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Green's function approach to nuclear reactions

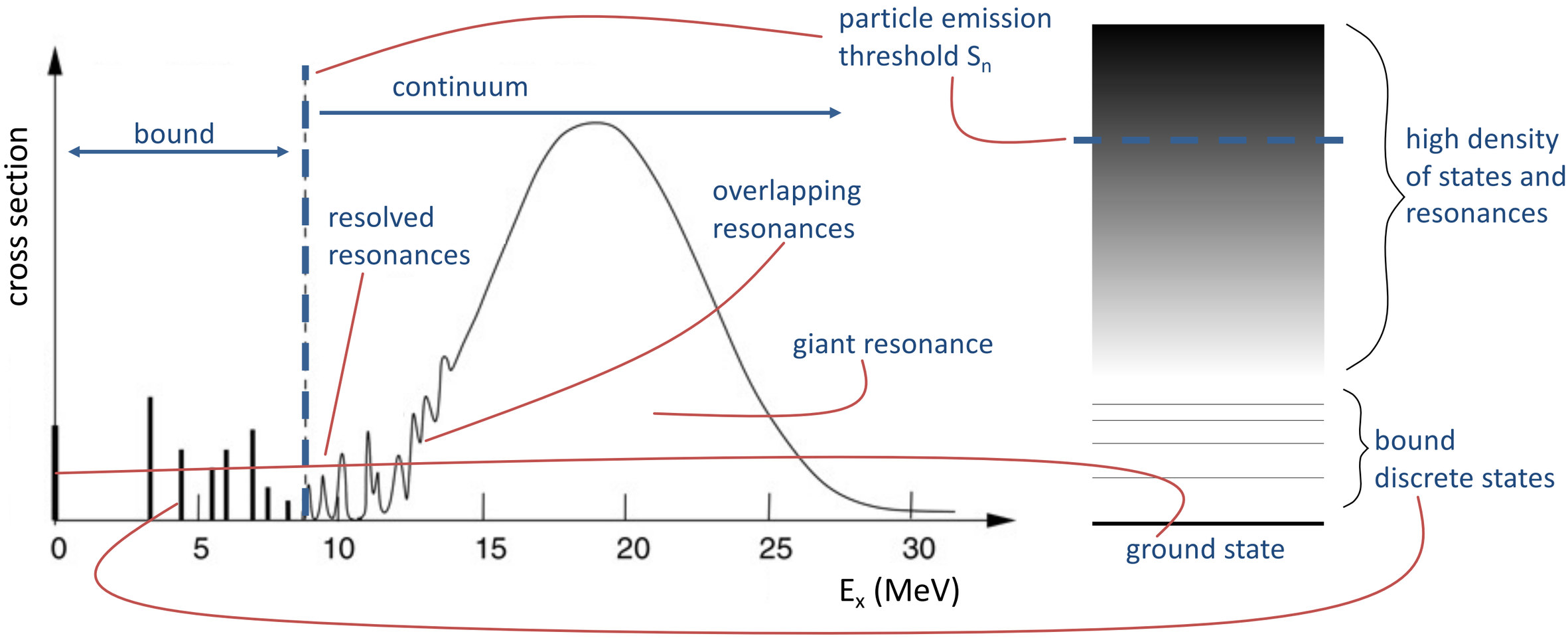
Gregory Potel Aguilar

Trento, May 18, 2026

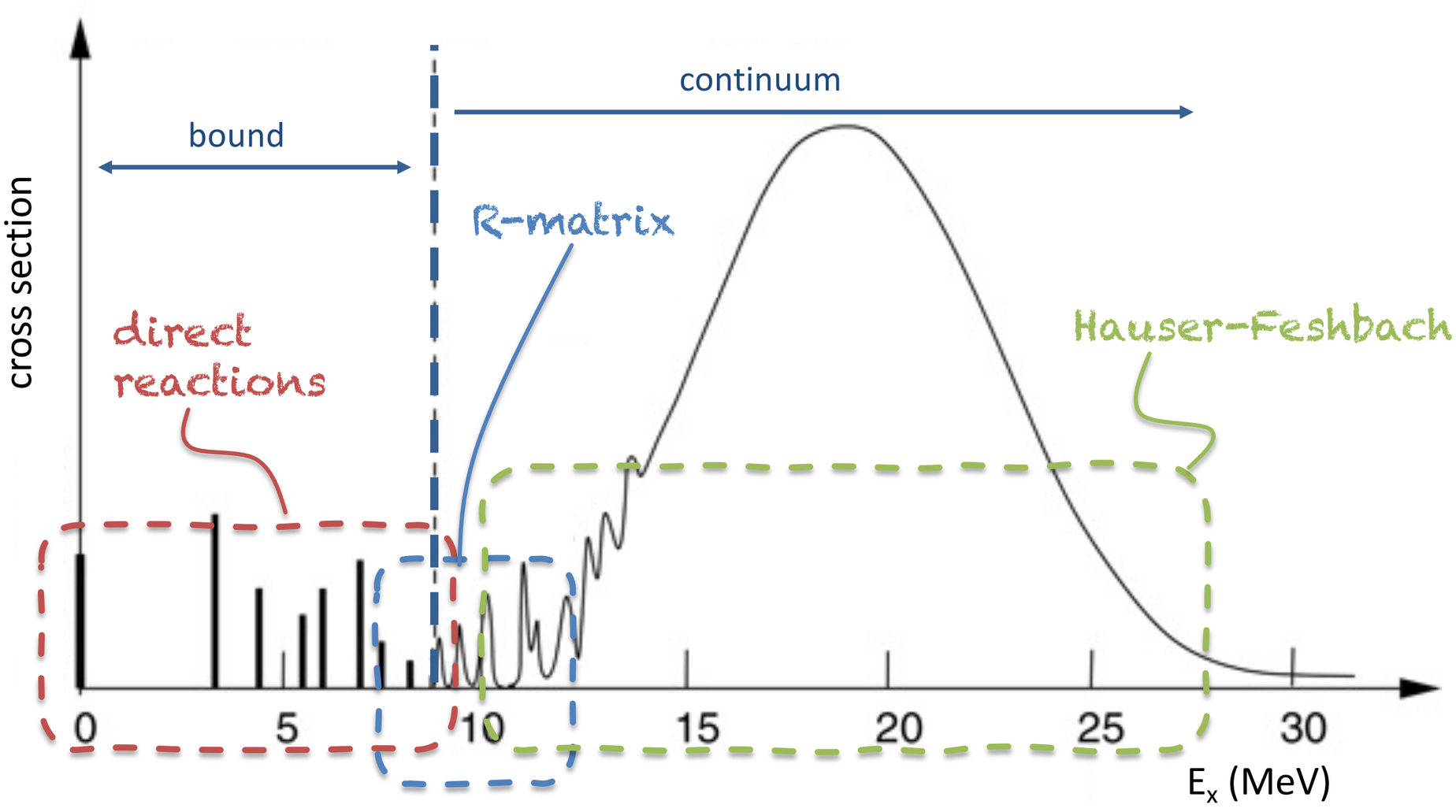
UNIVERSIDAD Ð SEVILLA 1505

Escuela Técnica Superior de INGENIERÍA DE SEVILLA

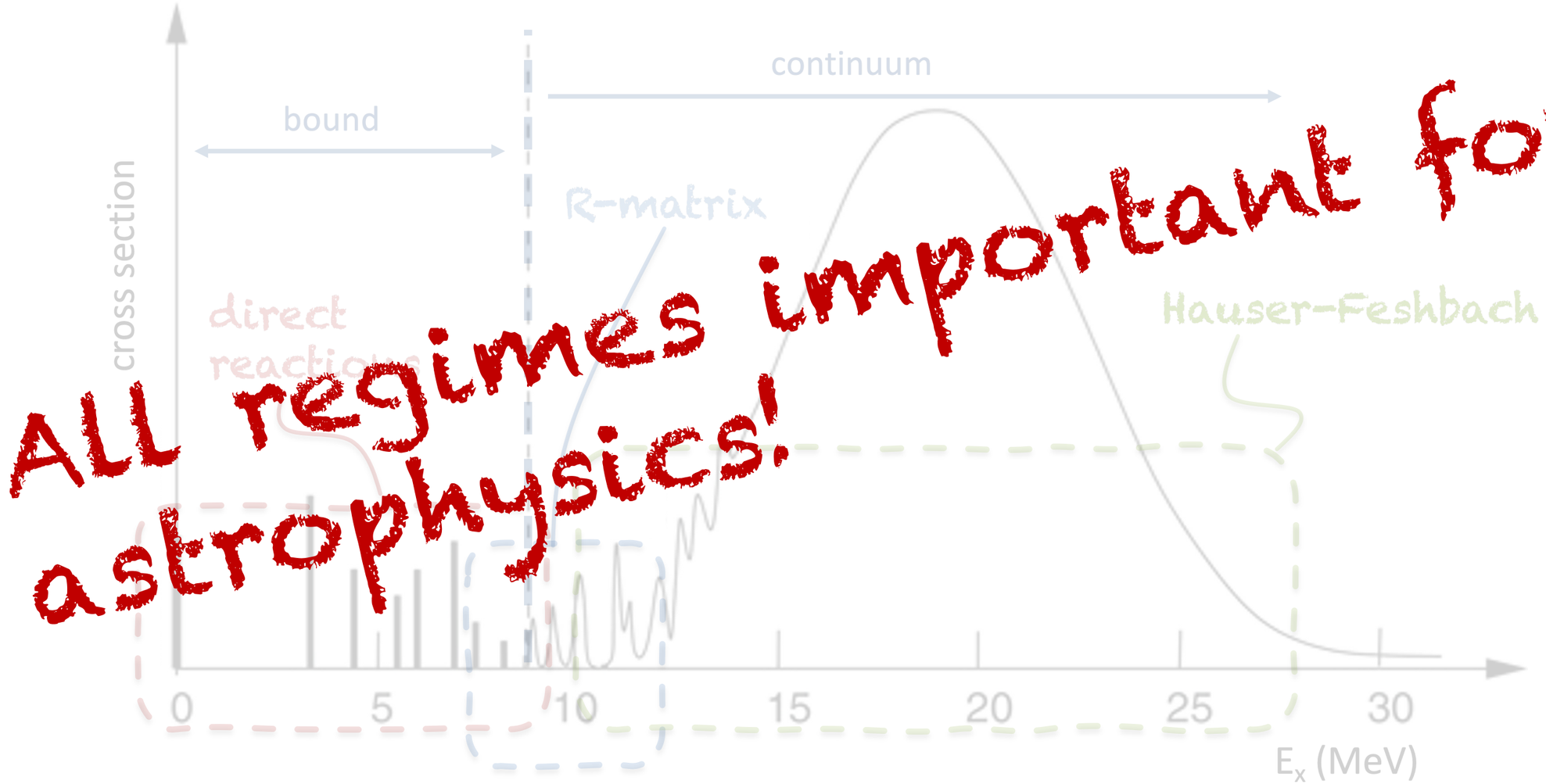
As theorists, we need to deal with all features of nuclear spectra



As theorists, we need to describe reactions across different regimes

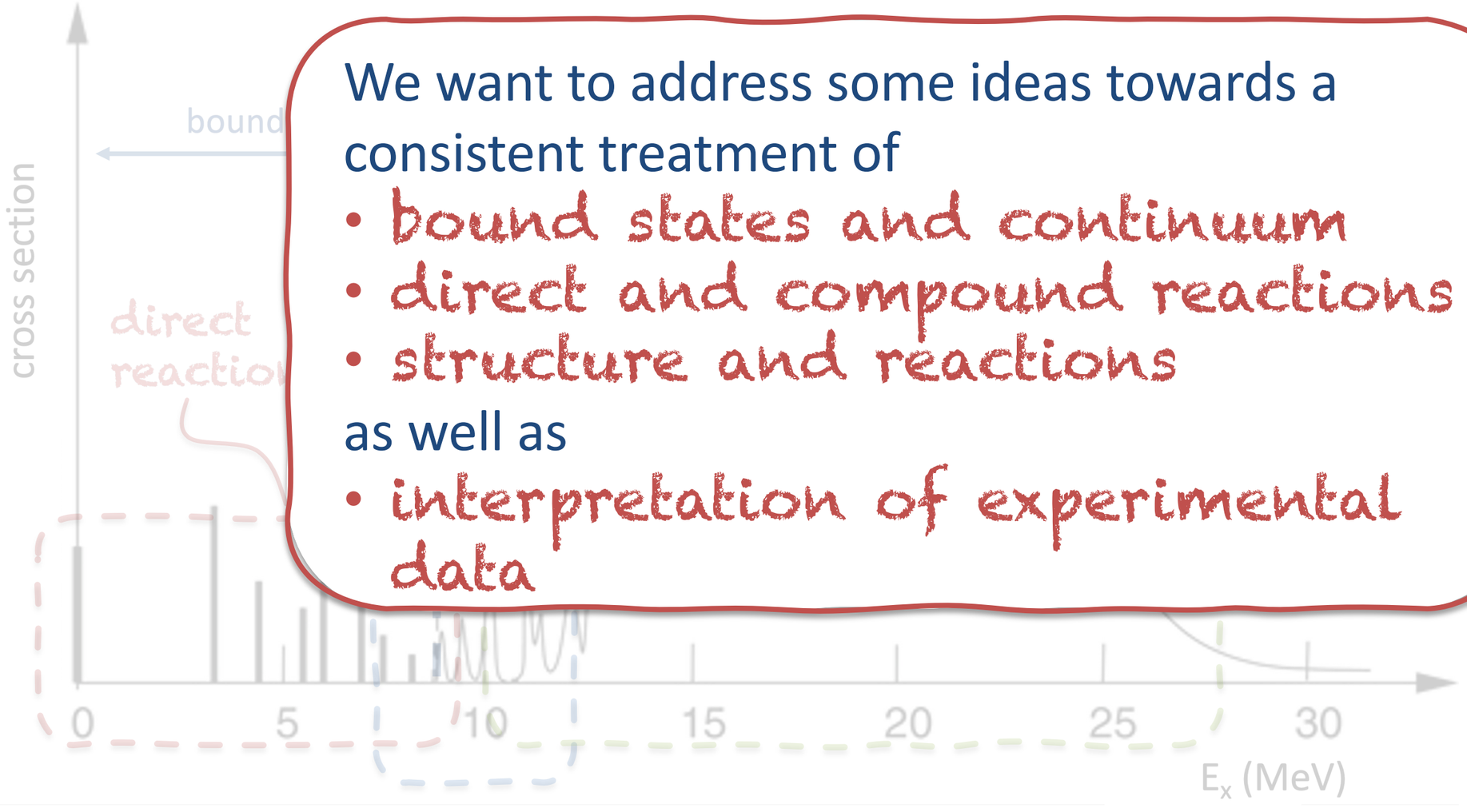


As theorists, we need to describe reactions across different regimes



ALL regimes important for astrophysics!

Can we treat everything on the same footing?



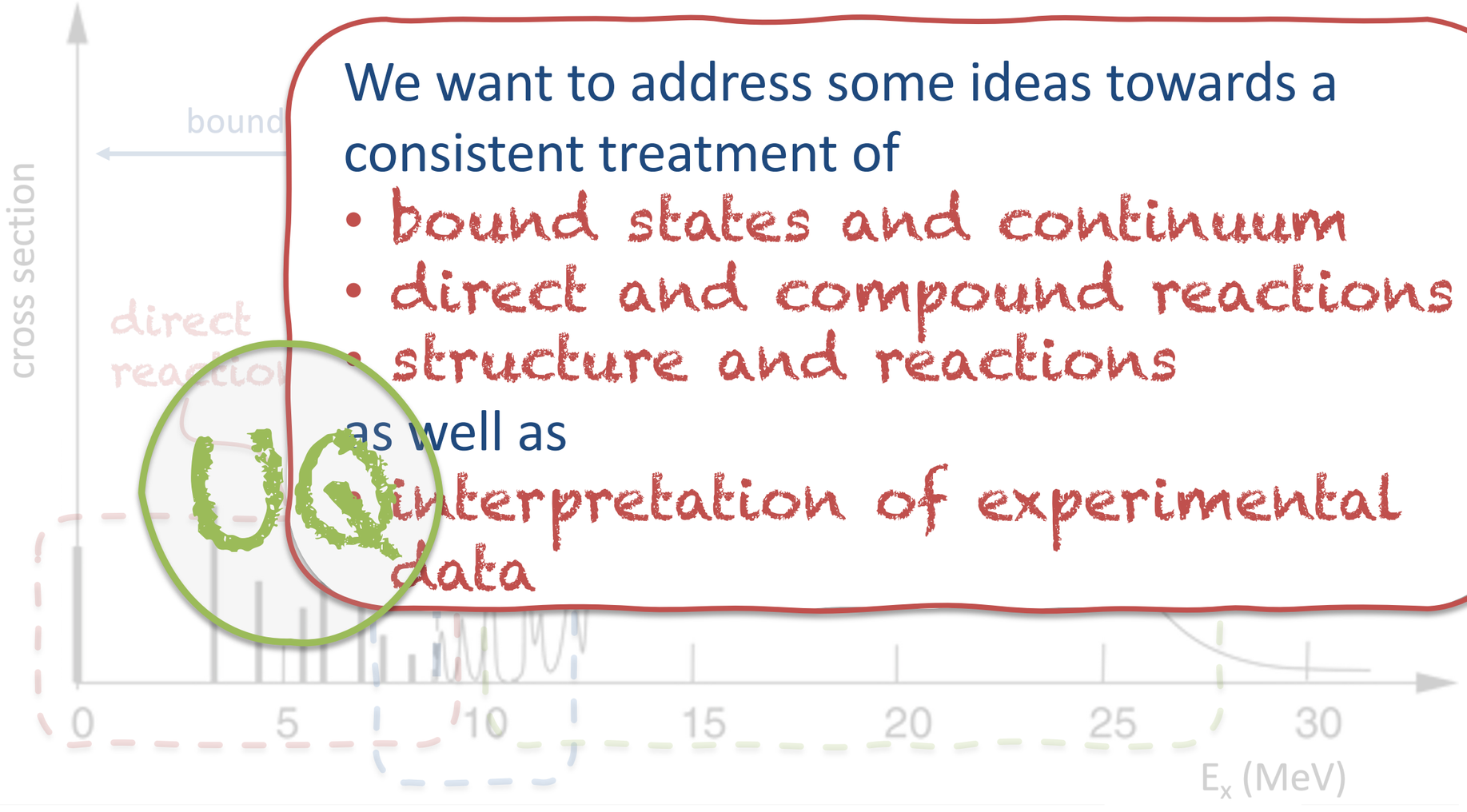
We want to address some ideas towards a consistent treatment of

- bound states and continuum
- direct and compound reactions
- structure and reactions

as well as

- interpretation of experimental data

Can we treat everything on the same footing?



We want to address some ideas towards a consistent treatment of

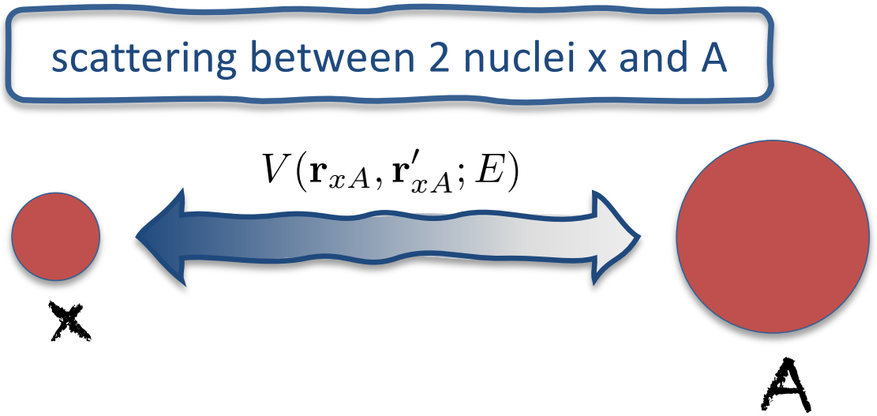
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UQ

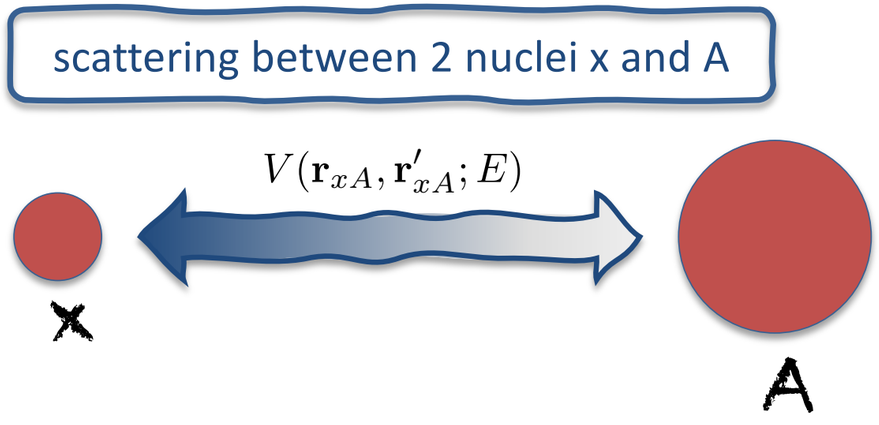
Green's Function Transfer (GFT) formalism in a nutshell



$$\psi_0(r_{xA}, E) = e^{i\mathbf{kr}_{xA}} + G(E)\,V(E)\,e^{i\mathbf{kr}_{xA}}$$

incoming free wave

Green's Function Transfer (GFT) formalism in a nutshell

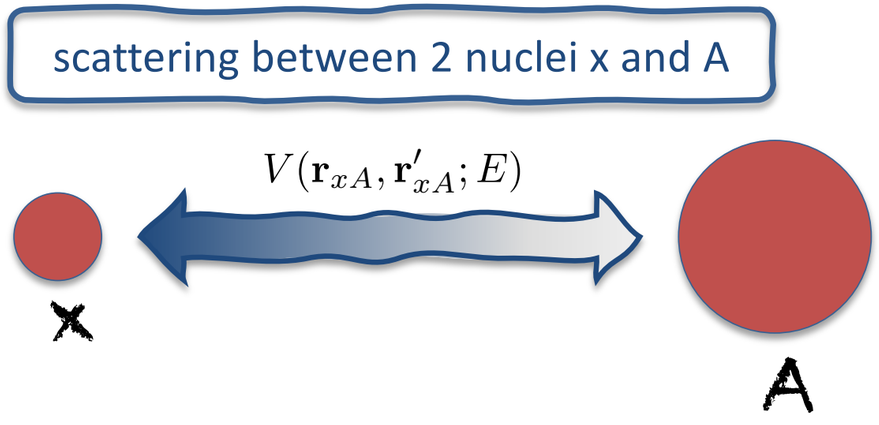


$$\psi_0(r_{xA}, E) = e^{i\mathbf{k}\mathbf{r}_{xA}} + G(E)\, V(E)\, e^{i\mathbf{k}\mathbf{r}_{xA}}$$

Green's function

$$G(E) = (E - T_x - V(E))^{-1}$$

Green's Function Transfer (GFT) formalism in a nutshell

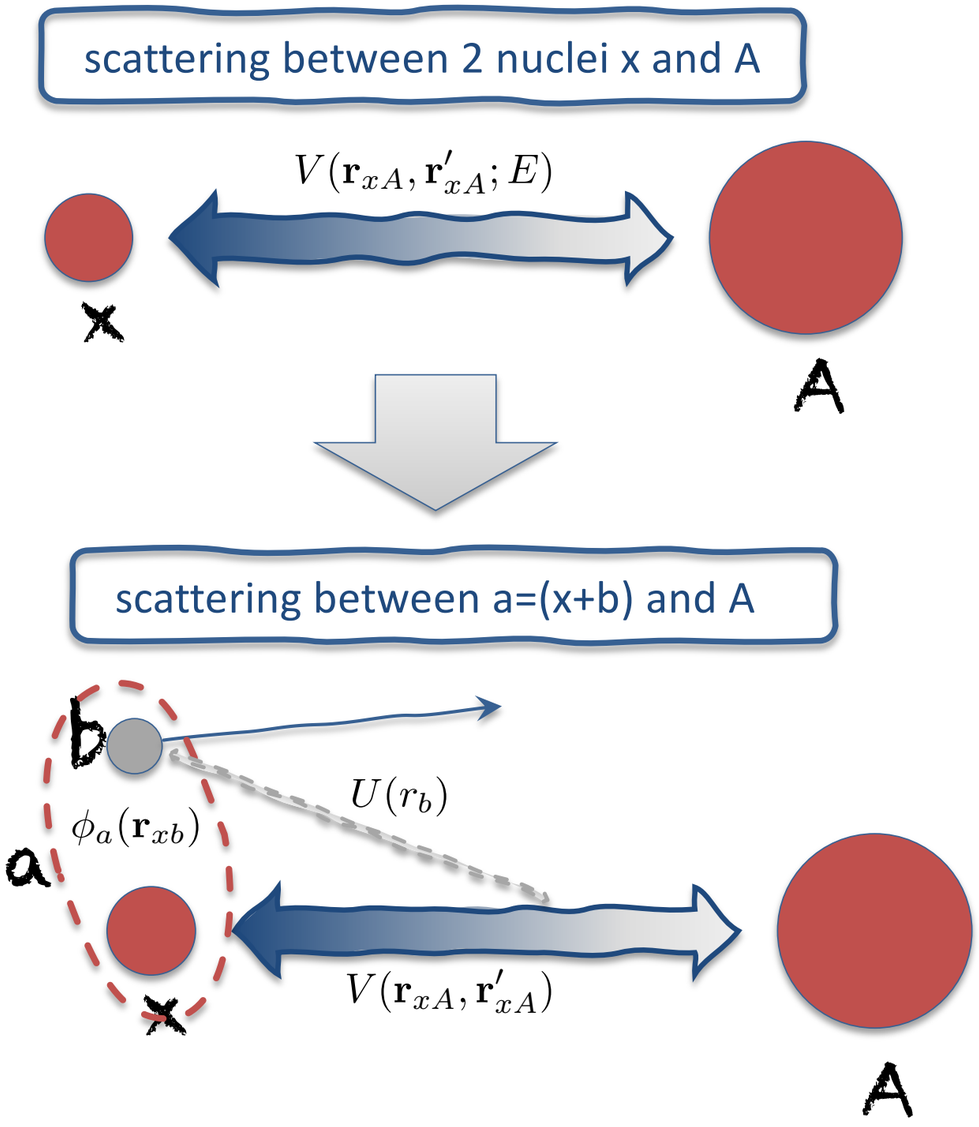


$$\psi_0(r_{xA}, E) = e^{i\mathbf{kr}_{xA}} + G(E)\, V(E)\, e^{i\mathbf{kr}_{xA}}$$

reaction cross section

$$\sigma_R = \frac{2\mu}{\hbar k_x} \langle \psi_0 | \mathrm{Im} V | \psi_0 \rangle$$

Green's Function Transfer (GFT) formalism in a nutshell



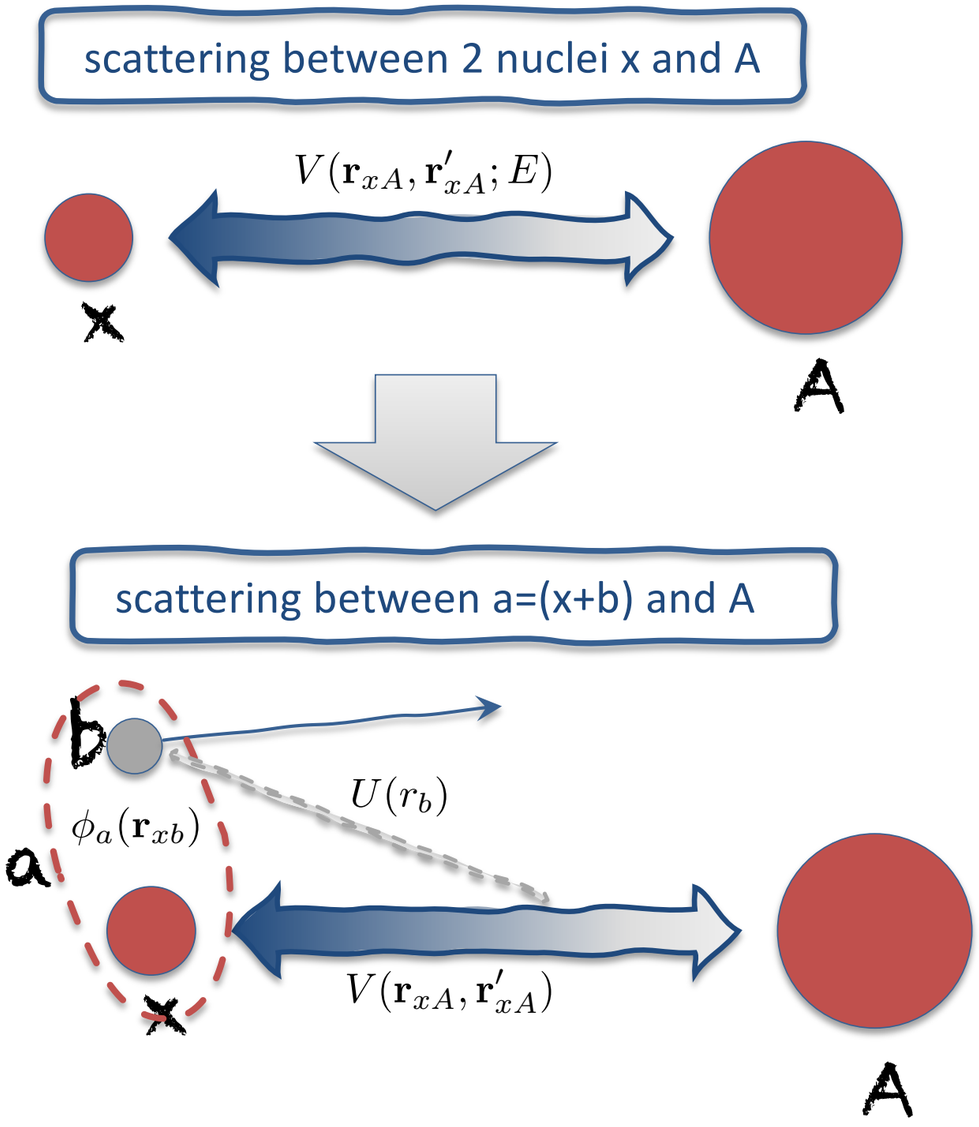
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Spectator approximation: *b* decoupled from *x-A*

Green's Function Transfer (GFT) formalism in a nutshell



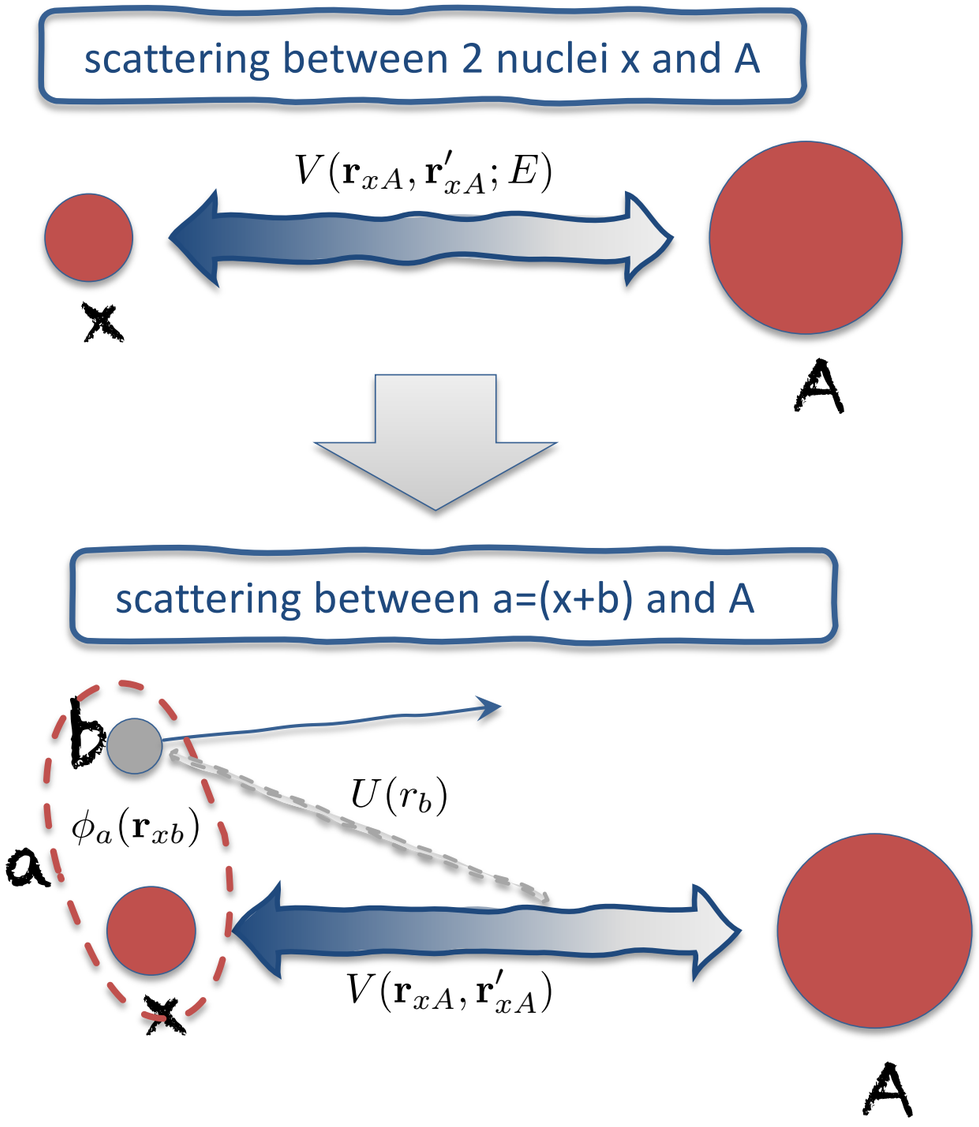
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$$\sigma_R = \frac{2\mu}{\hbar k_x}\langle\psi_0|\mathrm{Im}V|\psi_0\rangle$$

$$\psi_0^I(\mathbf{r}_{xA}, E) = \psi^{HM}(\mathbf{r}_a) + G(E - E_b)V(E - E_b)\psi^{HM}$$

Green's Function Transfer (GFT) formalism in a nutshell



$$\psi_0(r_{xA}, E) = e^{i\mathbf{k}\mathbf{r}_{xA}} + G(E)\, V(E)\, e^{i\mathbf{k}\mathbf{r}_{xA}}$$

reaction cross section

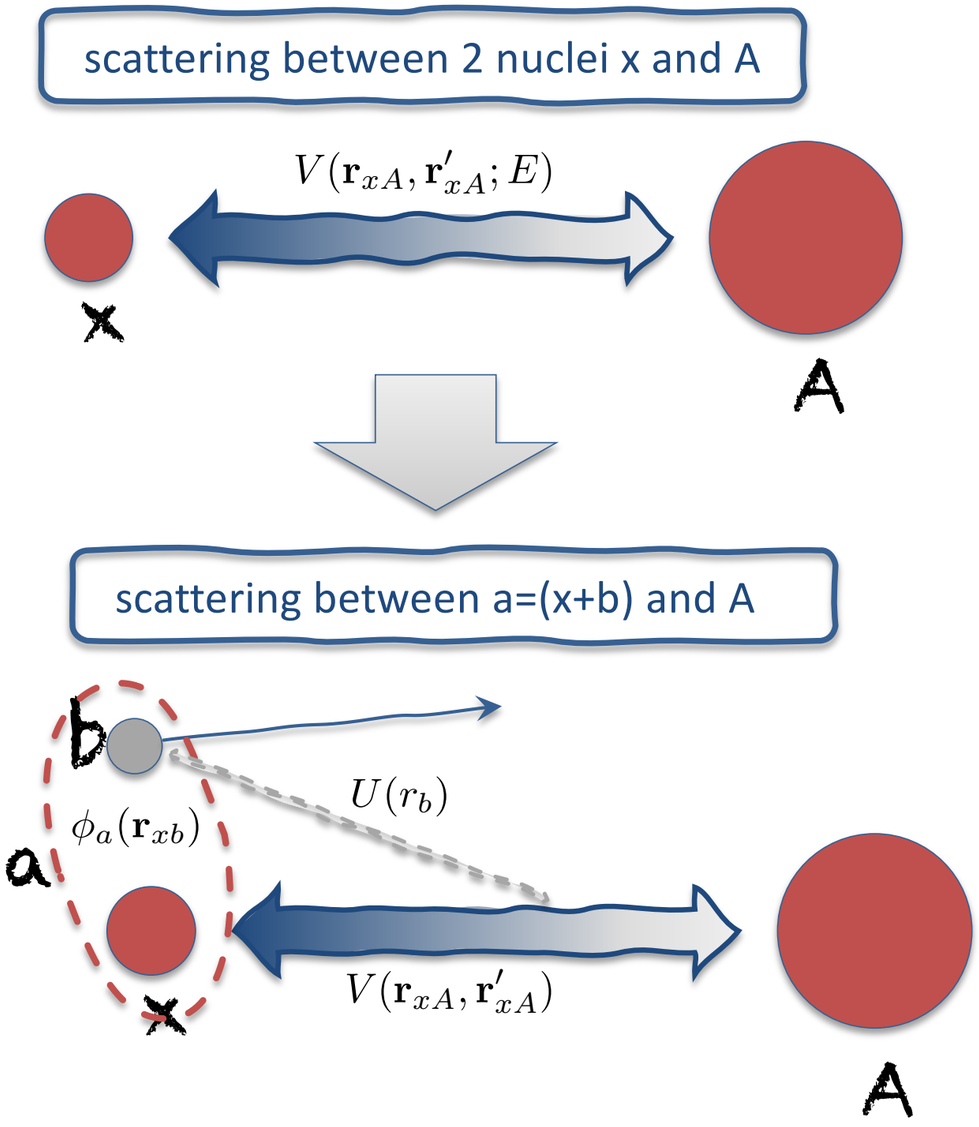
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$$\psi^{HM}(\mathbf{r}_{xA}) = \int \chi_b^*(\mathbf{r}_{bB}, \mathbf{k}_b) \phi_a(\mathbf{r}_{xb}) e^{i\mathbf{k}\mathbf{r}_a}\, d\mathbf{r}_{xb}$$

entrance channel

Green's Function Transfer (GFT) formalism in a nutshell



$$\psi_0(r_{xA}, E) = e^{i\mathbf{k}\mathbf{r}_{xA}} + G(E)\,V(E)\,e^{i\mathbf{k}\mathbf{r}_{xA}}$$

reaction cross section

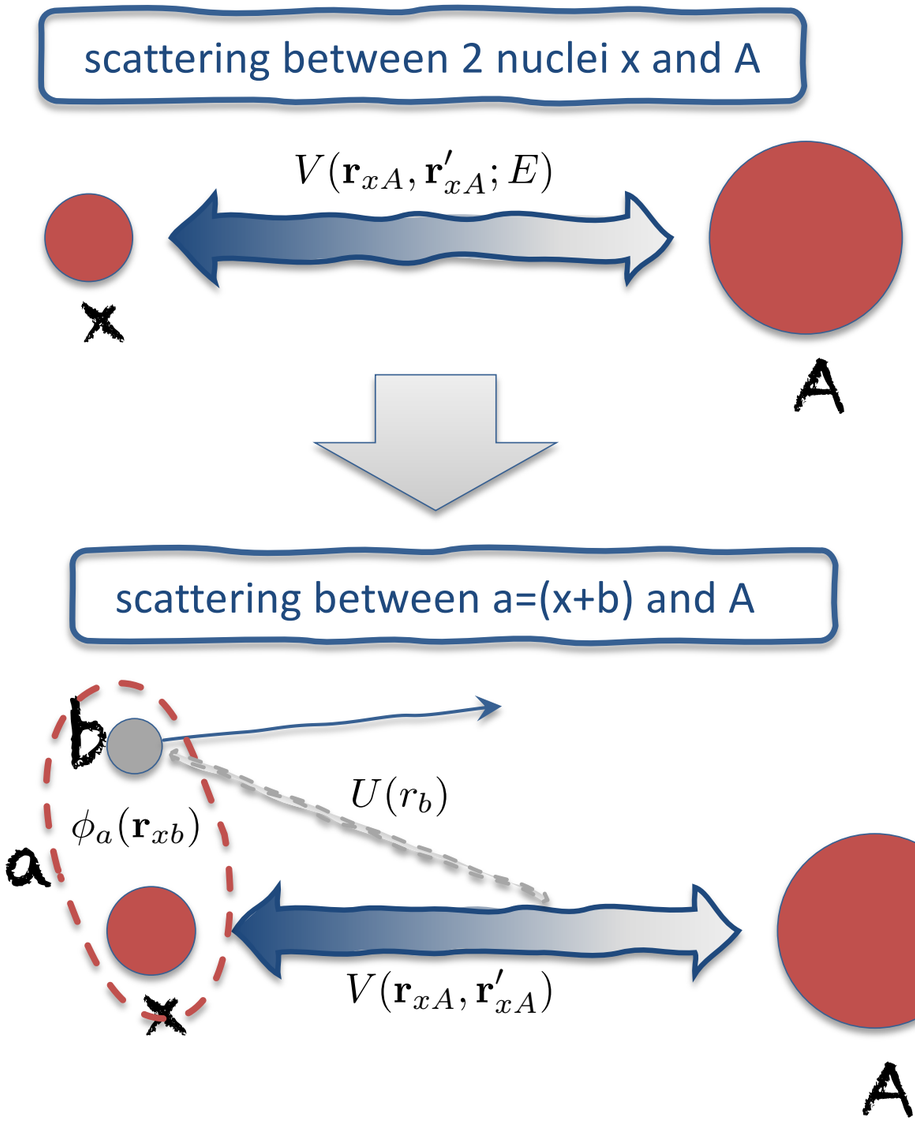
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$$\psi_0^I(\mathbf{r}_{xA}, E) = \psi^{HM}(\mathbf{r}_a) + G(E - E_b)V(E - E_b)\psi^{HM}$$

reaction cross section

$$\sigma_R^I(E, E_b) = \frac{2\mu}{\hbar k_x}\langle\psi_0^I|\mathrm{Im}V(E - E_b)|\psi_0^I\rangle$$

Green's Function Transfer (GFT) formalism in a nutshell



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$$\psi_0^I(\mathbf{r}_{xA}, E) = \psi^{HM}(\mathbf{r}_a) + G(E - E_b) V(E - E_b) \psi^{HM}$$

$$G(E) = (E - T_x - V(E))^{-1}$$

- *G* (reaction) consistent with *V* (structure)
- Same dynamics as x-A scattering
- Entrance channel modified

Treating structure and reaction on the same footing

$$\langle e^{i\mathbf{k}_f \mathbf{r}_{xA}} \phi_0(\xi) | \psi_0(r_{xA}, \xi, E) \rangle = \langle e^{i\mathbf{k}_f \mathbf{r}_{xA}} \phi_0(\xi) | G(E)\, \mathcal{V}(E) | e^{i\mathbf{k}\mathbf{r}_{xA}} \phi_0(\xi) \rangle$$

classification according to asymptotic behavior

$$\lim_{r_{xA}\to\infty} = 0 \rightarrow \text{structure}$$

$$\lim_{r_{xA}\to\infty} = \text{oscillatory} \rightarrow \text{reaction}$$

some possible implementations of structure (*V(E)*)

- ab-initio,
- EFT,
- QRPA+G-matrix,
- shell model,
- phenomenology,
- ... (Elster, Sargsyan, Barbieri)

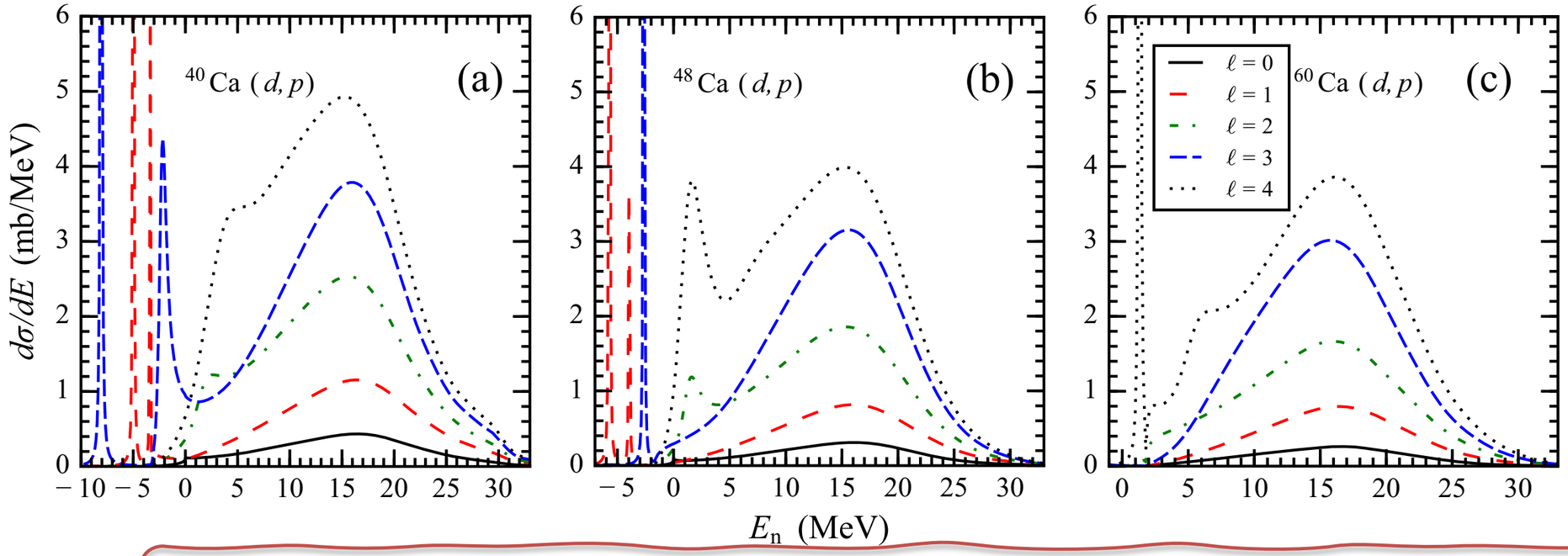
GFT with the Dispersive Optical Model (DOM); Ca(d,p)

Eur. Phys. J. A (2017) **53**: 178

THE EUROPEAN PHYSICAL JOURNAL A

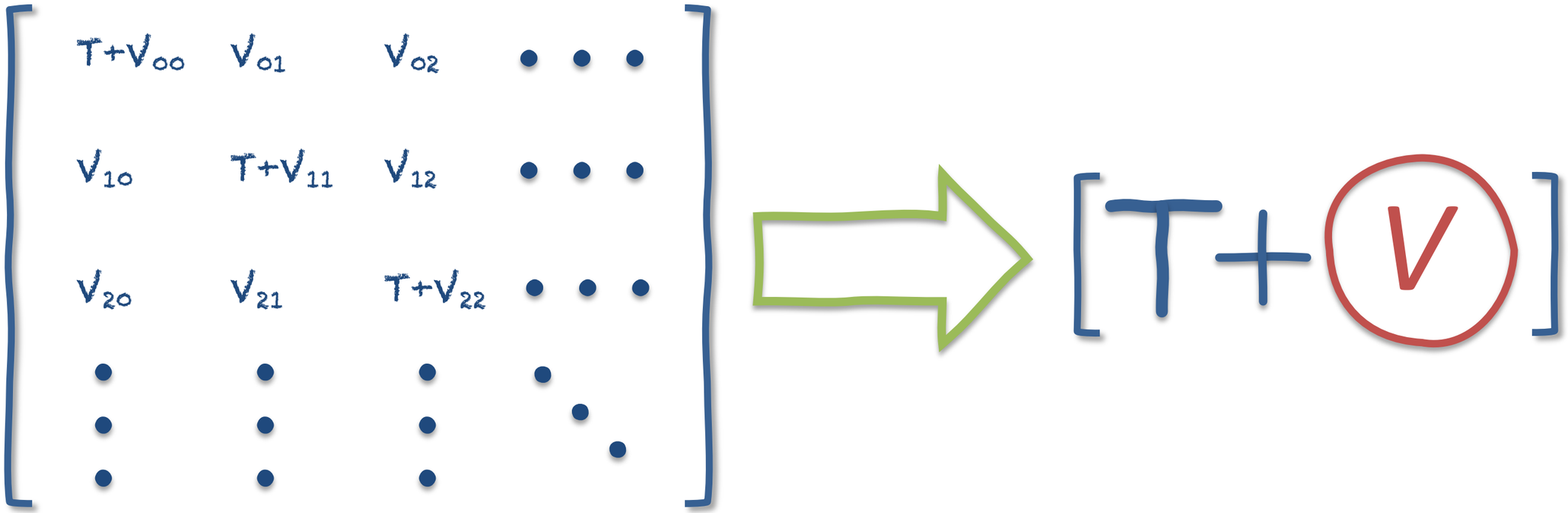
Toward a complete theory for predicting inclusive deuteron breakup away from stability

G. Potel[1,a], G. Perdikakis[1,2,3,b], B.V. Carlson[4,c], M.C. Atkinson[5], W.H. Dickhoff[5], J.E. Escher[6], M.S. Hussein[4,7,8], J. Lei[9,d], W. Li[1], A.O. Macchiavelli[10], A.M. Moro[9], F.M. Nunes[1,11], S.D. Pain[12], and J. Rotureau[1]



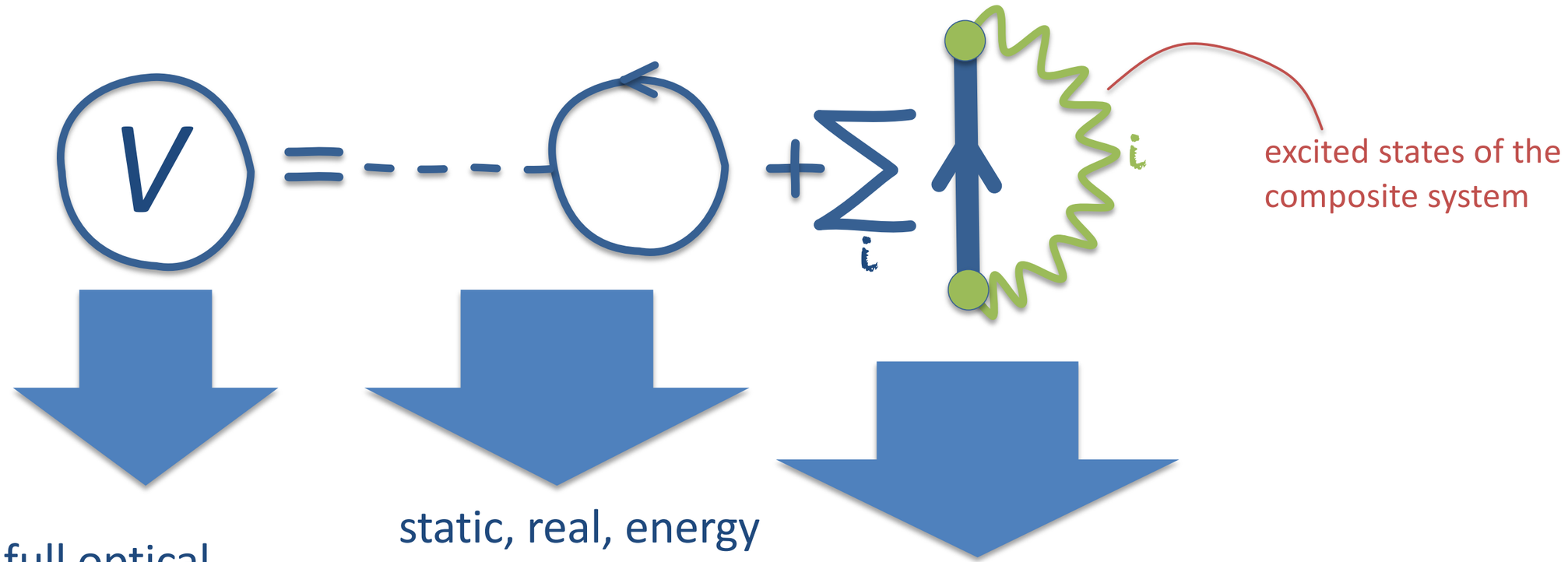
- Dispersive: reproduction of positive and negative energy cross section
- Controlled extrapolation to exotic nuclei

The Optical Potential is a projection of the many-body Hamiltonian on the elastic channel



- The “optical reduction” transforms a many-body operator into a one-body operator
- It is a well-defined, in principle exact, mathematical operation

The OP accounts for the composite nature of the target nucleus



^{24}Mg+n with valence shell model

excitation energy E_i

angular momentum

parity

spectroscopic factor S_i

0	0	2	1	0.584066
0.701	0	0	1	−0.716831
1.169	0	2	1	0.488705
2.033	0	2	1	−0.288311
2.529	0	0	1	0.318332
2.701	0	2	1	0.542416
3.859	0	2	1	0.0495903
3.926	0	1	−1	−0.0298132
4.118	0	1	−1	−0.584623
4.226	0	3	−1	−0.651056
4.46	0	2	1	0.100777
4.816	0	2	1	0.0975601
4.945	0	1	−1	0.516883
5.065	0	1	−1	−0.0511968
5.416	0	0	1	−0.283037
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5.785	0	2	1	−0.0103979
5.792	0	2	1	0.132477
5.935	0	3	−1	0.069005
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6.417	0	2	1	0.115995
6.609	0	2	1	0.100457
6.739	0	3	−1	0.157325
6.771	0	1	−1	0.419452
6.801	0	3	−1	0.160889

~600 states from E_i=0 to E_i=14.6 MeV

Shell model calculations by K. Kravvaris with PSDPF interaction M Bouhelal, *et al.*, Nucl. Phys. A **864** (2011)

^{24}Mg+n with valence shell model

excitation energy E_i

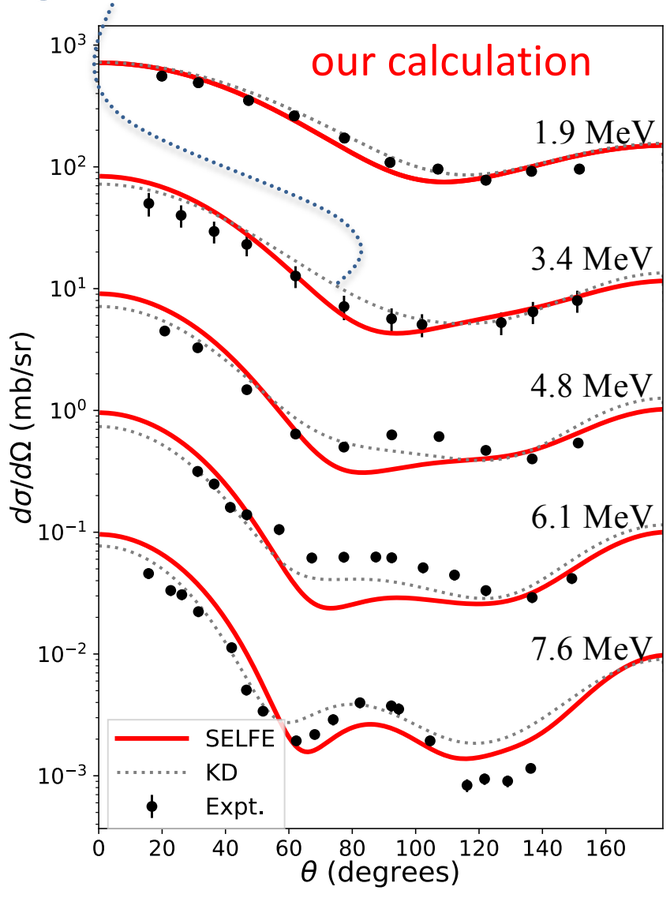
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Koning-Delaroche



Sargsyan, GP, Kravvaris, Escher PRC (2025)

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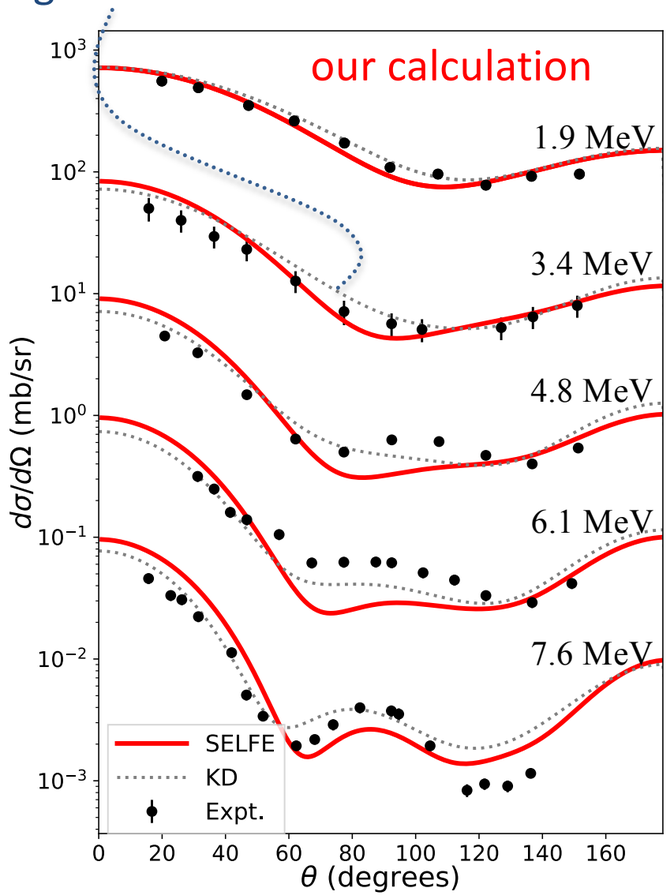
angular momentum

parity

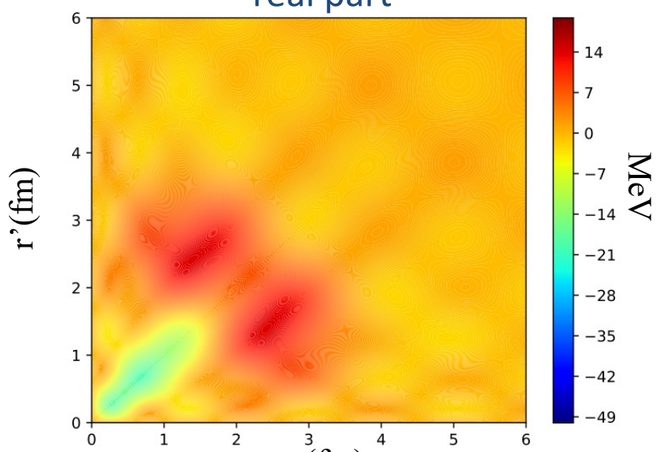
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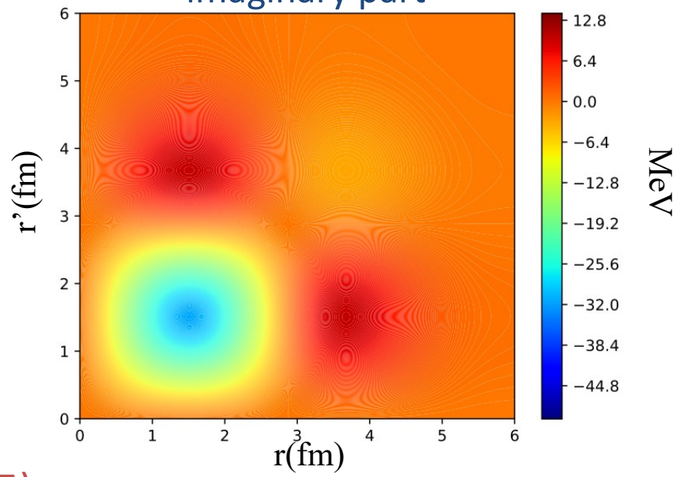
Koning-Delaroche



real part



imaginary part

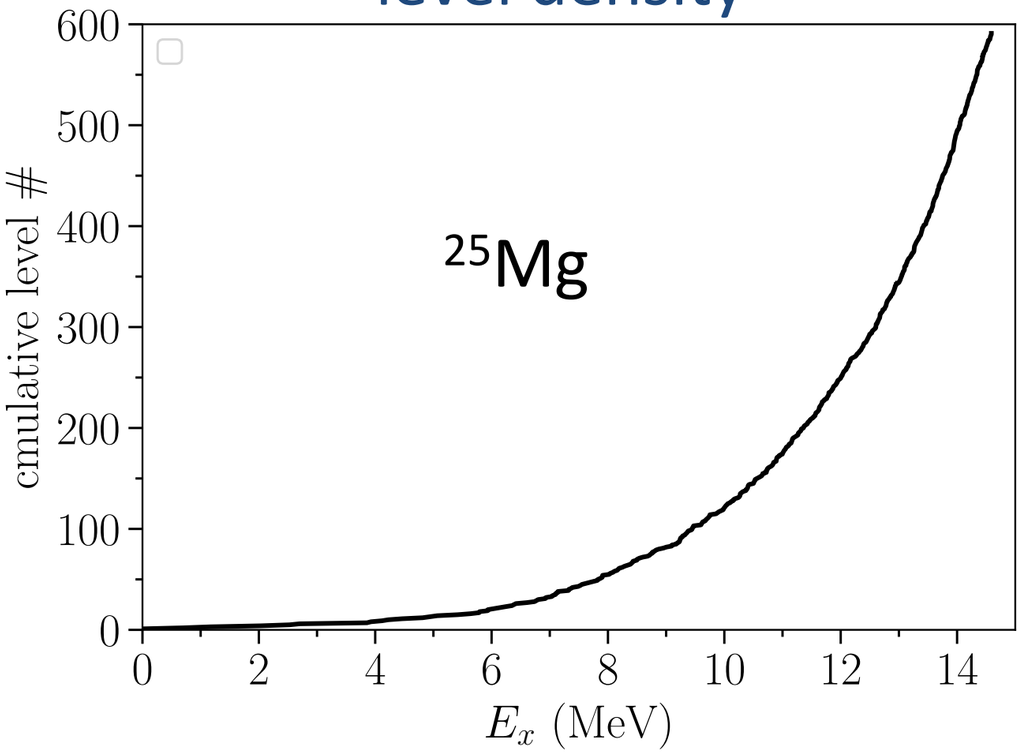


Sargsyan, GP, Kravvaris, Escher PRC (2025)

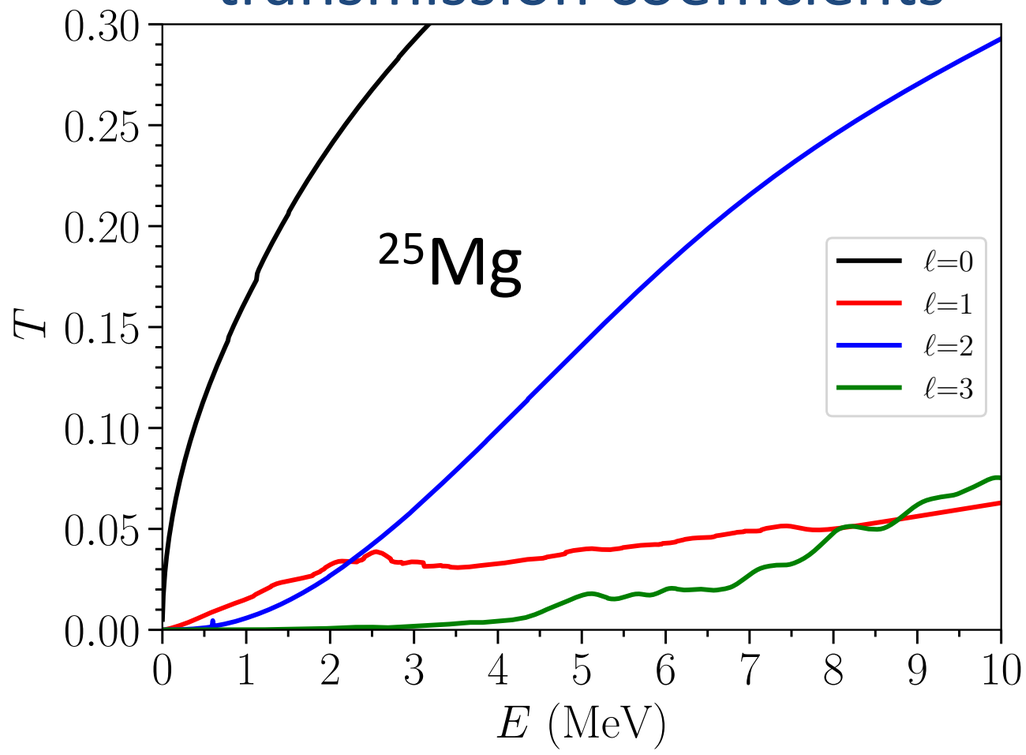
non-local, imaginary, energy-dependent

The OP, the level density, and the γ strength function are connected through the same underlying physics

level density

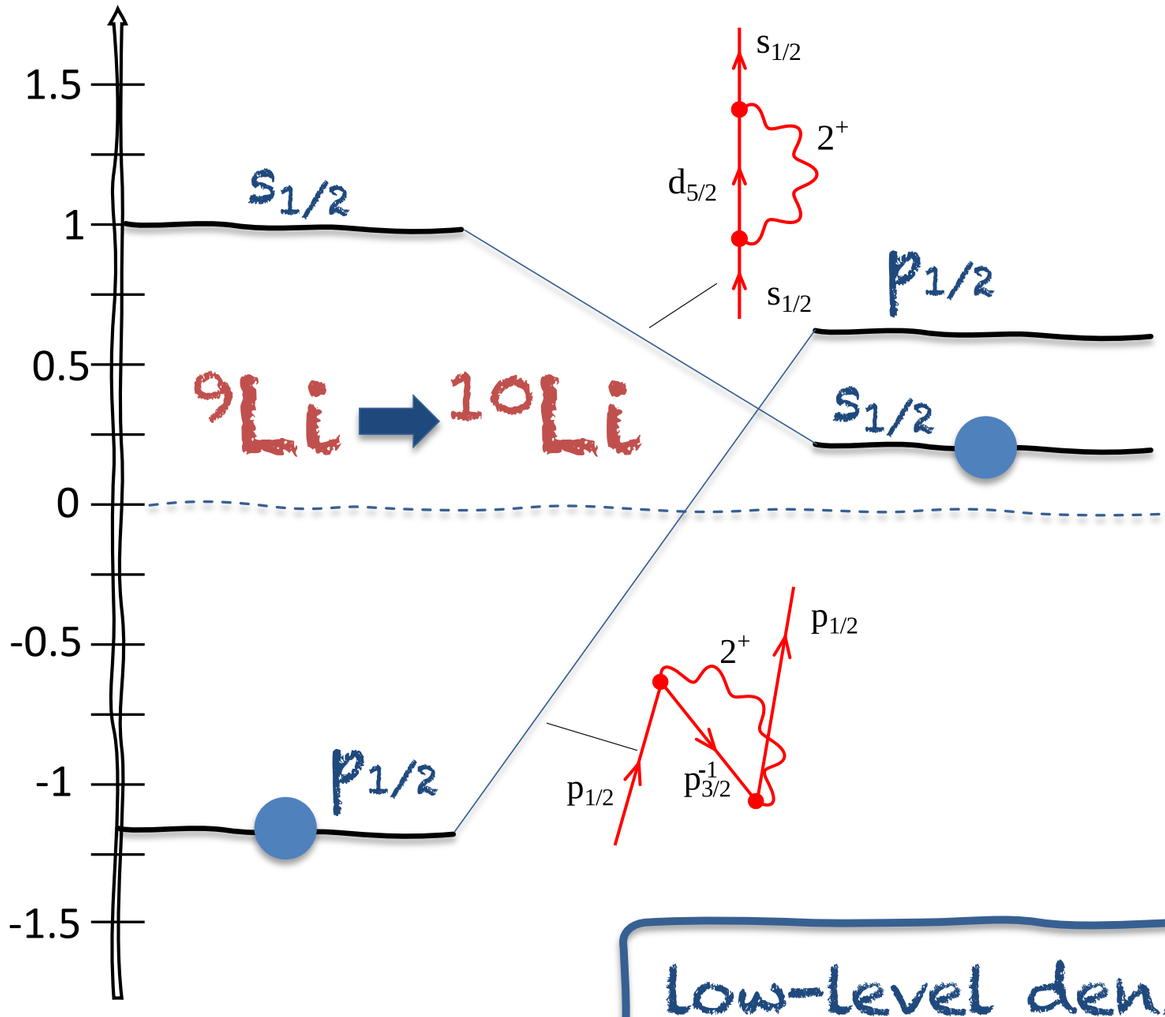


transmission coefficients



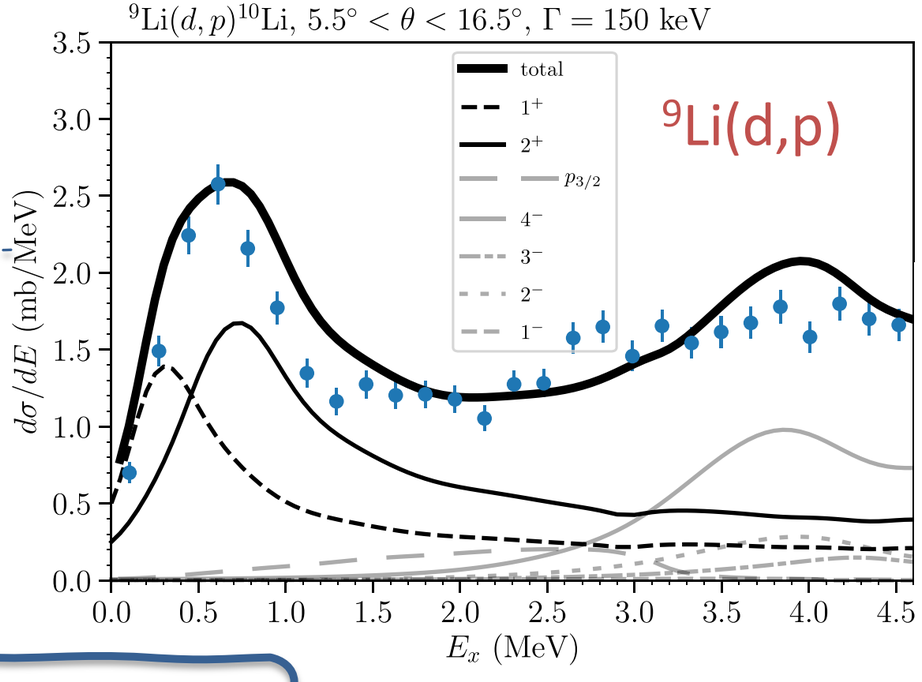
We can explicitly check the limits of Hauser-Feshbach within a consistent framework

GFT with Nuclear Field Theory (NFT): ^{9}Li(d,p)



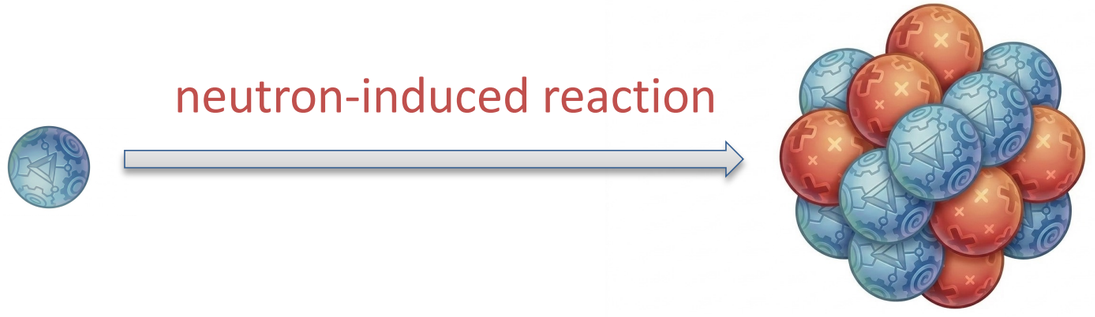
Cavallaro *et al.*, PRL **118**, 012701 (2017)

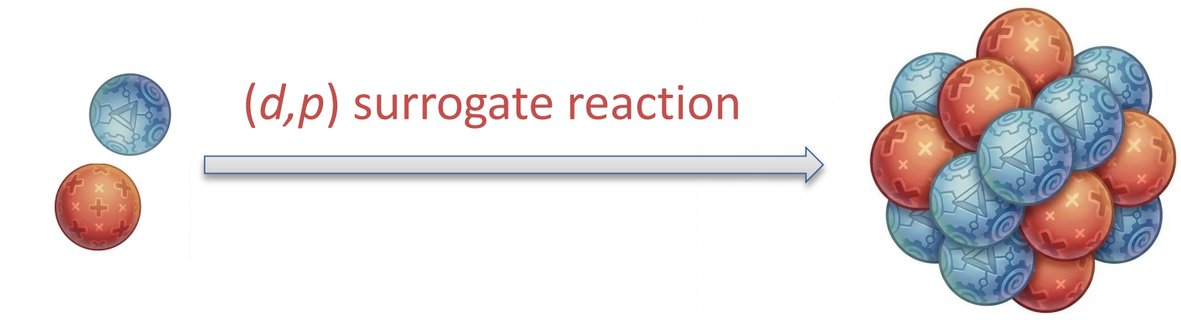
Barranco, GP, Vigezzi, Broglia PRC **101**, 031305(R) (2020)



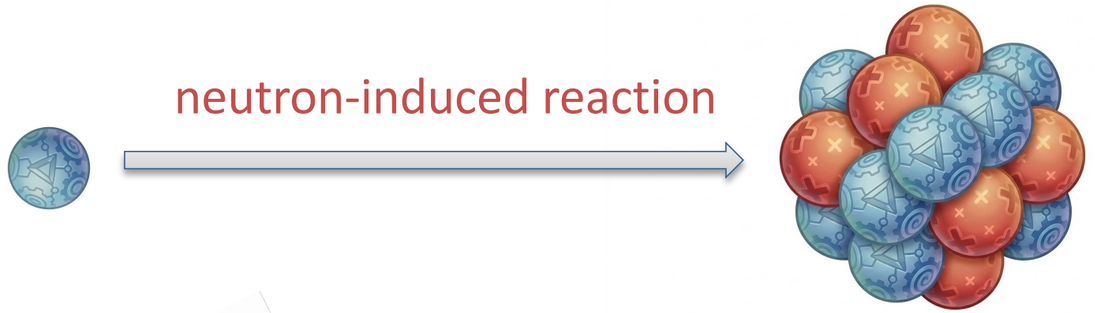
low-level density regime

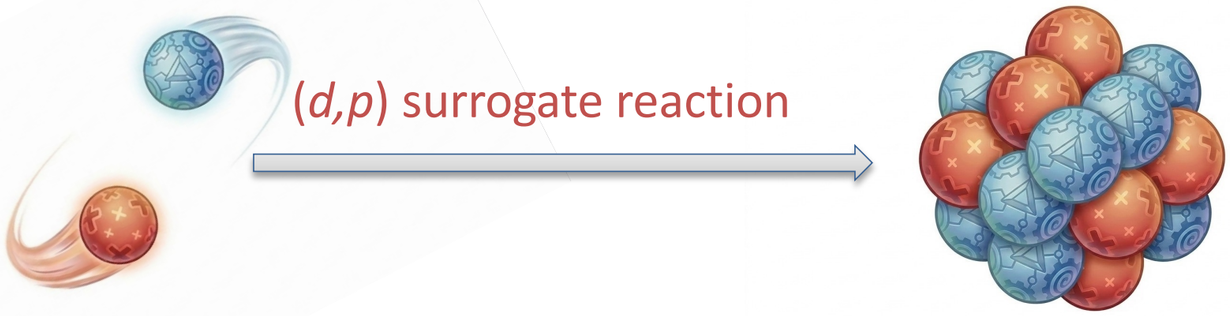
Which neutron energy in a surrogate reaction?





Which neutron energy in a surrogate reaction?





Which neutron energy in a surrogate reaction?



entrance channel :

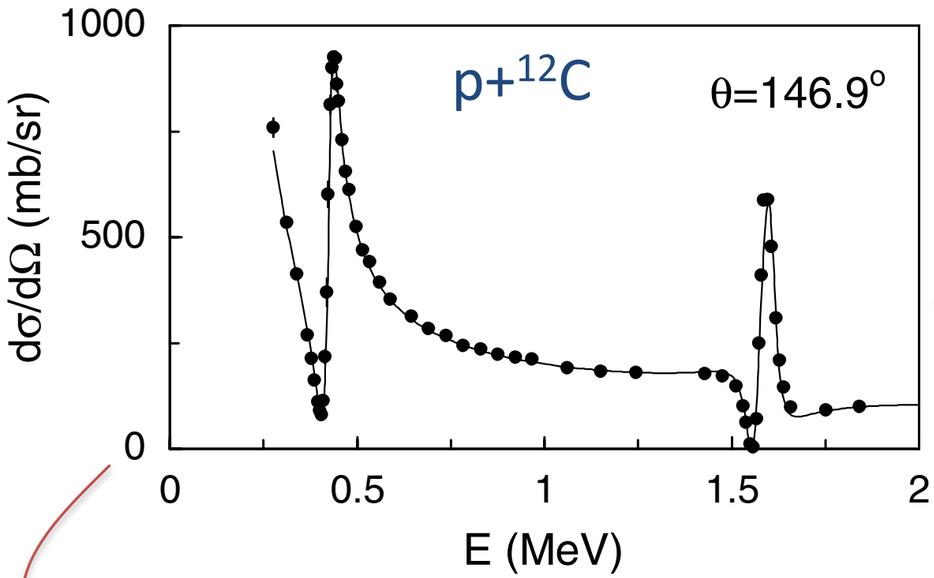
$$e^{i\mathbf{kr}_a}$$



entrance channel :

$$\psi^{HM}(\mathbf{r}_{xA}) = \int \chi_b^*(\mathbf{r}_{bB}, \mathbf{k}_b)\phi_a(\mathbf{r}_{xb})e^{i\mathbf{kr}_a}\, d\mathbf{r}_{xb}$$

R-matrix parametrization in a nutshell



Rep. Prog. Phys. **73** (2010) 036301 (44pp)

The *R*-matrix theory

P Descouvemont[1] and D Baye[1,2]

$$\frac{d\sigma(E)}{d\sigma} \propto |T_{i0}(E)|^2$$

$$T_{i0}(E) = \sqrt{P_i(E)P_0(E)} \sum_{pq} \frac{\gamma_{ip}\gamma_{0q}}{(E_p - E)\delta_{pq} - \sum_c \gamma_{ic}\gamma_{jc}(S_c(E) + iP_c(E))}$$

T-matrix partial widths and energy parameters fitted from data

Towards an R-matrix parametrization of (*d,p*) reactions



R-matrix parametrization of T-matrix:

$$T_{i0}(E) = \sqrt{P_i(E)P_0(E)} \sum_{pq} \frac{\gamma_{ip}\gamma_{0q}}{(E_p - E)\delta_{pq} - \sum_c \gamma_{ic}\gamma_{jc}(S_c(E) + iP_c(E))}$$



R-matrix parametrization of T-matrix:

$$T^I_{i0}(E) = \sqrt{P_i(E)} \sum_{pq} \frac{\gamma_{ip}\gamma_{0q}}{(E_p - E)\delta_{pq} - \sum_c \gamma_{ic}\gamma_{jc}(S_c(E) + iP_c(E))} \int \sqrt{P_0(E_k)}\, g(\mathbf{k})\, d\mathbf{k}.$$

Towards an R-matrix parametrization of (*d*,*p*) reactions



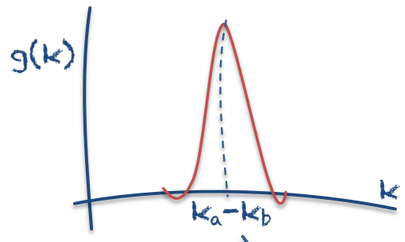
R-matrix parametrization of T-matrix:

$$T_{i0}(E) = \sqrt{P_i(E)P_0(E)} \sum_{pq} \frac{\gamma_{ip}\gamma_{0q}}{(E_p - E)\delta_{pq} - \sum_c \gamma_{ic}\gamma_{jc}(S_c(E) + iP_c(E))}$$



R-matrix parametrization of T-matrix:

$$T^I_{i0}(E) = \sqrt{P_i(E)} \sum_{pq} \frac{\gamma_{ip}\gamma_{0q}}{(E_p - E)\delta_{pq} - \sum_c \gamma_{ic}\gamma_{jc}(S_c(E) + iP_c(E))} \underbrace{\int \sqrt{P_0(E_k)}\, g(\mathbf{k})\, d\mathbf{k}.}$$



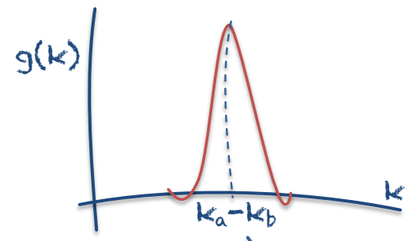
distortion of entrance channel

Towards an R-matrix parametrization of (*d,p*) reactions



R-matrix parametrization of T-matrix:

$$T_{i0}(E) = \sqrt{P_i(E)P_0(E)} \sum_{pq} \frac{\gamma_{ip}\gamma_{0q}}{(E_p - E)\delta_{pq} - \sum_c \gamma_{ic}\gamma_{jc}(S_c(E) + iP_c(E))}$$





R-matrix parametrization of T-matrix:

$$T^I_{i0}(E) = \sqrt{P_i(E)} \sum_{pq} \frac{\gamma_{ip}\gamma_{0q}}{(E_p - E)\delta_{pq} - \sum_c \gamma_{ic}\gamma_{jc}(S_c(E) + iP_c(E))} \int \sqrt{P_0(E_k)}\, g(\mathbf{k})\, d\mathbf{k}.$$

Use of R-matrix fitting techniques in (*d,p*) experiments?

Conclusions and some perspectives

- Phenomenology has to be complemented with theory for reliable extrapolation across different regimes.
- The calculation of the OP provides a flexible and versatile path, including 3-body reactions (with GFT).

what's next?

- Improve microscopic inputs, describe symmetry-breaking systems (deformation, pairing).
- Disentangle direct, pre-equilibrium, and compound reactions.
- Explore the limits of validity of the statistical model (r-process nuclei).
- Implement UQ.
- R-Matrix parametrization of indirect reactions.

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Thank you!

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