Analysis of Nucleon Spin Structures

Digital Twins for Nuclear and Particle Physics (NPTwins 2026)
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In collaboration with: T.A. Chowdhury, H. Dong, T. Izubuchi, M. Kamruzzaman, N. Karthik, T. Khan, A. Paul, J. Schoenleber, R.S. Sufian, P. Sun, Y. Zhao

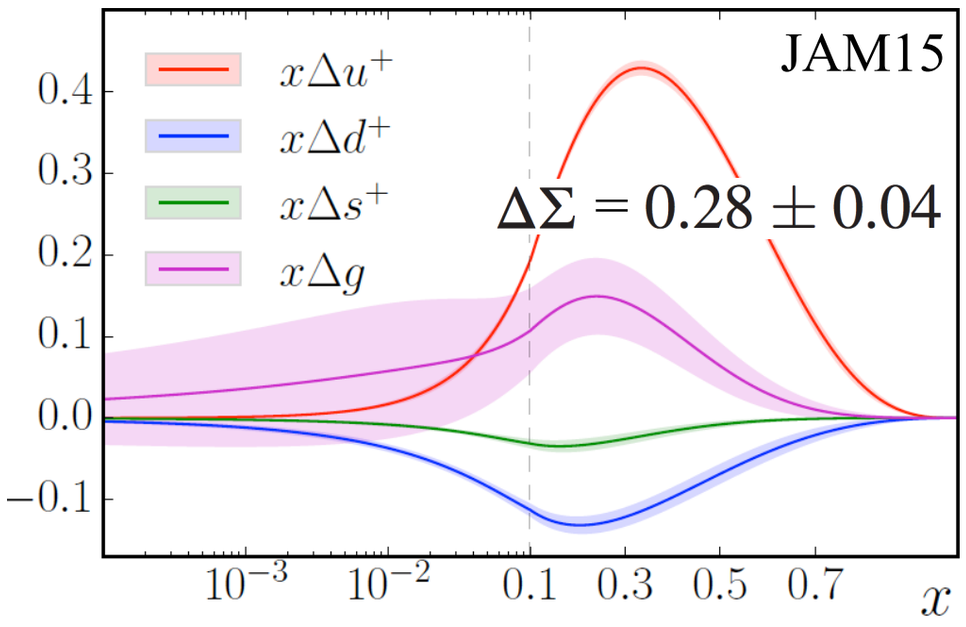
Nucleon Spin Structure

Proton spin puzzle

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s \sim 0.3$$

Spin decomposition

$$J = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$



JAM Collaboration, PR D 93, 074005 (2016).

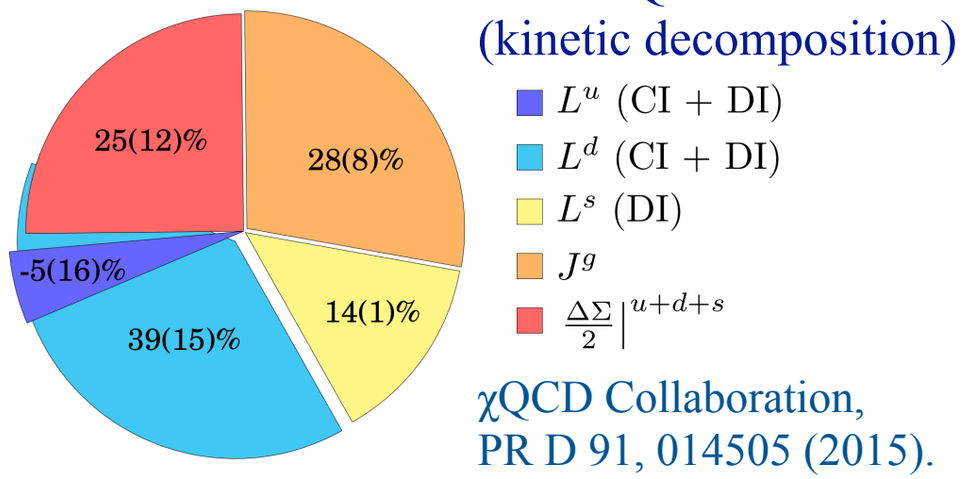
JAM17: $\Delta\Sigma = 0.36 \pm 0.09$

JAM Collaboration, PRL 119, 132001 (2017).

Quark spin only contributes a small fraction to the nucleon spin.

J. Ashman *et al.*, PLB 206, 364 (1988); NP B328, 1 (1989).

Lattice QCD
(kinetic decomposition)

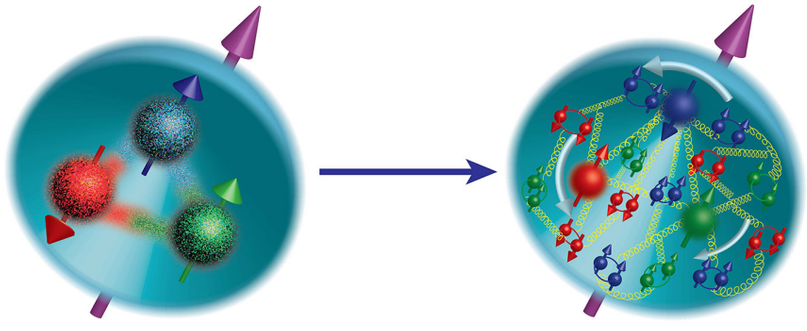


χQCD Collaboration,
PR D 91, 014505 (2015).

Gluon spin from LQCD: $S_g = 0.251(47)(16)$

50% of total proton spin

Y.-B. Yang *et al.* (χQCD Collaboration), PRL 118, 102001 (2017).



Access $\Delta g(x)$ in Lattice QCD

Matrix element to access polarized gluon distribution:

$$\Delta M_{\mu\alpha;\lambda\beta}(z,p,s) = \langle p,s\,|\,G_{\mu\alpha}(z)W[z,0]\tilde{G}_{\lambda\beta}(0)\,|\,p,s\rangle - (z \to -z)$$

X. Ji, PRL 110, 262002 (2013).

A proper combination: $\Delta\mathscr{M}_{00}(z,p_z) = \Delta M_{0i;0i}(z,p_z) + \Delta M_{ij;ij}(z,p_z)$

Renormalized matrix element: $\Delta\mathfrak{M}(z,p_z) \equiv i\dfrac{[\Delta\mathscr{M}_{00}(z,p_z)/p_z p_0]/Z_L(z/a_L)}{\mathscr{M}_{00}(z,p_z=0)/m_p^2}$

in terms of invariant amplitudes:

Ioffe time: $\omega = zp_z$

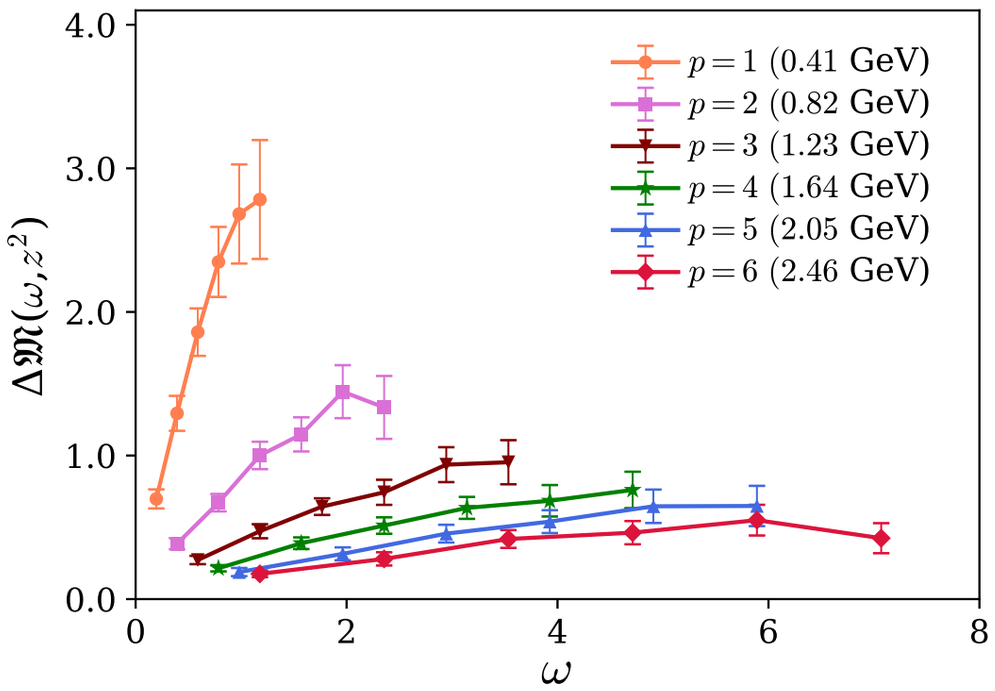
$$\Delta\mathfrak{M}(\omega,z^2) = [\Delta\mathscr{M}^{(+)}_{sp}(\omega,z^2) - \omega\Delta\mathscr{M}_{pp}(\omega,z^2)] - \frac{m_p^2}{p_z^2}\omega\Delta\mathscr{M}_{pp}(\omega,z^2)$$

I. Balitsky, W. Morris, A. Radyushkin, JHEP 02 (2022) 193.

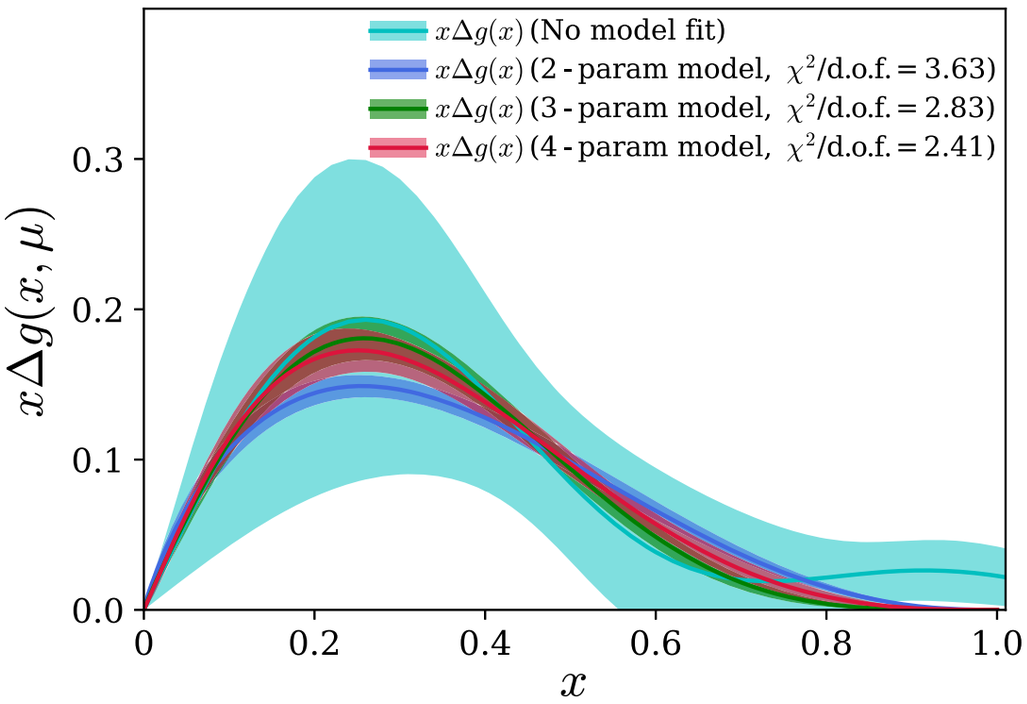
At the light-cone limit:

$$\Delta\mathfrak{M}_g(\omega) \xrightarrow{\text{perturbative matching}} \Delta\mathcal{F}(\omega,\mu) = \frac{i}{2}\int_{-1}^{1} \mathrm{d}x\, e^{-i\omega x} x\Delta g(x,\mu)$$

Access $\Delta g(x)$ in Lattice QCD



C. Egerer *et al.* (HadStruc Collboration), PRD 106 (2022) 094511.



T. Khan, TL, R.S. Sufian, PRD 108 (2023) 074502.

- Lattice data: 1901 samples (number of configurations), 6 lattice momenta p_z
- Challenge to determine $\Delta g(x)$:
  - need small z for controllable power corrections
  - need large boost p_z to access large Ioffe time, required to reconstruct x dependence
  - poor signal-to-noise ratio at large hadron boosts p_z and large z
  - potential danger of fits using some functional form ansatz

Remove Kinematic Power Correction

At some hadron boost momentum p_z:

$$p_k^2 \Delta\mathfrak{M}(\omega)\big|_{p_k} = p_k^2[\Delta\mathscr{M}_{sp}^{(+)}(\omega) - \omega\Delta\mathscr{M}_{pp}(\omega)]\big|_{p_k} - m_p^2\omega\Delta\mathscr{M}_{pp}(\omega)\big|_{p_k}.$$

Eliminate the second term across different p_z matrix elements:

$$\Delta\mathfrak{M}_g(\omega) \equiv \frac{r^2\Delta\mathfrak{M}(\omega)\big|_{p_k} - \Delta\mathfrak{M}(\omega)\big|_{p_l}}{r^2-1} \qquad r = \frac{p_k}{p_l} = \frac{k}{l}$$

$$\approx \Delta\mathscr{M}_{sp}^{(+)}(\omega) - \omega\Delta\mathscr{M}_{pp}(\omega),$$

- The approximate relation only holds at small z
- Strong z dependence when higher-twist effects become substantial
- A direct fit finds for current data it fails for $z > 0.36$ fm

Generating Synthetic Data

- Training on short-distance ($z \leq 0.36$ fm) data, learning the patterns where higher-twist contamination is minimal.
- Guide the shape with phenomenological ITD.
- Updating $\Delta\mathfrak{M}$ using gradient descent.
- Generate synthetic datasets of contamination-free matrix elements: 1901 samples, 6 momenta, each having 200 ω values.

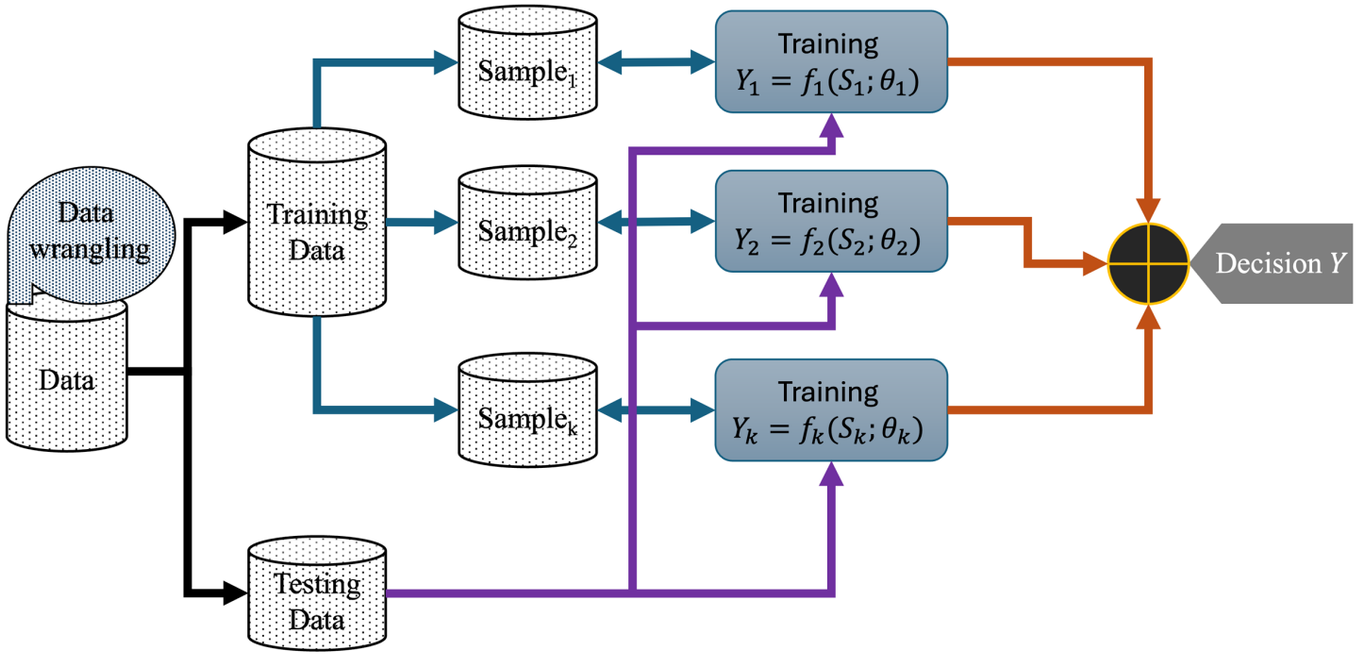
ALGORITHM 1. Find $\Delta\mathfrak{M}(\omega)$ values using gradient descent.

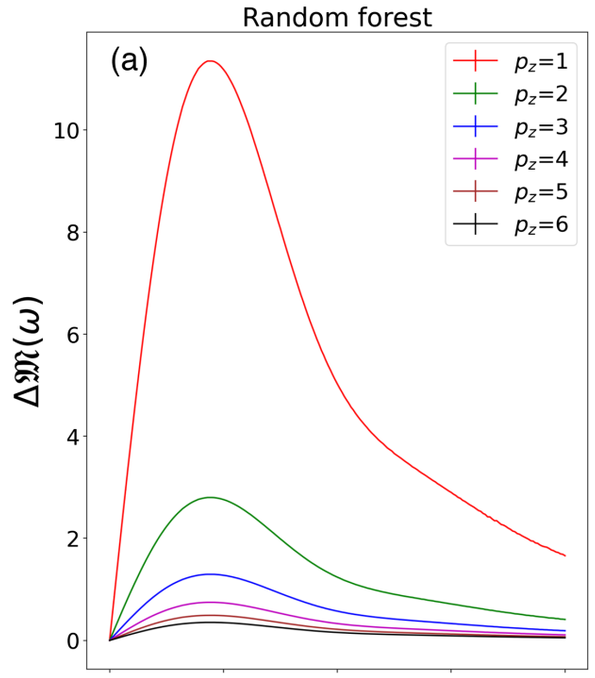
Input: Data $\Delta\mathcal{I}_g(\omega)$, index i for p_i, index j for p_j
Output: Data $\mathcal{D}_g$

$\alpha \leftarrow \frac{j^2}{j^2-i^2}$
$\beta \leftarrow \frac{-i^2}{j^2-i^2}$
Initialize learning rate, $\eta \leftarrow 0.01$
$\mathcal{D}_g \leftarrow \varnothing$
for each point $(\Delta\mathcal{I}_g, \omega) \in \Delta\mathcal{I}_g(\omega)$ **do**
  Initialize, $\Delta\mathfrak{M}_j \leftarrow 0$
  Initialize, $\Delta\mathfrak{M}_i \leftarrow n * \Delta\mathcal{I}_g, n > 0$
  for *epoch* $\in \{1, \ldots, 100\}$ **do**
    $\Delta\mathfrak{M}_g \leftarrow \alpha\Delta\mathfrak{M}_j + \beta\Delta\mathfrak{M}_i$
    $\mathcal{L} \leftarrow (\Delta\mathcal{I}_g - \Delta\mathfrak{M}_g)^2 = (\Delta\mathcal{I}_g - (\alpha\Delta\mathfrak{M}_j + \beta\Delta\mathfrak{M}_i))^2$
    $\frac{\partial\mathcal{L}}{\partial\Delta\mathfrak{M}_j} \leftarrow -2 \times \alpha \times (\Delta\mathfrak{M}_g - \Delta\mathcal{I})$
    $\frac{\partial\mathcal{L}}{\partial\Delta\mathfrak{M}_i} \leftarrow -2 \times \beta \times (\Delta\mathfrak{M}_g - \Delta\mathcal{I}')$
    $\Delta\mathfrak{M}_j \leftarrow \Delta\mathfrak{M}_j - \eta \times \frac{\partial\mathcal{L}}{\partial\Delta\mathfrak{M}_j}$
    $\Delta\mathfrak{M}_i \leftarrow \Delta\mathfrak{M}_i - \eta \times \frac{\partial\mathcal{L}}{\partial\Delta\mathfrak{M}_i}$
  $\mathcal{G} \leftarrow [\omega, \Delta\mathfrak{M}_i, \Delta\mathfrak{M}_j]$
  for $k \in \{\ell : \ell > i, \ell \neq j\}$ **do**
    $\gamma \leftarrow \frac{i^2}{k^2}$
    $\Delta\mathfrak{M}_k \leftarrow (1-\gamma) \times \Delta\mathcal{I}_g + \gamma \times \Delta\mathfrak{M}_i$
    $\mathcal{G} \leftarrow \text{Append}(\mathcal{G}, \Delta\mathfrak{M}_k)$
  $\mathcal{D}_g \leftarrow \text{Append}(\mathcal{D}_g, \mathcal{G})$

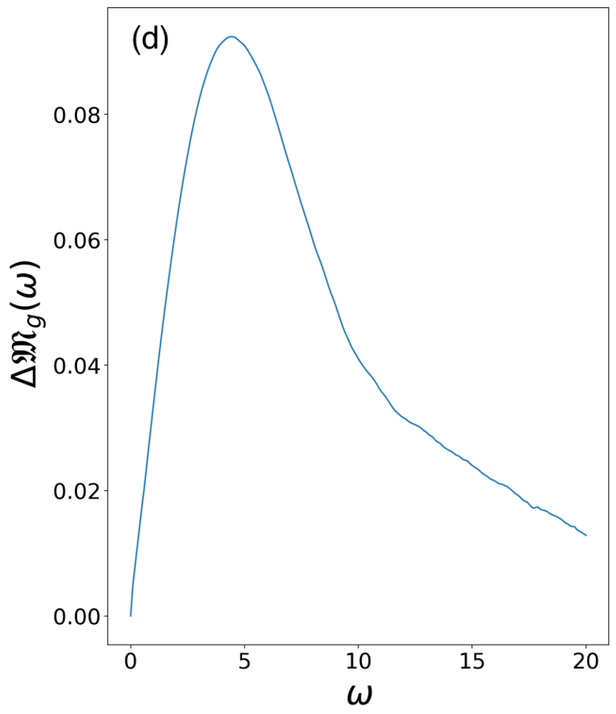
T.A. Chowdhury, T. Izubuchi, M. Kamruzzaman, N. Karthik, T. Khan, TL, A. Paul, J. Schoenleber, R.S. Sufian, PRD 111 (2025) 074509.

ML Extrapolating ITD





- Ensemble ML process: each model is a weak learner, combine all weak learners to produce strong decision
- Random forest regressor and each regressor is a decision tree
- Sliding-window approach: as a Bayesian time-series problem. Feed a sequence of 5 previous points, to predict the next point.
- RF smoothly extends ITD curves up to $\omega \approx 14$.

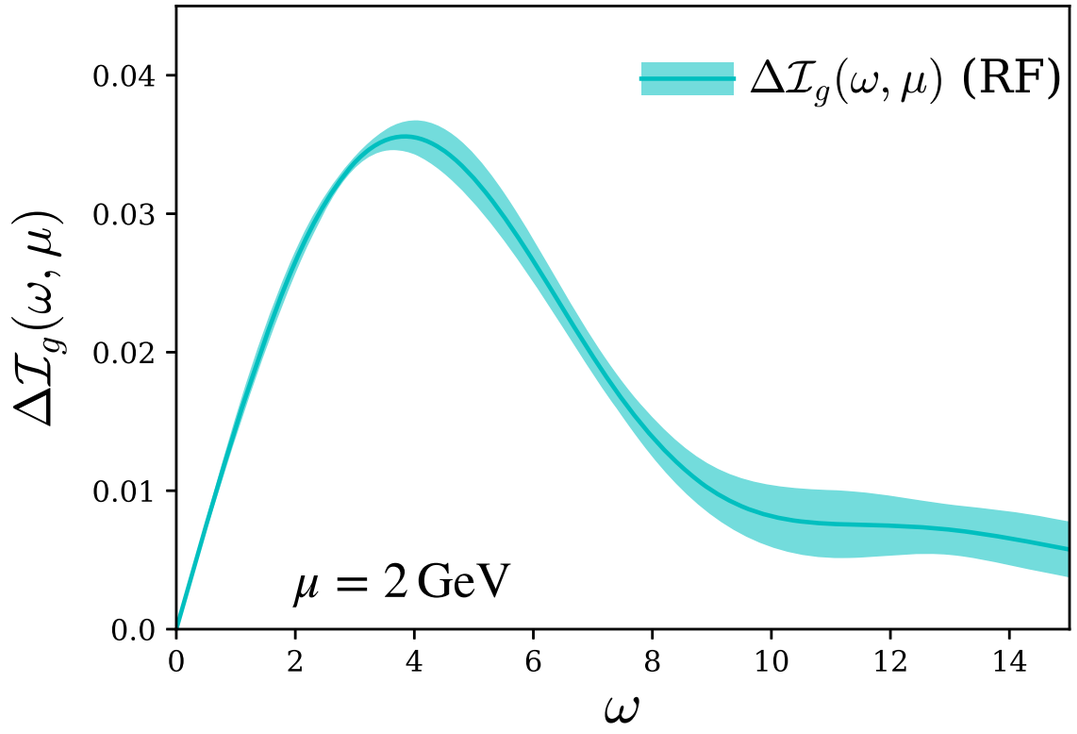


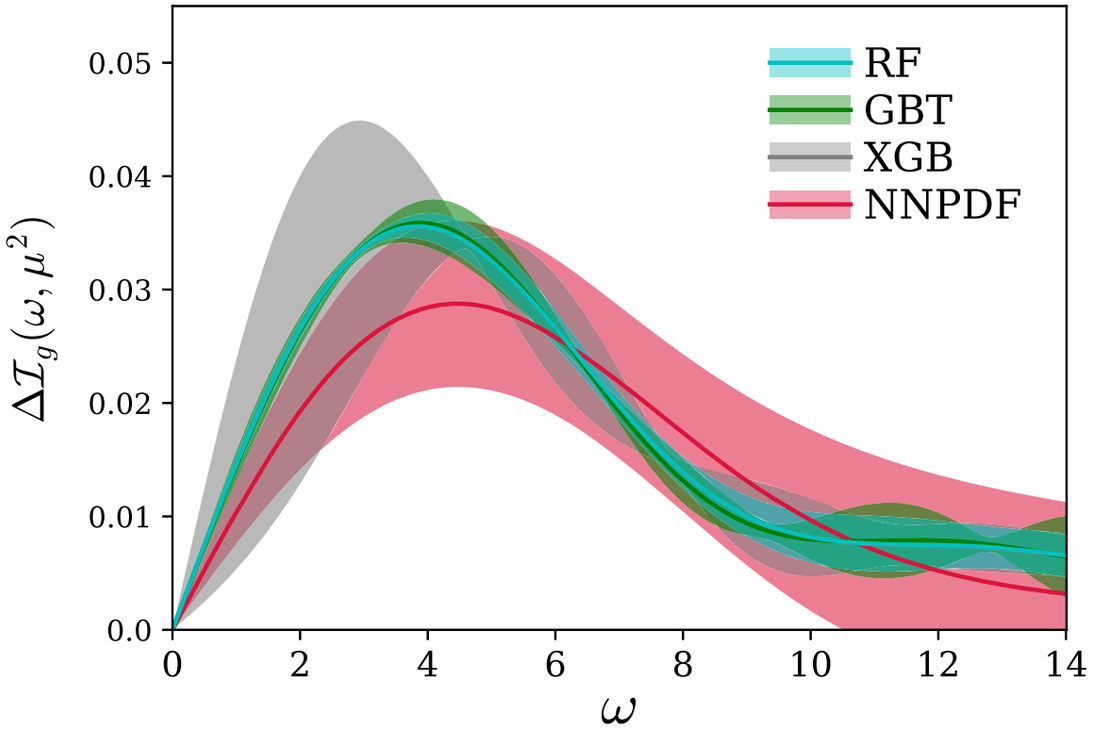
T.A. Chowdhury, T. Izubuchi, M. Kamruzzaman, N. Karthik, T. Khan, TL, A. Paul, J. Schoenleber, R.S. Sufian, PRD 111, (2025) 074509.

Matching to Light-cone ITD

$$\Delta\mathfrak{M}_g(\omega)\langle x\rangle_g(\mu) = \Delta\mathcal{I}_g(\omega,\mu) - \frac{\alpha_s N_c}{2\pi}\int_0^1 du\,\Delta\mathcal{I}_g(u\omega,\mu)\left\{\ln\left(z^2\mu^2\frac{e^{2\gamma_E}}{4}\right)\left(\left[\frac{2u^2}{\bar{u}}+4u\bar{u}\right]_+ - \left(\frac{1}{2}+\frac{4}{3}\frac{\langle x_S\rangle(\mu)}{\langle x_g\rangle(\mu)}\right)\delta(\bar{u})\right)\right.$$
$$\left. +4\left[\frac{u+\ln(1-u)}{\bar{u}}\right]_+ - \left(\frac{1}{\bar{u}}-\bar{u}\right)_+ - \frac{1}{2}\delta(\bar{u}) + 2\bar{u}u\right\}$$
$$-\frac{\alpha_s C_F}{2\pi}\int_0^1 du\,\Delta\mathcal{I}_S(u\omega,\mu)\left\{\ln\left(z^2\mu^2\frac{e^{2\gamma_E}}{4}\right)\Delta\mathcal{B}_{gq}(u) + 2\bar{u}u\right\}$$

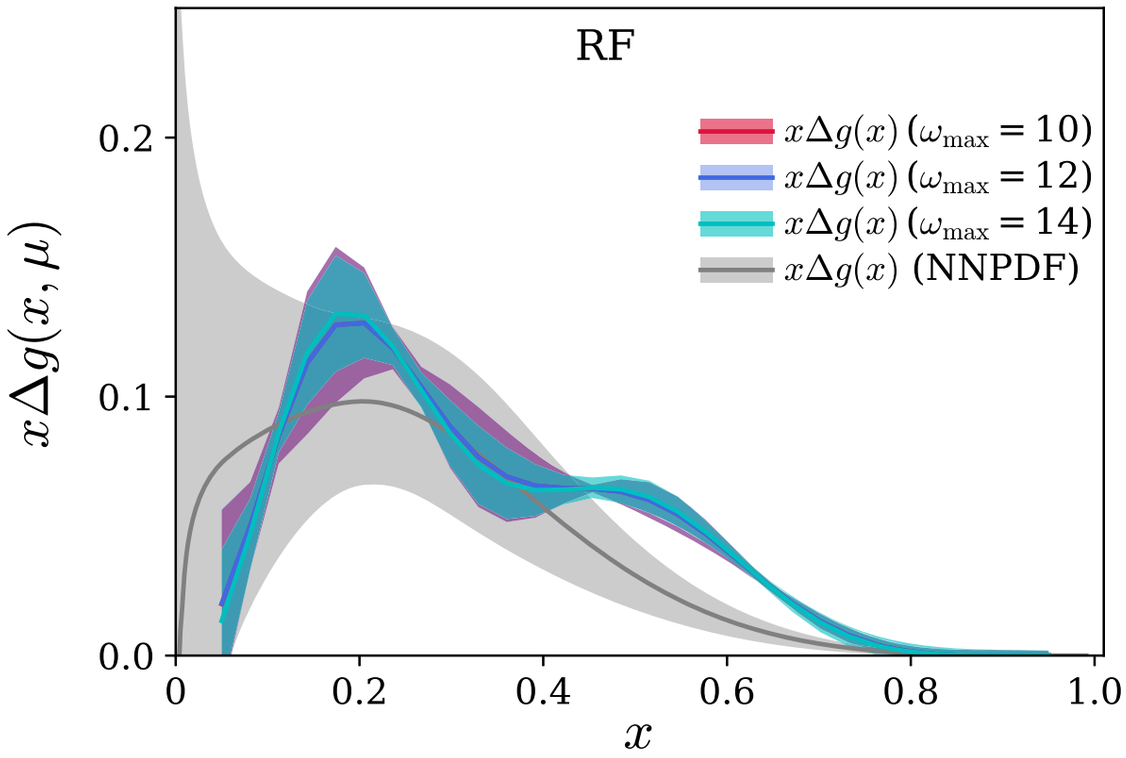
I. Balitsky, W. Morris, A. Radyushkin, JHEP 02 (2022) 193.





T.A. Chowdhury, T. Izubuchi, M. Kamruzzaman, N. Karthik, T. Khan, TL, A. Paul, J. Schoenleber, R.S. Sufian, PRD 111, (2025) 074509.

NN Reconstruction of $\Delta g(x)$



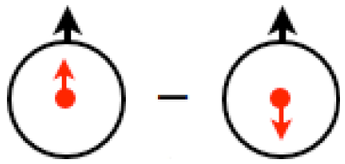
$$\Delta\mathcal{I}(\omega,\mu) = \frac{i}{2}\int_{-1}^{1} \mathrm{d}x\, e^{-i\omega x} x\Delta g(x,\mu)$$

- PDF is parametrized as a function × NN
- NN: densely connected, three hidden layers (32,32,8)
- Training transform the output to ITD and compare with the RF generated ITD up to $\omega_{\max} = 10, 12, 14$.

T.A. Chowdhury, T. Izubuchi, M. Kamruzzaman, N. Karthik, T. Khan, TL, A. Paul, J. Schoenleber, R.S. Sufian, PRD 111, (2025) 074509.

Transversity Distribution

Transversity distribution

h_1  (Collinear & TMD)

A transverse counter part to the longitudinal spin structure: helicity g_{1L}, but NOT the same.

Chiral-odd:

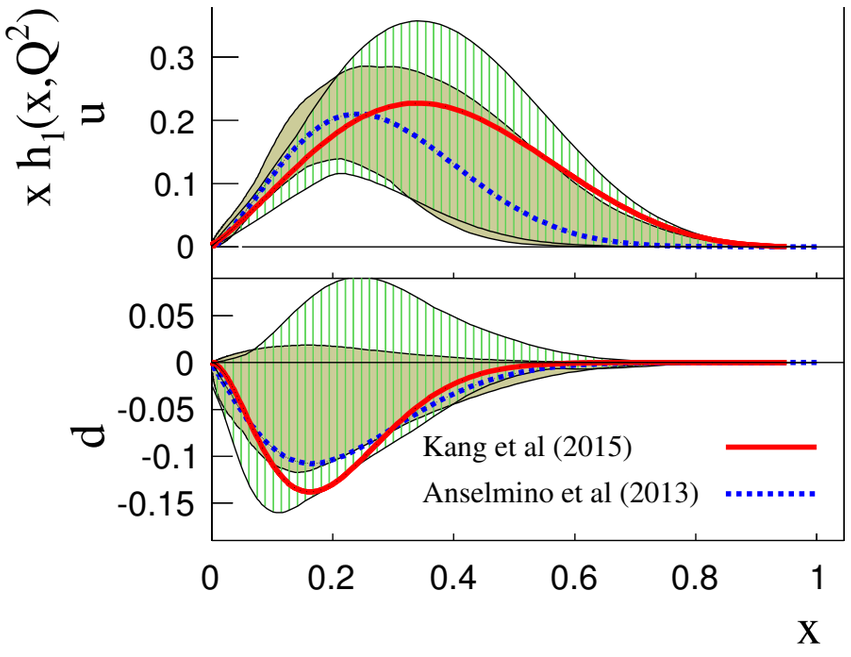
No mixing with gluons
Valence dominant
Couple to another chiral-odd function.

Effect in SIDIS:

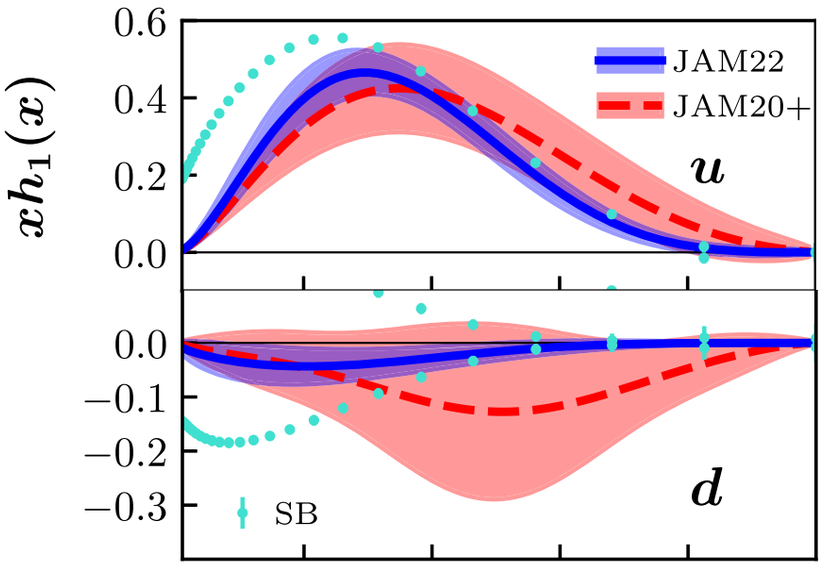
transverse single spin asymmetry (Collins asymmetry)

$$A_{UT}^{\sin(\phi_h+\phi_S)} \sim h_1(x,k_T^2) \otimes H_1^{\perp}(z,p_T^2)$$

Phenomenological extractions



Z.-B. Kang, A. Prokudin, P. Sun, F. Yuan, PRD 93, 014009 (2016).

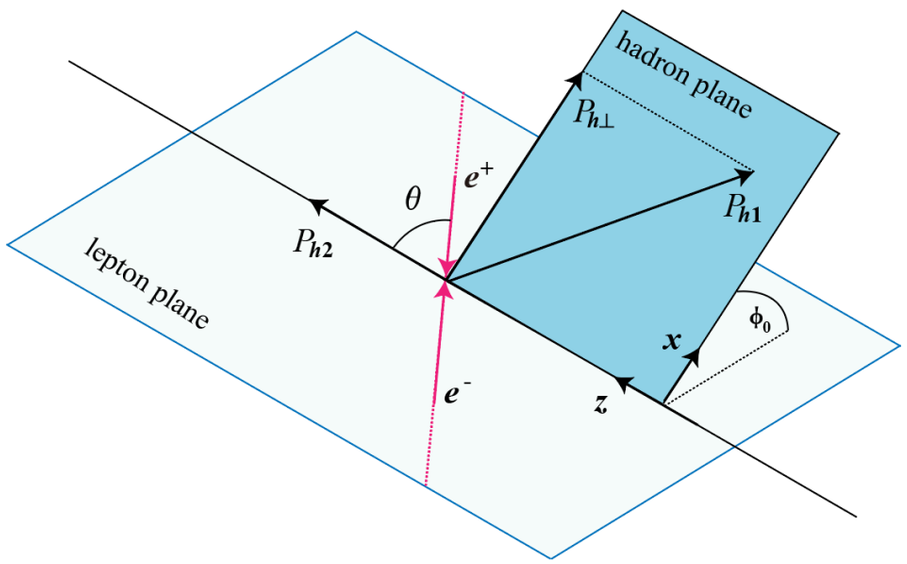


JAM Collaboration, PRD 104, 034014 (2022).

Assuming vanishing transverse polarization of sea quarks!

Complementary Process

Semi-inclusive e^+e^- annihilation: $e^+e^- \to h_1 h_2 X$



$$\frac{d^5\sigma}{dz_1 dz_2 d^2\boldsymbol{P}_{h\perp} d\cos\theta}$$

$$= \frac{3\pi\alpha^2}{2Q^2} z_1^2 z_2^2 \left[\left(1+\cos^2\theta\right) F_{UU}^{h_1 h_2} + \sin^2\theta \cos\left(2\phi_0\right) F_{\text{Collins}}^{h_1 h_2} \right]$$

In TMD region: h_1 and h_2 are near back-to-back, $P_{hT} \ll Q$ $\quad F_{\text{Collins}}^{h_1 h_2} \sim H_1^{\perp h_1} \otimes H_1^{\perp h_2}$

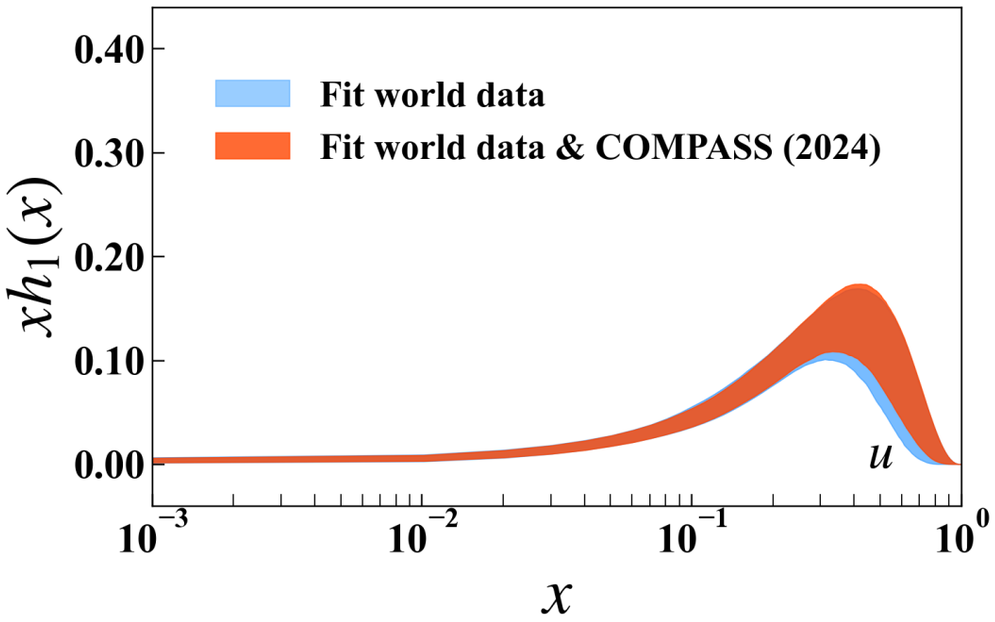
Experimental measurements:

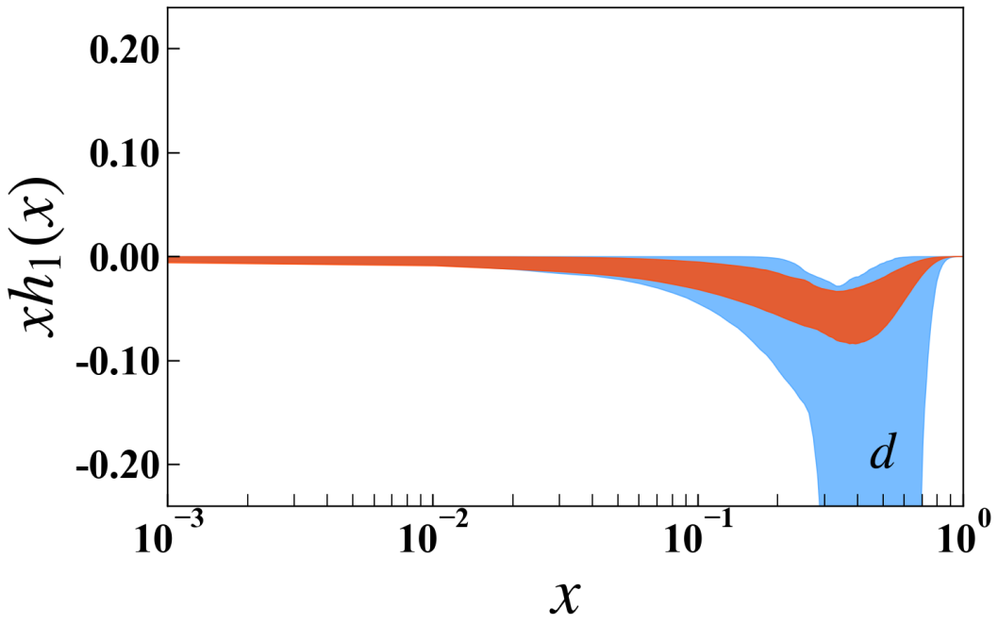
Belle: $\sqrt{s} = 10.58\,\text{GeV}$ Phys. Rev. D 78 (2008) 032011; 86 (2012) 039905(E).

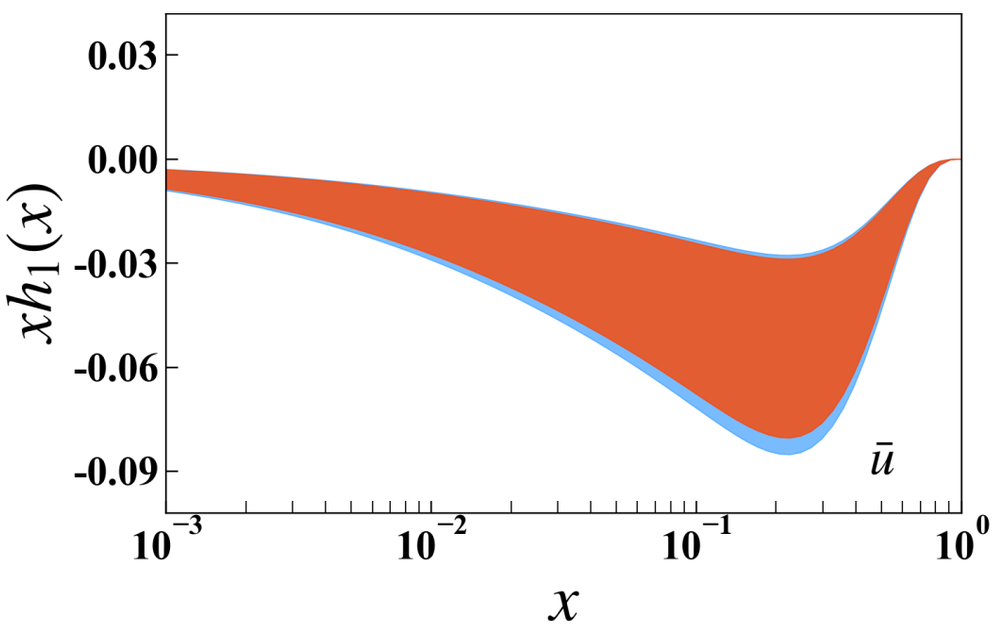
BaBar: $\sqrt{s} = 10.6\ \text{GeV}$ Phys. Rev. D 90 (2014) 052003; Phys. Rev. D 92 (2015) 111101.

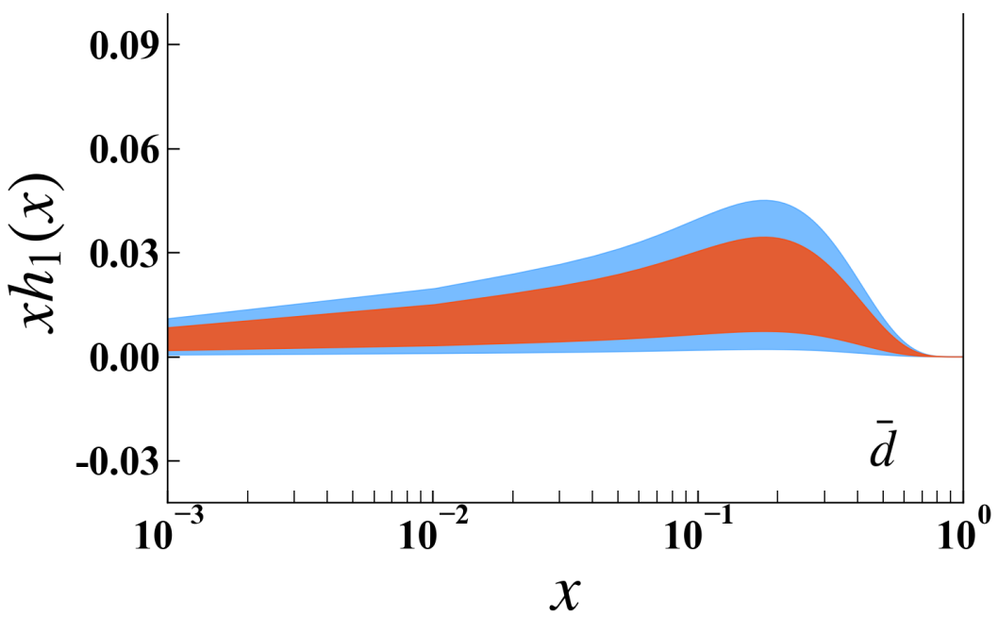
BESIII: $\sqrt{s} = 3.68\ \text{GeV}$ Phys. Rev. Lett. 116 (2016) 042001.

Transversity Distributions







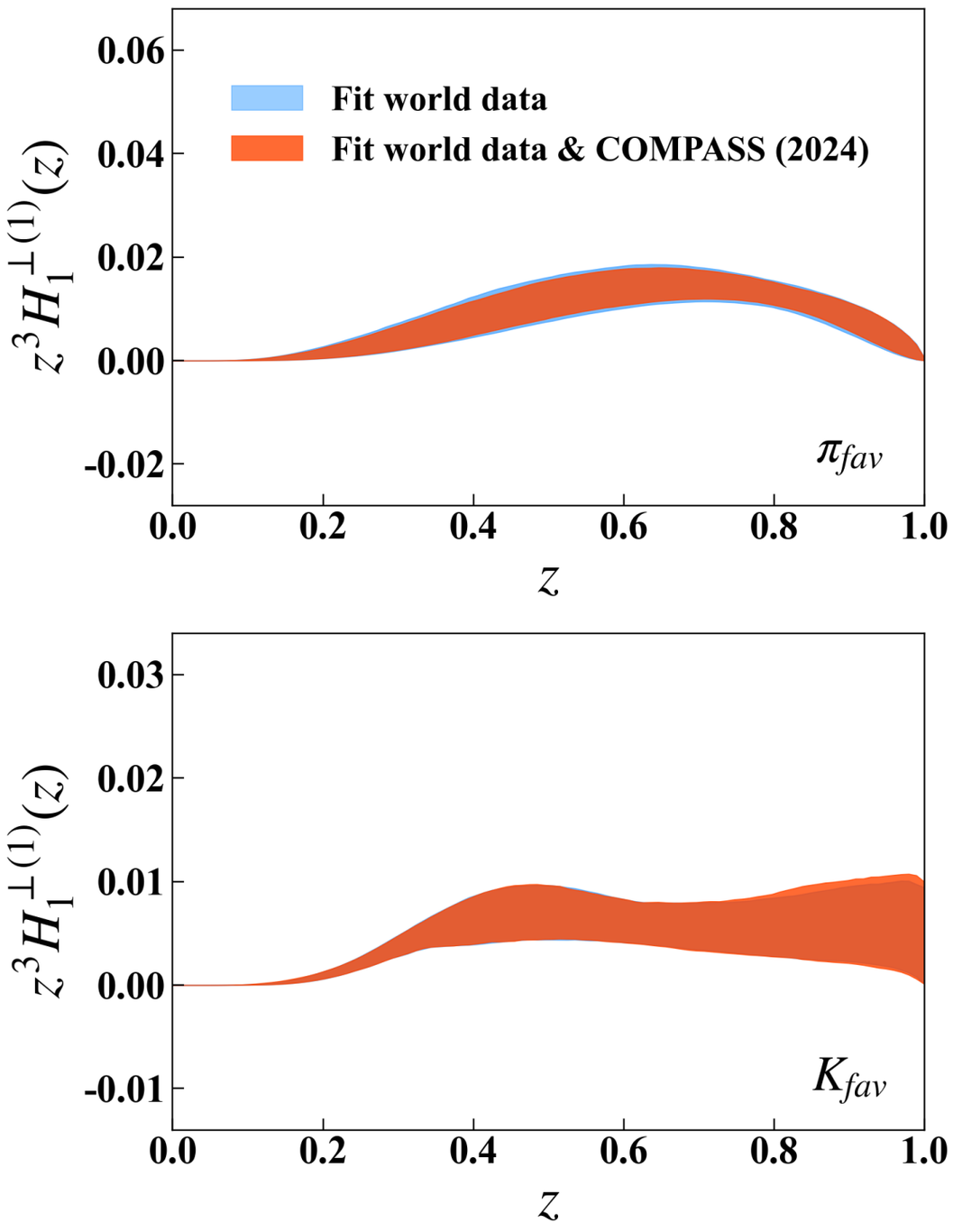


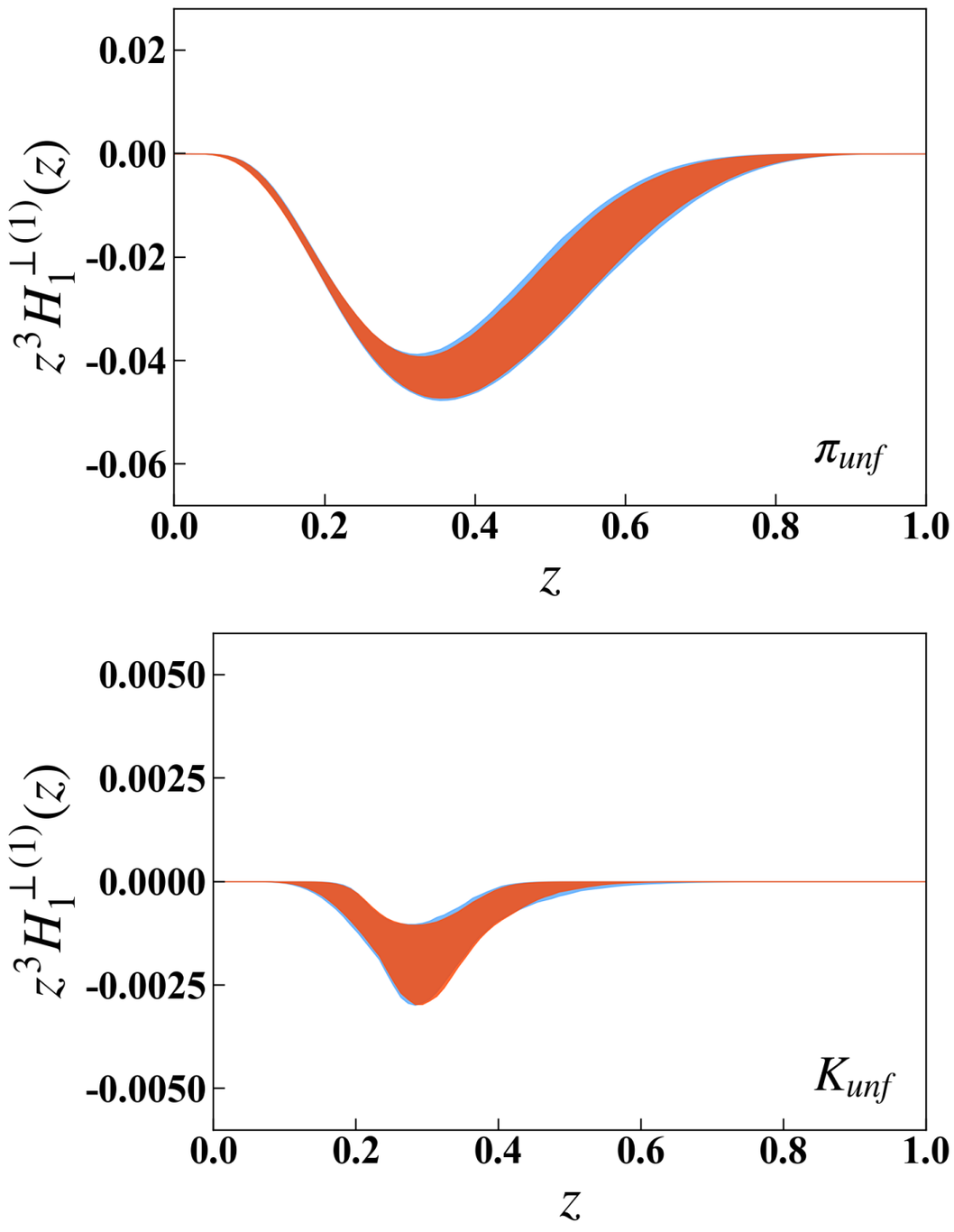
Reconfirmed by new COMPASS data.

Significant improvement on d and $\bar{d}$ distributions.

C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Collins Fragmentation Functions



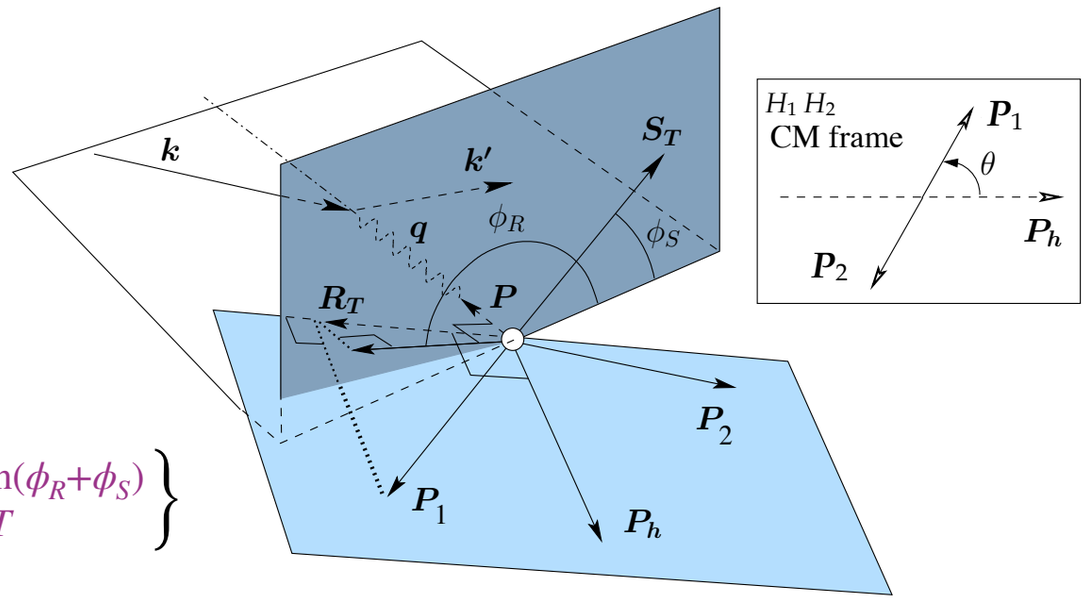


C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

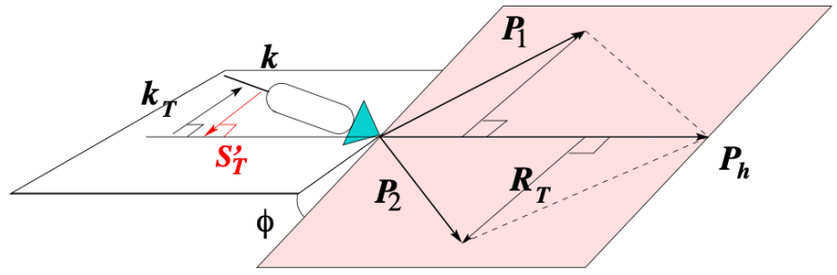
Collinear Transversity Distributions

Dihadron SIDIS

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}\phi_S\,\mathrm{d}z\,\mathrm{d}\phi_R\,\mathrm{d}M_h^2\,\mathrm{d}\cos\theta} = \frac{\alpha^2}{x_B y Q^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\left\{F_{UU} + S_T\,\varepsilon\,\sin(\phi_R+\phi_S)F_{UT}^{\sin(\phi_R+\phi_S)}\right\}$$



$$F_{UT}^{\sin(\phi_R+\phi_S)} \sim h_1 \otimes H_1^{\sphericalangle}$$



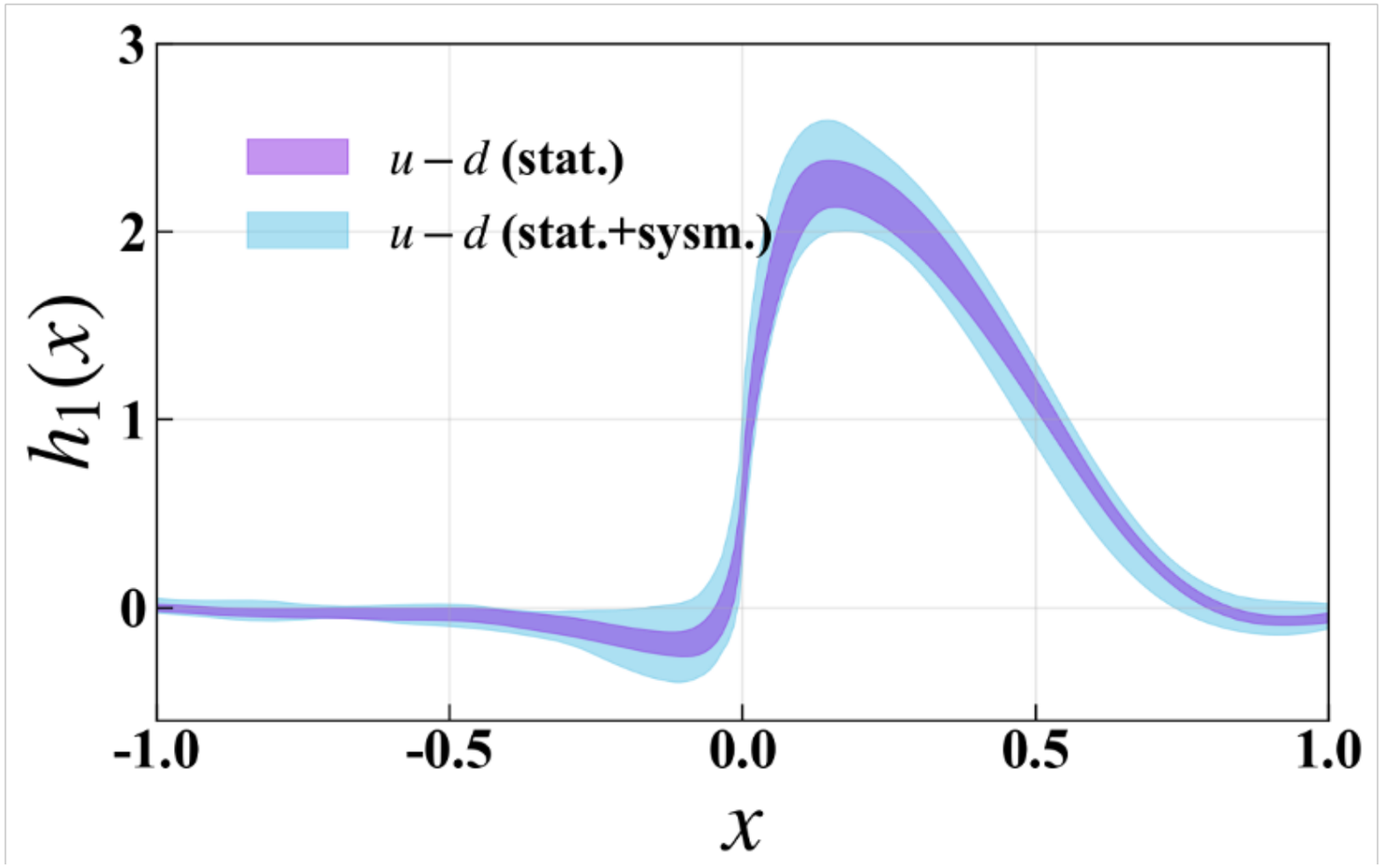
h_1: collinear transversity distribution function

$H_1^{\sphericalangle}$: a chiral-odd dihadron fragmentation function

describing the correlation between quark transverse polarization and the azimuthal orientation of the hadron pair plane

M. Radici, R. Jakob, and A. Bianconi, Phys. Rev. D 65 (2002) 074031;
A. Bacchetta, A. Courtoy, M. Radici, JHEP 03 (2013) 119.

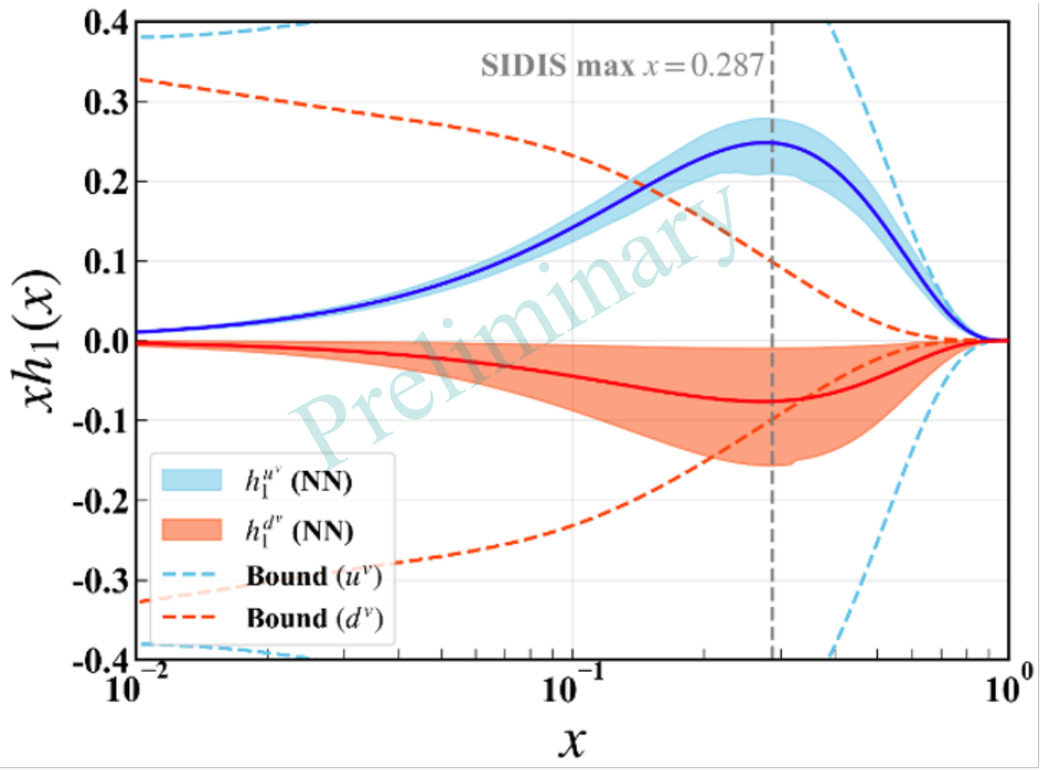
Transversity Distribution from Lattice QCD



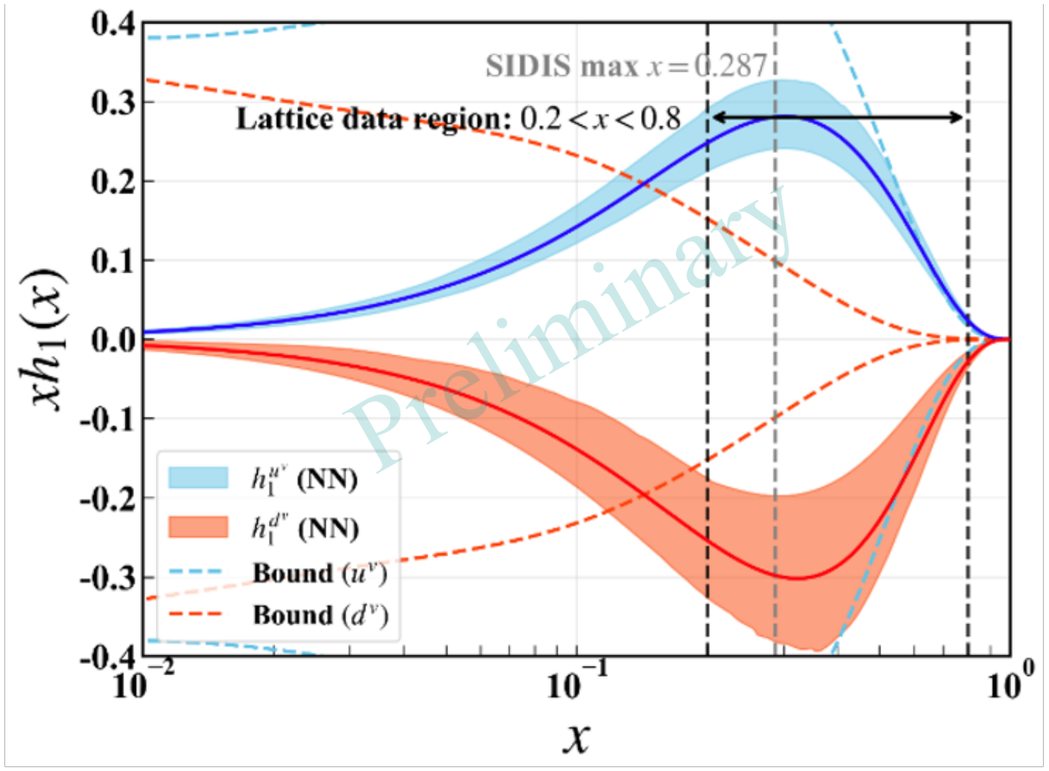
Lattice Parton Collaboration, Phys. Rev. Lett. 131 (2023) 261901.

A Combined Analysis of Transversity

SIDIS + e^+e^- dihadron data



SIDIS + e^+e^- + LQCD



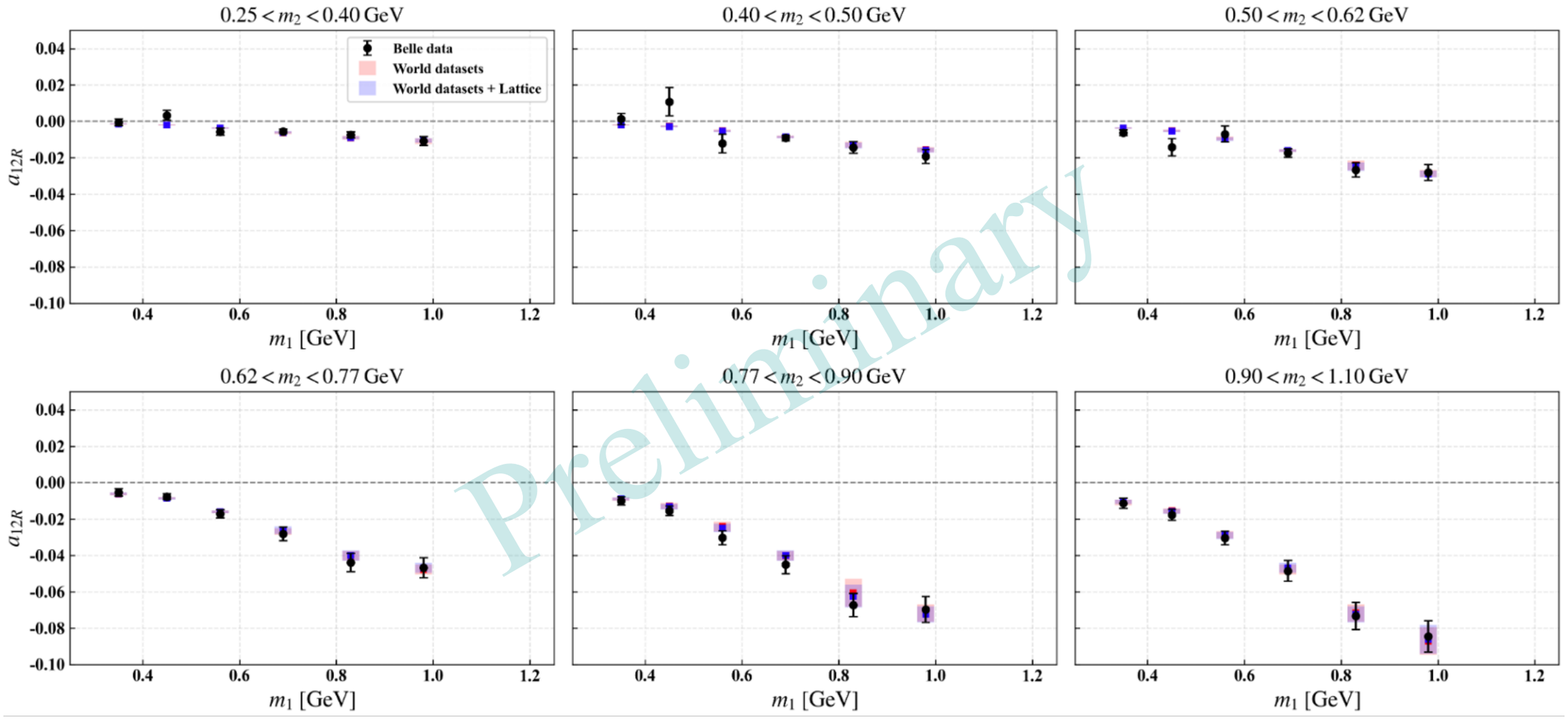
χ^2/N	Belle (dσ)	Belle (Asym)	HERMES (p)	COMPASS(p)	COMPASS(d)
without LQCD	0.799	0.793	0.894	0.903	0.821
with LQCD	0.799	0.722	0.988	0.939	0.795

RHIC-STAR data not included yet.

H. Dong, TL, P. Sun, Y. Zhao, 2026

A Combined Analysis of Transversity

Comparison with Belle data



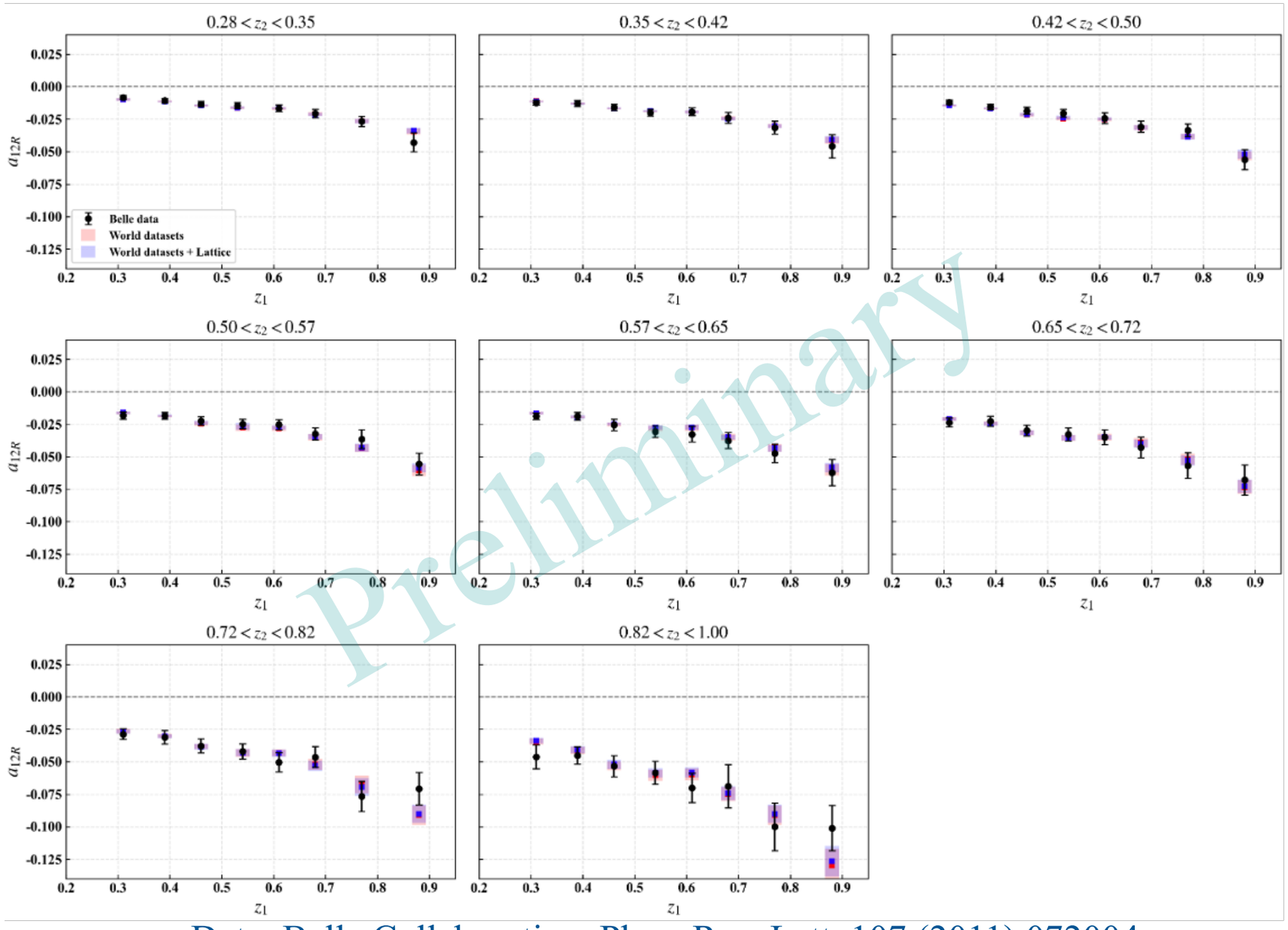
Preliminary

Data: Belle Collaboration, Phys. Rev. Lett. 107 (2011) 072004.

H. Dong, TL, P. Sun, Y. Zhao, 2026

A Combined Analysis of Transversity

Comparison with Belle data



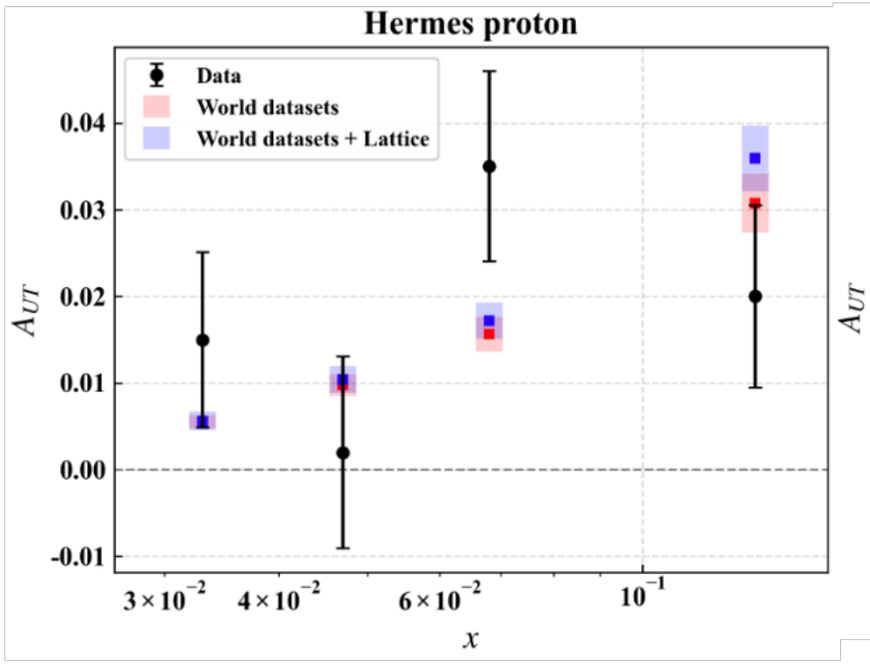
Data: Belle Collaboration, Phys. Rev. Lett. 107 (2011) 072004.

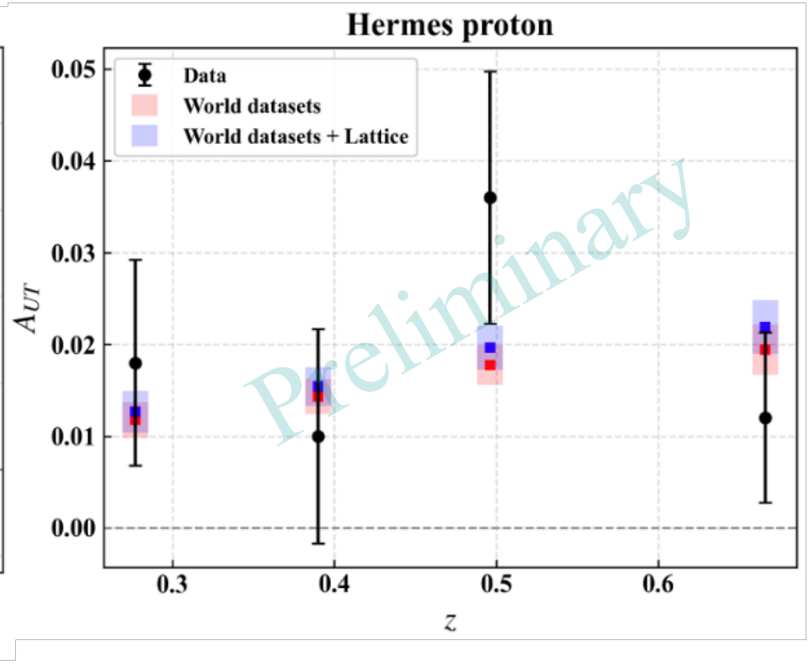
H. Dong, TL, P. Sun, Y. Zhao, 2026

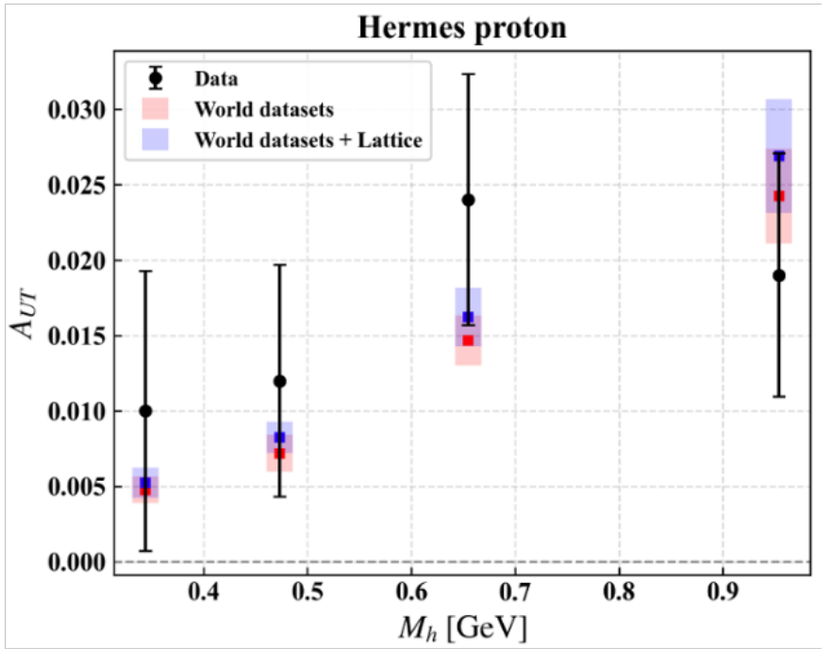
Preliminary

A Combined Analysis of Transversity

Comparison with HERMES data





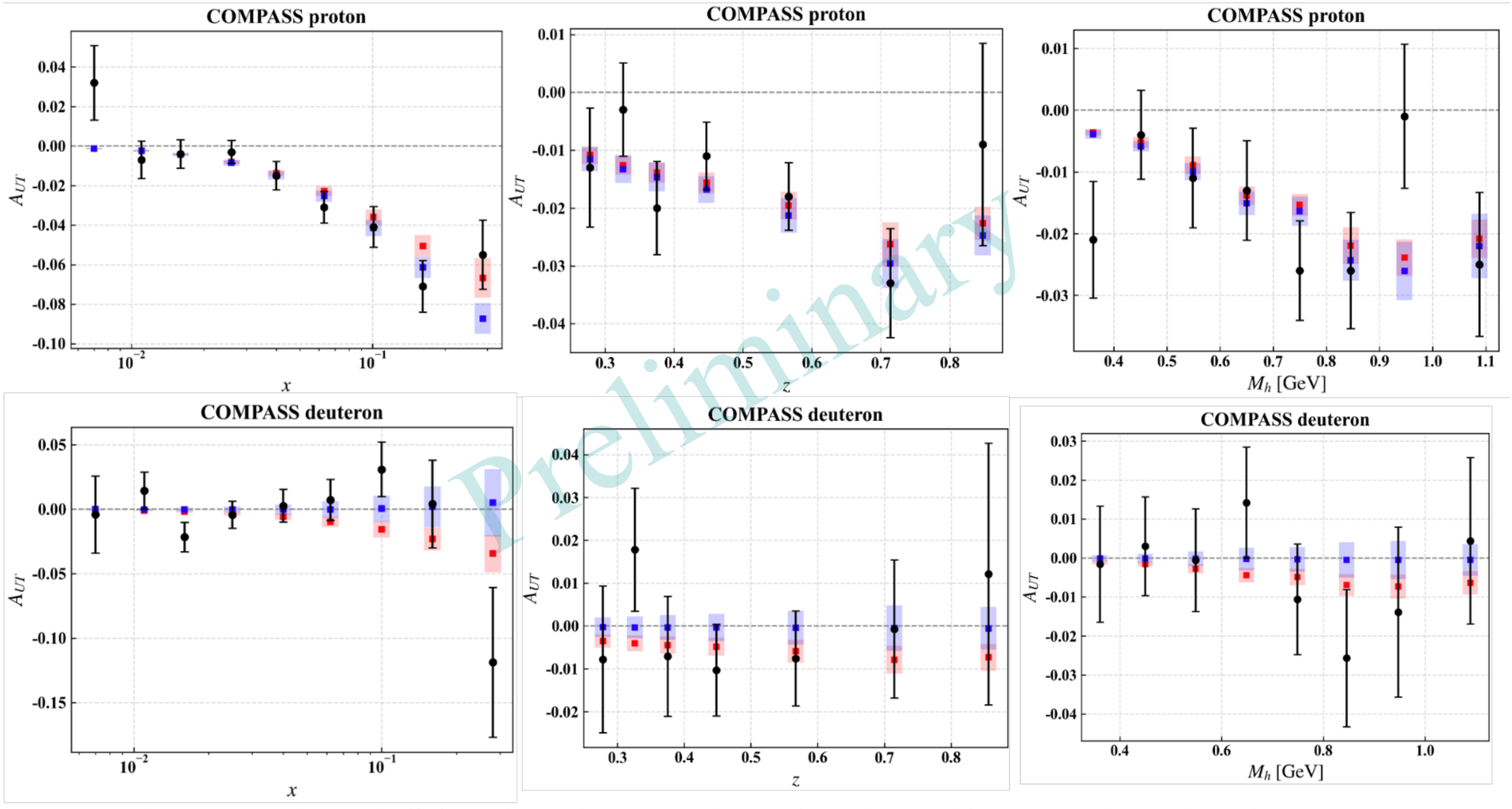


Data: HERMES Collaboration, JHEP 06 (2008) 017.

H. Dong, TL, P. Sun, Y. Zhao, 2026

A Combined Analysis of Transversity

Comparison with COMPASS data



Data: COMPASS Collaboration, Phys. Lett. B 736 (2014) 124; Phys. Lett. B 845 (2023) 138155.

H. Dong, TL, P. Sun, Y. Zhao, 2026

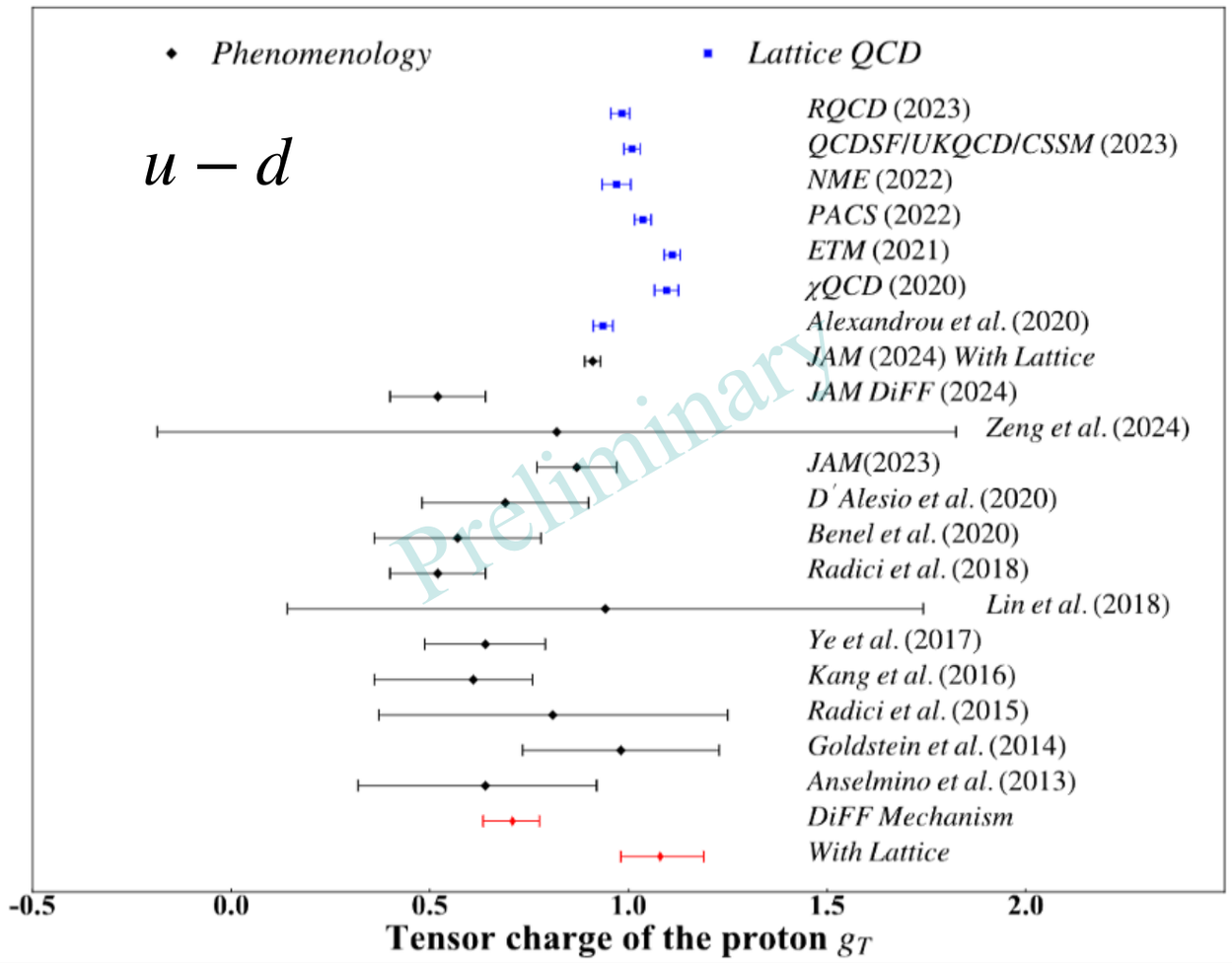
Preliminary

Tensor Charge

Tensor charge

$$\langle P,S|\bar{\psi}^q i\sigma^{\mu\nu}\gamma_5\psi^q|P,S\rangle = g_T^q \bar{u}(P,S) i\sigma^{\mu\nu}\gamma_5 u(P,S) \qquad g_T^q = \int_0^1 [h_1^q(x) - h_1^{\bar{q}}(x)]\,dx$$

- A fundamental QCD quantity: matrix element of local operators.
- Moment of the transversity distribution: valence quark dominant.
- Calculable in lattice QCD.



H. Dong, TL, P. Sun, Y. Zhao, 2026

Summary

- Nucleon spin structure is still not well understood.
- ML can help extract parton distributions from lattice QCD data
  - learn patterns from noisy data
  - reduce higher-twist contaminations
  - extend limited coverage: $z, \omega, \cdots$
  - reconstruct x-dependent distributions
  …
- Lattice QCD also provides complementary information to determine the distributions, especially when lacking precise experimental data.

Thank you!

Backup

Imposing a Bound?

Question:

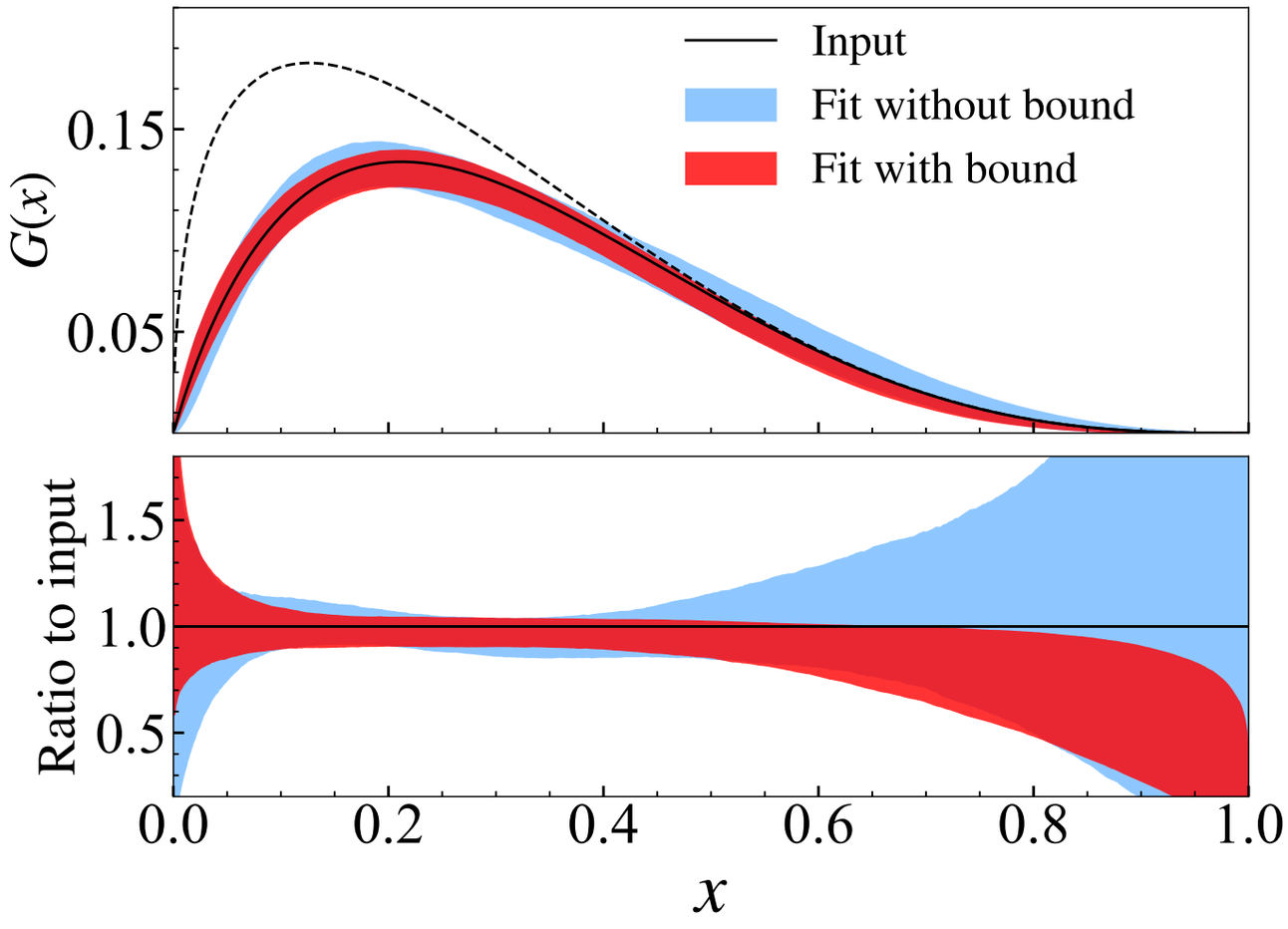
Whether we should impose the bound $|g_1(x)| \leq f_1(x)$ in the fit?
Similar situations in many other quantities.

A toy model test:

Input some "pol" function $G(x)$
and "unpol" function $F(x)$
Generate asymmetry "data"

Fit the "data" with or without imposing the bound $G(x) \leq F(x)$

Imposing a bound in the fit may bias the fit result!

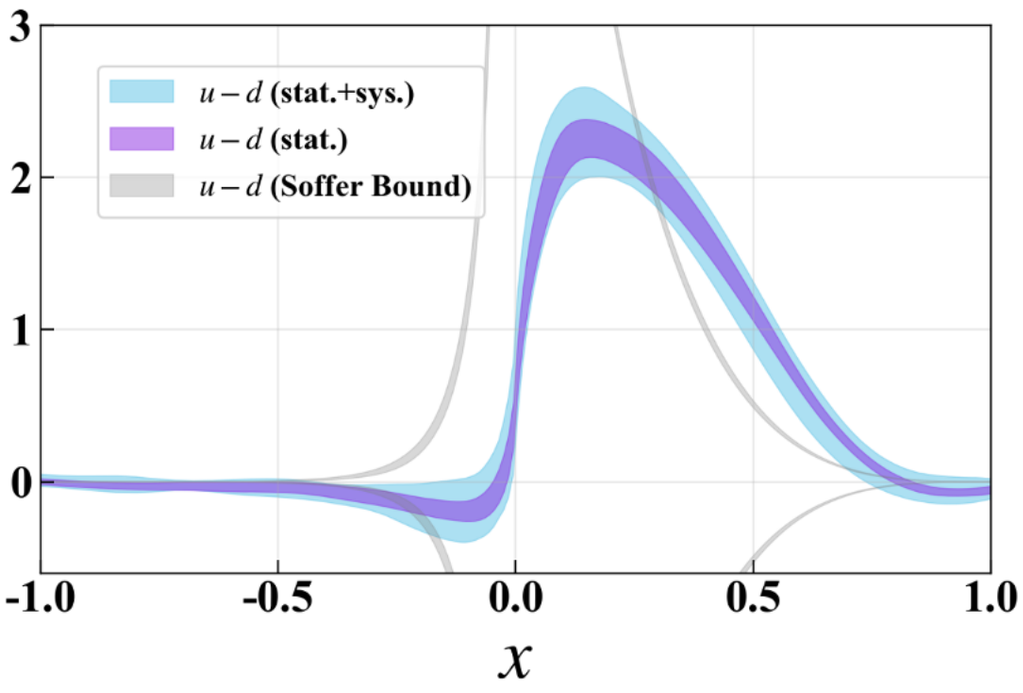


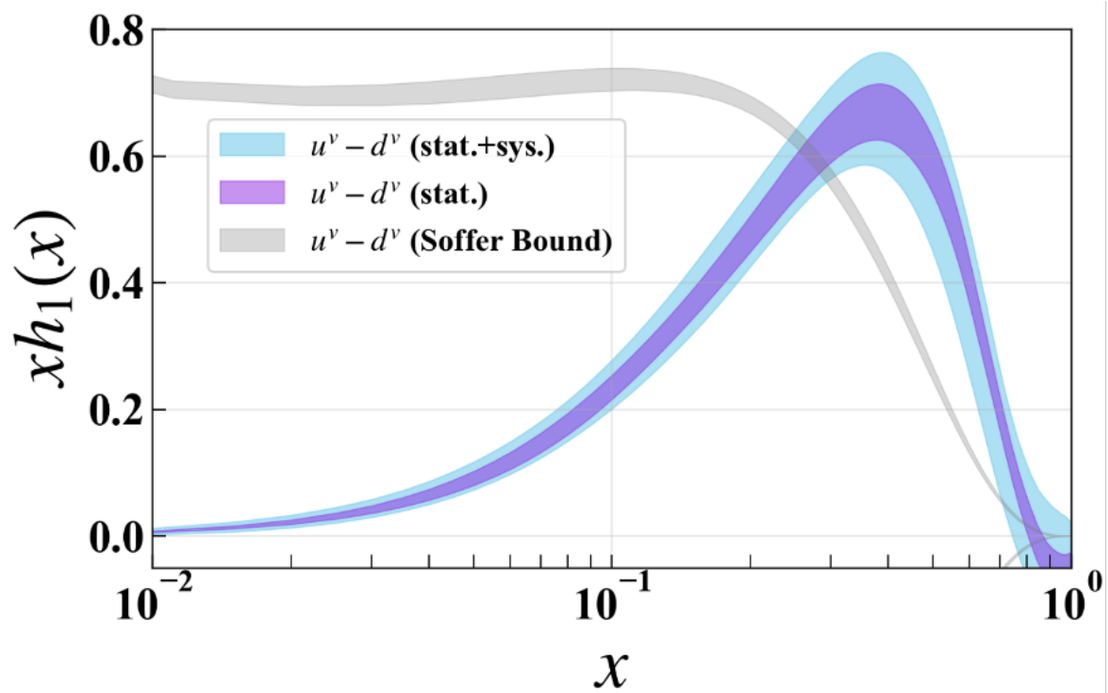
About Soffer's Bound

Soffer's bound on transversity distributions

$$|h_1(x)| \leq \frac{1}{2}[f_1(x) + g_1(x)]$$

It is broken in existing lattice QCD calculations





Lattice results from: LPC Collaboration, Phys. Rev. Lett. 131 (2023) 261901.

TMD Evolution

Evolution equations

$$\mu^2\frac{dF(x,b;\mu^2,\zeta)}{d\mu^2}=\frac{\gamma_F(\mu,\zeta)}{2}F(x,b;\mu^2,\zeta)$$

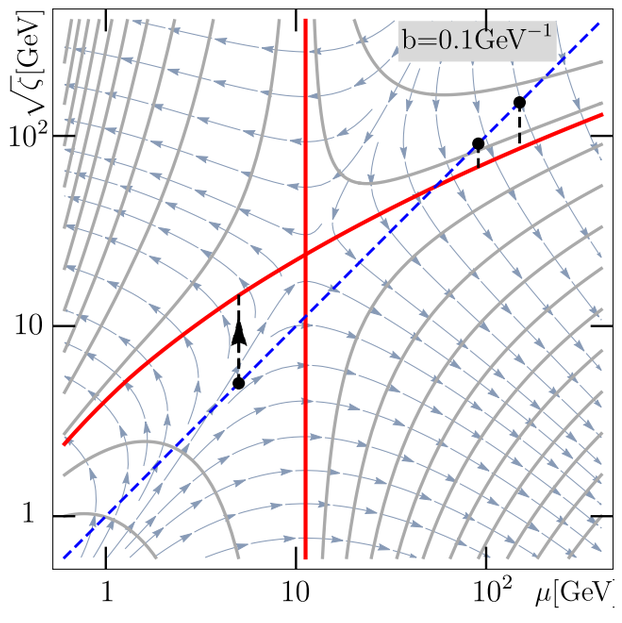
$$\zeta\frac{dF(x,b;\mu^2,\zeta)}{d\zeta}=-\mathscr{D}(\mu,b)F(x,b;\mu^2,\zeta)$$

$$-\zeta\frac{d\gamma_F(\mu,\zeta)}{d\zeta}=\mu\frac{d\mathscr{D}(\mu,b)}{d\mu}=\Gamma_{\rm cusp}(\mu)$$

$$\gamma_F(\mu,\zeta)=\Gamma_{\rm cusp}(\mu)\ \ln\frac{\mu^2}{\zeta}-\gamma_V(\mu)$$

$$F\left(x,b;\mu_f,\zeta_f\right)=\exp\left[\int_P\left(\gamma_F(\mu,\zeta)\frac{d\mu}{\mu}-\mathscr{D}(\mu,b)\frac{d\zeta}{\zeta}\right)\right]F\left(x,b;\mu_i,\zeta_i\right)$$

ζ-prescription



equipotential lines:

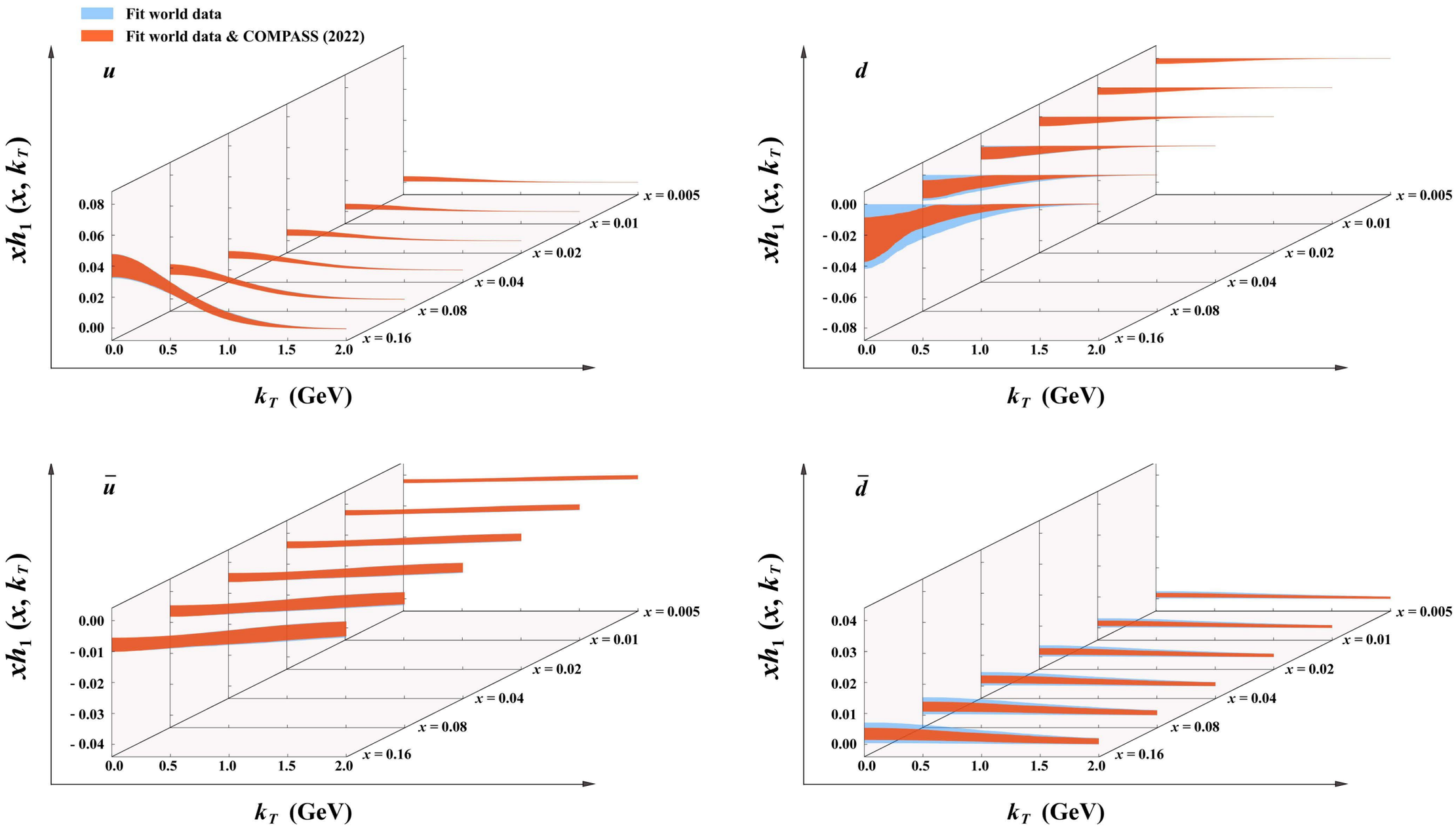
$$\frac{d\ln\zeta_\mu(\mu,b)}{d\ln\mu^2}=\frac{\gamma_F\left(\mu,\zeta_\mu(\mu,b)\right)}{2\mathscr{D}(\mu,b)}$$

$$\mathscr{D}\left(\mu_0,b\right)=0,\quad\gamma_F\left(\mu_0,\zeta_\mu\left(\mu_0,b\right)\right)=0$$

$$F\left(x,b;Q,Q^2\right)=\left(\frac{Q^2}{\zeta_Q(b)}\right)^{-\mathscr{D}(Q,b)}F(x,b),\quad\mu_f^2=\zeta_f=Q^2$$

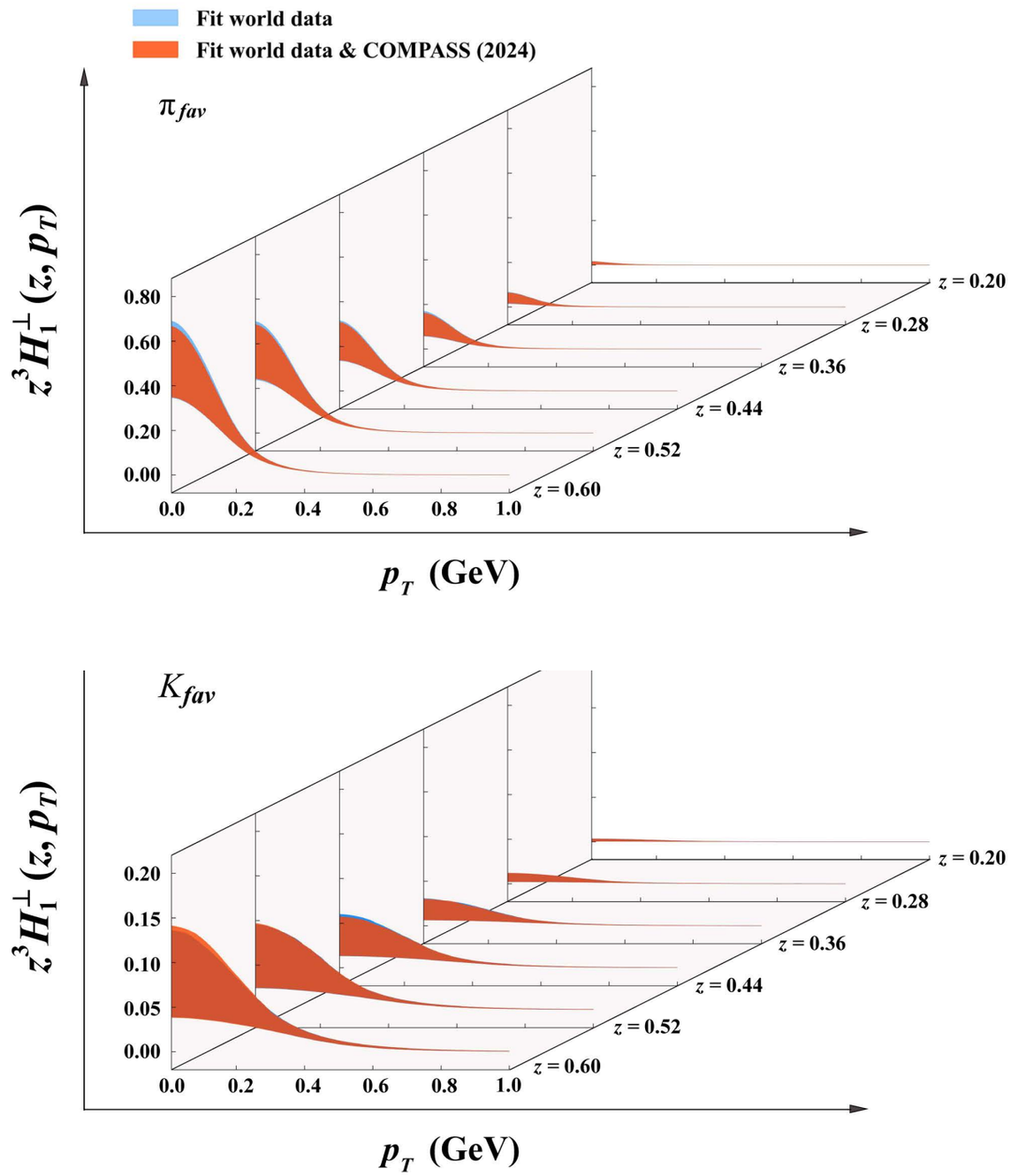
I. Scimemi, A. Vladmirov, JHEP 06 (2020) 137.

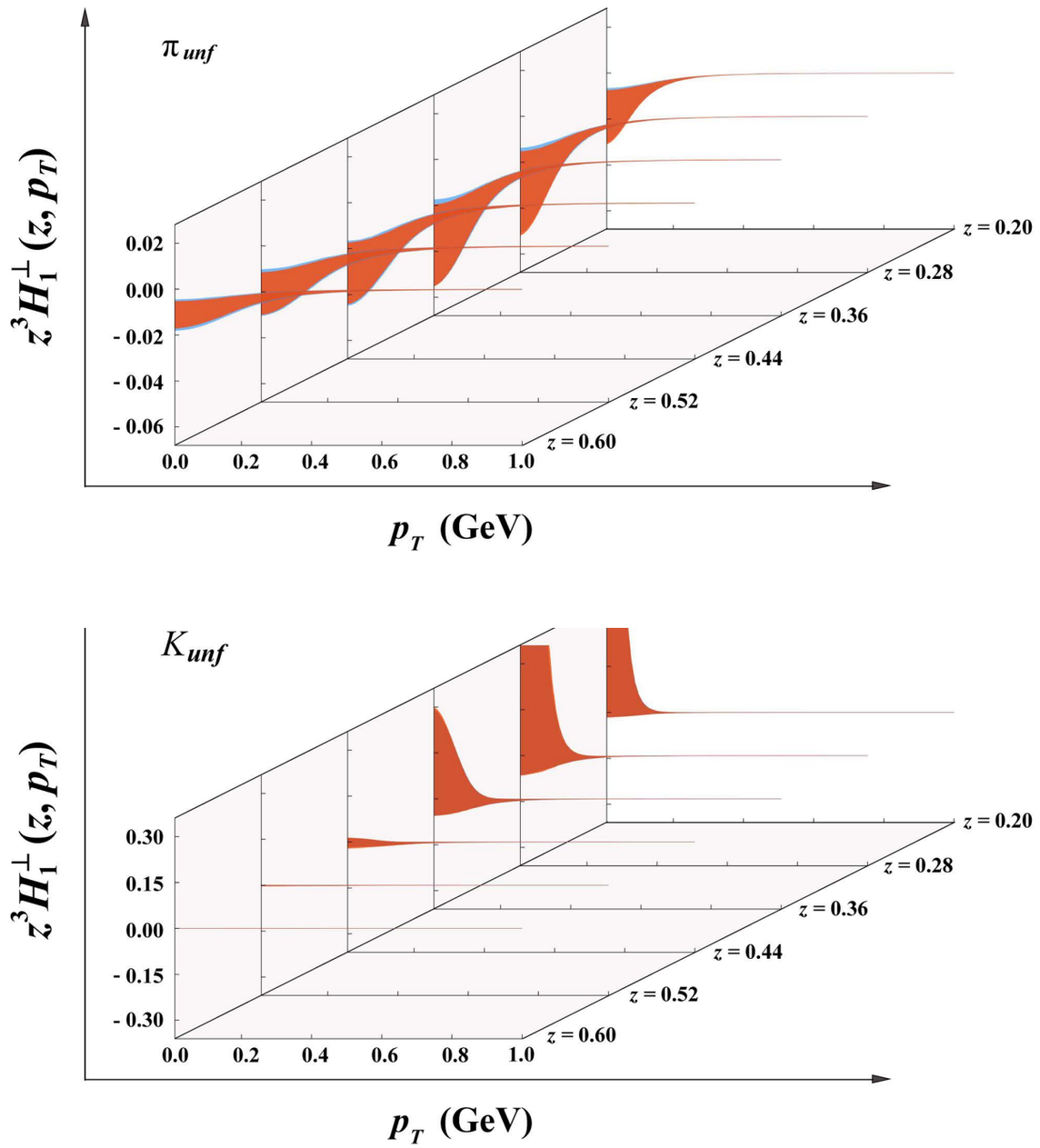
Transversity Distributions



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Collins Fragmentation Functions





C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324