Adjoint meson and tetraquark static energies using novel BOEFT interpolators

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Workshop on Glueballs and Charmonium: Lattice QCD, Experiment and Phenomenology, Trento, Italy

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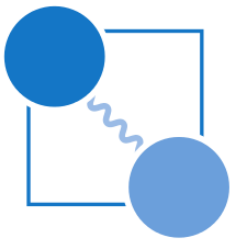

T30f
Theoretische Teilchen- und Kernphysik



Advance Grant



Motivation

- Explore the $I = 1$ hidden-heavy Z_b $(\bar{b}b\bar{q}q)$ tetraquark channel
- $J^{PC} = 1^{+[-]}$
- Relevant for $Z_b(10610)$ and $Z_b^{'}(10650)$ (near degenerate) exotic states discovered by Belle.
- Both states lie close to the $B\bar{B}^*$ and $B^*\bar{B}^*$ threshold, respectively

Belle, A. Bondar et al., [arXiv:1110.2251], Belle, A. Garmash et al., [arXiv:1403.0992]

- In particular, validate Born-Oppenheimer Effective field Theory (BOEFT) reasoning for this observed degeneracy.

M. Berwein, N.Brambilla, A. Mohapatra, A. Vairo, arXiv:2408.04719v2 [hep-ph]

Background

- BOEFT exploits the energy hierarchy between heavy quarks and the light degrees of freedom (LDF) (light quarks and gluons) of QCD.

See Antonio Vairo's talk on Tuesday at 15:30

M. Berwein, N.Brambilla, A. Mohapatra, A. Vairo, arXiv:2408.04719v2 [hep-ph]

- In the hidden-heavy sector, it provides an elegant alternative to extremely challenging Lüscher's method by facilitating the extraction of the static energies for infinitely heavy quarks in the presence of LDF on the lattice.
- BO approximation gives, in addition to the spectrum, a physical insight into the binding mechanisms of QCD in the form of the potentials. It can probe QCD at both short-distance and large-distance scales.

Background

- The first step is to calculate static energies with infinitely heavy quarks on the lattice.
- The heavy quarks are static and supply a color source.
- Different configurations of the LDF (light quarks and gluons) identify different static energies, which are characterised by the quantum numbers of the LDF.

Background

- The first step is to calculate static energies with infinitely heavy quarks on the lattice.
- The heavy quarks are static and supply a color source.
- Different configurations of the LDF (light quarks and gluons) identify different static energies, which are characterised by the quantum numbers of the LDF.
- The second step is to plug these static energies into the coupled Schrödinger equations in the framework of BOEFT to extract the spectrum of XYZ states comprising of two heavy quarks or one heavy quark-antiquark pair.

See Roberto Bruschini's talk on Friday at 9:00

See Antonio Vairo's talk on Tuesday at 15:30

M. Berwein, N.Brambilla, A. Mohapatra, A. Vairo, arXiv:2408.04719v2 [hep-ph]

Experimental motivation

- Z_b and Z_b' with $J^{PC} = 1^{+-}$ are superpositions of Z_1 and Z_2 with 1^{--} and 0^{-+} as LDF quantum numbers respectively:

$$|Z_b'\rangle = \frac{1}{\sqrt{2}}\Big(\underbrace{|S_{b\bar{b}} = 0\rangle|K^{PC} = 1^{--}\rangle}_{Z_1} - \underbrace{|S_{b\bar{b}} = 1\rangle|K^{PC} = 0^{-+}\rangle}_{Z_2}\Big)$$

$$|Z_b\rangle = \frac{1}{\sqrt{2}}\Big(|S_{b\bar{b}} = 0\rangle|K^{PC} = 1^{--}\rangle + |S_{b\bar{b}} = 1\rangle|K^{PC} = 0^{-+}\rangle\Big)$$

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- Experimental Observation:
  - Z_b' lies very close to $B^*\bar{B}^*$ threshold
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  - Suppression of Z_b' decay to $B\bar{B}^*$

BOEFT explanation

- $B\bar{B}^*$: $0 \otimes 1 = 1$
- $B^*\bar{B}^*$: $1 \otimes 1 = 0 \oplus 1 \oplus 2$
- $B\bar{B}$: $0 \otimes 0 = 0$

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  $(1/2)^- \otimes (1/2)^+ = 1^{--} \oplus 0^{-+}$

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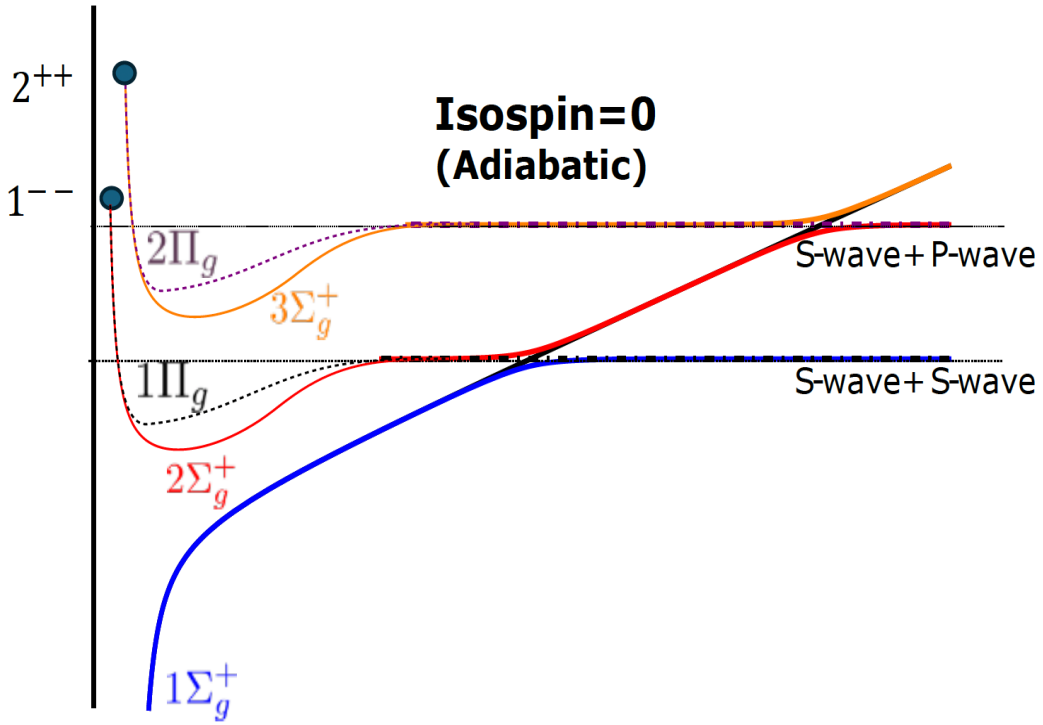
- $\langle Z_b'|B\bar{B}^*\rangle = 0$ if LDF (adjoint mesons) associated with Z_1 and Z_2 are degenerate.
- M.B. Voloshin arXiv:1601.02540v1 [hep-ph] called this Light Quark Spin Symmetry without any proof, and now BOEFT puts it on better ground. In BOEFT, this corresponds to the isovector, 1^{--}, and pesudoscalar, 0^{-+}, adjoint mesons being degenerate.

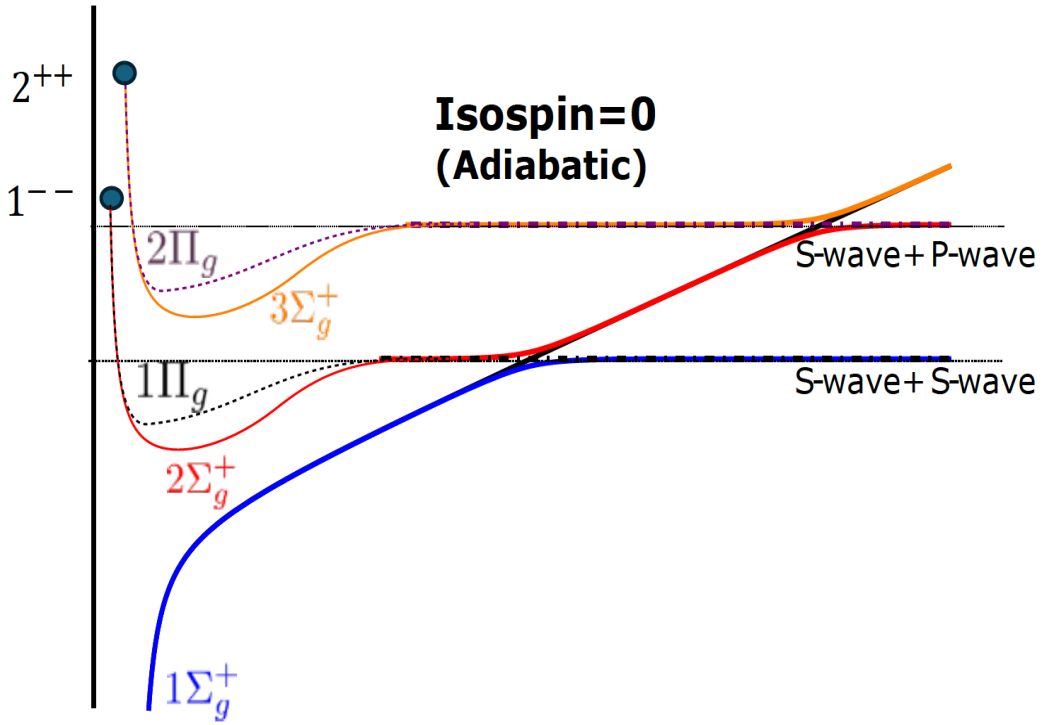
Tetraquark and adjoint meson operators



M. Berwein, N.Brambilla, A. Mohapatra, A. Vairo, arXiv:2408.04719v2 [hep-ph]

- As $r \to \infty$, states with 3 and $\bar{3}$ sources should become pairs of a $3-$hadron and a $\bar{3}-$hadron (or a pair of static-light mesons).
- As $r \to 0$, the 3 and $\bar{3}$ sources reduce to a linear combination of $1-$source and $8-$source.

Tetraquark and adjoint meson operators



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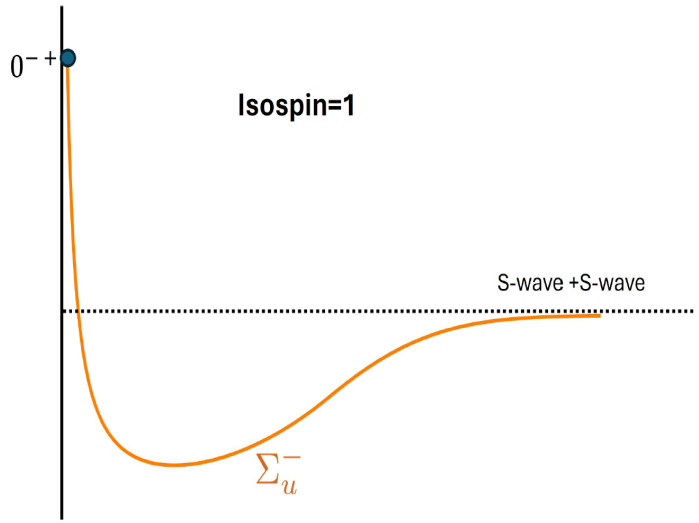
- For $Q\bar{Q}$ (Σ_g^+), BO potentials should smoothly connect $1-$potential at small r to $(3+\bar{3})-$potential at large r.
- Rest $(3+\bar{3})-$potentials at large r should connect to $8-$potential at small r.

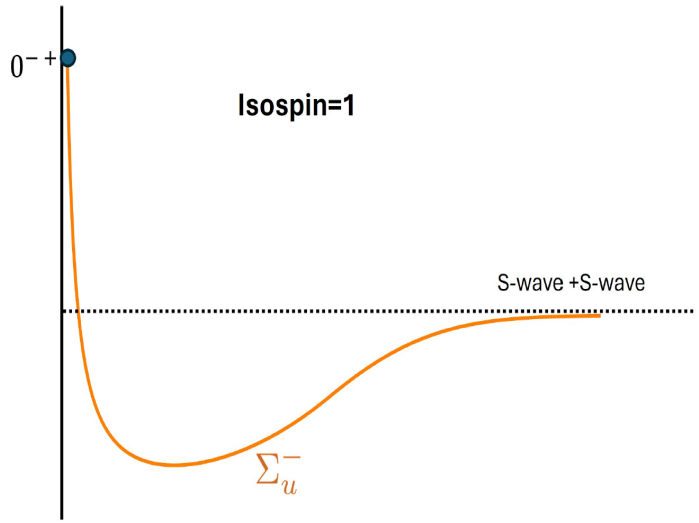
Tetraquark and adjoint meson operators



M. Berwein, N.Brambilla, A. Mohapatra, A. Vairo, arXiv:2408.04719v2 [hep-ph]

- $O_{BO} = \bar{Q}(0)\Gamma_1\, U(0, r/2)\, T^a\, \underbrace{\bar{q}(r/2)\Gamma_2 T^a q(r/2)}_{\text{adjoint meson} \equiv O^a_{AM}}\, U(r/2, r) Q(r)$

Tetraquark and adjoint meson operators



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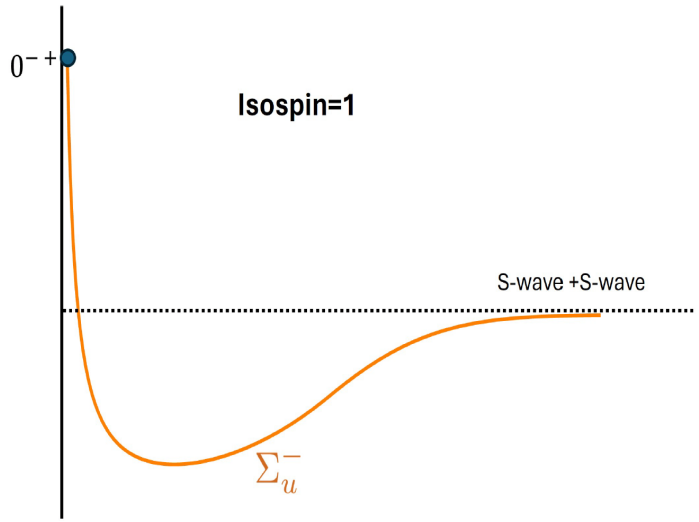
- For short distances, at the leading order, $\langle O_{BO} | \bar{Q}Q \rangle$ will result in a Tr $(T^a) = 0$.

Tetraquark and adjoint meson operators



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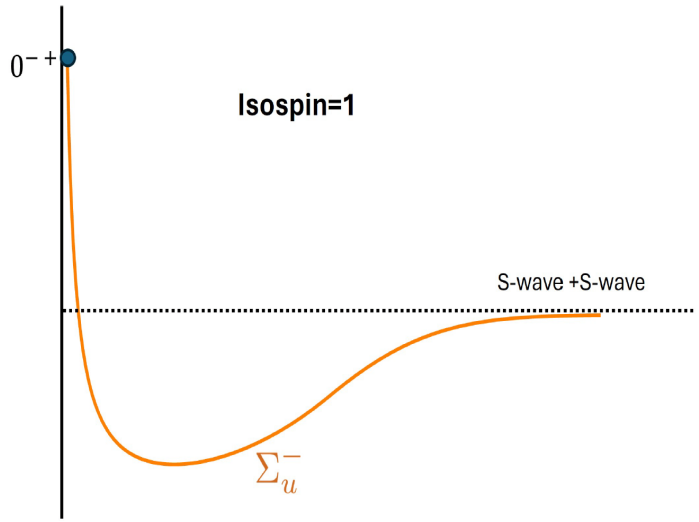
- At each r, ideally, $E_{O_{BO}}$ can be extracted from the effective mass plateaus of the correlator $\langle O_{BO}(t) | O^\dagger_{BO}(0) \rangle$

Tetraquark and adjoint meson operators



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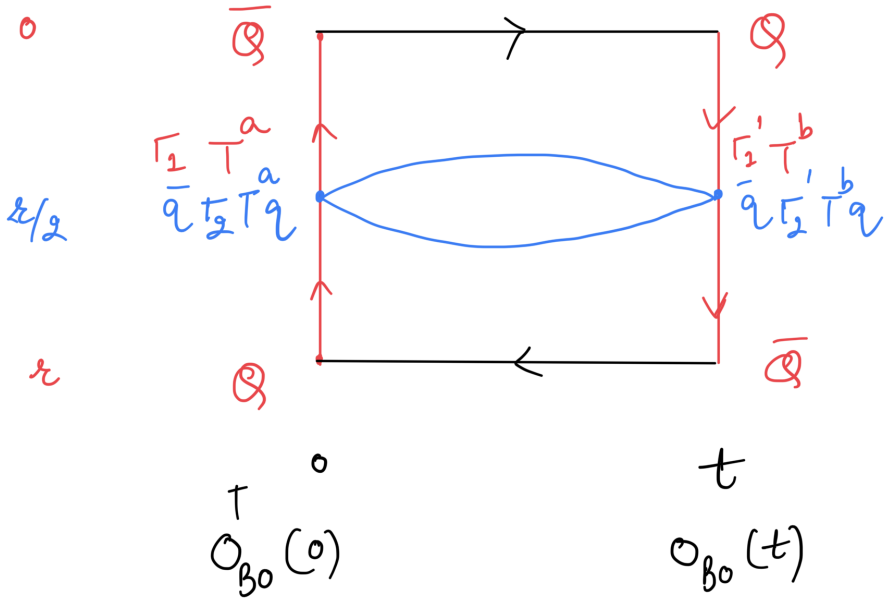
- For $r \to 0$, $E_{O_{BO}} = V_8 + \Lambda_{AM} + \mathcal{O}(r^2)$, where Λ_{AM} is the adjoint meson mass.

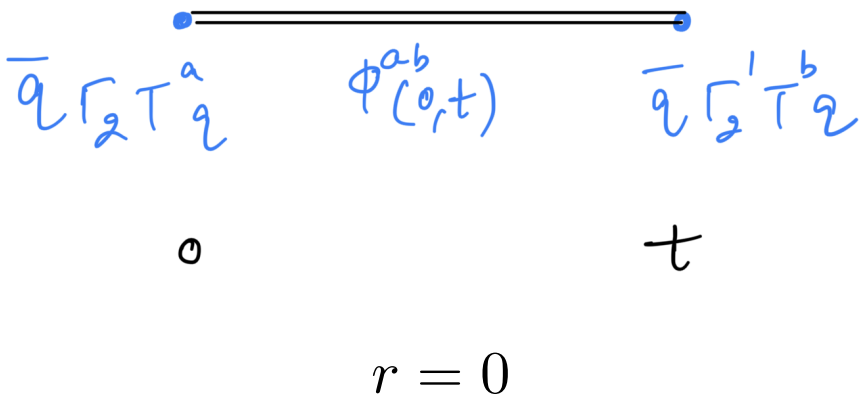
Tetraquark and adjoint meson operators

- Following novel tetraquark operator is proposed to have a zero overlap with quarkonium and pions:

$$O_{BO} = \bar{Q}(0)\Gamma_1\, U(0, r/2)\, T^a \underbrace{\bar{q}(r/2)\Gamma_2 T^a q(r/2)}_{\text{adjoint meson} \equiv O^a_{AM}} U(r/2, r) Q(r)$$

$$a \in [1, 8]$$





Tetraquark and adjoint meson operators

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- The adjoint meson correlator:

$$\langle O^a_{AM}(t)\phi^{ab}_{adj}(t,0)O^{b\dagger}_{AM}(0)\rangle,$$

where

$$\phi^{ab}_{adj}(t,0) = \mathsf{Tr}\left[T^a U(t,0) T^b U(0,t)\right]$$

Tetraquark and adjoint meson operators

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- $\Gamma_2 = \gamma_5 \Rightarrow 0^{-+}$ adjoint meson LDF associated with Z_2
  $\Gamma_2 = \gamma_i \Rightarrow 1^{--}$ adjoint meson LDF associated with Z_1 In the quenched case, vector and PS adjoint mesons were found to be -10(103) MeV and 34(161) MeV heavier than the lightest gluelump, M. Foster and C. Michael. Phys. Rev. D, 59:094509, 1999

Adjoint meson correlator

- $C(t) =< O^a_{AM}(T/2)\phi^{ab}_{adj}(T/2,-T/2)O^{b\dagger}_{AM}(-T/2) >$ is given by

$t \to T/2;\ 0 \to -T/2$

$$
\begin{aligned}
< 0| \quad & \bar{q}^{\alpha_1}_{c_1}(T/2,R)\ \Gamma^{\alpha_1\beta_1}_{snk}\ T^a_{c_1c_2} q^{\beta_1}_{c_2}(T/2,R) \\
& \phi^{ab}_{adj}(T/2,-T/2) \\
& \bar{q}^{\alpha_2}_{c_3}(-T/2,R)\ \Gamma^{\alpha_2\beta_2}_{src}\ T^b_{c_3c_4} q^{\beta_2}_{c_4}(-T/2,R) \qquad |0>
\end{aligned}
$$

with

$$\phi^{ab}_{adj}(T/2,-T/2) = \mathsf{Tr}[(U(-T/2,T/2)T^aU(T/2,-T/2)T^b)]$$

Adjoint meson correlator

- $< O^a_{AM}(T/2)\phi^{ab}_{adj}(T/2,-T/2)O^{b\dagger}_{AM}(-T/2) >$
- We use distillation M. Peardon, J. Bulava et. al., arXiv: 0905.2160 [hep-lat]

$$q^{\beta_1}_{c_2}(T/2,R)\bar{q}^{\alpha_2}_{c_3}(-T/2,R)$$
$$= \sum_{k_1,k_2} v^{k_1}_{R,c_2}(T/2)\tau^{k_1,k_2}_{\beta_1\alpha_2}(T/2,-T/2)v^{\dagger k_2}_{R,c_3}(-T/2)$$

- Color Fierz transformation:

$$T^a_{\alpha\beta}T^a_{\gamma\delta} = \delta_{\alpha\delta}\delta_{\beta\gamma} - \delta_{\alpha\beta}\delta_{\gamma\delta}/3$$

Adjoint meson correlator

- $< O^{a}_{AM}(T/2)\phi^{ab}_{adj}(T/2,-T/2)O^{b\dagger}_{AM}(-T/2) >$
-

$$\sum_{k_1,k_2,k_3,k_4} \Bigg(\Gamma^{\alpha_2\beta_2}_{src}\tau[-T/2,T/2]^{k_3k_4}_{\beta_2\alpha_1}\hat{\tau}[R;T/2,-T/2]_{k_4,k_3}$$
$$\Gamma^{\alpha_1\beta_1}_{snk}\tau[T/2,-T/2]^{k_1k_2}_{\beta_1\alpha_2}\hat{\tau}[R;-T/2,T/2]_{k_2,k_1}$$
$$-\frac{1}{3}\Gamma^{\alpha_2\beta_2}_{src}\tau[-T/2,T/2]^{k_3k_4}_{\beta_2\alpha_1}\hat{\tau}[R;T/2,T/2]_{k_4,k_1}$$
$$\Gamma^{\alpha_1\beta_1}_{snk}\tau[T/2,-T/2]^{k_1k_2}_{\beta_1\alpha_2}\hat{\tau}[R;-T/2,-T/2]_{k_2,k_3}\Bigg)$$

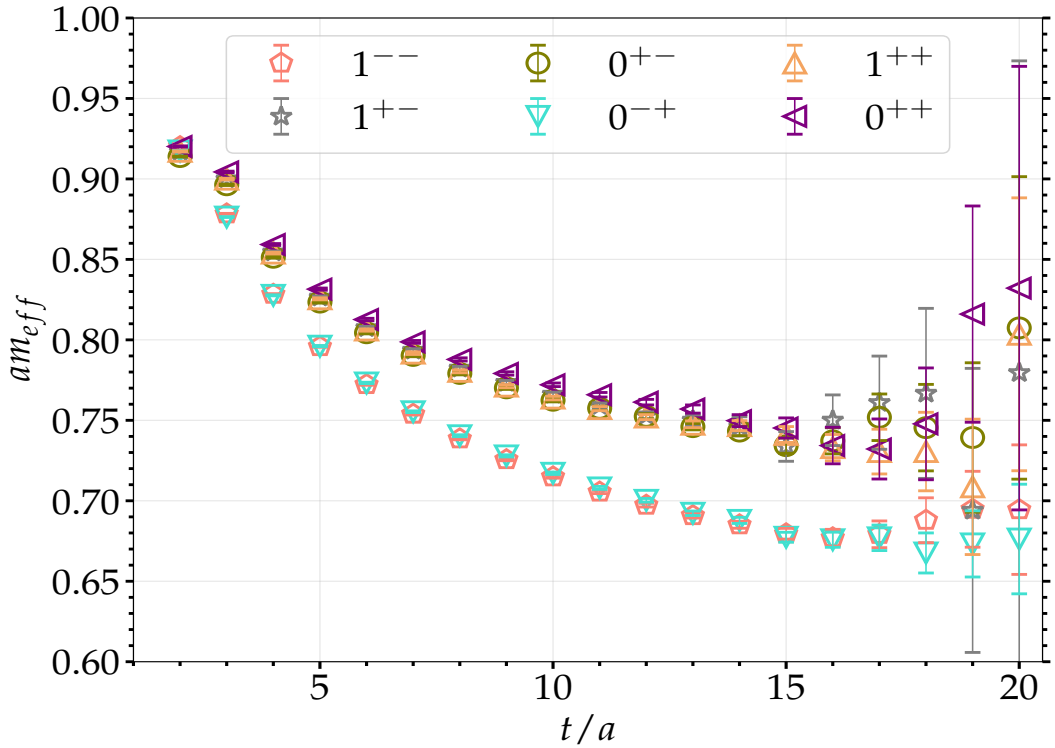
where,
$\hat{\tau}[R;T/2,-T/2]_{k_4,k_3} = v^{\dagger k_4}_{R,c_1}(T/2)U(T/2,-T/2)_{c_1c_4}v^{k_3}_{R,c_4}(-T/2)$

Lattice setup for adjoint meson correlators

- $N_f = 3 + 1$ ensembles produced with the openQCD package, with $m_\pi = 406$ MeV, $32^3 \times 96$ lattices with open boundary conditions in time, and $a = 0.05359$ fm ALPHA Collaboration, Eur. Phys. J. C 80 (2020) 349 [2002.02866].
- For the light quark, we use 100 distillation vectors and standard distillation on 4000 gauge configurations. The eigenvectors were computed from links that have 20 steps of 3D APE smearing with $\alpha = 0.5$. The temporal links entering the temporal lines have HYP2 smearing with parameters $\alpha_1 = 1$, $\alpha_2 = 1$, $\alpha_3 = 0.5$.

Preliminary analysis of adjoint mesons

$$am^{i}_{eff}(t, t+1) = \log \frac{C_{ii}(t)}{C_{ii}(t+1)}; \; C_{ij}(t) = \sum_{n=1}^{\infty} e^{-E_n t} \langle \hat{O}_i | n \rangle \langle n | \hat{O}_j \rangle$$



- 0^{-+} and 1^{--} agree validating the BOEFT prediction.
- Agreement between four out of six adjoint mesons associated with S-wave+P-wave threshold.

$k_{\bar{q}}^P \otimes k_q^P$	k^{PC}	BO quantum # Λ_η^σ
$(1/2)^- \otimes (1/2)^+$	0^{-+}	Σ_u^-
	1^{--}	Σ_g^+, Π_g
$(1/2)^- \otimes (1/2)^- + (1/2)^+ \otimes (1/2)^+$	0^{+-}	Σ_u^+
	1^{+-}	Σ_u^-, Π_u
$(1/2)^- \otimes (1/2)^- - (1/2)^+ \otimes (1/2)^+$	0^{++}	Σ_g^+
	1^{++}	Σ_g^-, Π_g
$(1/2)^- \otimes (3/2)^- + (3/2)^+ \otimes (1/2)^+$	1^{+-}	Σ_u^-, Π_u
	2^{+-}	$\Sigma_u^+, \Pi_u, \Delta_u$
$(1/2)^- \otimes (3/2)^- - (3/2)^+ \otimes (1/2)^+$	1^{++}	Σ_g^-, Π_g
	2^{++}	$\Sigma_g^+, \Pi_g, \Delta_g$

Tab X of M. Berwein, N.Brambilla, A. Mohapatra, A. Vairo, arXiv:2408.04719v2 [hep-ph]

- First block corresponds to S-wave $+S$-wave adjoint mesons.
- First block corresponds to S-wave $+P$-wave adjoint mesons.

Conclusions

- Preliminary results for the adjoint mesons associated with the Z_b tetraquark channel validate the predictions of BOEFT.
- Adjoint mesons associated with S-wave+S-wave threshold are degenerate.
- Four out of six adjoint mesons associated with S-wave+P-wave threshold are degenerate.

Outlook

- Do computation for the remaining two S-wave+P-wave threshold adjoint mesons.
- Insert distillation profiles, and do GEVP in each quantum number channel to get better plateaux.
- Define a scheme for adjoint meson masses computed at a fixed lattice spacing.

Ongoing work

- Calculation of the static energies in the Z_2 (Σ_u^-) tetraquark channel is in progress.
- We use $24^3 \times 128$ CLS ensembles with $a = 0.086$ fm.
- Our operator basis has seven operators, which fall into three operator classes:
  - Quarkonium plus pions
  - Static-light meson pair
  - Adjoint tetraquark operator

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- Our operator basis has seven operators, which fall into three operator classes:
  - Quarkonium plus pions
  - Static-light meson pair
  - Adjoint tetraquark operator
- We already have some preliminary results on static energies as a function of interquark separation, r. We perform GEVP at each r using 7×7 correlation matrix.
- We see from the off-diagonal correlation matrix elements that indeed the adjoint meson tetraquark operator does not couple to quarkonium plus pion at short distances.
- We use 4D HYP2 smearing to improve signal-to-noise ratio. We use the distillation technique for light propagators.