Connecting Quantum error correction and Lattice Gauge Theories

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TIFPA

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Motivation

Lattice Gauge Theories are an application where quantum computers may provide some advantage

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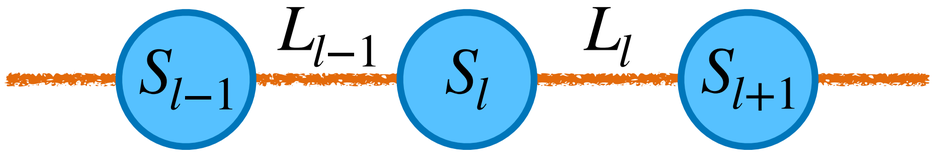
In order to do useful and complicated calculations, we need quantum error correction

Can we engineer a quantum error correcting code, specifically tailored for Lattice Gauge Theories?

Lattice Gauge Theories



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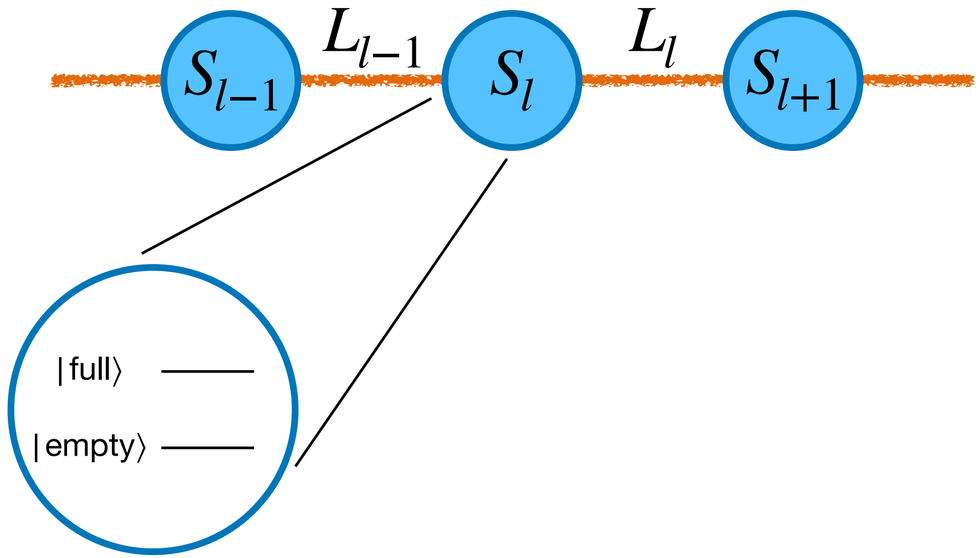

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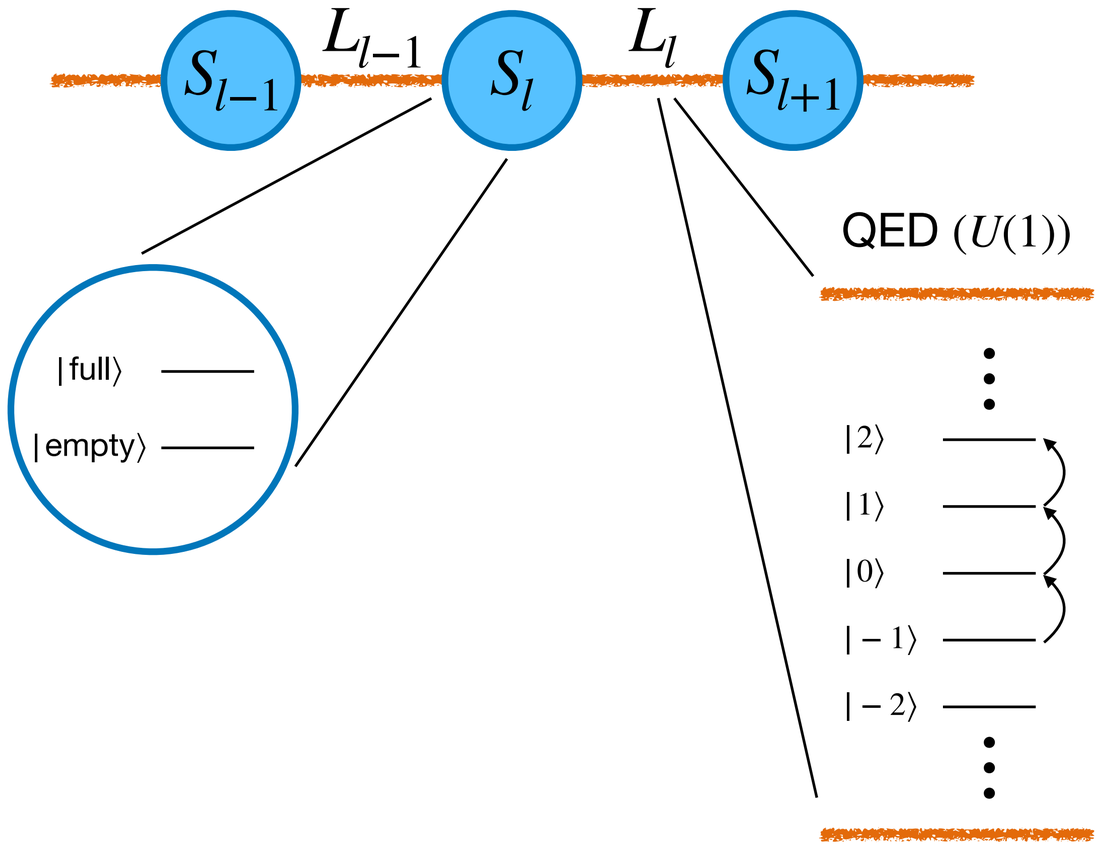

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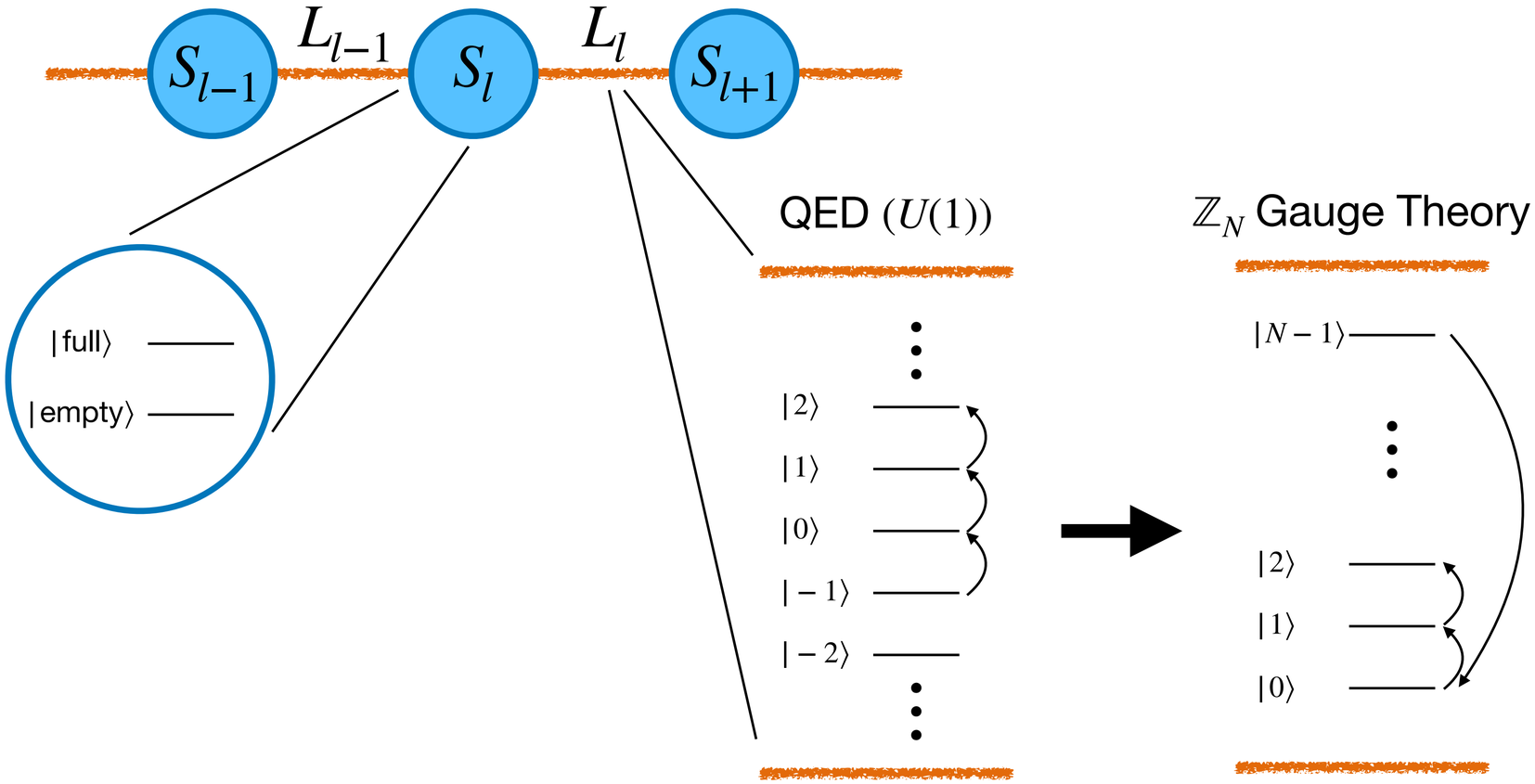

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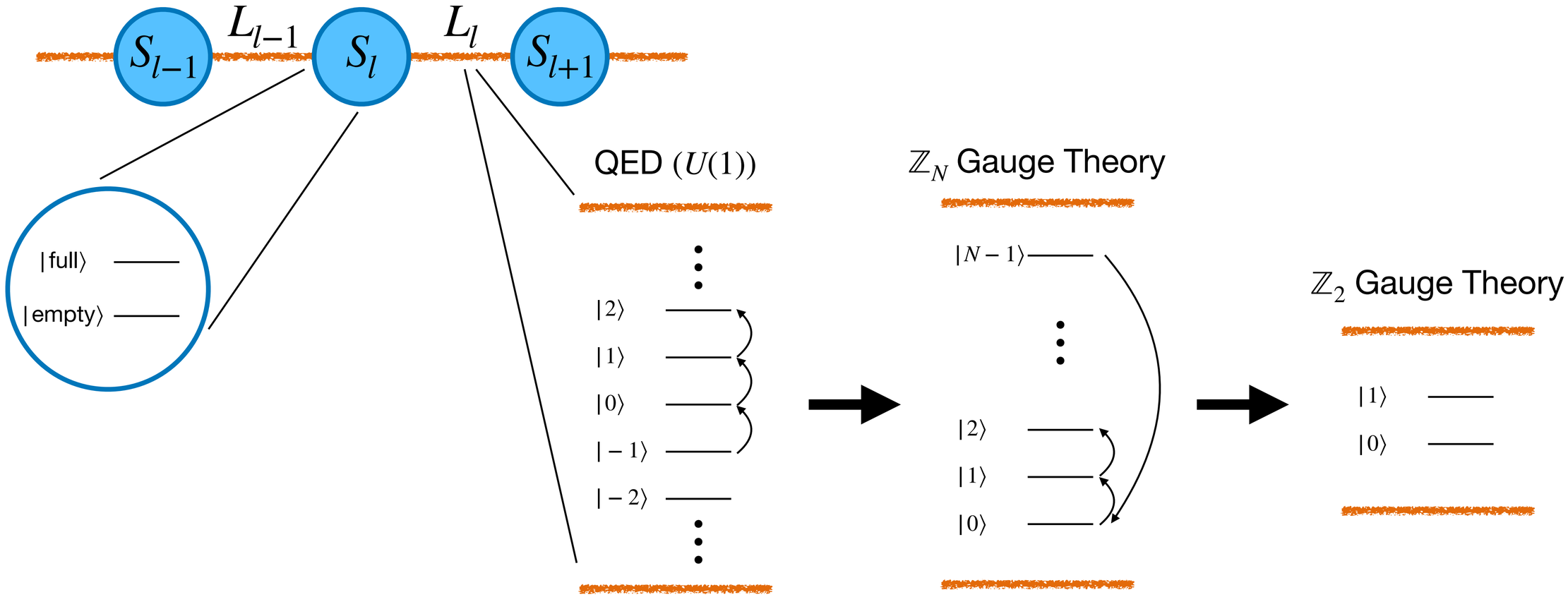

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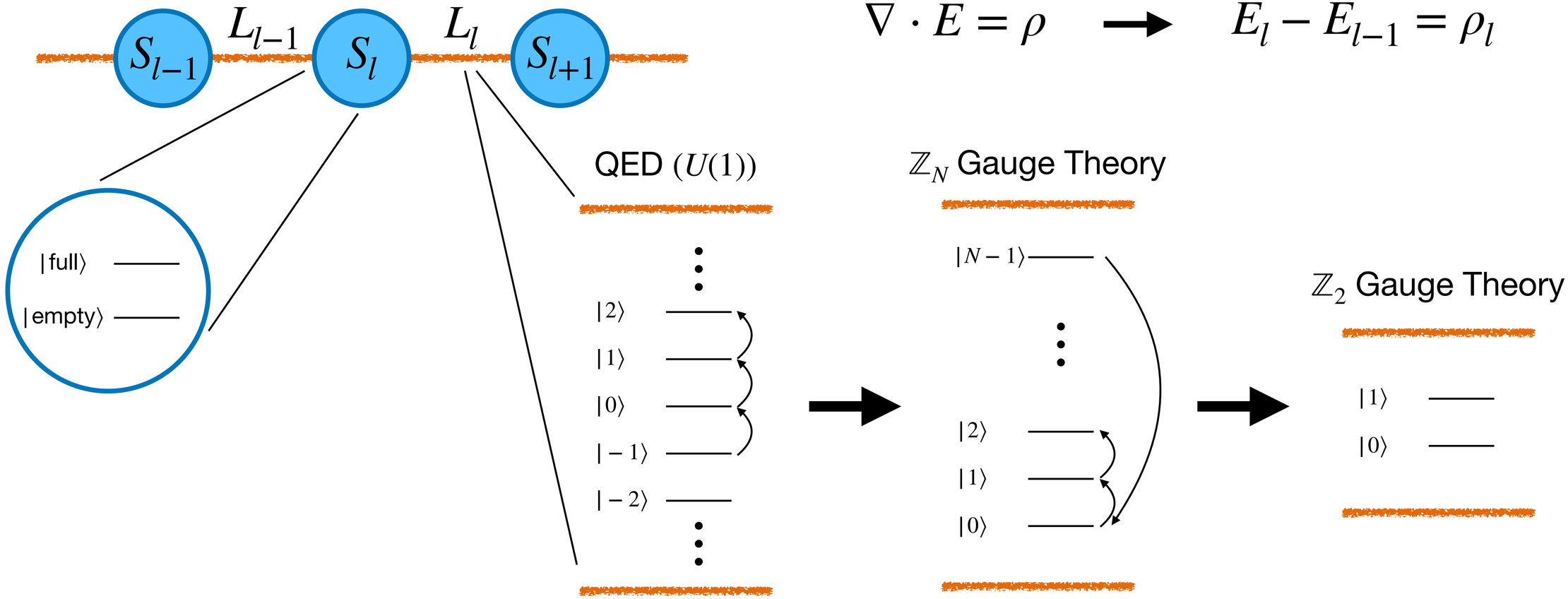

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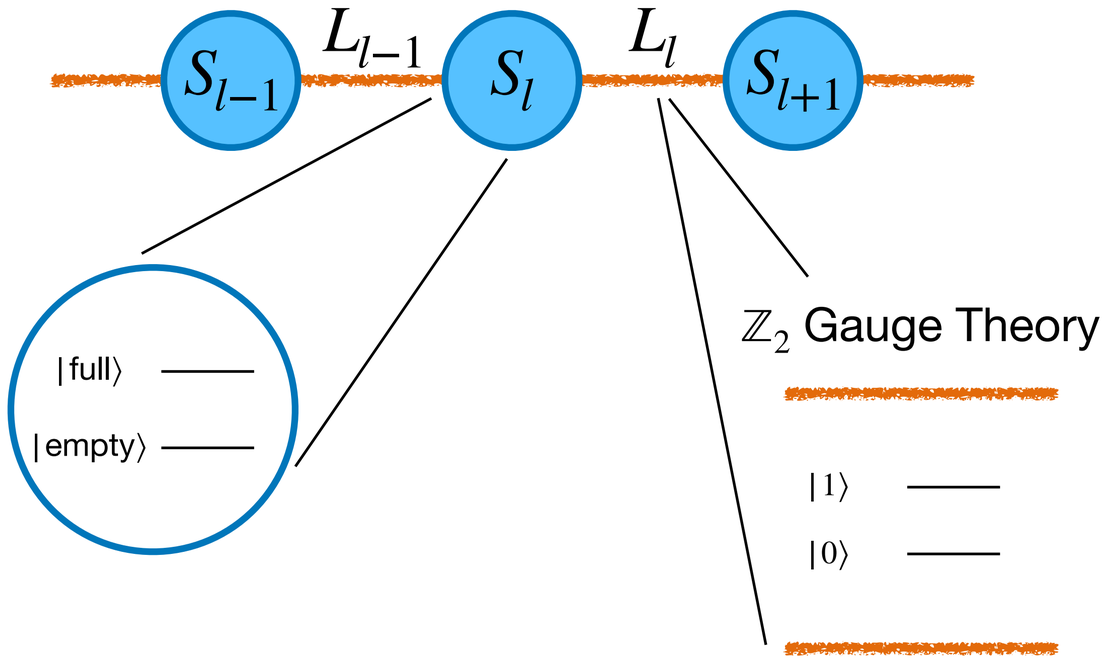

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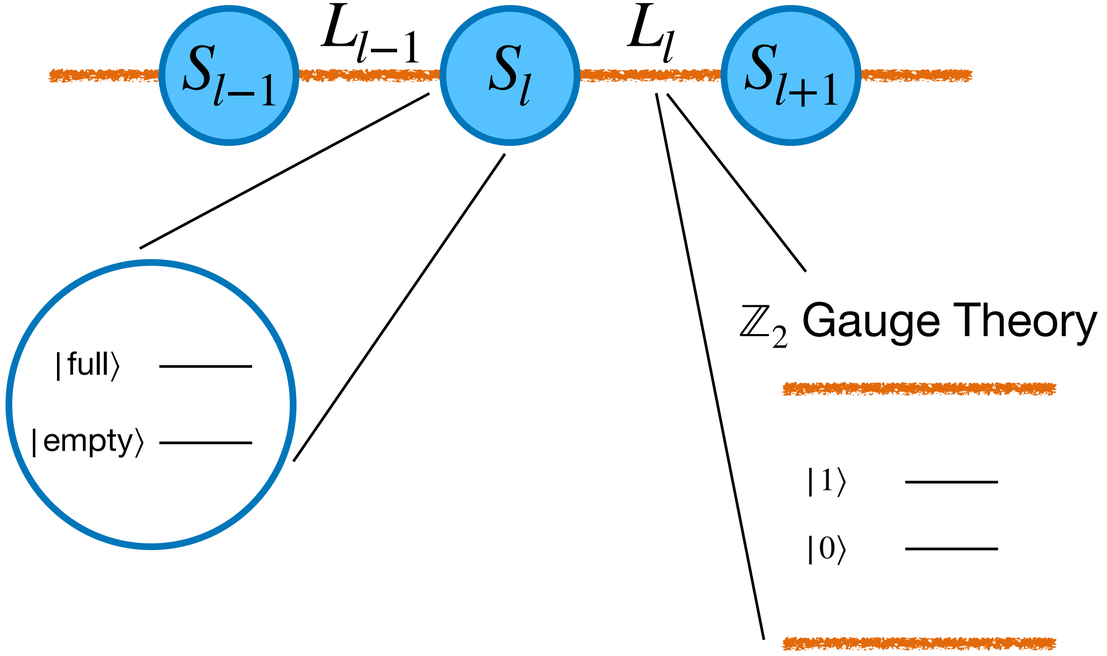

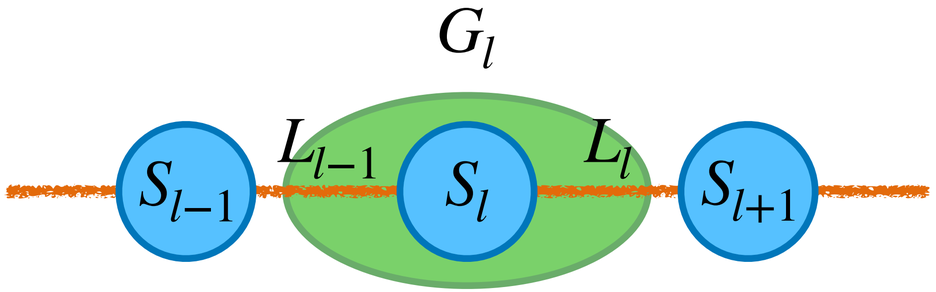

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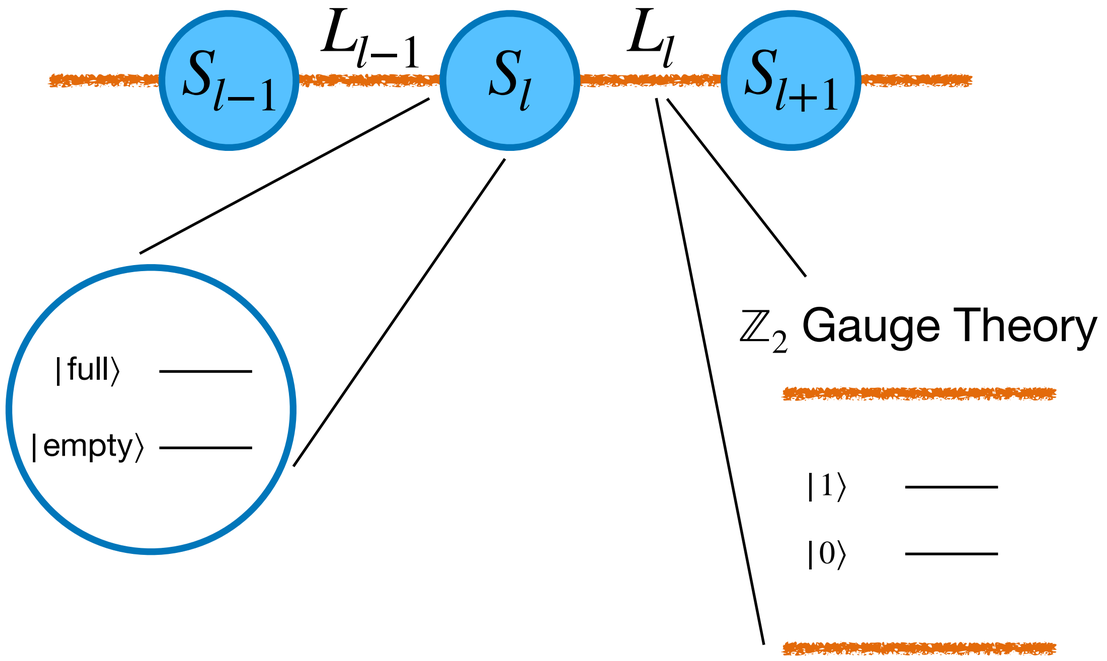

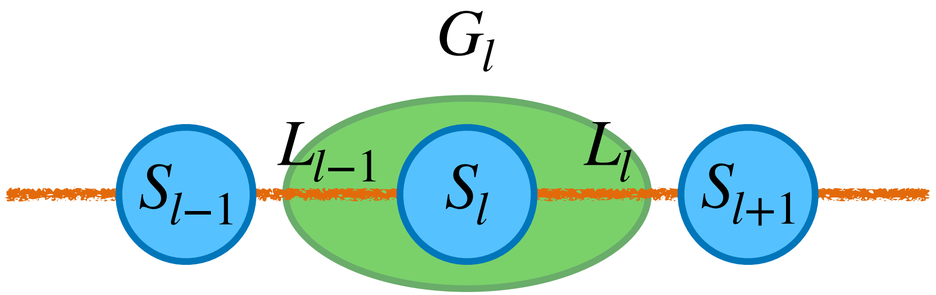

Gauss' law operator

$$G_l|phys\rangle = |phys\rangle$$

Quantum Error Correction

Quantum Error Correction

Classical computers

bit: $\{0, 1\}$

Errors: bit flip

$$0 \to 1$$
$$1 \to 0$$

Redundancy

$$0_L = 000, \quad 1_L = 111$$

$$010 \to 0_L$$

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Symmetries (stabilizers)

$$S_1 = Z_1 Z_2, \quad S_2 = Z_2 Z_3$$

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$S_2\,|1_L\rangle = |1_L\rangle$

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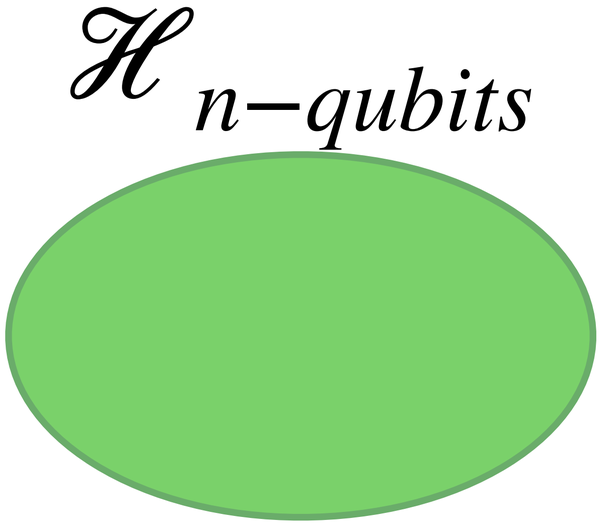

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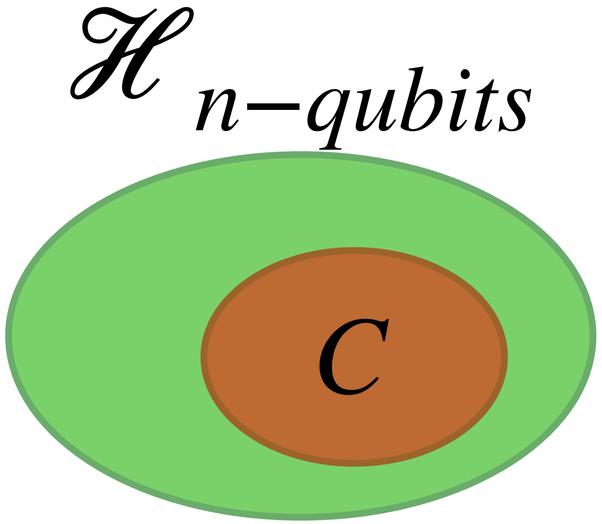
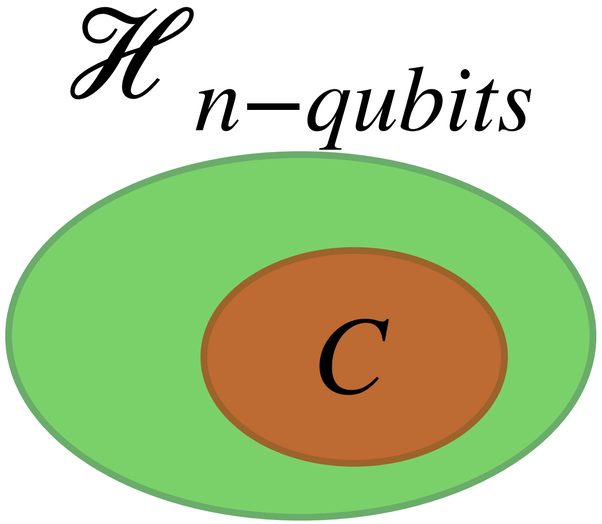


$$C = \{\, |\psi\rangle : S_i |\psi\rangle = |\psi\rangle \}$$

Quantum Error Correction



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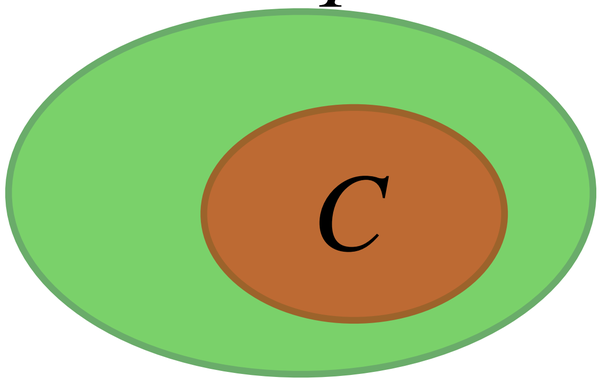
Quantum Error Correction

$\mathscr{H}_{n-qubits}$



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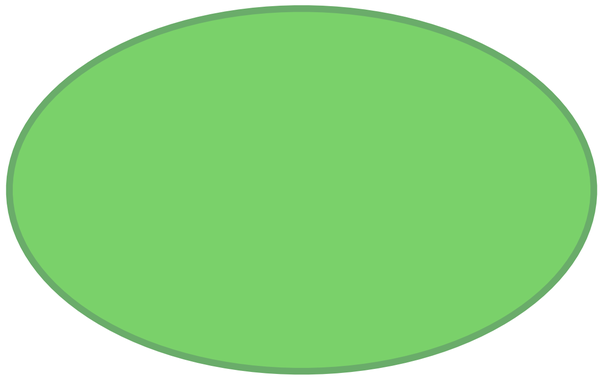
$\mathscr{H}_{lattice}$



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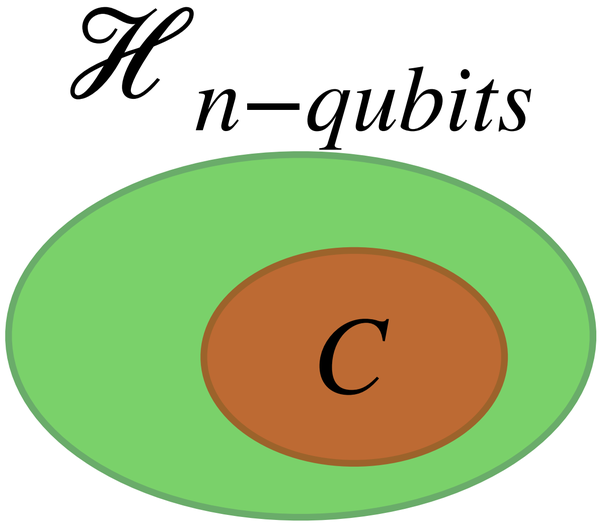

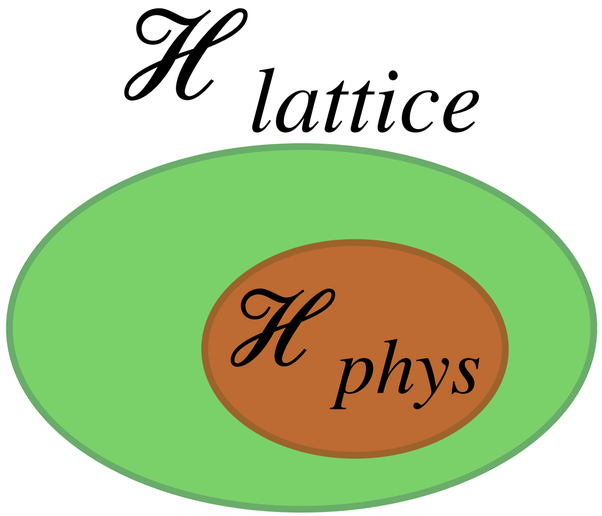

$$C = \{\, |\psi\rangle : S_i|\psi\rangle = |\psi\rangle \,\}$$

$$\mathscr{H}_{phys} = \{\, |phys\rangle : G_l|phys\rangle = |phys\rangle \,\}$$

$$S_1 = Z_1Z_2, \quad S_2 = Z_2Z_3$$

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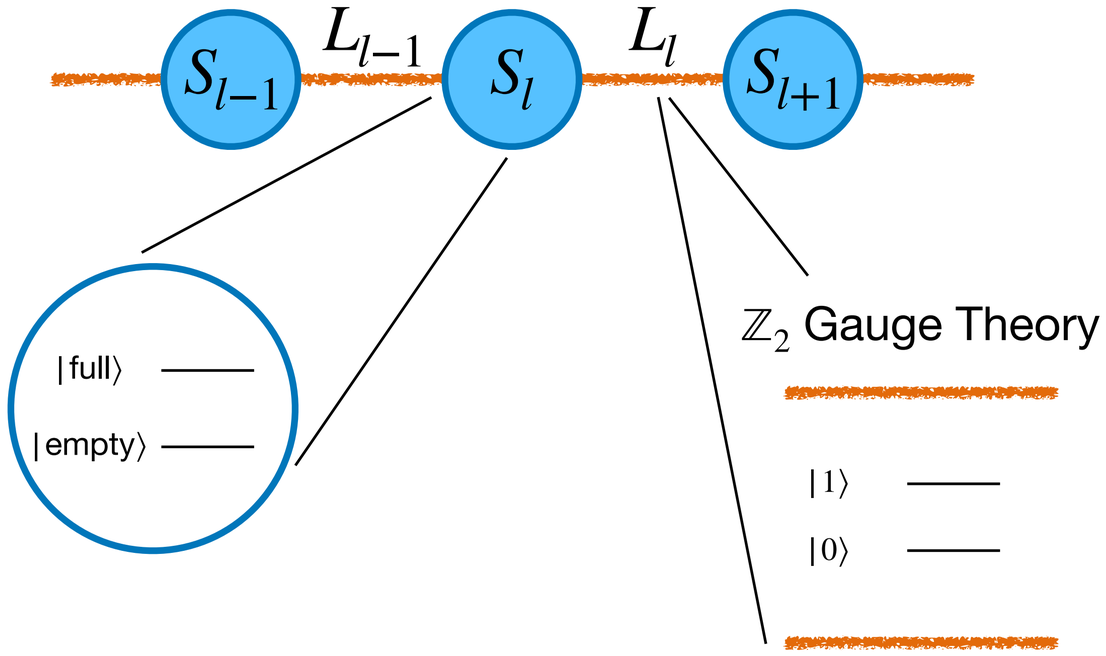
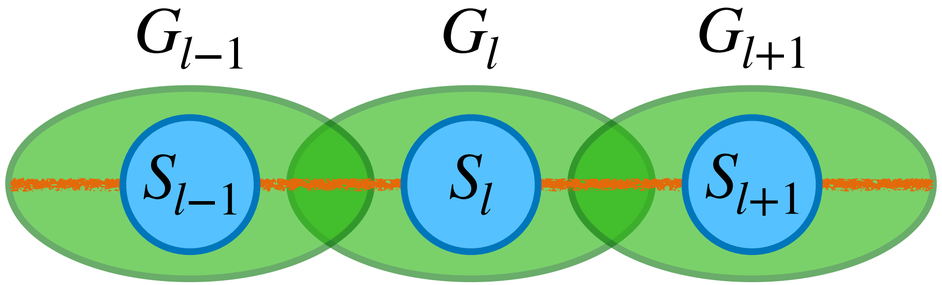
QEC for LGTs





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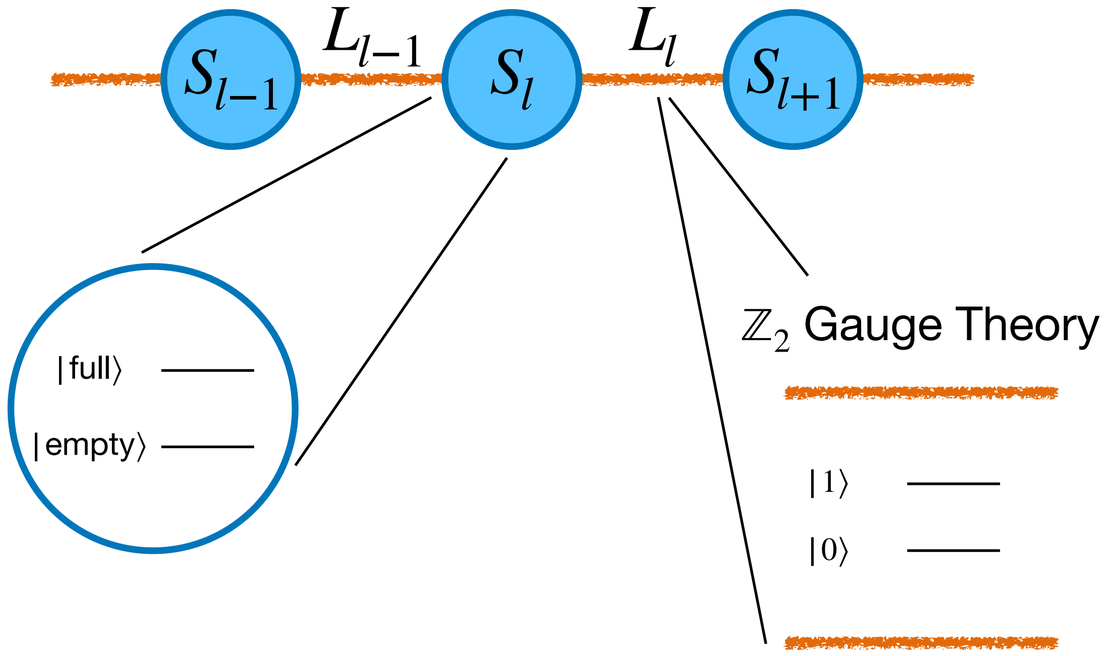

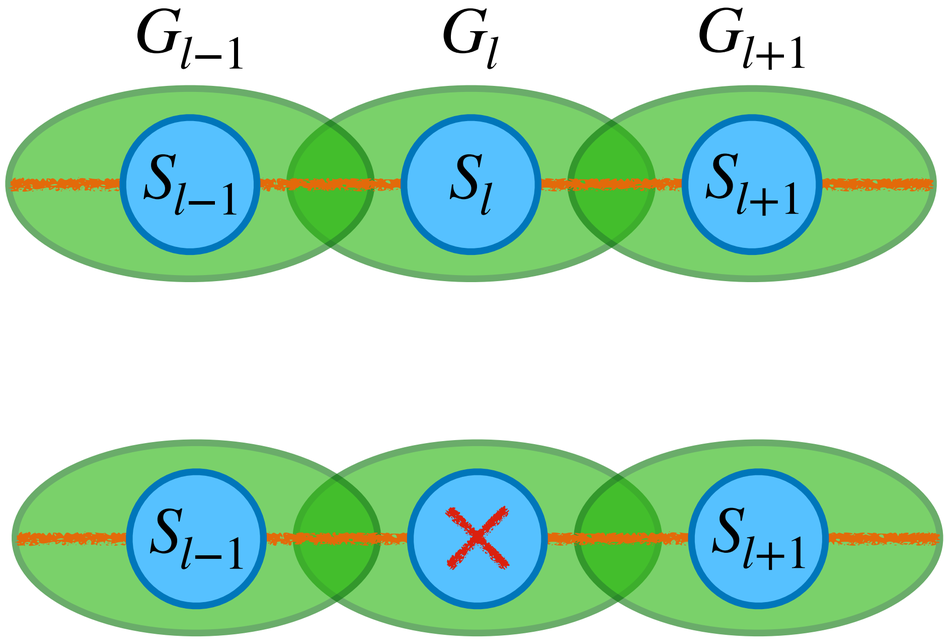

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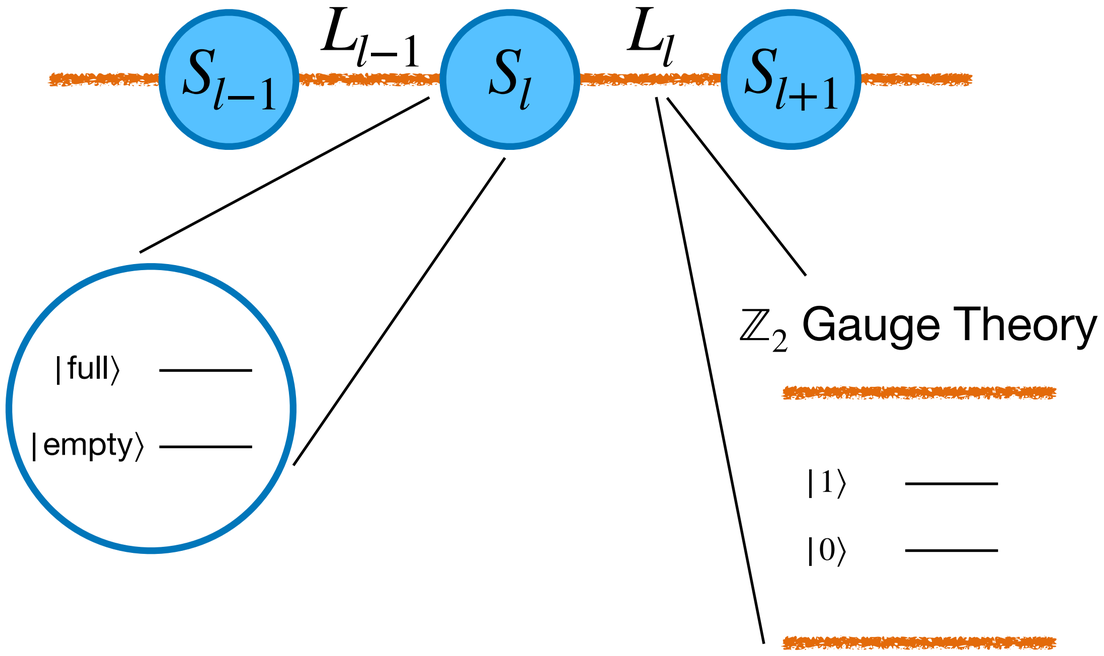

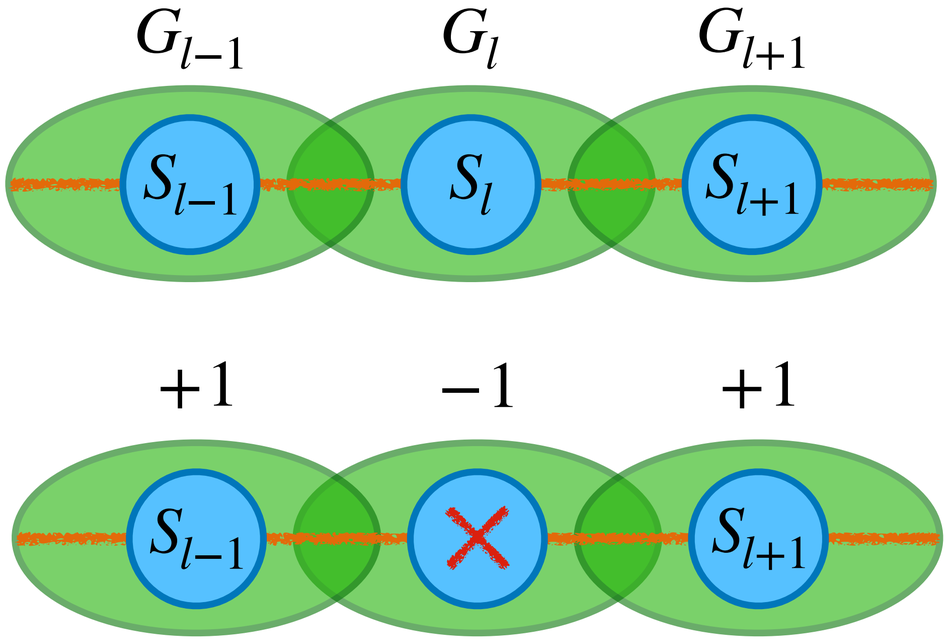

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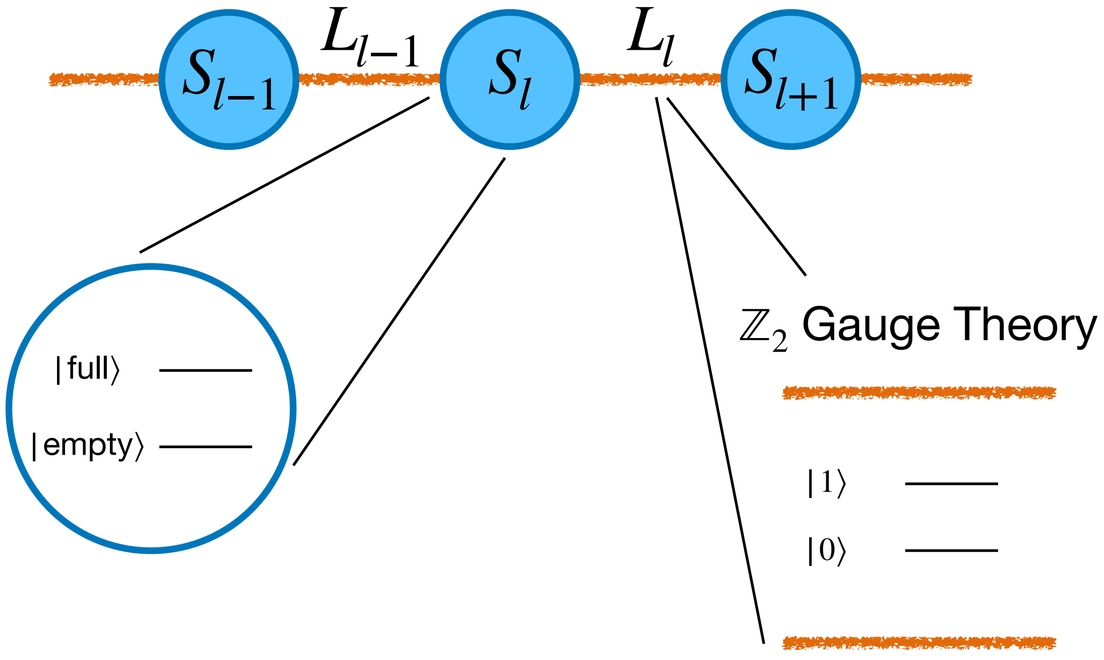

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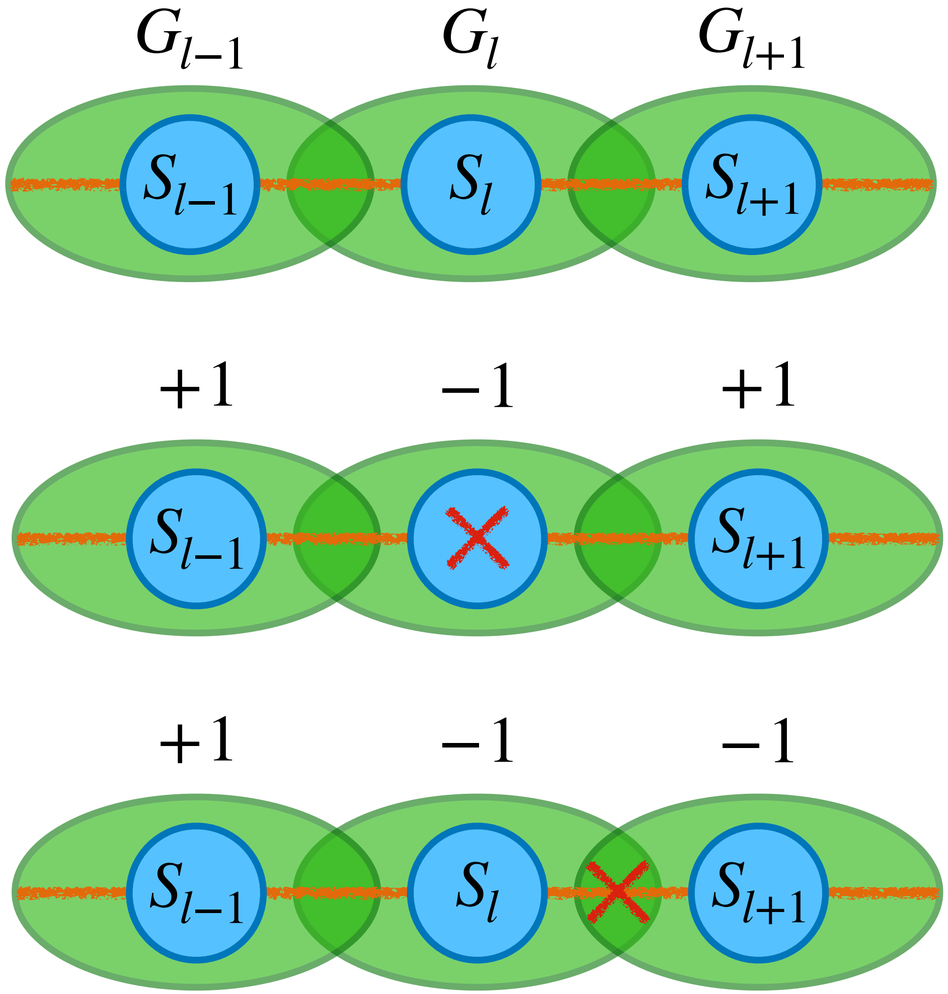
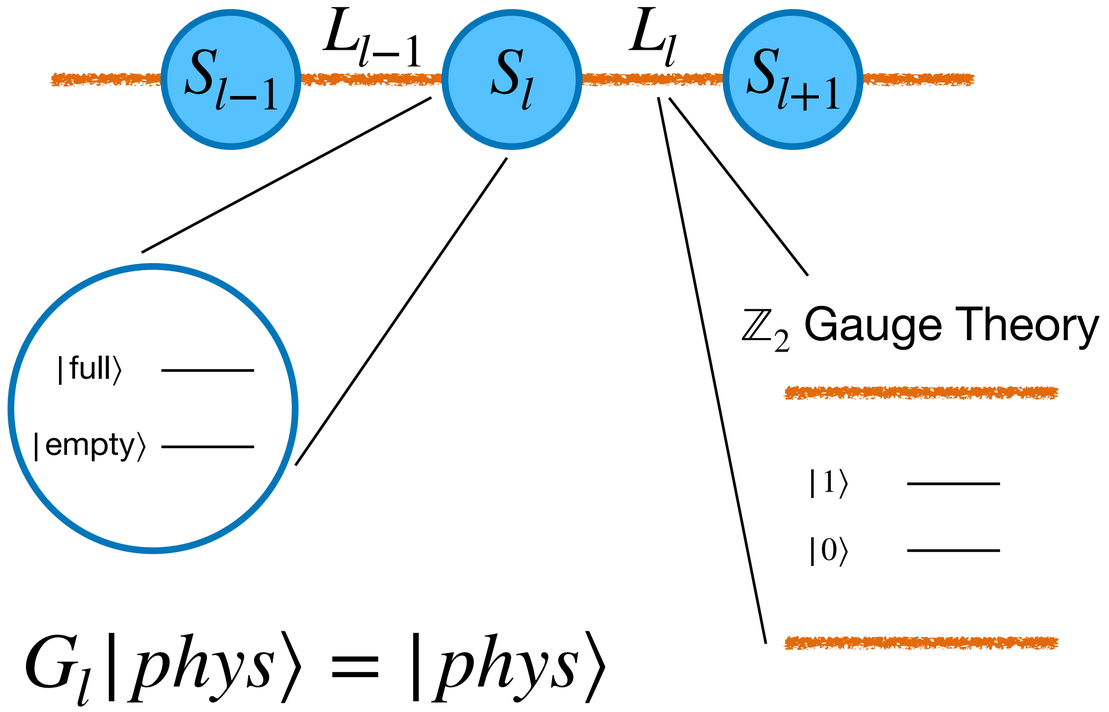


QEC for $\mathbb{Z}_2$ LGT



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Quantum error correction with gauge symmetries

Abhishek Rajput[1]✉, Alessandro Roggero[2,3,4]✉ and Nathan Wiebe[1,5,6]✉

Quantum simulations of lattice gauge theories (LGTs) are often formulated on an enlarged Hilbert space containing both physical and unphysical sectors in order to retain a local Hamiltonian. We provide simple fault-tolerant procedures that exploit such redundancy by combining a phase flip error correction code with the Gauss' law constraint to correct one-qubit errors for a $\mathbb{Z}_2$ or truncated U(1) LGT in 1+1 and 2+1 dimensions with a link flux cutoff of 1. Unlike existing work on detecting violations of Gauss' law, our circuits are fault tolerant and the overall error correction scheme outperforms a naïve application of the [5, 1, 3] code. The constructions outlined can be extended to LGT systems with larger cutoffs and may be of use in understanding how to hybridize error correction and quantum simulation for LGTs in higher space-time dimensions and with different symmetry groups.

npj Quantum Information (2023)9:41 ; https://doi.org/10.1038/s41534-023-00706-8

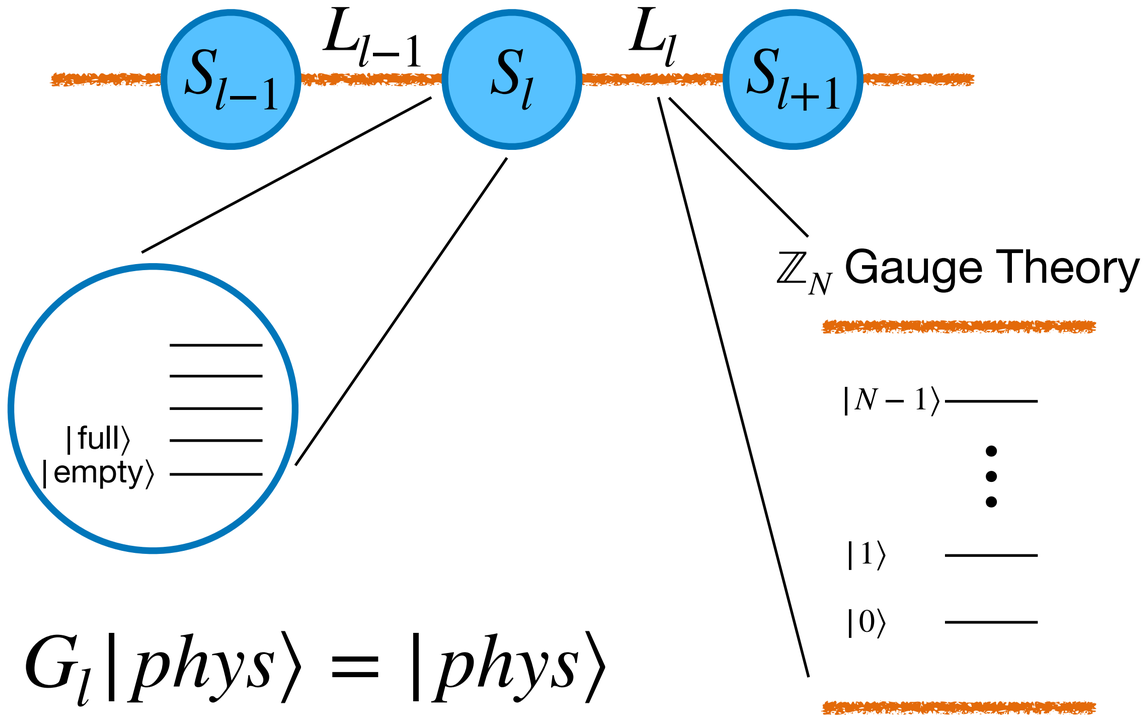
Fault-tolerant simulation of Lattice Gauge Theories with gauge covariant codes

Luca Spagnoli[1,2], Alessandro Roggero[1,2], and Nathan Wiebe[3,4,5]

Quantum 10, 1968

Gauge group → Abelian group → Finite group → $\mathbb{Z}_N$ group → $\mathbb{Z}_2$ group

QEC for $\mathbb{Z}_N$ LGT



$$G_l|phys\rangle = |phys\rangle$$

Qudit stabilizer codes for $\mathbb{Z}_N$ lattice gauge theories with matter

Luca Spagnoli* and Alessandro Roggero†
Dipartimento di Fisica, University of Trento, via Sommarive 14, I38123, Povo, Trento, Italy and INFN-TIFPA Trento Institute of Fundamental Physics and Applications, Trento, Italy

Nathan Wiebe‡
Department of Computer Science, University of Toronto, Toronto, ON M5S 2E4, Canada and Pacific Northwest National Laboratory, Richland, WA 99354, USA

(Dated: February 25, 2026)

In this work we extend the connection between Quantum Error Correction (QEC) and Lattice Gauge Theories (LGTs) by showing that a $\mathbb{Z}_N$ gauge theory with prime dimension N coupled to dynamical matter can be expressed as a qudit stabilizer code. Using the stabilizer formalism we show how to formulate an exact mapping of the encoded $\mathbb{Z}_N$ gauge theory onto two different bosonic models, uncovering a logical duality generated by error correction itself. From this perspective, quantum error correction provides a unifying language to expose dual descriptions of lattice gauge theories. In addition, we generalize earlier $\mathbb{Z}_2$ constructions on qubits to $\mathbb{Z}_N$ on N-level qudits and demonstrate how universal fault-tolerant gates can be implemented via state injection between compatible qudit codes.

arXiv:2602.20661

Gauge group → Abelian group → Finite group → $\mathbb{Z}_N$ group ✗ $\mathbb{Z}_2$ group

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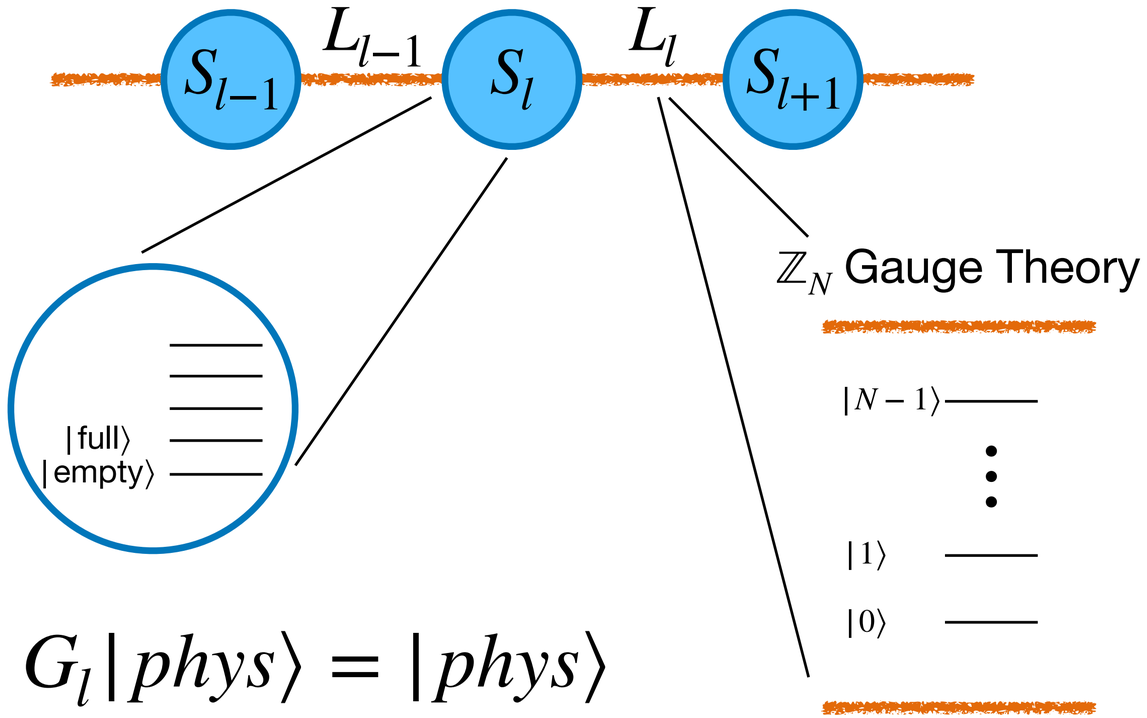

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- N prime number
- N-level sites

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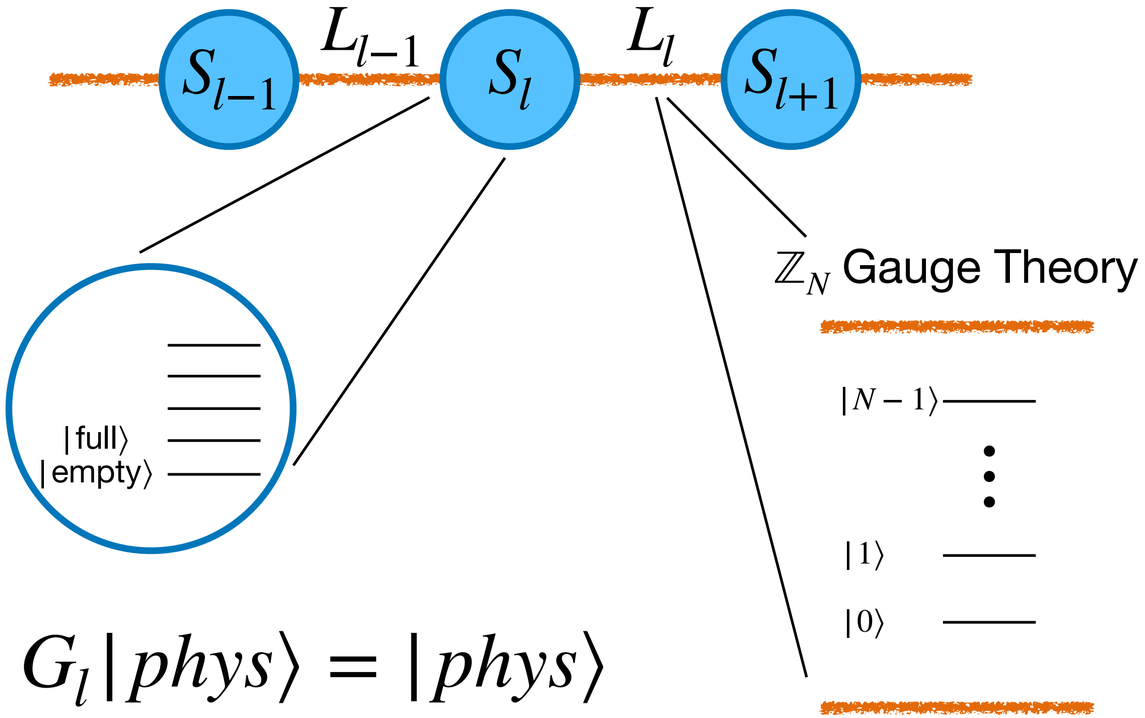

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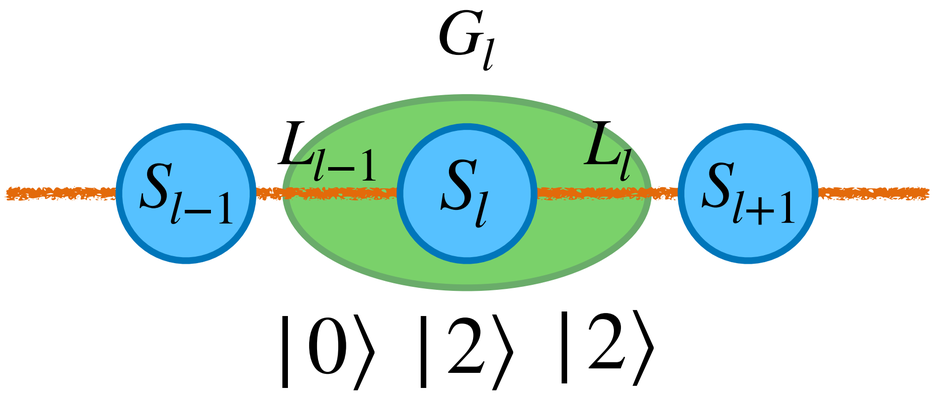

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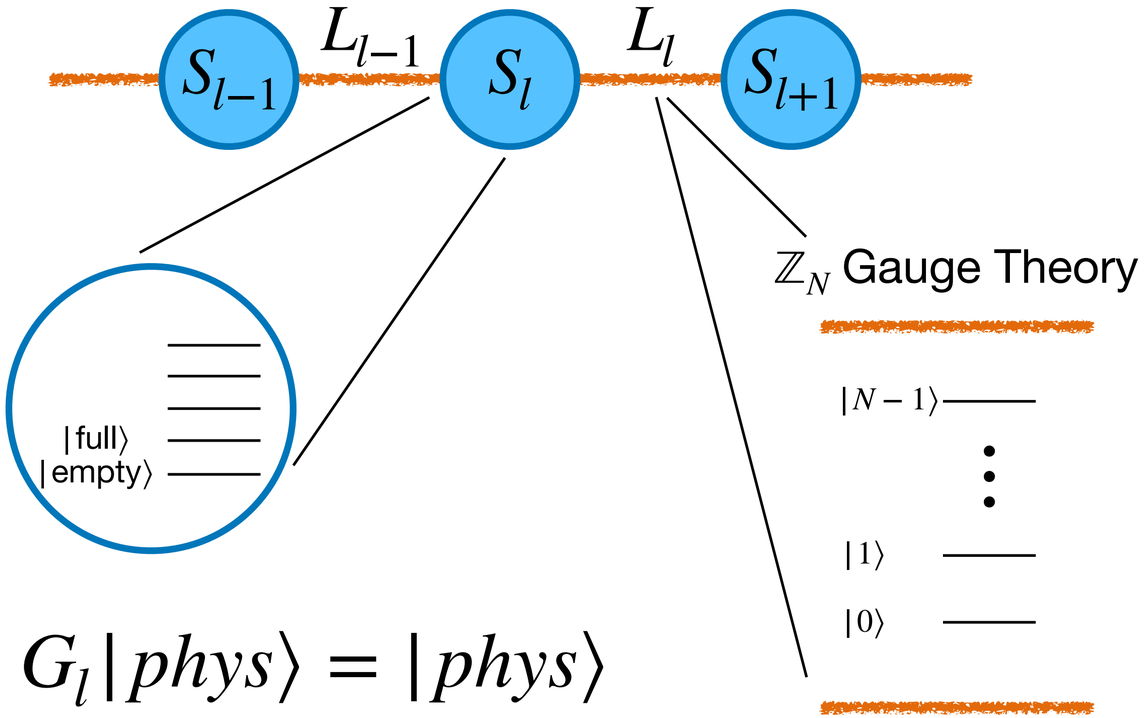

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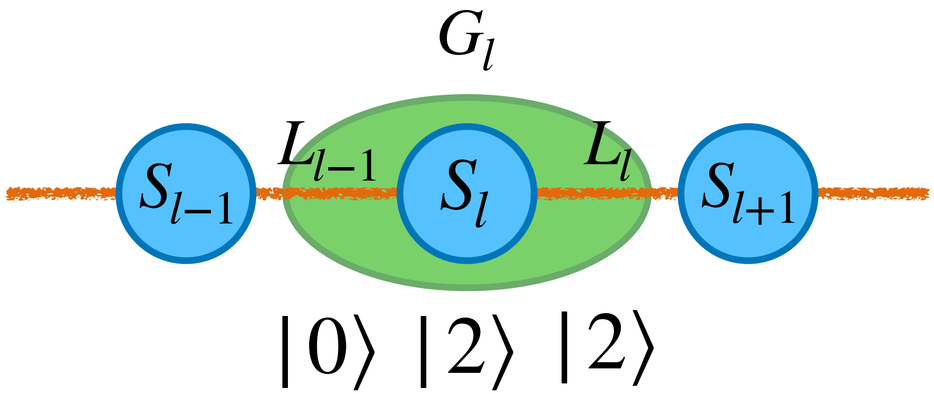

Non-Pauli stabilizers:

$$M_l|empty\rangle = |empty\rangle$$
$$M_l|full\rangle = |full\rangle$$
$$M_l|n\rangle = -|n\rangle, \quad n > 1$$

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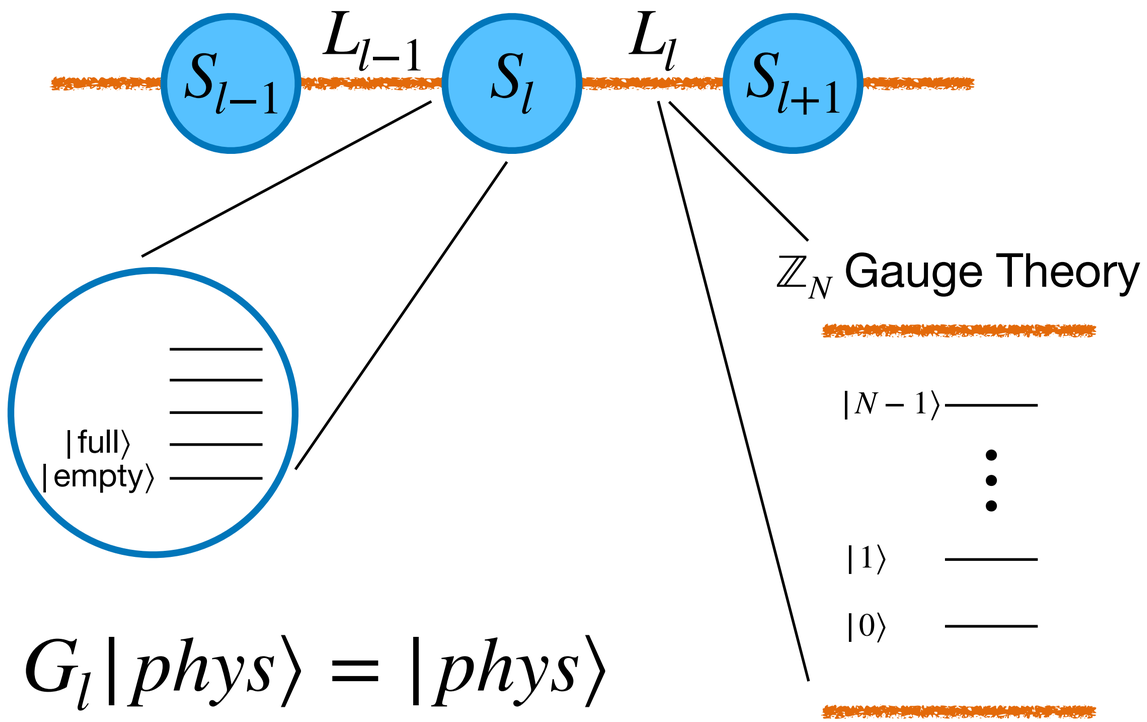
QEC for $\mathbb{Z}_N$ LGT



$$G_l|phys\rangle = |phys\rangle$$

Stabilisers: M_l, G_l

But G_l generate a $d = 3$ QECC

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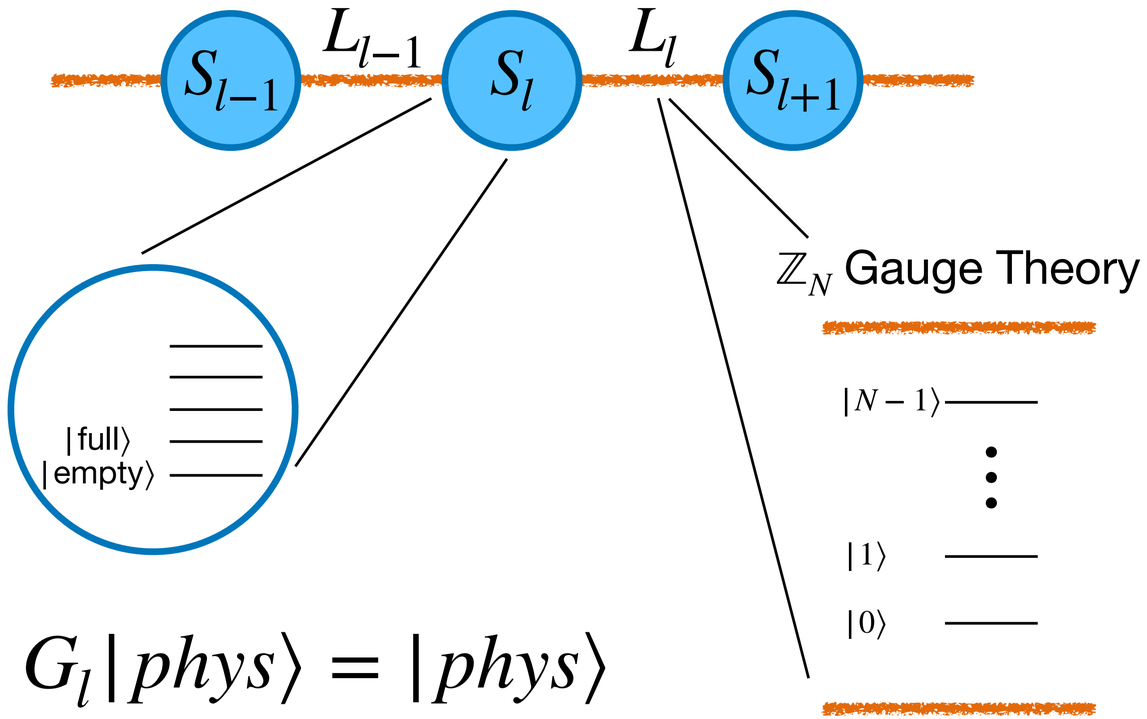

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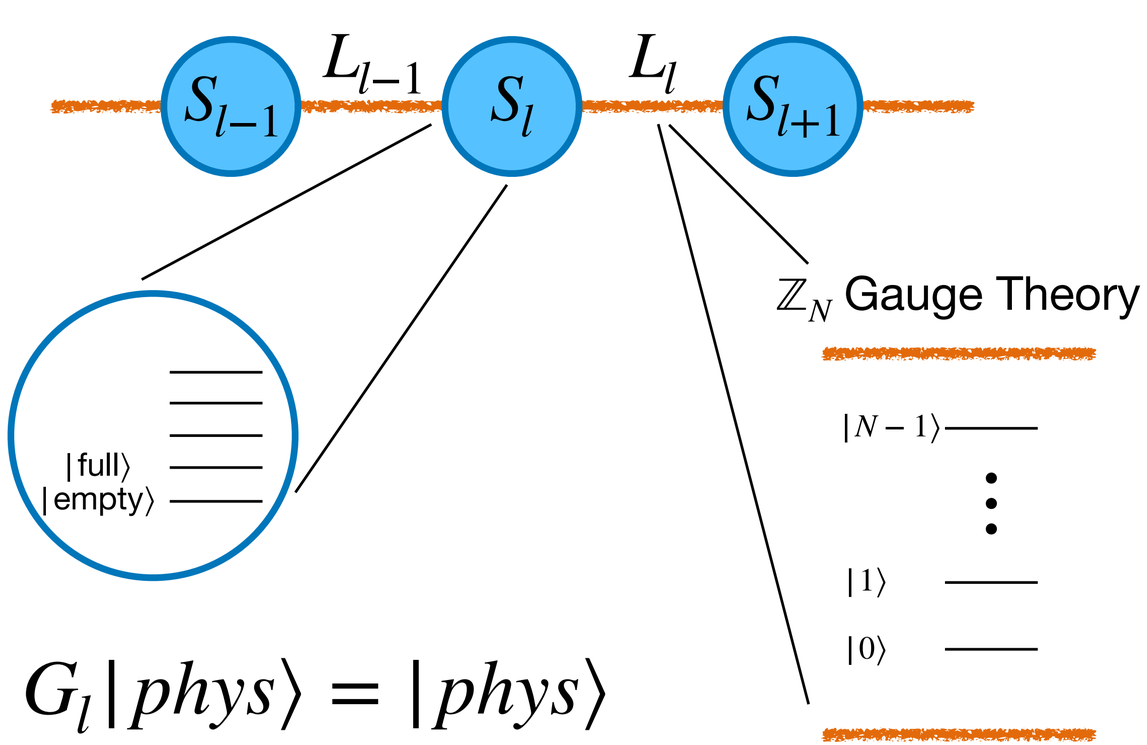
Stabilisers: ~~M_l~~, G_l

But G_l generate a $d = 3$ QECC

Gauge group → Abelian group → Finite group → $\mathbb{Z}_N$ group → ~~→~~ $\mathbb{Z}_2$ group

QEC for $\mathbb{Z}_N$ LGT



$$G_l|phys\rangle = |phys\rangle$$

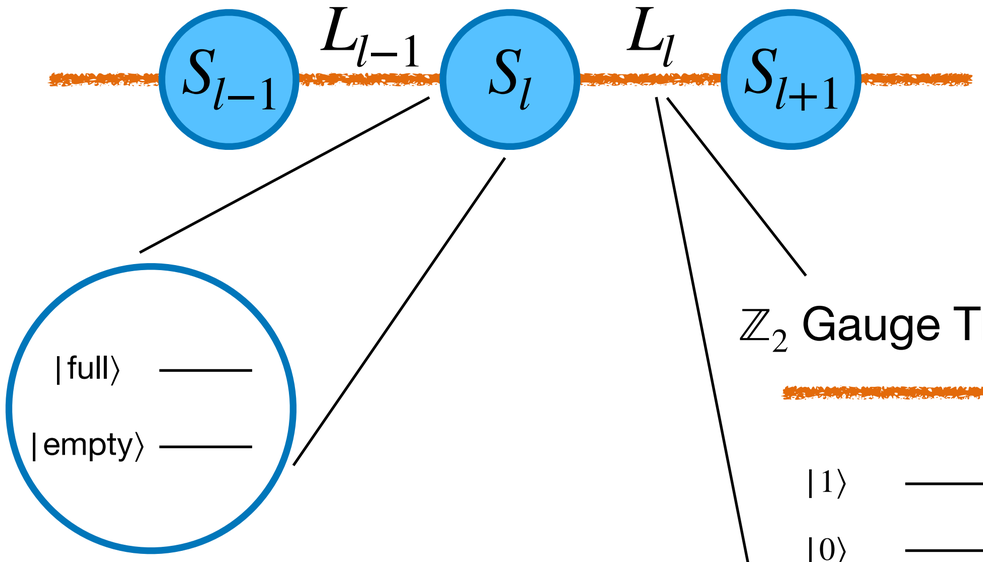
Stabilisers: ~~M_l~~, G_l

But G_l generate a $d = 3$ QECC

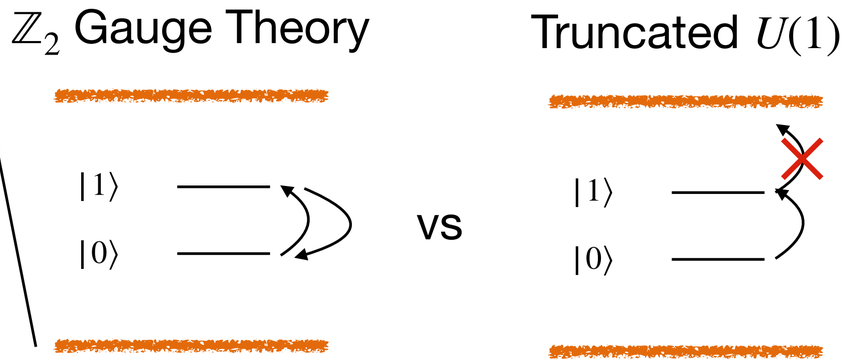
The constraints of the theory generate a non-Pauli stabiliser group, but we can use only Pauli stabilisers to correct errors

Gauge group → Abelian group → Finite group → $\mathbb{Z}_N$ group → $\mathbb{Z}_2$ group

Limit testing



$$G_l|phys\rangle = |phys\rangle$$



Trade-offs in Gauss's law error correction for lattice gauge theory quantum simulations

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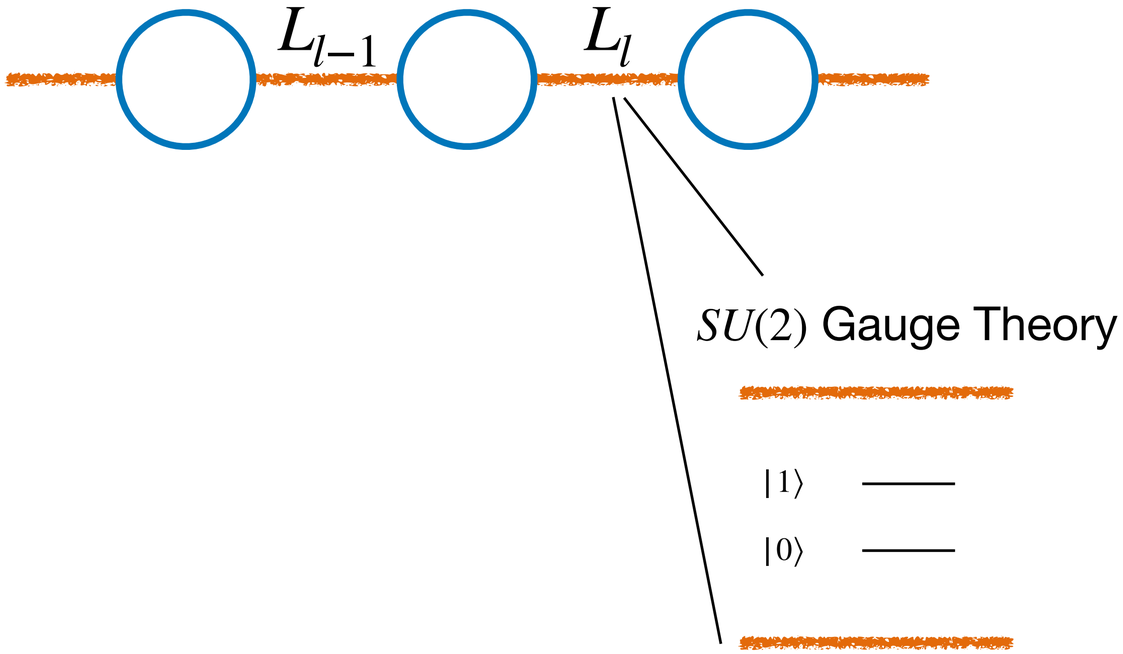
Gauss's law-based quantum error correction (GLQEC) offers a promising approach to reducing qubit overhead in lattice gauge theory simulations by leveraging built-in symmetries. For applications of GLQEC to 1+1D lattice quantum electrodynamics (QED), we identify two significant trade-offs. First, we prove via dimension-counting arguments that GLQEC requires periodic electric fields, thereby constraining the design space for lattice QED simulations. Second, we numerically compare GLQEC with a universal quantum error correction (UQEC) code, specifically the $d = 3$ bitflip repetition code, and find that while GLQEC can achieve lower logical error rates in single-round error correction, it exhibits faster decoherence to the steady-state mixed ensemble under multiple rounds. The mixing speed penalty is manifest in observables of interest for both memory experiments and Hamiltonian evolution. We identify a mixing speed threshold, $p_{th} = 0.277(2)$, above which using GLQEC exhibits even faster decoherence than without error correction. Our results highlight fundamental limitations of symmetry-based error correction schemes and inform corresponding constraints on formulations of lattice gauge theories compatible with error-robust quantum simulation techniques.

arXiv:2602.22121

Gauge group → Abelian group → Finite group → ? → $\mathbb{Z}_N$ group → ✗ → $\mathbb{Z}_2$ group

Other abelian groups



Quantum Error Correction Codes for Truncated SU(2) Lattice Gauge Theories

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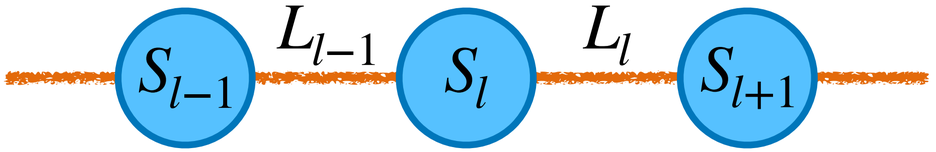
We construct two quantum error correction codes for pure SU(2) lattice gauge theory in the electric basis truncated at the electric flux $j_{\max} = 1/2$, which are applicable on quasi-1D plaquette chains, 2D honeycomb and 3D triamond and hyperhoneycomb lattices. The first code converts Gauss's law at each vertex into a stabilizer while the second only uses half vertices and is locally the carbon code. Both codes are able to correct single-qubit errors. The electric and magnetic terms in the SU(2) Hamiltonian are expressed in terms of logical gates in both codes. The logical-gate Hamiltonian in the first code exactly matches the spin Hamiltonian for gauge singlet states found in previous work.

arXiv:2511.13721

Gauge group → ? → Abelian group → Finite group → $\mathbb{Z}_N$ group → ✗ → $\mathbb{Z}_2$ group

Conclusions



$$G_l |phys\rangle = |phys\rangle$$

- Lattice Gauge Theories have redundancy
- That redundancy is enough to correct one type of error
- We did it for $\mathbb{Z}_2$ and $\mathbb{Z}_N$ using qudits

Future works

- What about non-abelian groups?
- What about Lie groups?