Structured Parameterization and Non-Stabilizerness in Hypergraph QAOA

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qamala

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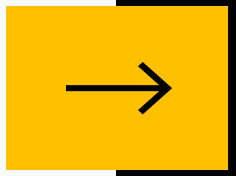
What is the problem?

- High gate noise
- Limited coherence times
- Barren Plateaus

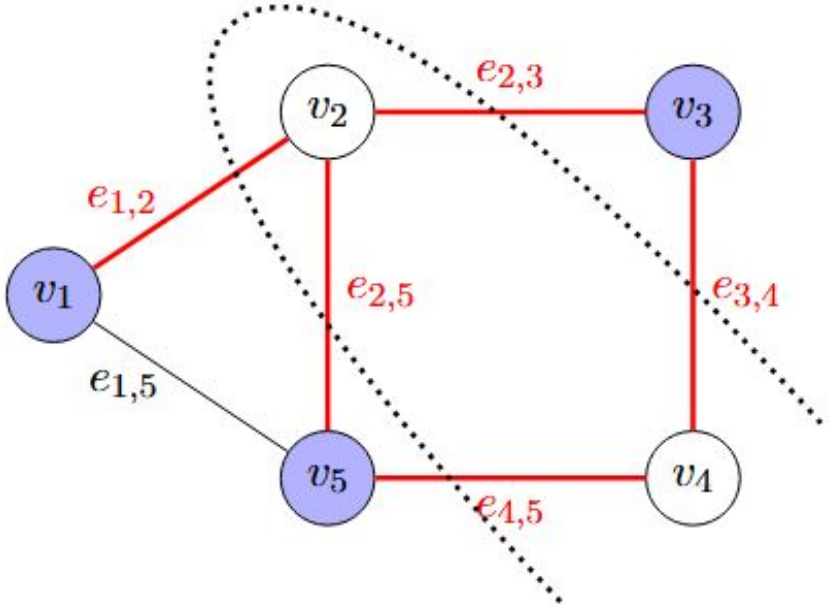


Our Solution

*k*A-QAOA: an expressive approach for combinatorial problems that does not exhaust classical and quantum resources

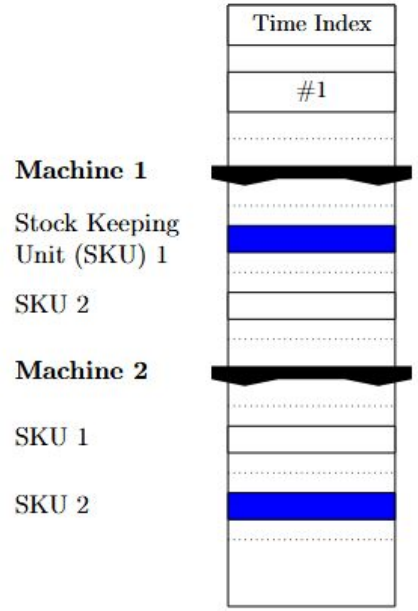
Real-World Problems

Max Cut



the Max Cut problem is equivalent to minimizing the Hamiltonian of a spin glass model

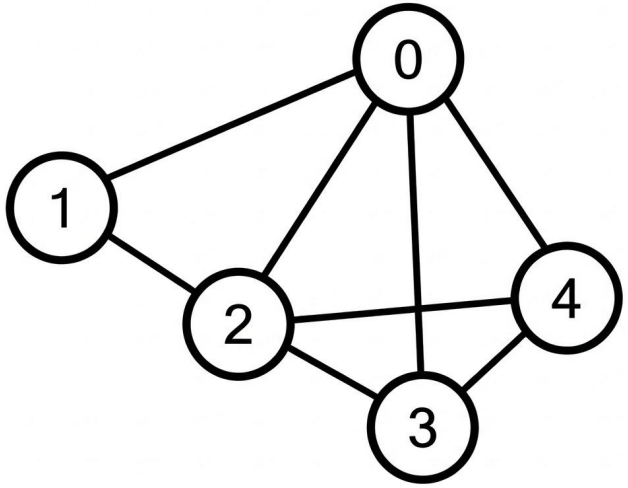
Job-Shop Scheduling Problem (JSSP)



Find the most efficient way to schedule a number of jobs and their respective operations while considering several constraints.

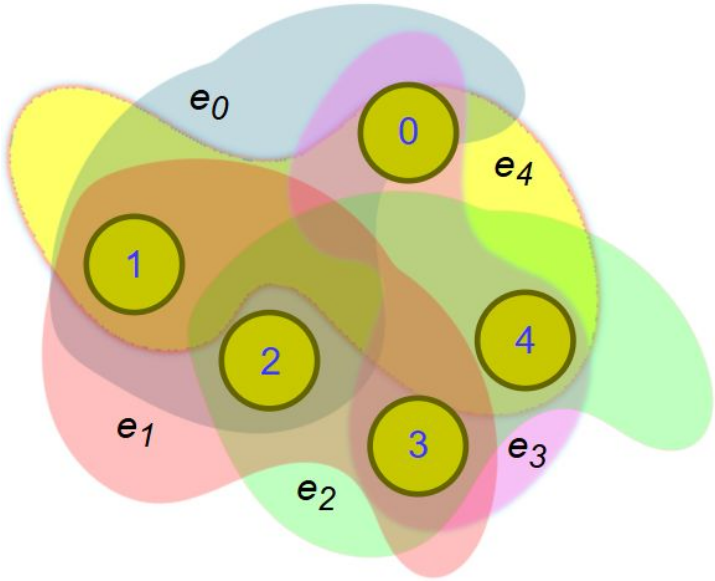
Topology of Real-World Constraints

Graphs



Multi-party constraints would require quadratisation, introducing ancilla qubits and deep CNOT staircases

Hypergraphs



Translucent shapes encompasses more than 2 nodes simultaneously, perfectly mapping *k*-body interactions without qubit overhead

QUBO and PUBO

$$C(\boldsymbol{x}) = \boldsymbol{x}^T \boldsymbol{Q} \boldsymbol{x} + \boldsymbol{q}^T \boldsymbol{x} = \sum_{i,j=1}^{n} Q_{ij} x_i x_j + \sum_{i=1}^{n} q_i x_i$$

$$x_i \to (1 - \sigma_i^z)/2$$

$$\begin{aligned} C(\boldsymbol{z}) &= \sum_{i,j=1}^{n} Q_{ij} x_i x_j + \sum_{i=1}^{n} q_i x_i \\ &= \sum_{i,j=1}^{n} \frac{Q_{ij}}{4} (1 - \sigma_i^z)(1 - \sigma_j^z) + \sum_{i=1}^{n} \frac{q_i}{2} (1 - \sigma_i^z) \\ &= \sum_{i,j=1}^{n} \frac{Q_{ij}}{4} \sigma_i^z \sigma_j^z - \sum_{i=1}^{n} \frac{1}{2} \left(q_i + \sum_{j=1}^{n} Q_{ij} \right) \sigma_i^z, \end{aligned}$$

To convert PUBO to QUBO we require quadratization which scales (worst case) as *O(mk)*, where *m* is the number of terms and *k* is the degree of the polynomial.

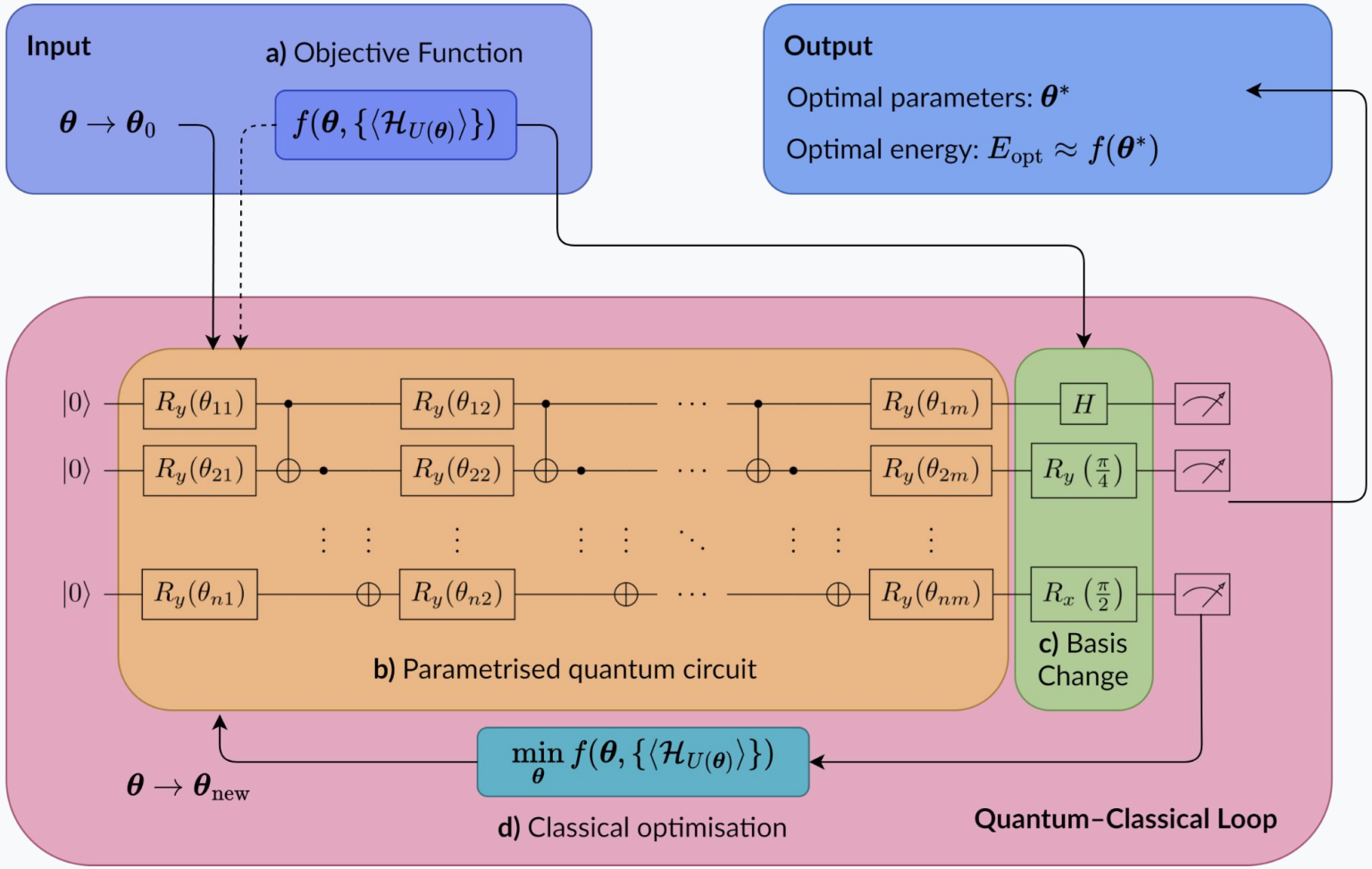
Example:

$$C(\boldsymbol{x}) = J x_0 x_1 x_2$$

$$\begin{aligned} C(\boldsymbol{x}, \boldsymbol{y}) = J y_0 x_2 &+ 2Q(x_0 x_1 \\ &- 2(x_0 x_1 - 2y_0(x_0 + x_1) + 3y_0) \end{aligned}$$

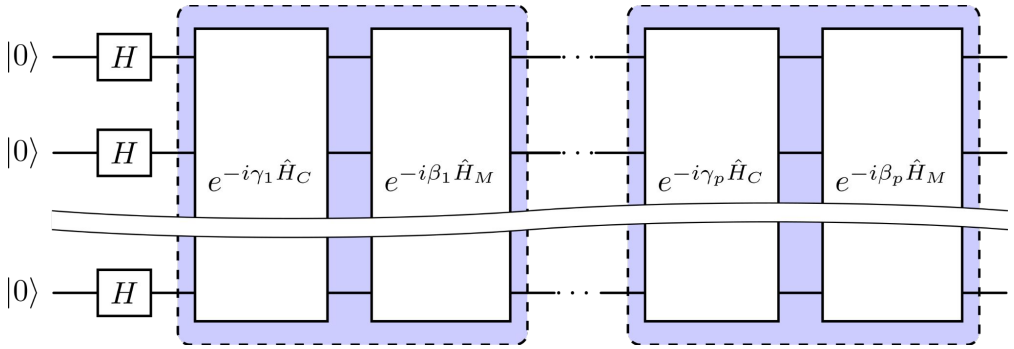
where Q is a penalty term and y_0 is an ancillary variable

Variational Quantum Algorithms



Parameterization Schemes

Single-Angle (SA-QAOA)



KEY DATA

Uses only $2p$ parameters.

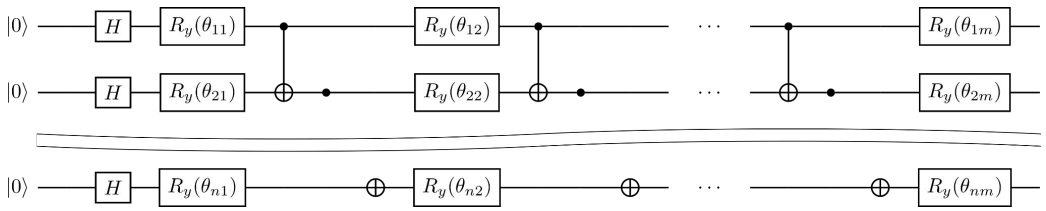
PROS

Easy to optimise.

CONS

Lacks expressivity for high-fidelity solutions.

Multi-Angle (MA-QAOA)



KEY DATA

A parameter for every single gate.

PROS

Highly Expressive.

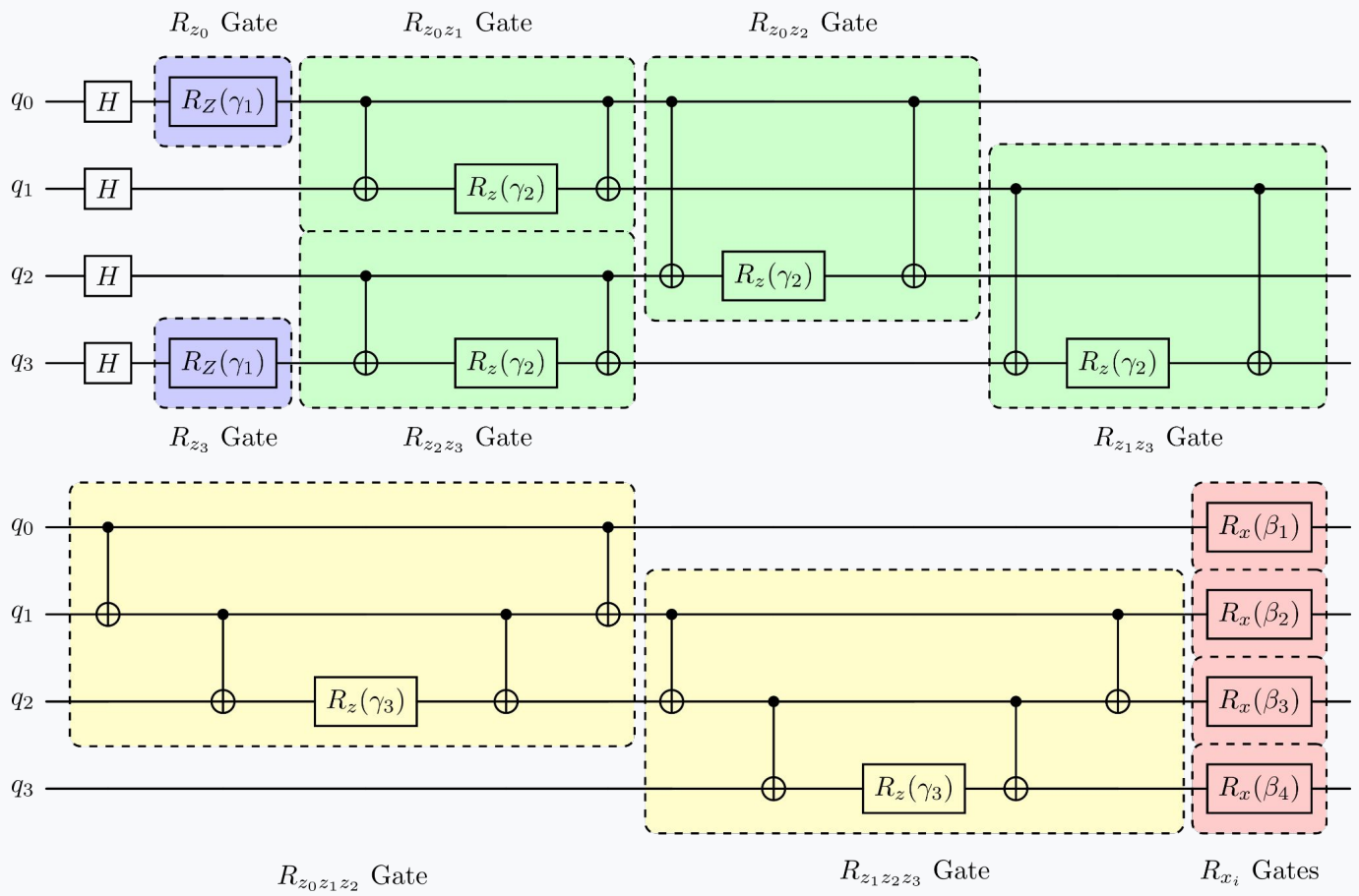
CONS

High complexity, susceptible to barren plateaus.

E. Farhi, J. Goldstone, and S. Gutmann, A Quantum Approximate Optimization Algorithm, arXiv:1411.4028 (2014).

The Middle Ground

Instead of a single angle, or an angle for every gate, *k*A-QAOA groups cost function terms by their *k*-body interaction order. It bridges the gap between abstract problem topology and physical hardware capabilities



QAOA Variants

Variant	Expressiveness	Classical Optimisation Complexity	Unstructured Problem Robustness	Hardware Alignment
SA-QAOA	Low	Low	High	Generic
MA-QAOA	High	Extreme	High	Disconnected
AA-QAOA	Variable	Moderate	Fails without symmetries	Disconnected
***k*A-QAOA**	**High**	**Moderate**	**High**	**Native k-body mapping**

While Automorphic-Angle (AA) QAOA reduces parameters, it fails on arbitrarily weighted graphs without symmetries. *k*A-QAOA provides robust efficiency regardless of graph symmetry.

Figures of Merit

Approximation Ratio (AR)

Measures how close the algorithm's output is to the optimal solution.

$$\mathrm{AR} = \frac{C_{\min}}{C_{\mathrm{opt}}}$$

Fidelity (F)

Quantifies the overlap between the prepared state and the target state.

$$F = |\langle \phi | \psi \rangle|^2$$

Function Evaluations (nfev)

Tracks classical computational resources required for optimization with e_k circuit evaluations per iteration.

$$\mathrm{nfev} = \sum_{k=1}^{N_{\mathrm{iter}}} e_k$$

Quantum Magic (M_2)

Evaluated using Rényi entropy to measure non-Clifford resources.

$$M_\alpha = \frac{1}{1-\alpha} \log_2 \left(\sum_{P \in \mathcal{P}_N} \frac{|\langle \psi | P | \psi \rangle|^{2\alpha}}{D^N} \right)$$

Performance Evaluation

3-uniform sign-alternating cyclic hypergraphs: $C(\boldsymbol{x}) = \sum_{i=0}^{n-3} (-1)^i x_i x_{i+1} x_{i+2}$

Random coefficient hypergraphs: $C(\boldsymbol{x}) = \sum_{i=0}^{n-1} J_i \prod_{j=0}^{2} x_{i+j} \qquad J_i \in \{-1, 0, +1\}$

Magic Barrier Investigation

We investigated the presence of a "magic barrier" when running MA-QAOA and kA-QAOA on hypergraphs compared to SA-QAOA.

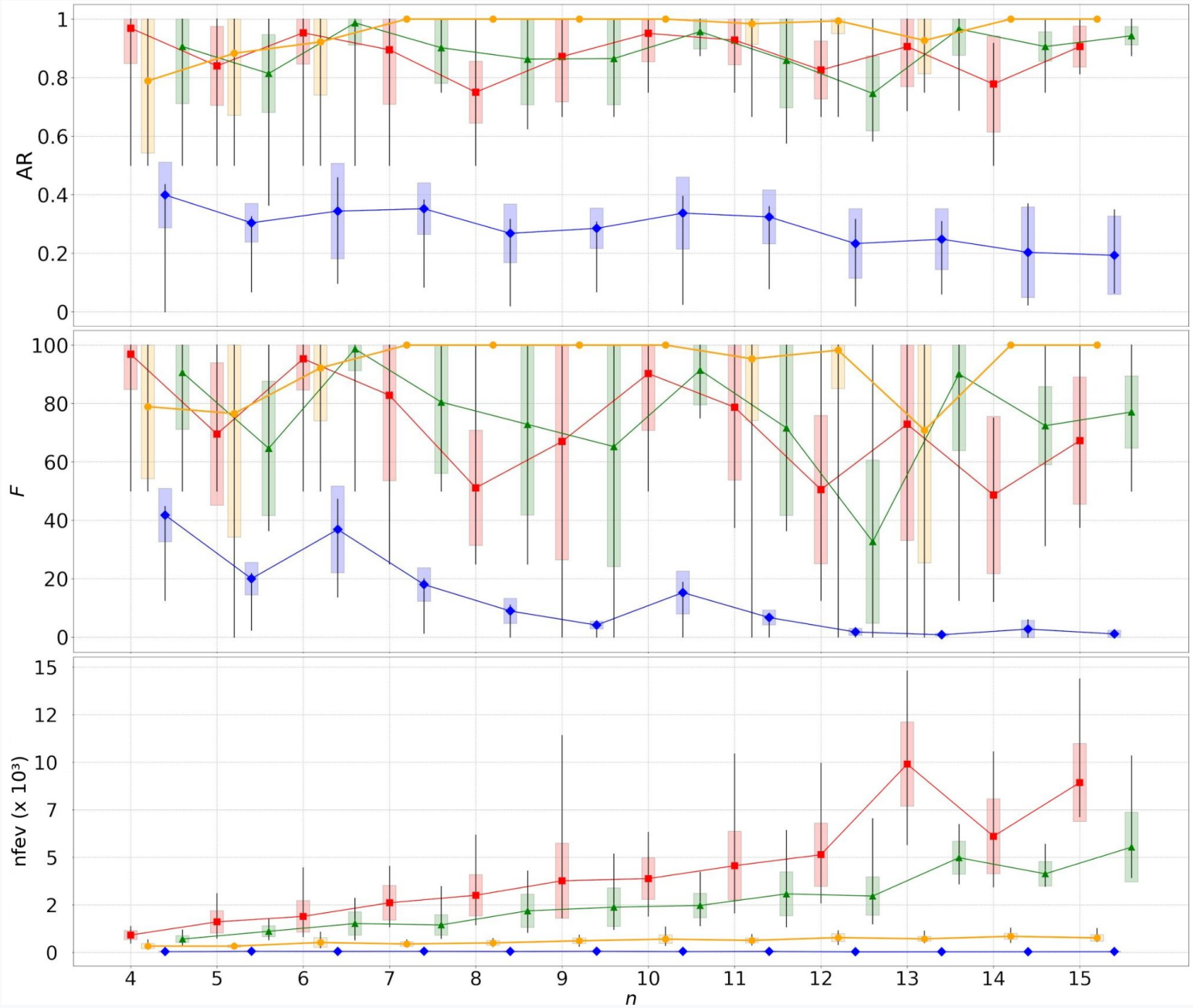
- **Setup:** 70 random 12-vertex hypergraphs with arbitrary connections with coefficients $J_i \in [-10, +10]$.
- **Depth:** Up to five layers, evaluating magic buildup M_2 throughout the circuit.
- **Constraint:** No ground-state degeneracies to link high-fidelity solutions with low magic.

C. Capecci, G. C. Santra, A. Bottarelli, E. Tirrito, and P. Hauke, Role of nonstabilizerness in quantum optimization (2025), arXiv:2505.17185 [quant-ph].

Performance Evaluation

3-uniform cyclic
sign-alternating hypergraphs
with one (1) QAOA layer

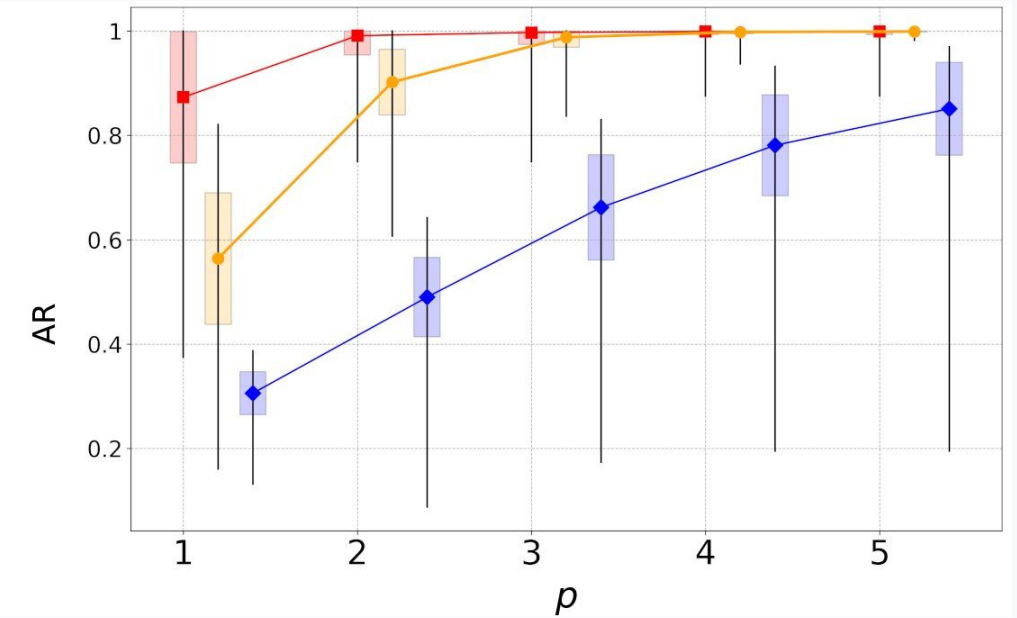
- MA-QAOA
- *k*A-QAOA
- SA-QAOA
- AA-QAOA

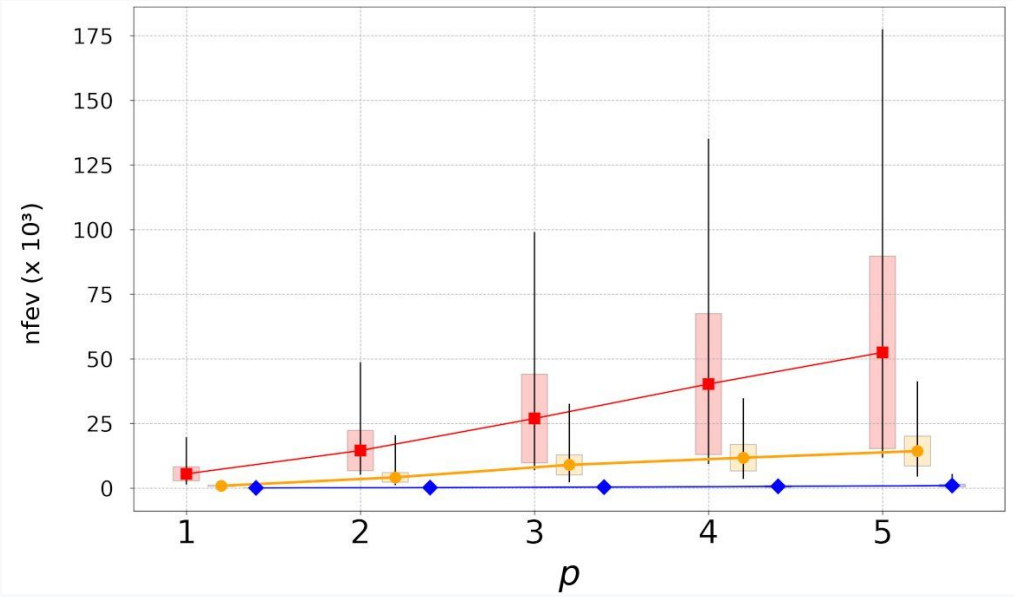


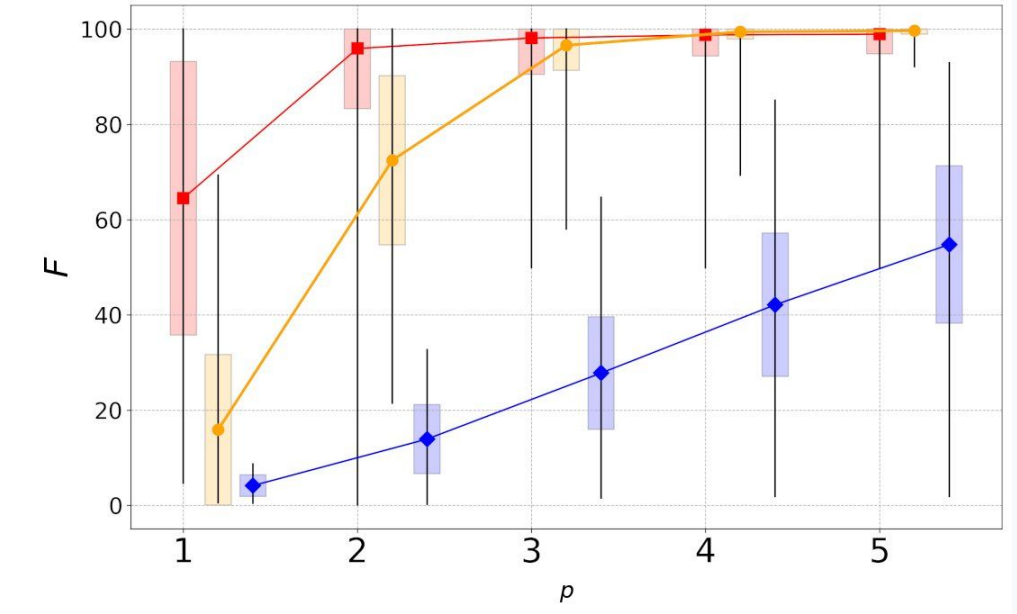
Performance Evaluation

3-uniform cyclic random coefficient hypergraphs with

- MA-QAOA
- *k*A-QAOA
- SA-QAOA



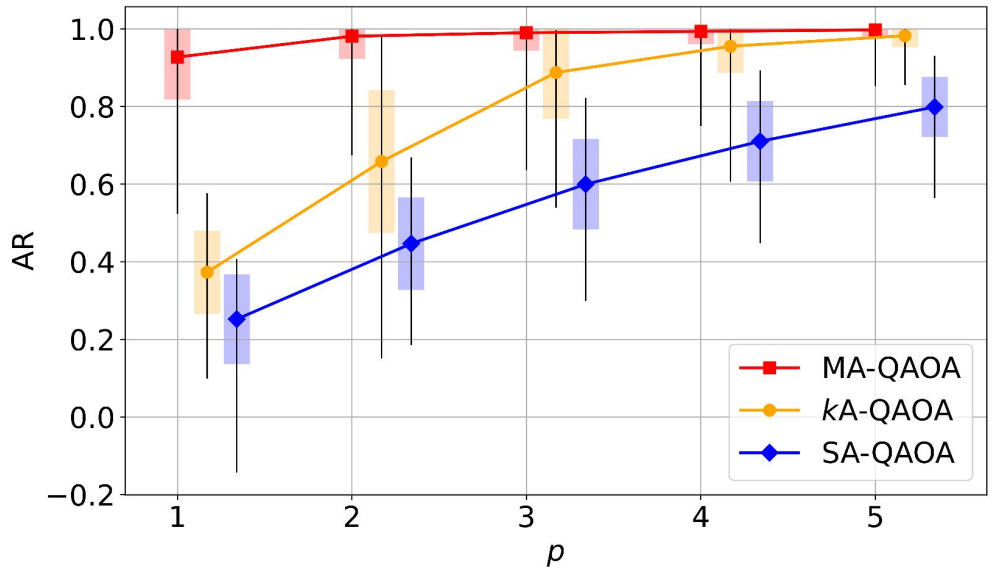




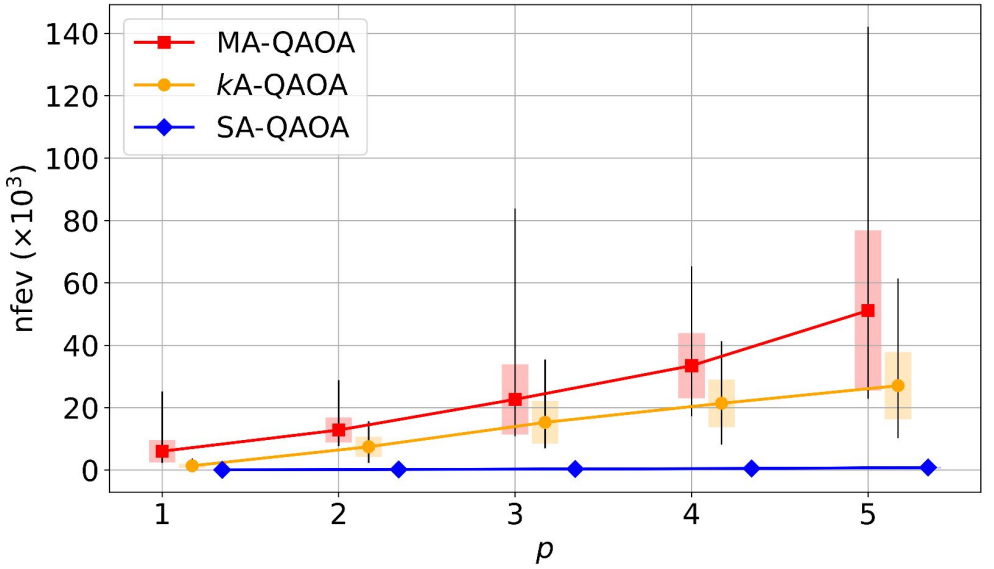
Performance Evaluation

Average of 70 random 12-vertex ($n = 12$) hypergraphs with arbitrary connections with integer coefficients $J_i \in [-10, +10]$.

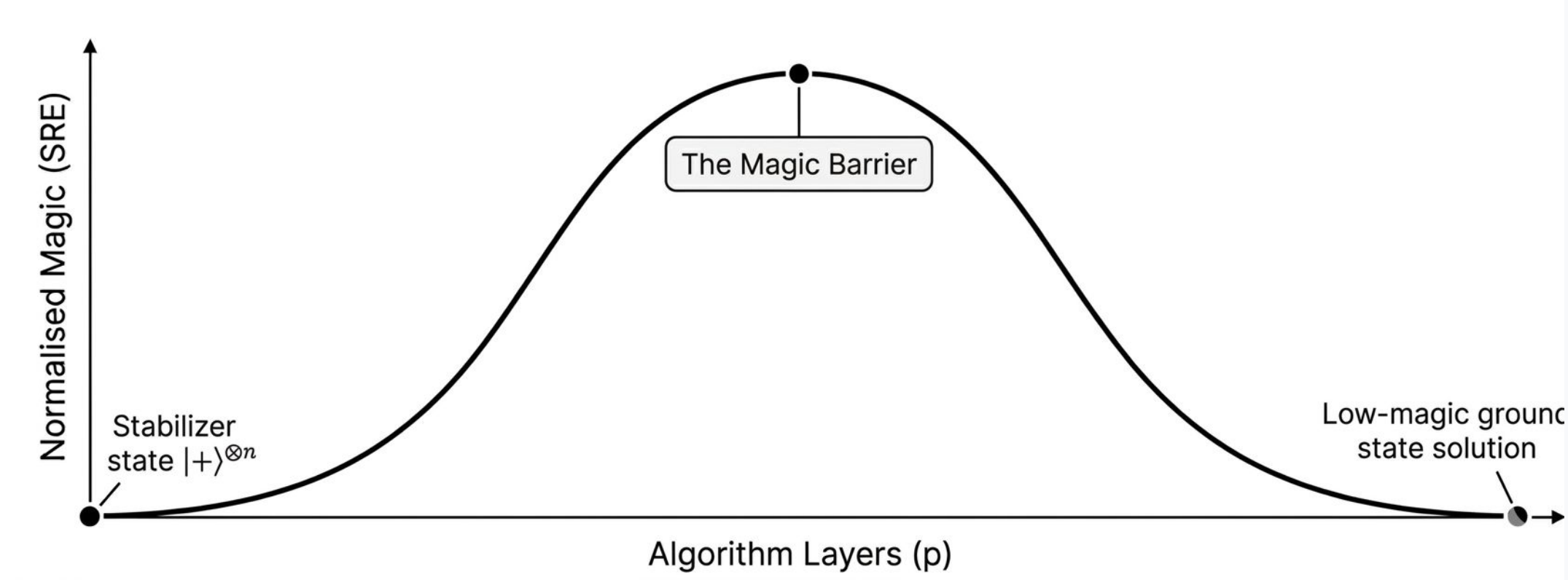
Approximation Ratio (AR) vs. Layers (p)



Function Evaluations (nfev) vs. Layers (p)



The Magic Barrier



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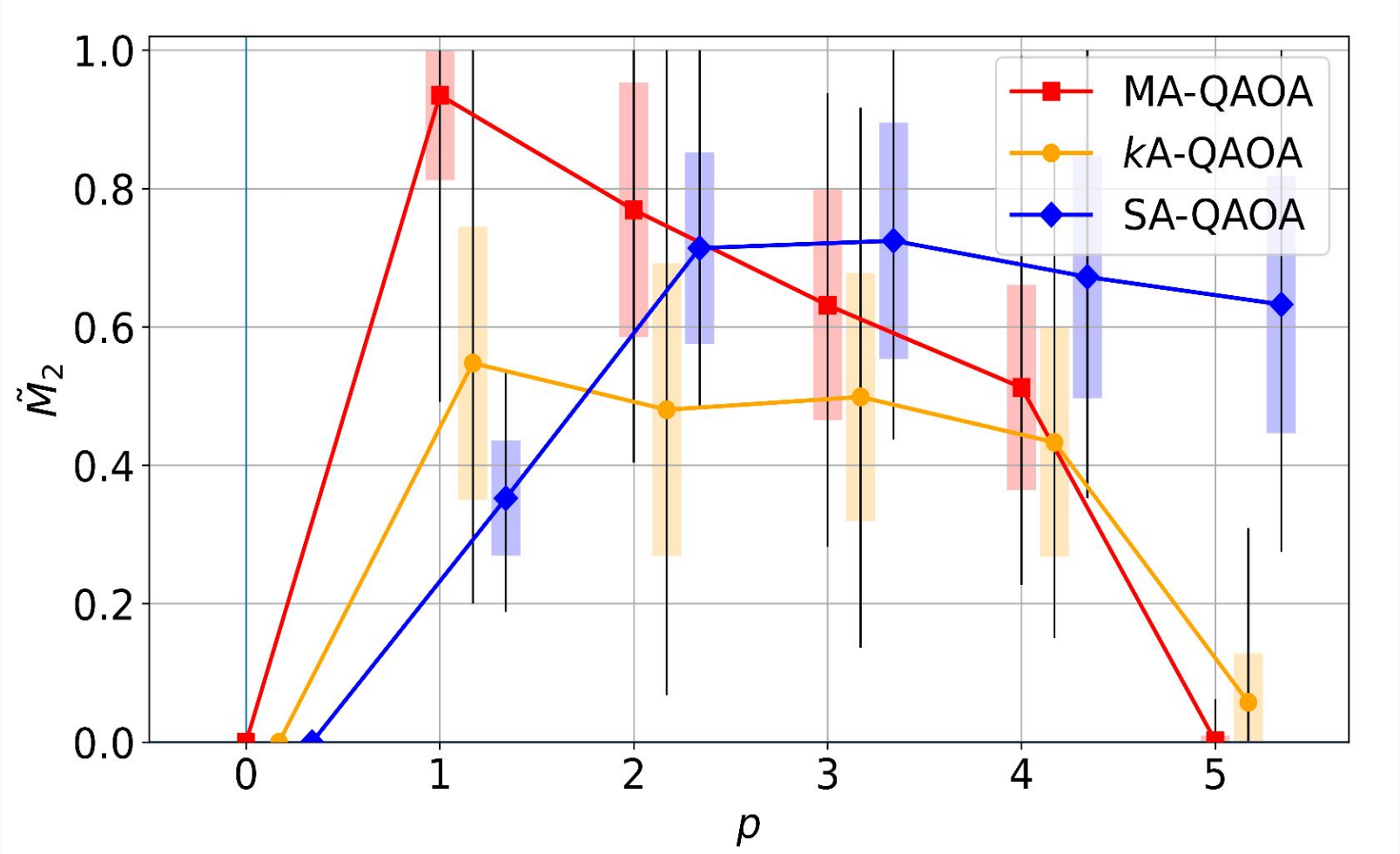
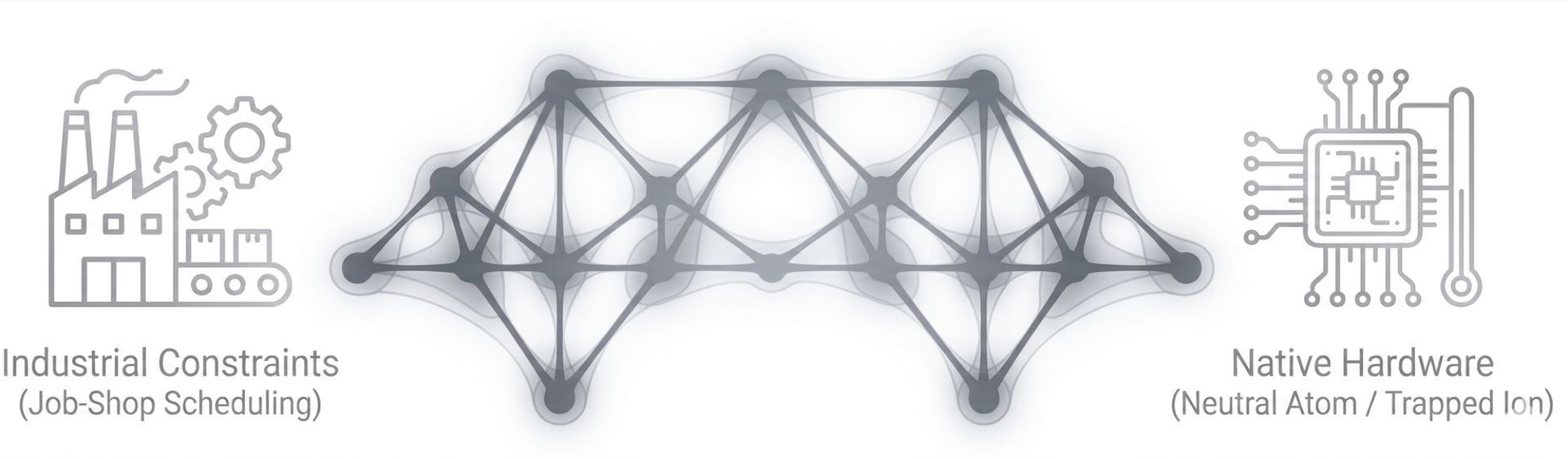


Conclusion



By reducing variational parameters while maintaining high approximation performance, ***k*A-QAOA** neutralises the primary bottleneck in scaling.

***k*A-QAOA is the optimal framework to exploit hardware-native multi-qubit gates for industrial optimisation.**

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Marco Tulio & Jessica Bavaresco

Quantum Thermodynamics
Marti Perarnau-Llobet & Mauro Paternostro

Quantum Gibbs Sampling and Boltzmann Machines
Alvaro Alhambra & Angela Capel