A general route to ab initio nuclear reactions in low-energy region

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5/20

Nuclear theory for astrophysics: Bridging bound-state and continuum physics @ ECT*

Collaborator: Serdar Elhatisari, Sebastian König, Dean Lee, Yuanzhuo Ma, and Takayuki Miyagi

Minimum states for reactions

ab-initio vs Pheno

Uncertainties

Interactions

Truncations and bases

0. Introduction

Ab initio nuclear theories

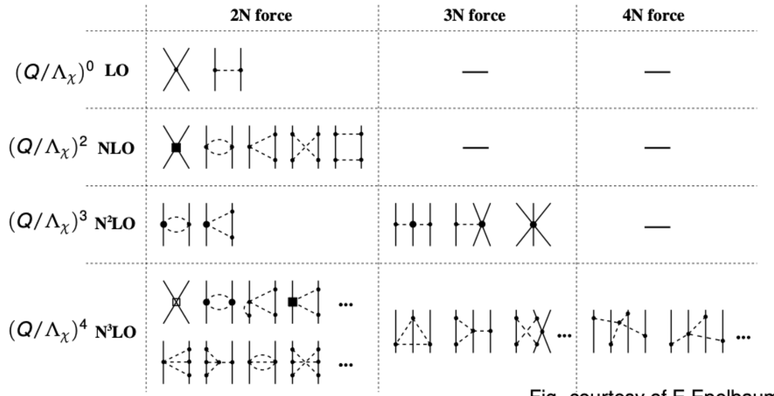


Fig. courtesy of E.Epelbaum

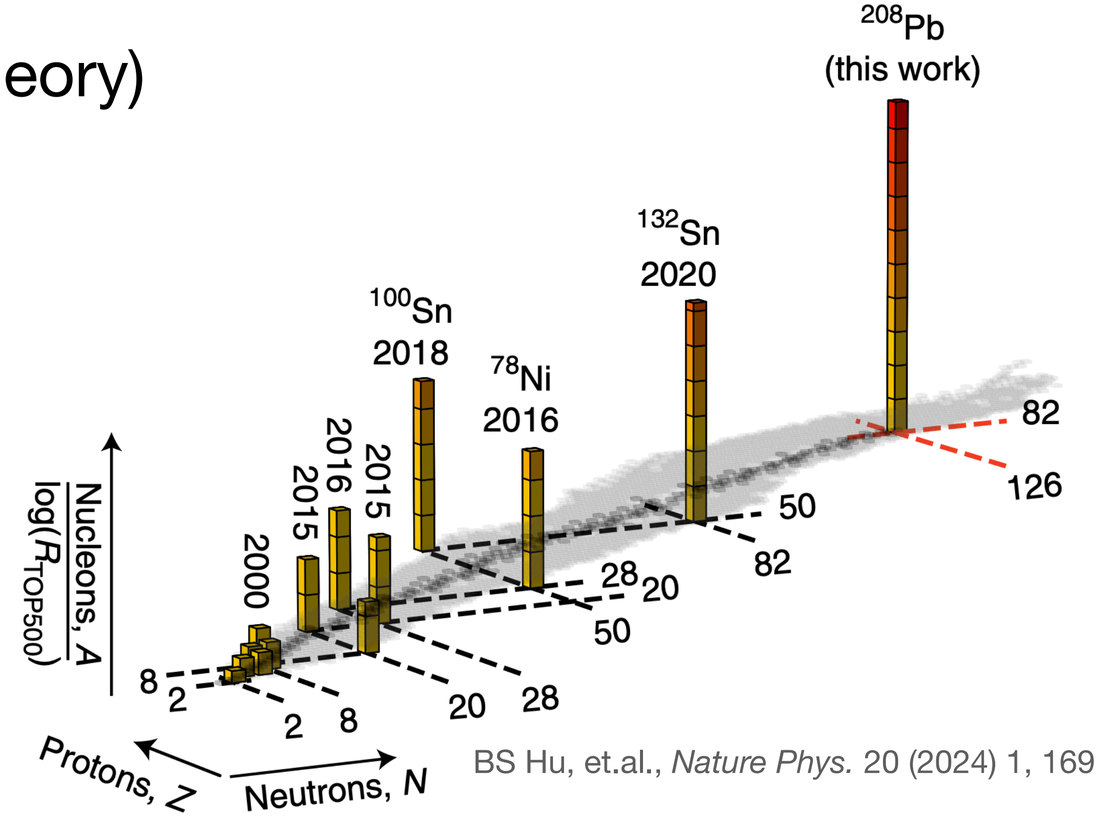
- NCSM, GF, AFDMC

- NLEFT (Nuclear Lattice Effective Field Theory)
  - Now valid through A ~ 132
  - with Spin orbit + Wigner-SU(4)

ZW Niu and BN Lu, PRL 135 (2025) 22, 222504

See talks by Teng Wang

- Coupled-Cluster, IMSRG

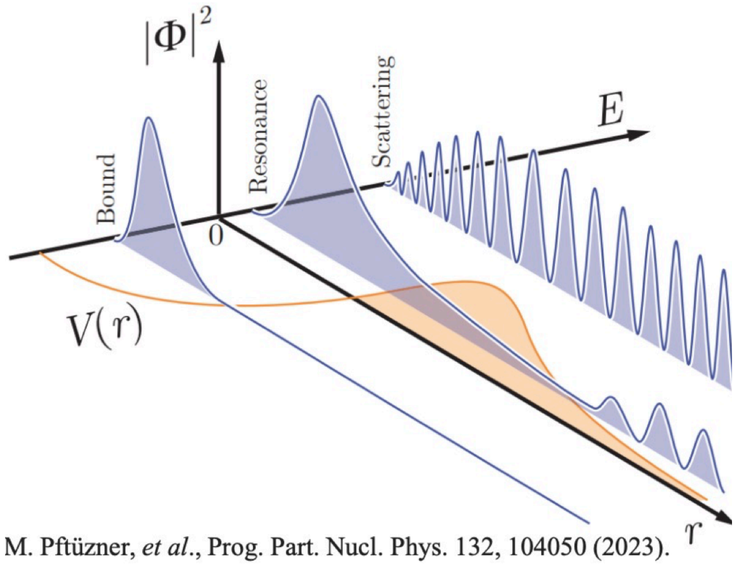


BS Hu, et.al., *Nature Phys.* 20 (2024) 1, 169

+ Continuum = Difficult

- The no-go theorem (Maiani-Testa) PLB 245 (1990) 585-590

- Continuum = Non-Hermitian physics



M. Pftüzner, *et al*., Prog. Part. Nucl. Phys. 132, 104050 (2023).

- Existing solutions:
  - Complex-scaling based methods (few-body CSM, Berggren basis…)
    By G.P. Aguilar and C. Barbieri **By G. Sargsyan**
  - Green's Functions, Adapted symmetry, etc
    By J. Herko
  - Ab initio effective Hamiltonian/Interaction (NCSMC, **Adiabatic projection**)
  - **Lüscher’s formalism**

Adiabatic Projection Method

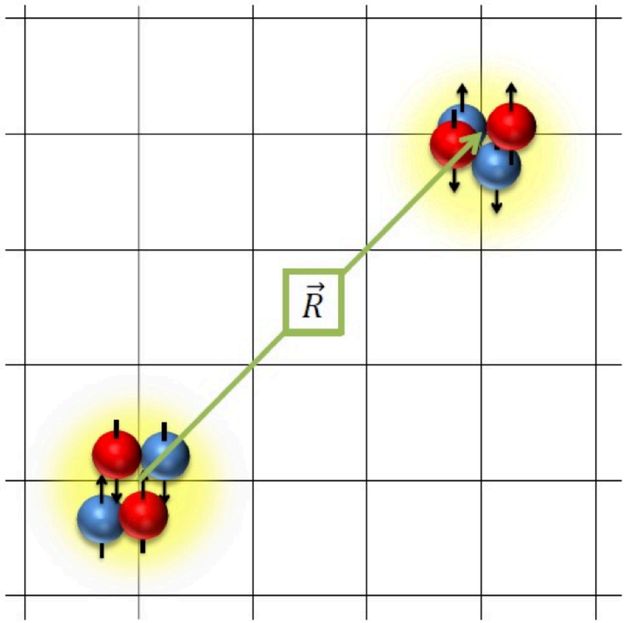
- Initial cluster states with relative separation $\vec{R}$

$$|\vec{R}\rangle = \sum_{\vec{r}} |\vec{r} + \vec{R}\rangle \otimes |\vec{r}\rangle$$

- Time evolved 'dressed cluster states'

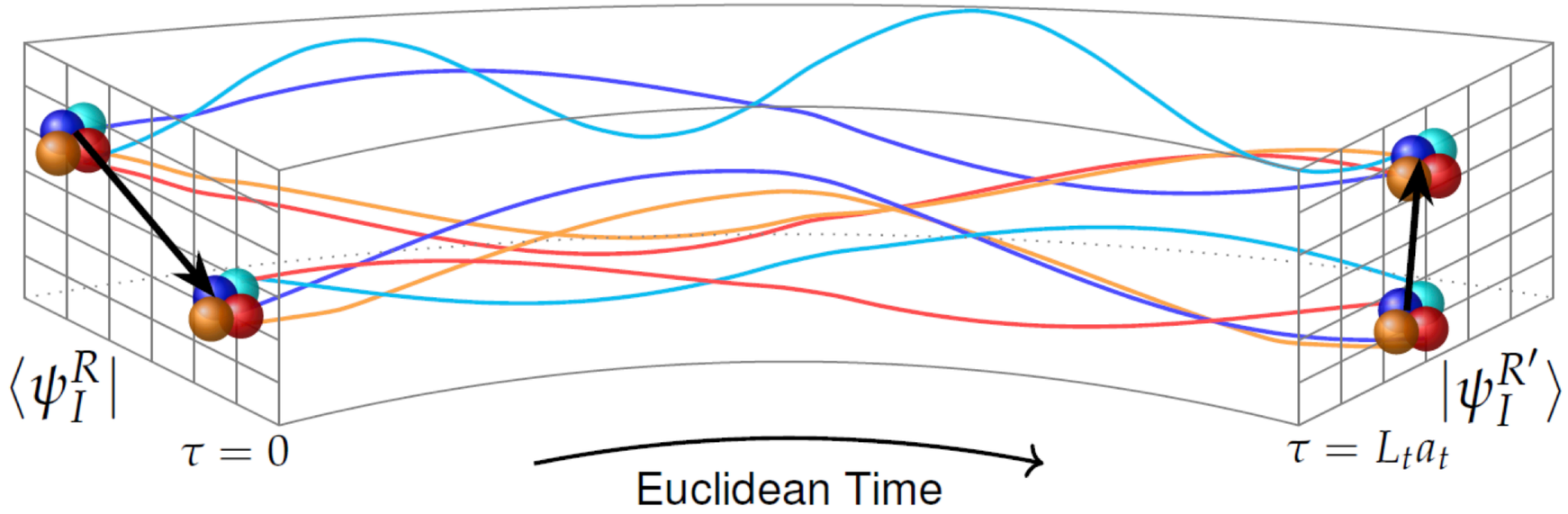
$$|\vec{R}\rangle_\tau = \exp(-H\tau)|\vec{R}\rangle$$

  - H is full A-body interacting microscopic Hamiltonian
  - $\tau = L_t a_t$
  - $\lim_{\tau\to\infty} \{|\vec{R}\rangle_\tau\}$ spans low-energy two-cluster subspace



[1] M. Pine, D. Lee, G. Rupak, *Eur. Phys. J. A* 49, 151 (2013)
[2] S. Elhatisari, D. Lee, Phys. Rev. C 90, 064001
[3] S. Elhatisari, D. Lee, G. Rupak, E. Epelbaum, H. Krebs, T. Lähde, T. Luu, & U-G. Meißner. Nature 528, 111-114 (2015).
[4] S. Elhatisari, D. Lee, U-G. Meißner, G. Rupak, Eur. Phys. J. A 52, 174 (2016)
[5] S. Elhatisari, T. Lähde, D. Lee, U-G. Meißner, T. Vonk, *J. High Energ. Phys.* 2022

Slides from
Avik Sarkar
@Sendai



- Projected Hamiltonian

$$[H_\tau]_{\vec{R},\vec{R}'} = {}_\tau\langle \vec{R}|H|\vec{R}'\rangle_\tau$$

Norm

$$[N_\tau]_{\vec{R},\vec{R}'} = {}_\tau\langle \vec{R}|\vec{R}'\rangle_\tau$$

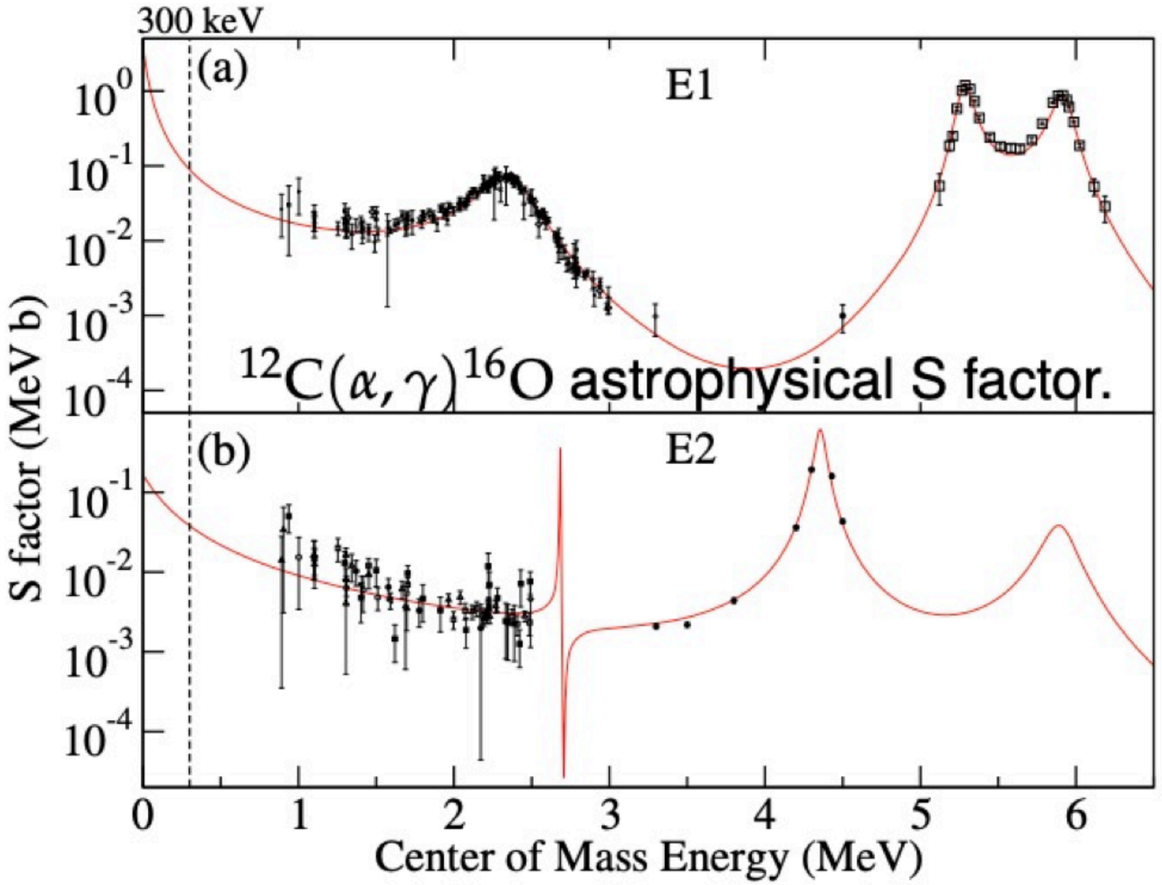
- Adiabatic Hamiltonian

$$[H_\tau^a]_{\vec{R},\vec{R}'} = \left[N_\tau^{-1/2} H_\tau N_\tau^{-1/2}\right]_{\vec{R},\vec{R}'}$$

Slides from
Avik Sarkar
@Sendai

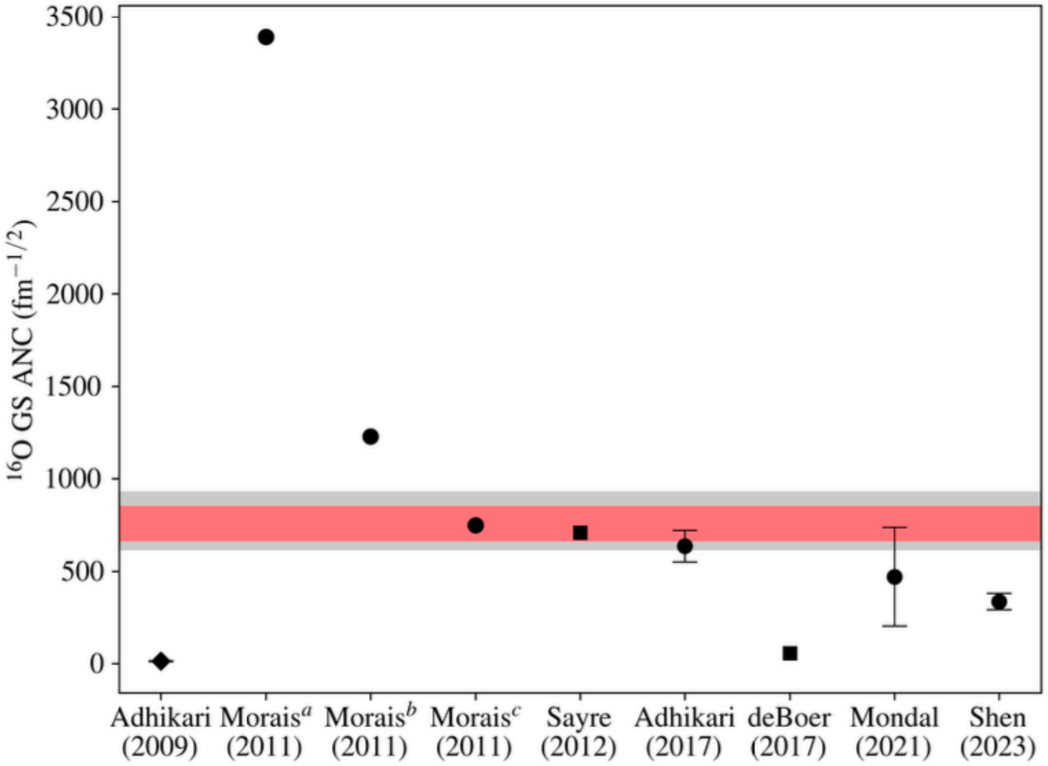
The Holly Grail

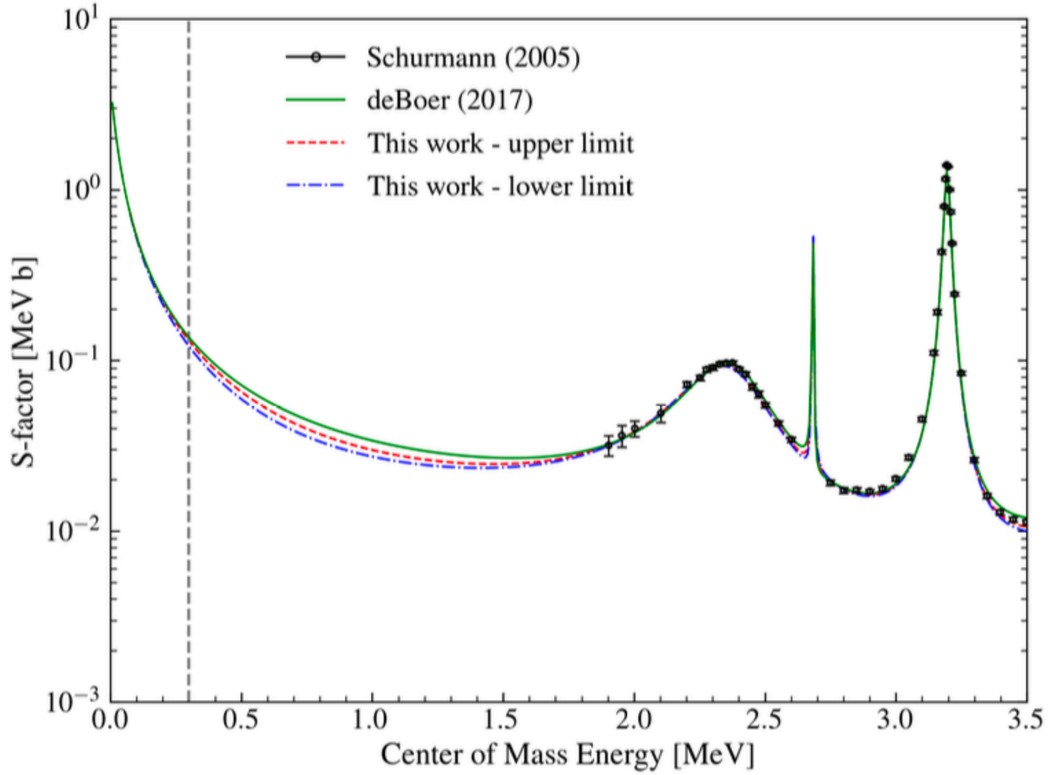
- Crucial reaction to heavier elements
- Low energy ambiguity (stellar env)
- From life formation to black holes
- Experimentally: 16O ANCs via transfer



deBoer et al., Rev. Mod. Phys. 89, 035007 (2017)

Almost 10 years later…





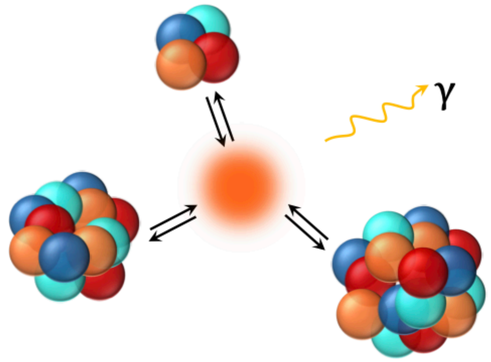
ANCs are all over the place….

Transfer reactions @ TAMU

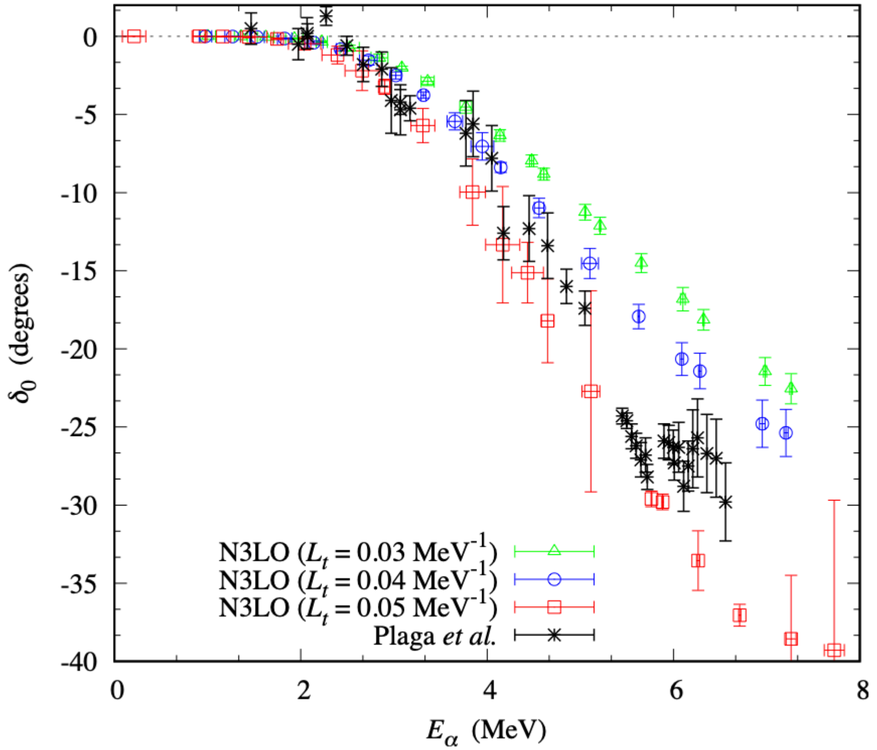
E. Harris, et. al., arXiv:2507.17059 [nucl-ex]

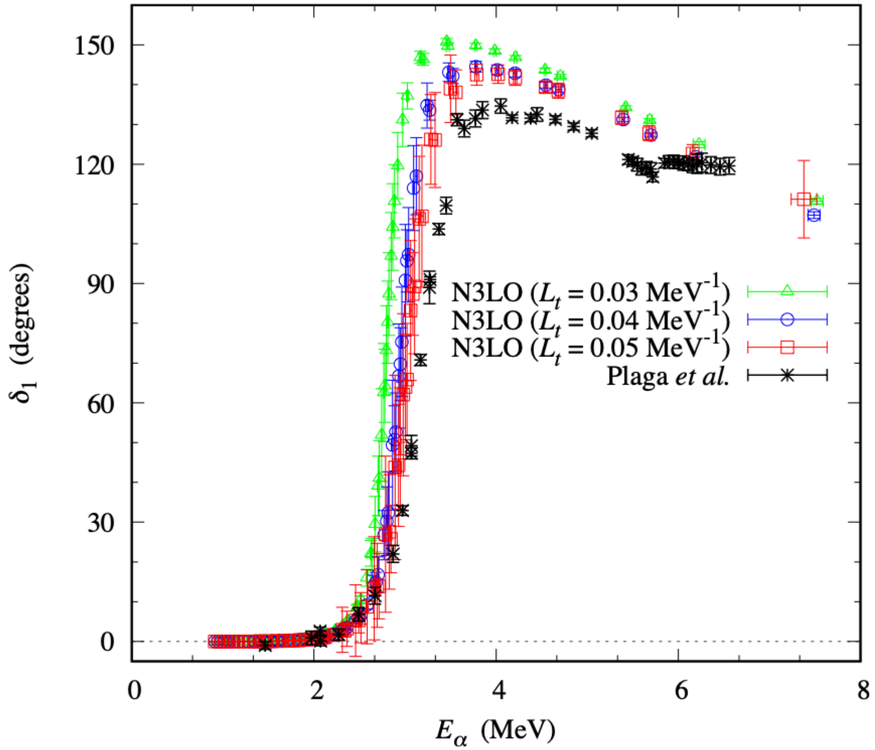
To claim the grail -> percentage level error

Prelim by S. Elhatisari



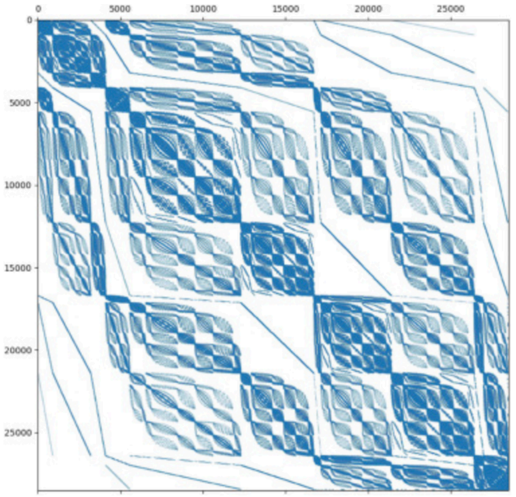
$a = 1.32$ fm and $p_{\max} = \pi/a = 471$ MeV





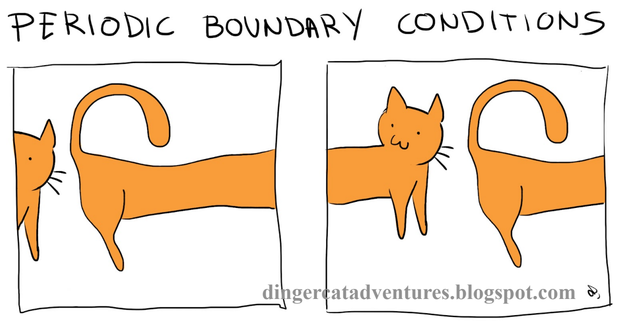
1. Lüscher formalism

$$H =$$

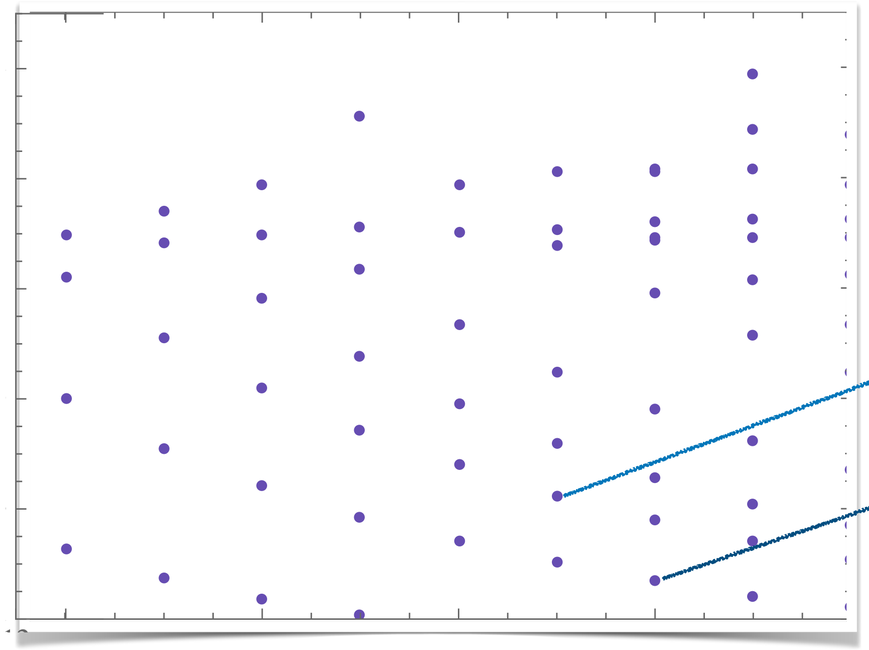


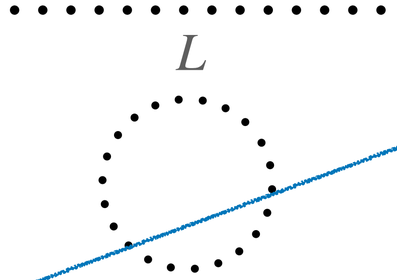
N. Shimizu Comp. Phys. Comm. 244, 372 (2019)

Lüscher, Nucl.Phys.B.**345** 531(1991)



$$\alpha \cos(p \cdot L/2) + \beta \sin(p \cdot L/2) = 0$$



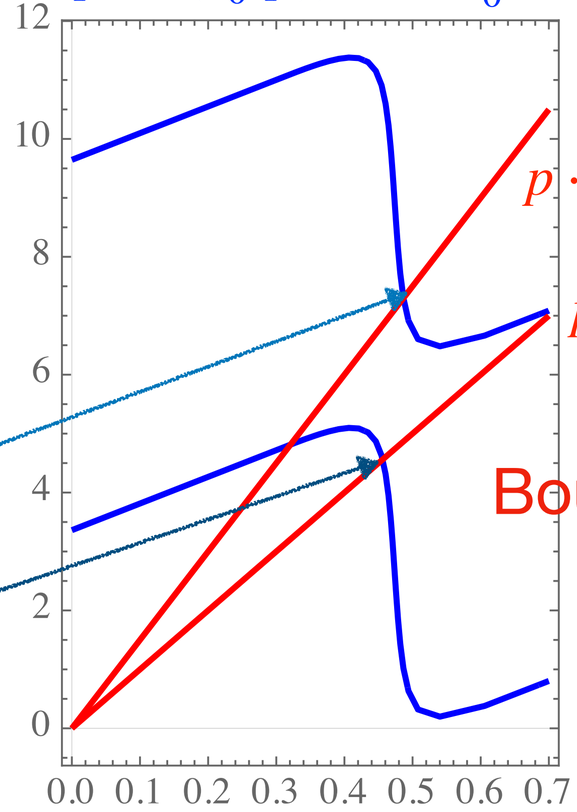


TRAP: (NCSM, IMSRG, QMC etc)
Xilin Zhang et. al., PRL **125** 112503 (2020)

HORSE: (HOB)
A.M. Shirokov et. al., PRC **94** 064320 (2016)

Potential QC

$$p \cot(\delta_0(p)) = -a_0^{-1} + \frac{1}{2} r_0 p^2 + \ldots$$



$$p \cdot L_1 = 2\phi_L(p)$$

$$p \cdot L_2 = 2\phi_L(p)$$

Boundary QC

Volume Dependence is an Inverse Problem

Volume Dependence...
Is it dark magic?



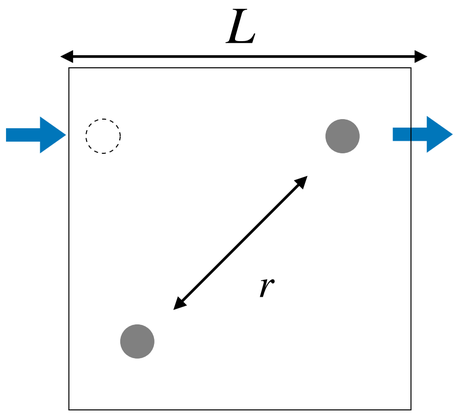
- Truncated states can be expressed as the sum of continuum states
- Volume-dependence techniques decipher continuum info
- Information is preserved if you know how

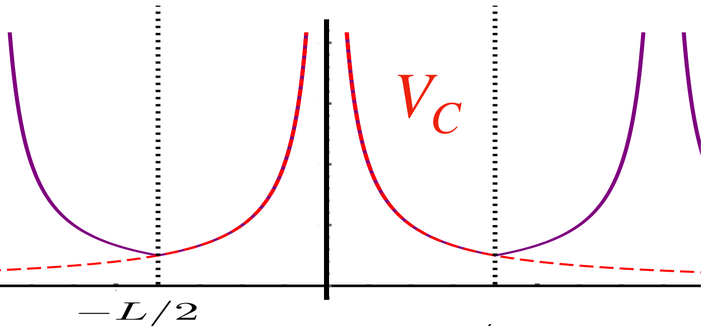
i.e., inverse problems to find the relevant states

Volume Dependence is a DIFFICULT Inverse Problem

HY, et al., PRL 131 212502 (2023)

In finite volume





*P. Guo, PRC **103** 064611 (2021),*
*P. Guo et. al., PRD **103** 094520 (2021).*

$$H = -\frac{1}{2\mu}\nabla^2 + V + V_C$$

$$V_L(x) = \sum_{\mathbf{n}} V(x + \mathbf{n}L)$$

$$H_L = -\frac{1}{2\mu}\nabla^2 + V_L + V_{C,\{L\}}$$

$$\psi_\infty(r) \to \frac{C_0}{\sqrt{4\pi r}} W_{-\bar{\eta},1/2}(2\kappa r)$$

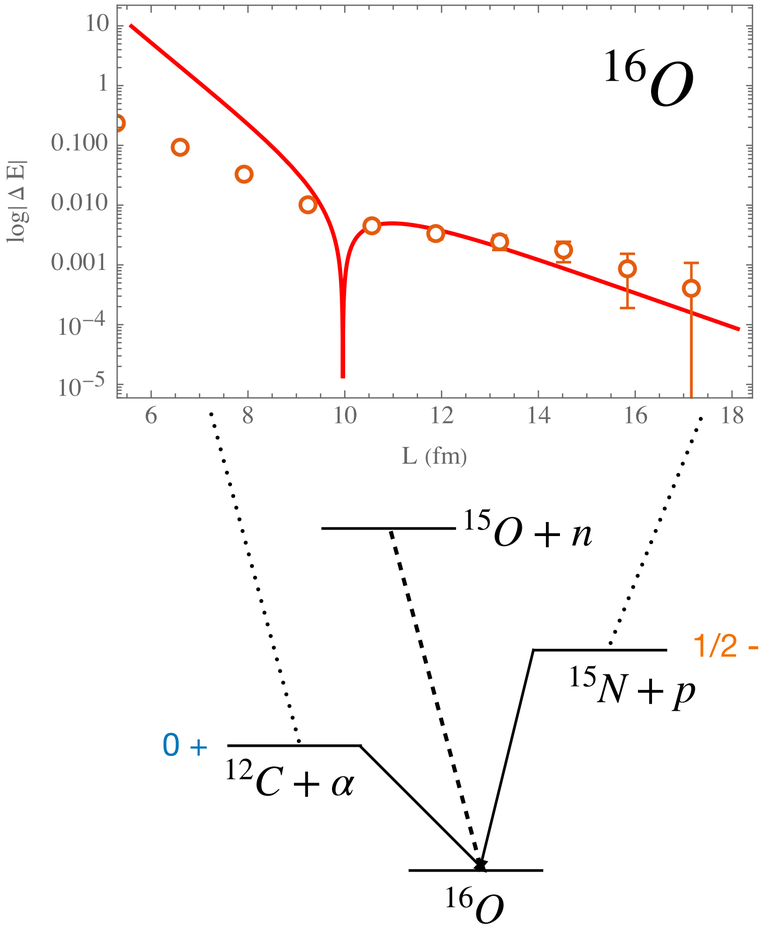
$$\bar{\eta} = \frac{\gamma}{2\kappa}$$

For Bound-states:

$$\Delta E(L) = -\frac{3C_0^2}{\mu L}\left[W_{-\bar{\eta},\frac{1}{2}}(\kappa L)\right]^2$$

$$+\Delta\tilde{E}(L) \quad +\Delta\tilde{E}'(L) \quad +\mathcal{O}(e^{-\sqrt{2}\kappa L})$$

+ Geometric Factors in Clusters
HY, arXiv:2511.06726 [nucl-th]



+ Multiple channels...

Parameterizations in FV

- General Field Theory results:
  - Relations that hold for **any** unitary quantum field theory
- Hamiltonian methods:
  - Fitting an EFT-inspired parameterization - energy scale $q^2 \sim \left(\frac{Lk}{2\pi}\right)^2$
- Plane wave method:
  - Chiral EFT

Plane wave basis: from Chiral/Pionless EFT

L. Meng et.al., JHEP 10 051 (2021)

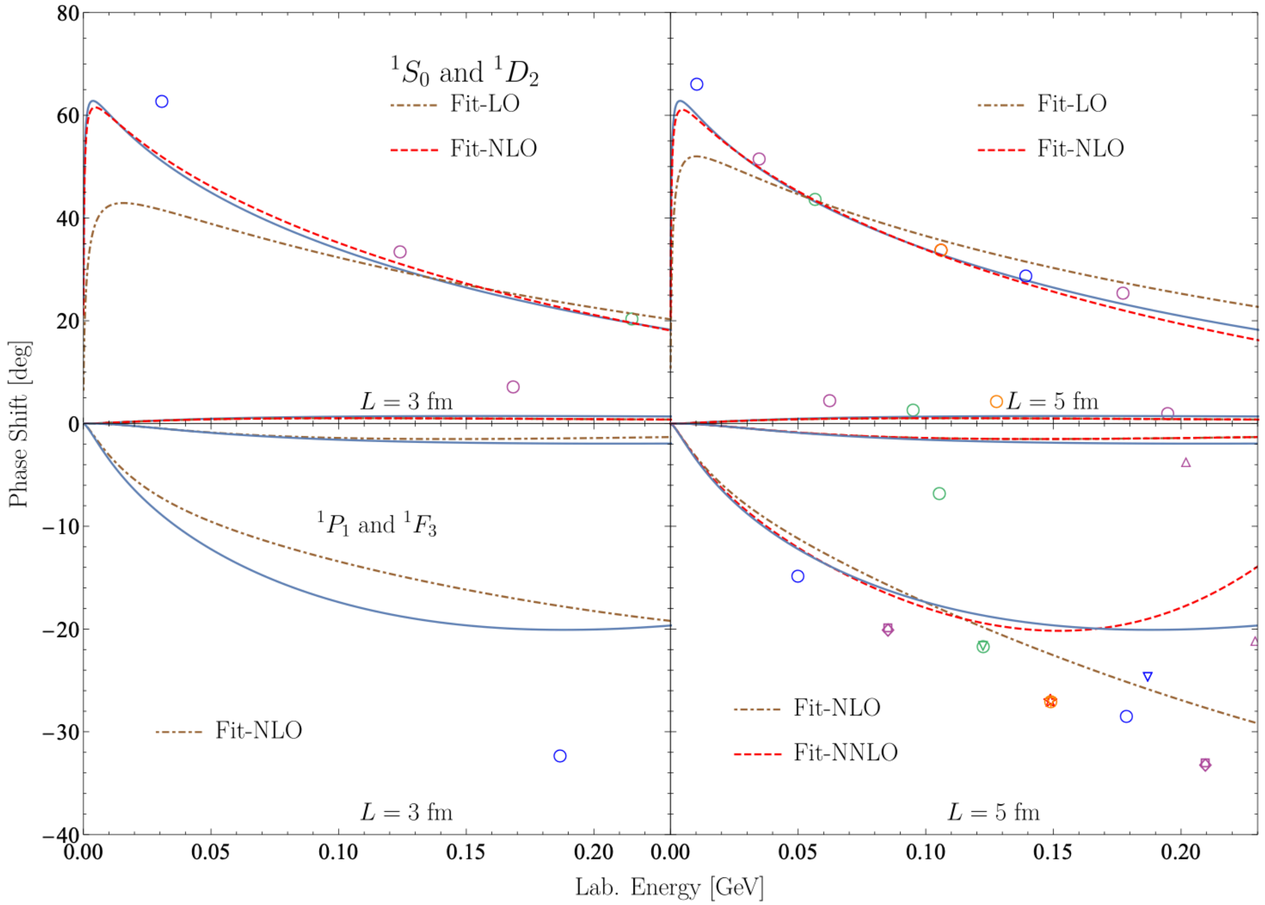
$$V_{\rm EFT} = V^{(0)}_{\rm OPE} + V^{(0)}_{\rm cont} + V^{(2)}_{\rm cont} + V^{(4)}_{\rm cont} + \dots \tag{5.18}$$

We use the same regulator as in eq. (5.17) and employ the same value for the cutoff parameter $\Lambda = 0.45$ GeV. In order to fit the FV energy levels with positive parity, we introduce the interaction at NLO,

$$V^{(0)}_{\rm cont} = \frac{1}{4\pi}\tilde{C}_{^1S_0}, \quad V^{(2)}_{\rm cont} = \frac{1}{4\pi}C_{^1S_0}(p^2 + p'^2)\,. \tag{5.19}$$

where $\tilde{C}_{^1S_0}$ and $C_{^1S_0}$ refer to the corresponding LECs. For the states with negative parity, we include the contact interactions up to NNLO,

$$V^{(2)}_{\rm cont}(p,p',z) = \frac{3}{4\pi}C_{^1P_1}pp'z, \quad V^{(4)}_{\rm cont}(p,p',z) = \frac{3}{4\pi}D_{^1P_1}pp'(p^2+p'^2)z\,, \tag{5.20}$$



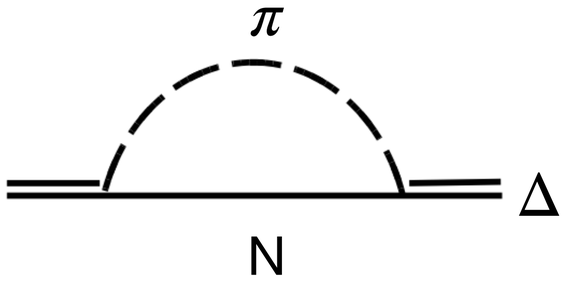
FV Hamitonian method: Standard View

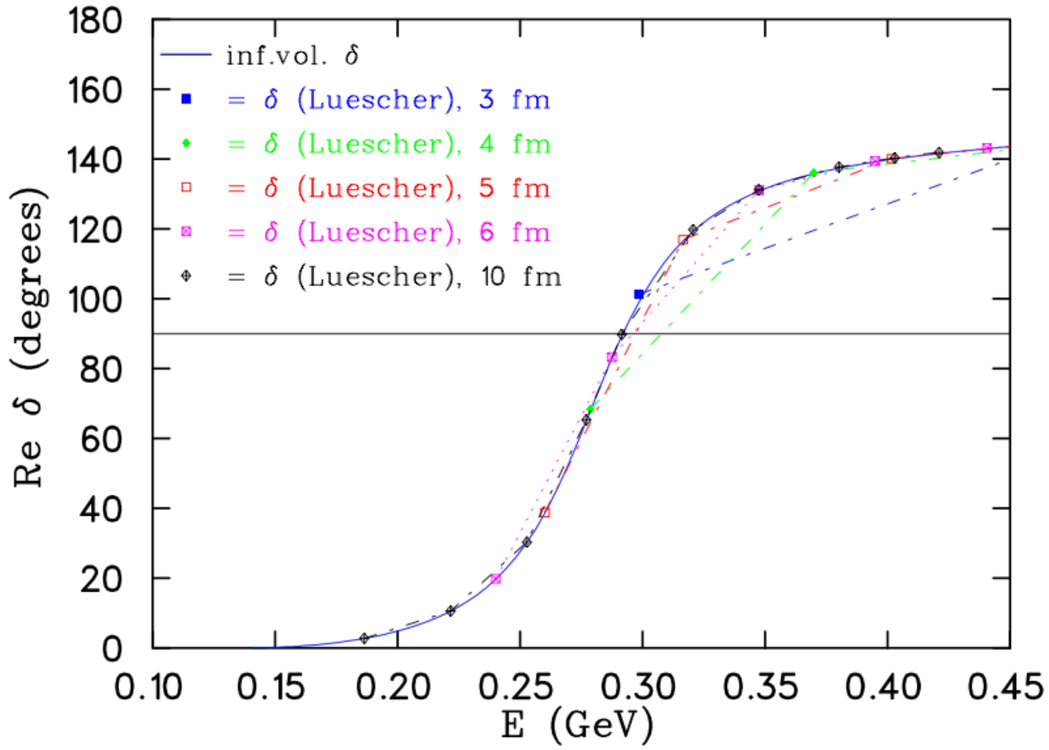
J.M.M Hall et.al., PRD 9, 094510 (2013)

$$H_0 = \begin{pmatrix} \Delta_0 & 0 & 0 & \cdots \\ 0 & \omega_\pi(k_1) & 0 & \cdots \\ 0 & 0 & \omega_\pi(k_2) & \\ \vdots & \vdots & & \ddots \end{pmatrix} \qquad \omega_\pi(k_n) \equiv \sqrt{k_n^2 + m_\pi^2}$$

$$H_I = \begin{pmatrix} 0 & g^{\rm fin}_{\Delta N}(k_1) & g^{\rm fin}_{\Delta N}(k_2) & \cdots \\ g^{\rm fin}_{\Delta N}(k_1) & 0 & 0 & \cdots \\ g^{\rm fin}_{\Delta N}(k_2) & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \qquad g^2_{\Delta N}(k) = \chi_\Delta \frac{2}{\pi} \frac{k^2\, u^2(k)}{\omega_\pi(k)},$$

$$u(k_n) = \left(1 + \frac{k_n^2}{\Lambda^2}\right)^{-2}$$



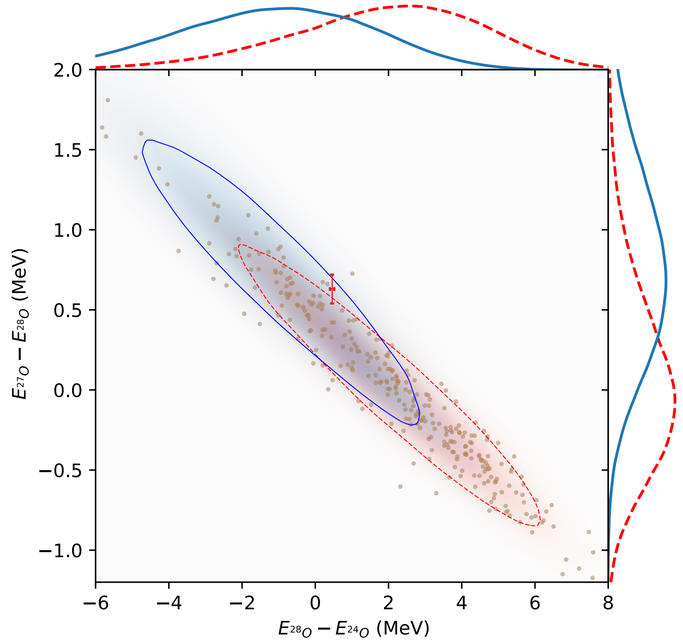


Relation to MEP:

MEP: fast Inverse problem solver **HY and Takayuki Miyagi, PRL 136, 202502 (2026)**

Finds $10^5 \sim 10^6$ set of LECs in minutes

Should work well on Finite Volume Hamiltonians



Multiparameter Eigenvalue Problems Emulator

The *Multiparameter* in MEP

means

Multiparametric Eigenvalues

$$\left(O_j + \sum_{i=1}^{m} \alpha_i A_{ij} \right) \mathbf{y}_j = 0\,, \quad 1 \le j \le m$$

Inputs **Outputs**

$$\left(H_0^{[j]} + \sum_{i=1}^{n_{\text{par}}} c_i H_i^{[j]} - E^{[j]} N^{[j]} \right) \mathbf{y}^{[j]} = 0$$

F. V. Atkinson and A. B.Mingarelli
Multiparameter eigenvalue problems: Sturm-Liouville theory
(2010)

$$(K_i - \alpha_i K_0)\mathbf{y}_{\otimes} = 0\,, \quad 1 \le i \le m$$

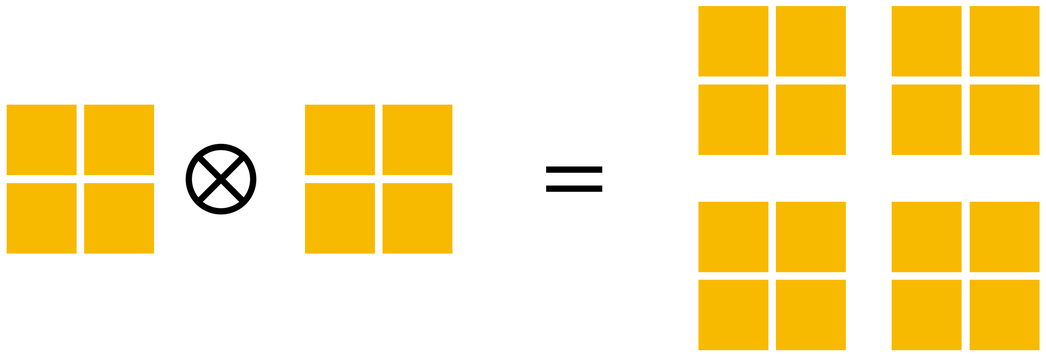
$$\mathbf{y}_{\otimes} = \bigotimes_{j=1}^{m} y_j$$

Kronecker Determinant

$$K_0 = \begin{vmatrix} A_{11} & \cdots & A_{1m} \\ \vdots & \ddots & \vdots \\ A_{m1} & \cdots & A_{mm} \end{vmatrix}_{\otimes} \qquad \begin{vmatrix} G_{11} & \cdots & G_{1m} \\ \vdots & \ddots & \vdots \\ G_{m1} & \cdots & G_{mm} \end{vmatrix}_{\otimes} \equiv \sum_{\sigma \in S_m} \operatorname{sgn}(\sigma) G_{1\sigma(1)} \otimes G_{2\sigma(2)} \cdots \otimes G_{m\sigma(m)}$$

$$K_i = \begin{vmatrix} A_{11} & \cdots & A_{1(i-1)} & O_1 & A_{1(i+1)} & \cdots & A_{1m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{m1} & \cdots & A_{m(i-1)} & O_m & A_{m(i+1)} & \cdots & A_{mm} \end{vmatrix}_{\otimes}$$

Kronecker Product



$$M \times M \qquad M^m \times M^m$$

*can be different sizes

Kronecker Determinant

$$K_0 = \begin{vmatrix} A_{11} & \cdots & A_{1m} \\ \vdots & \ddots & \vdots \\ A_{m1} & \cdots & A_{mm} \end{vmatrix}_{\otimes}$$

$$(K_i - \alpha_i K_0)\mathbf{y}_{\otimes} = 0 \qquad M^m \times M^m$$

Subspace Projection ↓ $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$ ↓

$$(\mathscr{K}_0)_{kl} = (\mathbf{y}_{\otimes,k})^* \cdot K_0 \cdot \mathbf{y}_{\otimes,l} = \begin{vmatrix} (A_{11})_{kl} & \cdots & (A_{1m})_{kl} \\ \vdots & \ddots & \vdots \\ (A_{m1})_{kl} & \cdots & (A_{mm})_{kl} \end{vmatrix}$$

$$(\mathscr{K}_i - \alpha_i \mathscr{K}_0)\mathbf{y} = 0 \qquad M \times M$$

Matrix Determinant!

2. FVH + MEP

Consider a (NLO) Cluster EFT $H_0 + C_0 V_0 + C_2 V_2 + \ldots \quad V_0 = \mathrm{e}^{-r^2/\sigma^2}, V_2 = r^2 \mathrm{e}^{-r^2/\sigma^2}$

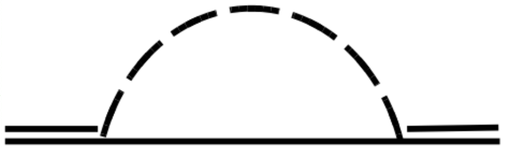
P. Capel et.al., PRC **98**, 034610 (2018)

Cross checked with FVH (dimer)

We solve the following MEP problem on FV:

$$[H_0(L_1) + C_0 V_0(L_1) + C_2 V_2(L_1) - E(L_1) N(L_1)] y(L_1) = 0$$

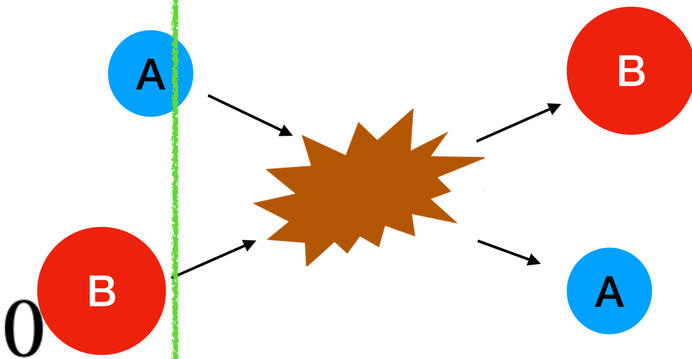
$$[H_0(L_2) + C_0 V_0(L_2) + C_2 V_2(L_2) - E(L_2) N(L_2)] y(L_2) = 0$$



By constructing a Reduced Basis system:

$$[K_1(L_1, L_2)]_{ij} = \begin{vmatrix} [H_0(L_1)]_{ij} - E(L_1)[N(L_1)]_{ij} & [V_2(L_1)]_{ij} \\ [H_0(L_2)]_{ij} - E(L_2)[N(L_2)]_{ij} & [V_2(L_2)]_{ij} \end{vmatrix}$$

$$K_0(L_1, L_2) = \begin{vmatrix} V_0(L_1) & V_2(L_1) \\ V_0(L_2) & V_2(L_2) \end{vmatrix}$$



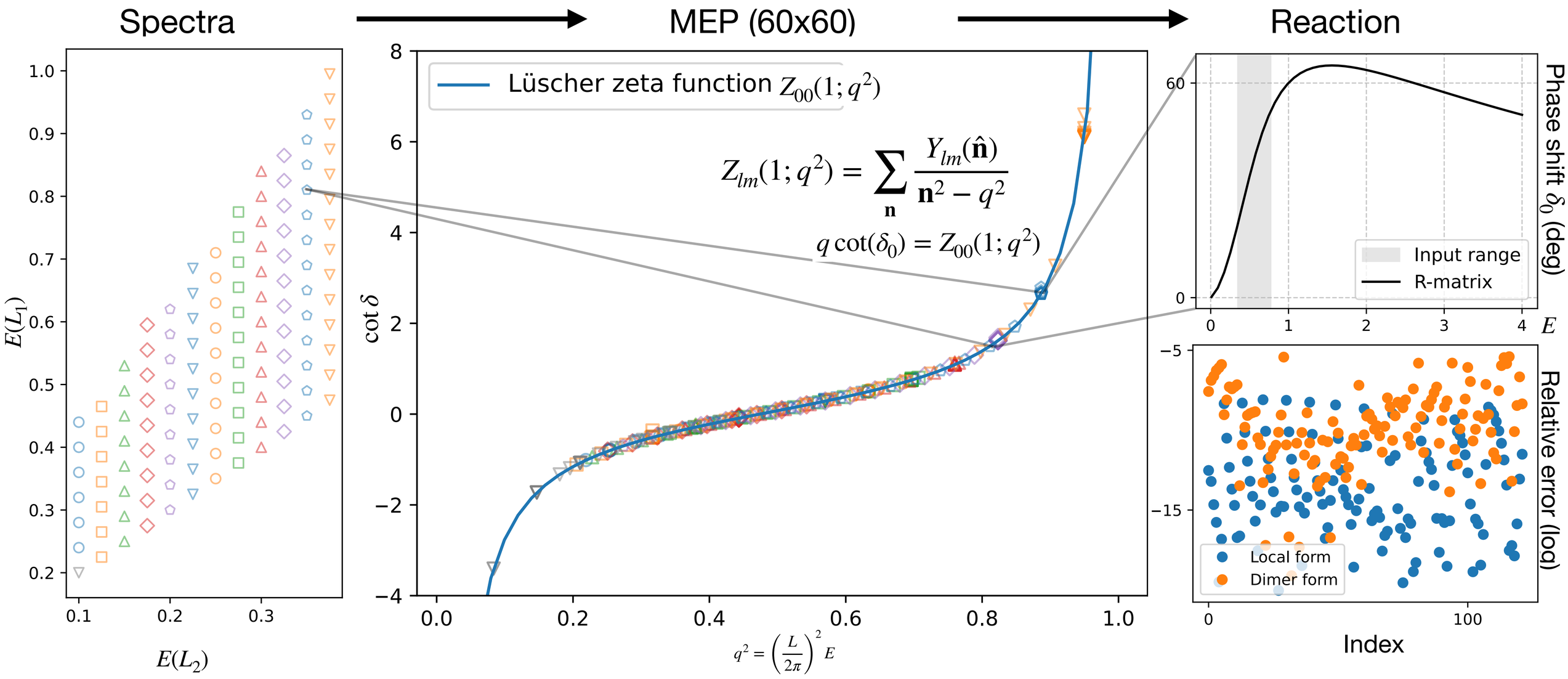
Ultimately, solving $[K_1(L_1, L_2) + C_0 K_0(L_1, L_2)] y(L_1, L_2) = 0$



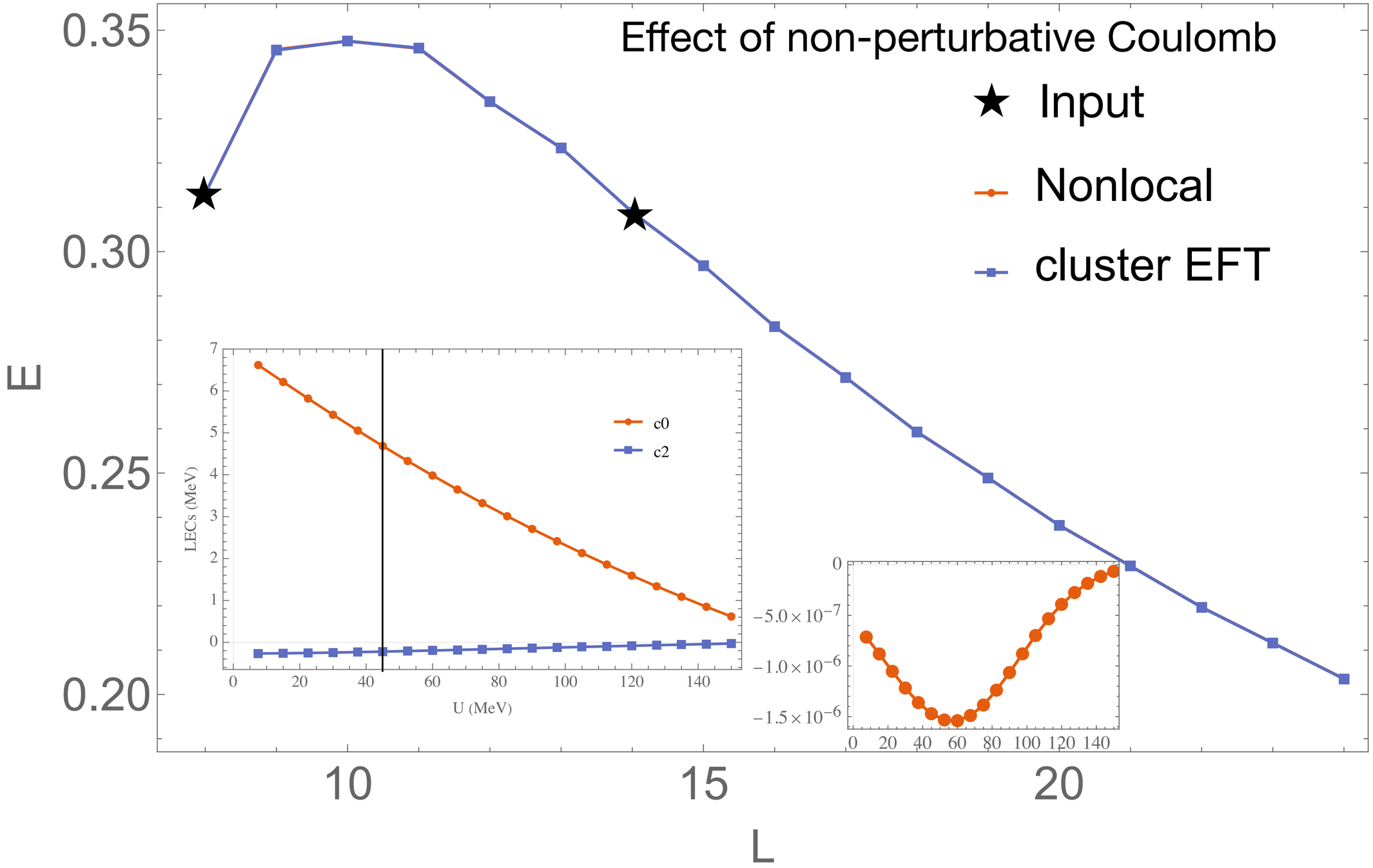
Finds $\mathcal{F} : \big(E(L_1), E(L_2)\big) \rightarrow (C_0, C_2)$

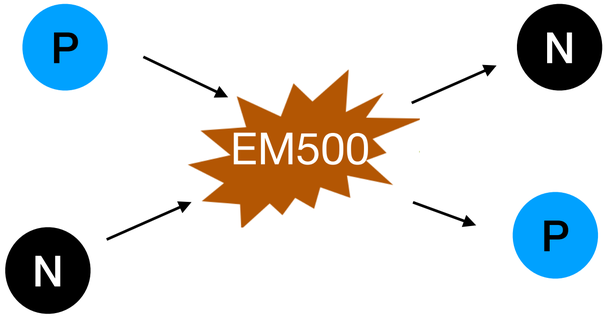
T-matrix **R-matrix** Spherical-Wall ...

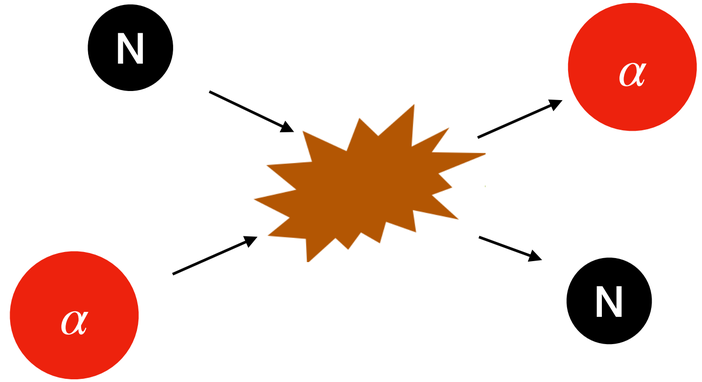
Fast Lüscher Substitution



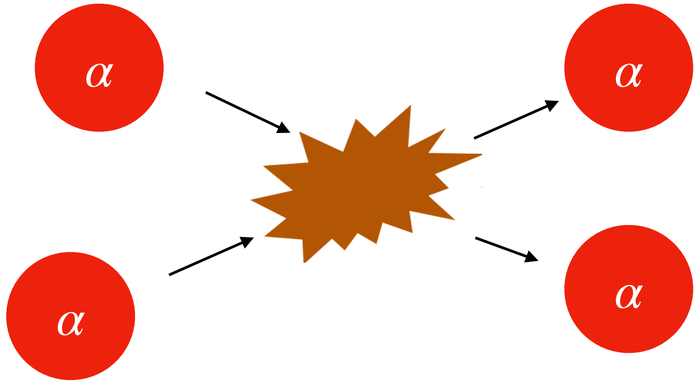
Training: cubic region of C_0, C_2 sampled by LHC sampler

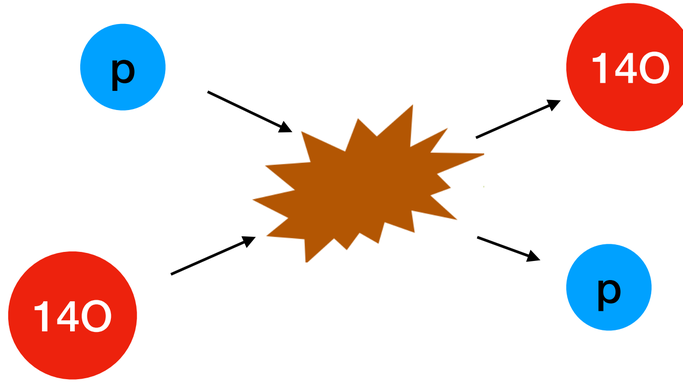

Effect of non-perturbative Coulomb
Input
Nonlocal
cluster EFT
E
L
0.35
0.30
0.25
0.20
10
15
20
LECs (MeV)
U (MeV)
c0
c2
0
20
40
60
80
100
120
140
1
2
3
4
5
6
7
-5.0×10^{-7}
-1.0×10^{-6}
-1.5×10^{-6}





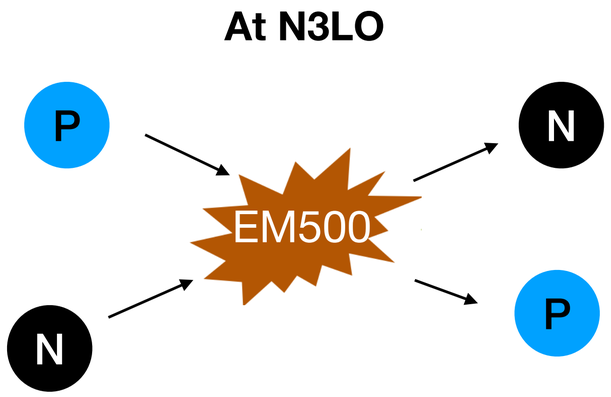
3. Realistic Interactions

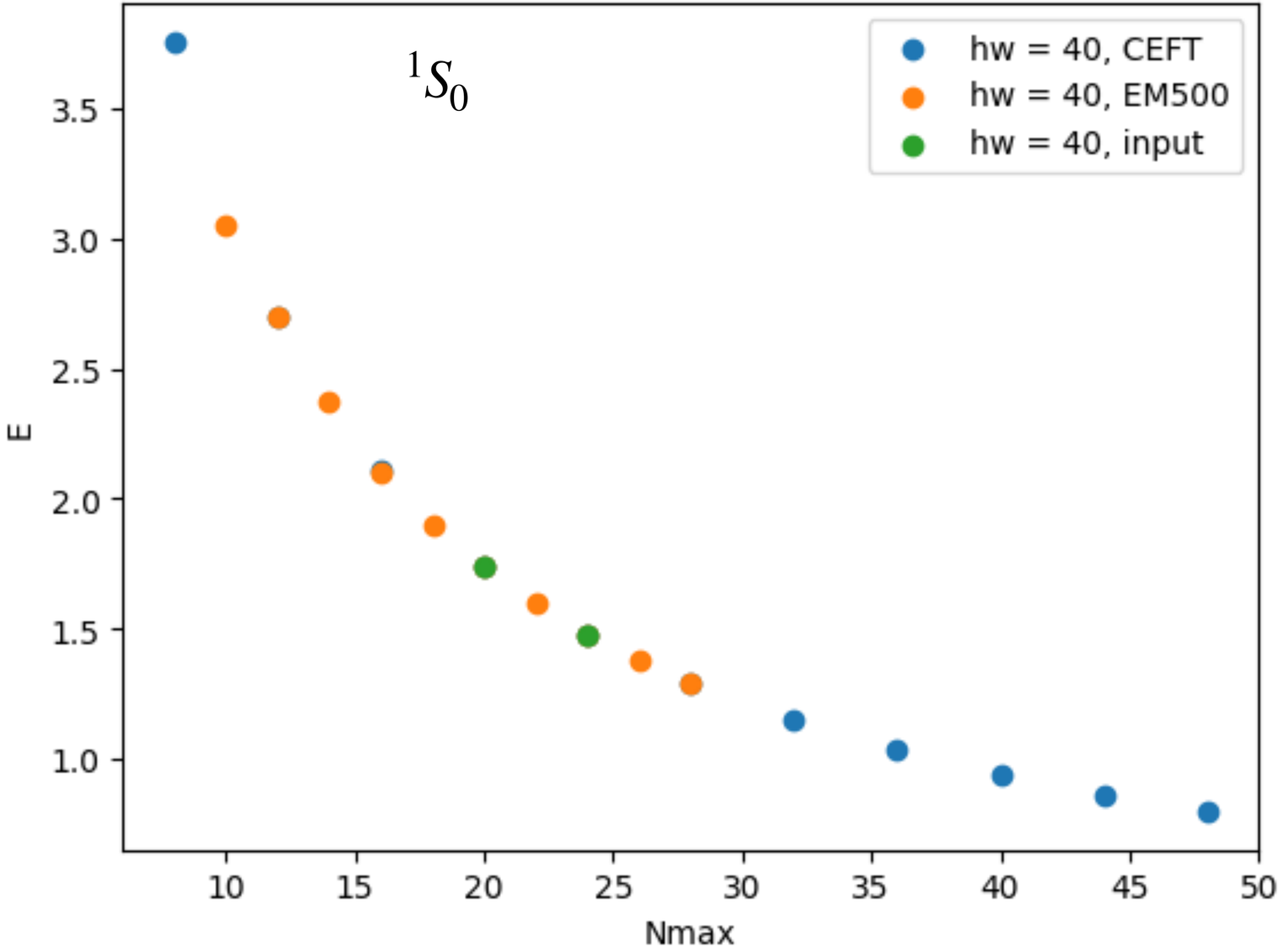




Simple test on n-p scattering

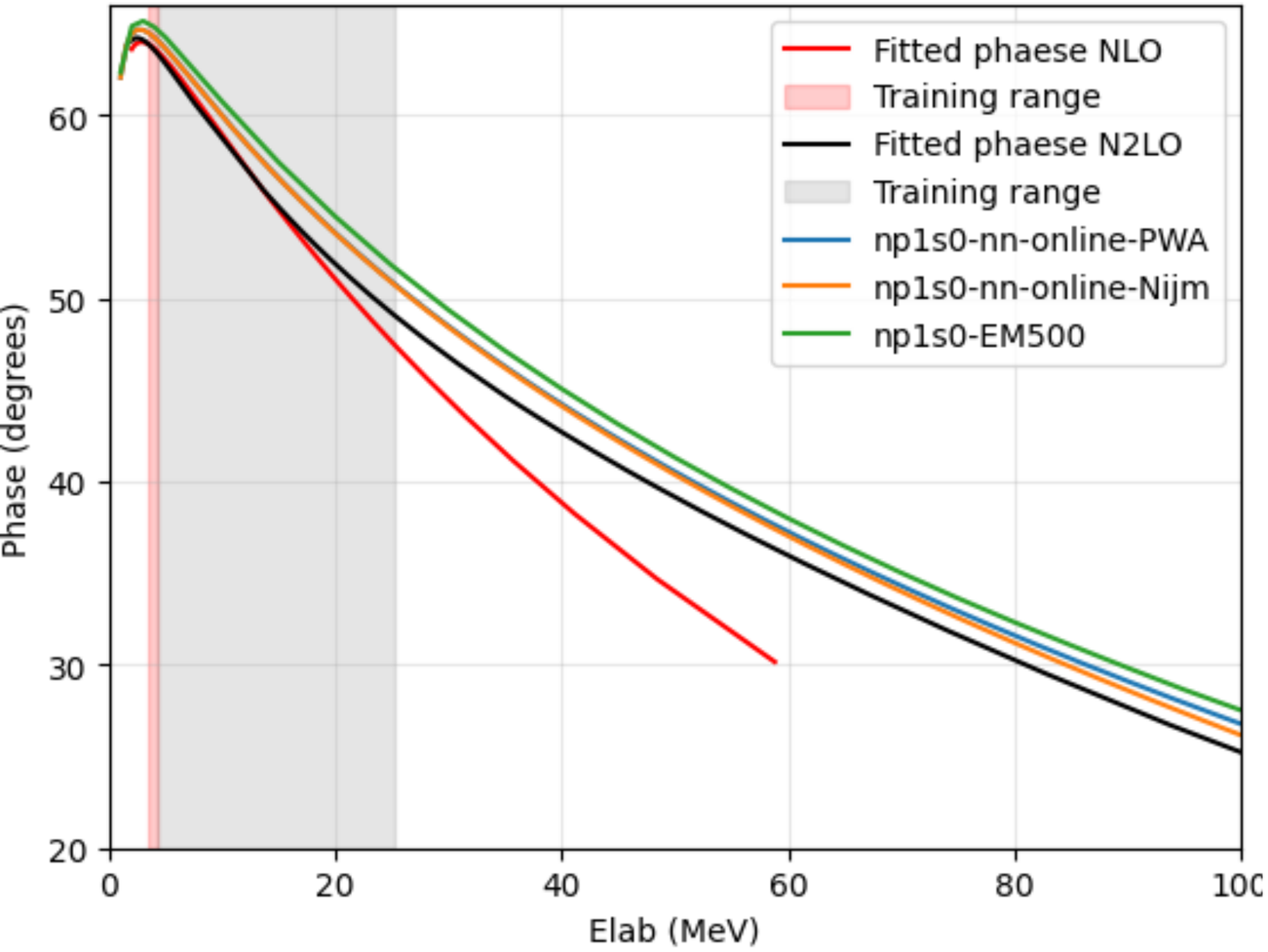
$$H_{n'l,n'l} = T_{n'l,n'l} + V_{n'l,n'l}$$



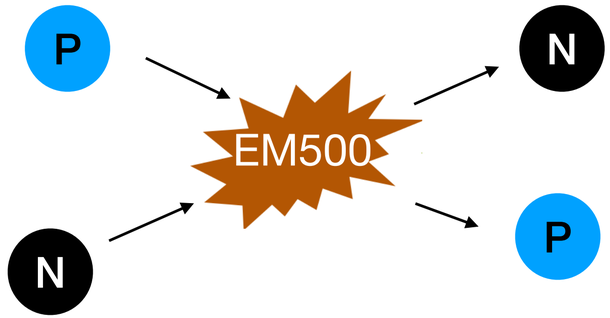


N2LO

N2LO CEFT: 3 Inputs, Power Counting Scheme



2 ground states + 1 excited

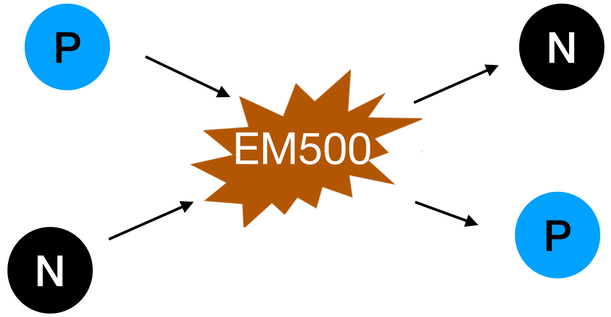


$^1S_0$

Nmax = 16, **24** , hw = 40 MeV

Cutoff $\sigma \sim 100\,\mathrm{MeV}$

N2LO

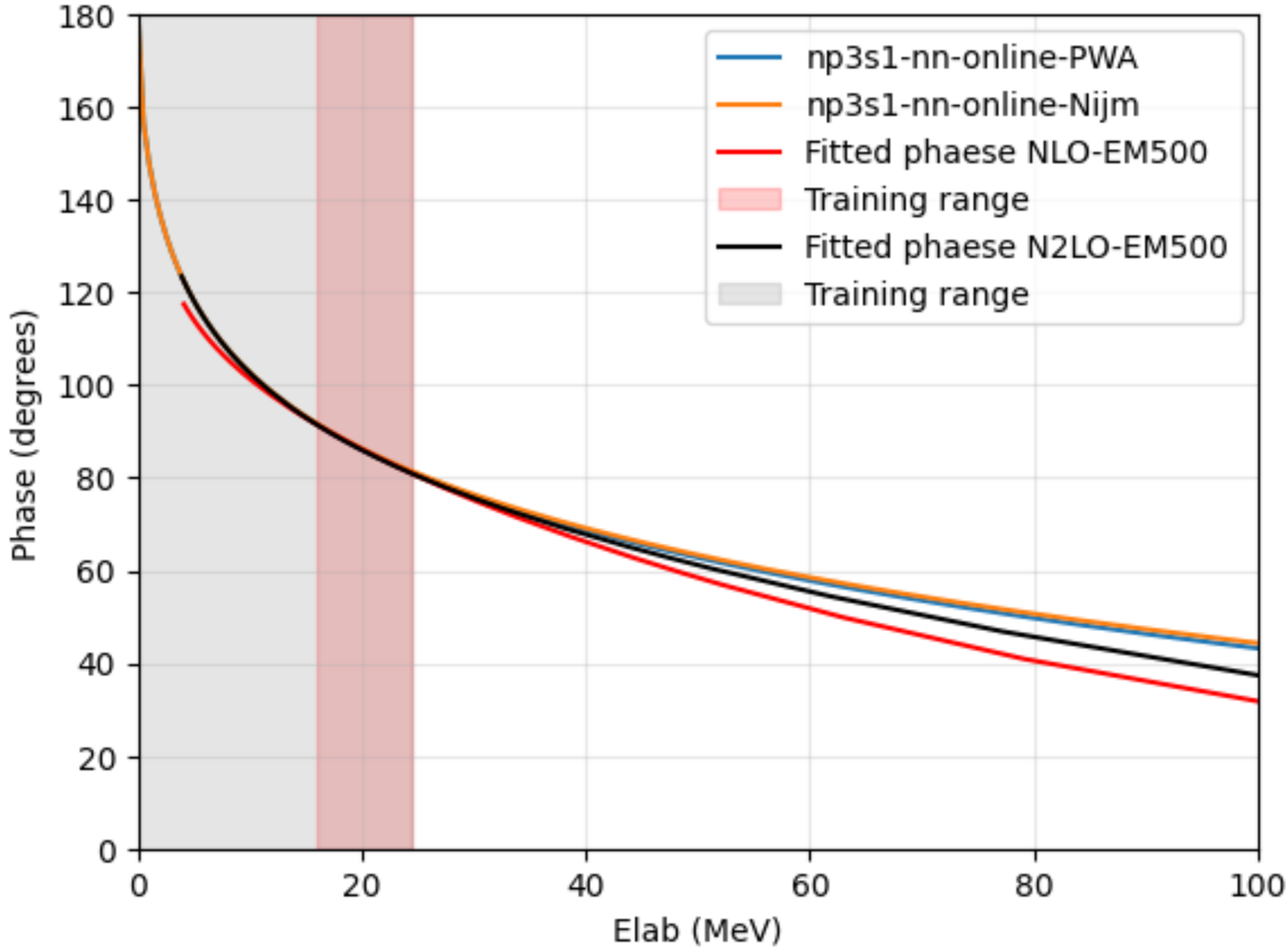


$^3S_1$

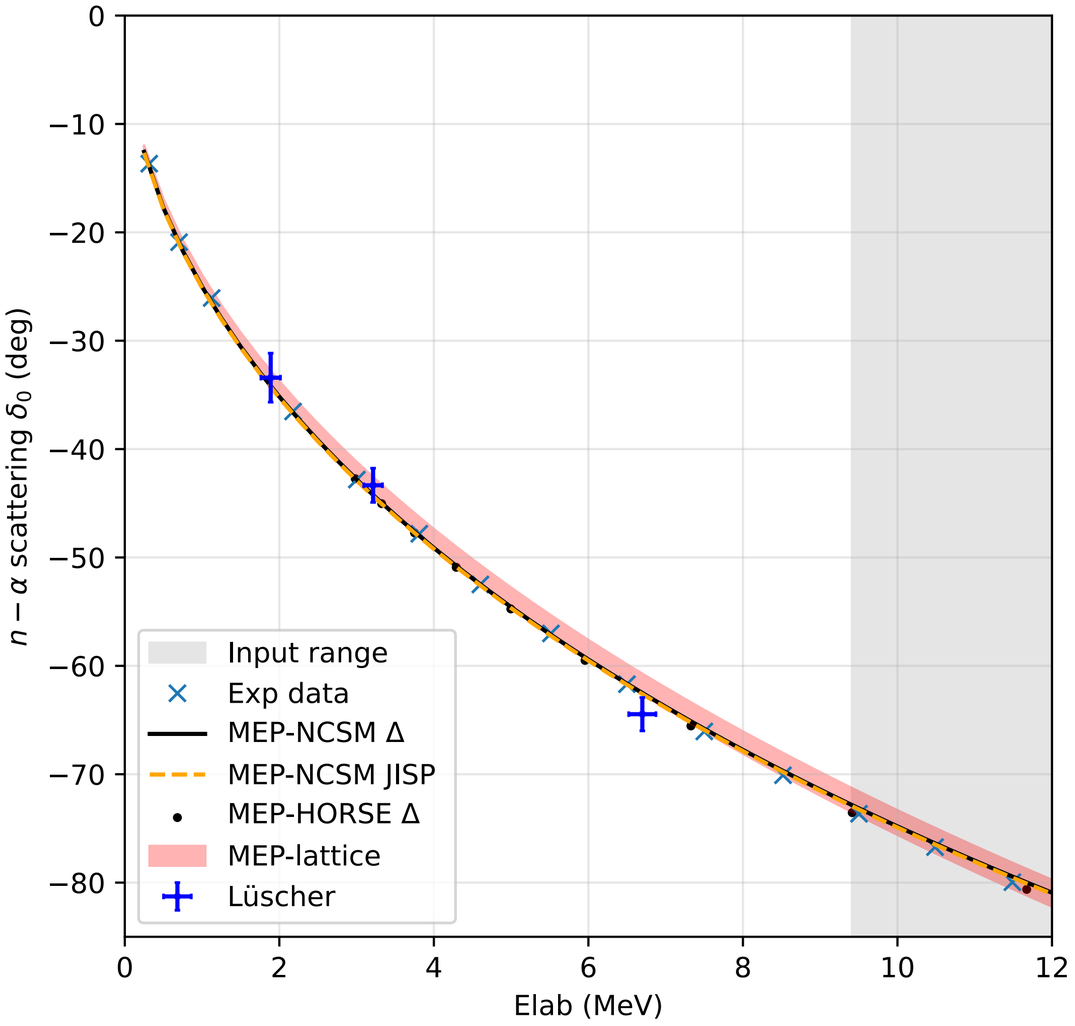
Nmax = 16, **24** , hw = 40 MeV

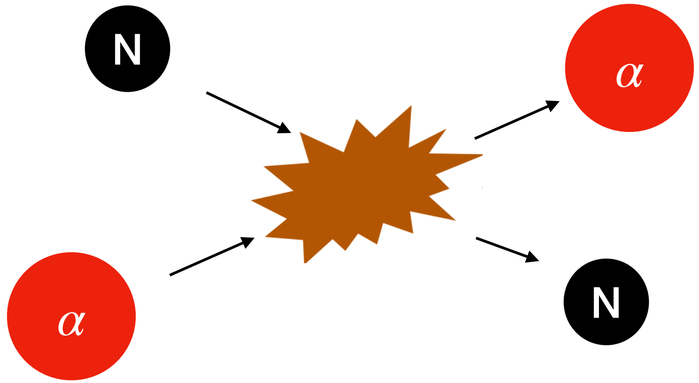
Cutoff $\sigma \sim 100\,\mathrm{MeV}$

Versatile: 1 bound-state + 2 scattering



Many-body validation...





NCSM (BIGSTICK)

Δ – full χ- EFT at NNLO (no 3N)

JISP16

Nmax = 9 - 15, hw = 25, 30 MeV

NLEFT

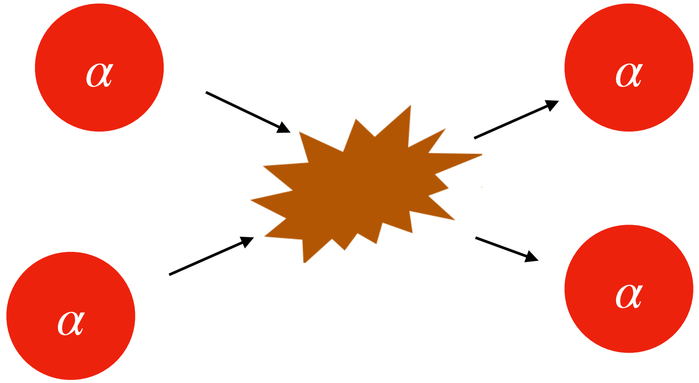
Chiral N3LO via wavefunction matching

S. Elhartisari et.al., *Nature* 630 (2024) 8015, 59-63

L = 6, 7, 8, $a_{\mathrm{latt}} = 1.315\,\mathrm{fm}$

Cutoff

$\sigma \sim 70 - 150\,\mathrm{MeV}$



NCSM (BIGSTICK)

JISP16

Nmax =10, 12, hw = 25 MeV

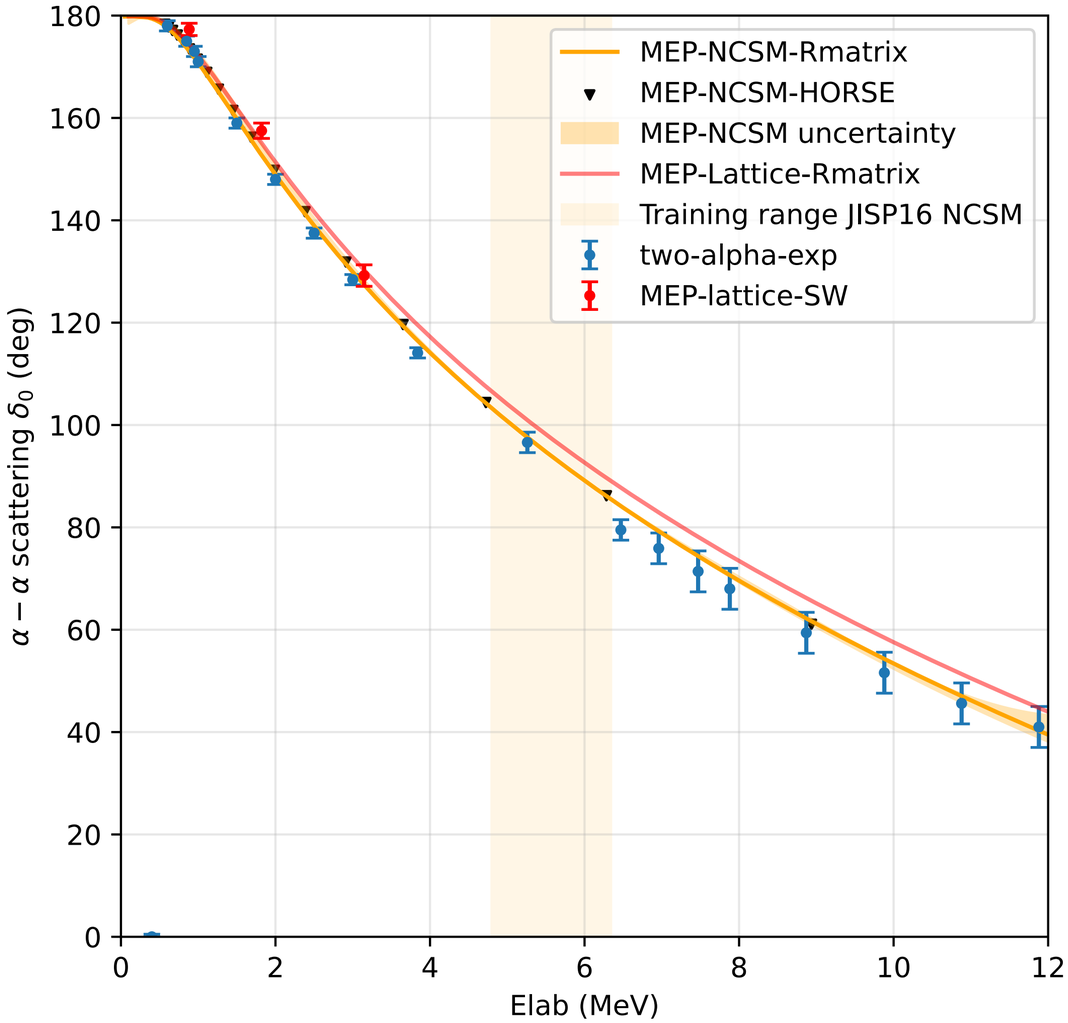
NLEFT

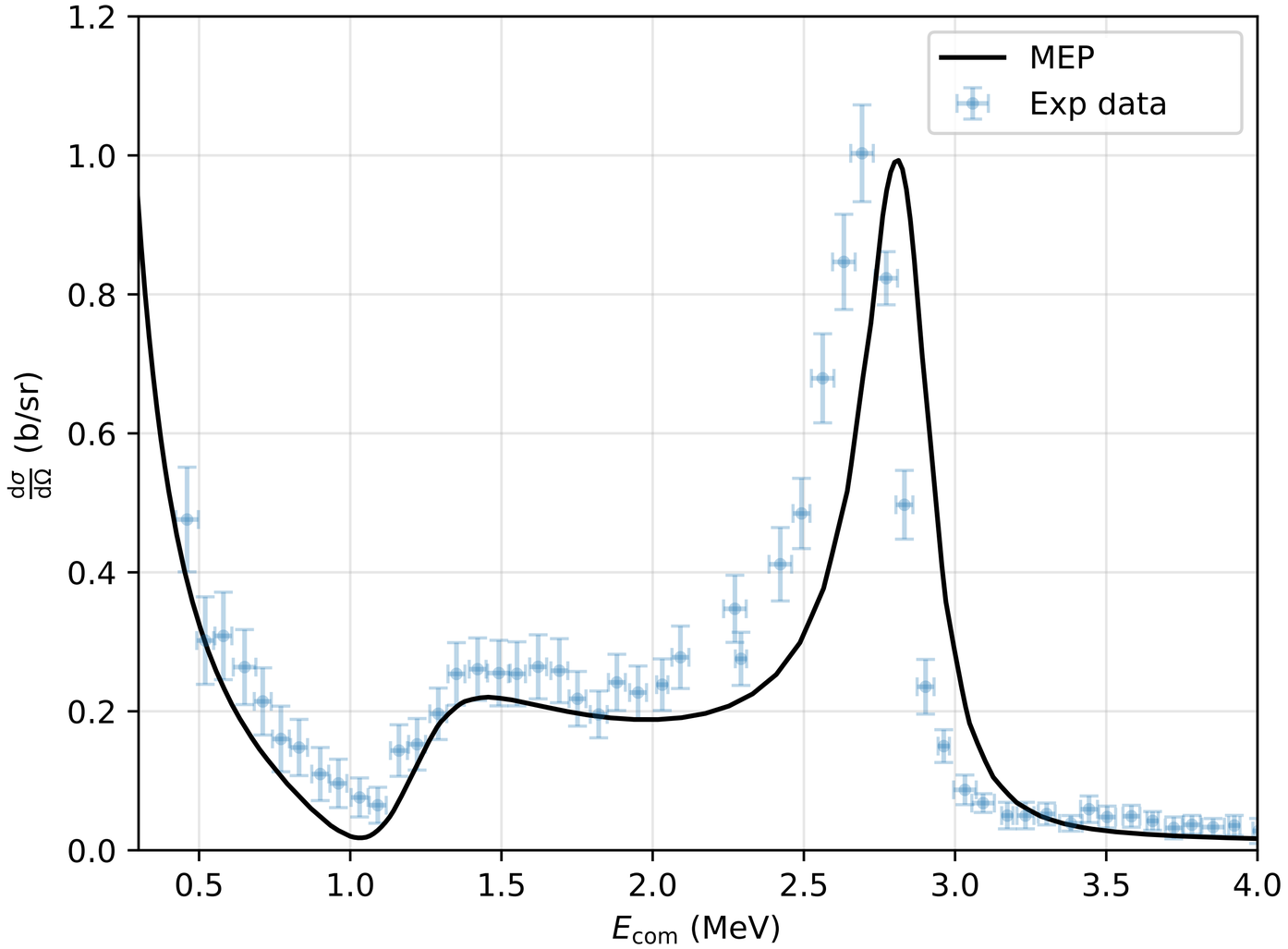
Chiral N3LO via wavefunction matching

L = 10, 12, $a_{\mathrm{latt}} = 1.315\,\mathrm{fm}$

Cutoff

$\sigma \sim 90 - 130\,\mathrm{MeV}$

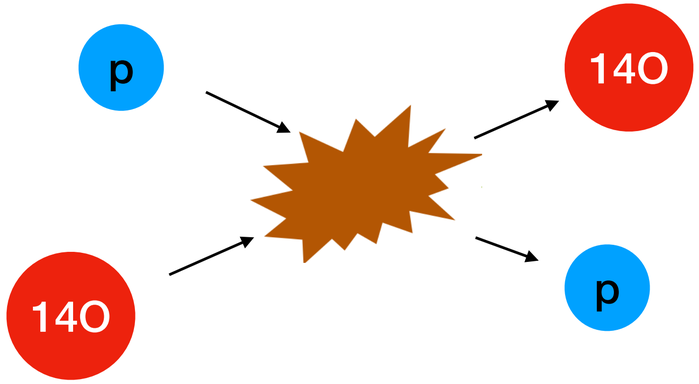




J^π	E_R^{exp} (MeV)	Γ^{exp} (keV)	E_R^{MEP} (MeV)	Γ^{MEP} (keV)
$1/2^+$	1.27(2)(2)	374(70)(+200)	1.25(3)(10)	379(14)(50)
$5/2^+$	2.81(12)	251(26)	2.83(5)(20)	265(20)(50)

Grancey et al., PLB 758, 26 (2016)
Girard-Alcindor et al., PRC 105, 51301 (2022)

@ GANIL



Δ – full χ- EFT at NNLO

VS-IMSRG **by imsrg++**

14O core + proton sd

at $\hbar\omega = 16\,\mathrm{MeV}$, $e_{max} = 10 - 14$, $E_{3max} = 24$

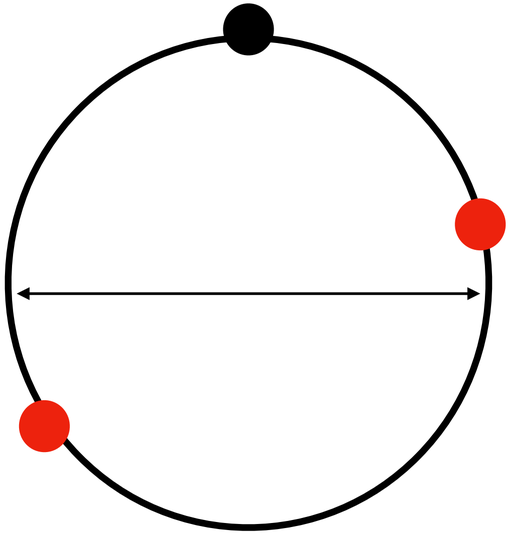
differential cross section at

180 deg (c.m.)

Cutoff $\sigma \sim 70 - 150\,\mathrm{MeV}$

4. Translational Invariance?

MCSM truncation



$$L \sim \sqrt{2N_{\text{shell}} + 3/2}$$

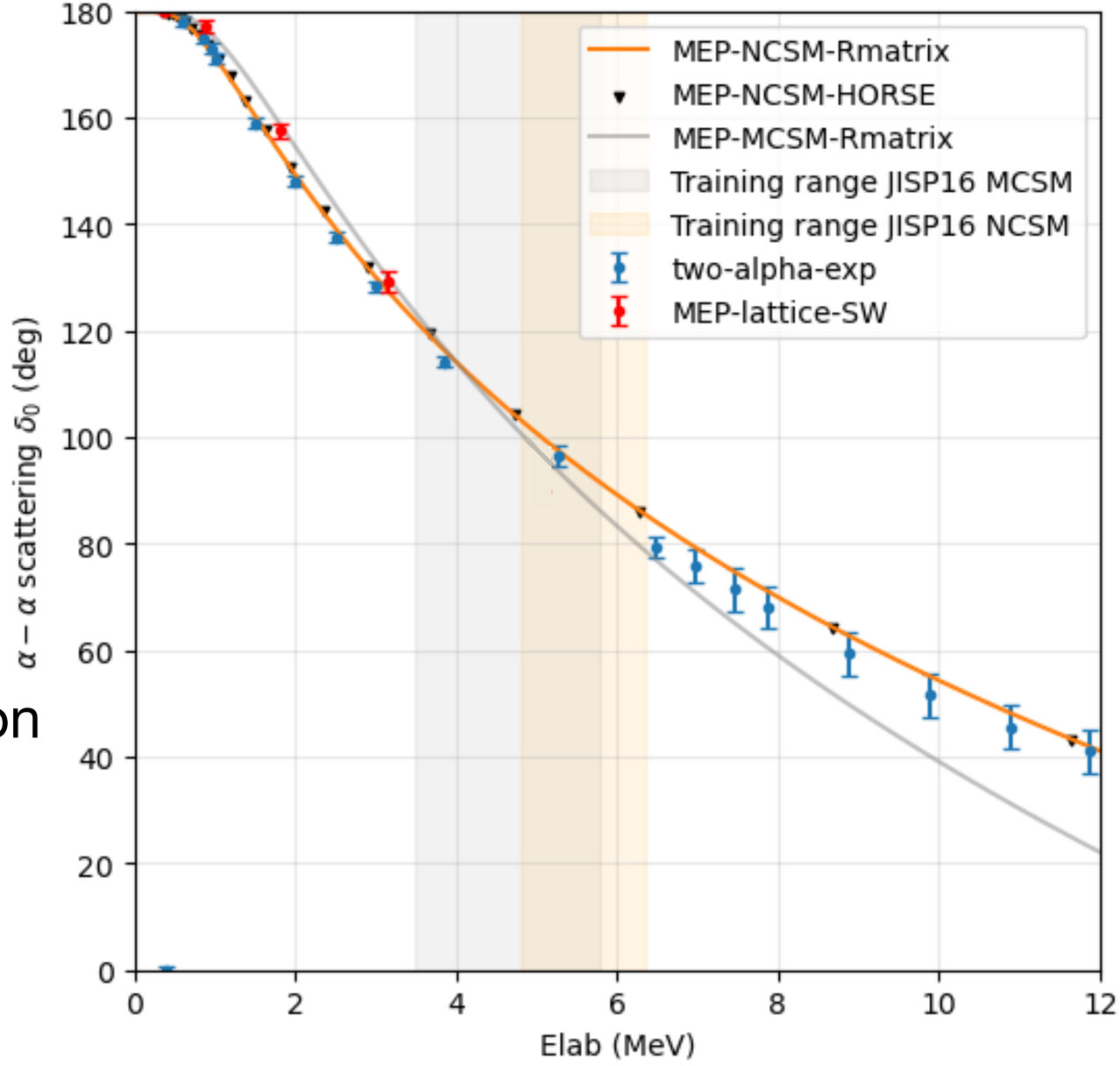
But no clean Talmi-Moshinsky factorization

$N_{\text{shell}} = 6,7$ **hw = 25, 30 MeV**

JISP16

T. Abe, et.al., Phys. Rev. C 104, 054315 (2021)

NCSM, MCSM, NLEFT



Summary

- FVH + MEP works in many ab initio methods (NCSM, NLEFT, VSIMSRG)
- Nuclear physics version of the Lüscher formalism
- UQ compatible from design
- **Inverse Problem** solver for non-linear systems
- Different parameterization options?

Outlook

- Lattices
- HO basis
- TRAP method, other bases, *in progress*
- ....

- Formal EFT matching
- Beyond two body
- Coupled channels ...

A General *ab initio* Reaction Method*

Many thanks...

- To my collaborators, S. Elhatisari, S. König, D. Lee, Y.Z. Ma, and T. Miyagi...
- 筑波大学 計算科学研究センター Center for Computational Sciences TOMOE
- And to you!





Backup slides