Towards an effective theory of the nuclear shell model

In-medium two-body potential from functional renormalization group

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Nuclear theory for astrophysics: Bridging bound-states and continuum physics

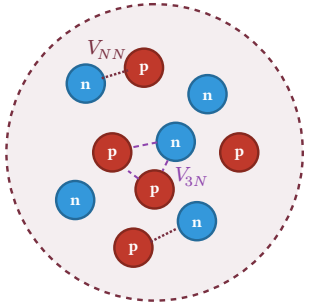
Limitations of ab initio theory and empirical shell model

Ab Initio

- Connected to QCD via Chiral EFT
  - High predictive power[1]
- Sensitive to details of nuclear forces in medium-mass and heavy nuclei[2]
  - Valence space decoupling approaches fall under ab initio

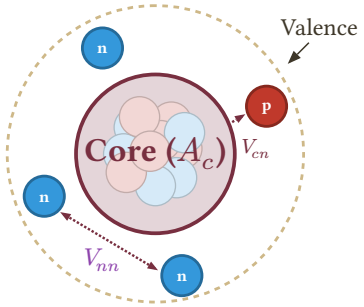
Empirical Shell model

- Effective core, fit of many-body data, no direct connection to QCD
- Precise where data available but limited predictive power beyond[3]



Microscopic Hamiltonian

$$H = T + V_{NN} + V_{3N} + \ldots \quad (1)$$



Shell Model Hamiltonian

$$H = T_n + V_{cn} + V_{nn} \quad (2)$$

Goal: we want to build an EFT of the nuclear shell model to have both precision and predictive power in a given mass range, and for large N/Z imbalance.

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[1] Machleidt and Entem, "Chiral Effective Field Theory and Nuclear Forces"; Bedaque and van Kolck, "Effective Field Theory for Few-Nucleon Systems".

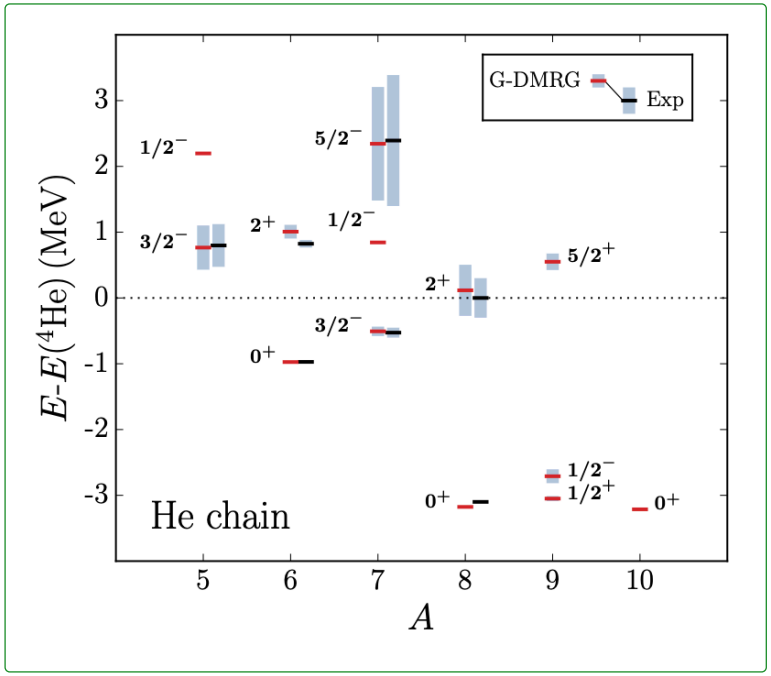
[2] Hergert, "A Guided Tour of \textit{Ab Initio} Nuclear Many-Body Theory".

[3] Brown, "The Nuclear Shell Model Towards the Drip Lines".

Motivation: physics of exotic nuclei

- Practical difficulties near drip lines
  - Scarce data, many active nucleons, and strong continuum couplings

- **Circumstantial evidence** for shell model EFT:
  1. SVD Analysis: shell-model interactions dominated by **few operators**[1]
  2. Contact + pion exchange $\longrightarrow$ reasonable **sd-shell spectra**[2]
  3. Single-parameter contact $\longrightarrow$ $^{5-10}$He spectra[3]



- Closest precedent: halo/cluster EFT (generalized pionless)
- **Problem**: no clean scale separation, we need **in-medium EFT**

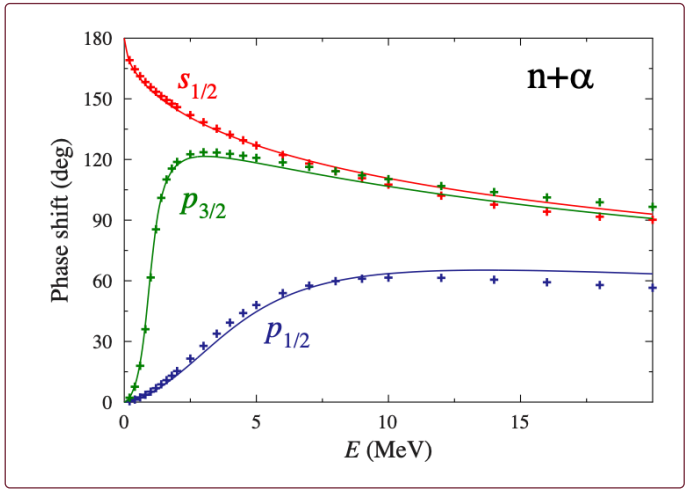
[1] Jaganathen et al., "Quantified Gamow Shell Model Interaction for { psd }-Shell Nuclei".

[2] Huth et al., "Shell-Model Interactions from Chiral Effective Field Theory".

[3] Fossez et al., "Energy Spectrum of Neutron-Rich Helium Isotopes: Complex Made Simple".

Modest roadmap to an EFT of the nuclear shell model

- Based on circumstantial evidence we believe the valence space interaction should look like a pionless EFT
- We want to start from nucleons and dimers as degrees of freedom, non-perturbatively introduce the core using **functional renormalization group (FRG)**



- For simplicity, we will assume:
  - a contact core-nucleon potential $V(r)$
  - smooth connection between in-medium and vacuum theories
  - only LO two-body terms at the moment (no LO three body terms)
  - In the future we want a core-nucleon that reproduces phase shift

See how a realistic low-energy NN interaction is modified in the presence of a core.

Functional renormalization group (FRG)

- An exact method to continuously and unitarily connect a high energy (UV) theory to a lower energy (IR) theory (similar to IMSRG in ab initio theory)
- The central ingredient is the effective action Γ_k which represents a coarse-grain version of full quantum action
- We want to evolve the effective action with certain degrees of freedom using a flow equation (Wetterich equation[1])

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \left[\frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k} \right] \tag{3}$$

  - $t \equiv \ln(k/\Lambda)$: flow parameter
  - STr: supertrace over every indices
  - $\Gamma_k^{(2)}$: second derivative with respect to each d.o.f.
  - R_k: regulator suppressing modes below k[2]

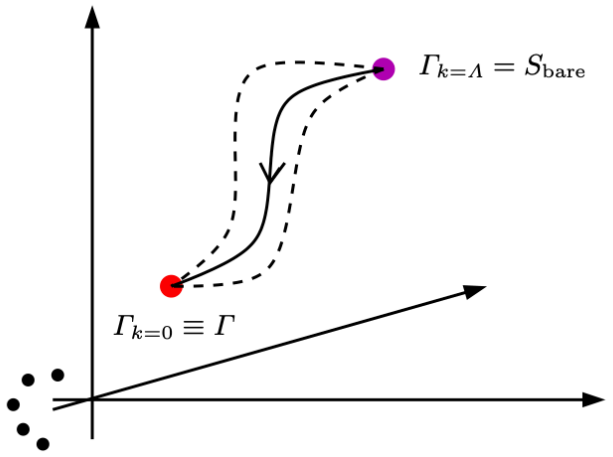


Figure 1: visualization of the flow in FRG[3]

[1] Wetterich, "Exact Evolution Equation for the Effective Potential".

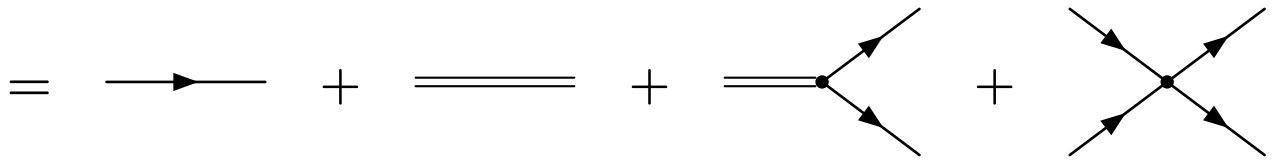
[2] suppressing low energy modes result in survival of lower energy modes after Legendre transformation to obtain the effective action from the action

[3] Gies, "Introduction to functional RG and applications to gauge theories"

Dimer truncation and effective potential

- Pionless-like truncation with dimer[1]

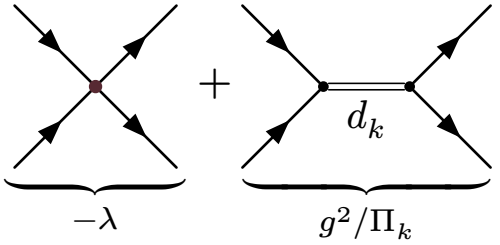
$$\Gamma_k\left[\psi,\psi^\dagger,d_k,d_k^\dagger\right]=\int d^4x\left[\psi^\dagger\left(i\partial_0-\frac{\nabla^2}{2m}\right)\psi+d_k^\dagger\Pi_k d_k\right.$$
$$\left.+g\left(\psi\hat{P}\psi\right)^\dagger d_k+gd_k^\dagger\left(\psi\hat{P}\psi\right)+\lambda\left(\psi\hat{P}\psi\right)^\dagger\left(\psi\hat{P}\psi\right)\right] \tag{4}$$



Effective potential

Integrating out d_k at the saddle point gives an exact relation for two-body effective potential:

$$V_{\text{eff}}(P_0,\boldsymbol{P})=-\lambda+\frac{g^2}{\Pi_k(P_0,\boldsymbol{P})}= \tag{5}$$



- The full interaction has two contributions: **direct** contact λ and **dimer-mediated** g^2/Π_k
- Near unitarity λ diverges $\rightarrow$ need to remove it from the flow

[1] Birse, "Functional Renormalization Group for Two-Body Scattering".

Dimer and dynamic bosonization

Since λ diverges, we need to remove it throughout the RG flow:

1. **Static dimer**: removes diverging λ at one point near unitarity
2. **Problem**: one RG step regenerates λ via a fermion loop — λ comes back
3. **Dynamic dimer** (Floerchinger & Wetterich): absorb the regenerated λ at every step by making d_k scale-dependent

- Result at the fixed point $\partial_t \lambda = \partial_t g = 0$: $\quad \lambda = 0$
- Effective potential simplifies to

$$V_{\text{eff}}(P_0, \boldsymbol{P}) = \frac{g^2}{\Pi_k(P_0, \boldsymbol{P})} \tag{6}$$

Flow identity

- At fixed point $\partial_t \lambda = \partial_t g = 0$, all running of V_{eff} lives in Π_k:

$$\frac{\partial_t V_{\text{eff}}}{V_{\text{eff}}} = -\frac{\partial_t \Pi_k}{\Pi_k} \tag{7}$$

Recovery of power countings

- At $P_0 = \boldsymbol{P} = 0$, the flow equation from dimer propagator in vacuum[1] will be

$$\nu(k) \equiv \frac{\partial_t V_{\text{eff}}}{V_{\text{eff}}} = -\frac{4k}{4k - 3\pi/a}. \tag{8}$$

- The power counting will be $V_{\text{eff}} \propto k^\nu$ at constant ν.
- At $k|a| \ll 1$, $\nu \to 0$:
  recovers Weinberg power counting[2]
- At $k|a| \gg 1$, $\nu \to -1$:
  recovers Kaplan-Savage-Wise (KSW) power counting[3]
- *Functional RG can recover both power countings in LO and connect them into one picture.*

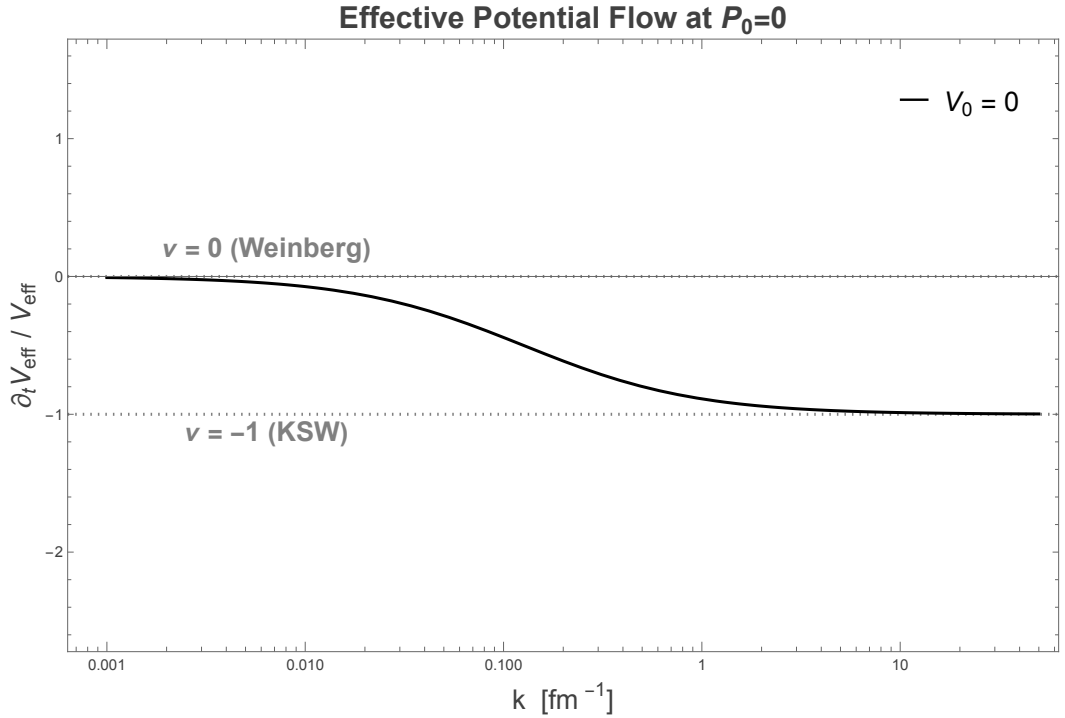


Figure 2: Vacuum effective potential flow in NN scattering channel ($a \sim -18.7$ fm)

[1] Birse, "Functional Renormalization Group for Two-Body Scattering".

[2] Weinberg, "Nuclear Forces from Chiral Lagrangians"; Weinberg, "Effective Chiral Lagrangians for Nucleon-Pion Interactions and Nuclear Forces".

[3] Kaplan et al., "A New Expansion for Nucleon-Nucleon Interactions".

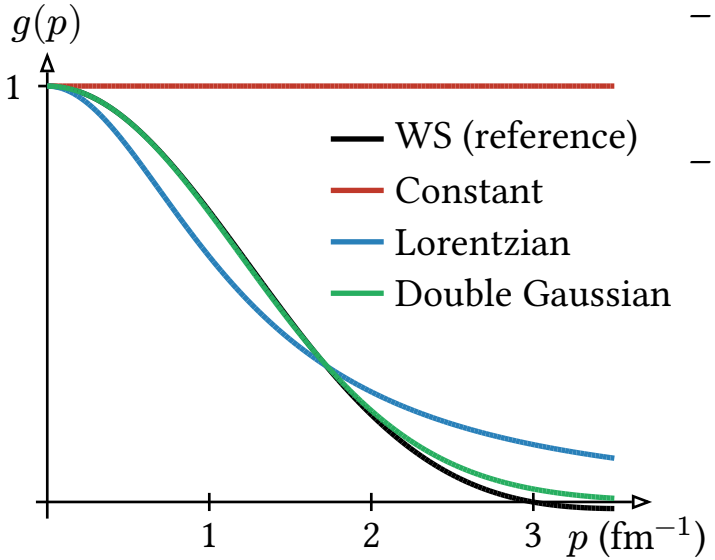
In-medium regulator

- We put the effect of core through the regulator

$$R_k(p) = R_k^{\mathrm{std}}(p) + R_k^{\mathrm{c}}(p) \tag{9}$$

- $R_k^{\mathrm{std}}(p)$: standard regulator for FRG, in this presentation, Litim regulators.
- $R_k^{\mathrm{c}}(p)$: extra term interpolates **vacuum theory at UV** and **in-medium theory at IR**
  - Why R_k^{c}?: Better for interpolation, calculation is easier, and non-local effect can be implemented.
  - Ansatz of $R_k^{\mathrm{c}}(p)$:

$$R_k^{\mathrm{c}}(p) = V_0 g(p)(1 - f(k)) \tag{10}$$



  - $f(k)$: Interpolating between vacuum theory at $k = \Lambda$ and in-medium theory at $k = 0$
  - Setting of $g(p)$
    - Contact ($g(p) = 1$) : contact-like interaction to core. (this presentation)
    - Lorentzian ($g(p) = p_F^2/(p^2 + p_F^2)$) : naive, analytically tractable approximation of Woods-Saxon potential (future)
    - Double gaussian ($g(p) = Ae^{-p^2/a^2} - Be^{-p^2/b^2}$) : More accurate approximation of Woods-Saxon potential. (future)

Three regimes from one equation

- With the in-medium regulator, we can calculate $\nu = \partial_t V_{\text{eff}} / V_{\text{eff}}|_{P_0=\boldsymbol{P}=0}$

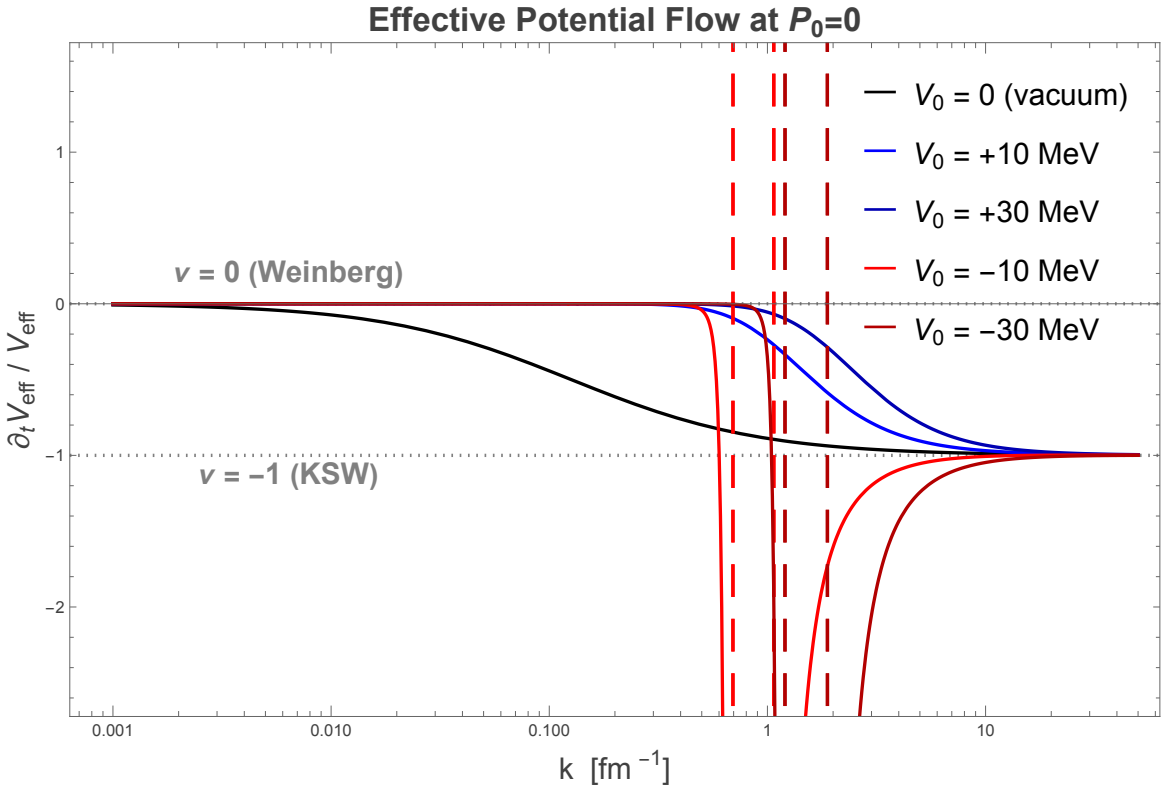


Figure 3: plot of $a_{\text{nn}} \sim -18.7$ fm channel at $P_0 = \boldsymbol{P} = 0$

- **Vacuum**:
  - for $a < 0$, continuously interpolating the KSW and Weinberg limit.
  - for $a > 0$, gives pole at $k_\star = 3\pi/(4a)$, recovering Birse's result
- **Repulsive**: shifted inverse propagator, pole is suppressed.
- **Attractive**: new critical scale k_c where $k_c^2 + \beta_{k_c} = 0$

Inclusion of P_0

After renormalization, the inverse propagator can be written as

$$\Pi_k(P_0) = F_k(P_0) + \frac{\pi}{2a}. \tag{11}$$

Here we keep $\boldsymbol{P} = 0$ and study the energy dependence through nonzero P_0.

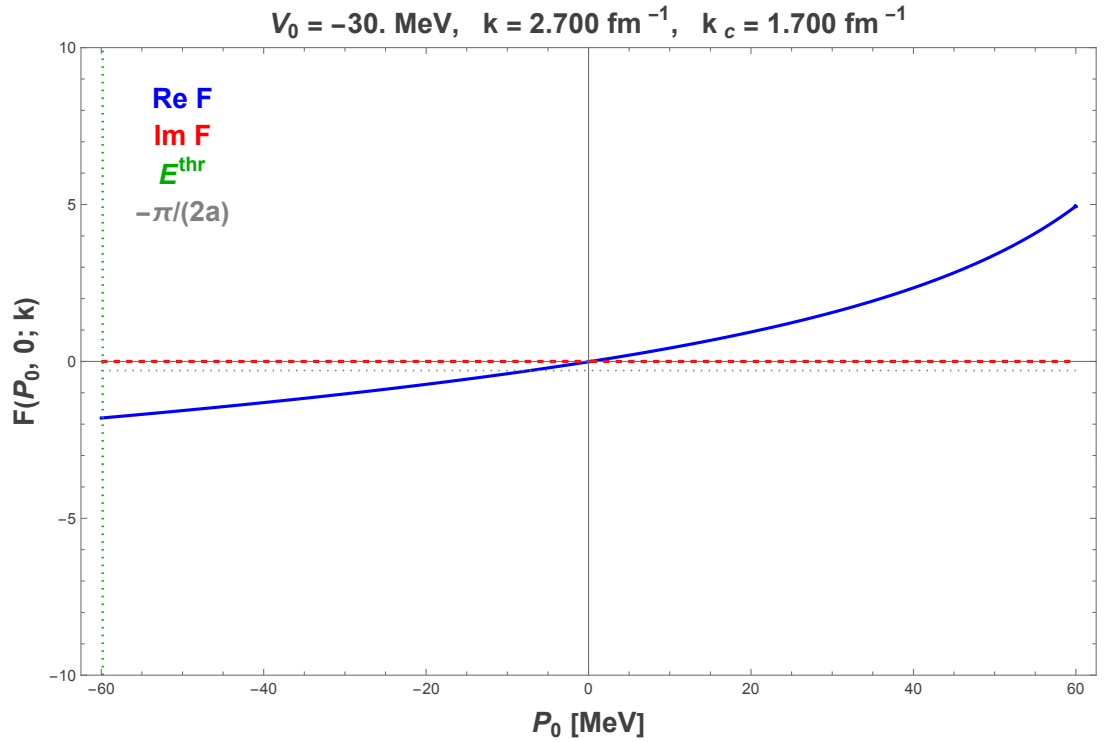


Figure 4: $F_k(P_0)$ in P_0 axis. Only LO matching applied.

At large k, $F_k(P_0)$ is almost smooth and weakly energy-dependent.

Inclusion of P_0

After renormalization, the inverse propagator can be written as

$$\Pi_k(P_0) = F_k(P_0) + \frac{\pi}{2a}. \tag{12}$$

Here we keep $\boldsymbol{P} = 0$ and study the energy dependence through nonzero P_0.

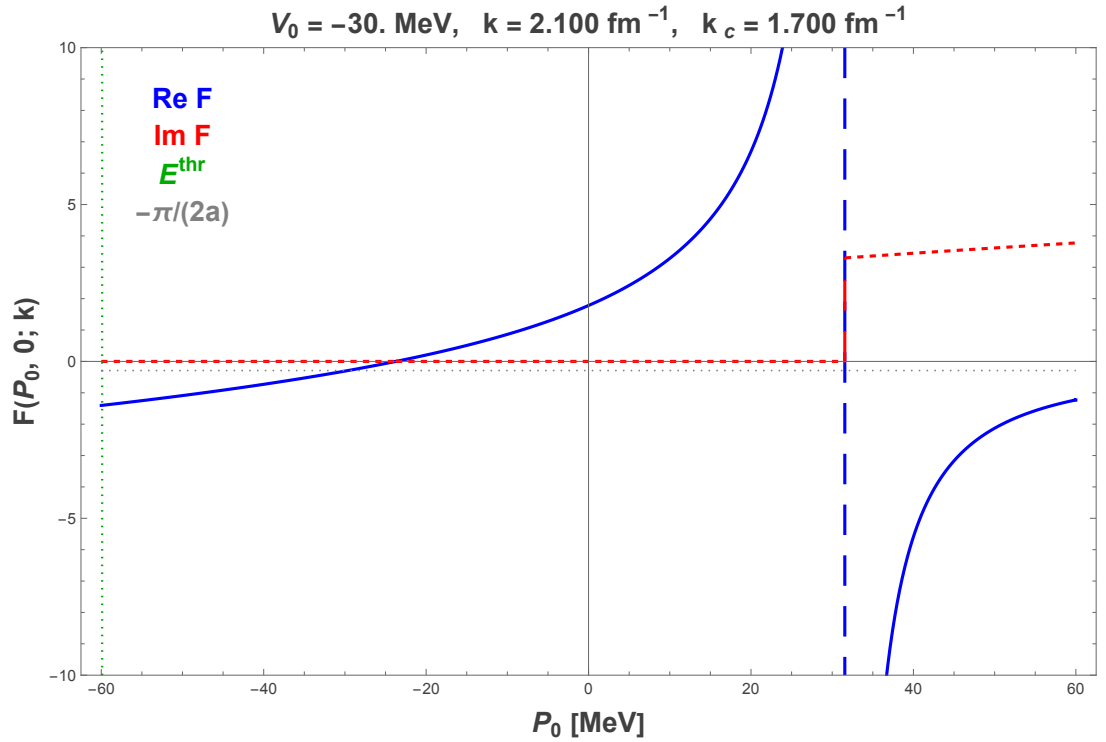


Figure 5: $F_k(P_0)$ in P_0 axis. Only LO matching applied.

As k approaches k_c, a diverging structure in Re F_k approaches to $P_0 = 0$. Im F_k emerging from the pole structure $i\varepsilon$.

Inclusion of P_0

After renormalization, the inverse propagator can be written as

$$\Pi_k(P_0) = F_k(P_0) + \frac{\pi}{2a}. \tag{13}$$

Here we keep $\boldsymbol{P} = 0$ and study the energy dependence through nonzero P_0.

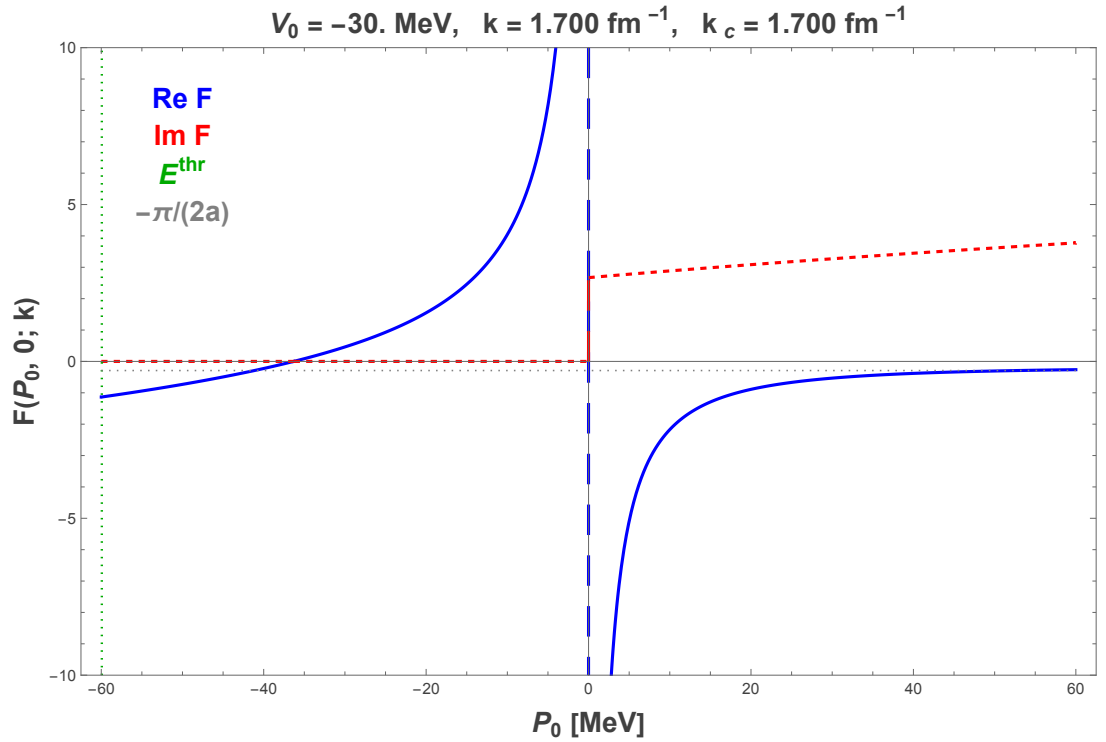


Figure 6: $F_k(P_0)$ in P_0 axis. Only LO matching applied.

Near $k_c = 1.7\text{fm}^{-1}$, the deformation becomes strongest around small positive P_0.

Inclusion of P_0

After renormalization, the inverse propagator can be written as

$$\Pi_k(P_0) = F_k(P_0) + \frac{\pi}{2a}. \tag{14}$$

Here we keep $\boldsymbol{P} = 0$ and study the energy dependence through nonzero P_0.

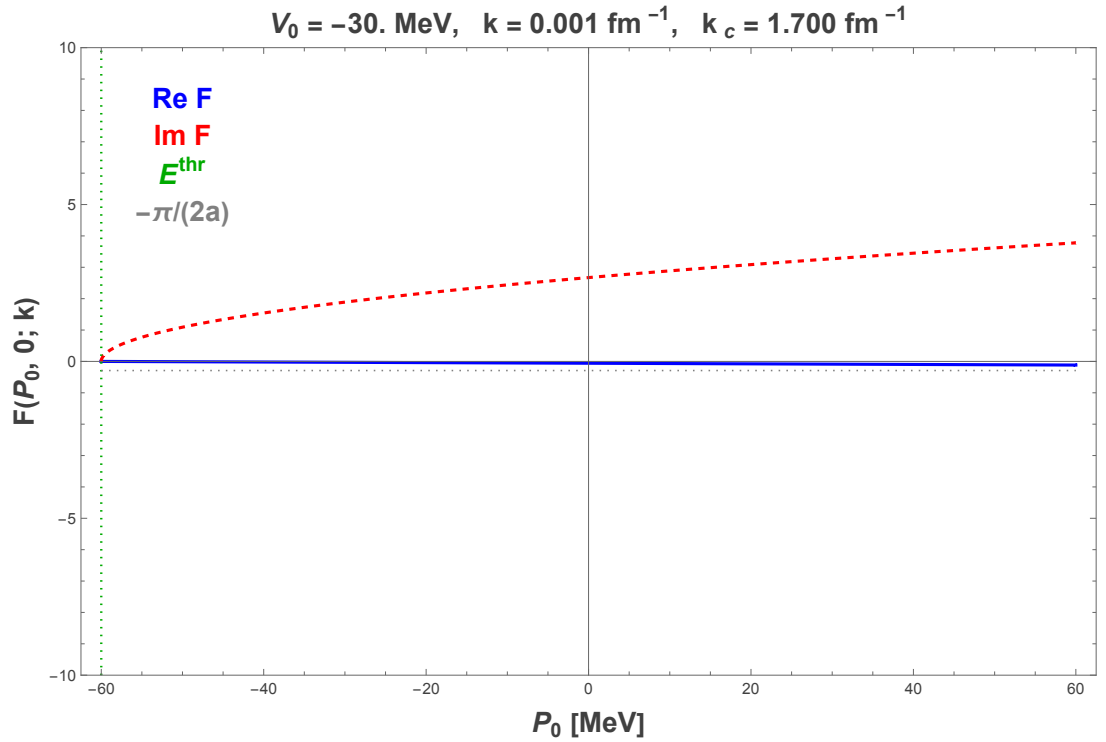


Figure 7: $F_k(P_0)$ in P_0 axis. Only LO matching applied.

At small k, the flow has included the low-energy fluctuations and the curve becomes smooth again.

Summary and outlook

Summary

- We proposed FRG setup toward an EFT of the nuclear shell model
- Dimer truncation reduces the two-body interaction to a single scale-dependent object Π_k
- **Vacuum**: framework reproduces both expected power-counting limits (Weinberg ($\nu \to 0$) and KSW ($\nu \to -1$)) from single flow equation
- **In-medium regulator**: observed V_{eff} flow changing in vacuum, repulsive core, attractive core different.
  - **Attractive core**: new critical scale k_c where Π_k acquires spectral content at $P_0 = 0$

Outlook

- Replace contact core $V_0 g(p)$ with realistic momentum-dependent profiles (Lorentzian, double Gaussian) $\longrightarrow$ spatial structure of core
- Match Im Π_k to scattering input with momentum-dependent profiles $\longrightarrow$ partial optical potential for the valence sector
- Promote core to a dynamical field $c \longrightarrow$ core excitation, recoil, and full structure–reaction interface

- **Long-term goal**: renormalizable valence-space interaction with controlled power counting, bridging shell-model structure and few-body reaction observables

Paper in preparation

Bibliography

[1] R. Machleidt and D. R. Entem, "Chiral effective field theory and nuclear forces," *Phys. Rep.*, vol. 503, p. 1, 2011, [Online]. Available: https://doi.org/10.1016/j.physrep.2011.02.001

[2] P. F. Bedaque and U. van Kolck, "Effective field theory for few-nucleon systems," *Annu. Rev. Nucl. Part. Sci.*, vol. 52, p. 339, 2002, [Online]. Available: https://doi.org/10.1146/annurev.nucl.52.050102.090637

[3] H. Hergert, "A guided tour of \textit{ab initio} nuclear many-body theory," *Front. Phys.*, vol. 8, p. 379, 2020, [Online]. Available: https://doi.org/10.3389/fphy.2020.00379

[4] B. A. Brown, "The nuclear shell model towards the drip lines," *Physics*, vol. 4, p. 525, 2022, [Online]. Available: https://doi.org/10.3390/physics4020035

[5] Y. Jaganathen, R. M. Id Betan, N. Michel, W. Nazarewicz, and M. Płoszajczak, "Quantified Gamow shell model interaction for { psd }-shell nuclei", *Phys. Rev. C*, vol. 96, p. 54316, 2017, [Online]. Available: https://doi.org/10.1103/PhysRevC.96.054316

[6] L. Huth, V. Durant, J. Simonis, and A. Schwenk, "Shell-model interactions from chiral effective field theory," *Phys. Rev. C*, vol. 98, p. 44301, 2018, [Online]. Available: https://doi.org/10.1103/PhysRevC.98.044301

[7] K. Fossez, J. Rotureau, and W. Nazarewicz, "Energy spectrum of neutron-rich helium isotopes: Complex made simple," *Phys. Rev. C*, vol. 98, p. 61302(R), 2018, [Online]. Available: https://doi.org/10.1103/PhysRevC.98.061302

[8] C. Wetterich, "Exact evolution equation for the effective potential," *Phys. Lett. B*, vol. 301, p. 90, 1993, [Online]. Available: https://doi.org/10.1016/0370-2693(93)90726-X

[9] M. C. Birse, "Functional renormalization group for two-body scattering," *Phys. Rev. C*, vol. 77, p. 47001, 2008, [Online]. Available: http://dx.doi.org/10.1103/PhysRevC.77.047001

[10] S. Weinberg, "Nuclear forces from chiral lagrangians," *Phys. Lett. B*, vol. 251, p. 288, 1990, [Online]. Available: https://doi.org/10.1016/0370-2693(90)90938-3

[11] S. Weinberg, "Effective chiral Lagrangians for nucleon-pion interactions and nuclear forces," *Nucl. Phys. B*, vol. 363, p. 3, 1991, [Online]. Available: https://doi.org/10.1016/0550-3213(91)90231-L

[12] D. B. Kaplan, M. J. Savage, and M. B. Wise, "A new expansion for nucleon-nucleon interactions," *Phys. Lett. B*, vol. 424, p. 390, 1998, [Online]. Available: https://doi.org/10.1016/S0370-2693(98)00210-X

Backup

Deuteron channel

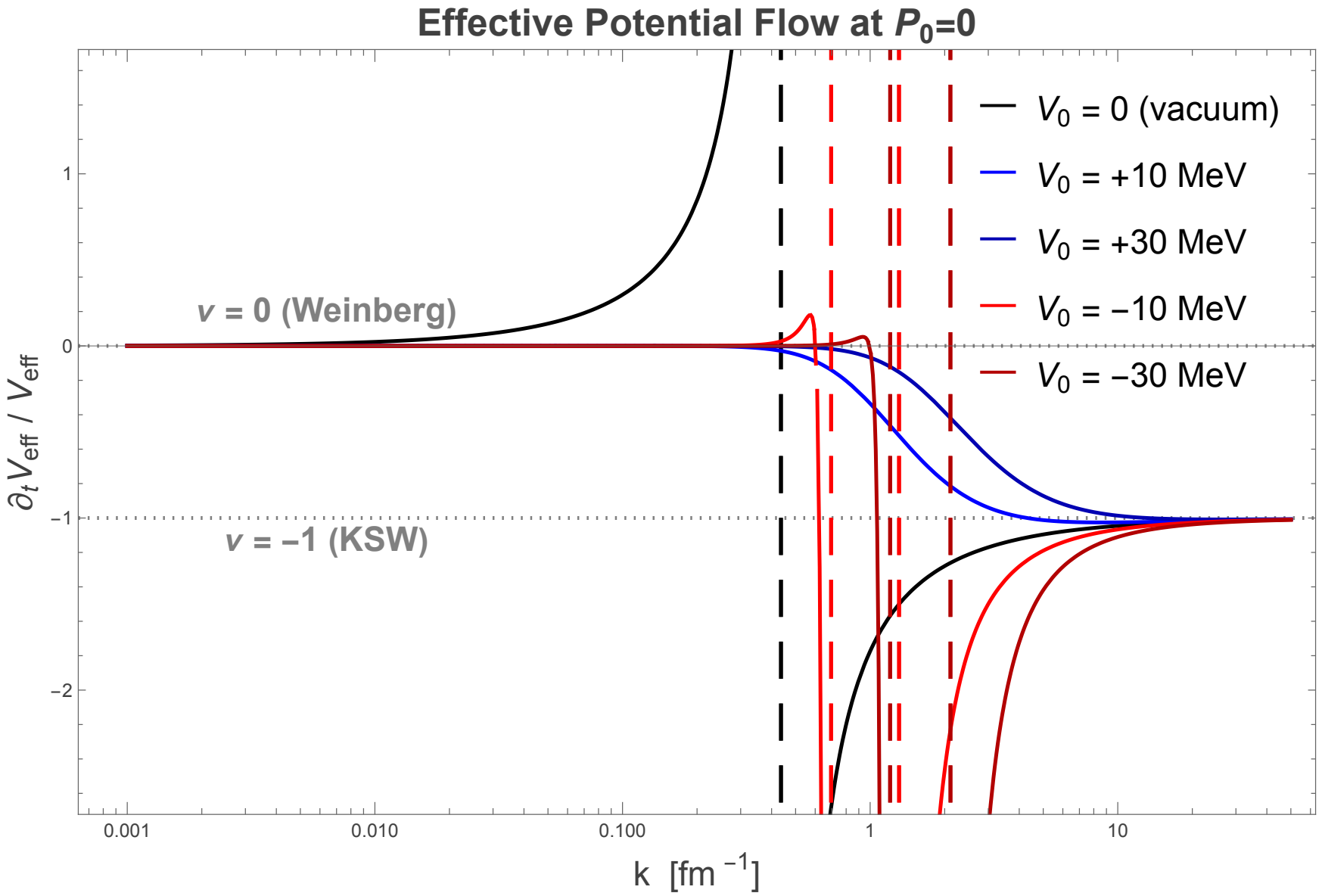


Figure 8: plot of deuteron channel at $P_0 = \boldsymbol{P} = 0$

Effective potential and Schur complement

The two-body effective potential is read off the nucleon-only effective action $\overline{\Gamma}_k$, obtained by integrating out d_k at the saddle point:

$$\overline{\Gamma}_k\left[\psi,\psi^\dagger\right]=\Gamma_k\left[\psi,\psi^\dagger,d_{\star[\psi]},d_\star^{\dagger[\psi]}\right],\quad \left.\frac{\delta\Gamma_k}{\delta d^\dagger}\right|_{d=d_\star}=0 \tag{15}$$

Solving the dimer EoM gives $d_\star=-g\left(\psi\hat{P}\psi\right)/\Pi_k$; substituting back,

$$V_{\text{eff}}(P_0,\boldsymbol{P})=-\frac{\delta^2\overline{\Gamma}_k}{\delta\left(\psi\hat{P}\psi\right)^\dagger\delta\left(\psi\hat{P}\psi\right)}=-\lambda+\frac{g^2}{\Pi_k(P_0,\boldsymbol{P})} \tag{16}$$

In Schur complement form: In the channel basis $\left\{\psi\hat{P}\psi,d\right\}$ the Hessian is block-structured:

$$\Gamma_k^{(2)}=\begin{pmatrix}\lambda & g\\ g & \Pi_k\end{pmatrix}\quad\overset{\text{eliminate } d}{\longrightarrow}\quad\overline{\Gamma}_k^{(2)}=\lambda-\frac{g^2}{\Pi_k} \tag{17}$$

Identical structure to Feshbach: Projecting Schrödinger onto P-space:

$$\begin{pmatrix}E-H_{\text{PP}} & -H_{\text{PQ}}\\ -H_{\text{QP}} & E-H_{\text{QQ}}\end{pmatrix}\begin{pmatrix}|P\rangle\\ |Q\rangle\end{pmatrix}=0\quad\overset{\text{eliminate } |Q\rangle}{\longrightarrow}\quad H_{\text{eff}}(E)=H_{\text{PP}}+H_{\text{PQ}}\left(E-H_{\text{QQ}}\right)^{-1}H_{\text{QP}} \tag{18}$$

Correspondence:

$$\Pi_k\longleftrightarrow\left(E-H_{\text{QQ}}\right),\qquad g\longleftrightarrow H_{\text{PQ}},\qquad\lambda\longleftrightarrow H_{\text{PP}},\qquad\psi\hat{P}\psi\longleftrightarrow|P\rangle,\qquad d\longleftrightarrow|Q\rangle \tag{19}$$

Integrating out the dimer in FRG **is** the Feshbach projection.

Backup: Path to reaction observables

Step 1. this work

- Constant core $g(p) = 1$
- Re $\Pi_k \longrightarrow$ Hermitian shell-model V_{eff}
- Vacuum power counting (Weinberg, KSW)
- Three regimes from the in-medium flow

Step 2. momentum profile

- $g(p)$: Lorentzian, double Gaussian, $\rightarrow$ WS
- Spatial structure of the core
- Singularities of $g(p)$ generate new branch points in Π_k
- Im Π_k acquires structure beyond shifted threshold

Step 3. reaction input

- Keep complex Π_k
- Match residual Im Π_k to $T(p)$
- Energy-dependent complex V_{eff}
- Partial optical potential (valence sector)
- ANC from pole of V_{eff} at $k \rightarrow 0$

Beyond Step 3: dynamical core $\rightarrow$ core excitation & full optical potential

Backup: Validity of the static-core approximation

Promote the core to a dynamical field c and write the Hessian on the basis (ψ, c):

$$\Gamma_k^{(2)} = \begin{pmatrix} A_\psi & V_{c\psi} \\ V_{c\psi}^\dagger & A_c \end{pmatrix}, \qquad A_c = i\partial_0 - \frac{\nabla^2}{2m_A} \tag{20}$$

Eliminate c by Schur complement:

$$\overline{A}_\psi = A_\psi - V_{c\psi} A_c^{-1} V_{c\psi}^\dagger \tag{21}$$

Heavy-core expansion — for processes with the core near its ground state ($q_0 \lesssim$ valence energy $\sim k_F^2/2m_n$):

$$A_c^{-1}(q_0, \boldsymbol{q}) = \frac{1}{q_0 - \boldsymbol{q}^2/2m_A} = \frac{1}{q_0}\left[1 + \frac{\boldsymbol{q}^2}{2m_A q_0} + \mathcal{O}\big(1/m_A^2\big)\right] \tag{22}$$

Leading order in $1/m_A$:

$$V_{c\psi} A_c^{-1} V_{c\psi}^\dagger \longmapsto V_0 g(p)(1 - f(k)) \tag{23}$$

$\to$ exactly the **static, instantaneous one-body potential** in R_k^c.

Recoil correction: By using $q_0 \sim k_F^2/2m_n$,

$$\frac{\boldsymbol{q}^2/2m_A}{q_0} \sim \frac{m_n}{m_A} \tag{24}$$

Core	^{4}He	^{16}O	^{40}Ca
m_n/m_A	≈ 0.25	≈ 0.06	≈ 0.025
Static-core LO	modest	good	excellent

Static core = LO of a controlled $1/m_A$ expansion (nuclear analog of HQET / Born–Oppenheimer)