(E1) measurements of ^{22}C are able to place further constraints on

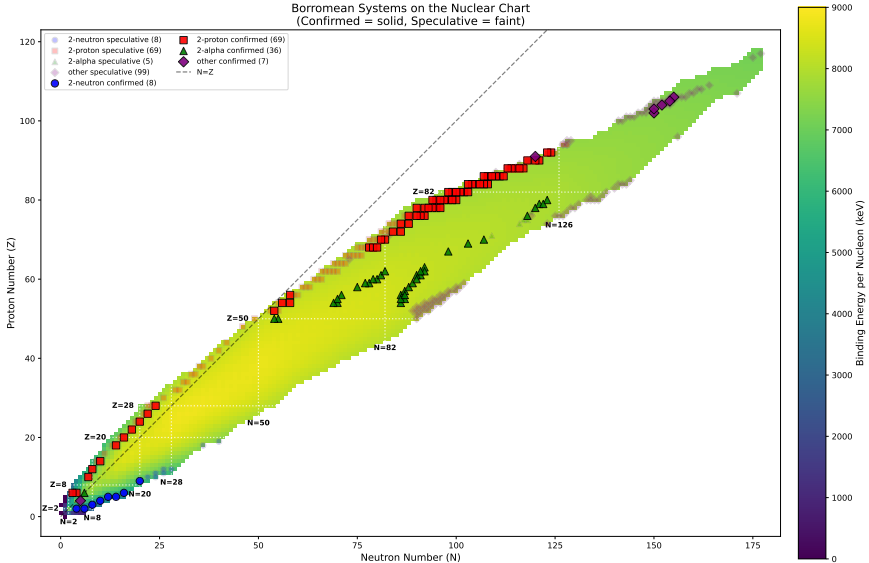
- The S_{2n} and partial-wave composition of ^{22}C
- The n+^{20}C scattering length
- The energy of the $d_{3/2}$ resonance in ^{21}C

This depends on a good extraction of B(E1) strength from breakup measurements.

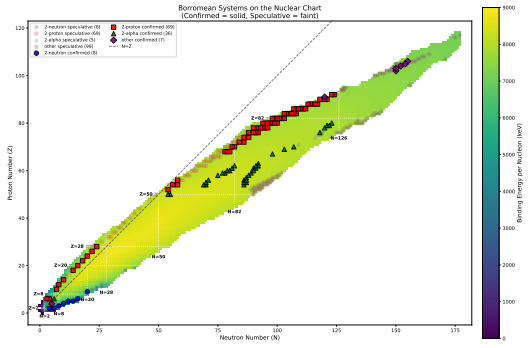
Continuing development

- Breakup calculations with eikonal model (in preparation)
- Looking at more Borromean systems
- Core excitation

There are heavy Borromean systems



There are different types of Borromean triple



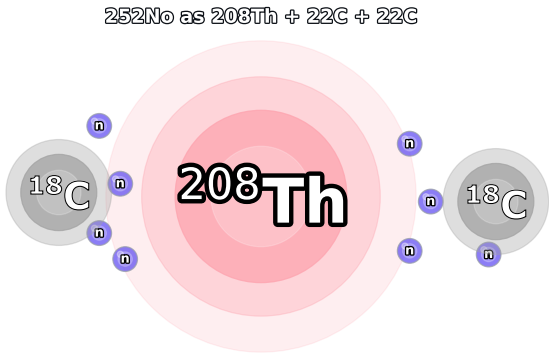
Type	Conf.	Spec.	Total
2-n	8	8	16
2-p	69	69	138
2-α	36	5	41
other	7	99	106
TOTAL	**120**	**181**	**301**

Light Clusters	Compound
$^{1}n + {}^{4}He$	^{9}Be
$^{11}Li + {}^{22}C$	^{211}Pa
$^{22}C + {}^{22}C$	^{252}No
$^{22}C + {}^{23}N$	^{253}Lr
$^{22}C + {}^{26}O$	^{256}Rf
$^{22}C + {}^{29}F$	^{259}Db
$^{17}B + {}^{29}F$	^{261}Sg

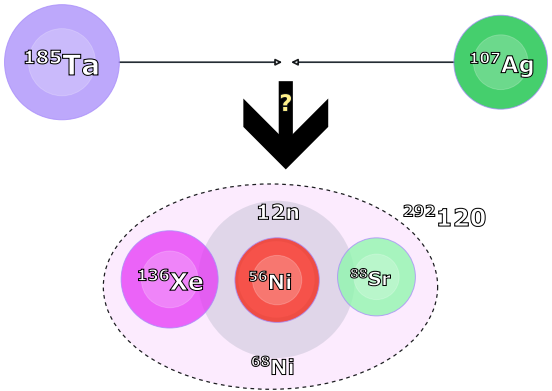
Many of the heavy examples could build molecular structures involving halo nuclei

Speculative other types:

Light Clusters	Freq.	Example Composites
^{40}Mg + ^{51}Ca	18	^{142}Te, ^{143}Te, ^{144}Te
^{43}Al + ^{50}Ca	18	^{143}I, ^{144}I, ^{145}I
^{37}Na + ^{52}Ca	15	^{141}Sb, ^{142}Sb, ^{143}Te
⋮	⋮	⋮
^{1}n + ^{4}He	8	^{208}Pt, ^{210}Au, ^{213}Hg
⋮	⋮	⋮

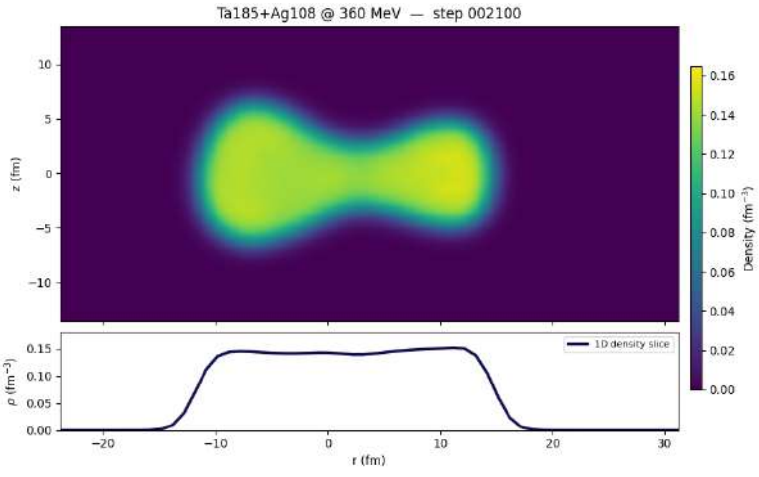


Current many-body approaches in the heavy region are often stuck with a mean-field approach



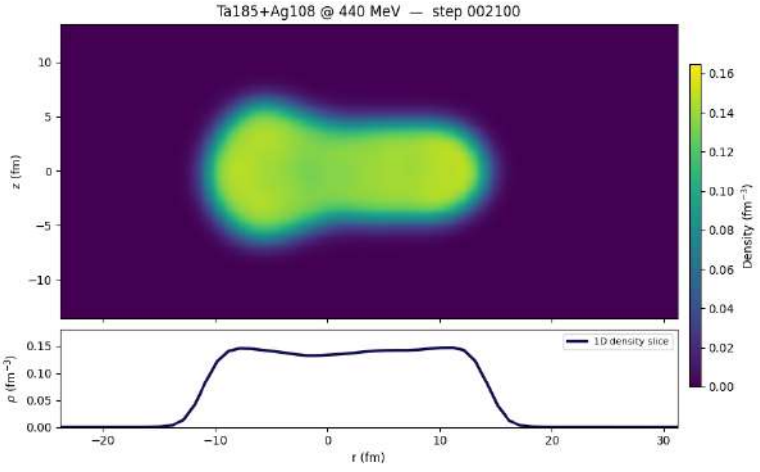
Optimal energy $E_{cm} \sim 375$ MeV (Coulomb barrier is $E_b = 348.5$ MeV).

TDHF doesn't explore the continuum



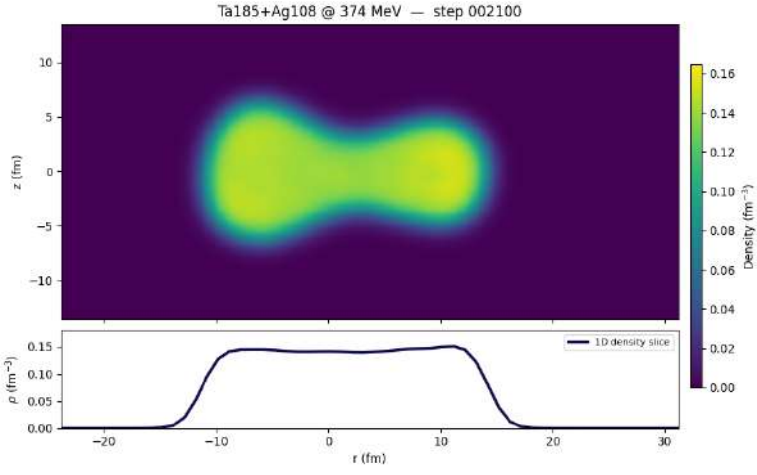
Below ideal energy

TDHF doesn't explore the continuum



00000056 Above ideal energy

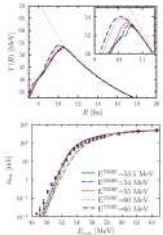
TDHF doesn't explore the continuum



Near ideal energy

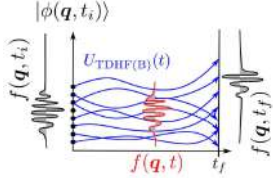
To model these kinds of reactions will require huge advancements in few- and many-body methods

Density-constrained Hartree-Fock calculations can give a real internuclear potential.



Scamps & Hashimoto, Phys. Rev. C, 2019

The generator coordinate method has been useful for fission calculations.



Verriere & Regnier, Front. Phys., 2020

Question

How can we compute the imaginary part of the internuclear (optical) potential?

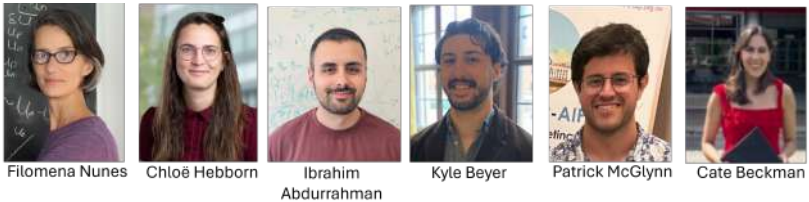
Question

How do we compute reactions that are not adiabatic?

Conclusions

- Reaction rates rely on precise and accurate calculations of structure and continuum properties of loosely bound nuclei.
- Only few-body methods offer exact* solubility.
- Few-body structures, including Borromean and Borromean-like systems are present throughout the nuclear chart, especially near driplines.
- With appropriate care taken to quantify uncertainty, we can make meaningful comparison to experimental data for light systems.
- Development is needed on both the few- and many-body sides to be able to build few-body models for heavy systems.

Acknowledgements



This work was supported in part by the CNRS through the AIQI-IN2P3 project.

Extra slides