CRC-TR 211
Strong-interaction matter under extreme conditions

ExoHad

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NEUE WEGE. SEIT 1607.

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

HFHF
Helmholtz Forschungsakademie Hessen für FAIR



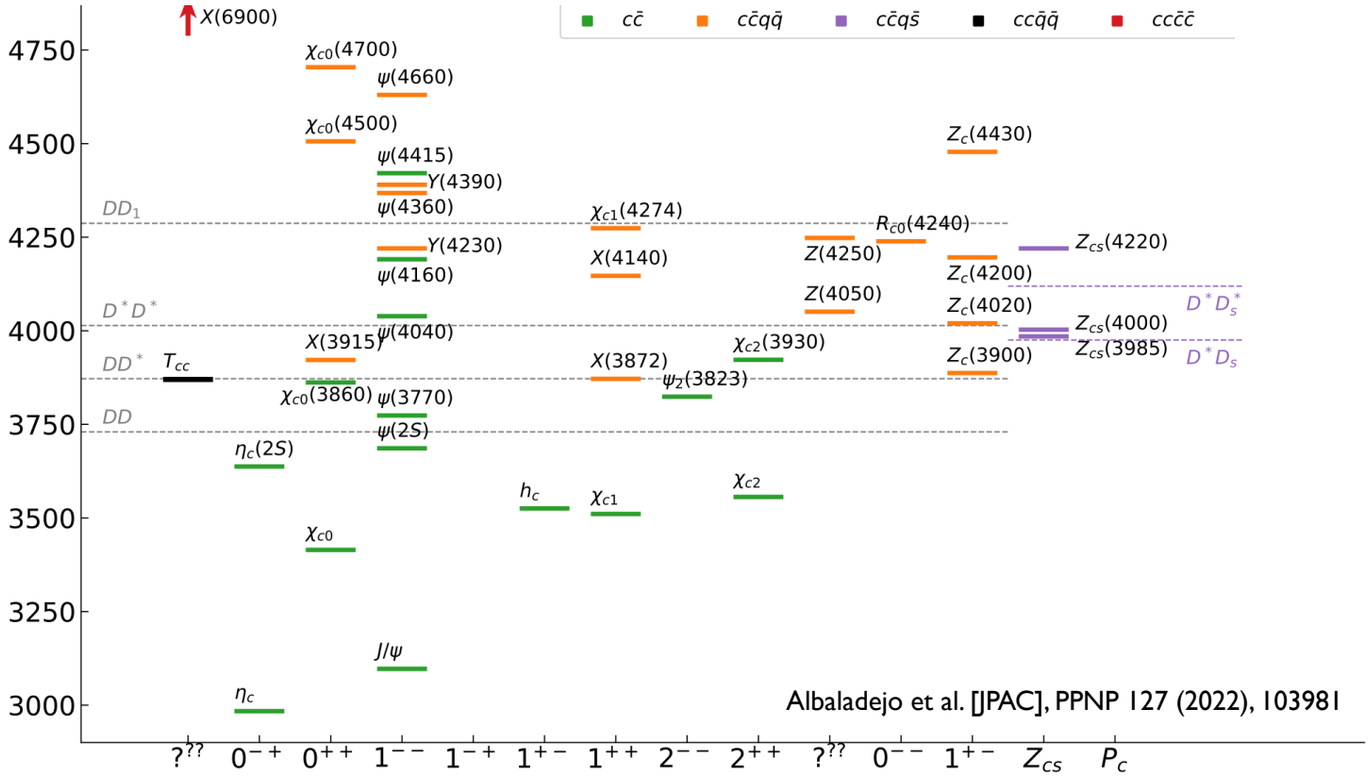
ECT*, Trento, 2026

# Four-quark states with functional methods

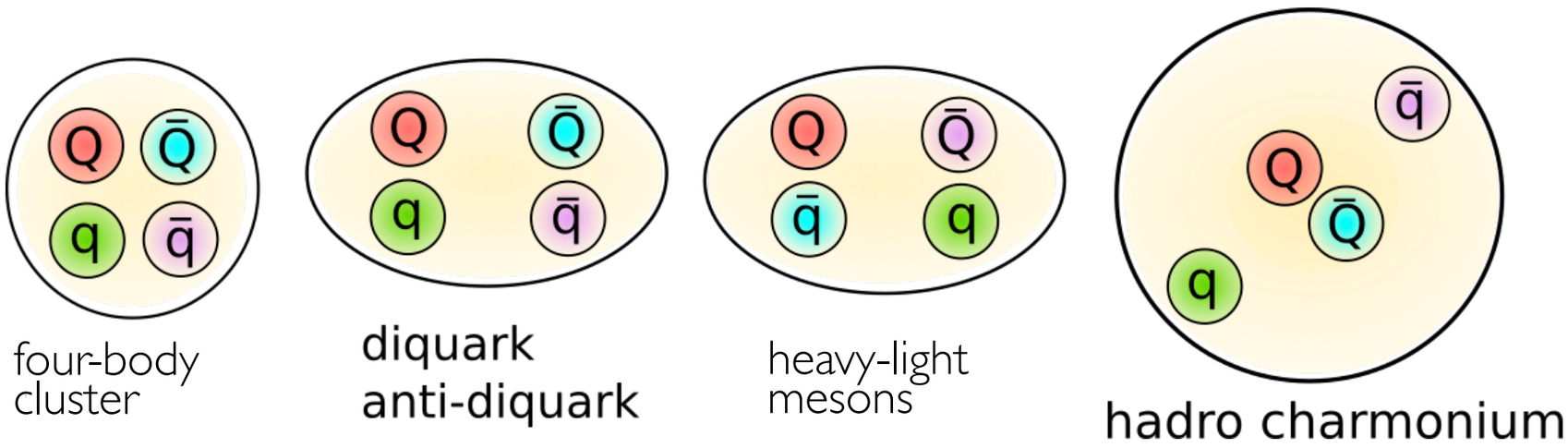
with Gernot Eichmann and Joshua Hoffer

Hoffer, Eichmann, CF, PRD 109 (2024) 7 074025
Hoffer, Eichmann, CF, PRD 111 (2025) 5 054028
Hoffer, Eichmann, CF, in preparation

# Exotic hadrons at Belle, BABAR, BES, LHCb,…



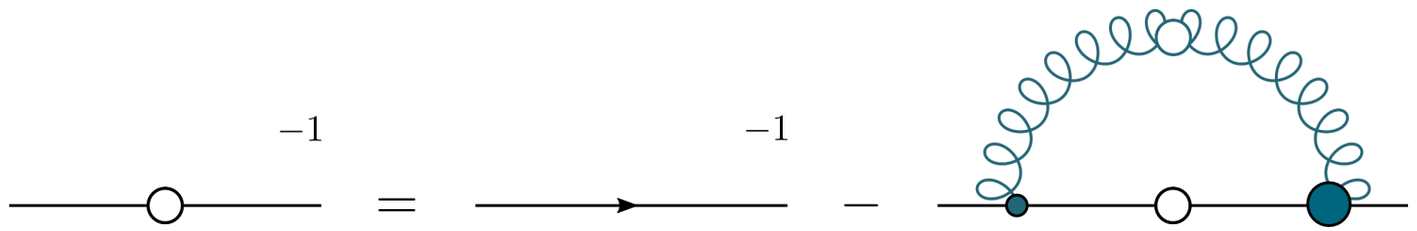

- same internal structure?
- flavour variations?
  $$b \leftrightarrow c \leftrightarrow s$$
- heavy vs light?
  $$\bar{c}c\bar{q}q \leftrightarrow \bar{q}q\bar{q}q$$
- thresholds?




Related to details of underlying QCD forces

# Overview

1. The formalism: gluons, quarks and two-quark states




2. Light quark sector:

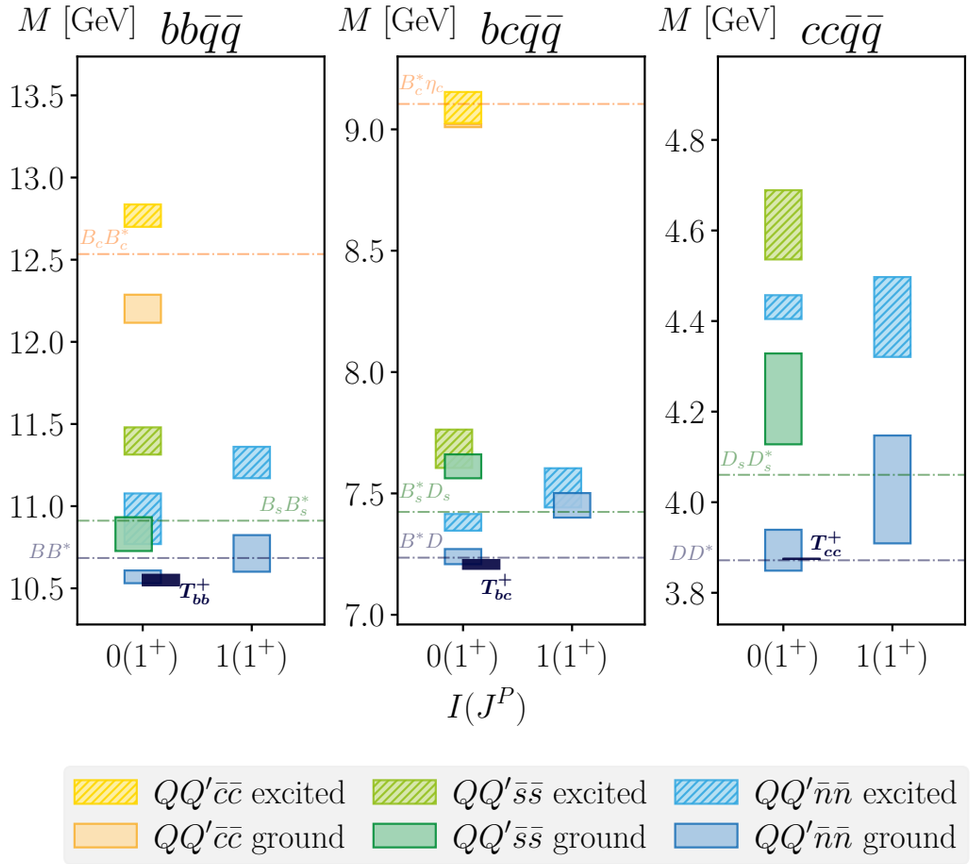
scalar mesons

3. Heavy four-quark states:

open flavour

4. Heavy four-quark states:

hidden flavour

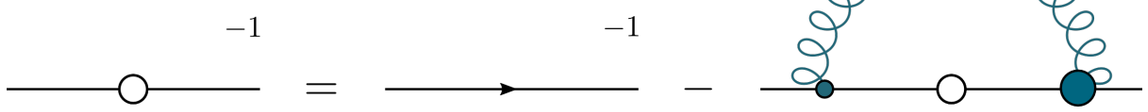


# Dyson-Schwinger equations - 'full' vs RL

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left( \bar{\Psi} \left( i \not{D} - m \right) \Psi - \frac{1}{4} \left( F^a_{\mu\nu} \right)^2 \right) \right\}$$

Euclidean, gauge fixed

propagators



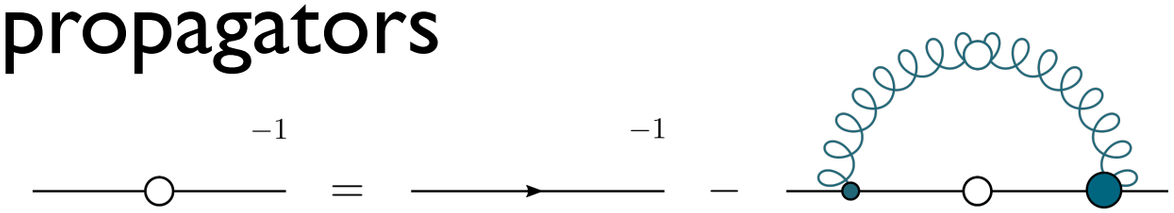

CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

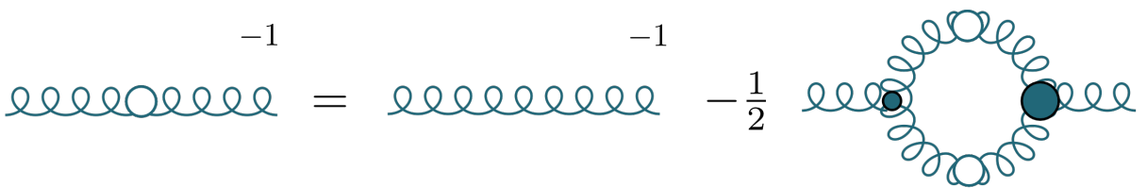
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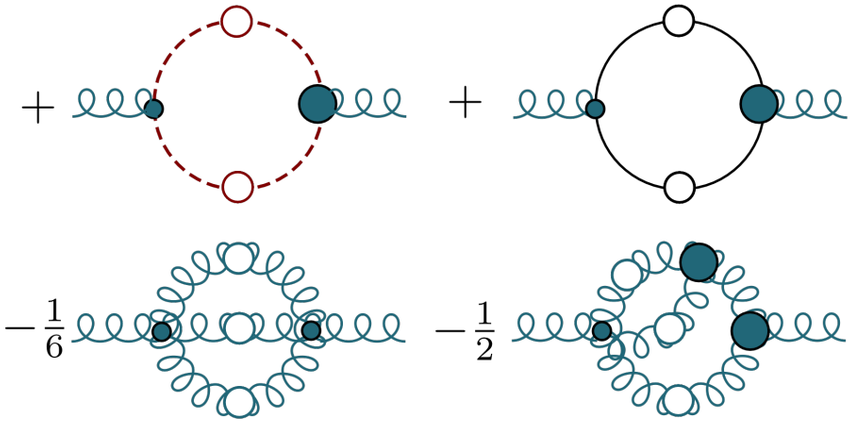
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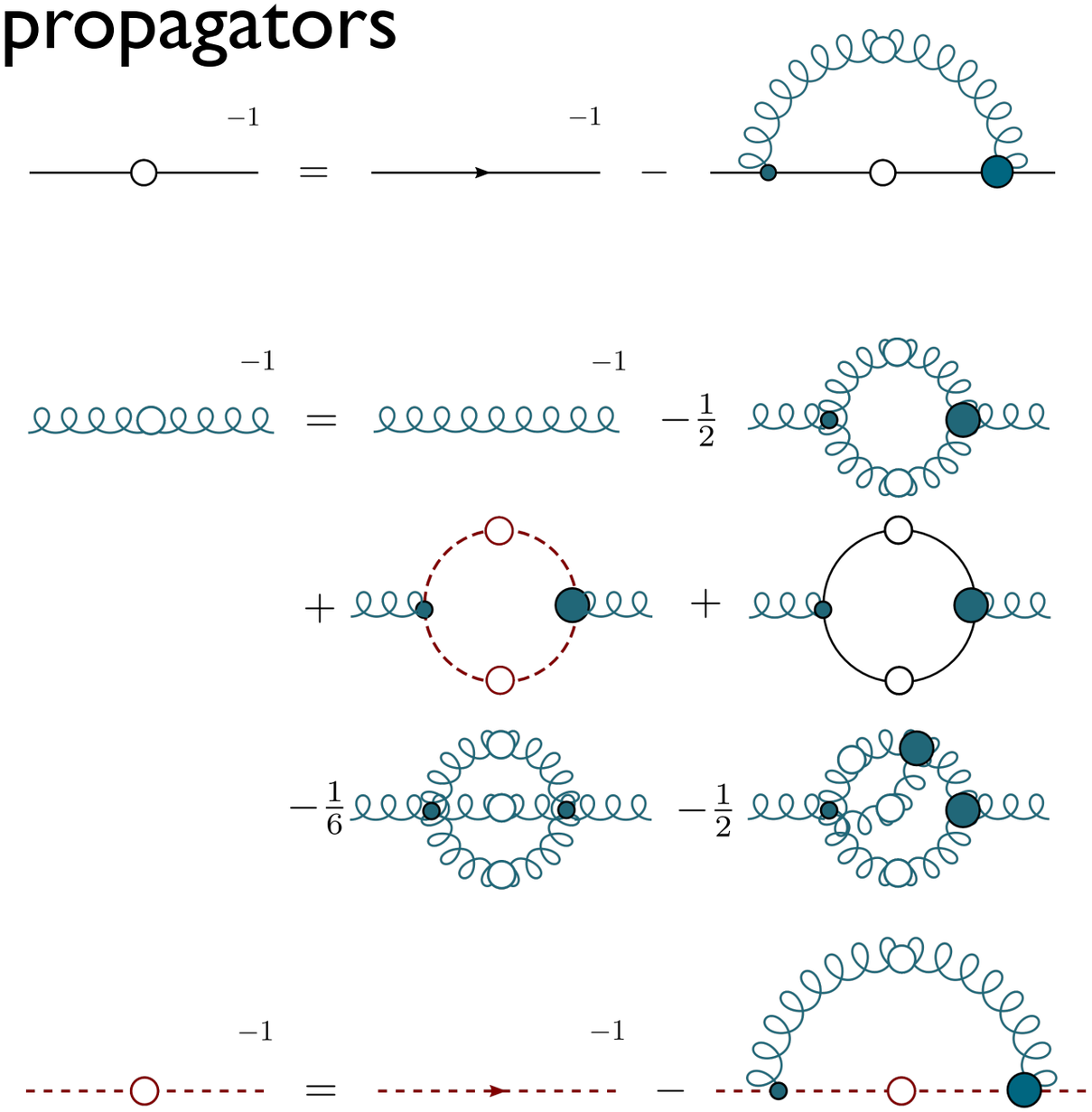
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Euclidean, gauge fixed

## propagators




## vertices



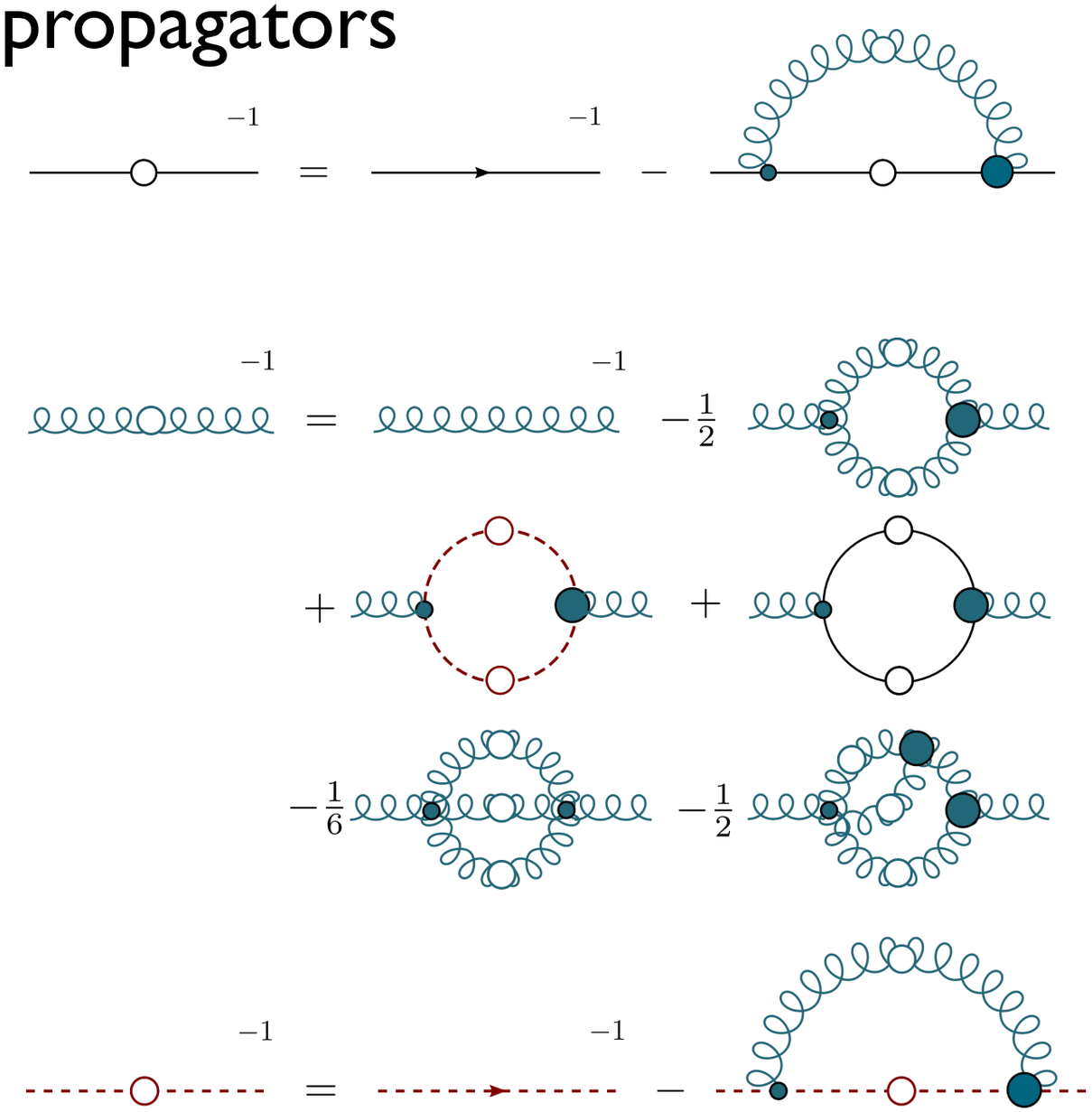

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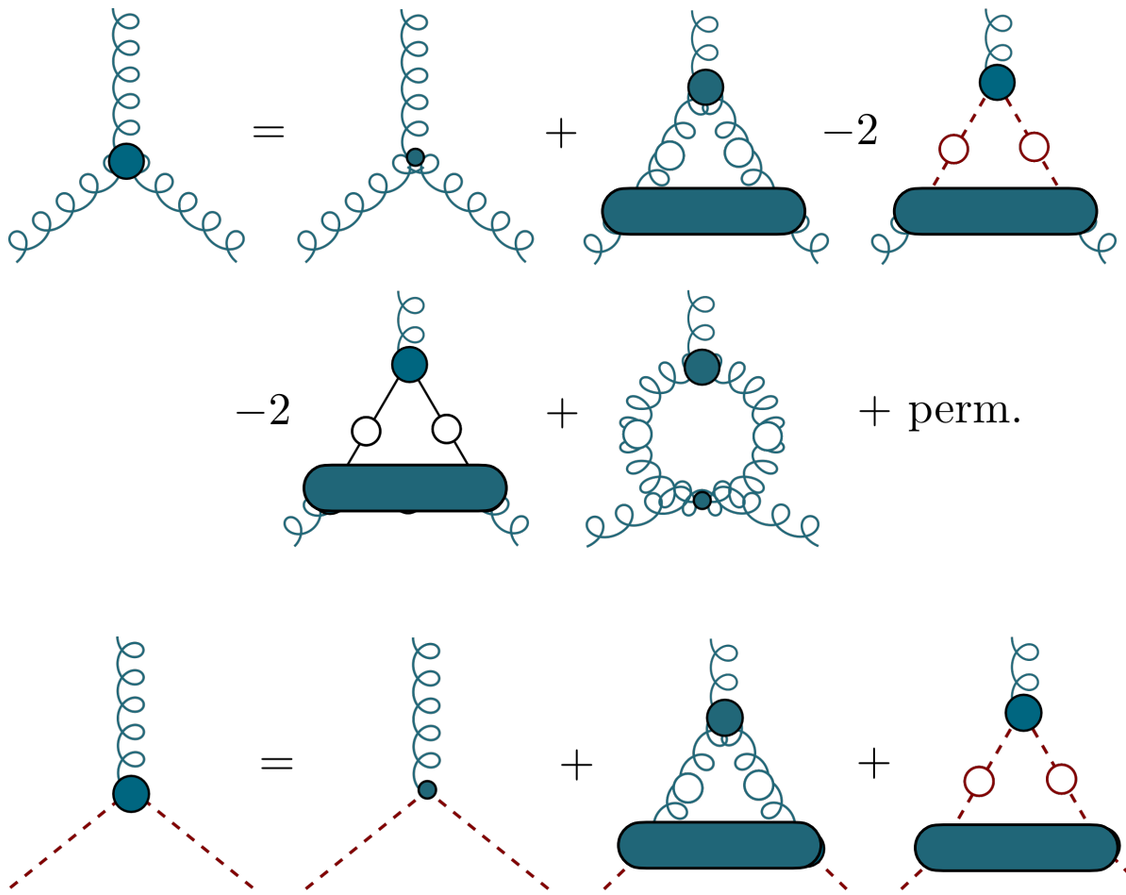
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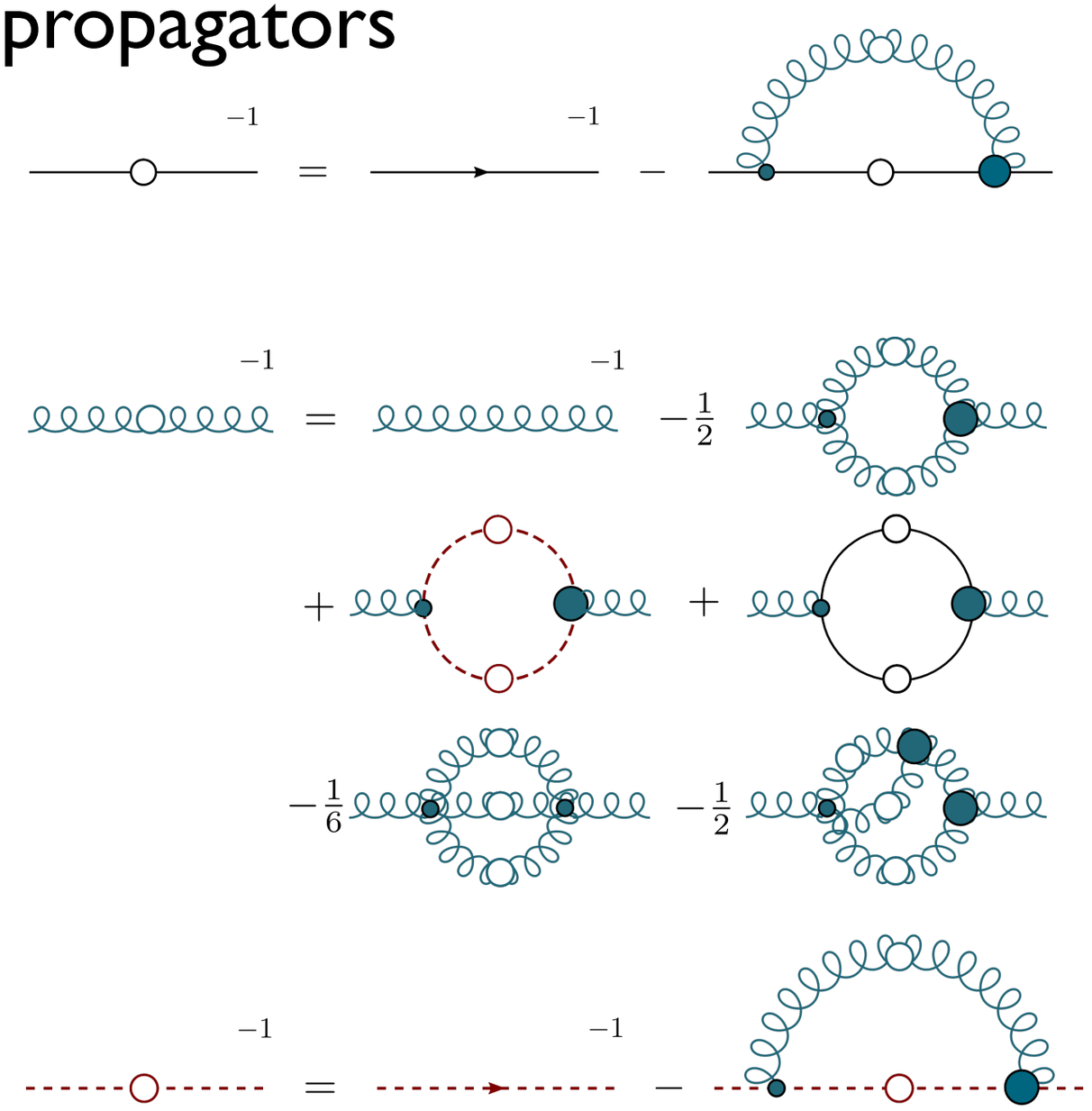

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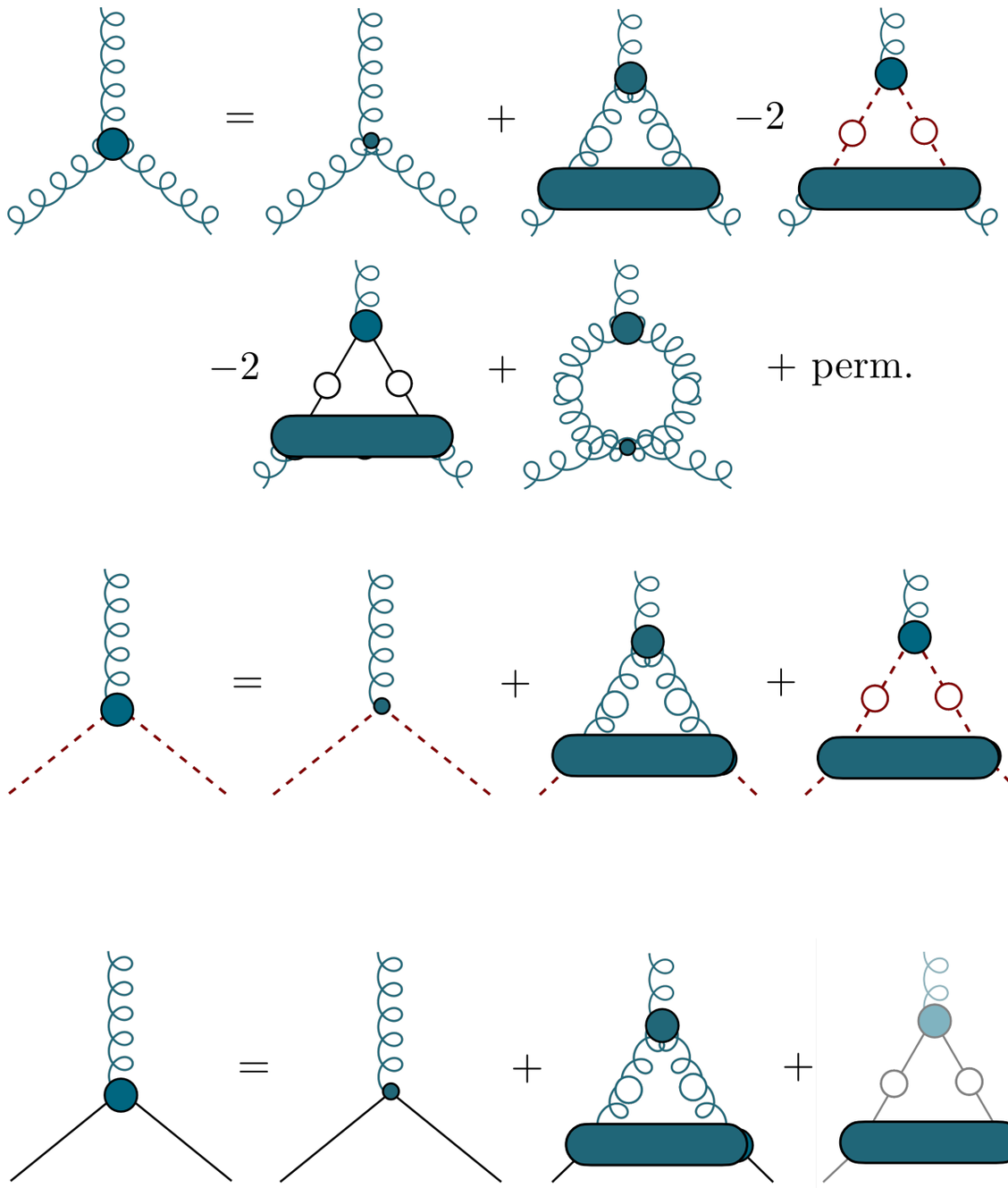
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Euclidean, gauge fixed

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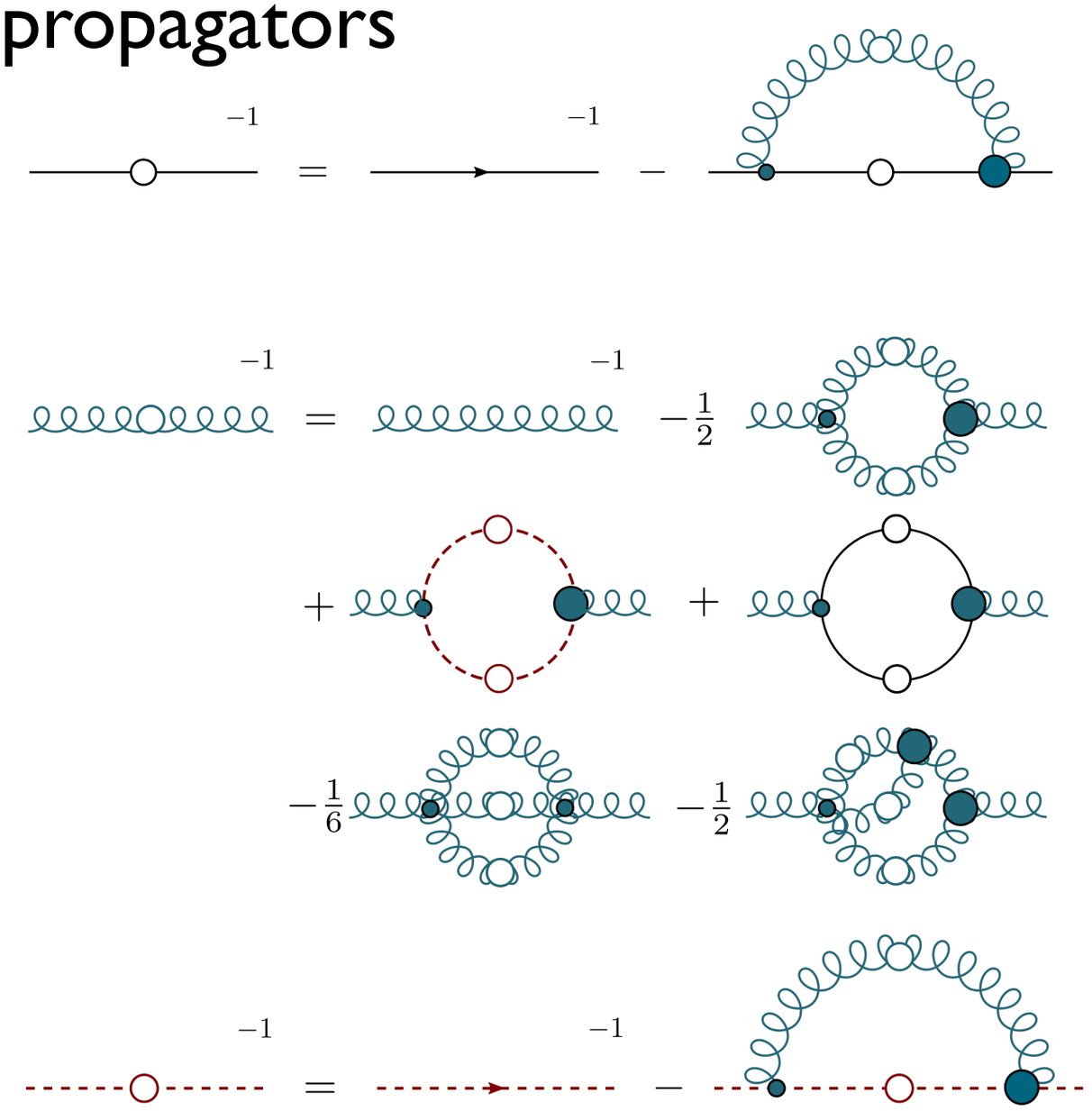

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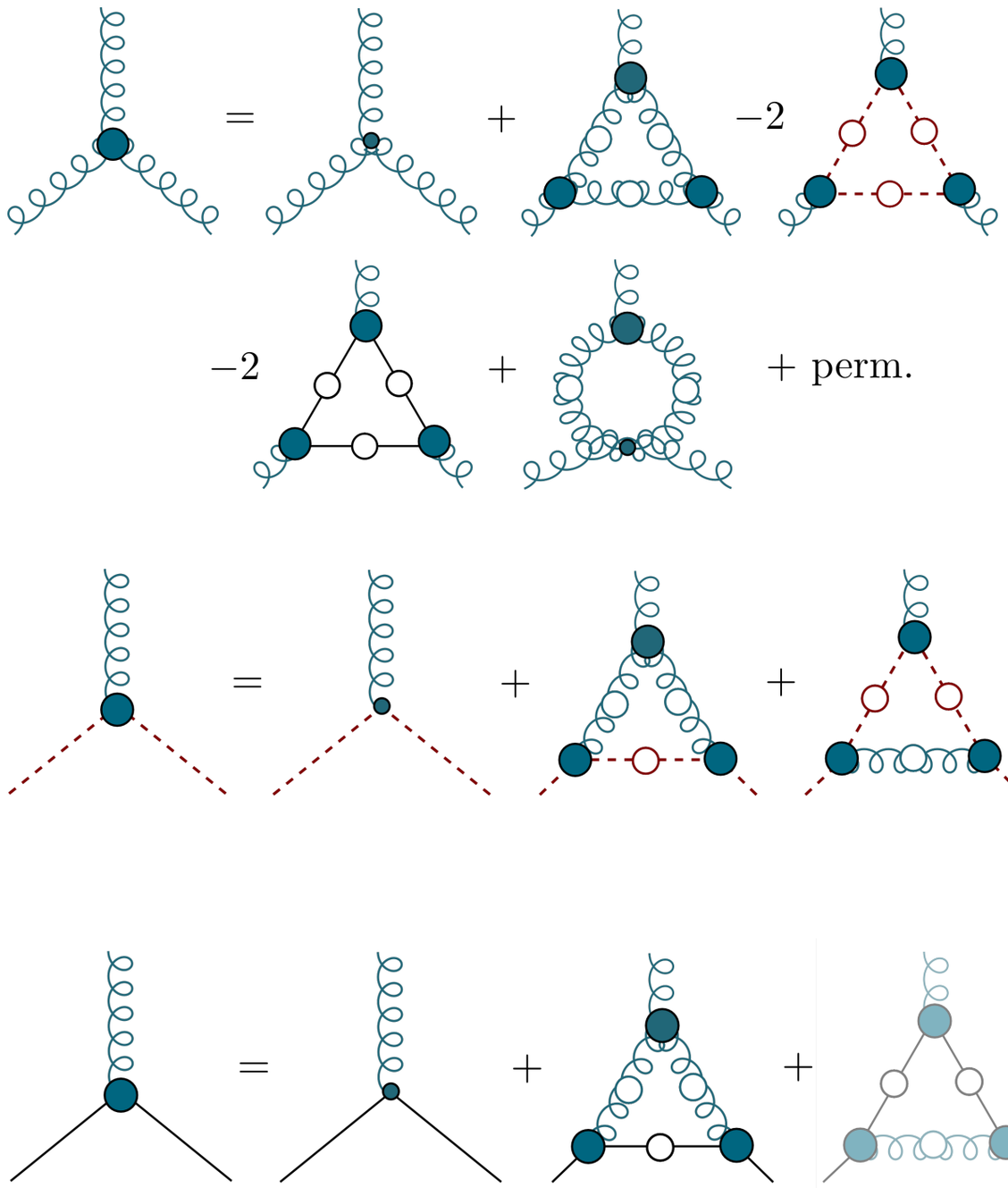
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## propagators




## vertices




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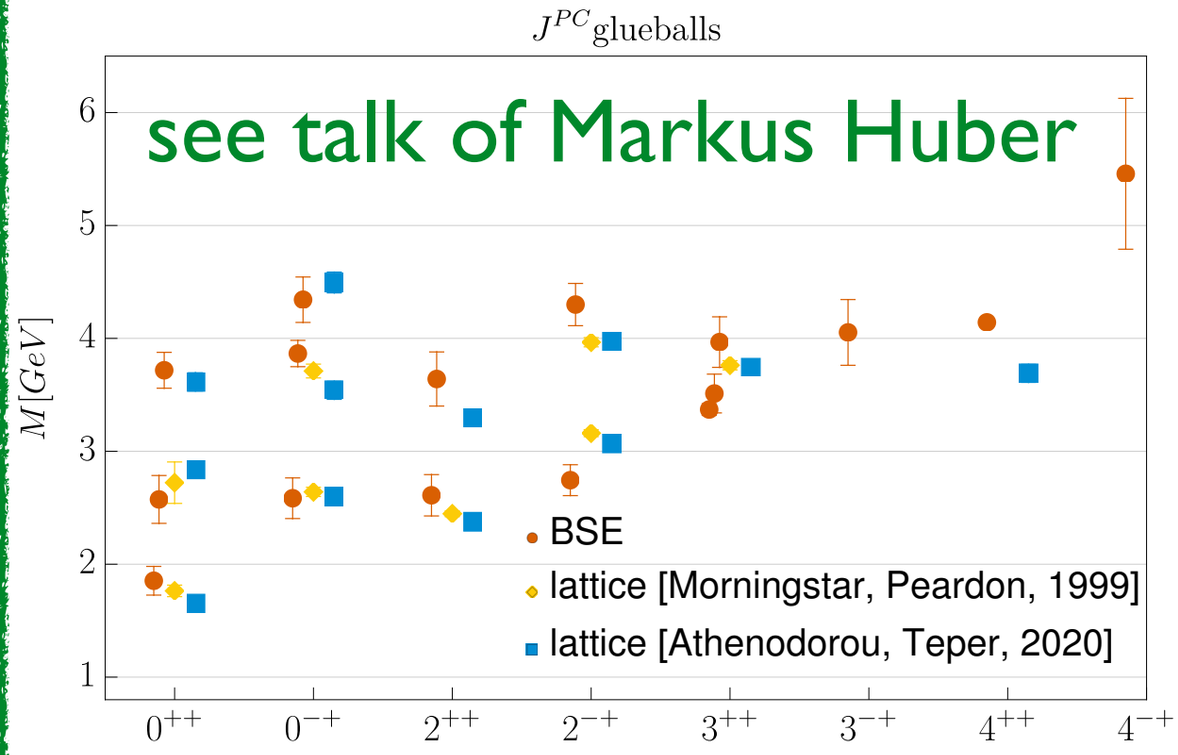
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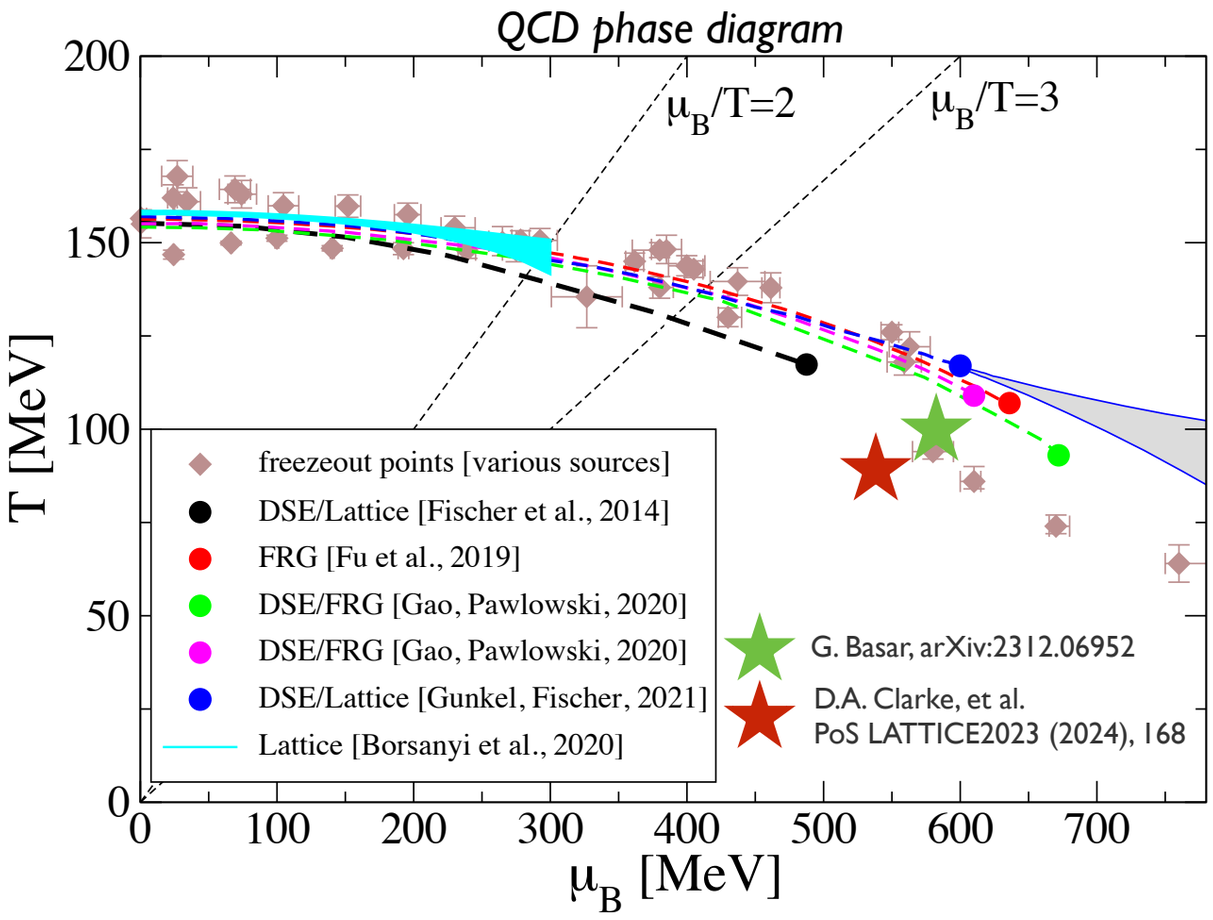
Euclidean, gauge fixed

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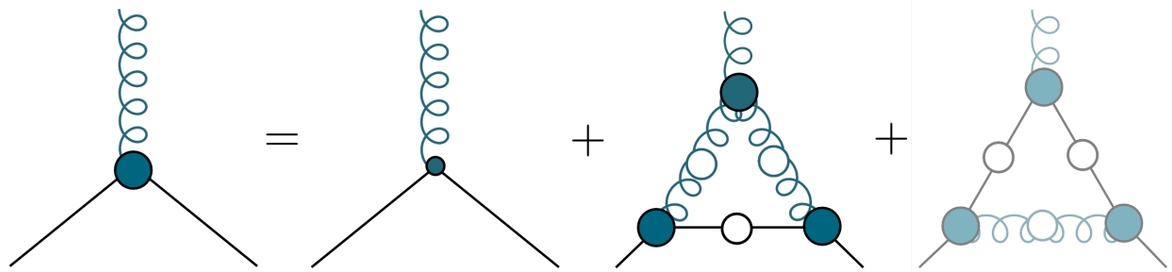

CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]
Huber, CF, Sanchis-Alepuz, EPJC 81 (2021) [arXiv:2110.09180]




Bernhardt, CF and Isserstedt, PLB 841 (2023)



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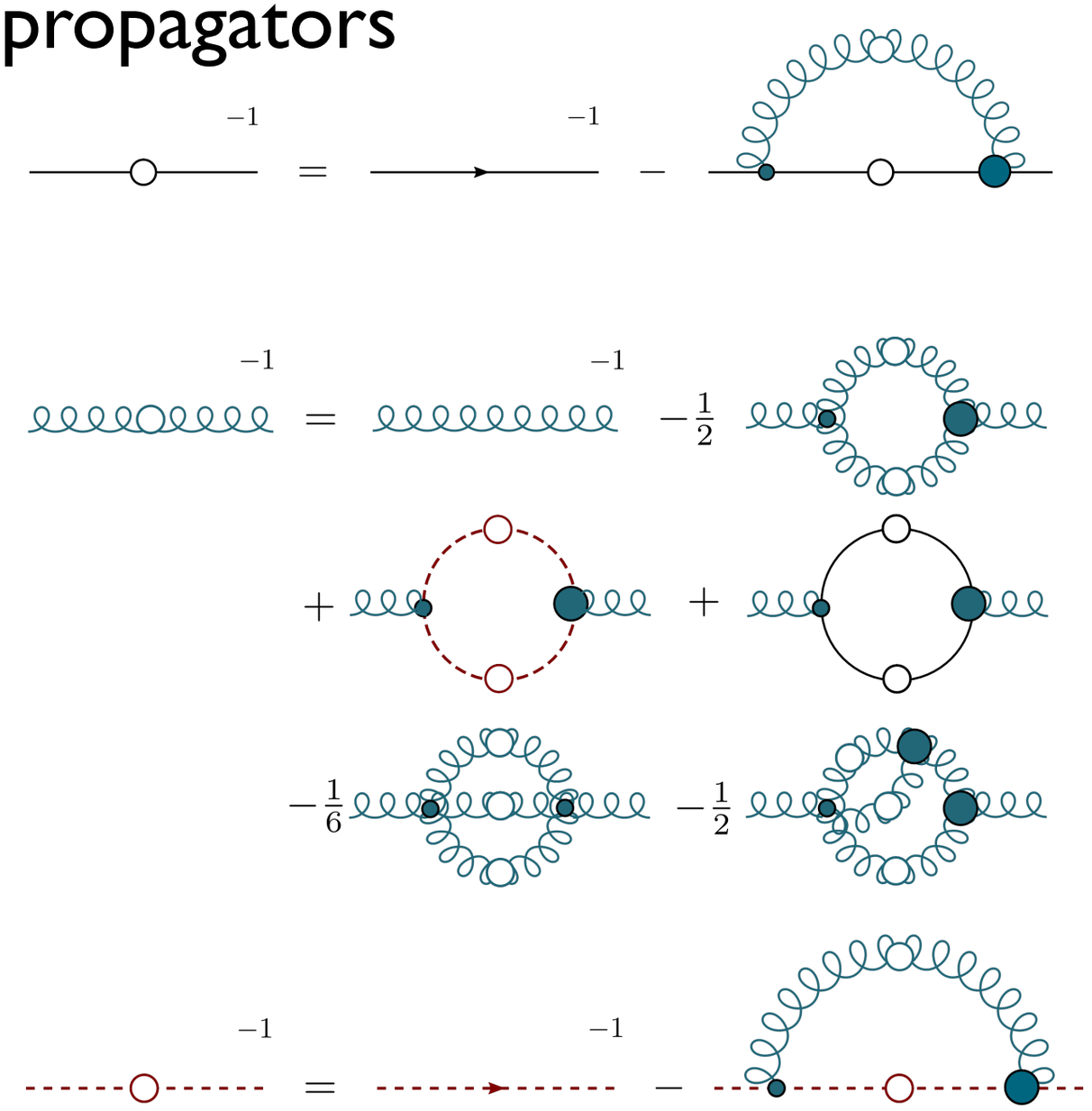


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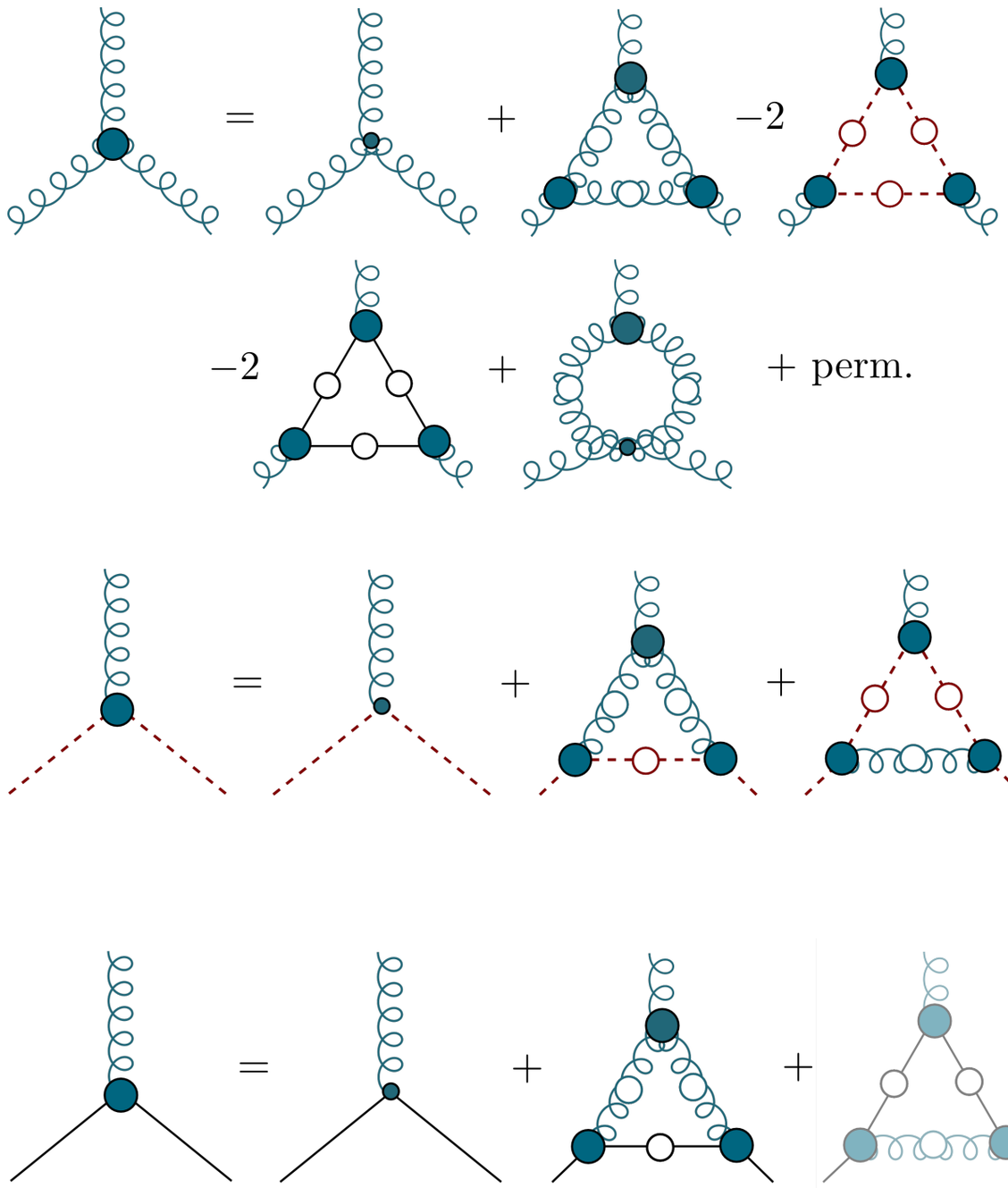
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Euclidean, gauge fixed

## propagators




## vertices



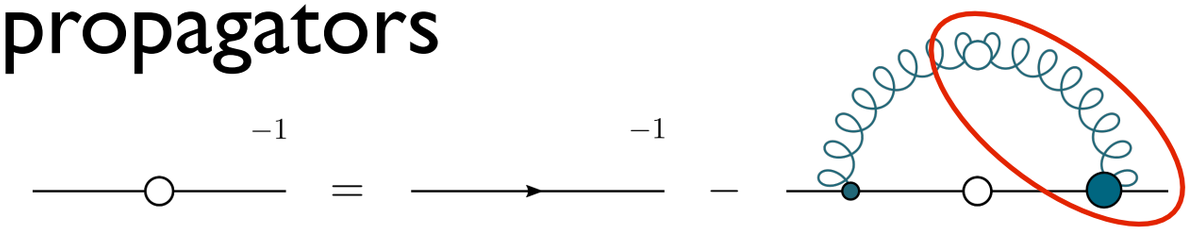

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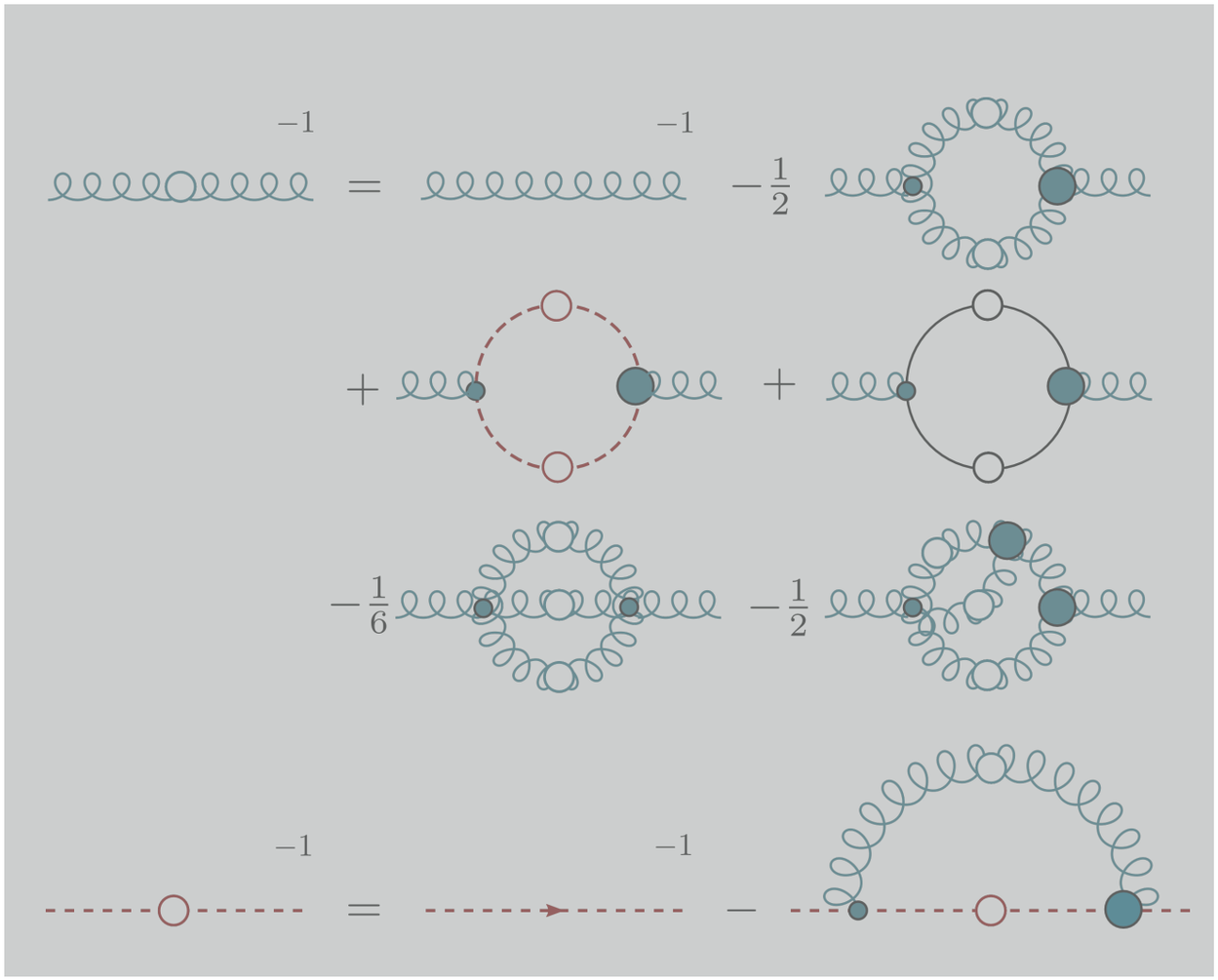
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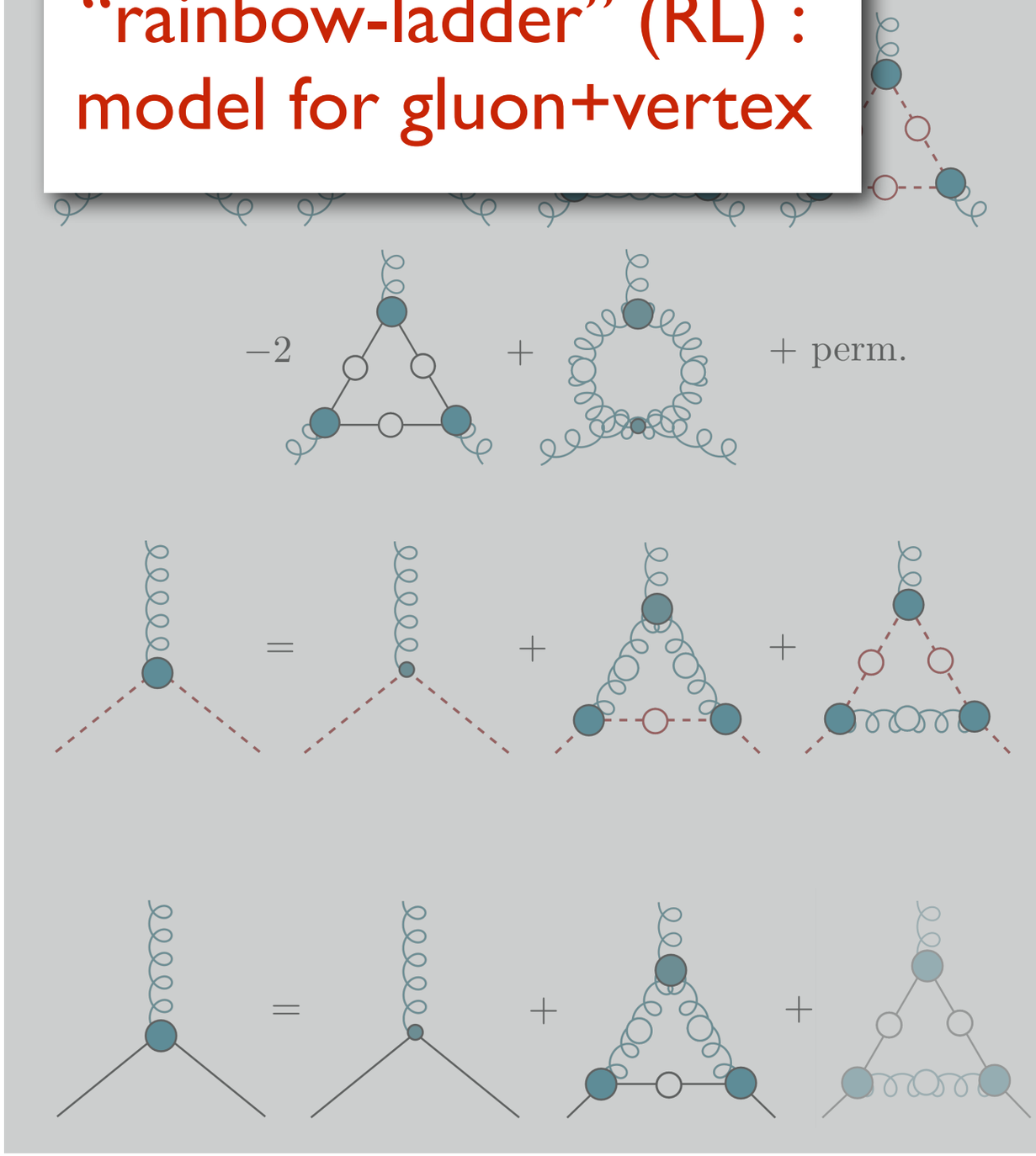


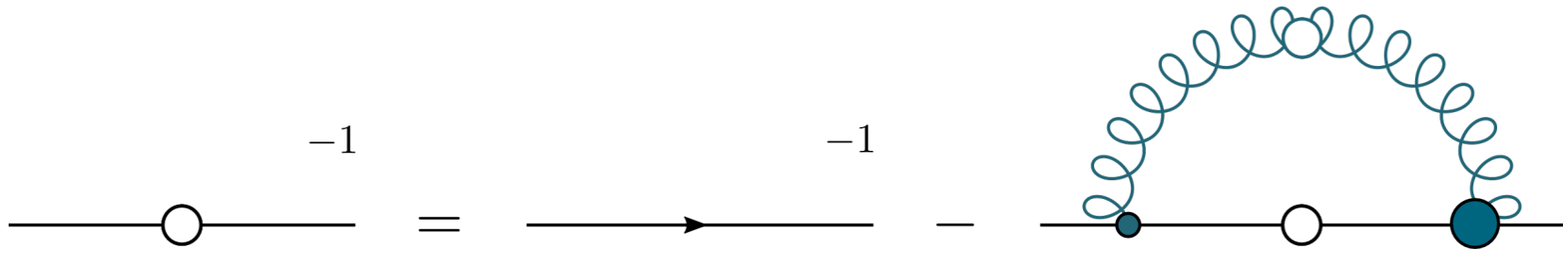

CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

## vertices

“rainbow-ladder” (RL) : model for gluon+vertex

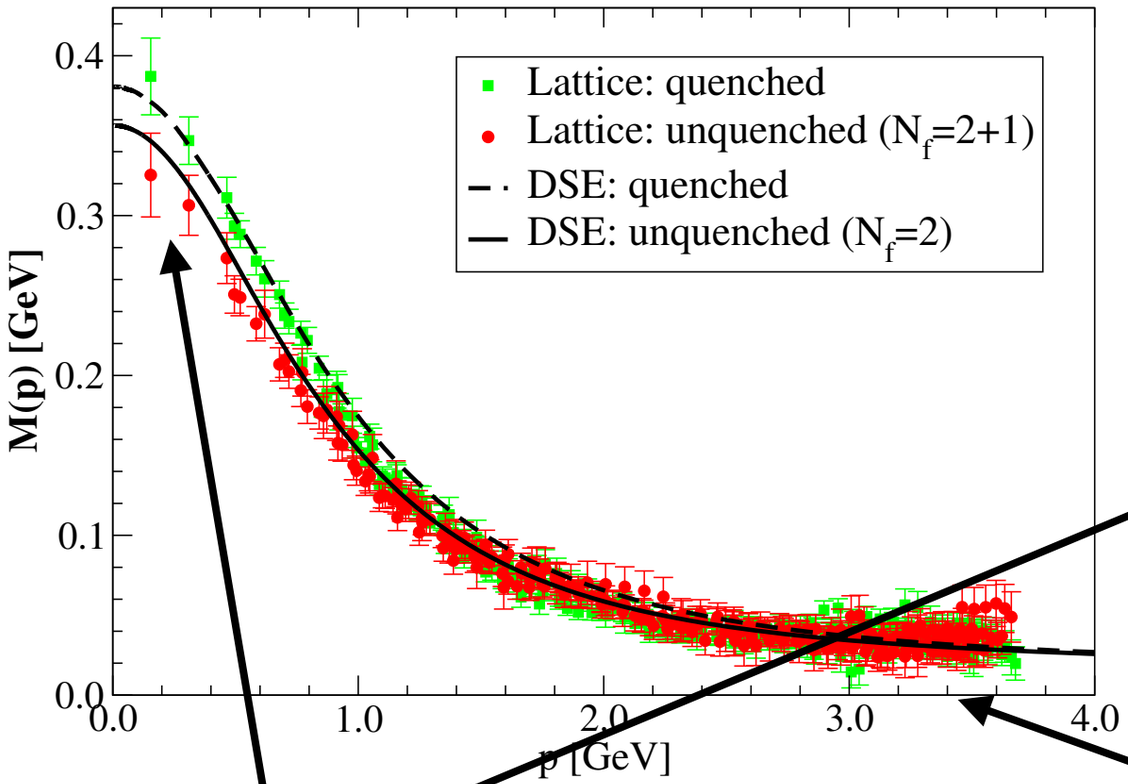


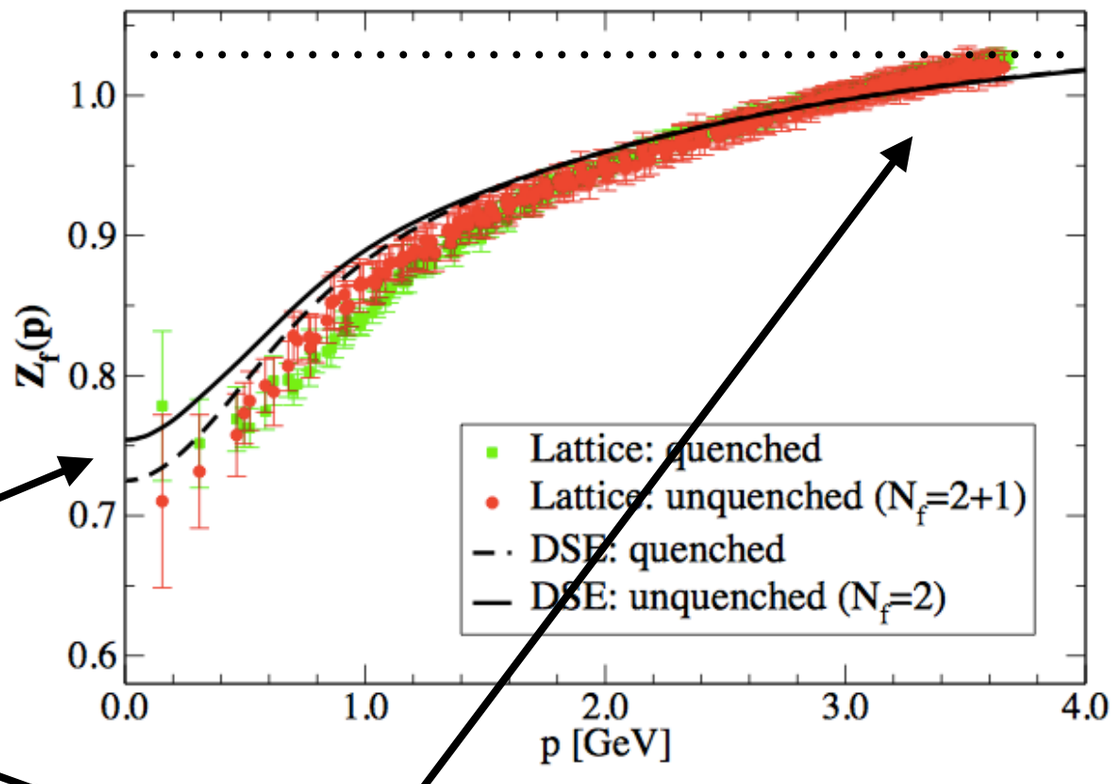
# Quarks: mass from interaction




$$S(p) = Z_f(p^2)\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Williams, CF, Heupel, PRD 93 (2016) 034026
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507



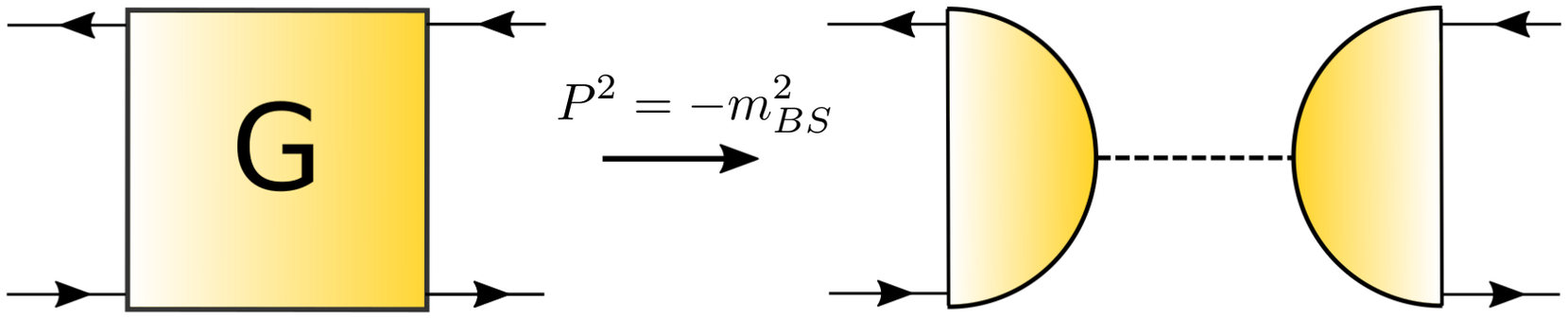




'constituent quark':
large mass; very composite

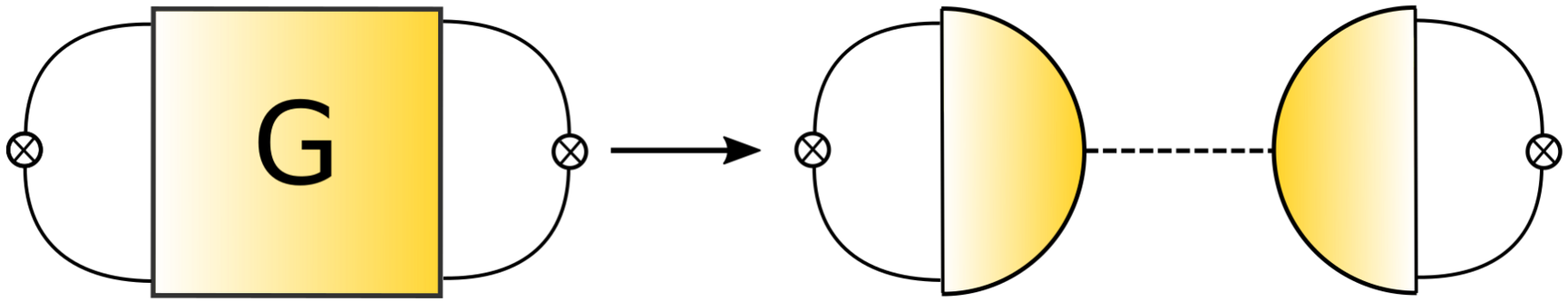
'current quark':
- small mass; non-composite

# Extracting spectra from QCD-correlators

functional:

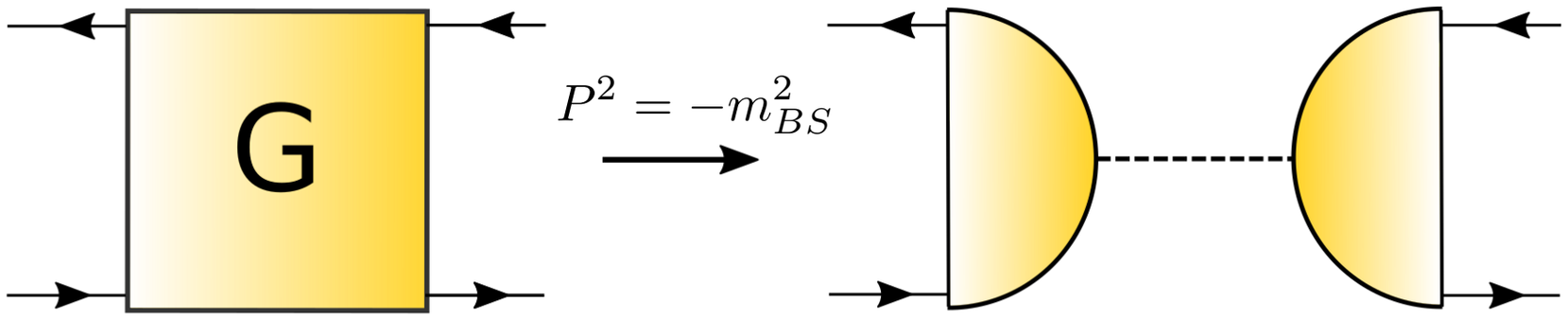



Lattice:

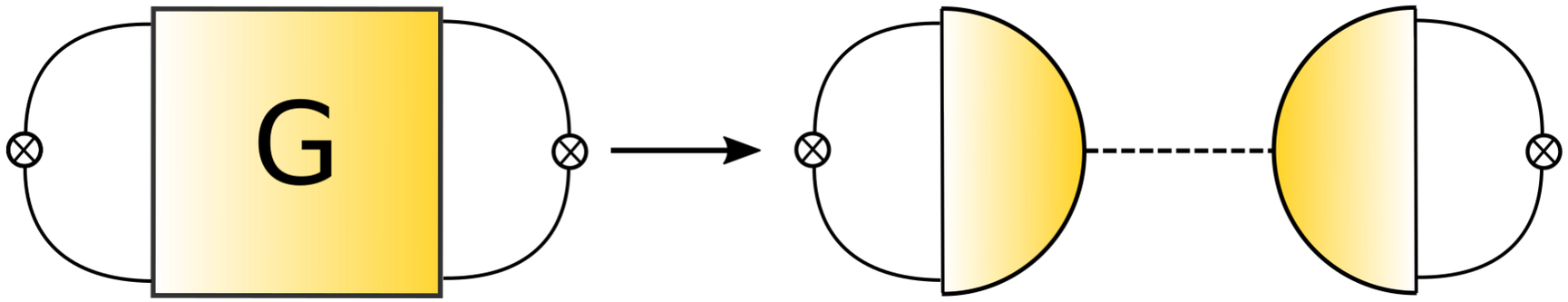


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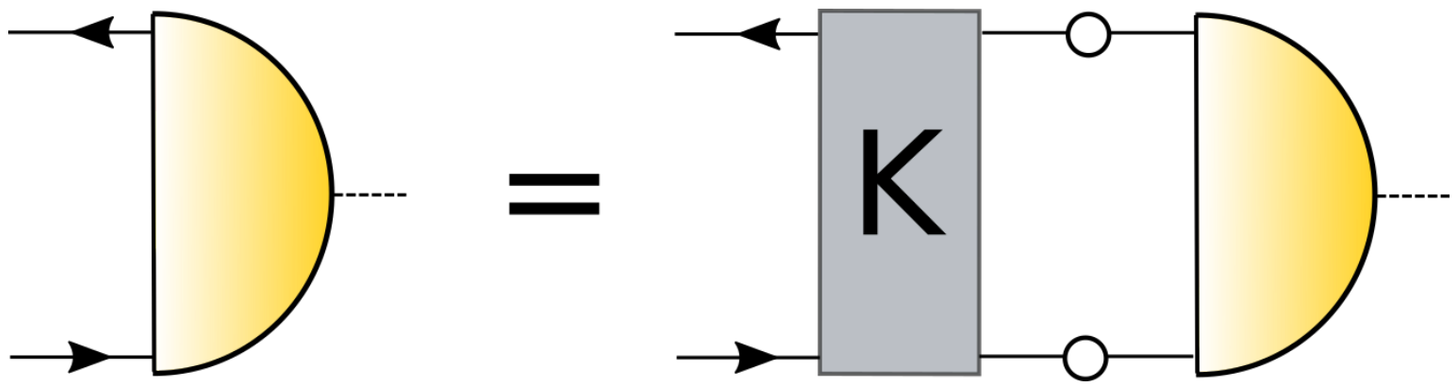
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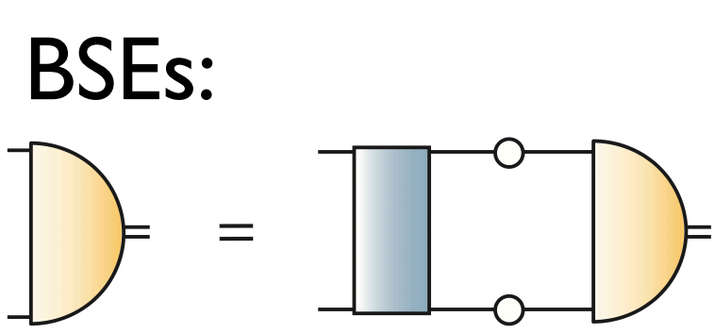


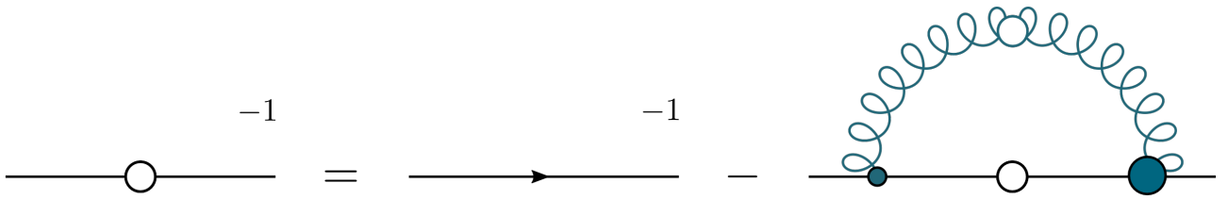

exact BSE:

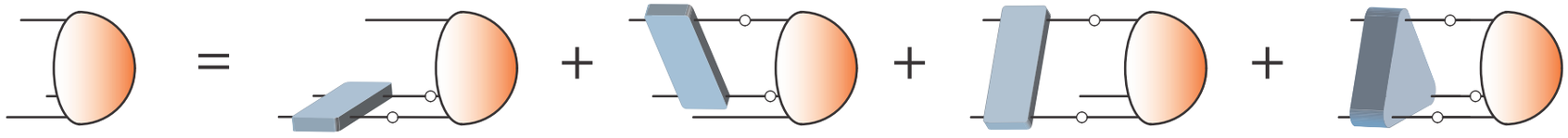


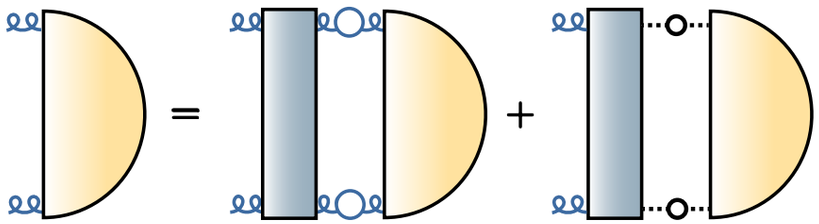
# Bound states and Bethe-Salpeter equations

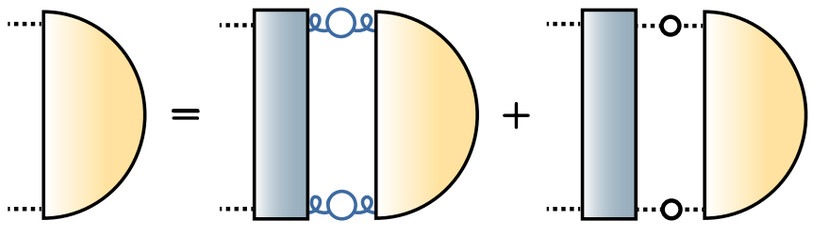
BSEs:

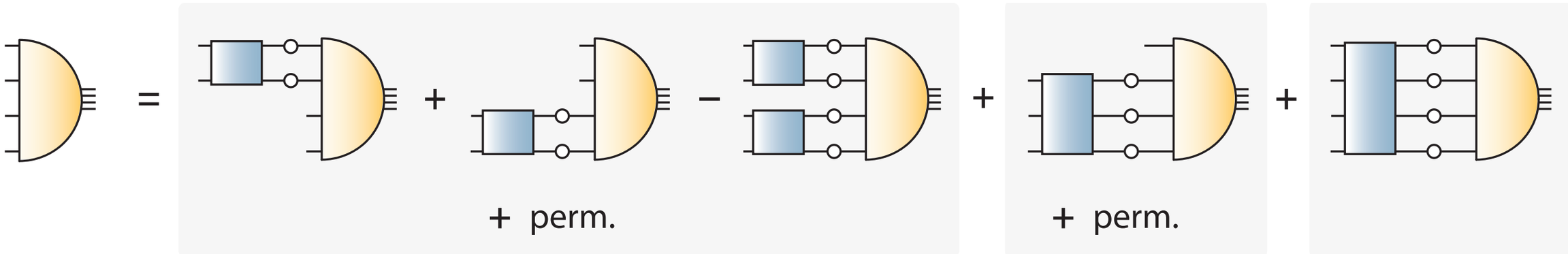






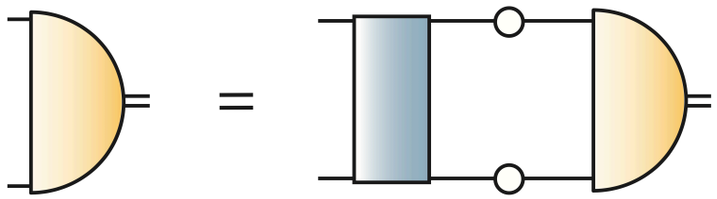


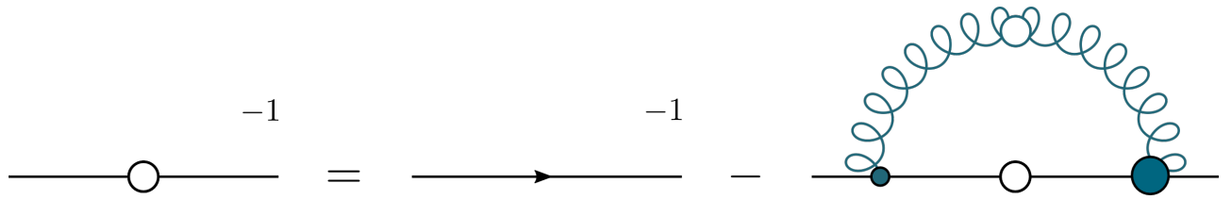


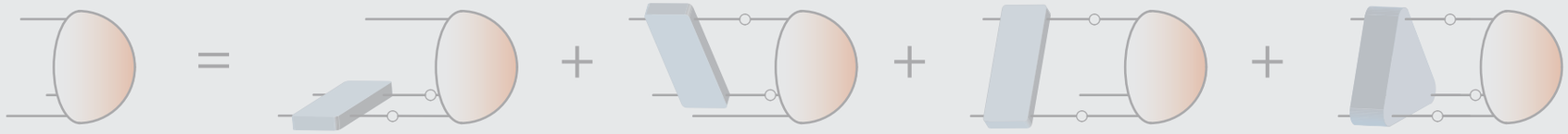

Eigenvalue equations: masses and wave functions

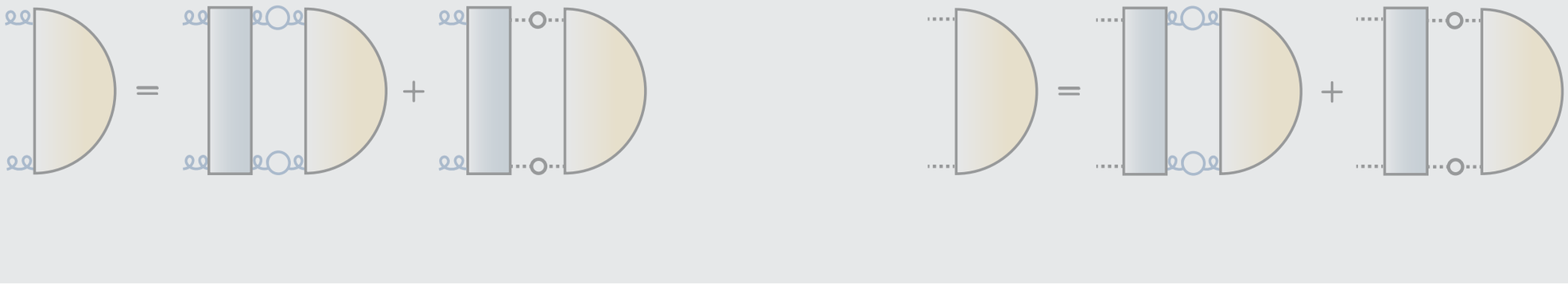
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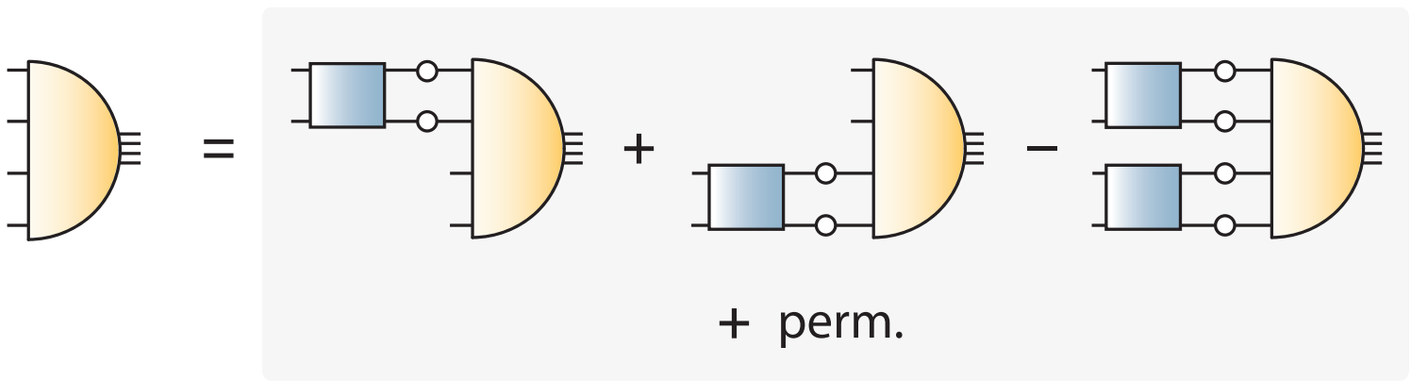


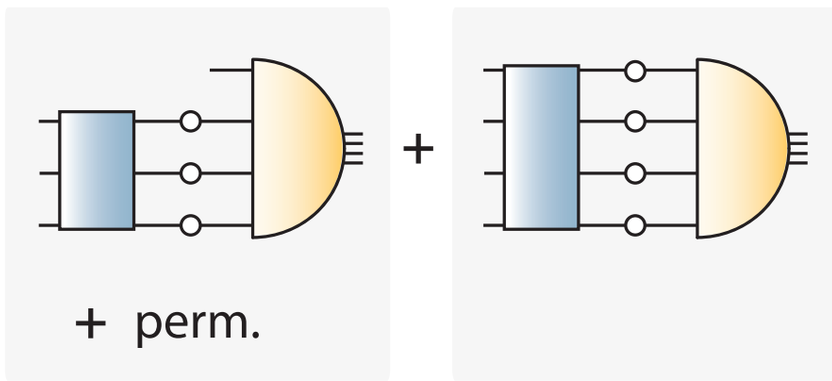






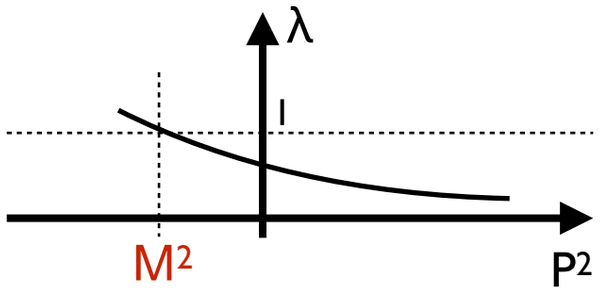


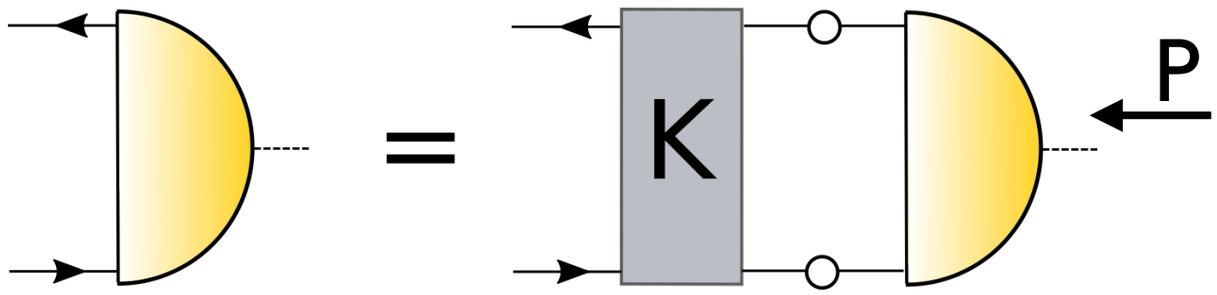




Eigenvalue equations: masses and wave functions

# The complex $P^2$-plane

$$\lambda(P^2) \stackrel{!}{=} 1$$

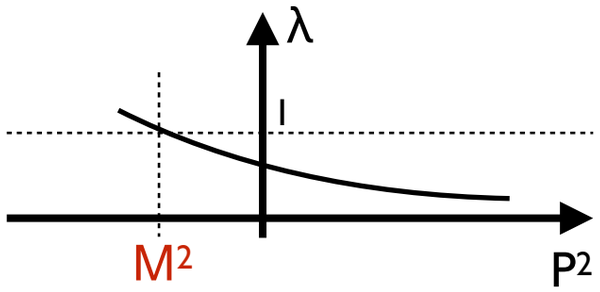


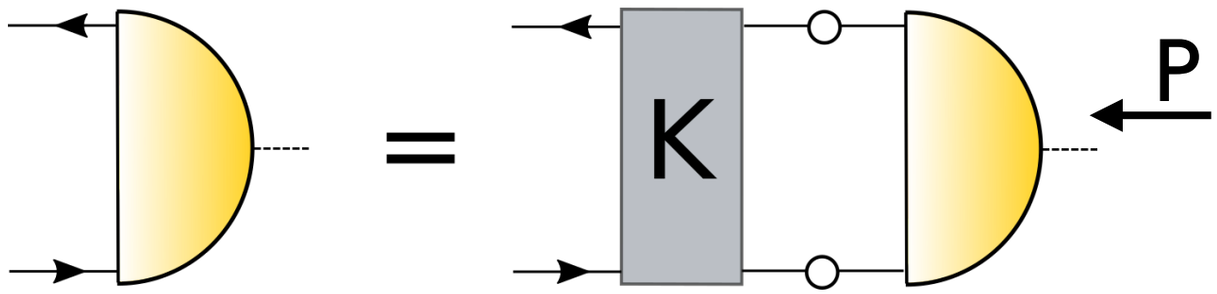




$$\lambda(P^2) \times \mathrm{BSA} = \mathrm{kernel} \times \mathrm{BSA}$$

# The complex $P^2$-plane

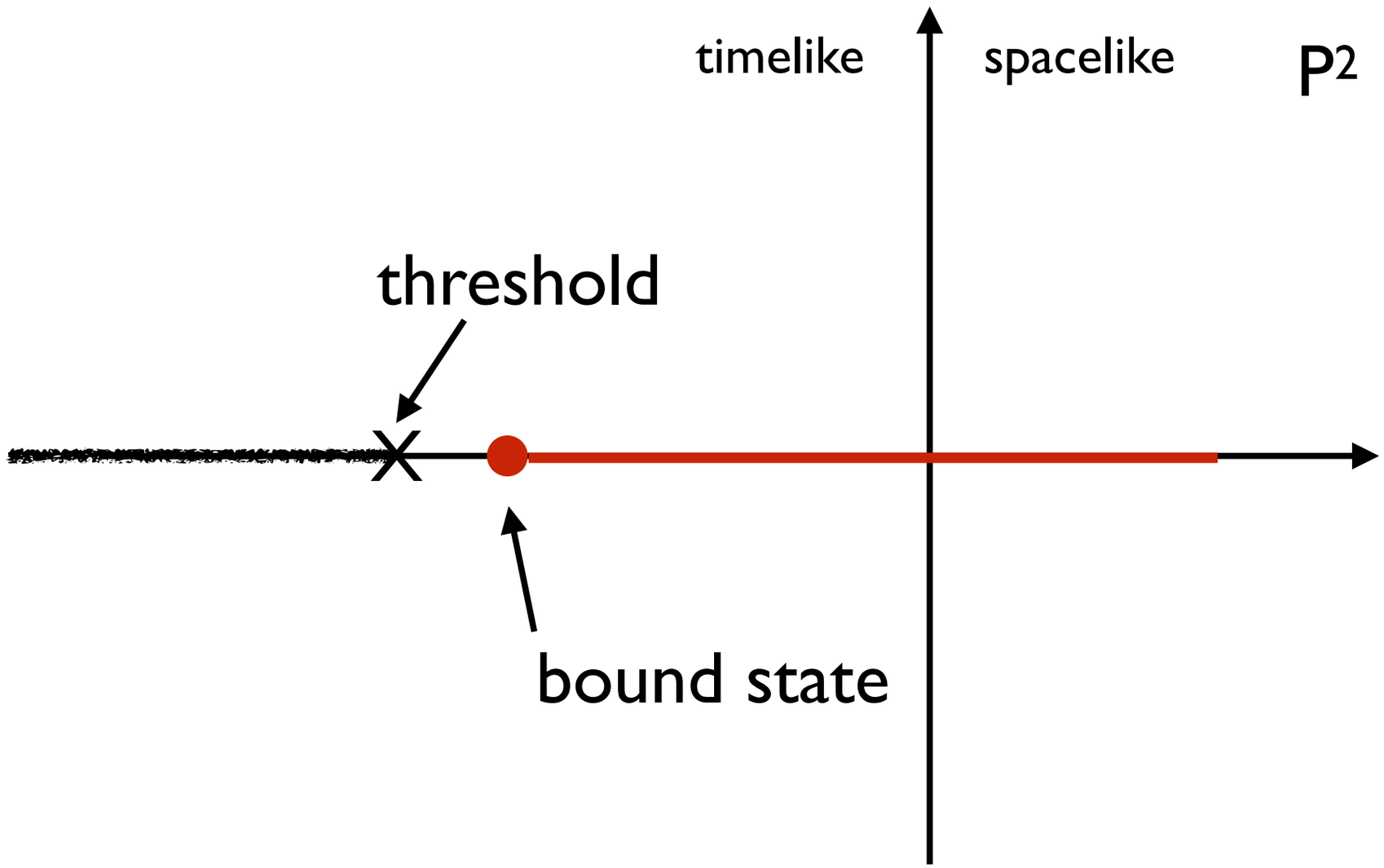
$$\lambda(P^2) \stackrel{!}{=} 1$$



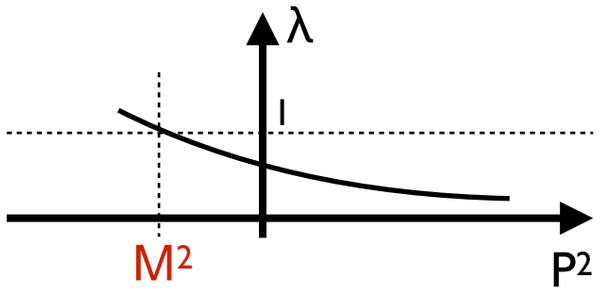

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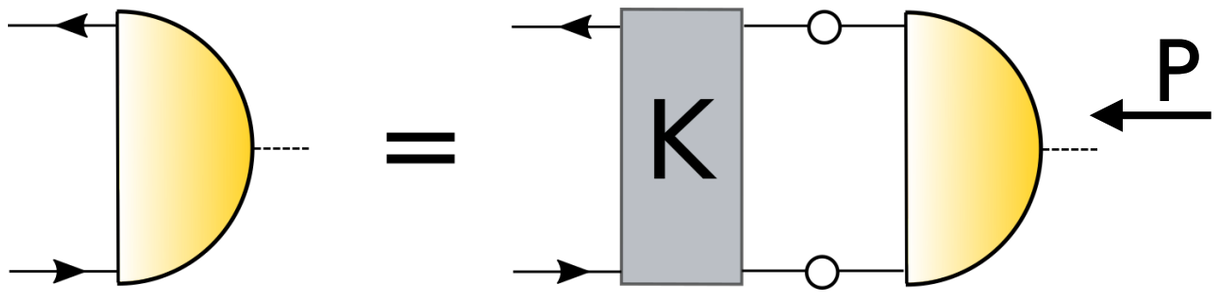
generic situation



# The complex $P^2$-plane

$\lambda(P^2) \stackrel{!}{=} 1$



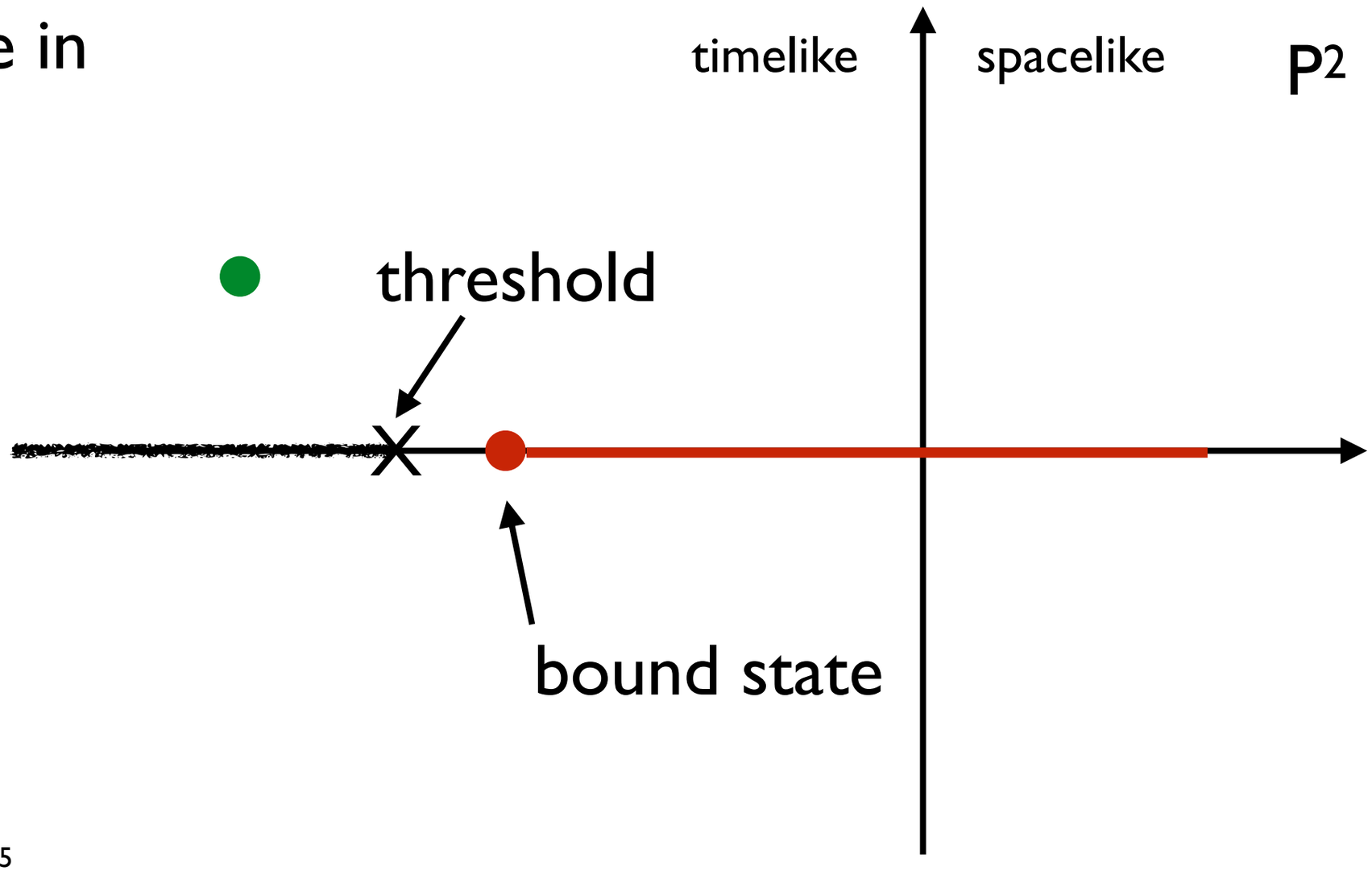

$\lambda(P^2) \times \mathrm{BSA} = \mathrm{kernel} \times \mathrm{BSA}$

---

extrapolation to pole in 2nd Riemann sheet

$\rho \to \pi\pi$

$\sigma \to \pi\pi$





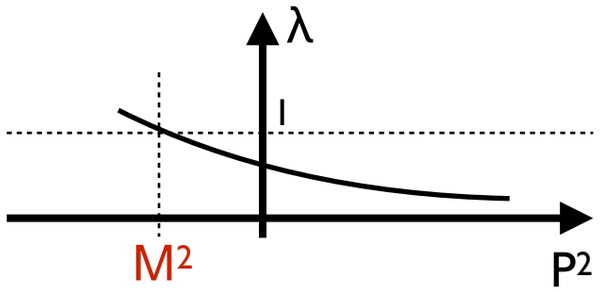
Williams, PLB 798 (2019) 134943, [arXiv:1804.11161]

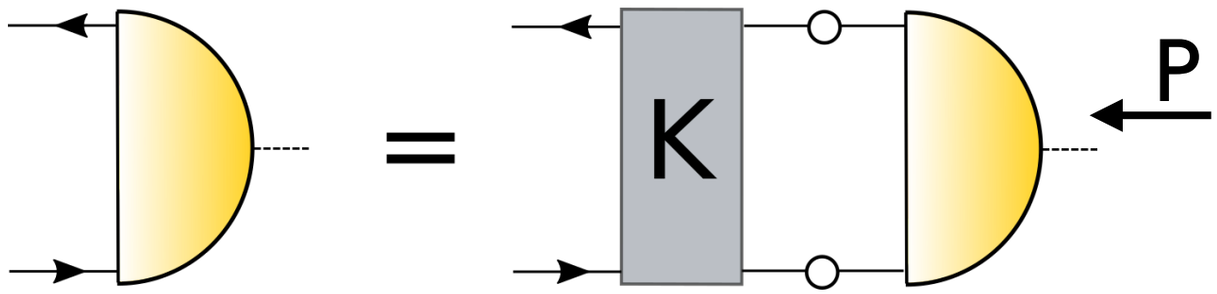
Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014, arXiv:2007.06495.

Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

# The complex $P^2$-plane

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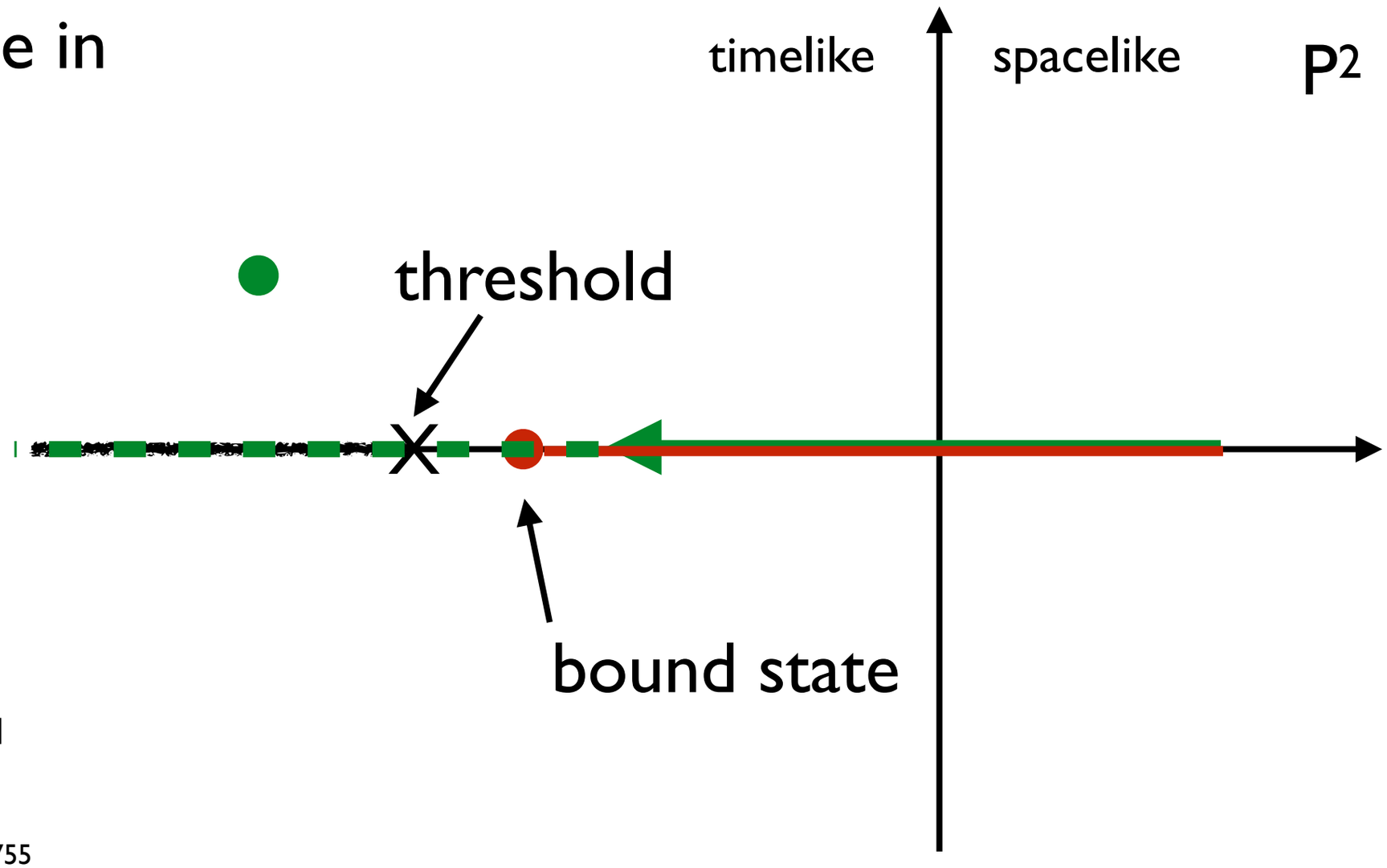

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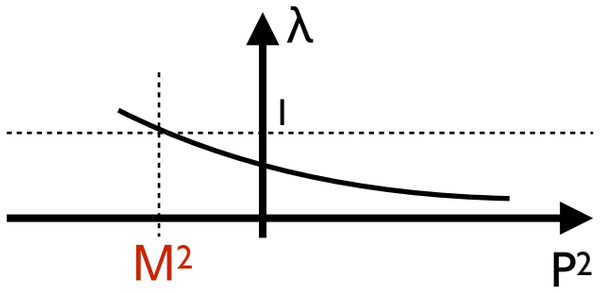
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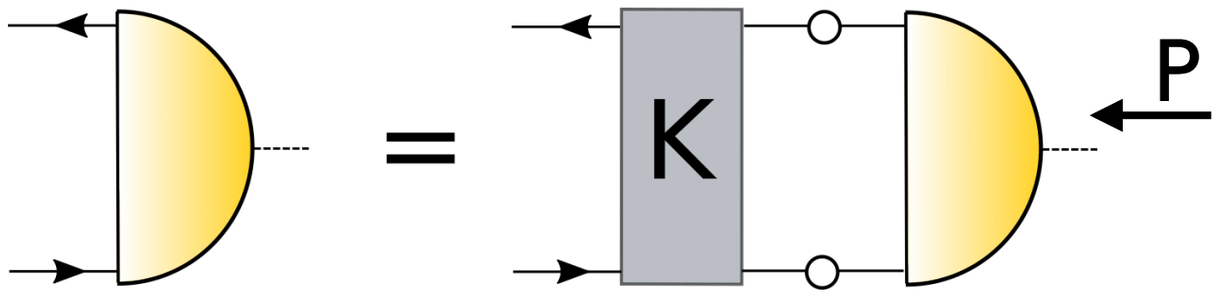
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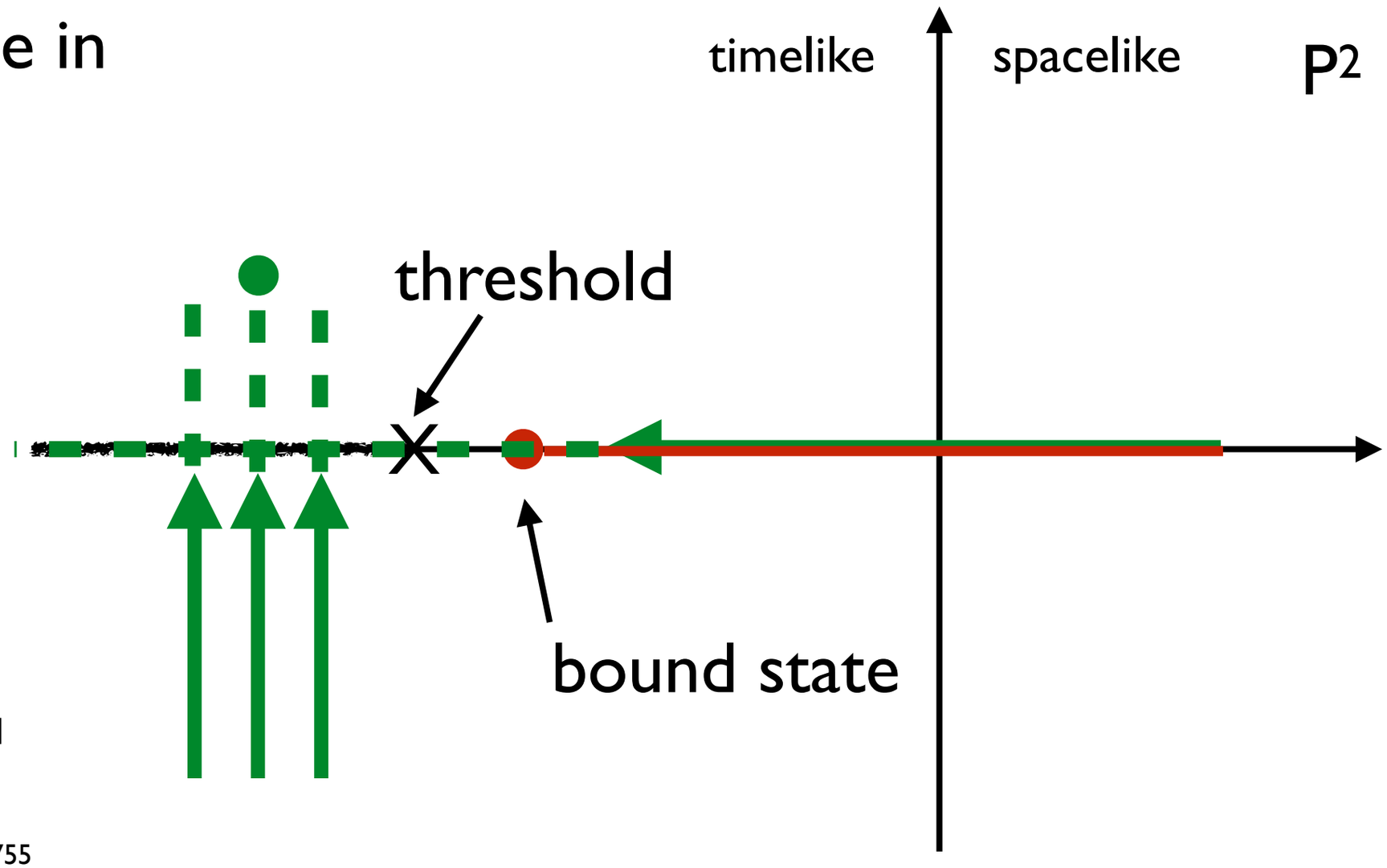

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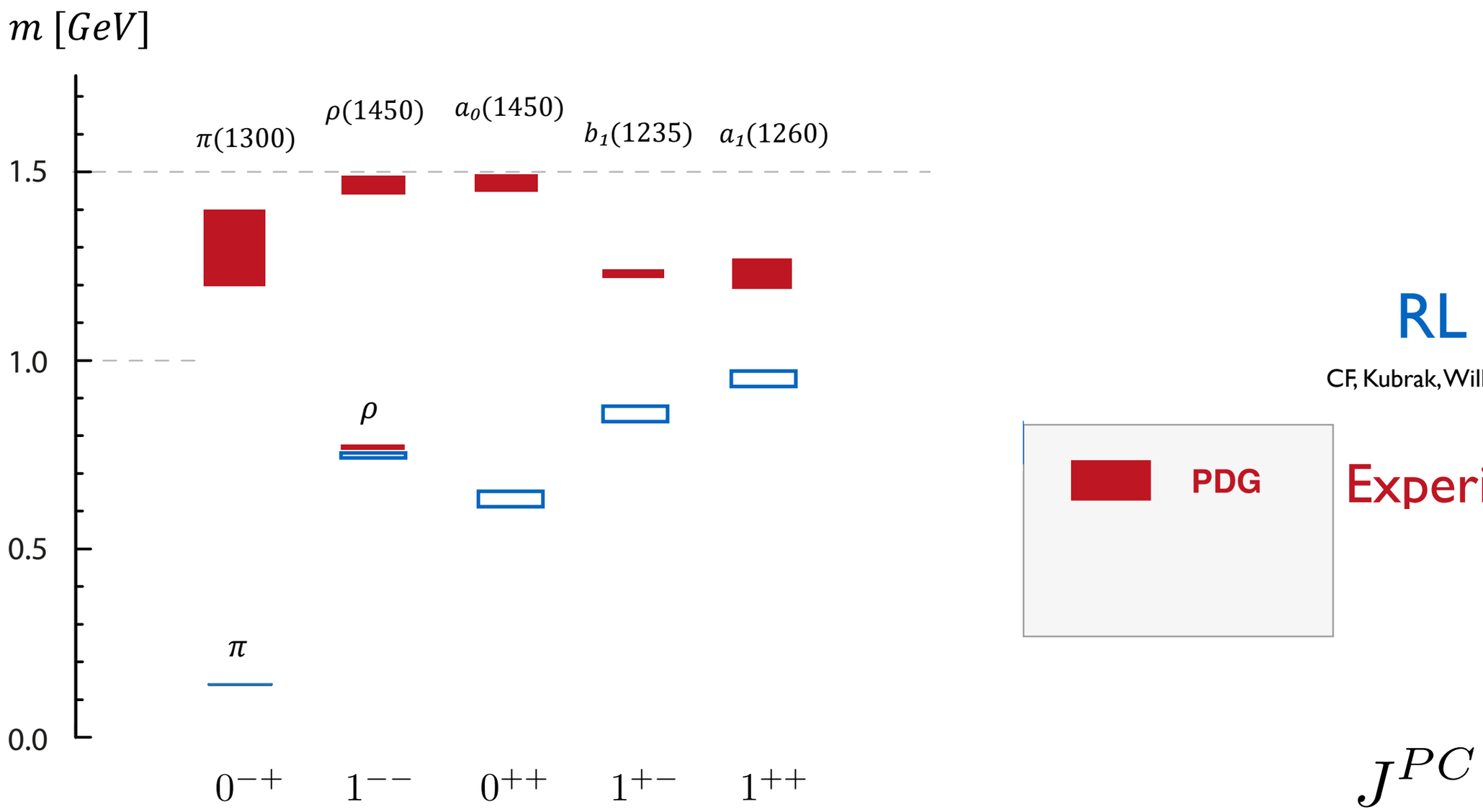


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Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

# Light meson spectrum - RL vs full 3PI-calculation



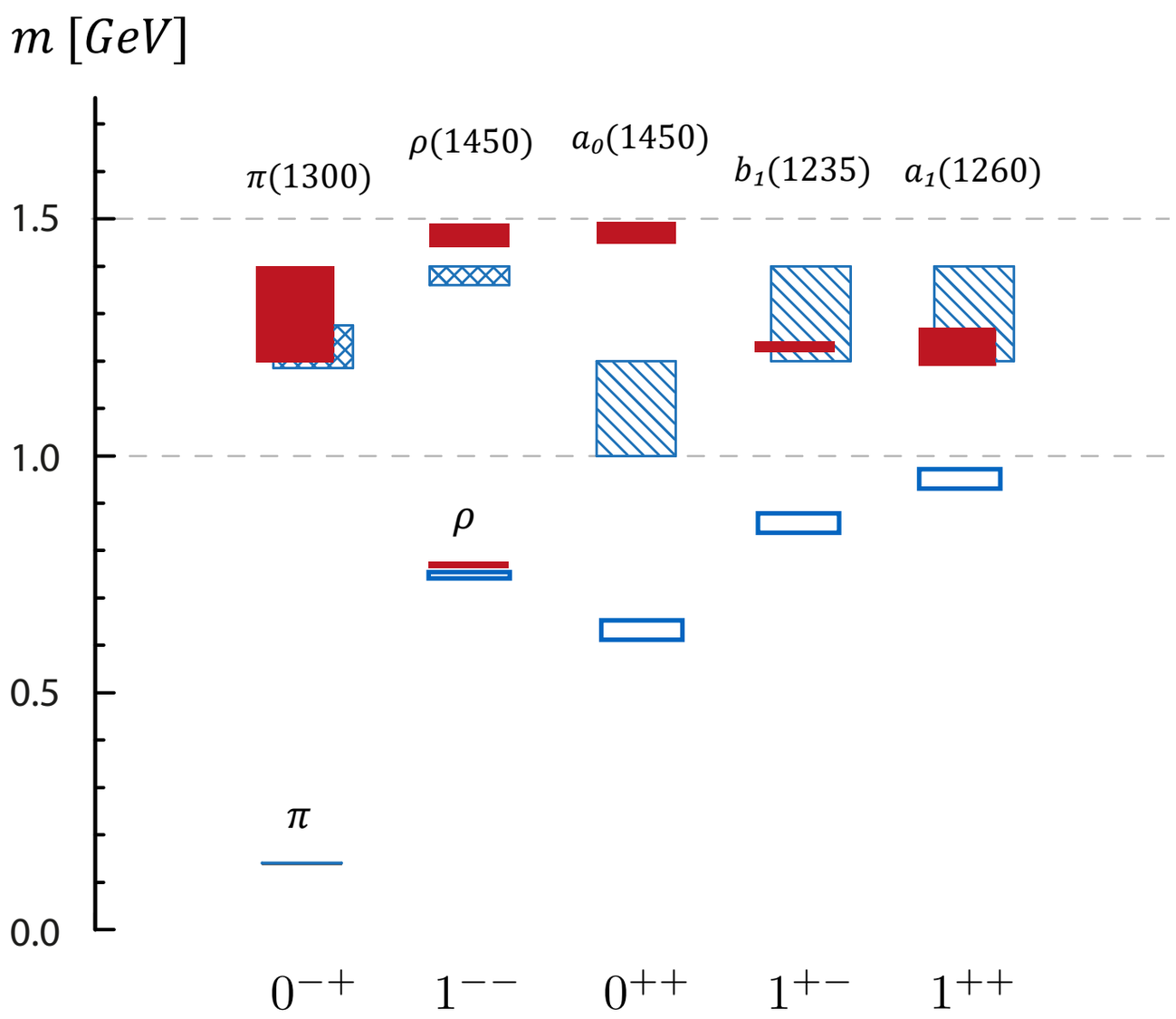

RL

CF, Kubrak, Williams, EPJA (2014) 126

PDG

Experiment

# Light meson spectrum - RL vs full 3PI-calculation




RL

CF, Kubrak, Williams, EPJA (2014) 126

PDG
3PI-3L
2PI-3L

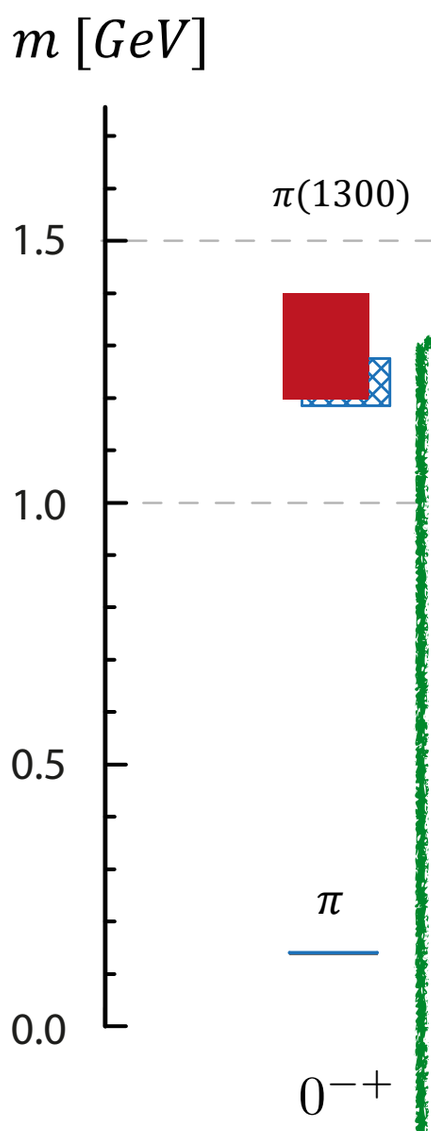
Experiment
our results

Williams, CF, Heupel, PRD93 (2016) 034026

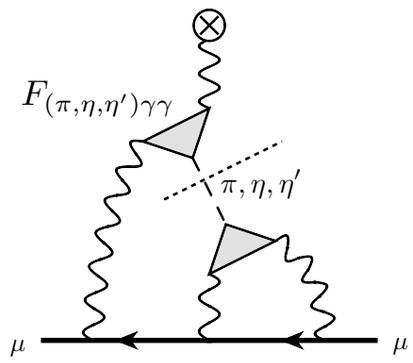
$J^{PC}$

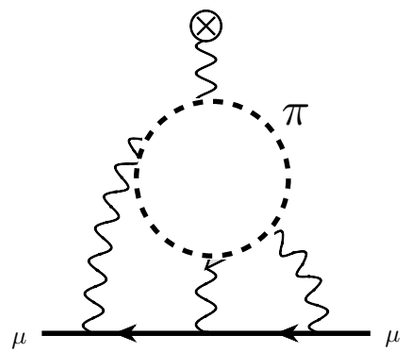
- good agreement with experiment in most channels
- special channels:
  pseudoscalar $0^{-+}$ : (pseudo-) Goldstone bosons
  scalar $0^{++}$ : complicated channel…

# Light meson spectrum - RL vs full 3PI-calculation




- good agreem
- special chann
  pseudoscalar
  scalar




T. Aoyama, et al., Phys. Rept. 887 (2020), 1-166
R. Aliberti, et al. Phys. Rept. 1143 (2025), 1-158

(2014) 126

) 034026

| | DSE | Dispersive |
|---|---|---|
| $a_\mu^{\pi-pole}$ | $62.6(1.3)$ | $63.0^{+2.7}_{-2.1}$ |
| $a_\mu^{\eta-pole}$ | $15.8(1.1)$ | $14.7(9)$ |
| $a_\mu^{\eta'-pole}$ | $13.3(8)$ | $13.5(7)$ |
| $a_\mu^{\pi-box}$ | $-15.7\,(2)(3)$ | $-15.9(2)$ |
| $a_\mu^{K-box}$ | $-0.48\,(2)(4)$ | $-0.48(1)$ |

# Diquarks

| | $J^P$ | color x flavour | Mass [MeV] |
|---|---|---|---|
| The good: | $0^+$ | $3_C \times 3_F$ | ~800 ✓ RL |
| | | $6_C \times 6_F$ | |

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| The bad: | $1^+$ | $3_C \times 6_F$<br>$6_C \times 3_F$ | ~1000 ✓ RL |

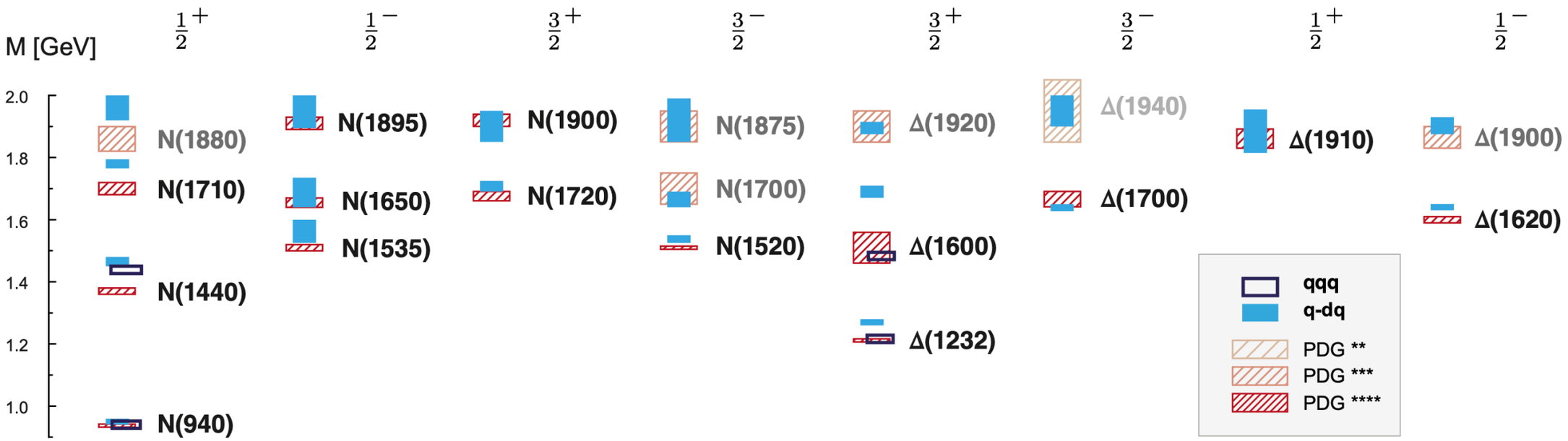
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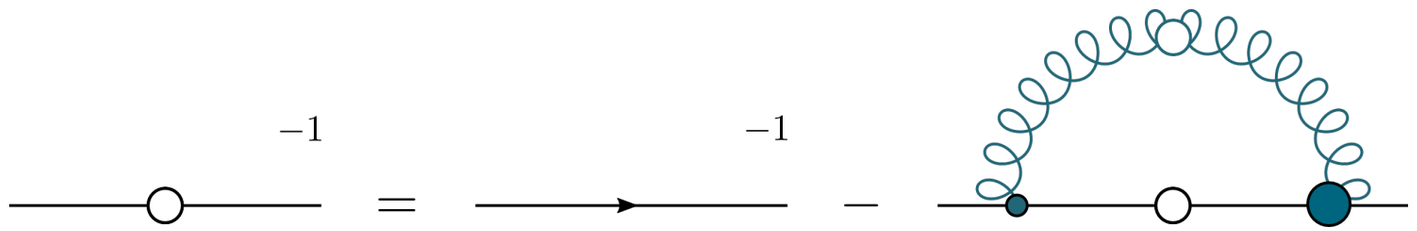
# Successful concept in baryon sector:




Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

# Overview

1. The formalism: gluons, quarks and two-quark states




2. Light quark sector:

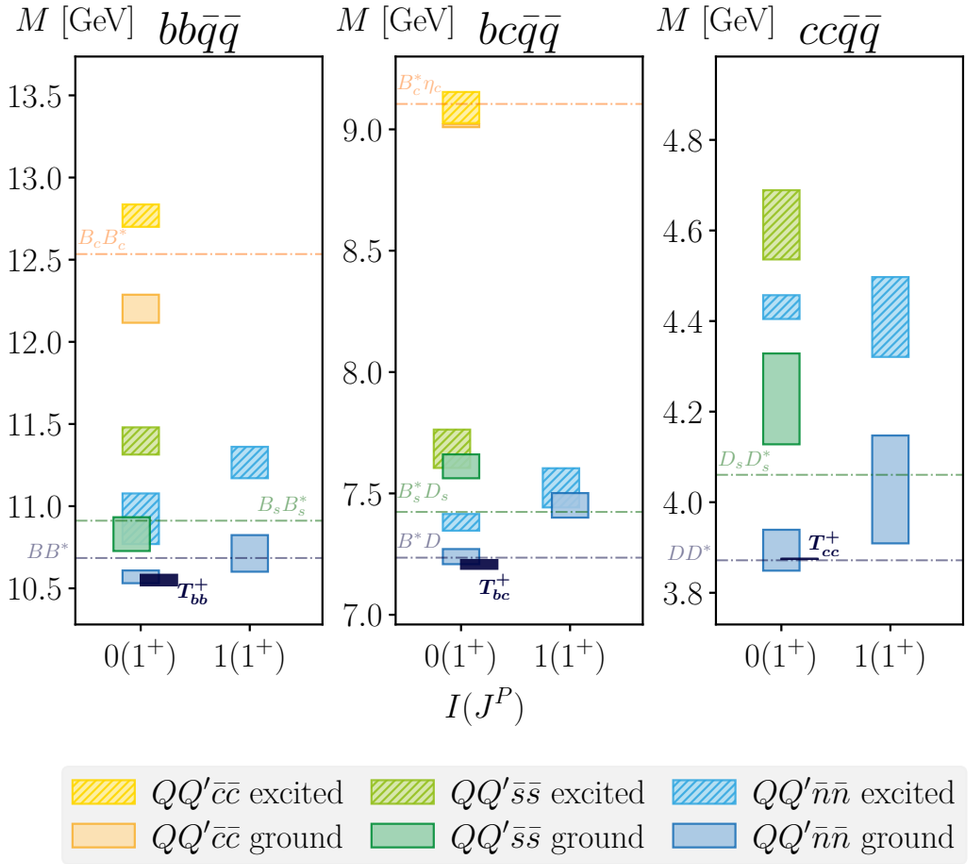
scalar mesons

3. Heavy four-quark states:

open flavour

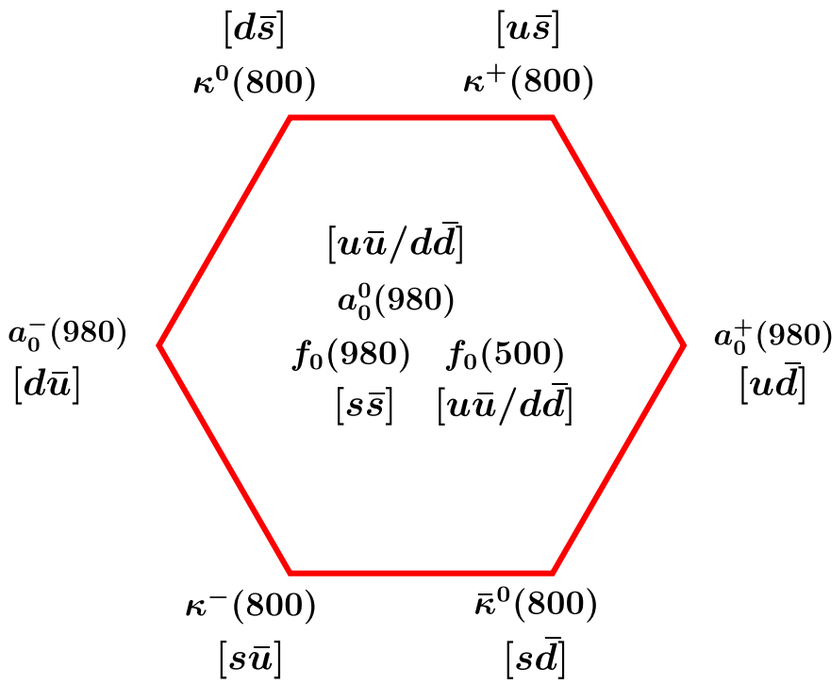
4. Heavy four-quark states:

hidden flavour



# Light mesons with $\bar{q}\bar{q}qq$-content

Light scalar mesons:

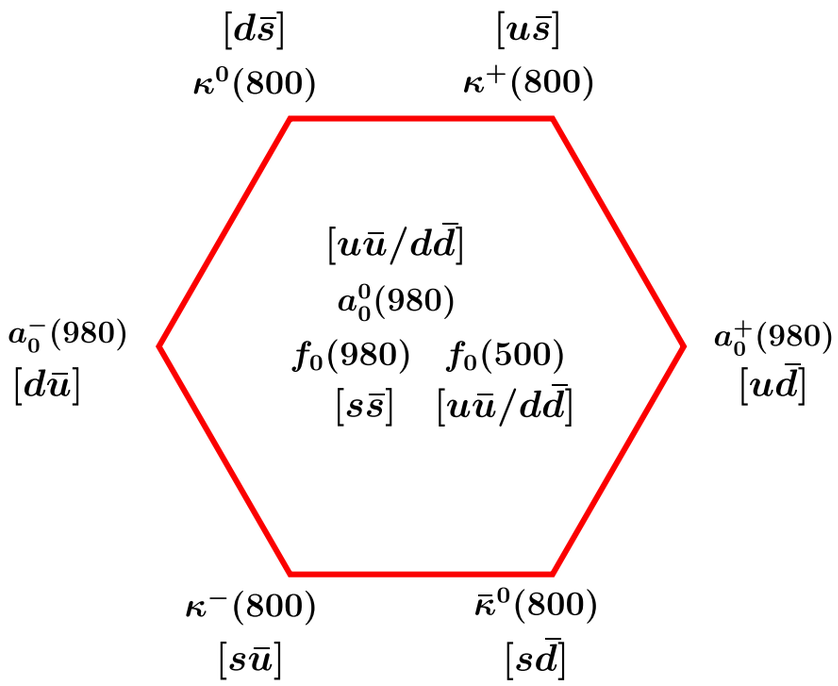



wrong level ordering

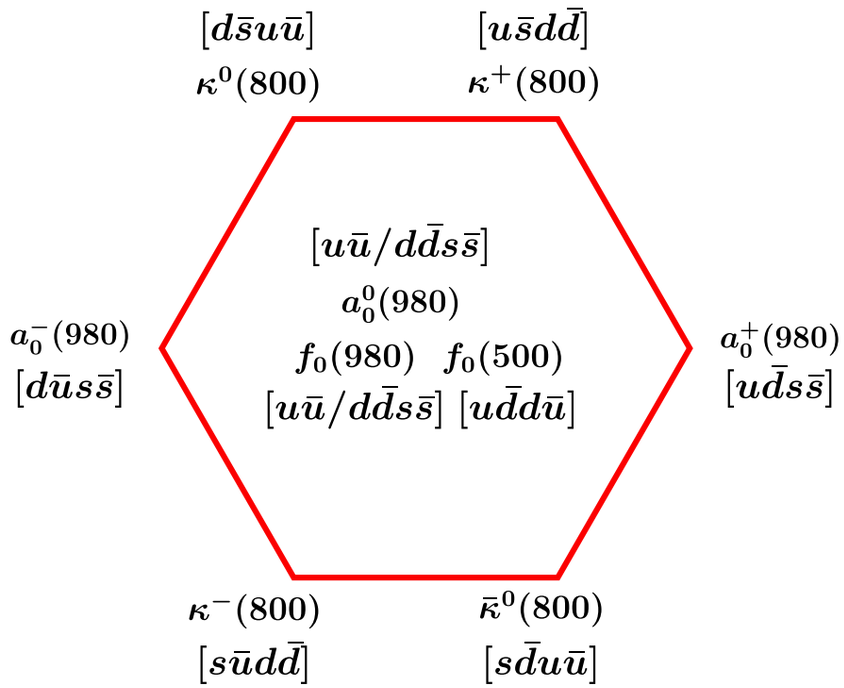
R. L. Jaffe, Phys. Rev. D 15, 267 (1977)

# Light mesons with $\bar{q}\bar{q}qq$-content

## Light scalar mesons:




wrong level ordering



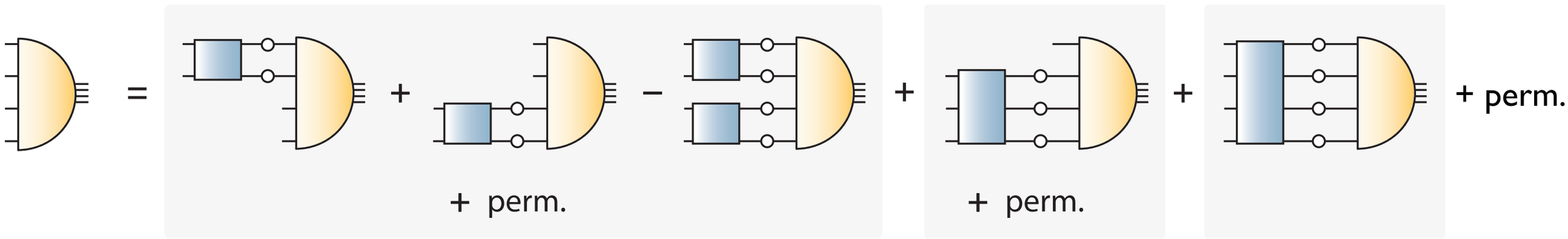

correct level ordering

R. L. Jaffe, Phys. Rev. D 15, 267 (1977)

# Tetraquarks from the four-body equation

## Exact equation:

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287




Two-body interactions

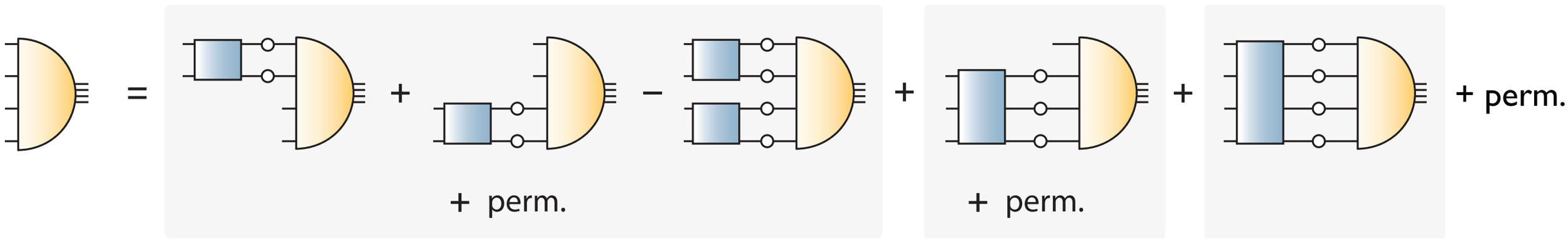
Three- and four-body interactions

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Santowsky, CF, PRD 105 (2022) 4,313

# Tetraquarks from the four-body equation

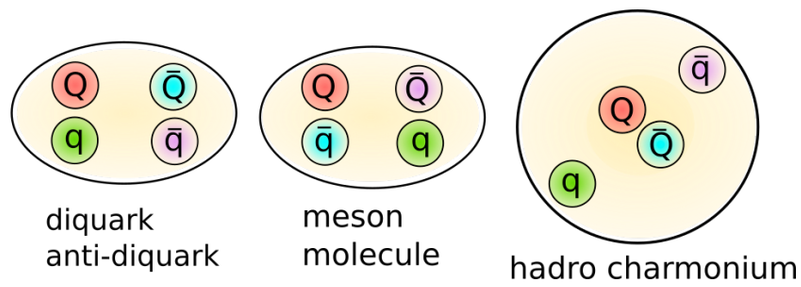
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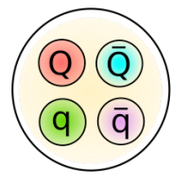
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Two-body interactions

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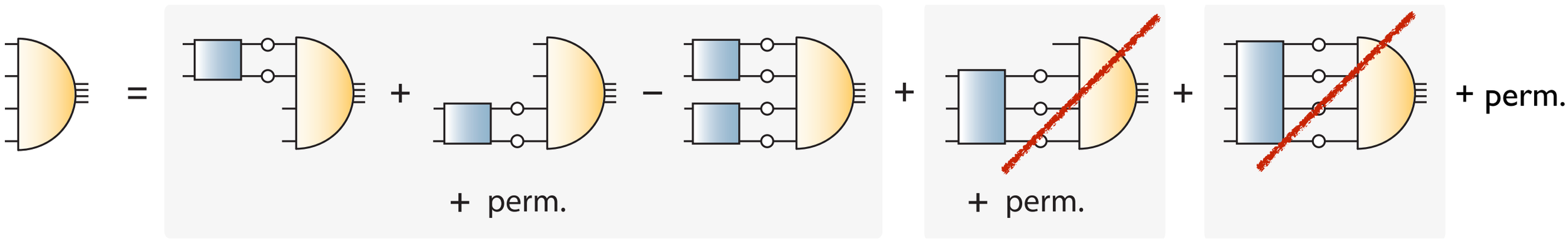

- Two-body interactions: allow for internal clustering
- use rainbow-ladder approximation…

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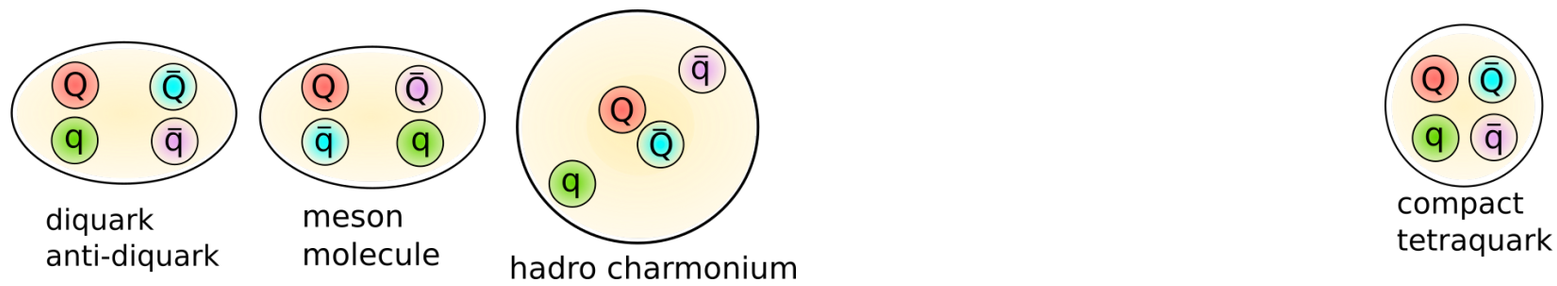
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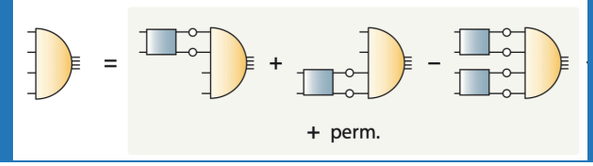
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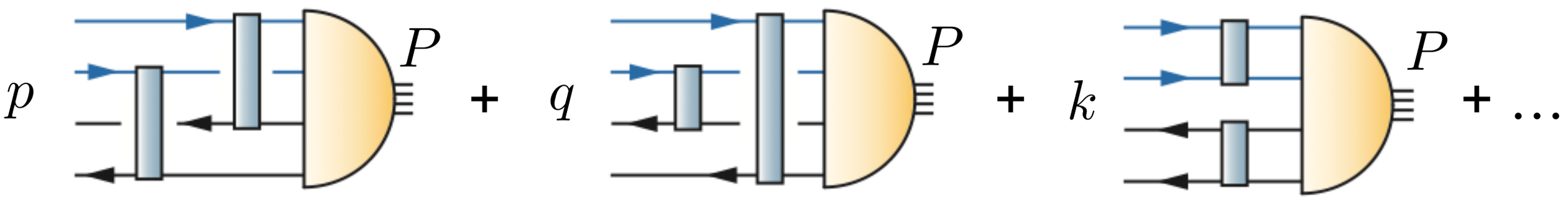



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# Structure of the amplitude




Scalar four-quark state:

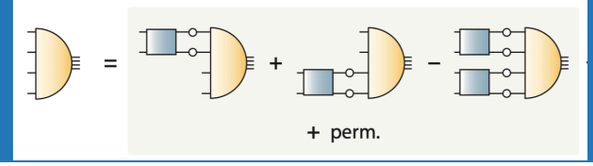
$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\cdots,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

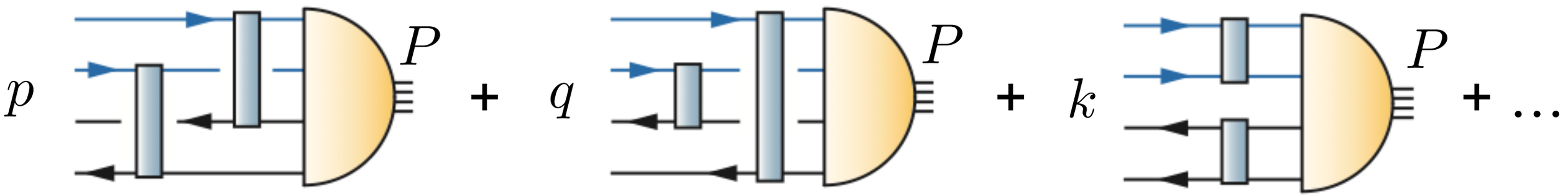
9 Lorentz scalars (built from P,p,q,k)

256 tensor structures (scalar)

$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

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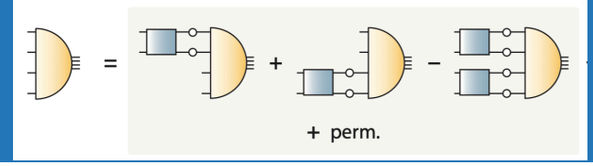
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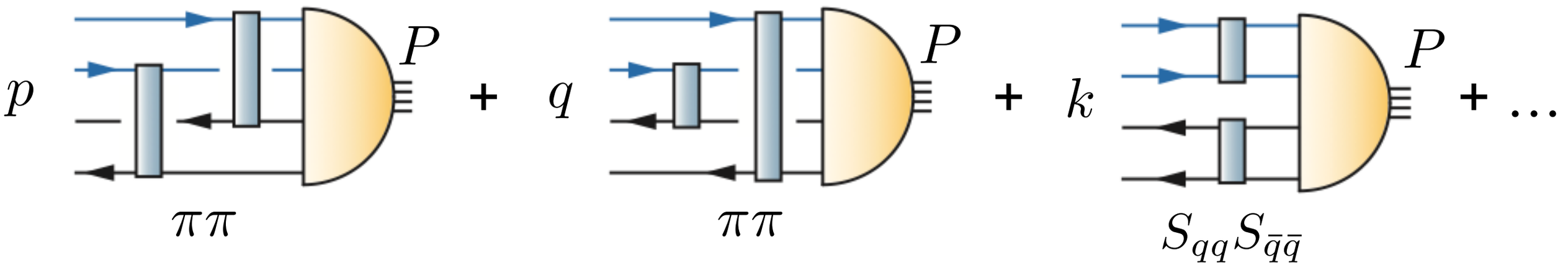
- reduce # tensor structures guided by physics

light scalars:

| | |
|---|---|
| $\pi\pi$ | $1 \otimes 1$ |
| $S_{qq} S_{\bar{q}\bar{q}}$ | $3 \otimes \bar{3}$ |

# Structure of the amplitude




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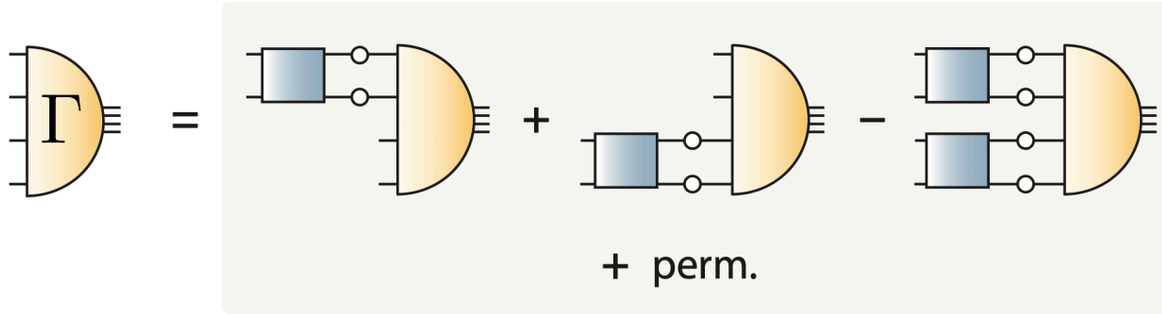
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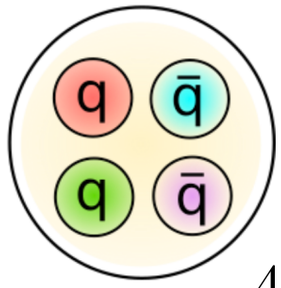
| | |
|---|---|
| $\pi\pi$ | $1 \otimes 1$ |
| $S_{qq}S_{\bar{q}\bar{q}}$ | $3 \otimes \bar{3}$ |

# Bound state vs resonance: scalar four-quark states

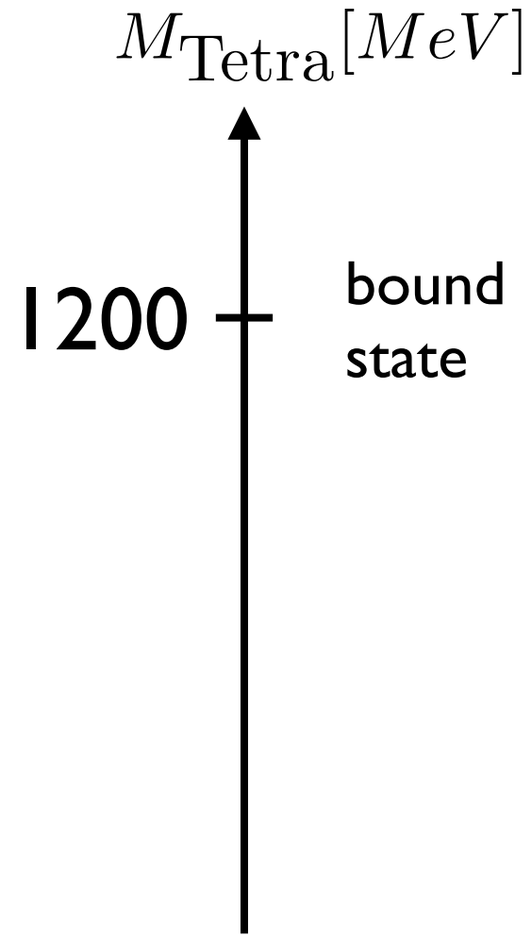
$$\Gamma(P,p,k,q)$$




very crude approx $\Gamma(P,\cancel{p,q,k})$ :

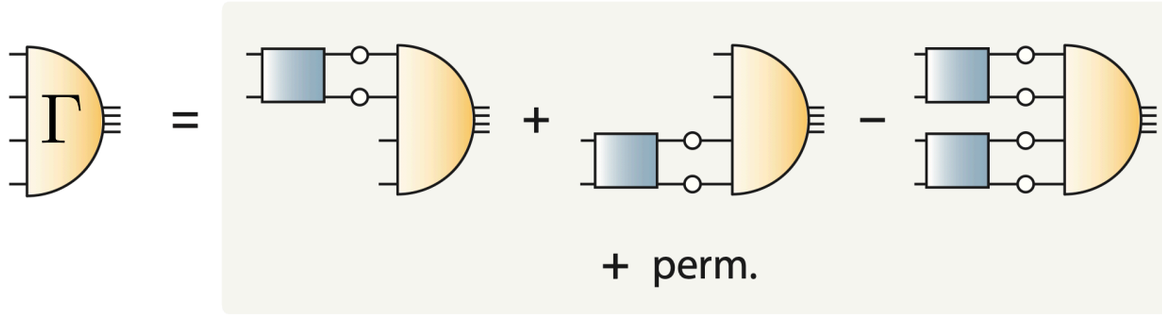


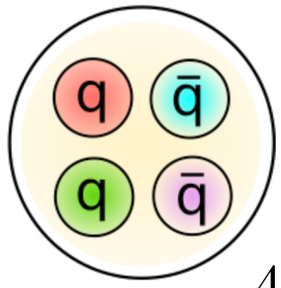

$4 \times 350\,\mathrm{MeV}$



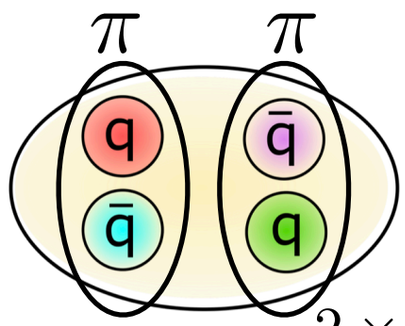

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

# Bound state vs resonance: scalar four-quark states

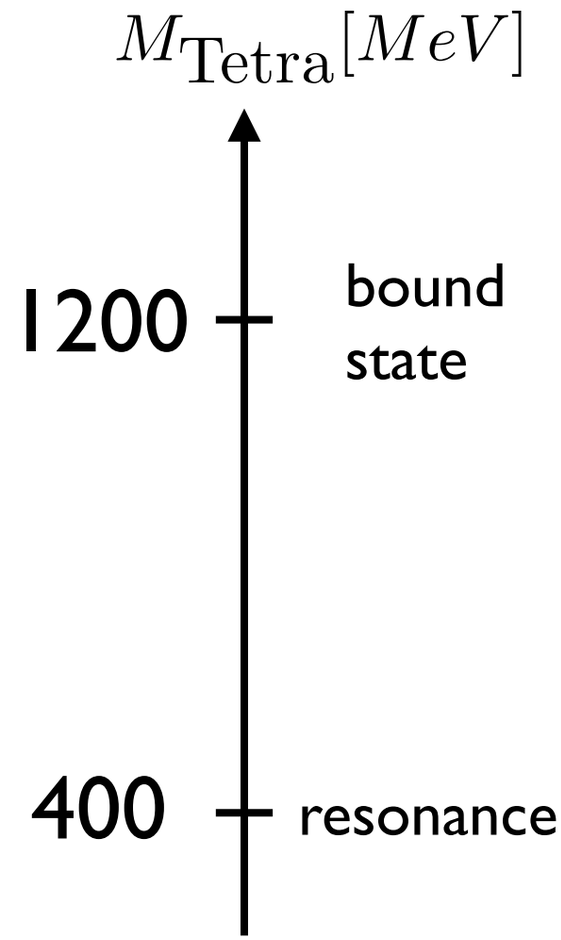
$\Gamma(P,p,k,q)$ 


very crude approx $\Gamma(P,\cancel{p,q,k})$ : 


$4 \times 350\,\mathrm{MeV}$

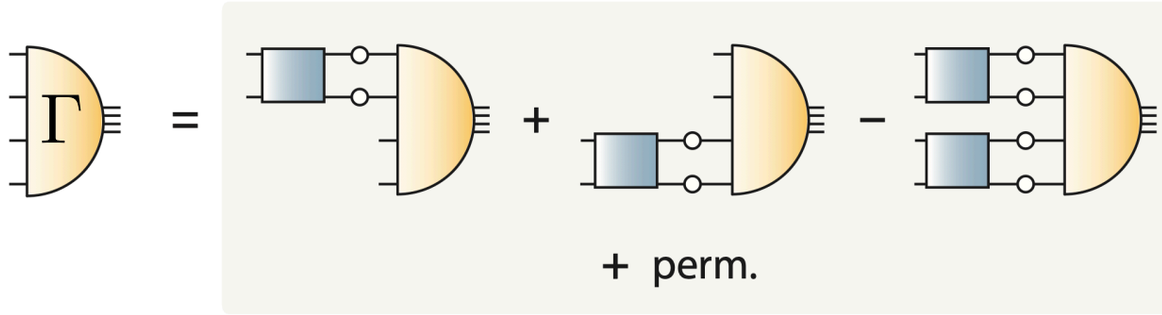
full rel. momenta $\Gamma(P,p,q,k)$ : 

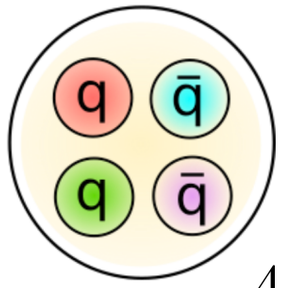

$2 \times 140\,\mathrm{MeV}$



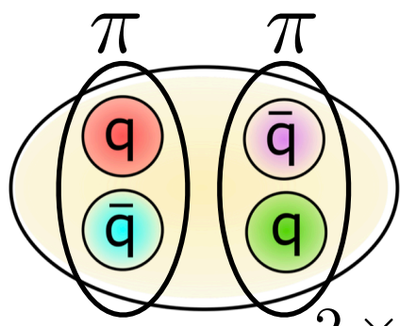

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

# Bound state vs resonance: scalar four-quark states

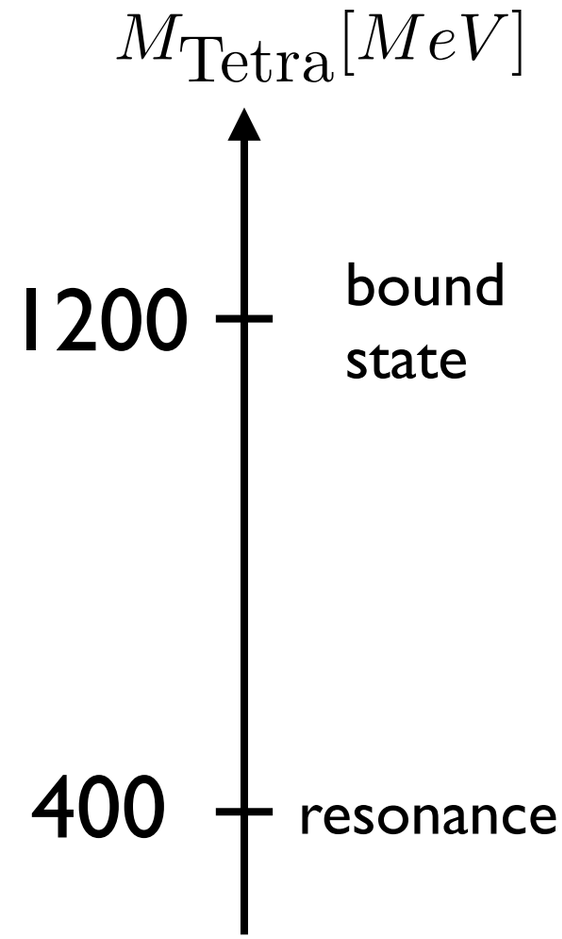
$\Gamma(P,p,k,q)$ 


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$4 \times 350\,\mathrm{MeV}$

full rel. momenta $\Gamma(P,p,q,k)$ : 


$2 \times 140\,\mathrm{MeV}$




⟶ π π dominates (Dq negligible)

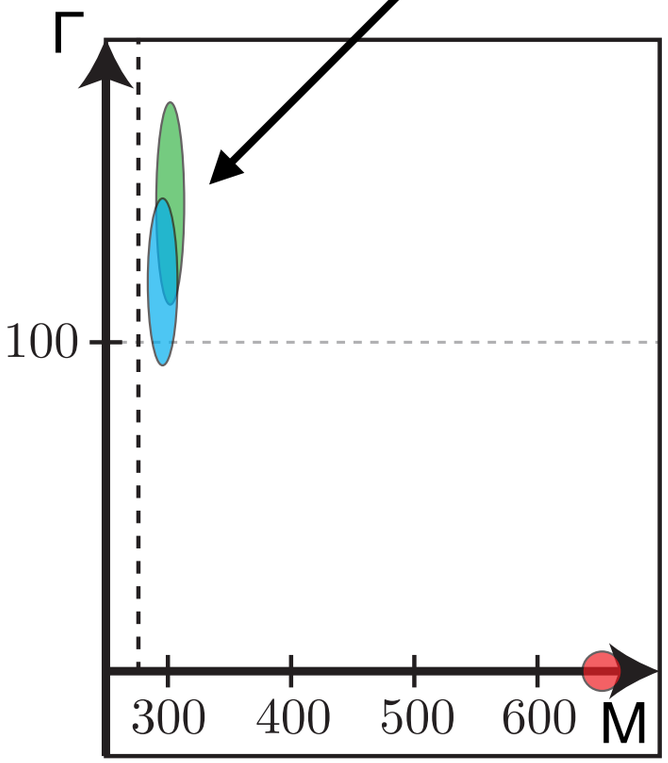
⟶ identify with $f_0(500)$ ('σ-meson')

with strange quarks: $M_{a_0,f_0} \approx 1\,\mathrm{GeV}$

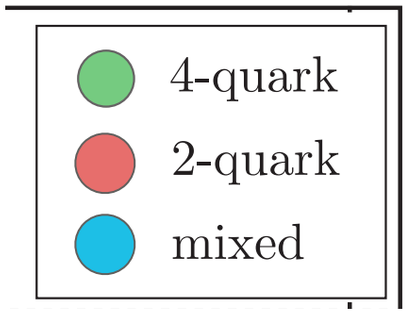
Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

# Mass evolution of four-quark state: 0++

$$f_0(500) : \pi\pi - \text{component dominates!}$$



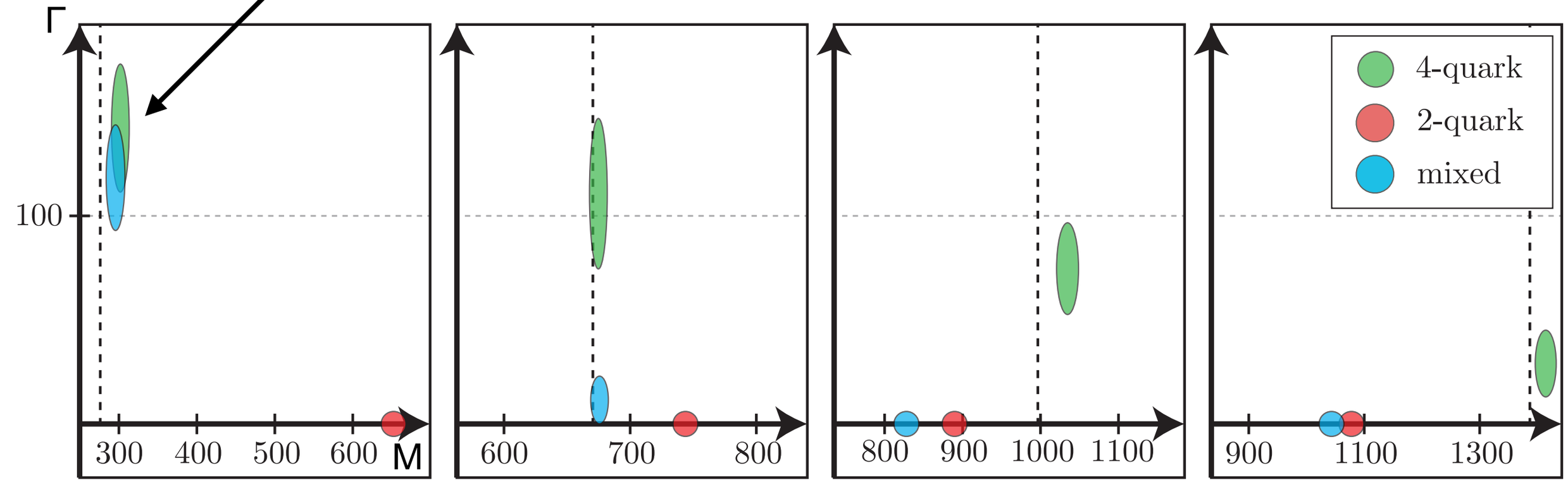

$$m_q = 3.8\,\mathrm{MeV}$$




Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

# Mass evolution of four-quark state: 0++

$f_0(500) : \pi\pi -$ component dominates!

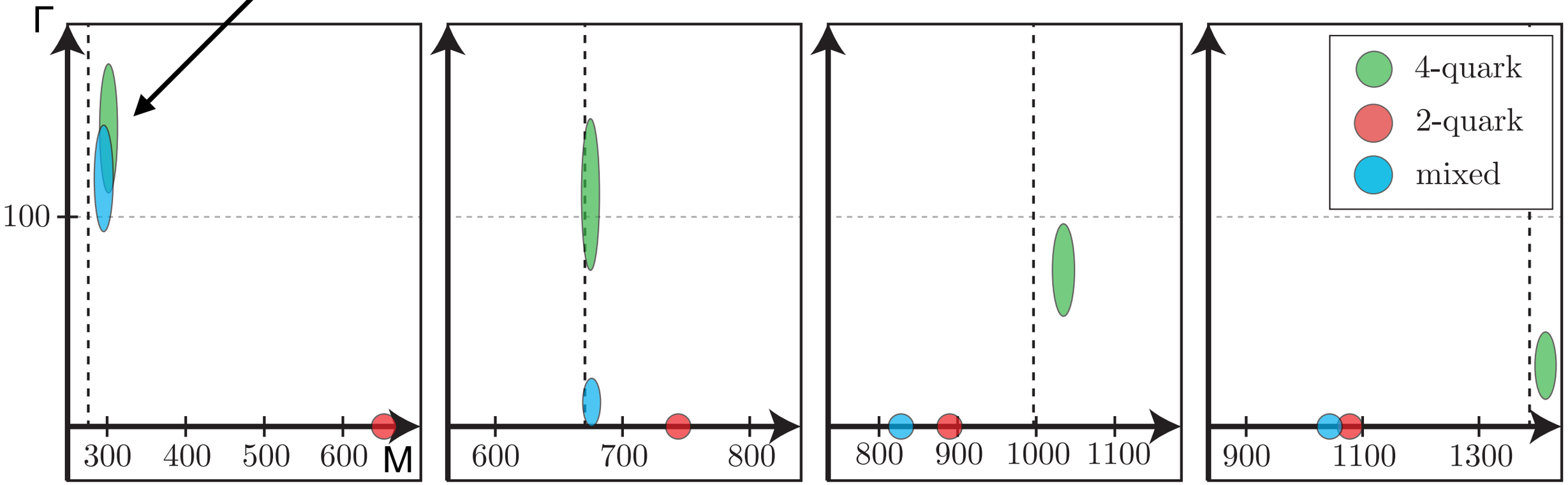



$m_q = 3.8\,\mathrm{MeV}$ $m_q = 20.0\,\mathrm{MeV}$ $m_q = 50.0\,\mathrm{MeV}$ $m_q = 85.5\,\mathrm{MeV}$

Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

# Mass evolution of four-quark state: 0++

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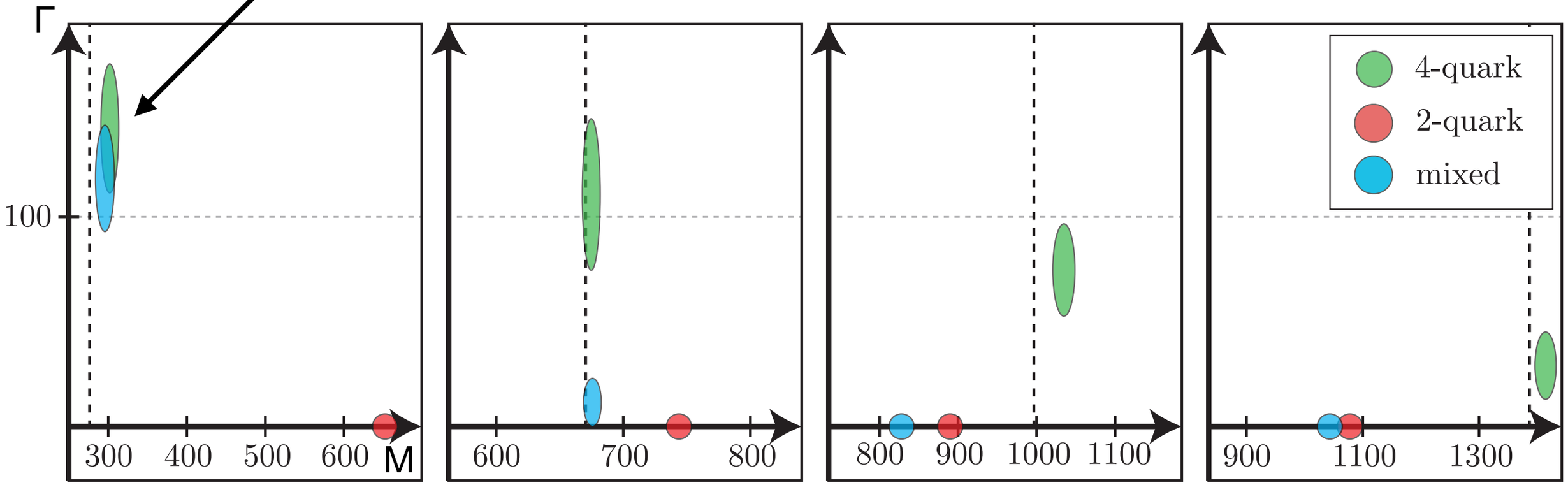

$m_q = 3.8$ MeV $\quad m_q = 20.0$ MeV $\quad m_q = 50.0$ MeV $\quad m_q = 85.5$ MeV

Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

- mixed state becomes qq-dominated for large $m_q$
- dynamical decision !

# Mass evolution of four-quark state: 0++

$f_0(500) : \pi\pi -$ component dominates!




$m_q = 3.8$ MeV  $m_q = 20.0$ MeV  $m_q = 50.0$ MeV  $m_q = 85.5$ MeV

Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

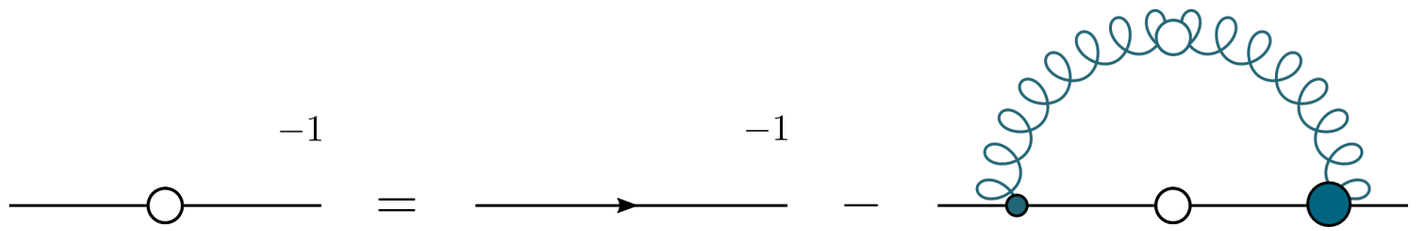
- mixed state becomes qq-dominated for large $m_q$
- dynamical decision !

⟶ consequences for $c\bar{c}q\bar{q}$, $c\bar{c}s\bar{s}$, $b\bar{b}q\bar{q}$, $b\bar{b}s\bar{s}$, $b\bar{b}c\bar{c}$ ?

work to be done!

# Overview

1. The formalism: gluons, quarks and two-quark states




2. Light quark sector:

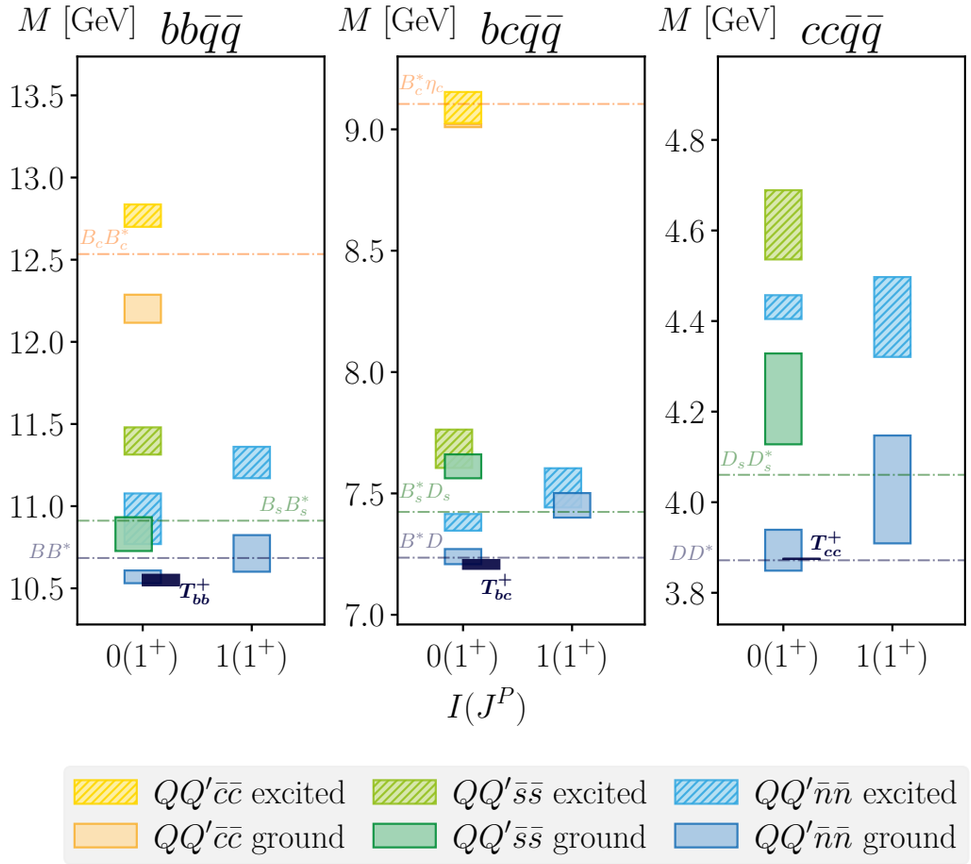
scalar mesons

3. Heavy four-quark states:

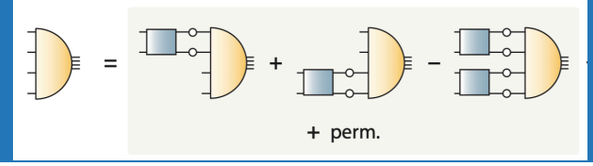
open flavour

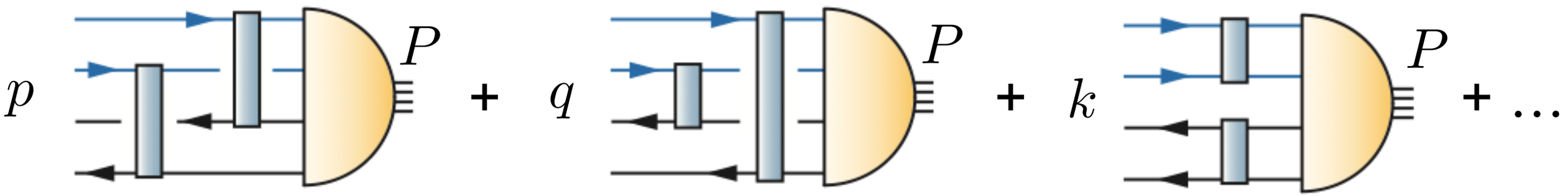
4. Heavy four-quark states:

hidden flavour



# Structure of the amplitude




Vector four-quark state:

$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\cdots,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

9 Lorentz scalars (built from P,p,q,k)

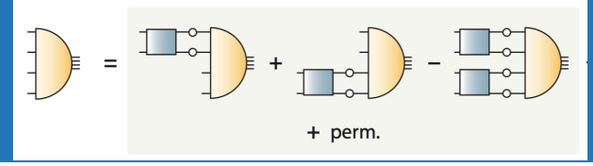
768 tensor structures (vector)

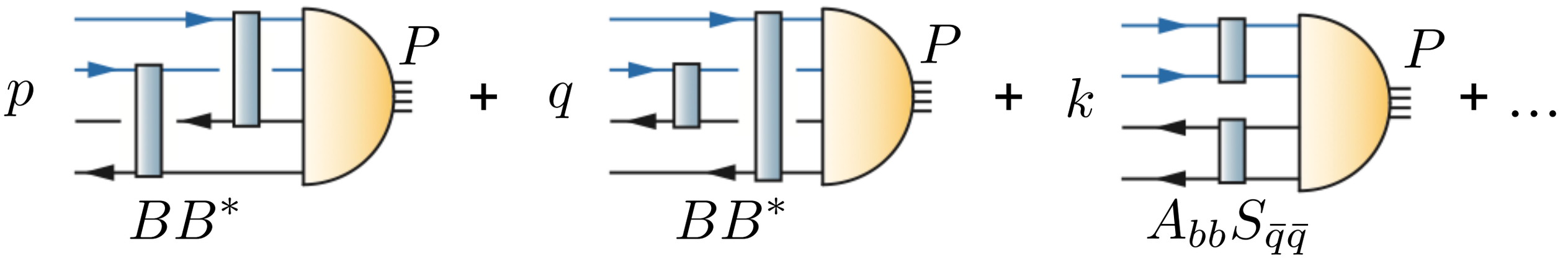
$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

- reduce # tensor structures guided by physics

$n \in \{u, d\}$

# Structure of the amplitude




Vector four-quark state:

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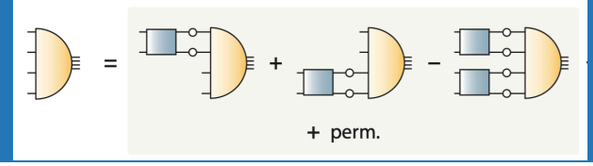
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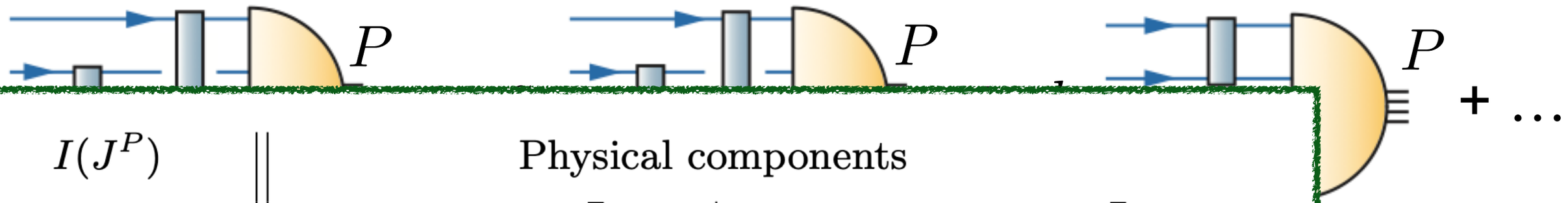
- reduce # tensor structures guided by physics

$0(1^+)\ bb\bar{n}\bar{n}$ : $n \in \{u,d\}$

| | |
|---|---|
| $BB^*$ | $1 \otimes 1,\ 8 \otimes 8$ |
| $A_{bb}S_{\bar{q}\bar{q}}$ | $\bar{3} \otimes 3,\ 6 \otimes \bar{6}$ |

# Structure of the amplitude




$p$

$V$

$flavor$

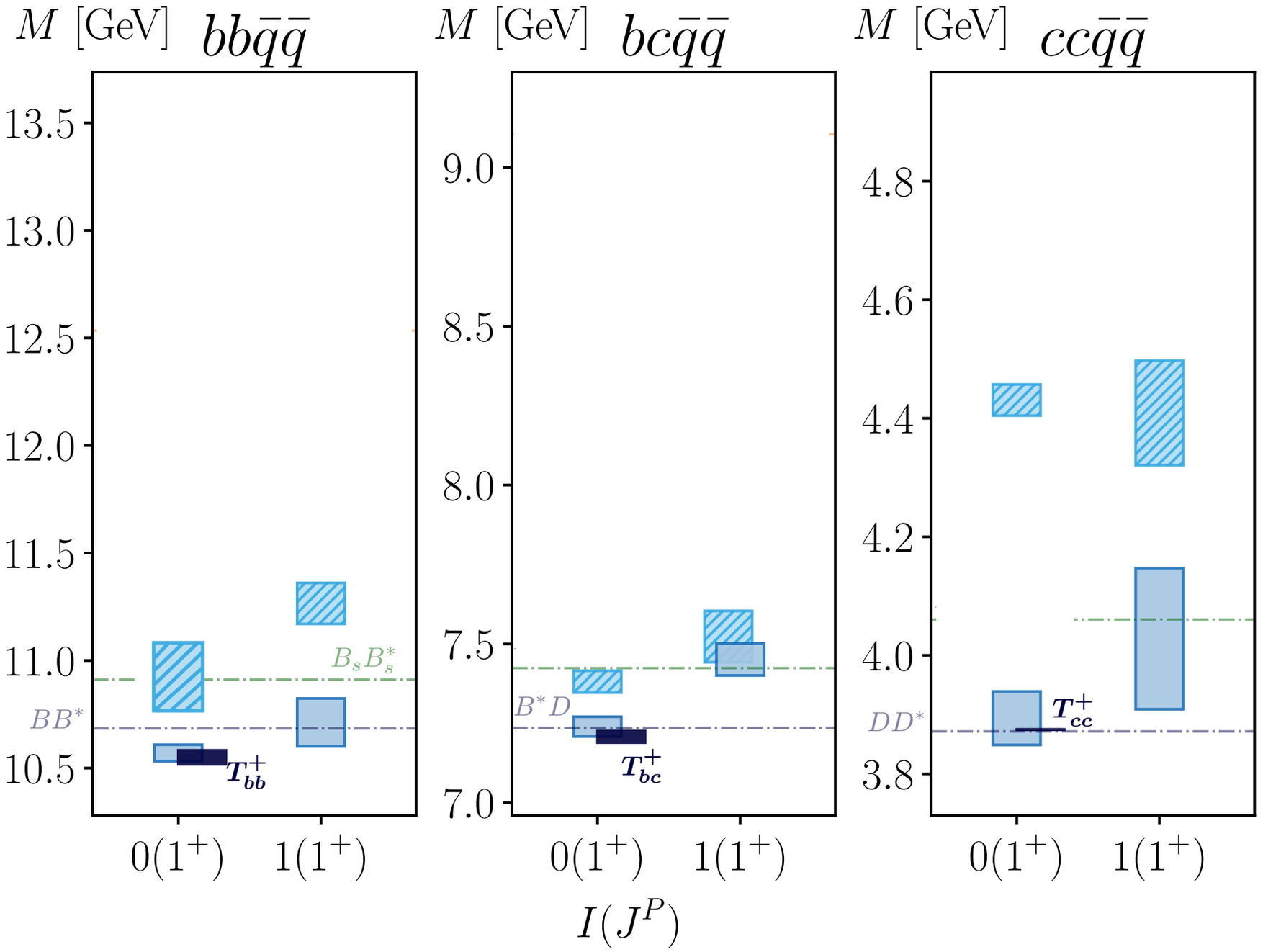
$\otimes\ \bar{6}$ or

$\otimes\ 8$

| $I(J^P)$ | Physical components | | | | | |
|---|---|---|---|---|---|---|
| | $\mathbf{1}\otimes\mathbf{1}$ | | $\bar{\mathbf{3}}\otimes\mathbf{3}$ | $\mathbf{8}\otimes\mathbf{8}$ | | $\mathbf{6}\otimes\bar{\mathbf{6}}$ |
| | $f_0$ | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ |
| $0(1^+)\ bb\bar{n}\bar{n}$ | $BB^*$ | $B^*B^*$ | $A_{bb}S$ | $BB^*$ | $B^*B^*$ | $S_{bb}A$ |
| $bc\bar{n}\bar{n}$ | $BD^*$ | $B^*D$ | $A_{bc}S$ | $BD^*$ | $B^*D$ | $S_{bc}A$ |
| $cc\bar{n}\bar{n}$ | $DD^*$ | $D^*D^*$ | $A_{cc}S$ | $DD^*$ | $D^*D^*$ | $S_{cc}A$ |
| $bb\bar{s}\bar{s}$ | $B_sB_s^*$ | – | $A_{bb}A_{ss}$ | $B_sB_s^*$ | – | – |
| $bc\bar{s}\bar{s}$ | $B_sD_s^*$ | $B_s^*D_s$ | $S_{bc}A_{ss}$ | $B_sD_s^*$ | $B_s^*D_s^*$ | $A_{bc}S_{ss}$ |
| $cc\bar{s}\bar{s}$ | $D_sD_s^*$ | – | $A_{cc}A_{ss}$ | $D_sD_s^*$ | – | – |
| $1(1^+)\ bb\bar{q}\bar{q}$ | $BB^*$ | – | $A_{bb}A$ | $BB^*$ | – | – |
| $bc\bar{q}\bar{q}$ | $BD^*$ | $B^*D$ | $S_{bc}A$ | $BD^*$ | $B^*D^*$ | $A_{bc}S$ |
| $cc\bar{q}\bar{q}$ | $DD^*$ | – | $A_{cc}A$ | $DD^*$ | – | – |

$n \in \{u, d\}$

# bb, bc and cc four-quark-states

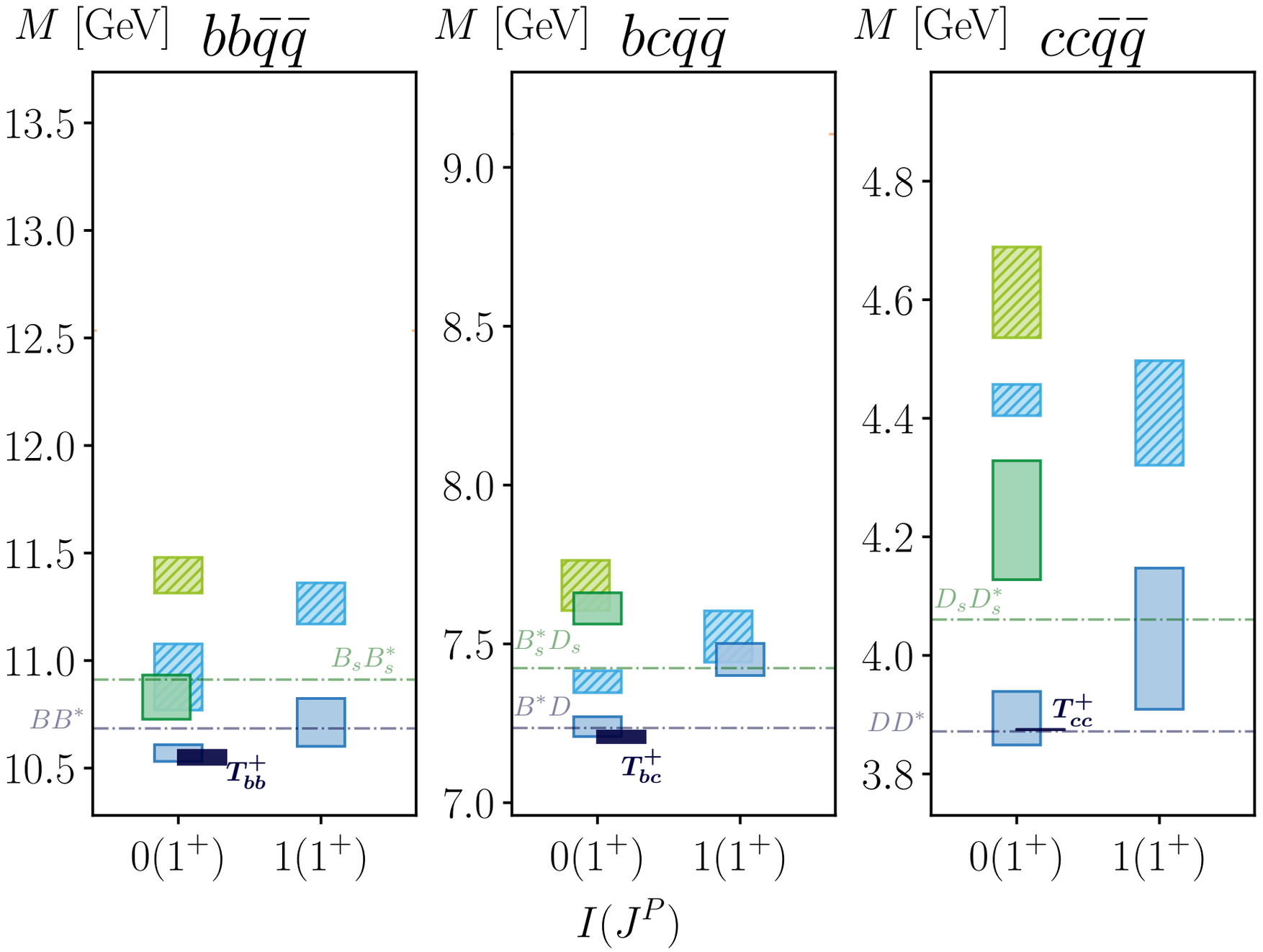



- I=0: the heavier the bound
- I matters
- bc: lower radial excitations

Hoffer, Eichmann, CF, PRD 111 (2025) 5 054028
Hoffer, Eichmann, CF, in preparation

Lattice:
see e.g. Francis, PPNP 140 104143 (2025)

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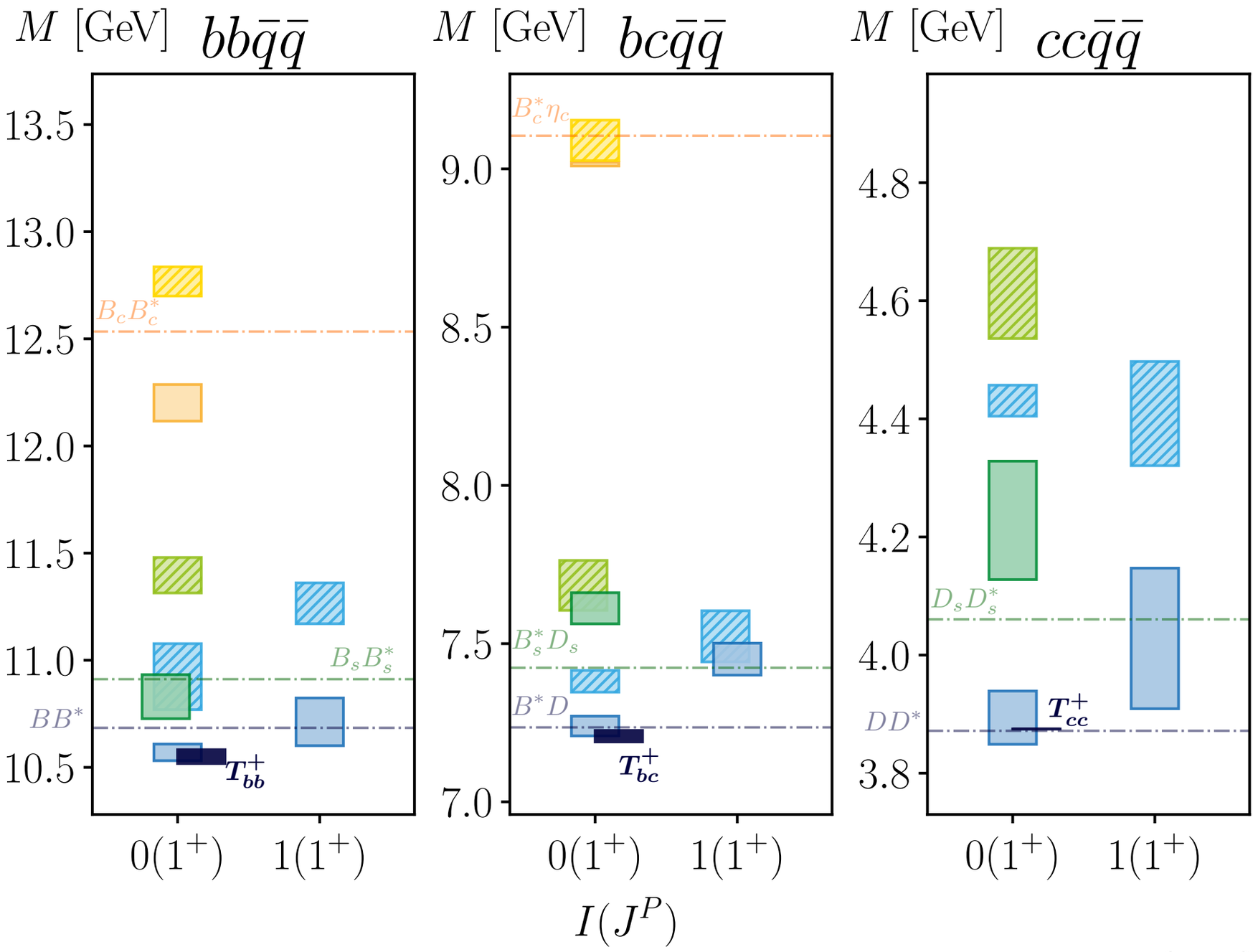



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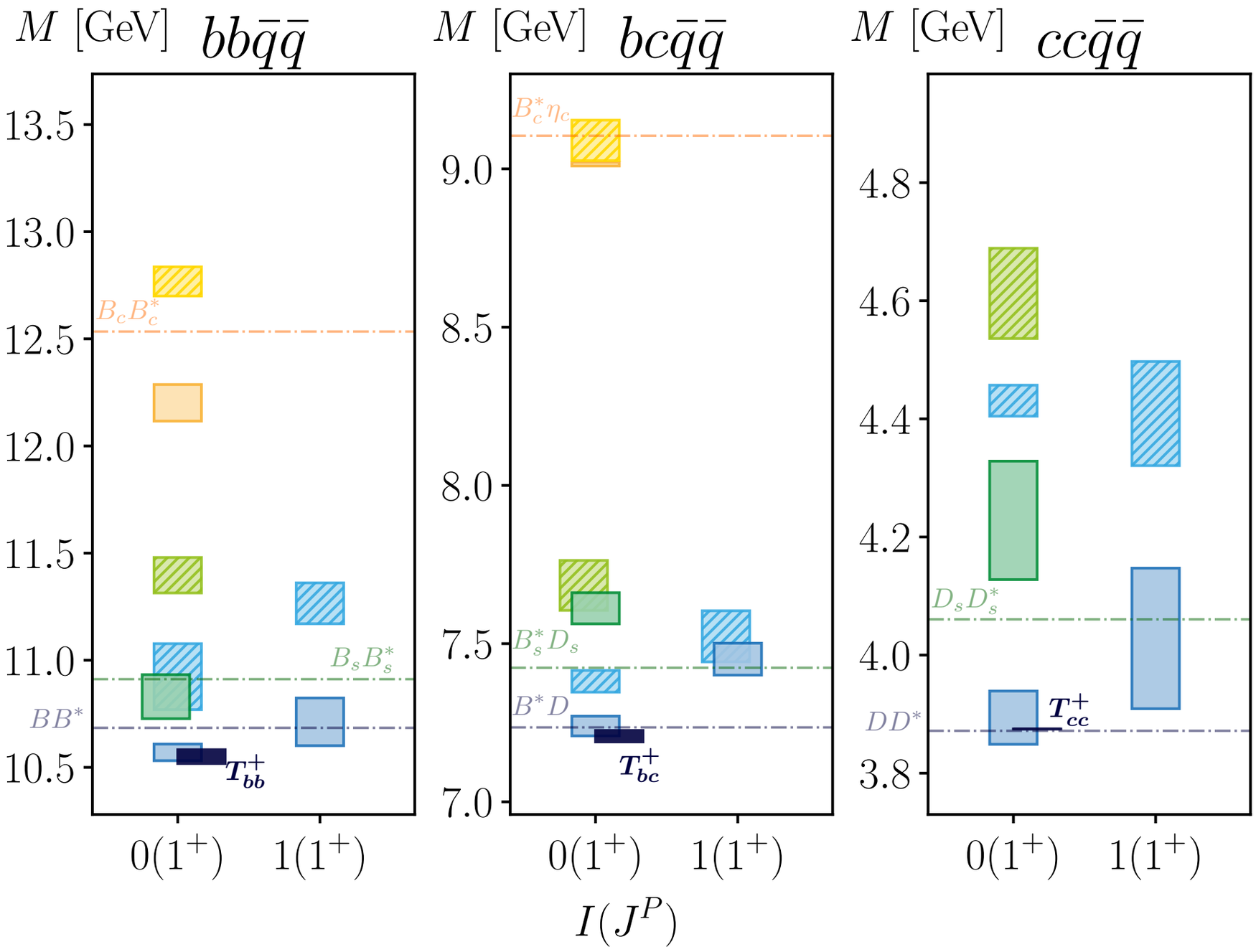



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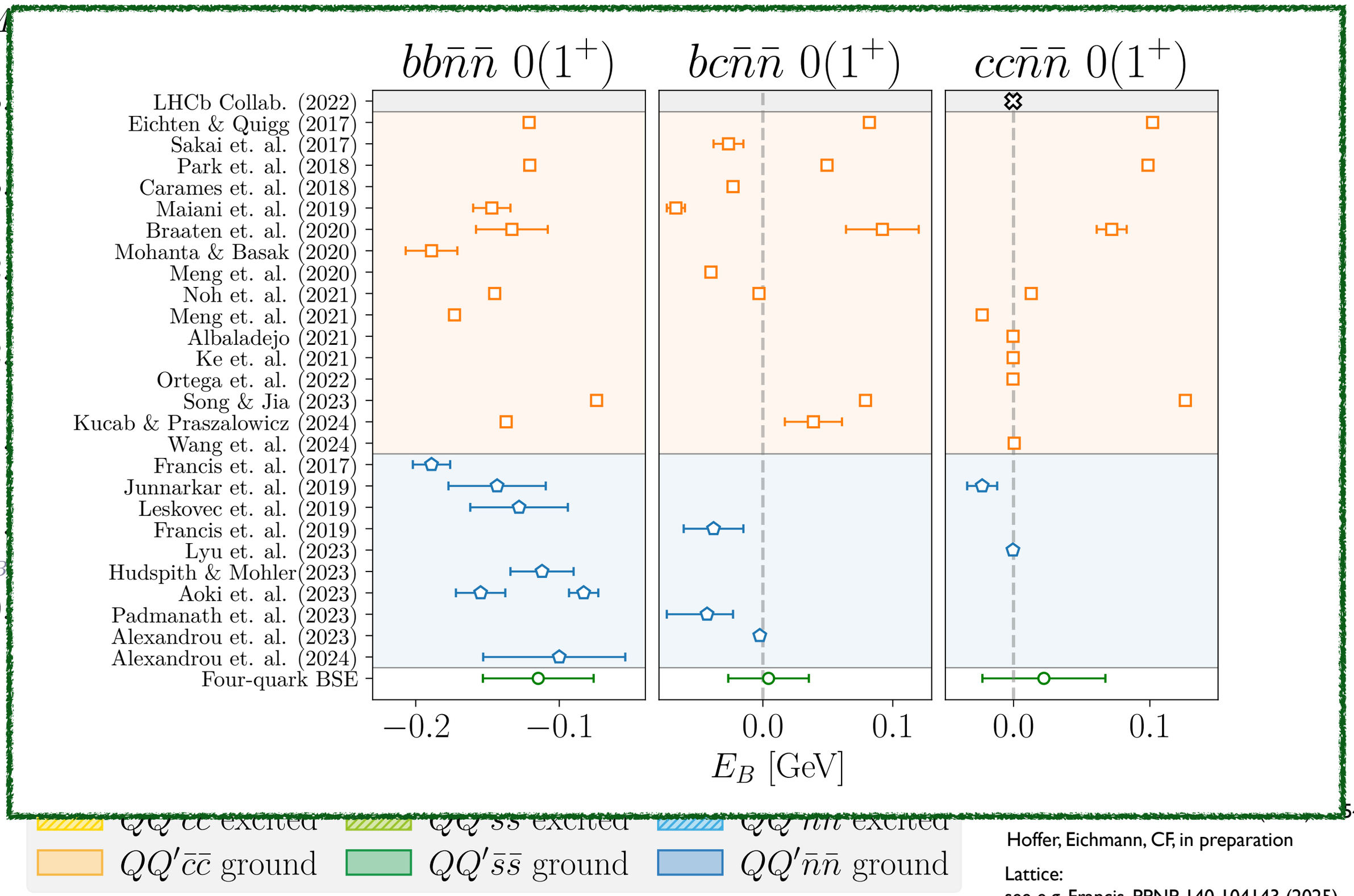
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Hoffer, Eichmann, CF, PRD 111 (2025) 5 054028
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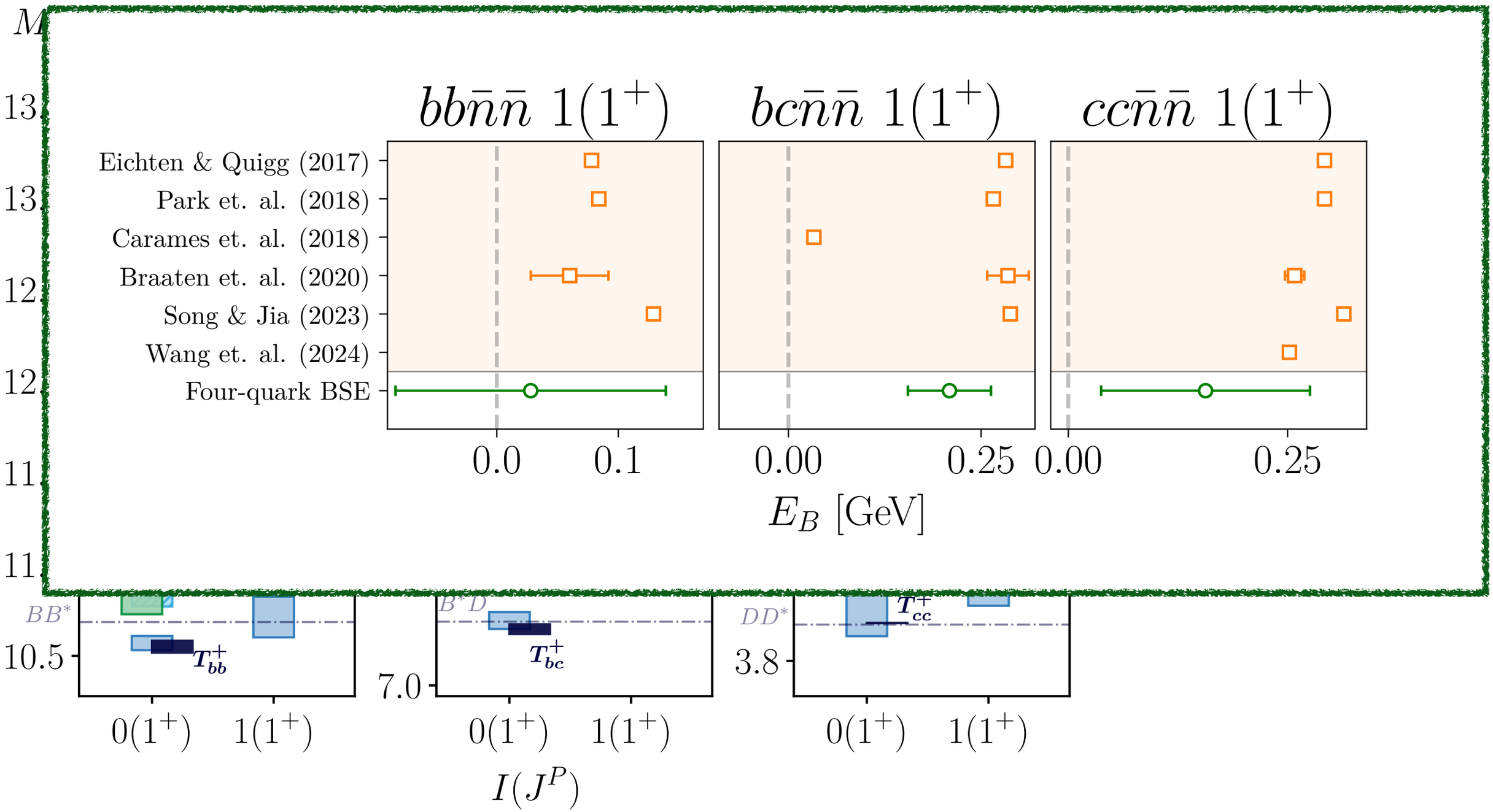



54028

Hoffer, Eichmann, CF, in preparation

Lattice:
see e.g. Francis, PPNP 140 104143 (2025)

# bb, bc and cc four-quark-states




$QQ'\bar{c}\bar{c}$ excited $QQ'\bar{s}\bar{s}$ excited $QQ'\bar{n}\bar{n}$ excited

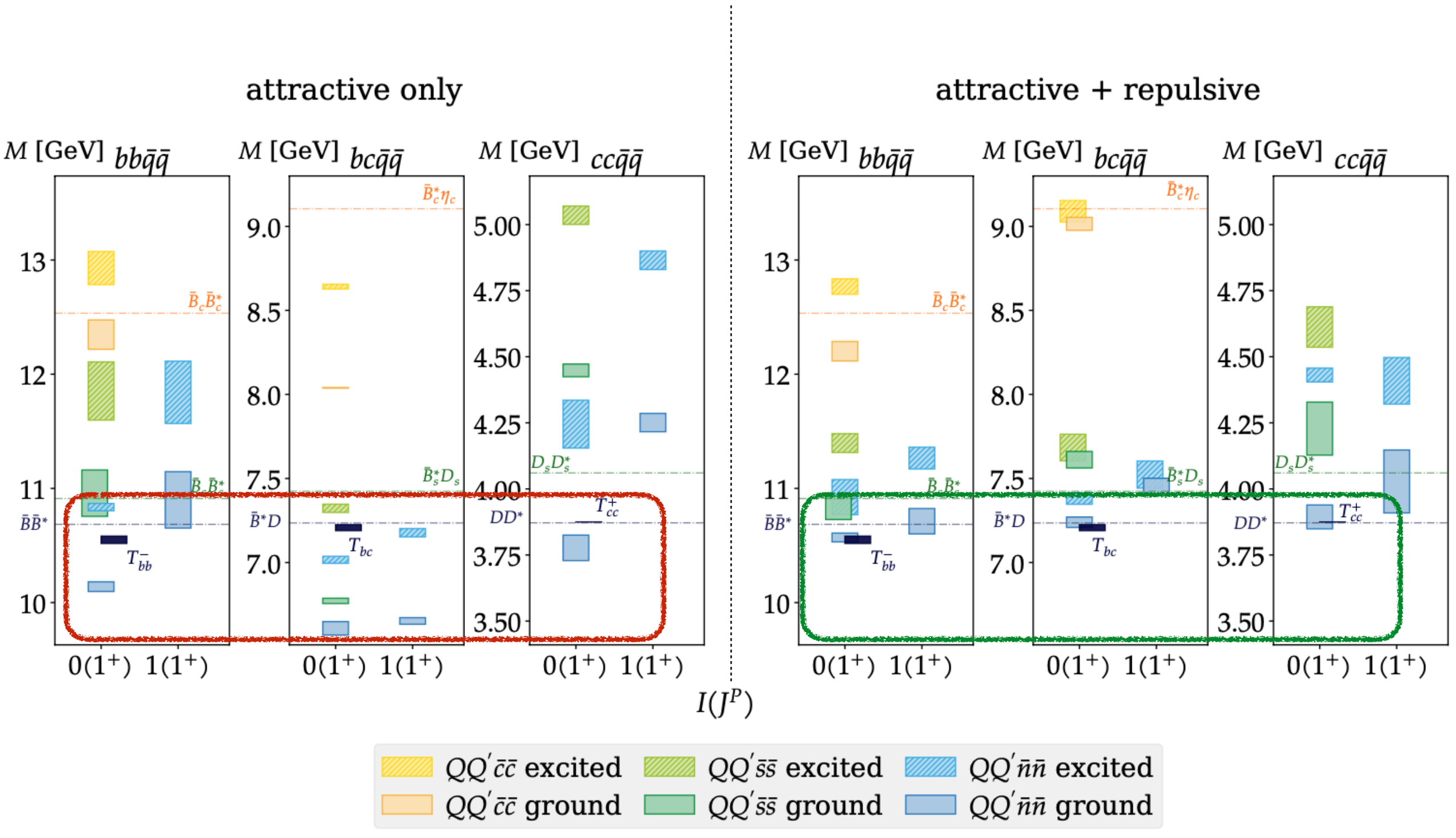
$QQ'\bar{c}\bar{c}$ ground $QQ'\bar{s}\bar{s}$ ground $QQ'\bar{n}\bar{n}$ ground

Hoffer, Eichmann, CF, PRD 111 (2025) 5 054028

Hoffer, Eichmann, CF, in preparation

Lattice:
see e.g. Francis, PPNP 140 104143 (2025)

# Effect of repulsive forces $8 \otimes 8, 6 \otimes \bar{6}$




## Repulsive forces are important!

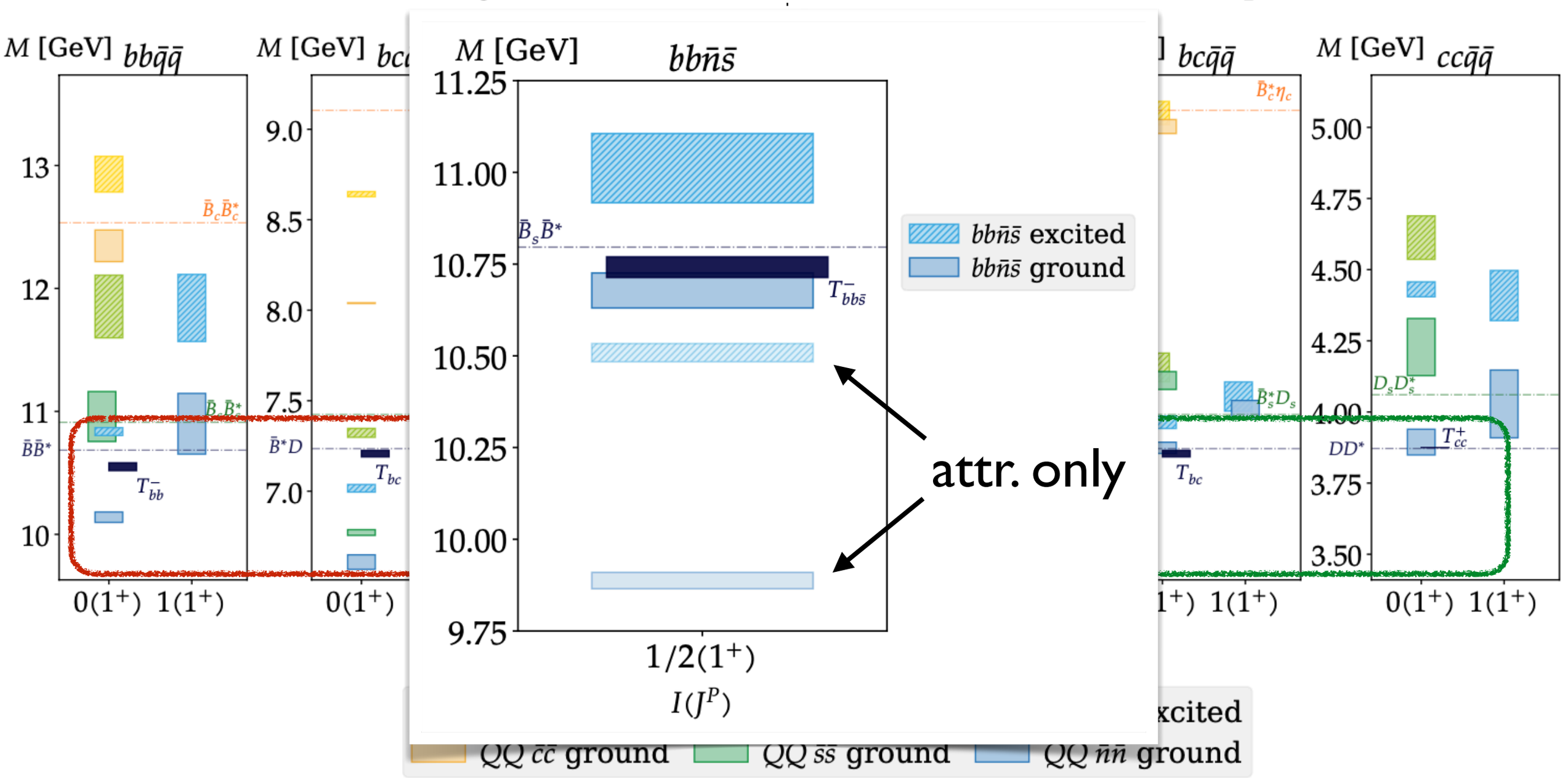
Hoffer, Eichmann, CF, PRD 111 (2025) 5 054028

Lattice: Bicudo, Cardoso, Peters, Pflaumer and Wagner, PRD 96 (2017) 5, 054510

# Effect of repulsive forces $8 \otimes 8, 6 \otimes \bar{6}$

attractive only

attractive + repulsive

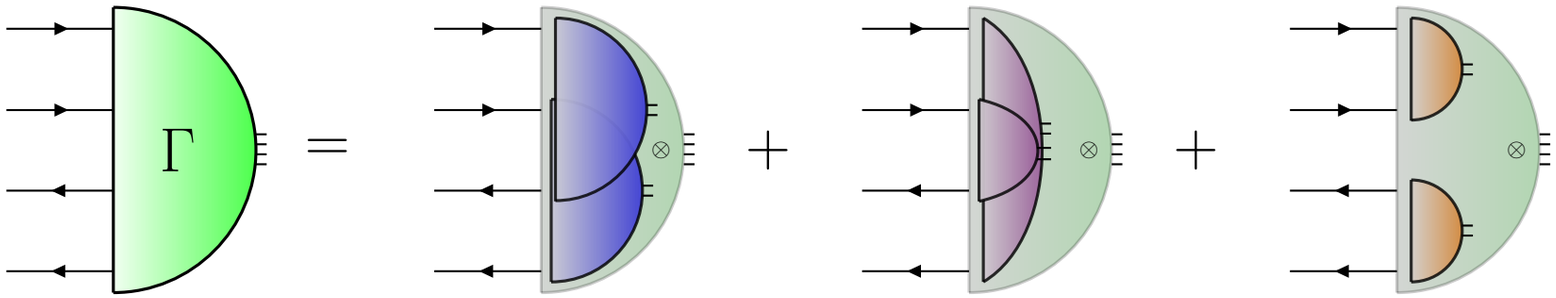



## Repulsive forces are important!

Lattice: Bicudo, Cardoso, Peters, Pflaumer and Wagner, PRD 96 (2017) 5, 054510

Hoffer, Eichmann, CF, PRD 111 (2025) 5 054028
Hoffer, Eichmann, CF, in preparation

# Identifying leading structures…

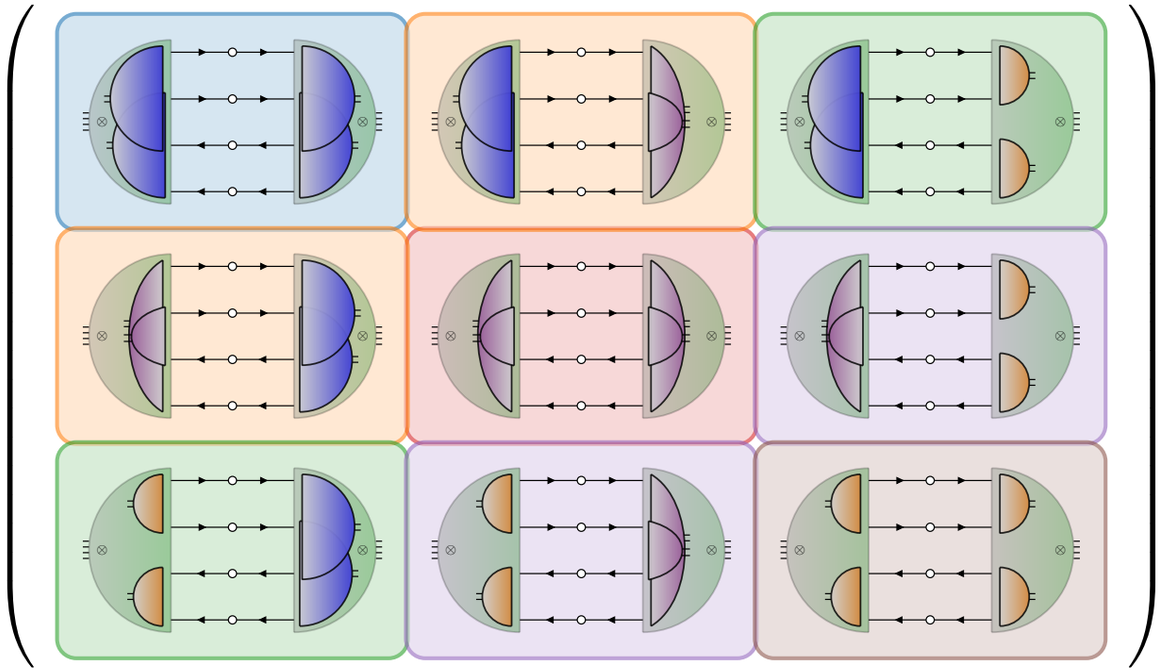


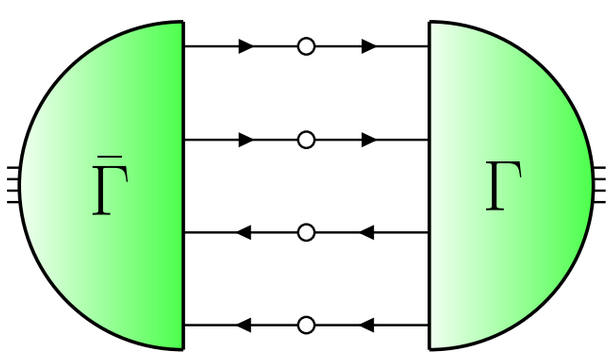

meson-meson

hadro-onium

diquark-antidiquark

- norm contributions

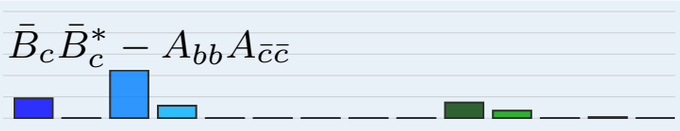




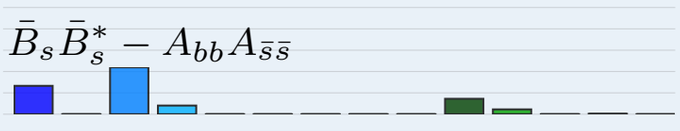
# Internal structure

$n \in \{u, d\}$

$bb\bar{c}\bar{c}$




$bb\bar{s}\bar{s}$



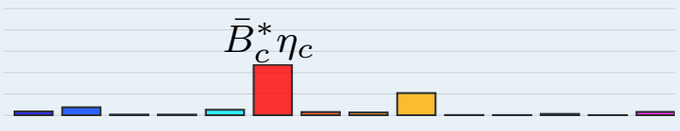

$bb\bar{n}\bar{n}$



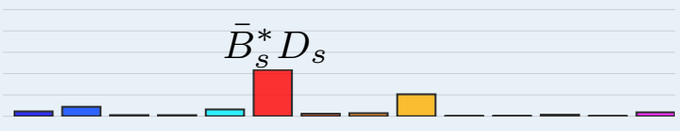




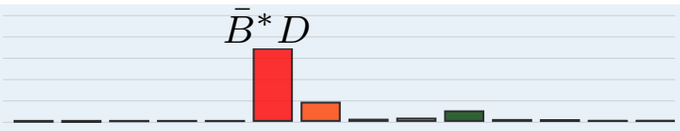
$bc\bar{c}\bar{c}$

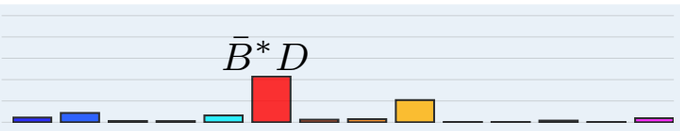



$bc\bar{s}\bar{s}$

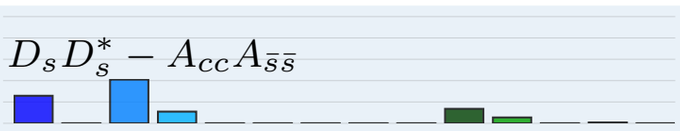



$bc\bar{n}\bar{n}$

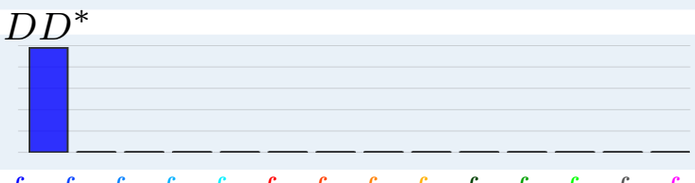


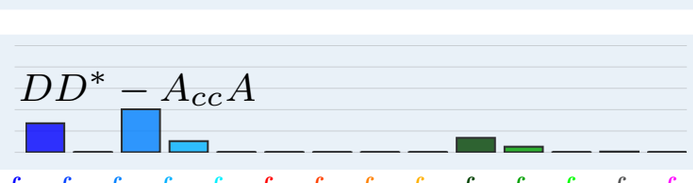




$cc\bar{s}\bar{s}$




$cc\bar{n}\bar{n}$



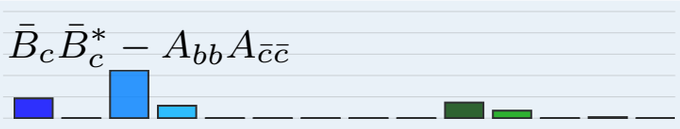

$0(1^+)$ $1(1^+)$

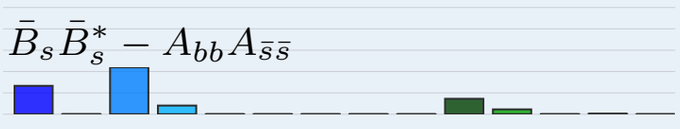
- decided dynamically !
- flavour and spin dependent…prediction for bc

Hoffer, Eichmann, CF, PRD 111 (2025) 5 054028

# Internal structure

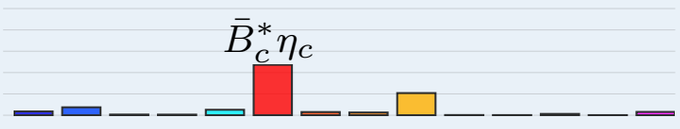
$n \in \{u, d\}$

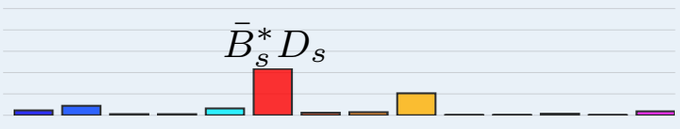
$bb\bar{c}\bar{c}$ 

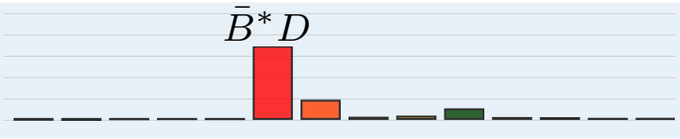

$bb\bar{s}\bar{s}$ 

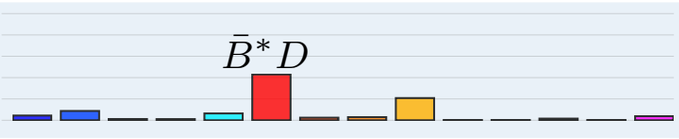

$bb\bar{n}\bar{n}$ 

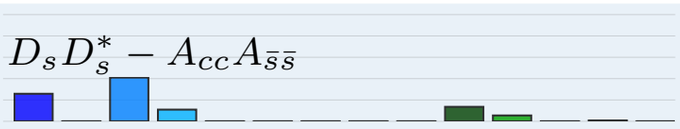


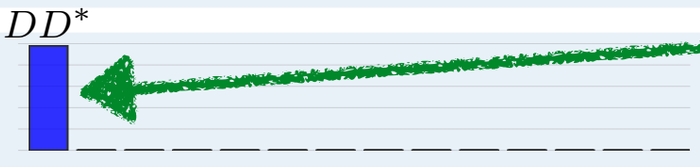

$bc\bar{c}\bar{c}$ 

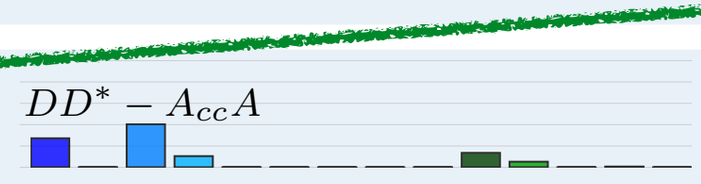

$bc\bar{s}\bar{s}$ 


$bc\bar{n}\bar{n}$ 




$cc\bar{s}\bar{s}$ 


$cc\bar{n}\bar{n}$ 



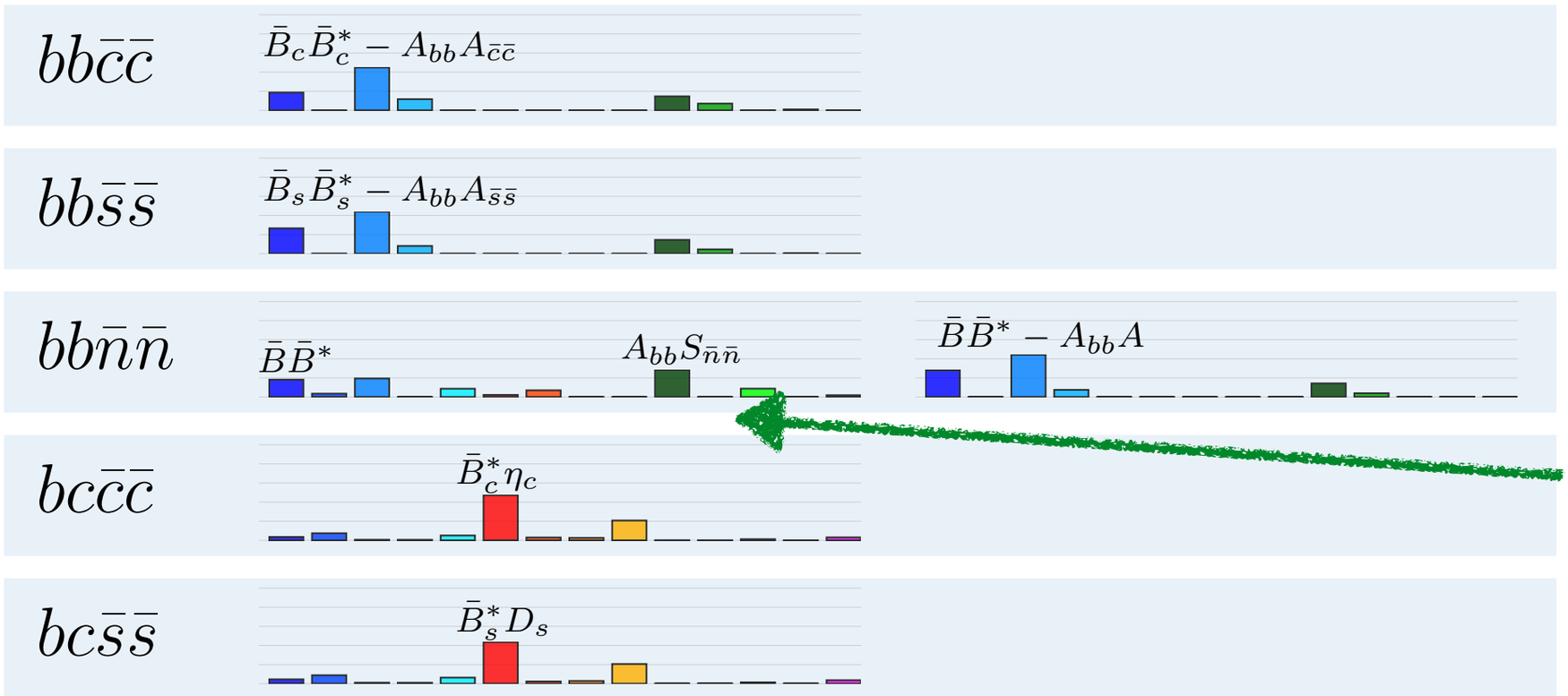

$T_{cc}^+$

- decided dynamically !
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# Internal structure

$n \in \{u, d\}$




## Diquark dominated!

also seen on lattice

Bicudo et al, PRD D103 (2021)
—> Talk of Sasa !

Review:
Francis, PPNP 140 (2025) 104143

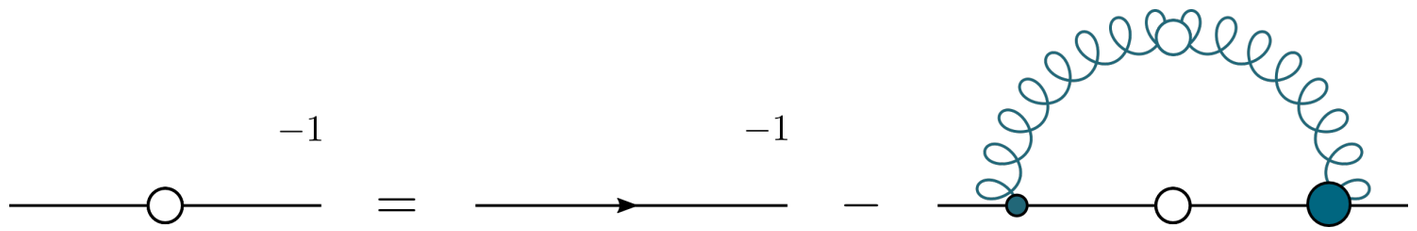
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# Overview

1. The formalism: gluons, quarks and two-quark states




2. Light quark sector:

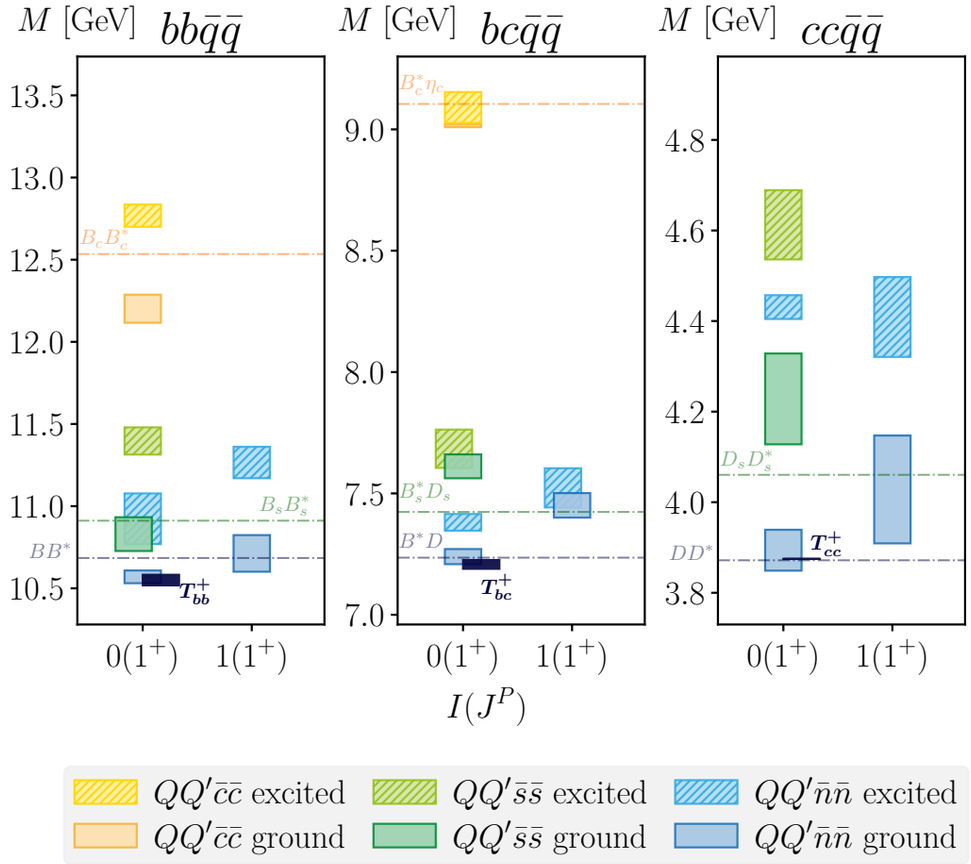
scalar mesons

3. Heavy four-quark states:

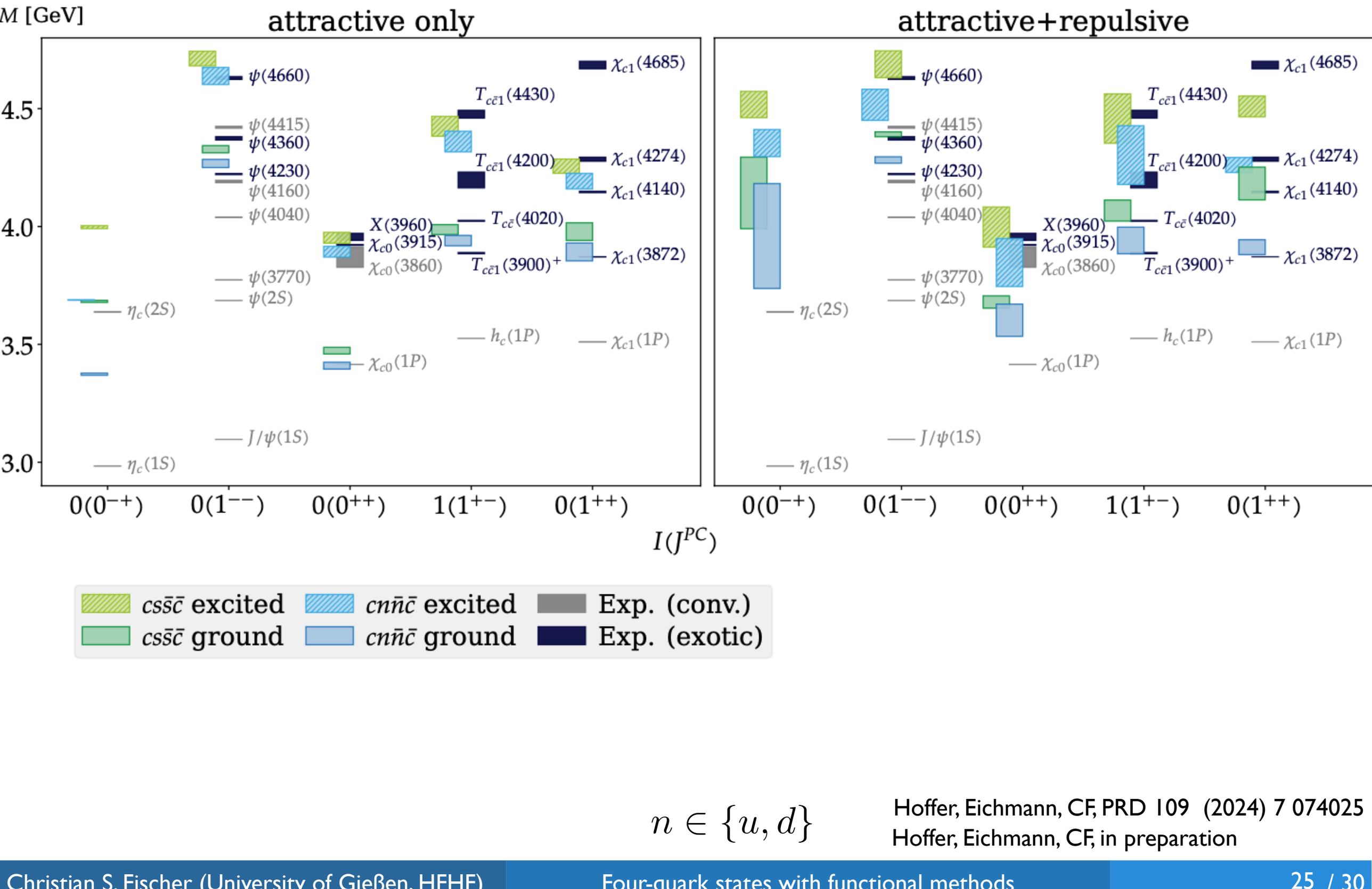
open flavour

4. Heavy four-quark states:

hidden flavour



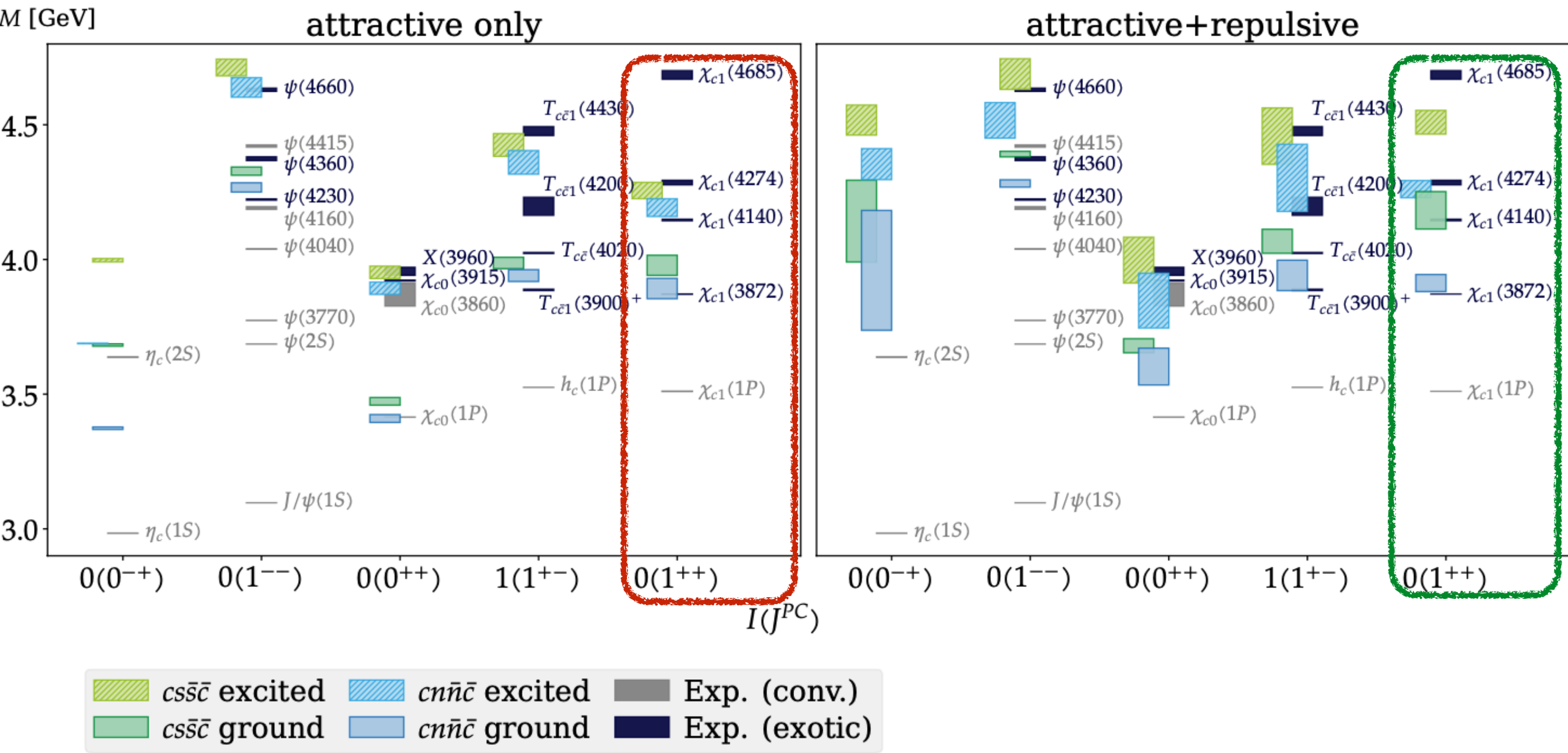
# Hidden flavour four-quark states with charm




$n \in \{u, d\}$

Hoffer, Eichmann, CF, PRD 109 (2024) 7 074025
Hoffer, Eichmann, CF, in preparation

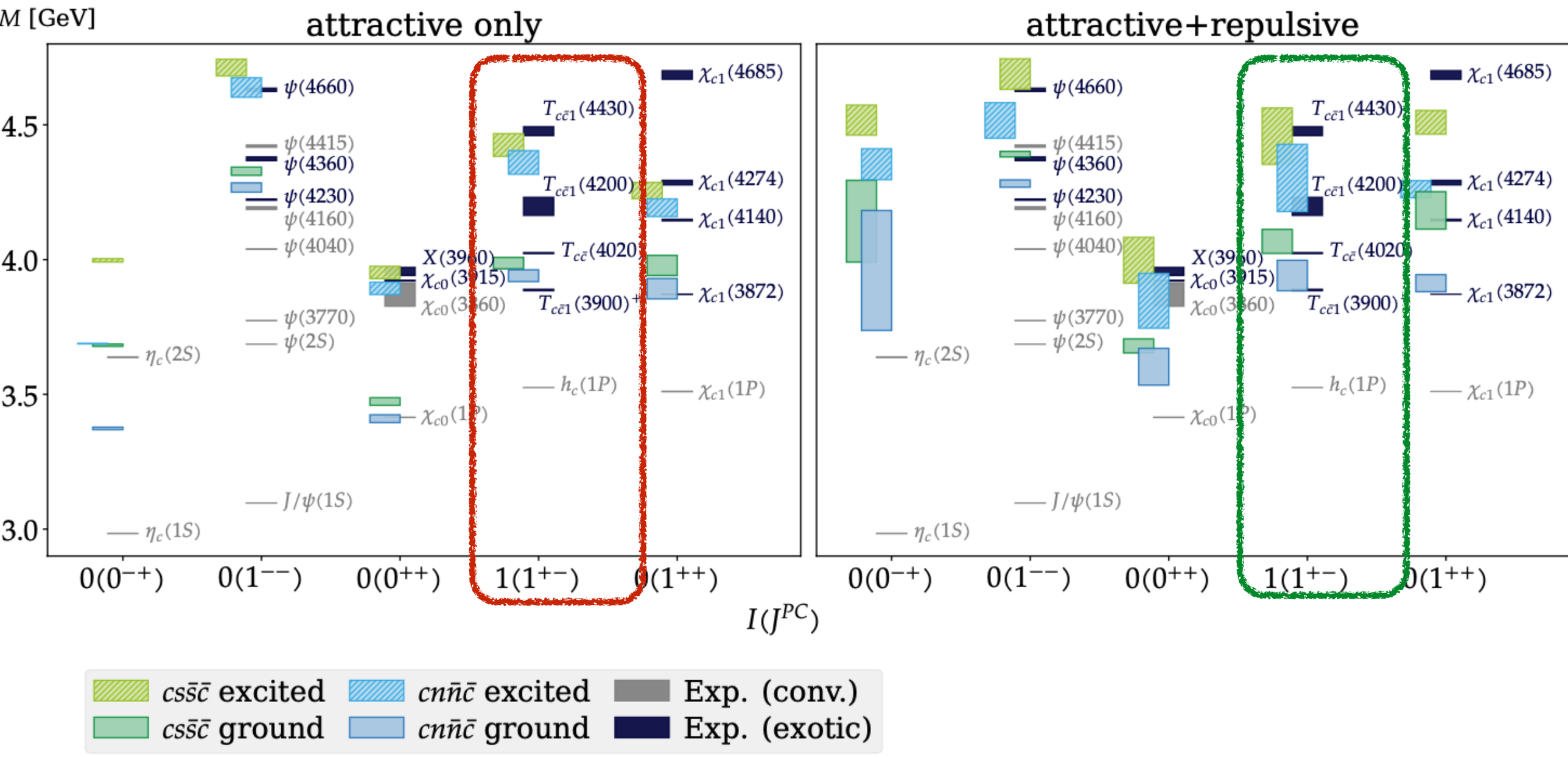
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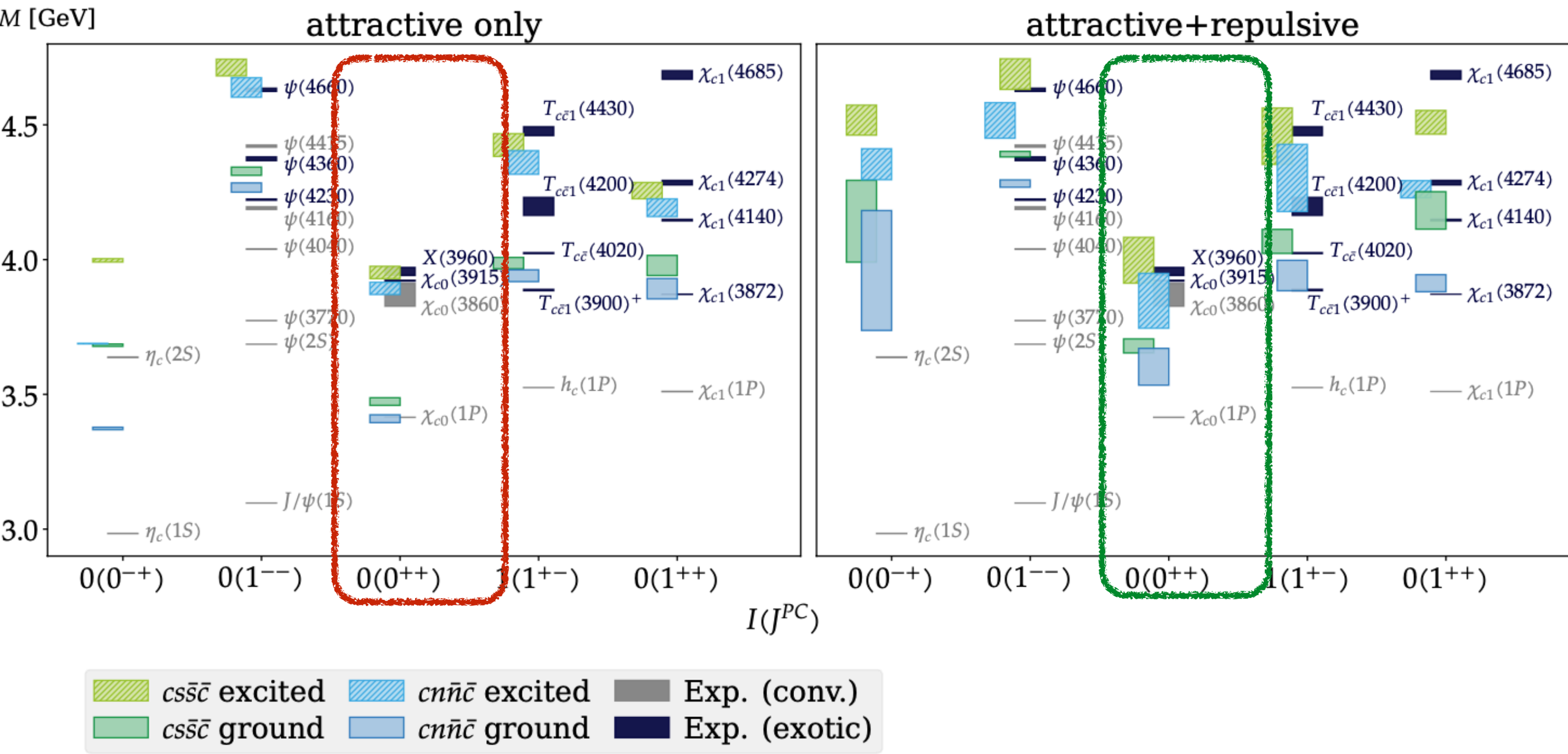
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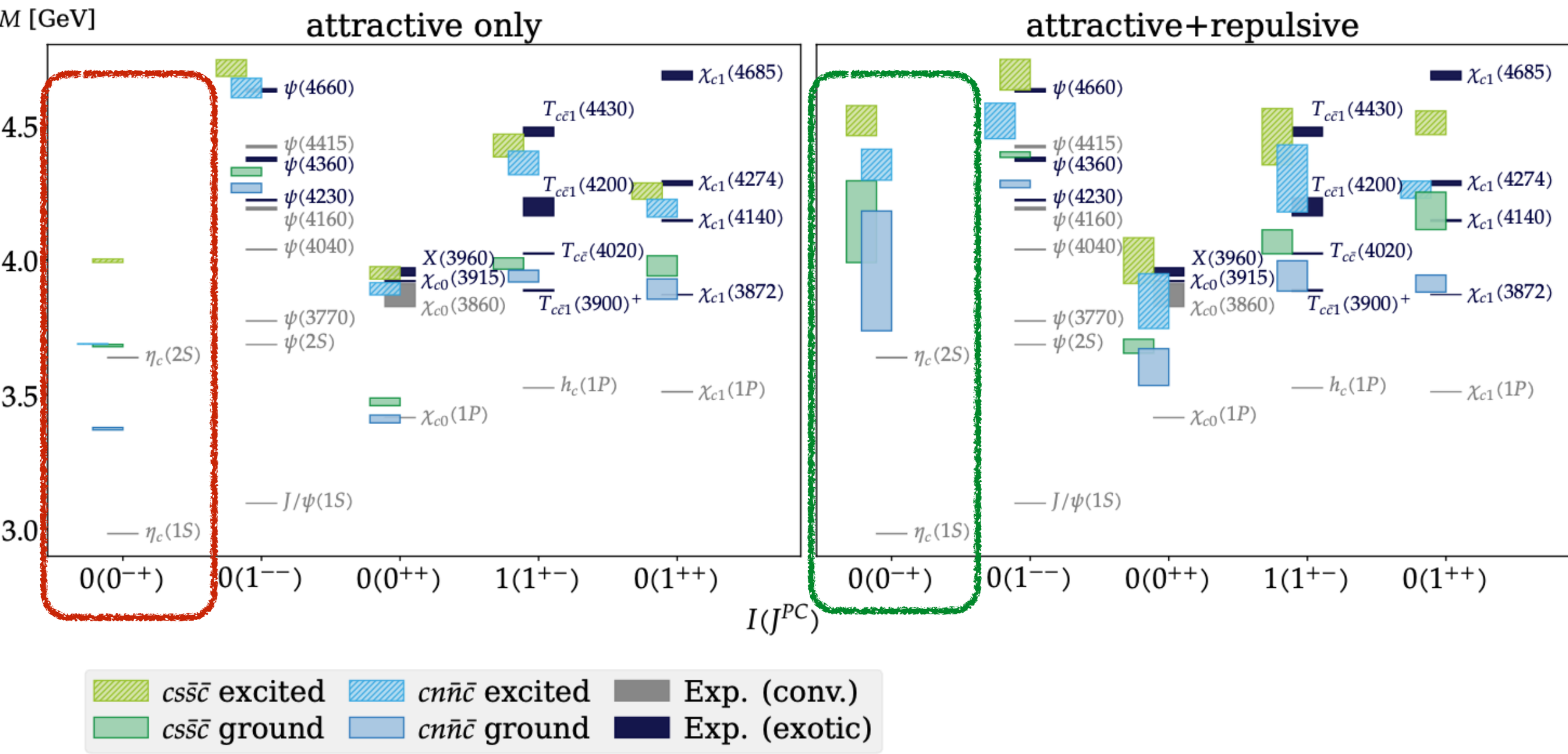
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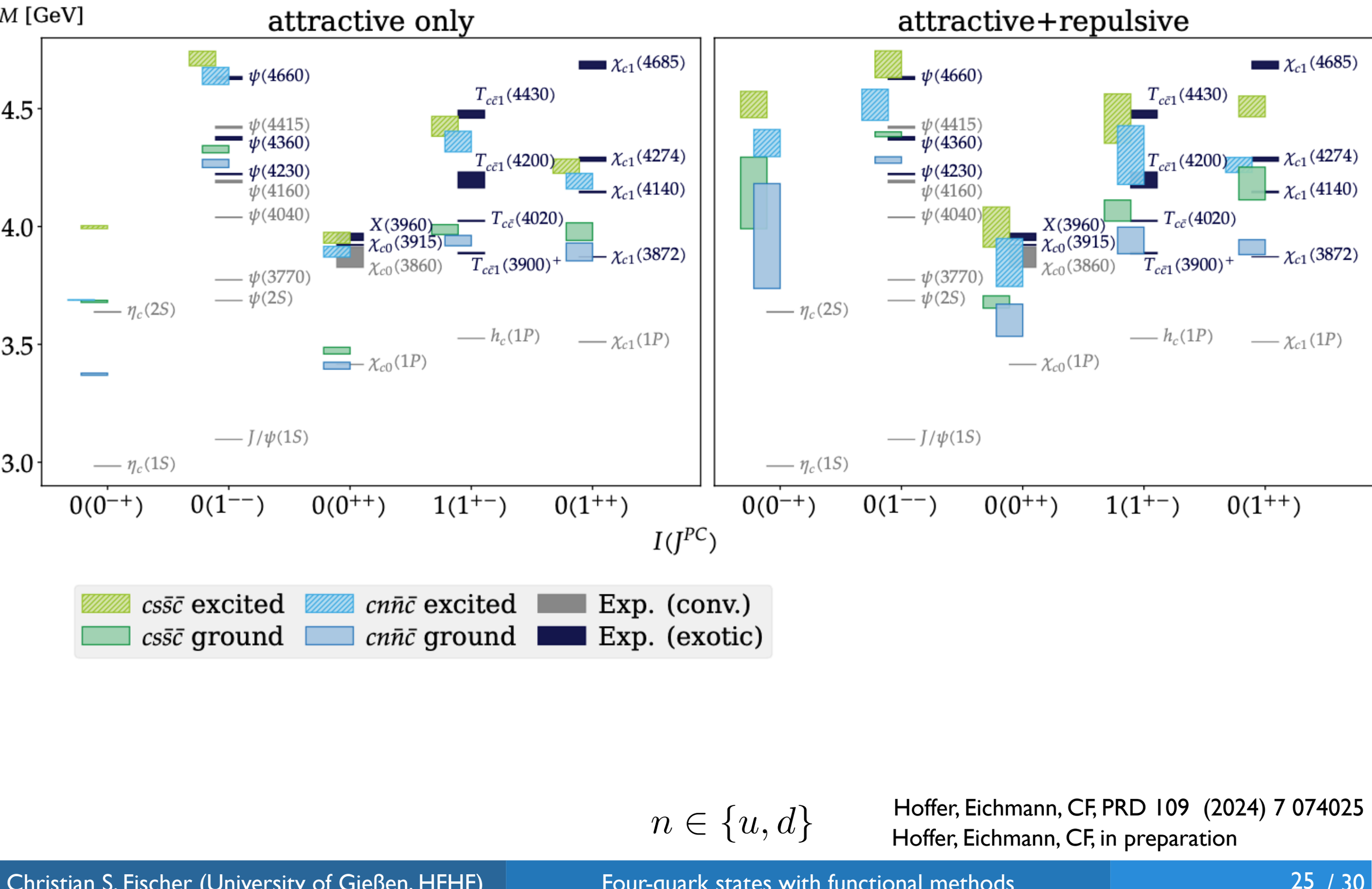
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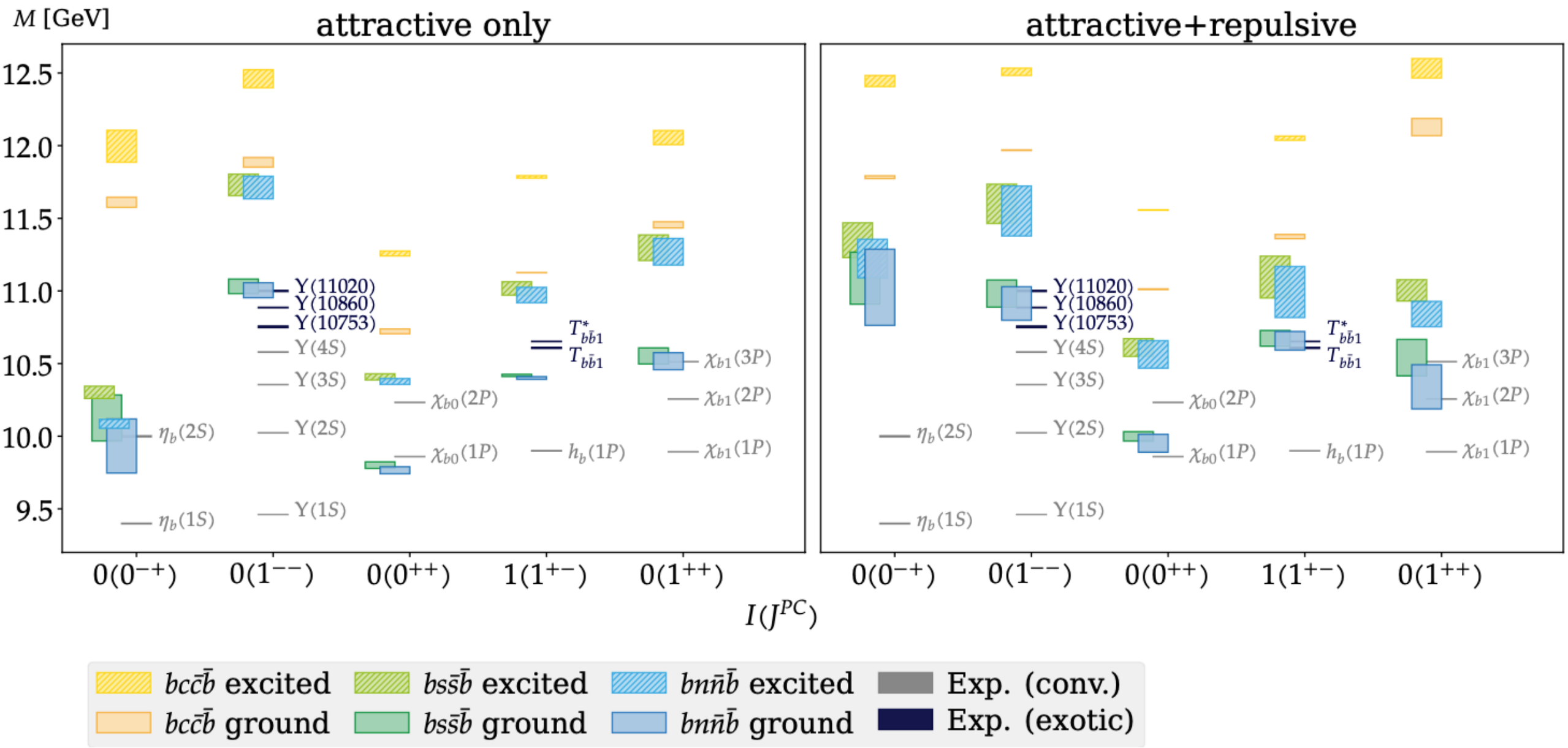
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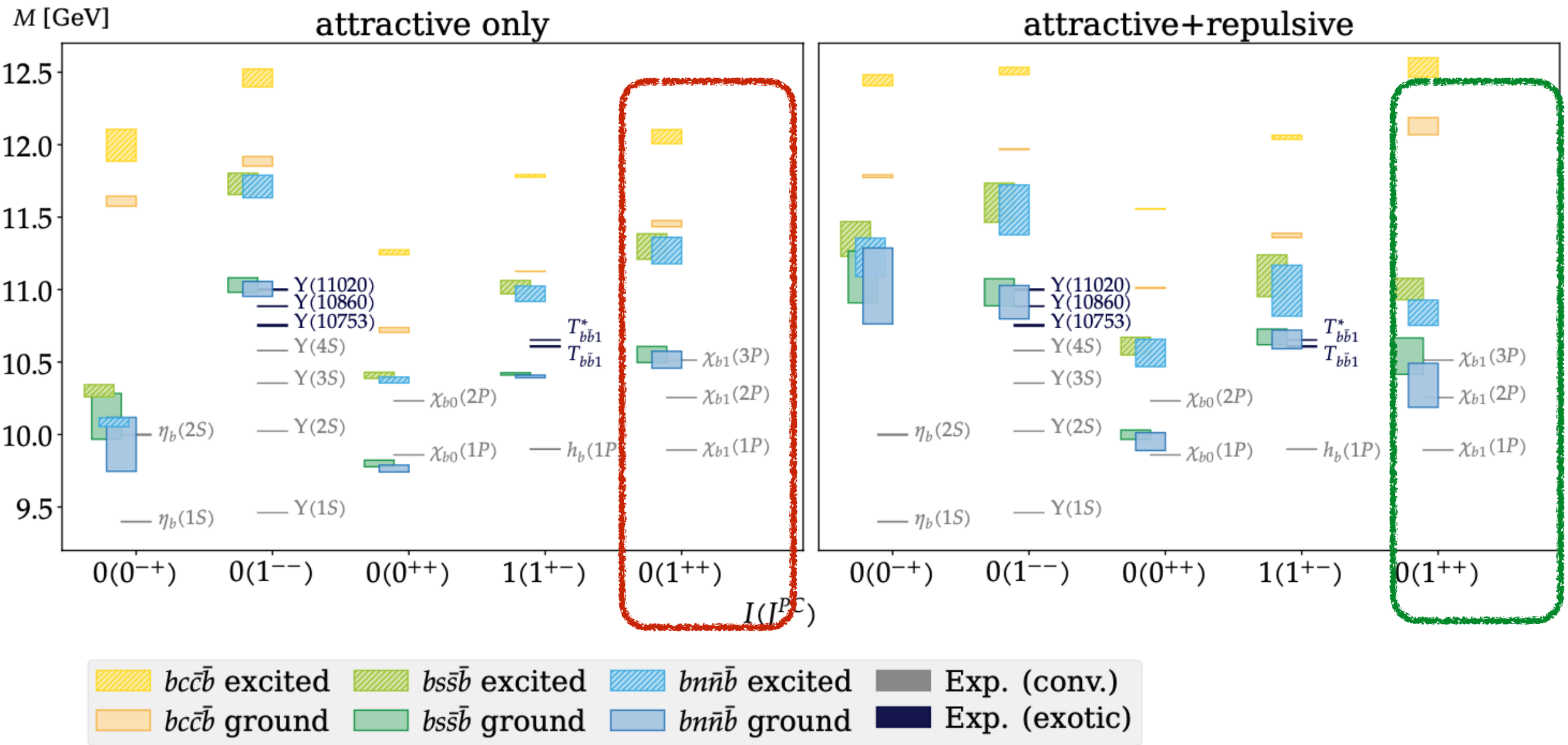
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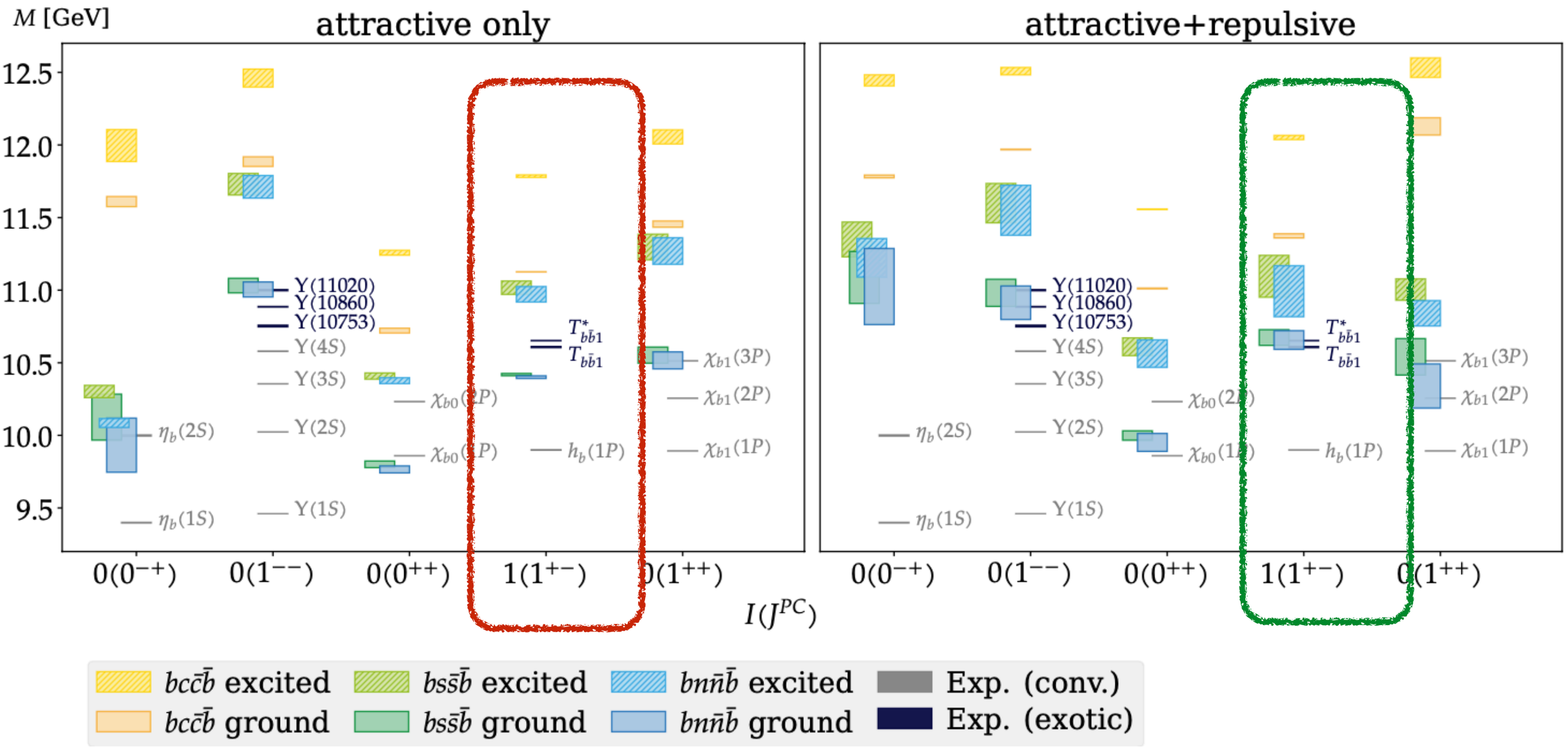
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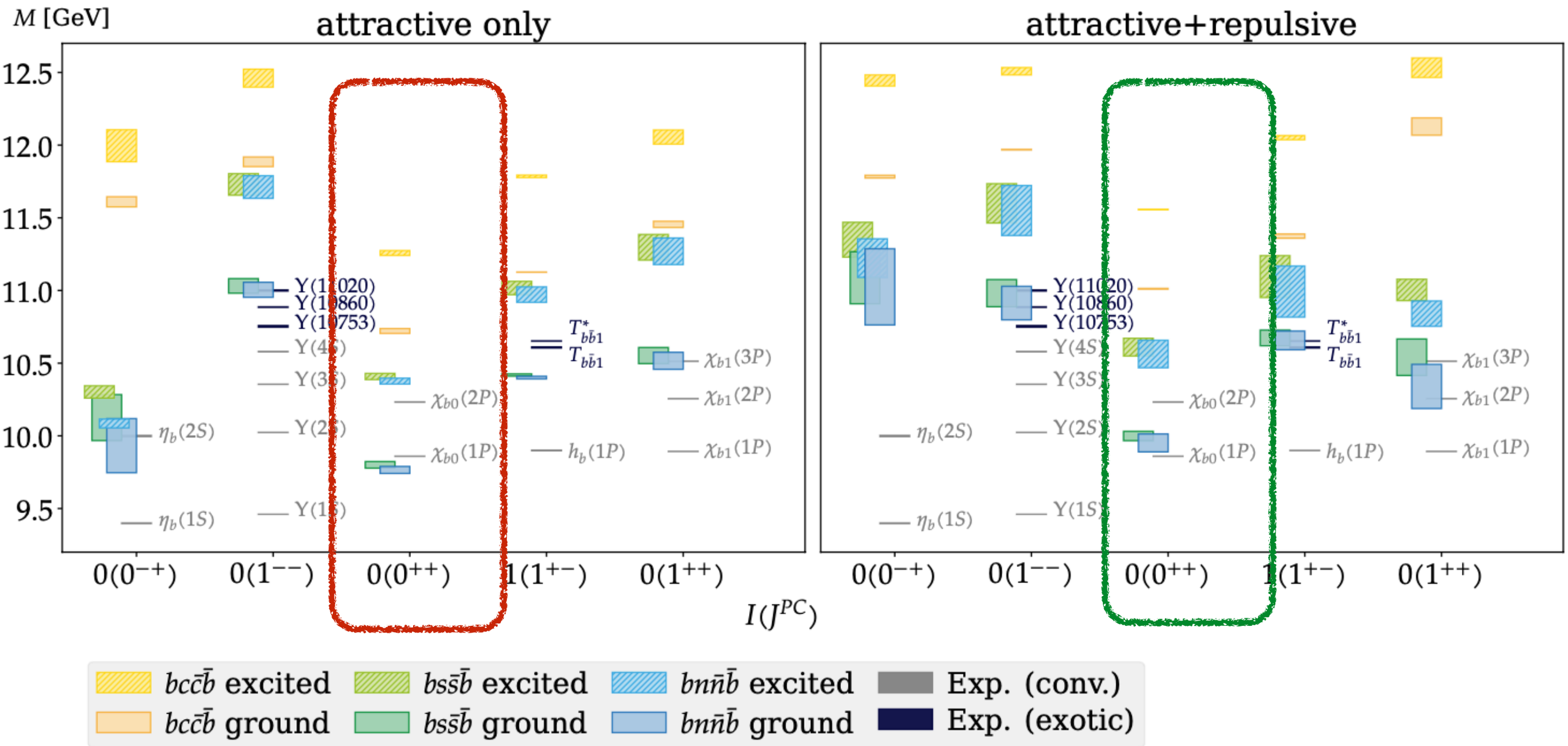
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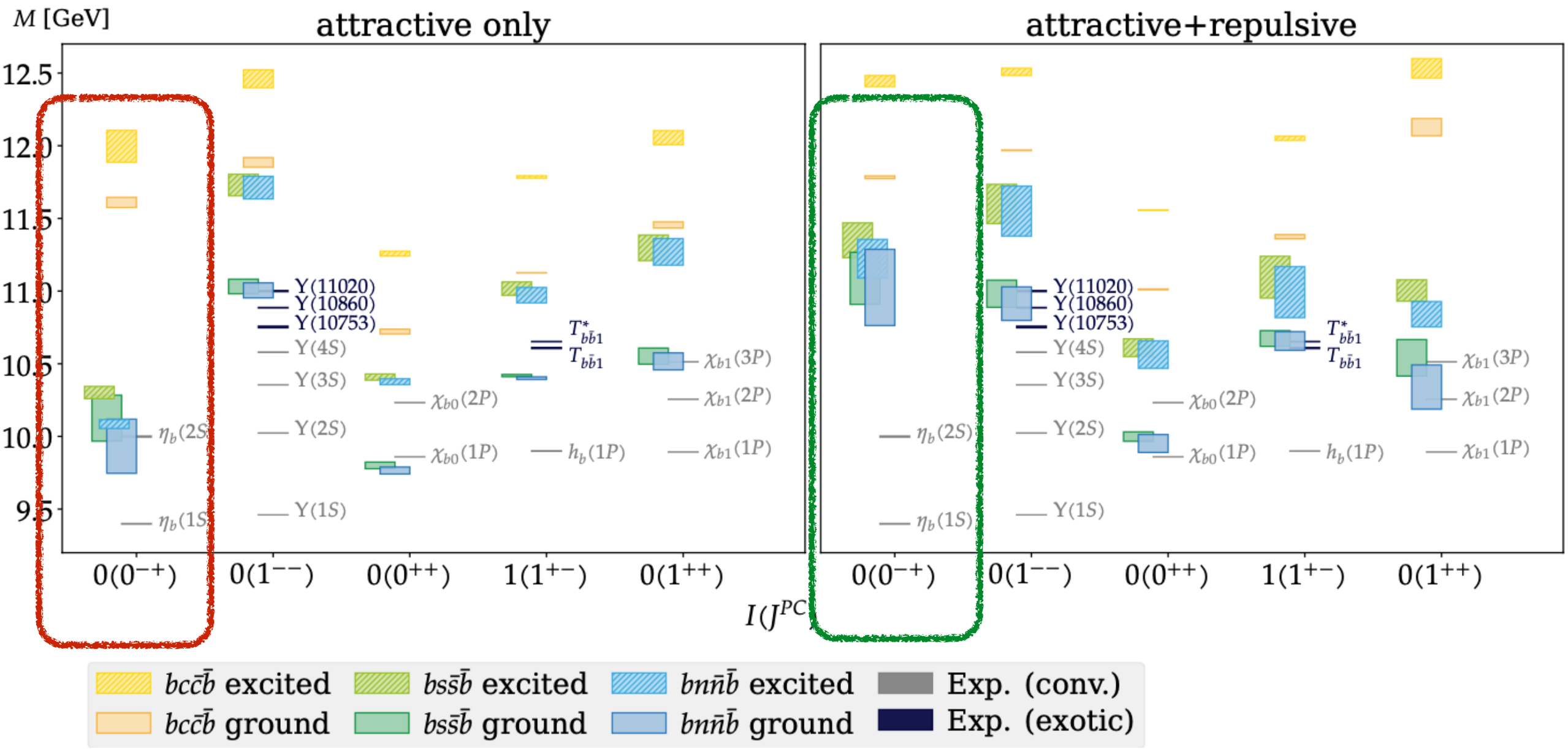
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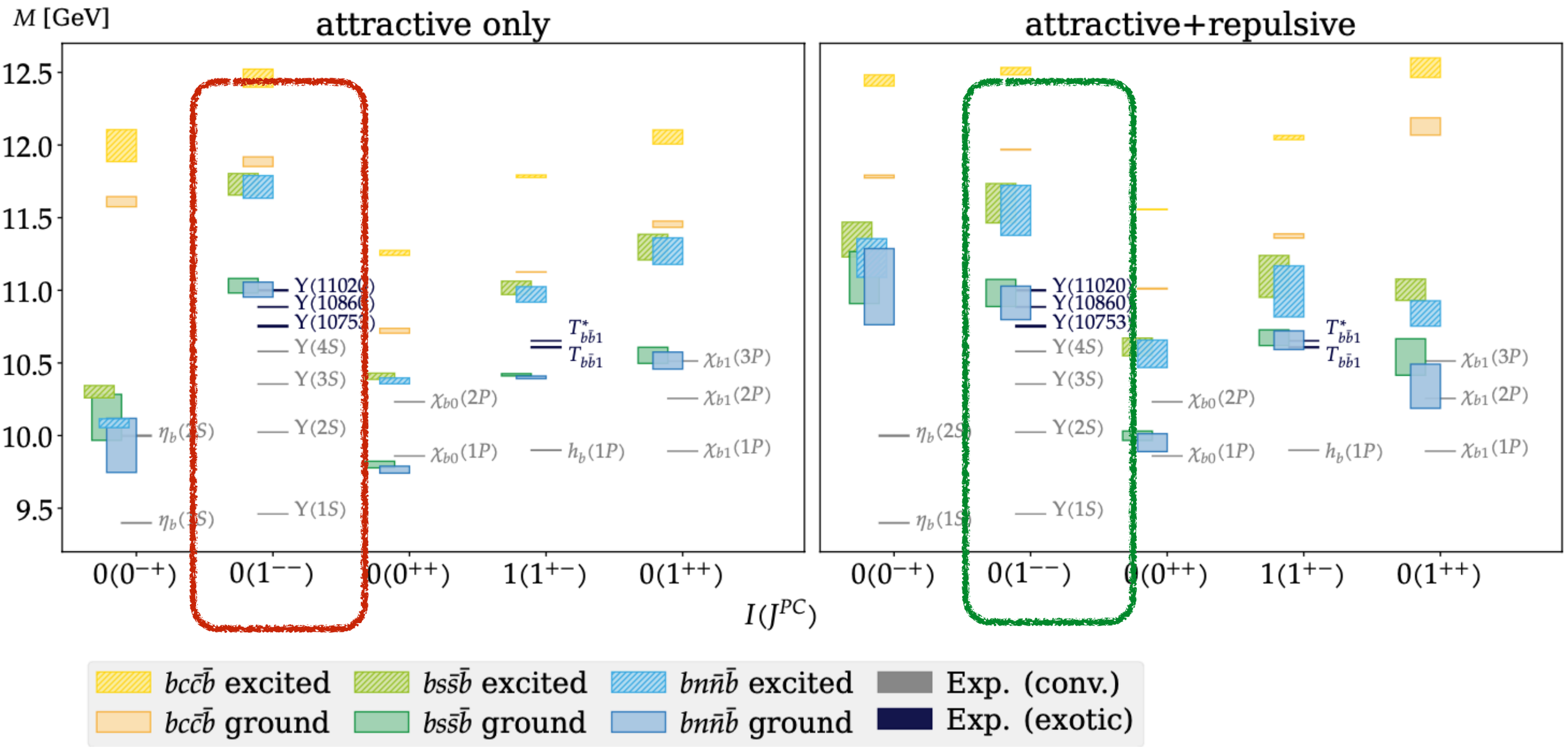
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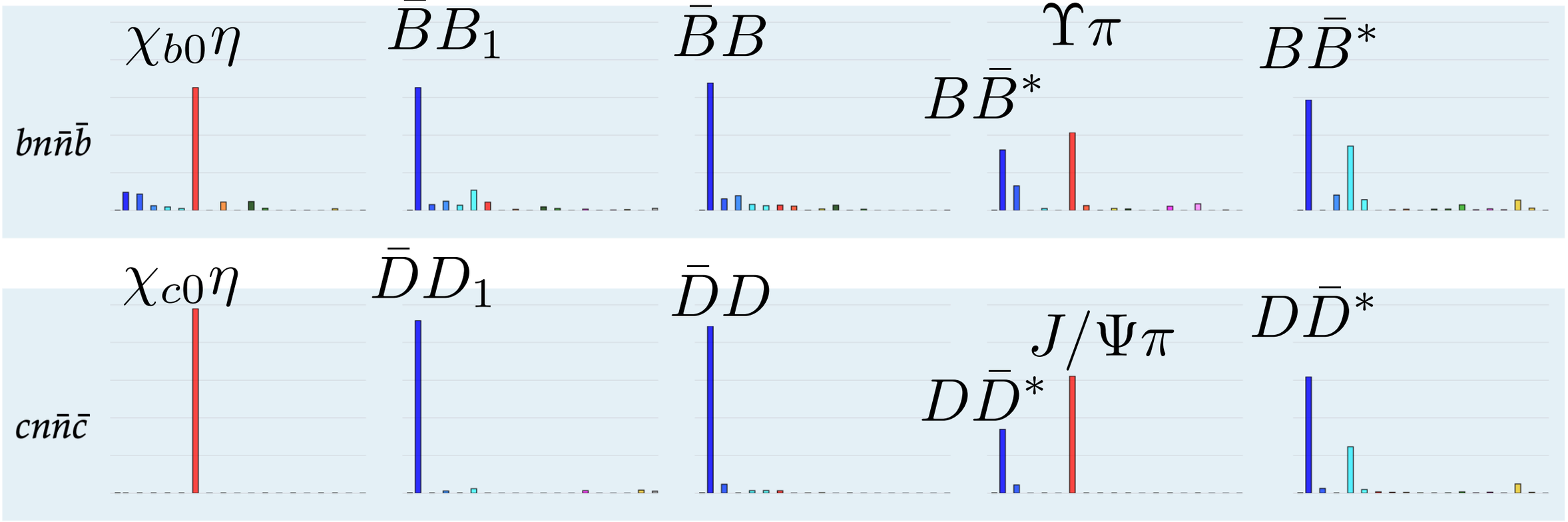



Taken at face value: Υ(10753) may not be four-quark state

$n \in \{u, d\}$

Hoffer, Eichmann, CF, PRD 109 (2024) 7 074025
Hoffer, Eichmann, CF, in preparation

# Internal structure




- decided dynamically !
- flavour and spin dependent…

Hoffer, Eichmann, CF, PRD 109 (2024) 7 074025
Hoffer, Eichmann, CF, in preparation

# Summary

Charm and bottom energy region:

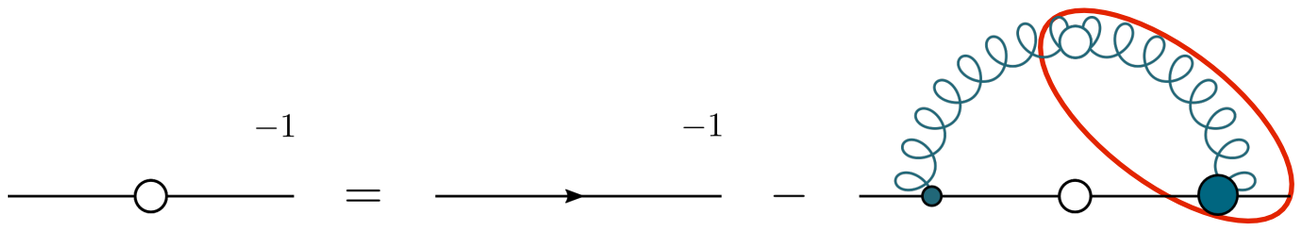
- Spectrum for hidden and open flavor
  four-quark states
- Interesting interplay:  attractive and repulsive forces
- Internal structure is flavour and spin dependent!
  - meson-meson is dominating/important
    in most states

Hoffer, Eichmann, CF, PRD 109 (2024) 074025
Hoffer, Eichmann, CF, PRD 111 (2025) 054028
Hoffer, Eichmann, CF, in preparation

To do:
- improve two-body forces
- include three and four-body forces
- visit second Riemann sheet
- hidden flavor: include mixing with two-quark states
- work together with lattice and EFT

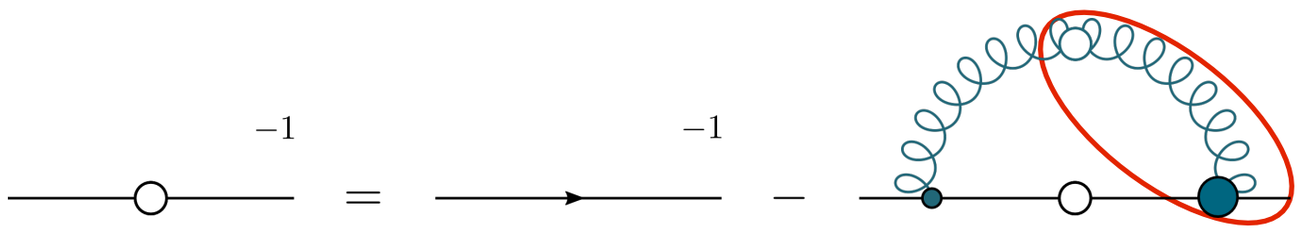
# Backup Slides

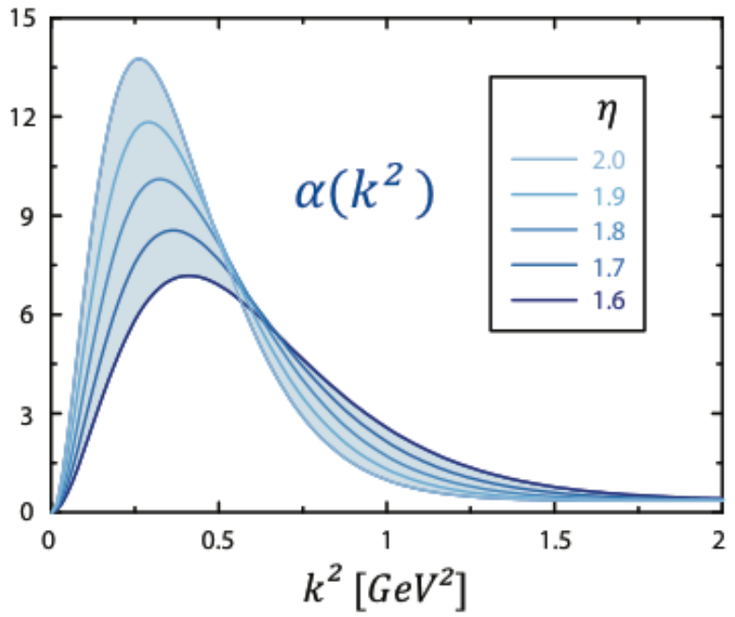
# Rainbow-ladder model for quark-gluon interaction




Combine gluon with quark-gluon vertex:

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Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

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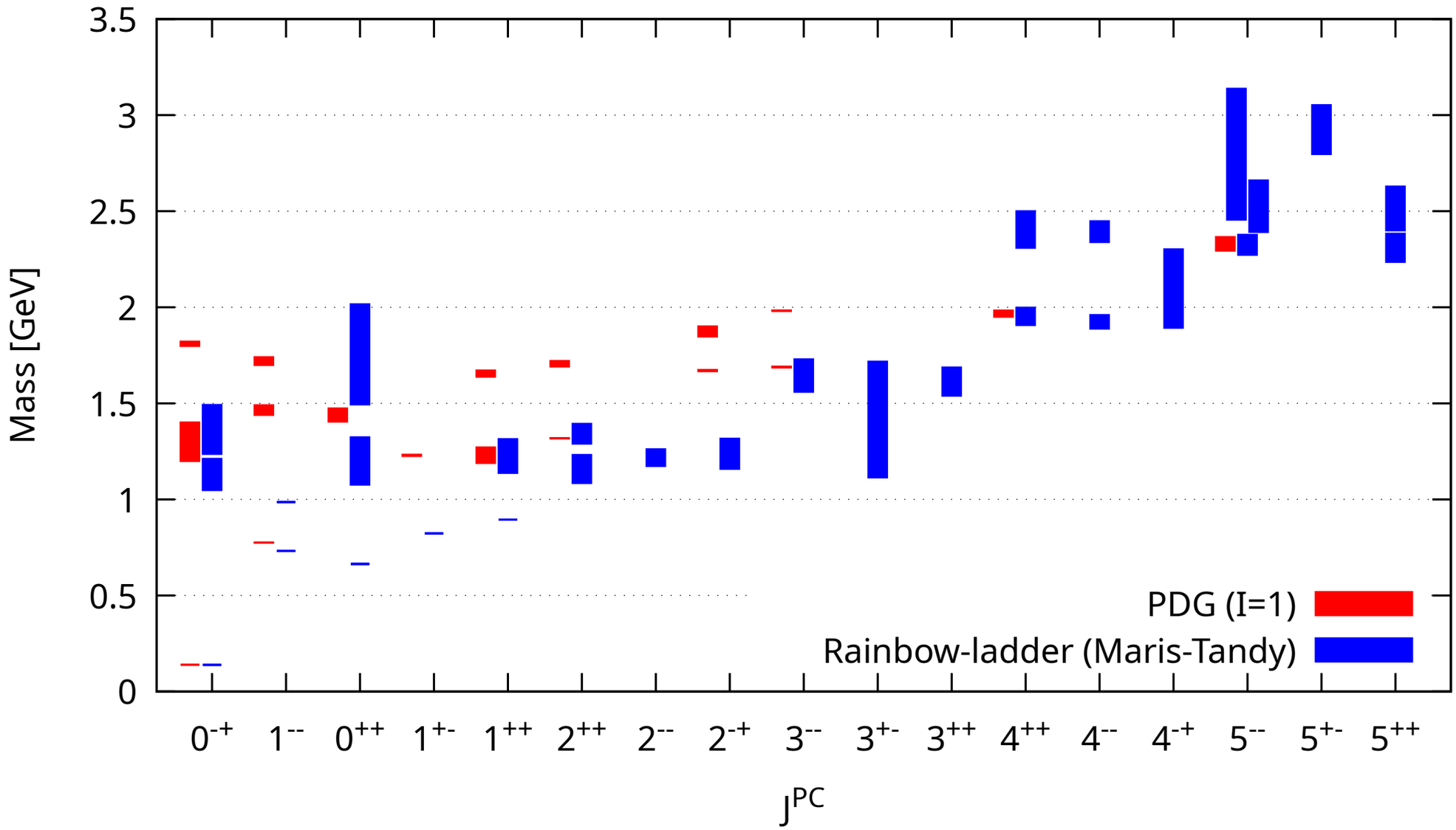
effective coupling

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

- scale $\Lambda$ from $f_\pi$, masses $m_u$=$m_d$, $m_s$ from $m_\pi$, $m_K$
- $\alpha_{UV}$ from perturbation theory
- parameter $\eta$ : results almost independent
- qualitatively similar to explicit calc.

Williams, EPJA 51 (2015) 5, 57.
Sanchis-Alepuz, Williams, PLB 749 (2015) 592;
Mitter, Pawlowski and Strodthoff, PRD 91 (2015) 054035
Williams, CF, Heupel, PRD93 (2016) 034026, and refs. therein

# Light meson spectrum (S=0): J=1,..,5

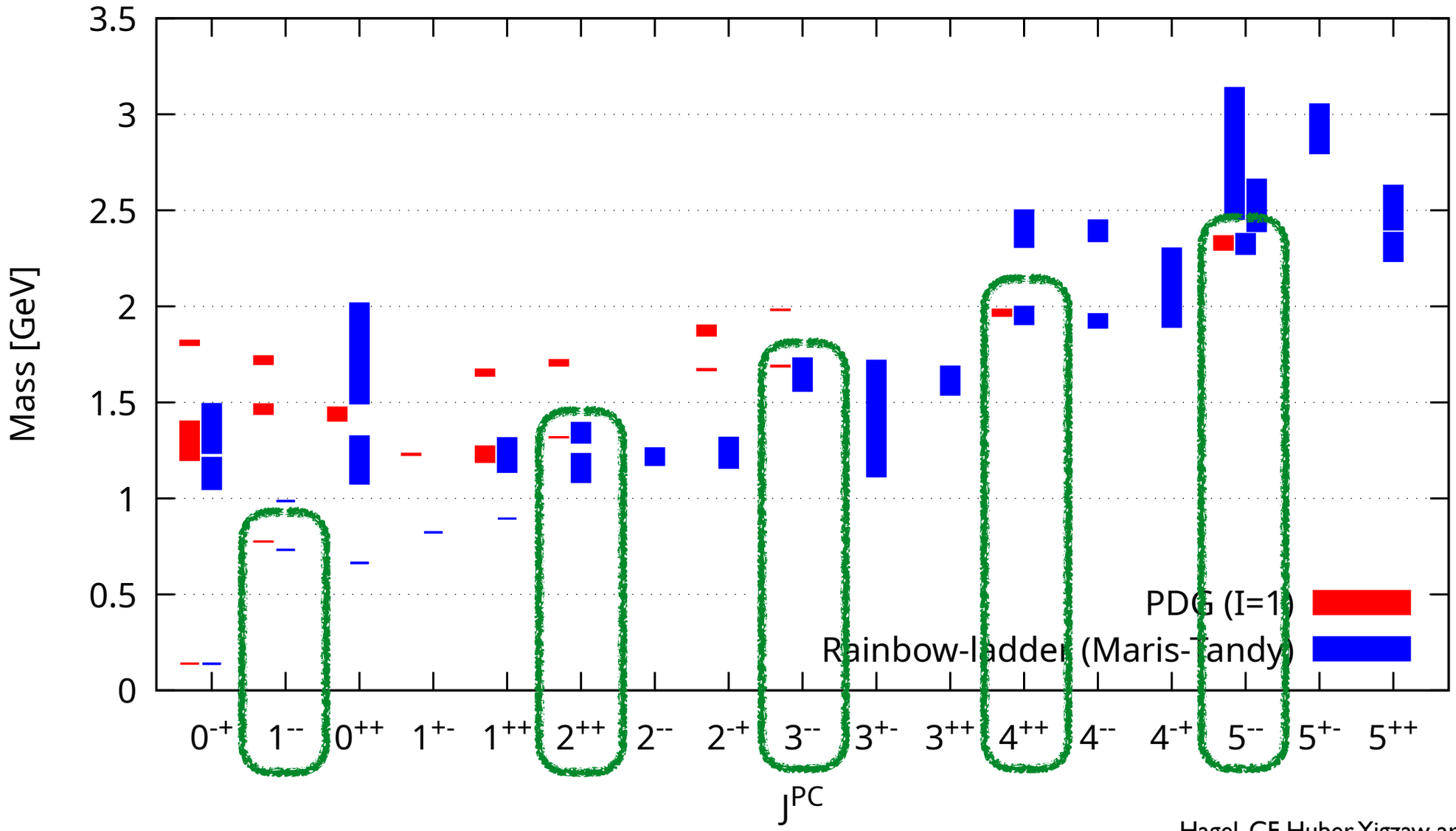



- RL good for ground states on Regge-trajectory:

$$1^{--}, 2^{++}, 3^{--}, 4^{++}, 5^{--}$$

- not competitive in other channels

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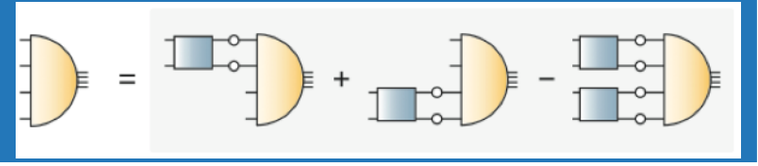

Hagel, CF, Huber, Yigzaw arXiv:2510.27423

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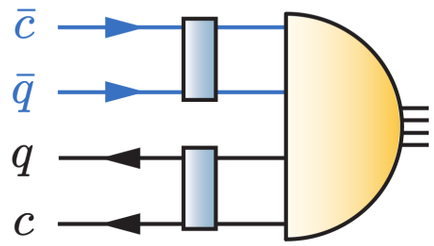
# Four-body equation: permutations



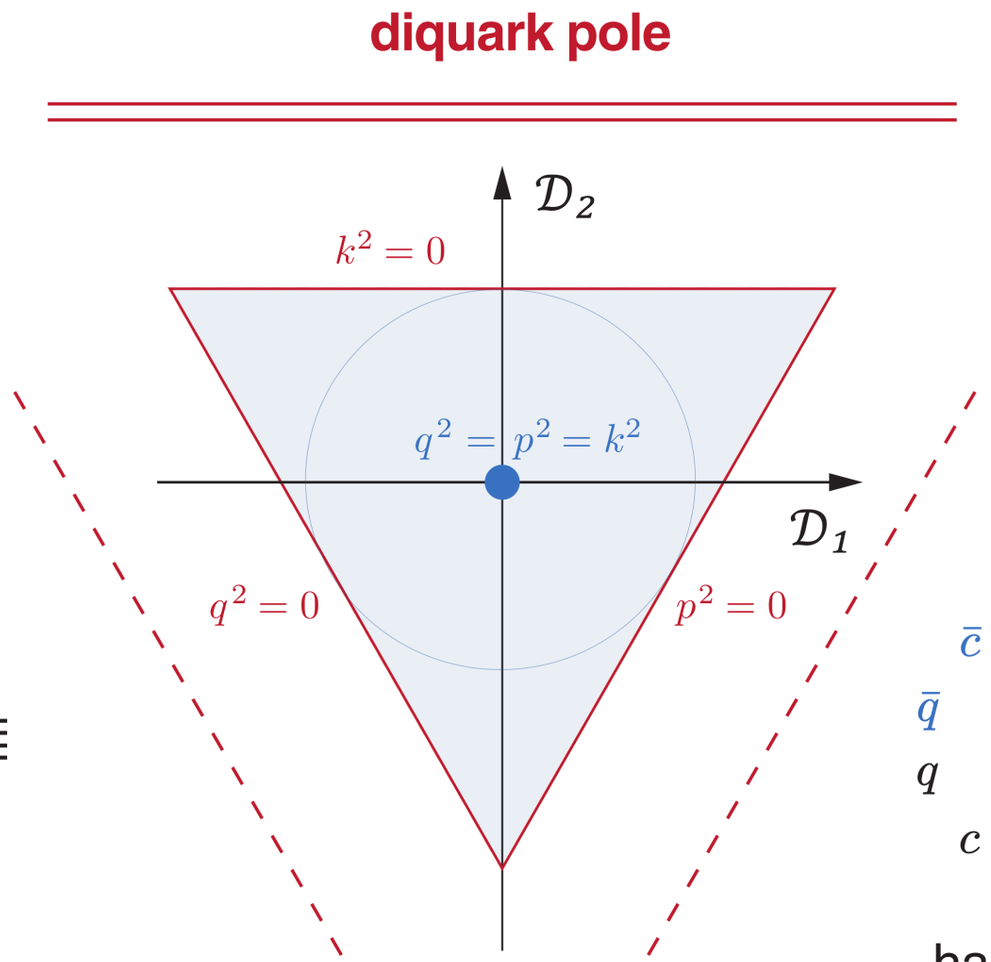
- **Singlet:** $S_0 = (p^2 + q^2 + k^2)/4$
- **Doublet:** $\mathcal{D}_1 \sim p^2 + q^2 - 2k^2$

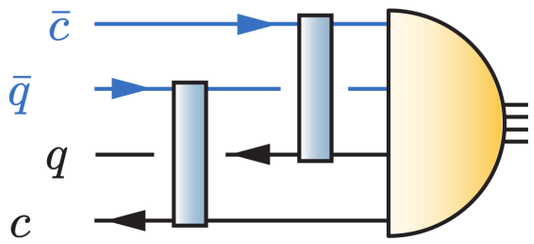
  $\mathcal{D}_2 \sim q^2 - p^2$

$p, q, k$ : relative momenta

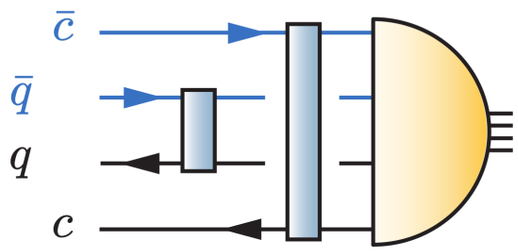



diquark-antidiquark



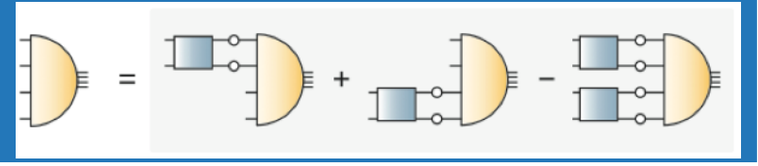




heavy-light mesons




hadrocharmonium

# Four-body equation: permutations

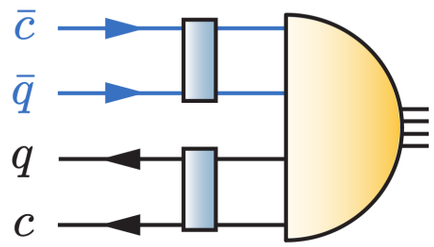


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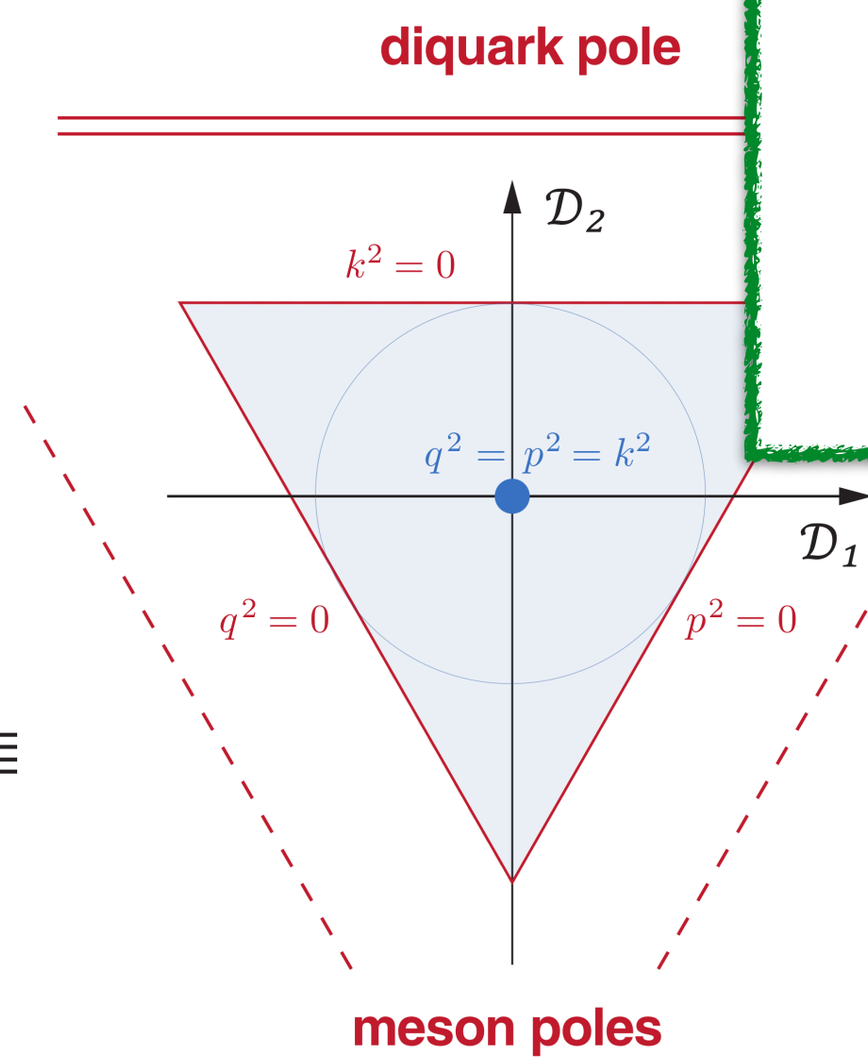
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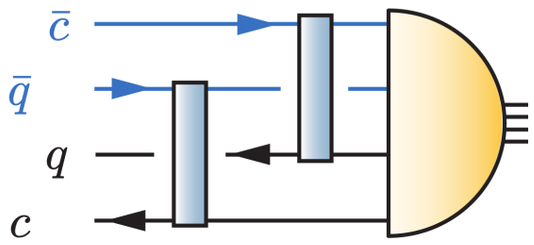
$p, q, k$ : relative momenta

- model independent: heavy-light meson poles more important than diquark poles (color factor !)

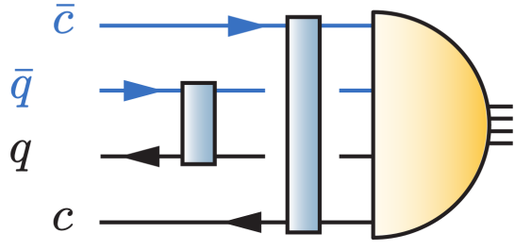



diquark-antidiquark



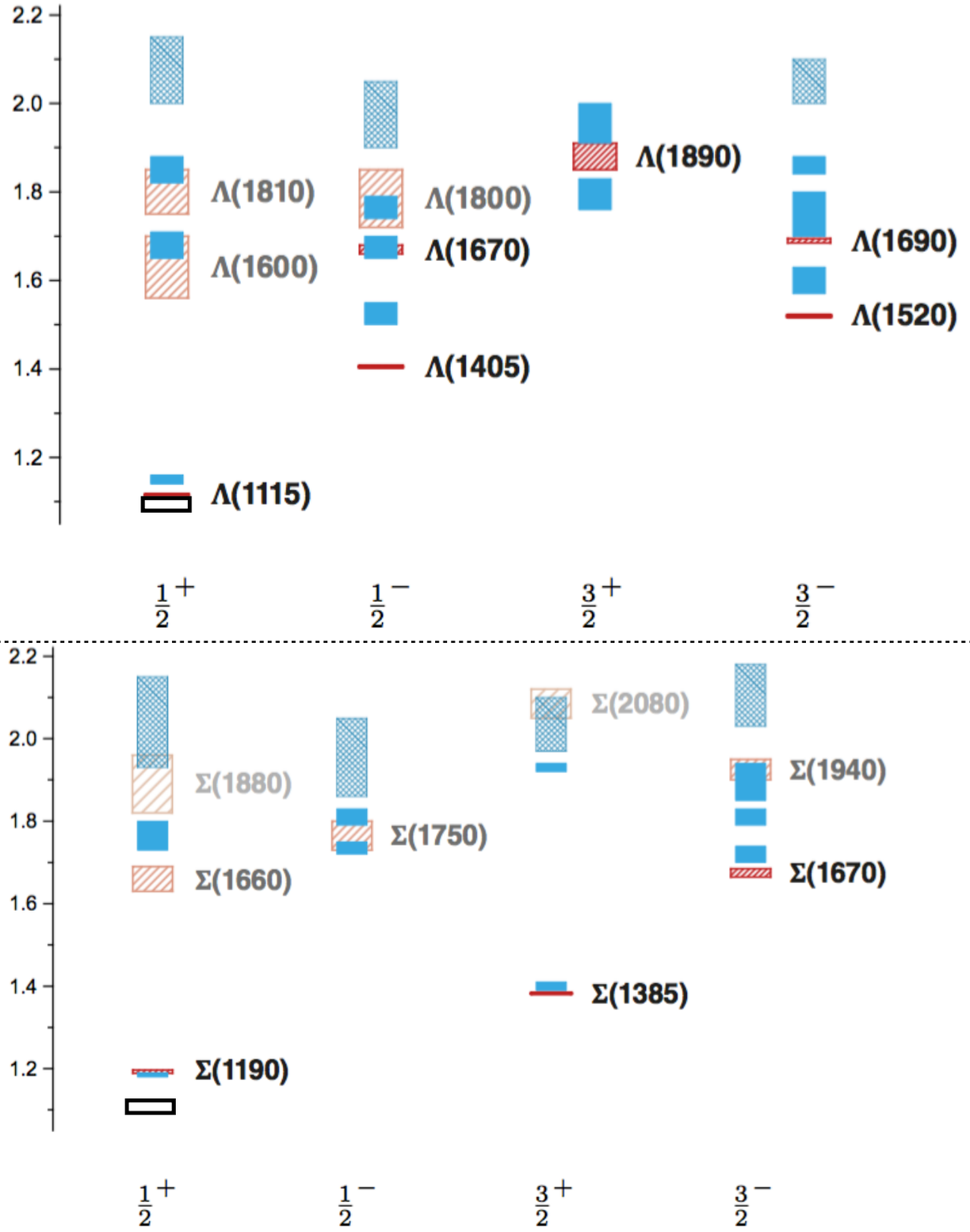



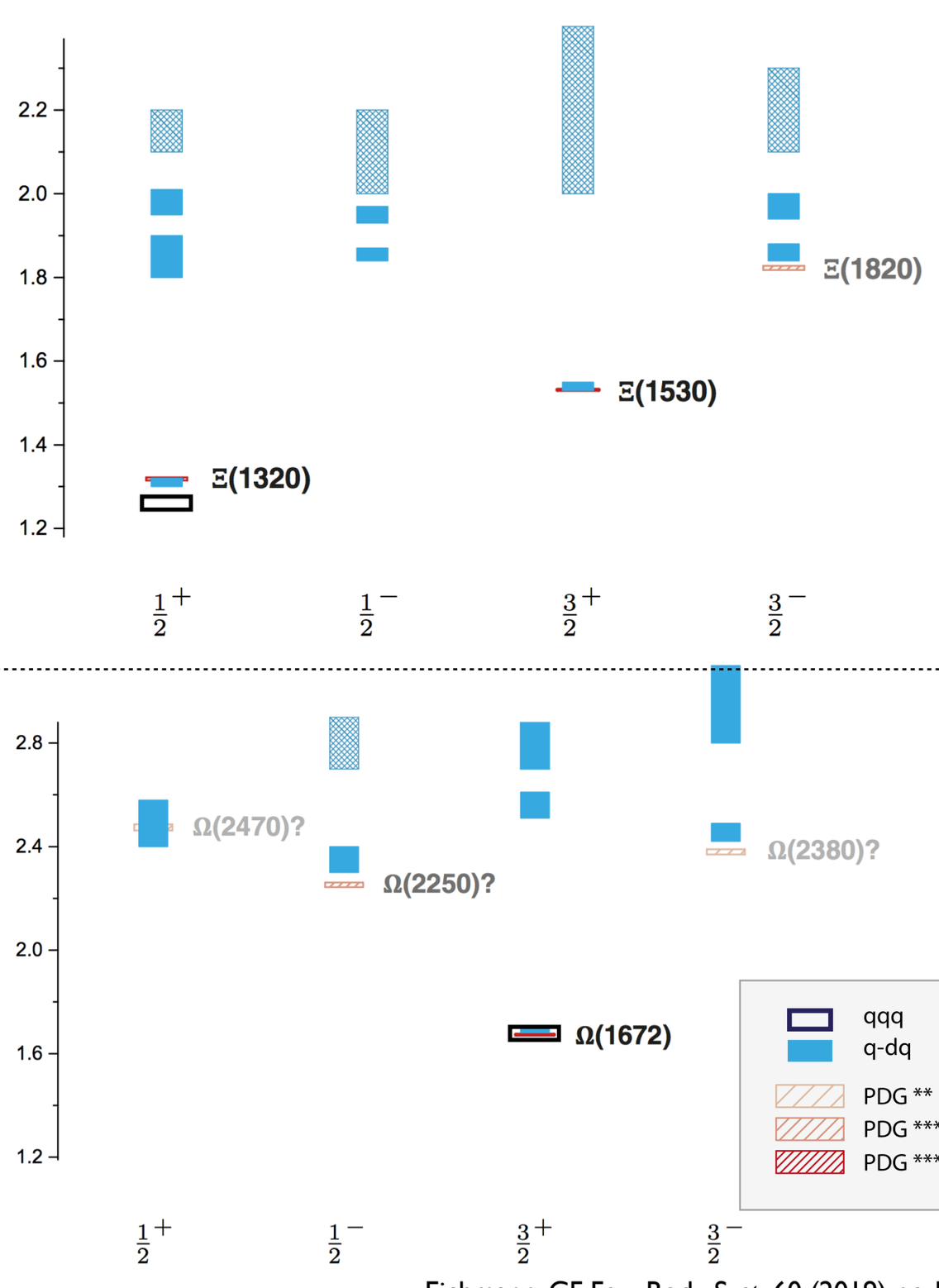

heavy-light mesons




hadrocharmonium

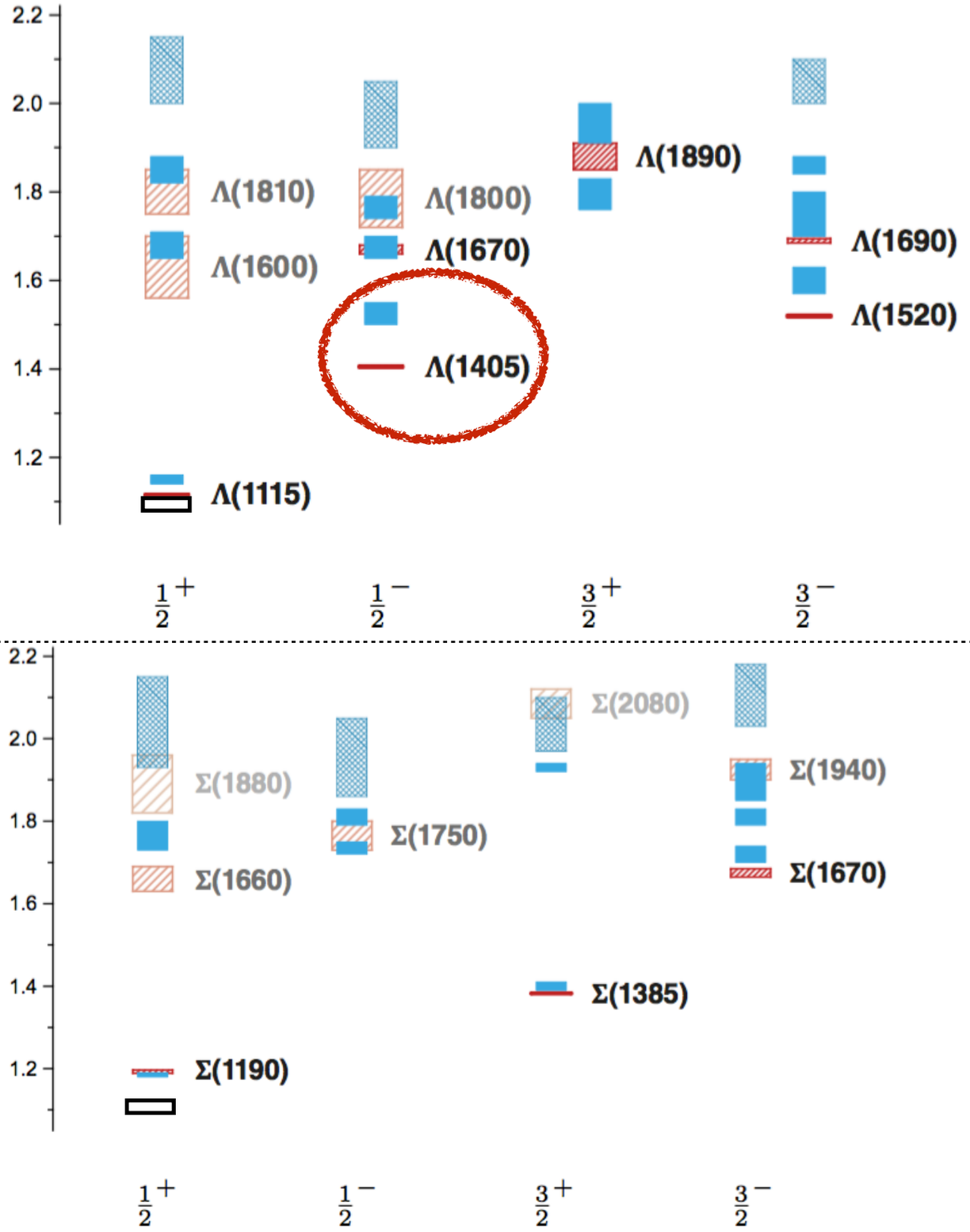
# Strange baryon spectrum: DSE-RL (preliminary !)

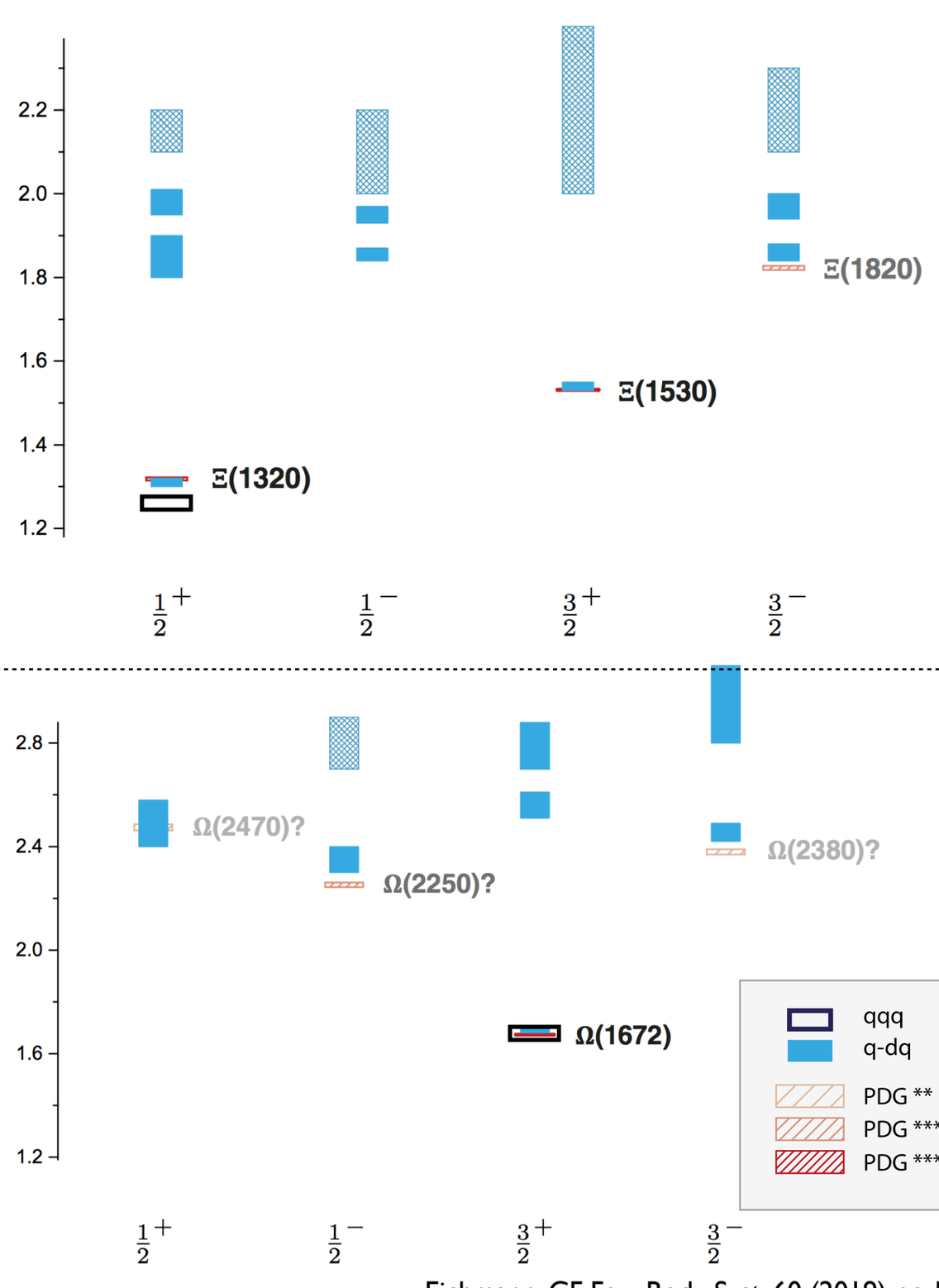



Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
CF, Eichmann PoS Hadron 2017 (2018) 007
Sanchis-Alepuz, CF, PRD 90 (2014) 096001

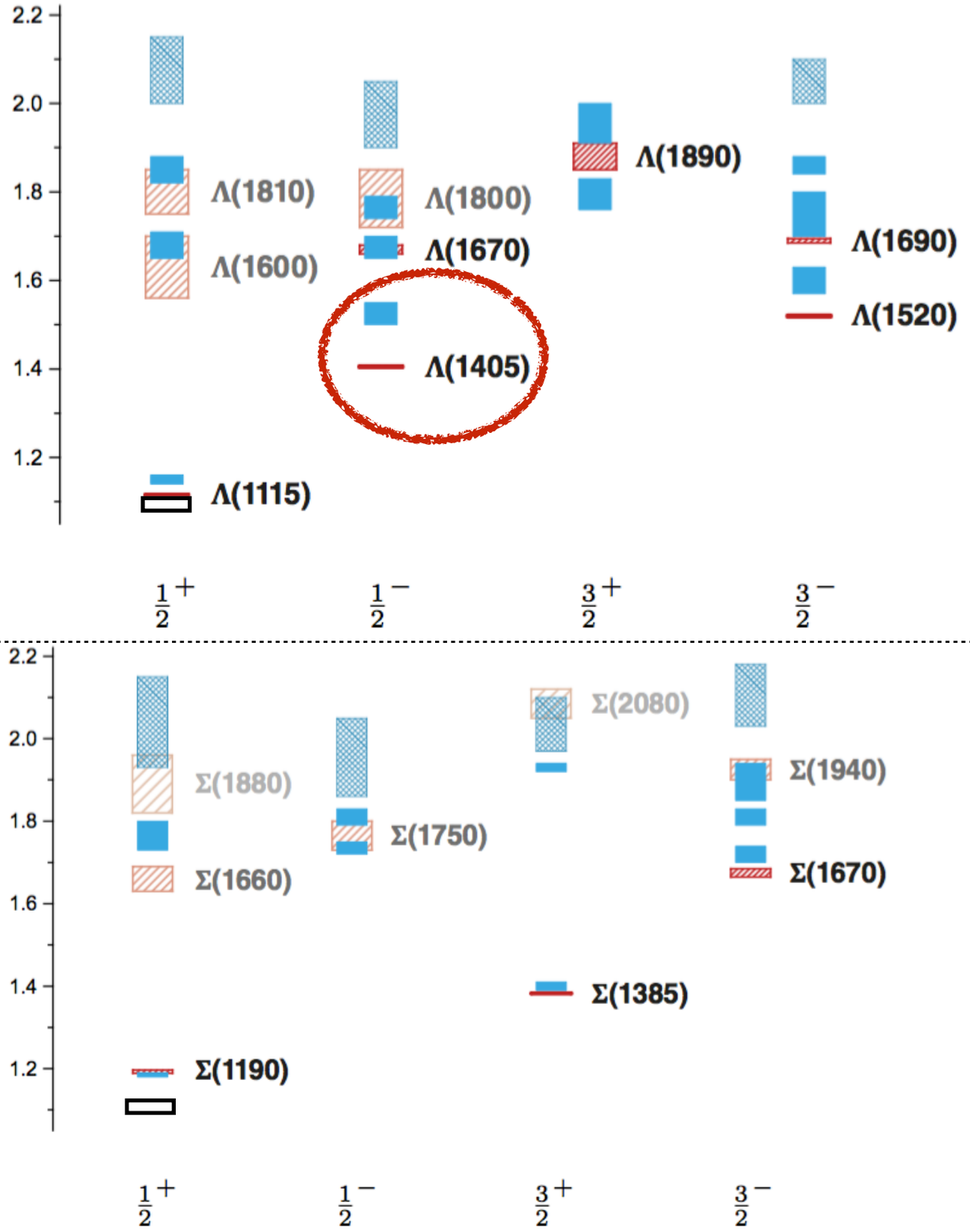
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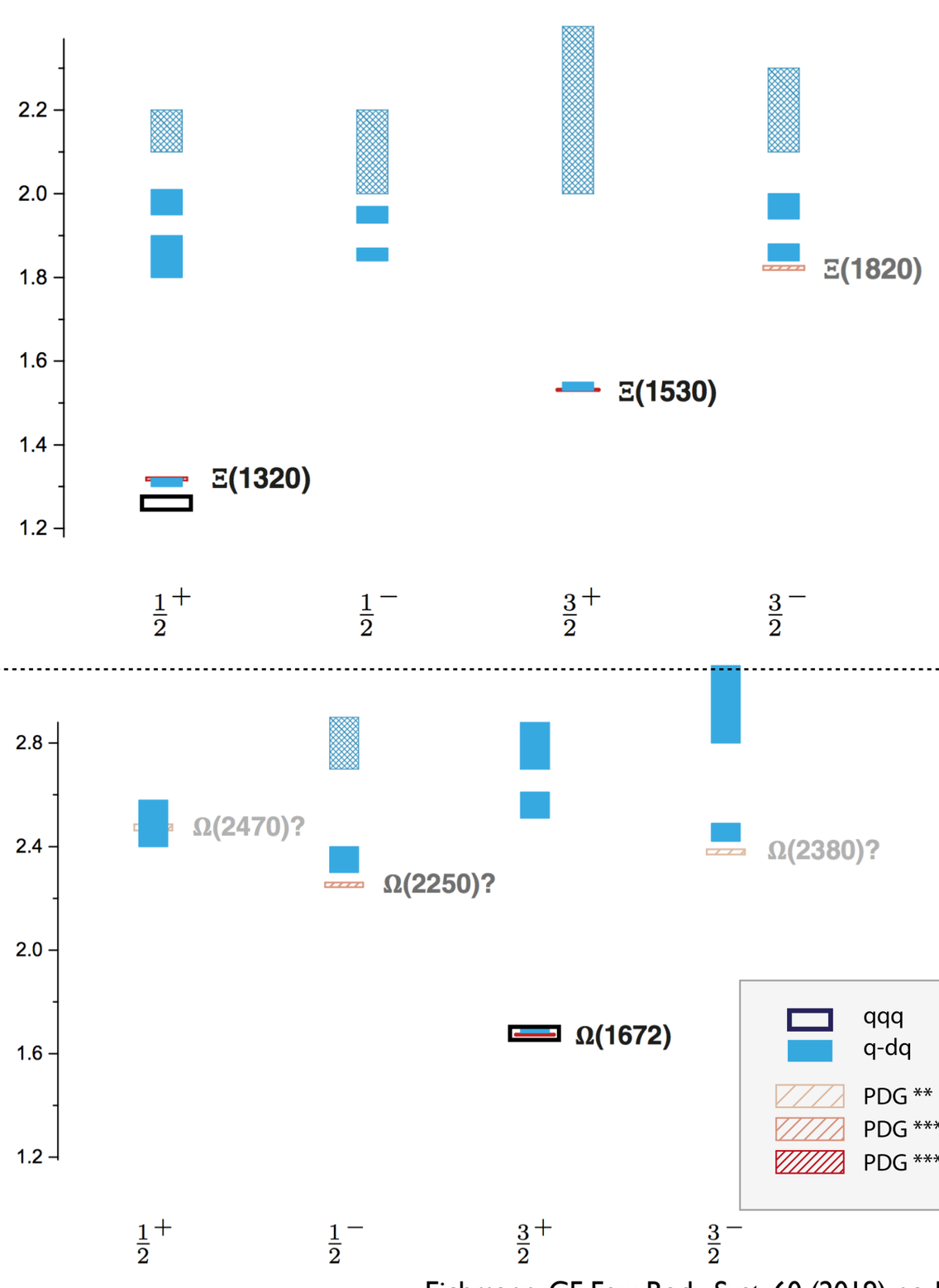



Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
CF, Eichmann PoS Hadron 2017 (2018) 007
Sanchis-Alepuz, CF, PRD 90 (2014) 096001

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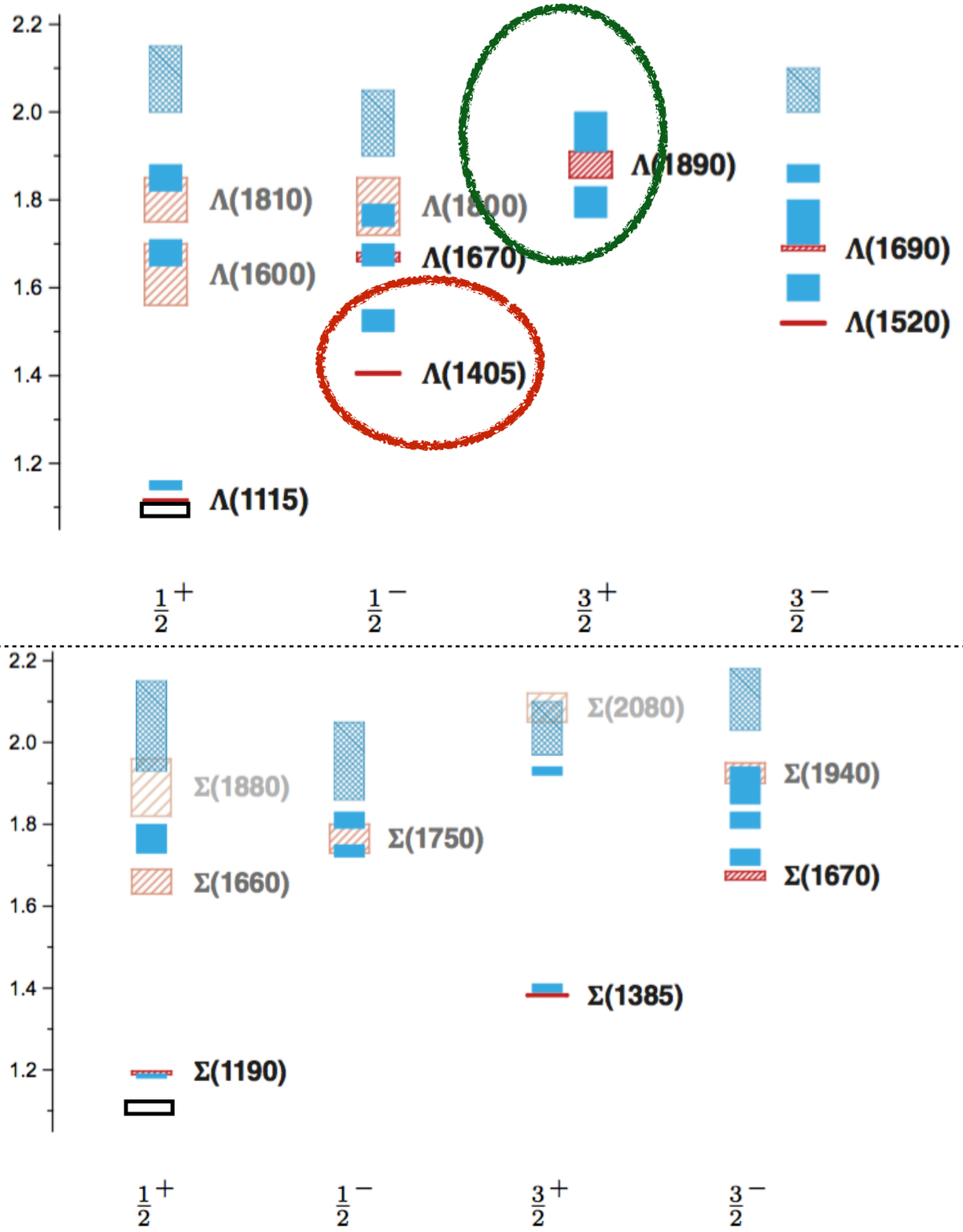


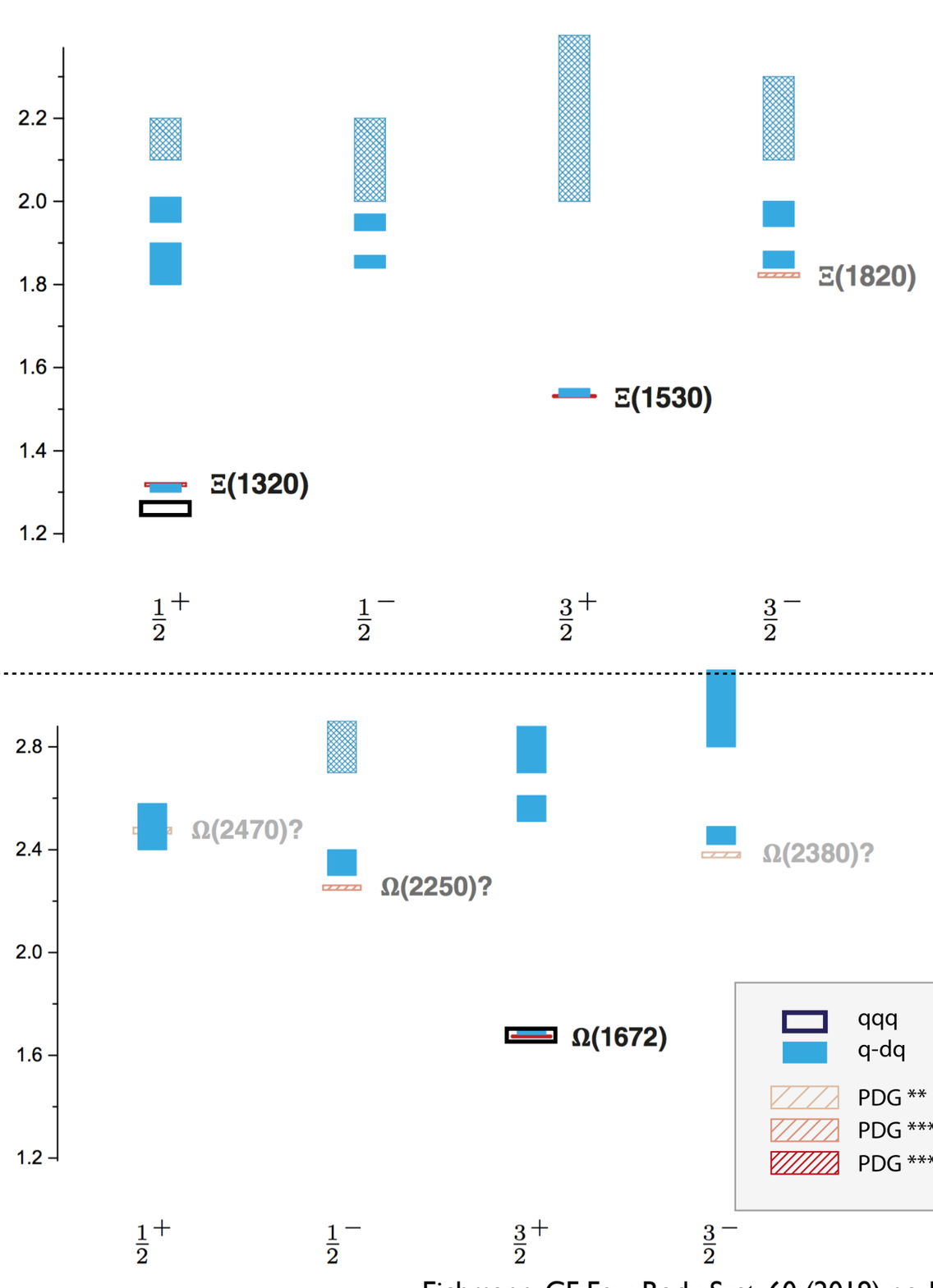




New states: Bonn-Gatchina (talk of M. Matveev)

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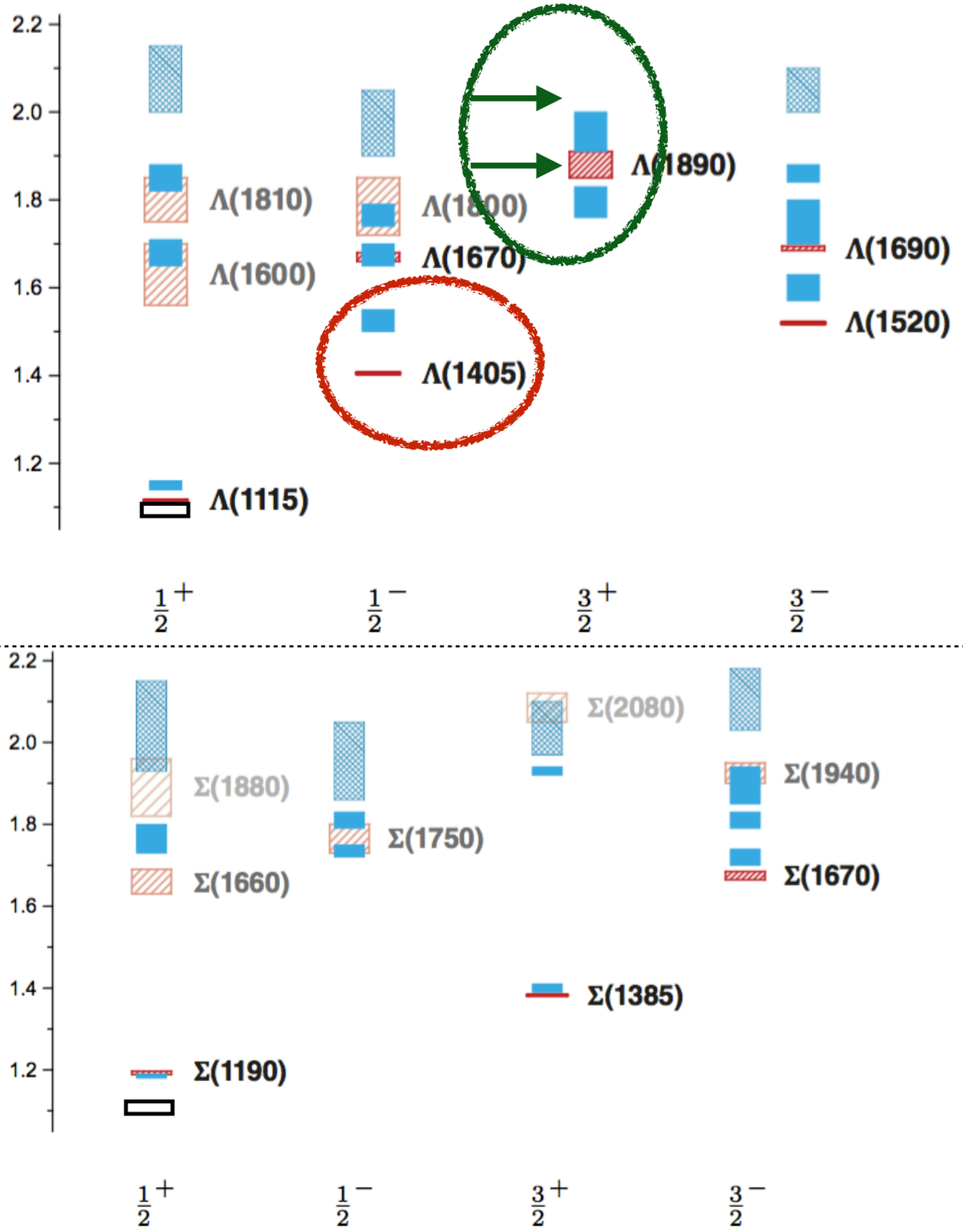


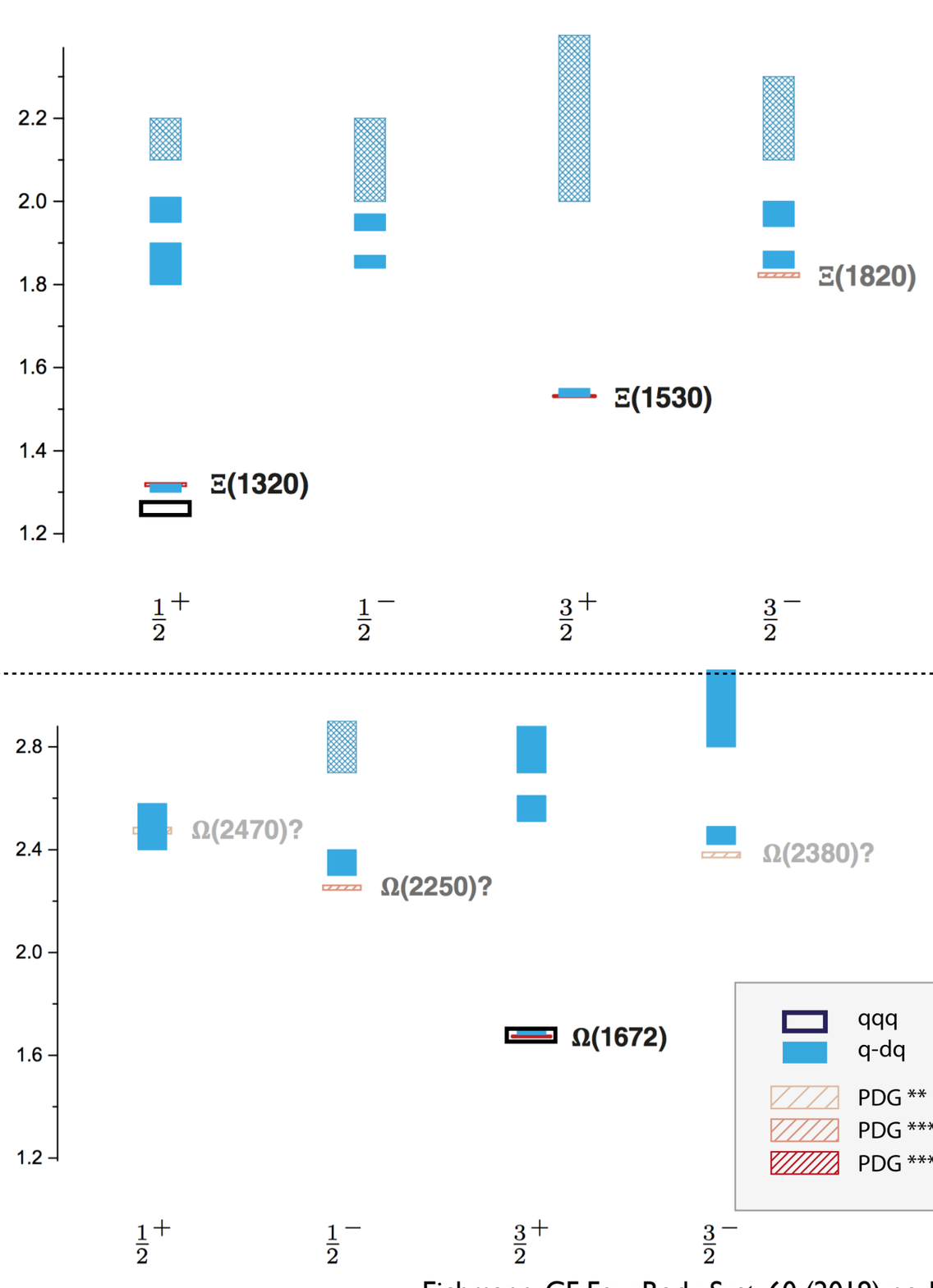




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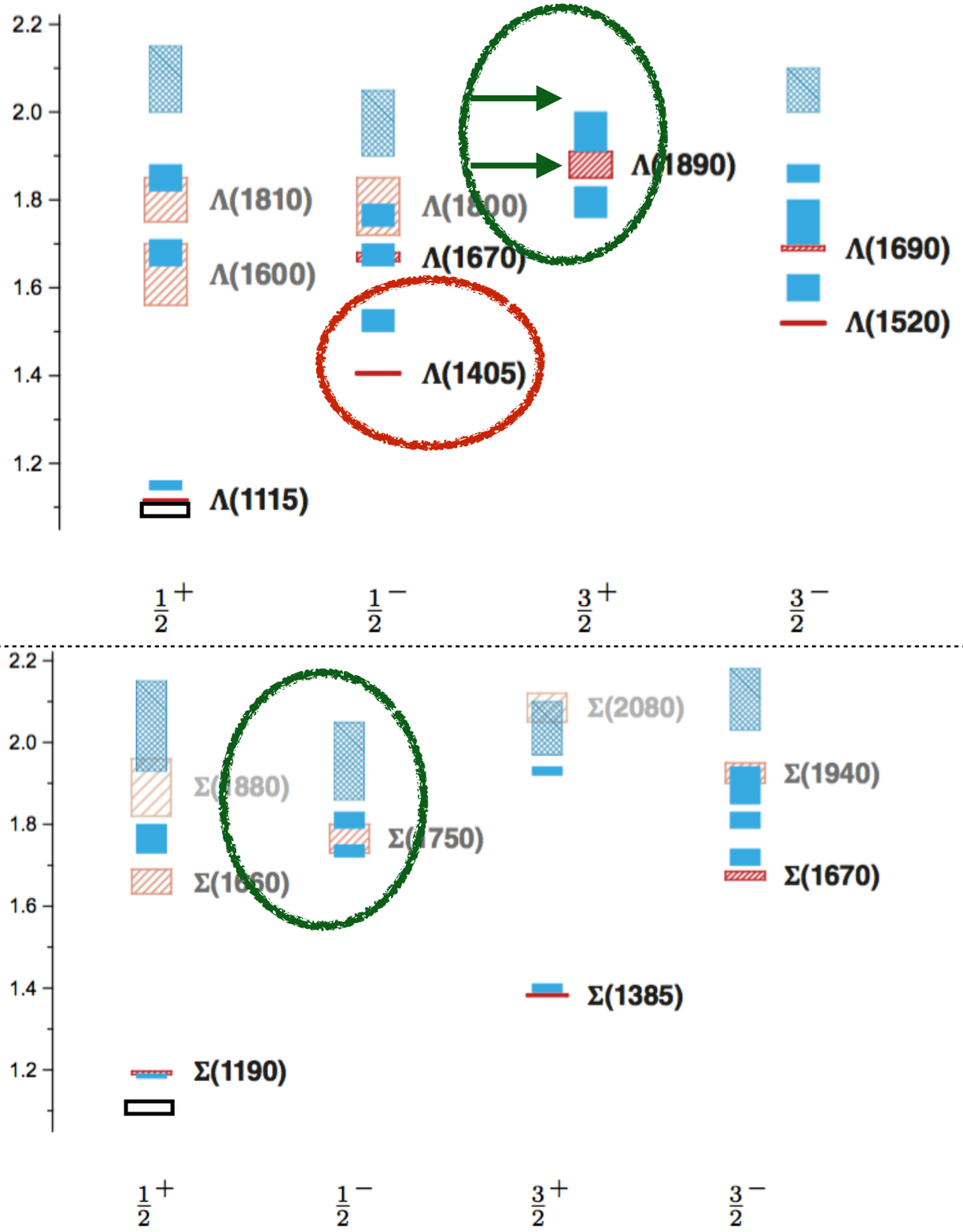


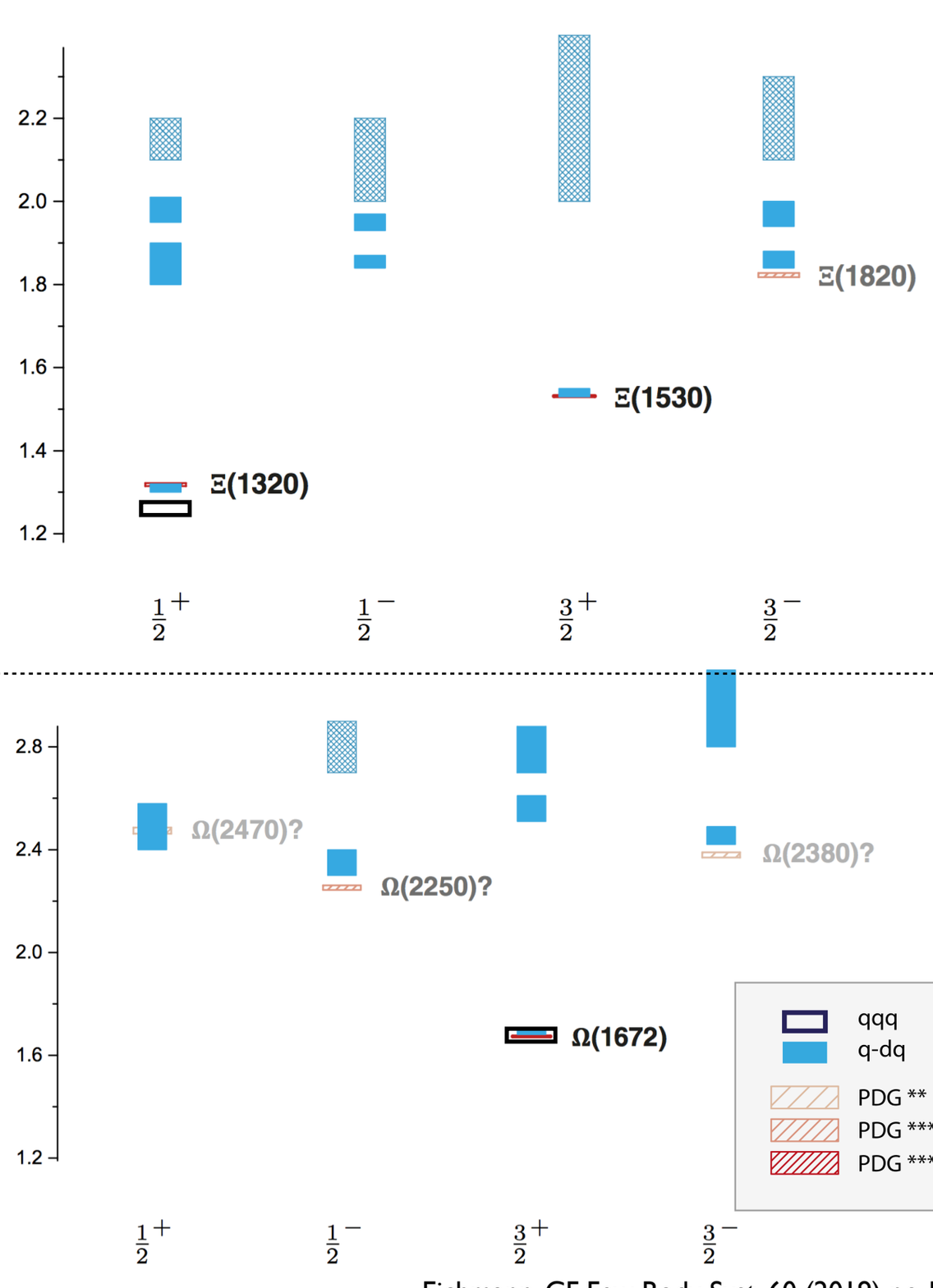




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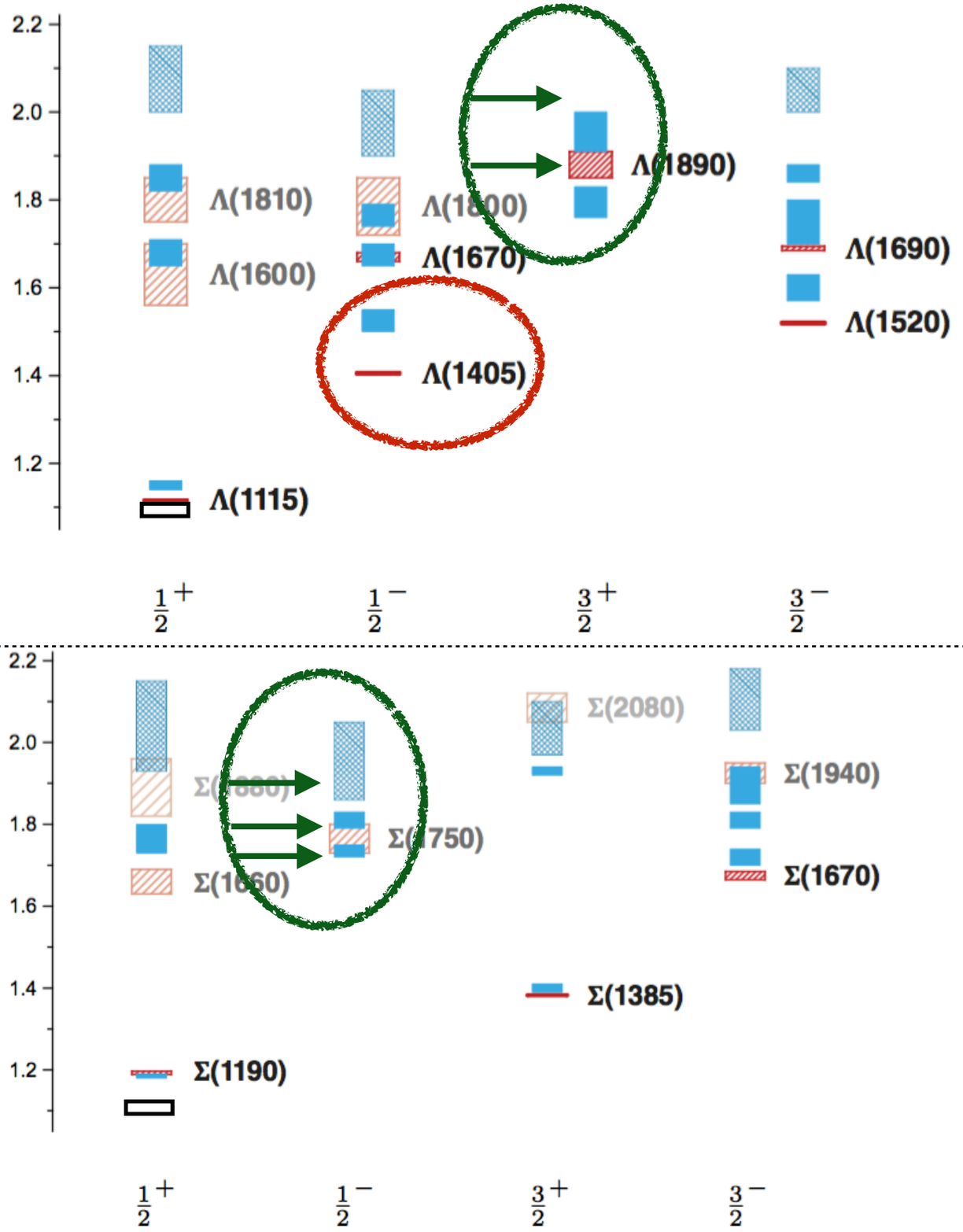


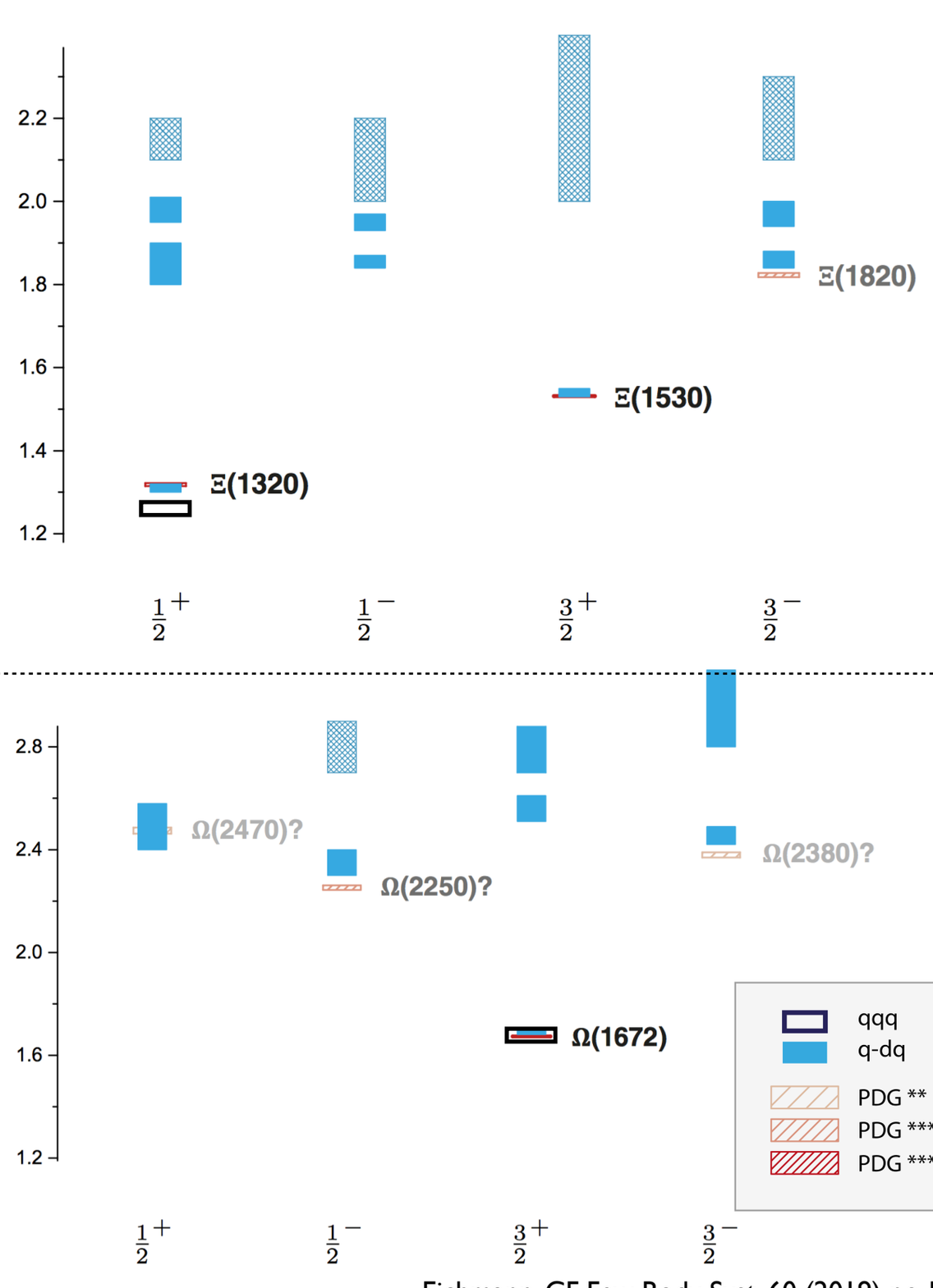




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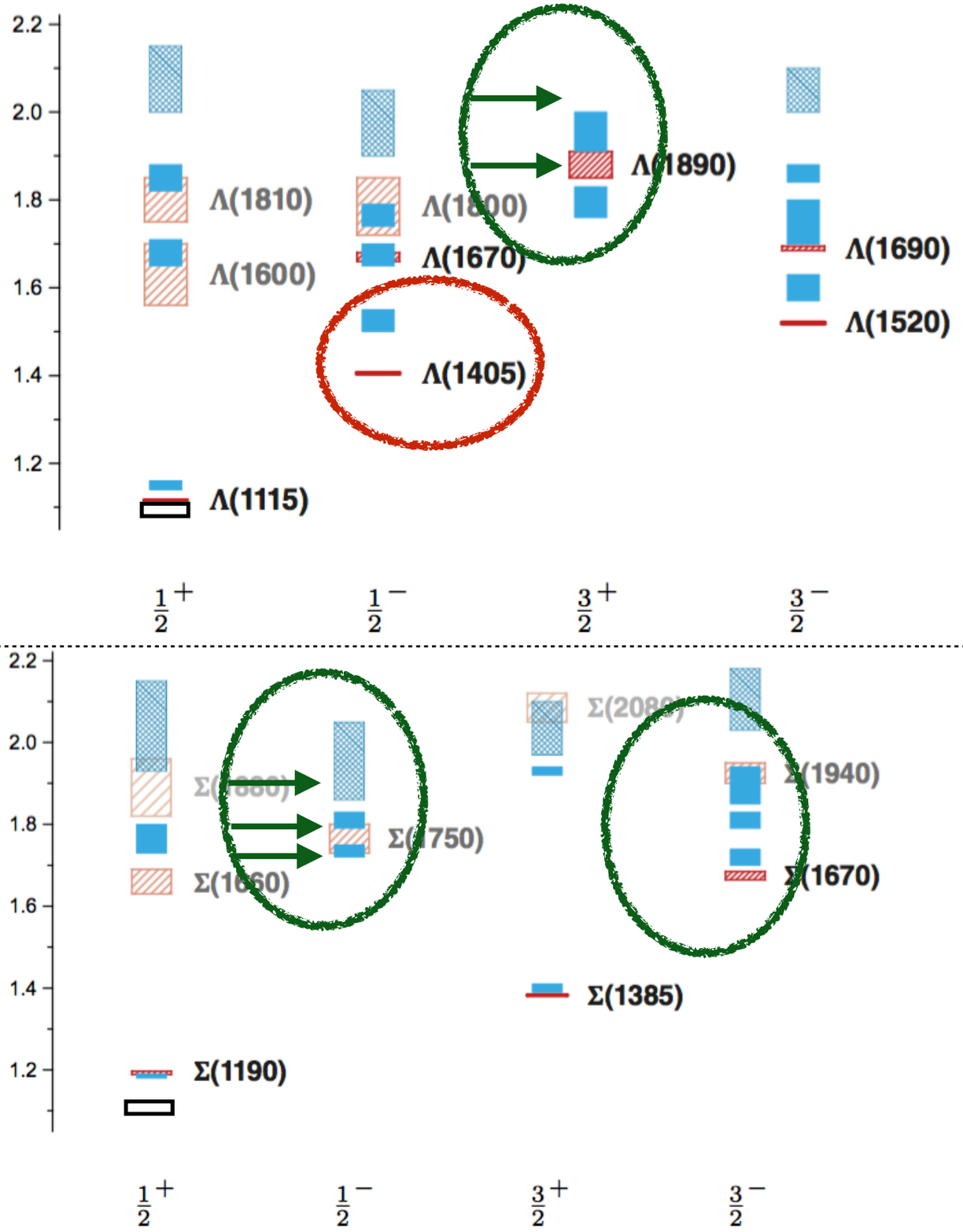


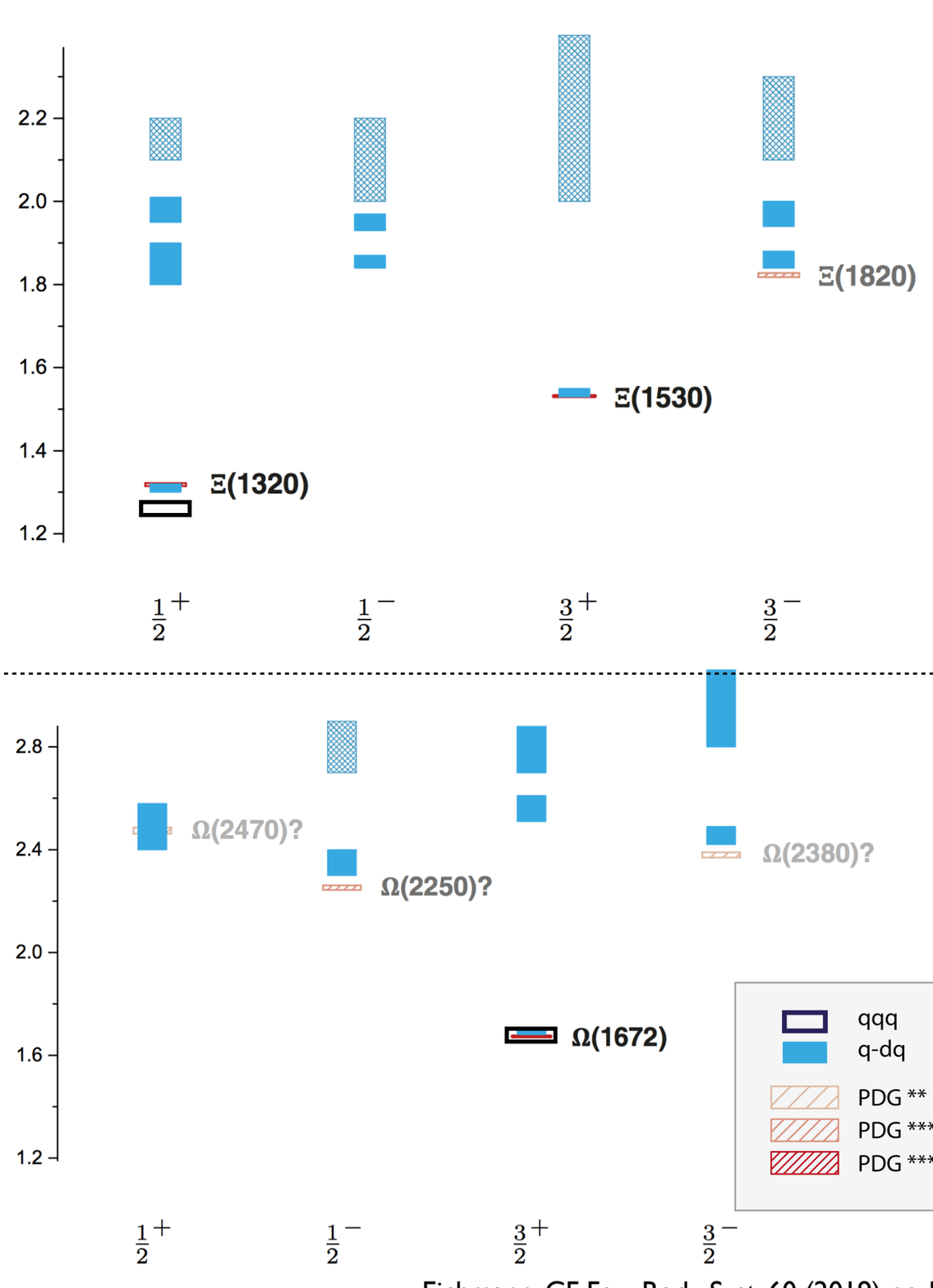




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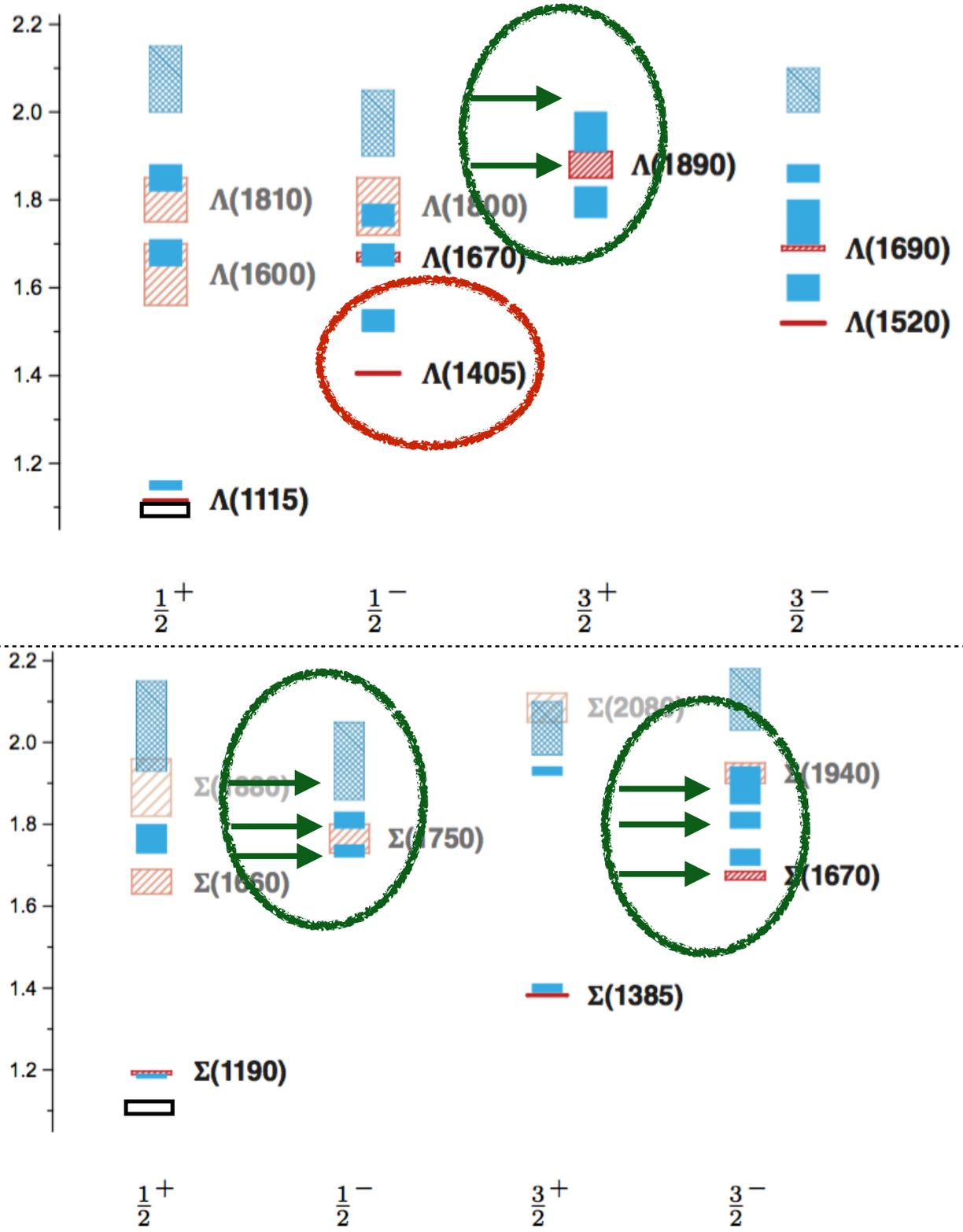


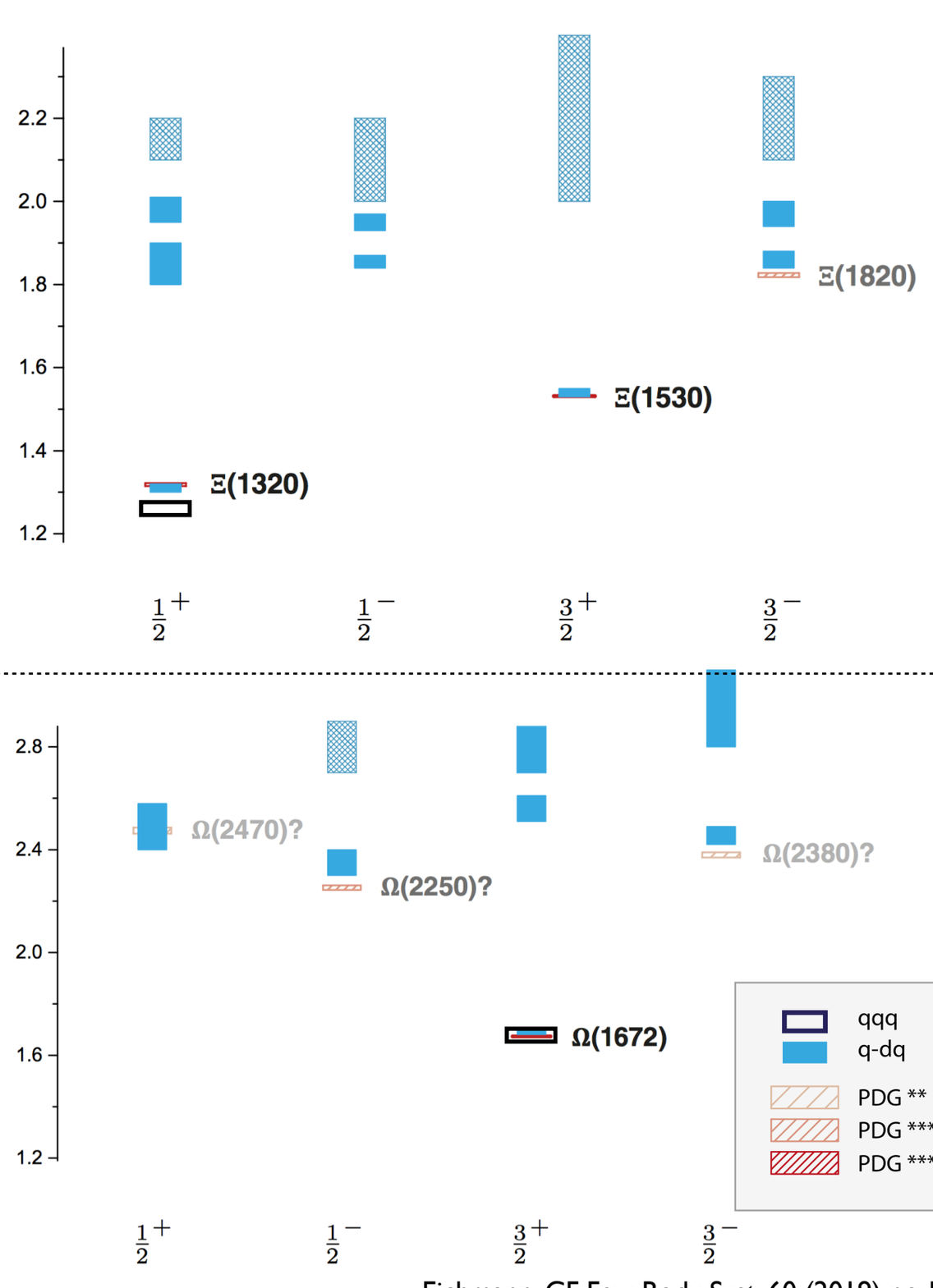




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