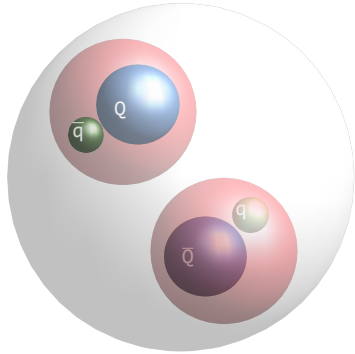
CHIRAL EFT FOR DOUBLY HEAVY EXOTICS

Christoph Hanhart | IAS Forschungszentrum Jülich

Hadronic Molecules



Guo et al., Rev.Mod.Phys.90(2018)015004

- are few-hadron states, bound by the strong force
- do exist: light nuclei.
  e.g. deuteron as pn & hypertriton as Λd bound state
- are located typically close to relevant continuum threshold;
  e.g., for $E_B = m_1 + m_2 - M$ and $\gamma = \sqrt{2\mu E_B}$
  - $E_B^{\text{deuteron}} = 2.22$ MeV $(\gamma = 45$ MeV$)$
  - $E_B^{\text{hypertriton}} = (0.11 \pm 0.03)$ MeV (to Λd) $(\gamma = 13$ MeV$)$

Are there hadronic molecules in the double heavy sector?

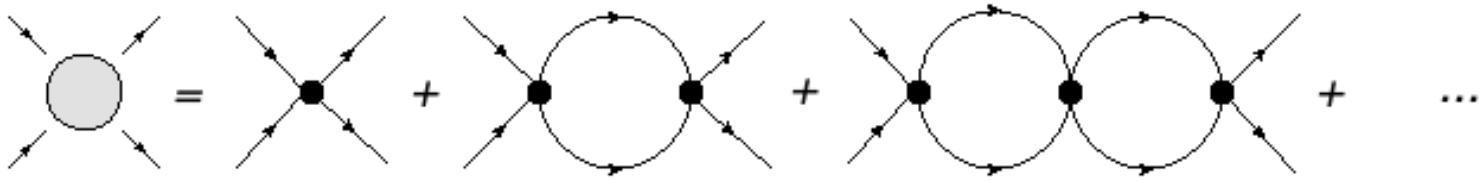
To address this we need to understand, how

- to describe two-hadron states theoretically
- the picture would change for more compact structures
- to exploit QCD symmetries to get a more global view

Two Particle Scattering

following C.H., arxiv: 2504.06043 [hep-ph]

EFT for near threshold poles:



$$T = C + C\Pi(E)C + C\Pi(E)C\Pi(E)C + \ldots$$

$$= C\sum_{n=0}^{\infty}(C\Pi(E))^n \quad \Longrightarrow \quad \boxed{T(E) = \frac{1}{C^{-1} - \Pi(E)}}$$

with $\Pi(E) = \int \frac{d^3q}{(2\pi)^3} \frac{F_\Lambda^2(q)}{E - q^2/(2\mu) + i\epsilon} = -\frac{\mu}{2\pi}\left(\Lambda + i\sqrt{2\mu E + i\epsilon}\right) + \mathcal{O}(\Lambda^{-1})$

$T(E)$ develops pole at $E = -E_B$ for $C^{-1} = \Pi(-E_B) = -\mu/(2\pi)\,(\Lambda - \gamma)$

... with binding momentum $\gamma = -i\sqrt{2\mu(-E_B) + i\epsilon} = \sqrt{2\mu E_B}$ (first sheet!)

Thus we have for the renormalised T matrix:

$$\boxed{T(E) = \left(\frac{2\pi}{\mu}\right) \frac{1}{\gamma + i\sqrt{2\mu E + i\epsilon}}} = \frac{g_{\text{eff}}^2}{E + E_B} + \text{reg. terms;} \qquad \boxed{g_{\text{eff}}^2 = \frac{2\pi\gamma}{\mu^2}}$$

Introducing compact component

Weinberg, Phys.Rev.137(1965)B672

For this introduce coupled channel ansatz:

$$|\Psi\rangle = \begin{pmatrix} \lambda|\psi_0\rangle \\ \chi(\mathbf{p})|h_1 h_2\rangle \end{pmatrix} \quad \text{solving} \quad \hat{\mathcal{H}}|\Psi\rangle = E|\Psi\rangle, \;\; \hat{\mathcal{H}} = \begin{pmatrix} \hat{H}_c & \hat{V} \\ \hat{V} & \hat{H}^0_{hh} \end{pmatrix}$$

where $\hat{H}^0_{hh} = p^2/(2\mu)$ (hh-interactions absorbed into pole term)

Weinberg, Phys.Rev.130 (1963)776

Thus $\boxed{\lambda^2 = |\langle\psi_0|\Psi\rangle|^2 = \text{Probability to find } \psi_0 \text{ in } \Psi}$ is quantity of interest!

Schrödinger eq. gives $\chi(p) = \lambda f(p^2)/(E - p^2/(2\mu))$ with $f(p^2) = \langle h_1 h_2|\hat{V}|\psi\rangle$

$$1 = \langle\Psi|\Psi\rangle = \lambda^2\left(1 + \int \frac{d^3p}{(2\pi)^3}\frac{f^2(p^2)}{(E_B + p^2/(2\mu))^2}\right) \text{ using } f^2(p^2) = g_0^2 + \mathcal{O}\left(p^2/\beta^2\right)$$

$$= \lambda^2\left(1 + g_0^2\int \frac{d^3p}{(2\pi)^3}\frac{1}{(E_B + p^2/(2\mu))^2} + \mathcal{O}\left(\frac{\gamma}{\beta}\right)\right)$$

$$= \lambda^2\left(1 + \frac{g_0^2\mu^2}{2\pi\gamma} + \mathcal{O}\left(\frac{\gamma}{\beta}\right)\right) \longrightarrow \boxed{g_0^2 = \frac{2\pi\gamma}{\mu^2}\left(\frac{1}{\lambda^2} - 1 + \mathcal{O}\left(\frac{\gamma}{\beta}\right)\right)}$$

and: $\lambda^2 = Z$ =wave function renormalisation

Discussion

We had $\boxed{g_0^2 = \frac{2\pi\gamma}{\mu^2}\left(\frac{1}{\lambda^2} - 1 + \mathcal{O}\left(\frac{\gamma}{\beta}\right)\right)}$ with $\boxed{\lambda^2 = \text{Probability to find } \psi_0 \text{ in } \Psi}$

g_0 also appears in the scattering amplitude (without range corrections):

$$T = \frac{g_0^2}{E + E_B + g_0^2 \Pi^{\text{reg.}}(E)} = \frac{g_0^2}{E + E_B + g_0^2 \mu/(2\pi)(ip + \gamma)} \quad \left[E = \frac{p^2}{2\mu}\right]$$

$$= \frac{g_{\text{eff}}(\lambda^2)^2}{E + E_B} + \text{regular terms} \quad \Longrightarrow \quad \boxed{g_{\text{eff}}(\lambda^2)^2 = Z\, g_0^2 = \frac{2\pi\gamma}{\mu^2}\left(1 - \lambda^2\right)}$$

$\lambda^2 = 0$ (pure mol.) $\rightarrow$ earlier results. Matching to effective range expansion:

$$T(E) = -\frac{2\pi}{\mu}\frac{1}{1/a + (r/2)p^2 - ip} \Longrightarrow \begin{cases} a = -2\frac{1-\lambda^2}{2-\lambda^2}\left(\frac{1}{\gamma}\right) + \mathcal{O}\left(\frac{1}{\beta}\right) \\ r = -\frac{\lambda^2}{1-\lambda^2}\left(\frac{1}{\gamma}\right) + \mathcal{O}\left(\frac{1}{\beta}\right) \end{cases}$$

$\lambda^2 = 0 \Longrightarrow a = -1/\gamma + \mathcal{O}(1/\beta)$; $\boxed{r = \mathcal{O}(1/\beta) > 0}$ (for single channel case)

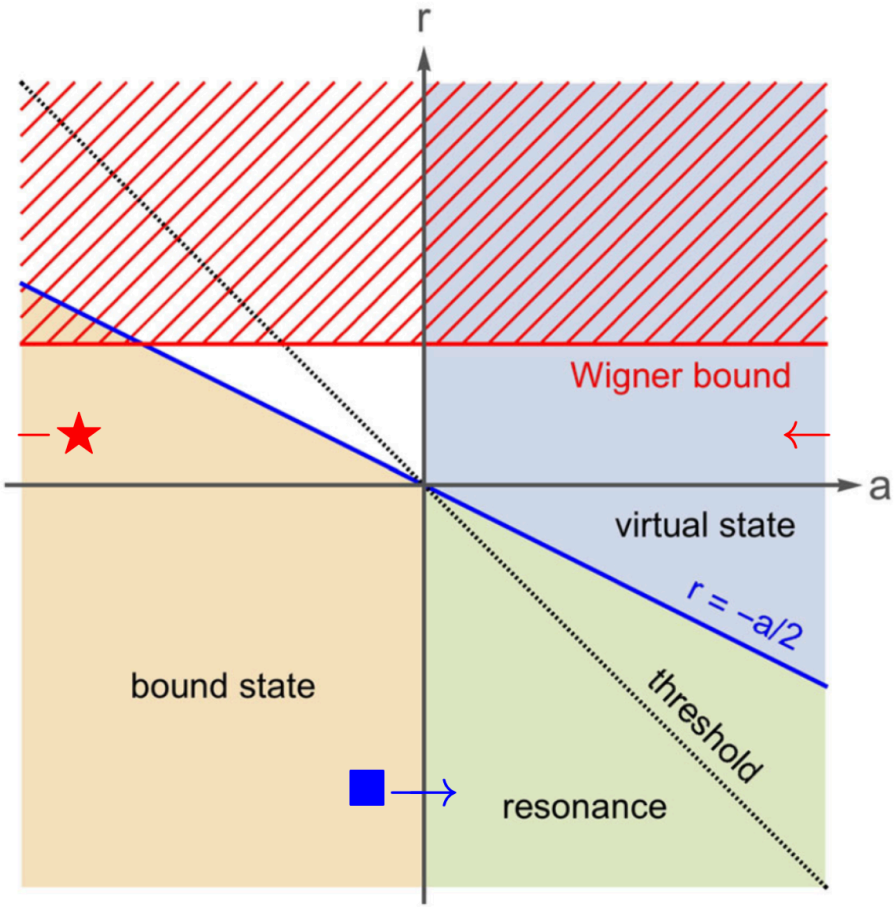
Admixture of compact component reduces a and gives sizeable neg. r

$\Longrightarrow$ would lead to bad convergence of EFT!

Virtual states and Resonances

Matuschek et al., EPJA57(2021)101

What happens to shallow bound state, when interaction weakened slightly?



e.g. reduction of quark mass

pure molecule
($a = -1/\gamma; r = \mathcal{O}(1/\beta) > 0$)

$\Longrightarrow$ goes to virtual state

purely compact state
($a = \mathcal{O}(1/\beta) < 0; |r| \gg \mathcal{O}(1/\beta); r < 0$)

$\Longrightarrow$ goes to resonance

In particular:

Sizeable compact component

$\rightarrow$ no transition through virtual state

see also: Baru et al., PLB586(2004)53; Gamermann et al., PRD81(2010)014029; Aceti, Oset, RD86(2012)014012; Nagahiro, Hosaka, PRC90(2014)065201; Guo, Oller, PRD93(2016)096001; Kamiya, Hyodo, PTEP(2017)023D02; Kang, Oller, EPJC77(2017)399; Sekihara, PRC95(2017)025206; Oller, AnnalsPhys.396(2018)429; Li et al., PRD105(2022)L071502; Dai, Oset, Nuovo Cim.C 47(2024)4; Albaladejo, Nieves, EPJC82(2022)724; Kinugawa, Hyodo, EPJA61(2025)154; ...

Summary: Properties of Molecules

- Are located in the vicinity of thresholds of the constituents
- For bound states the coupling to the constituents gets maximal
- The preferred decay channel is into the constituents,
  regardless phase space suppression
  $\Longrightarrow$ explains why Γ small, regardless open channels
- Scattering length is large
  $\Longrightarrow$ Near threshold enhancements!
- Positive, natural effective range
  $\Longrightarrow$ Important exception: Large coupled channel effects
- Dominant isospin violation: Mass differences of constituents
- Virtual states are always molecular

Disclaimers and Outline

The method presented is 'diagnostic' — especially,

- it does not allow for conclusions on the binding force;
- it allows one only to study individual states;
- quantitative interpretation gets lost when states get bound too deeply ('uncertainty' $\sim R\gamma$)

To go beyond tailor made effective field theories needed

In this talk I present how a chiral unitary theory can be applied to systems of two heavy hadrons and allows for a study of the $T_{cc}(3875)$, $X(3872)$ (& its isovector partners) and the LHCb pentaquarks

EFT for doubly heavy molecules

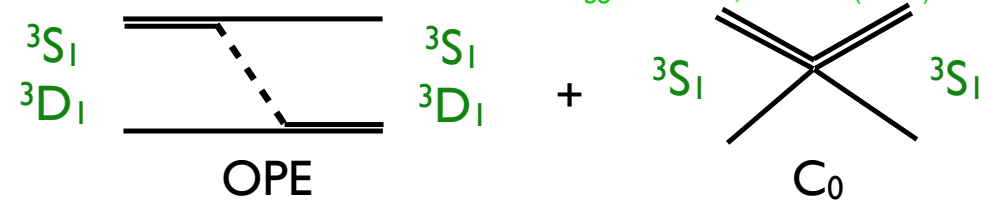
Schemes currently proposed (only initial work cited)—analogous to *NN*

- Contact EFT: no pions — AlFiky et al., PLB640(2006)238
- *X*-EFT: pert. pions — Fleming et al., PRD76(2007)034006
- chiral EFT: non.-pert. pions — Baru et al., PRD84(2011)074029
  - broadest range of applicability
  - Pion dynamics fully under control (3-body unitarity) $\Longrightarrow$ we use this

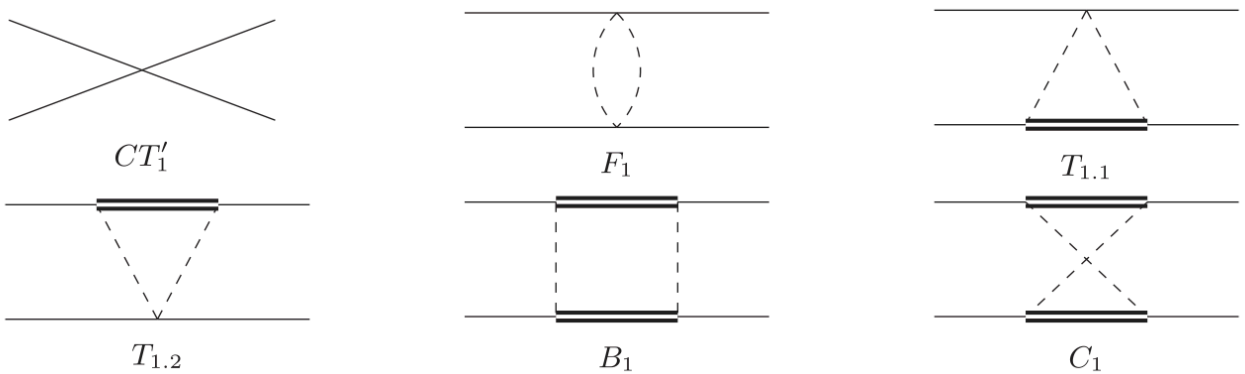
Thus we solve LS-equation

For T_{cc}: Du et al., PRD105(2022)014024

$T = V + VGT$ with $V_{LO} =$



$V_{NLO} =$



Note:

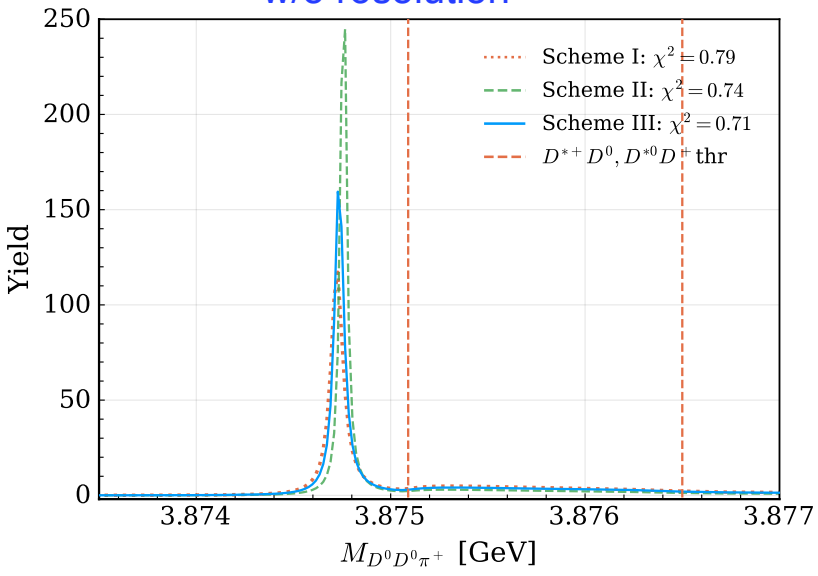
reducible box enhanced by $\pi\mu/p$

Observation: Loops can be absorbed into counter terms — Chacko et al. PRD 111 (2025)034042

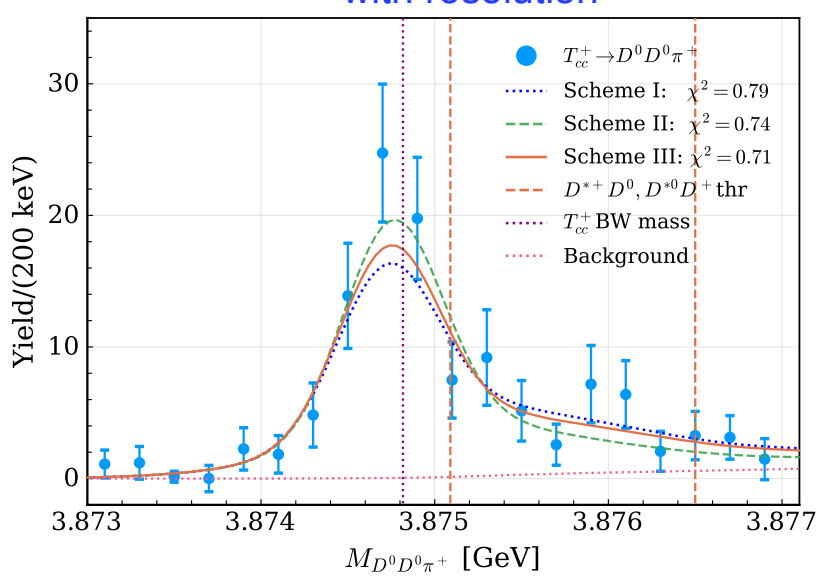
Example I: $T_{cc}(3875)$

data: LHCb, Nature Physics 18(2022)751

w/o resolution



with resolution



Scheme	I	II	III
Description	2-body unitarity: No OPE, static D^* width	Incomplete 3-body unitarity: No OPE, dynamical D^* width	full 3-body unitarity: OPE + dynamical D^* width
Pole [keV]	$-368^{+43}_{-42} - i(37 \pm 0)$	$-333^{+41}_{-36} - i(18 \pm 1)$	$-356^{+39}_{-38} - i(28 \pm 1)$
χ^2	0.79	0.74	0.71

Mass fitted

Width predicted

- Precision needs 3 body dynamics (problem: experimental resolution)
- $r = -2.4 \pm 0.01 \pm 0.85$ fm, but $r_0 = +1.38 \pm 0.01 \pm 0.85$ fm*

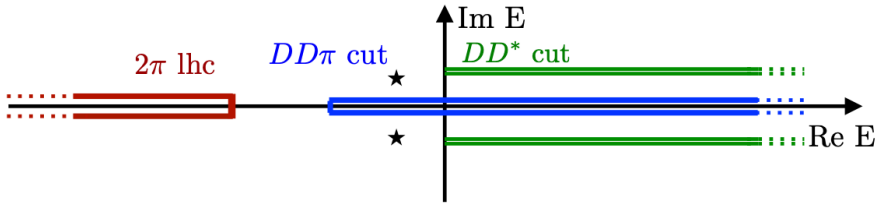
$\Longrightarrow$ T_{cc}^+ qualifies as isoscalar DD^* molecule

*: isospin viol. effect from D^+D^{*0} removed

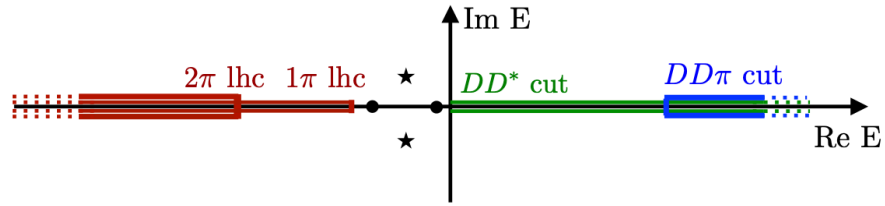
Baru et al., PLB833(2022)137290

Lattice study - chiral extrapolation

$m_\pi < \Delta M$ (physical situation)



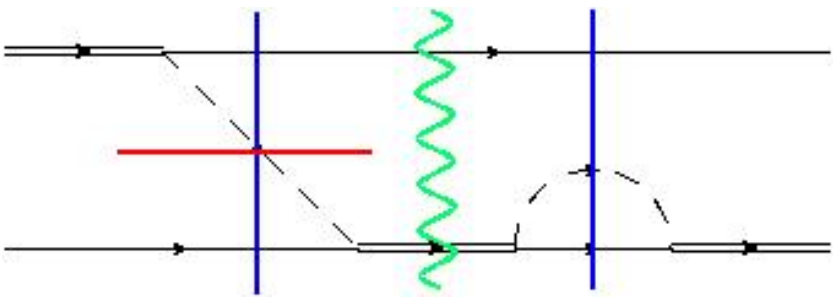
$m_\pi > \Delta M$ (lattice set up)



For example DD^* Lüscher analysis @ $m_\pi = 280$ MeV

Padmanath & Prelovsek, PRL129(2022)032002

Then D^* is stable ($m_\pi > \Delta M = M_{D^*} - M_D$) $\rightarrow$ cut structure:



right-hand three-body cuts

right-hand two-body cut

NEW: left-hand cut (lhc)
from one π exchange

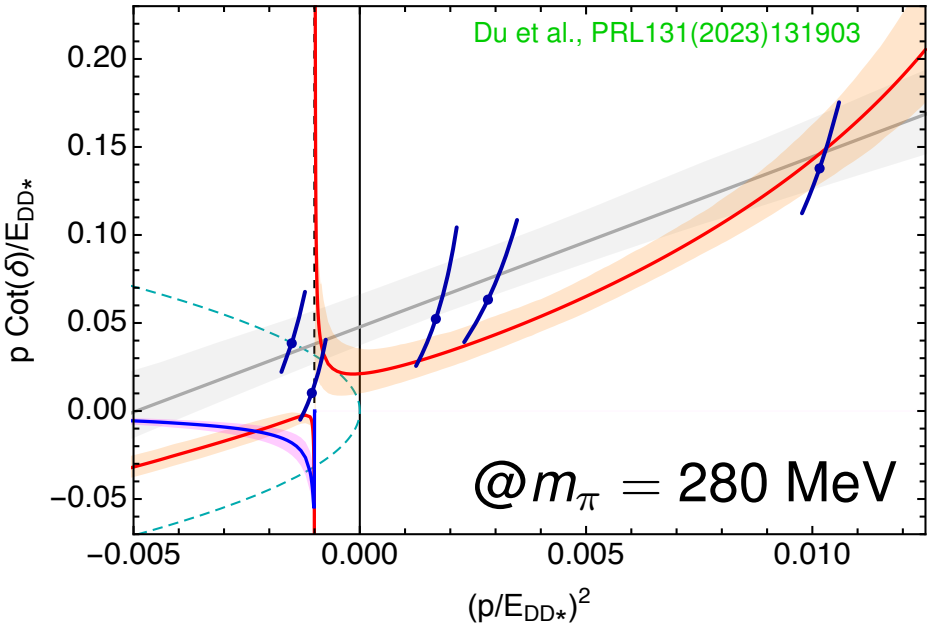
Left-hand cut appears as new non-analyticity very near-by

$\Longrightarrow$ Study its impact on phase-shifts

Du et al., PRL131(2023)131903

Additional non-analyticity calls for careful analytic continuation

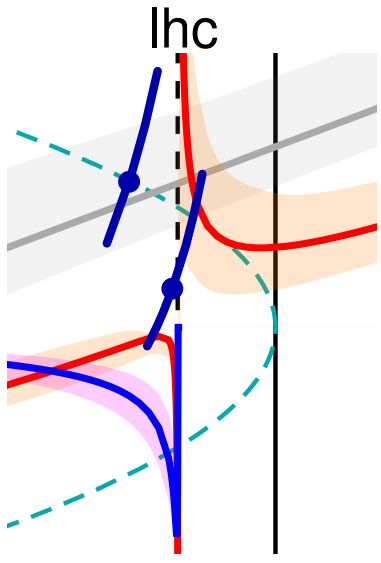
Pole Trajectories



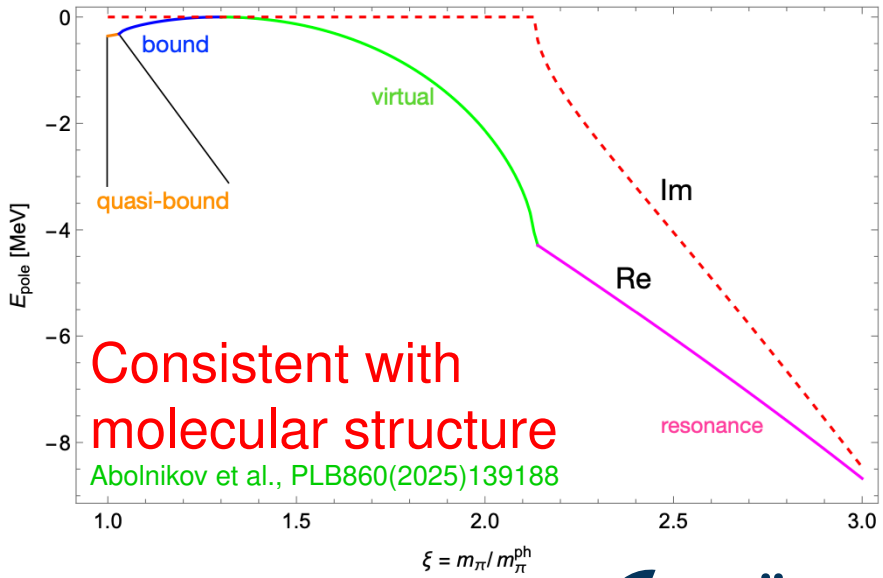
Zoom:

virtual state

bound state



- Lattice analysis w/o lhc
  $\Longrightarrow$ ignore lowest levels
- Subtle interplay long and short ranged interactions
  $\Longrightarrow$ New ERE needed
  Du et al., PRL135 (2025) 1, 011903
- Lüscher needs modification
  Raposo and Hansen, JHEP08(2024)075;
  Meng et al., PRD109(2024)L071506;
  Bubna et al., JHEP05(2024)168;
  Dawid et al., PLB864 (2025) 139442



Consistent with molecular structure
Abolnikov et al., PLB860(2025)139188

Example II: $\chi_{c1}(3872)$ aka $X(3872)$

Allow for $I = 0$ and $I = 1$ interaction; add inelastic channels + ρ-ω mixing

Ji et al., [arXiv:2502.04458 [hep-ph]]

- Excellent fit
- X as bound state

  $E_X = \left(-160^{+57}_{-74} - i\,125^{+23}_{-38}\right)$ keV

  with 2.7 σ below $D^0\bar{D}^{*0}$ threshold
- W_{c1} with

  $I^G(J^{PC}) = 1^-(1^{++})$

  confirmed with

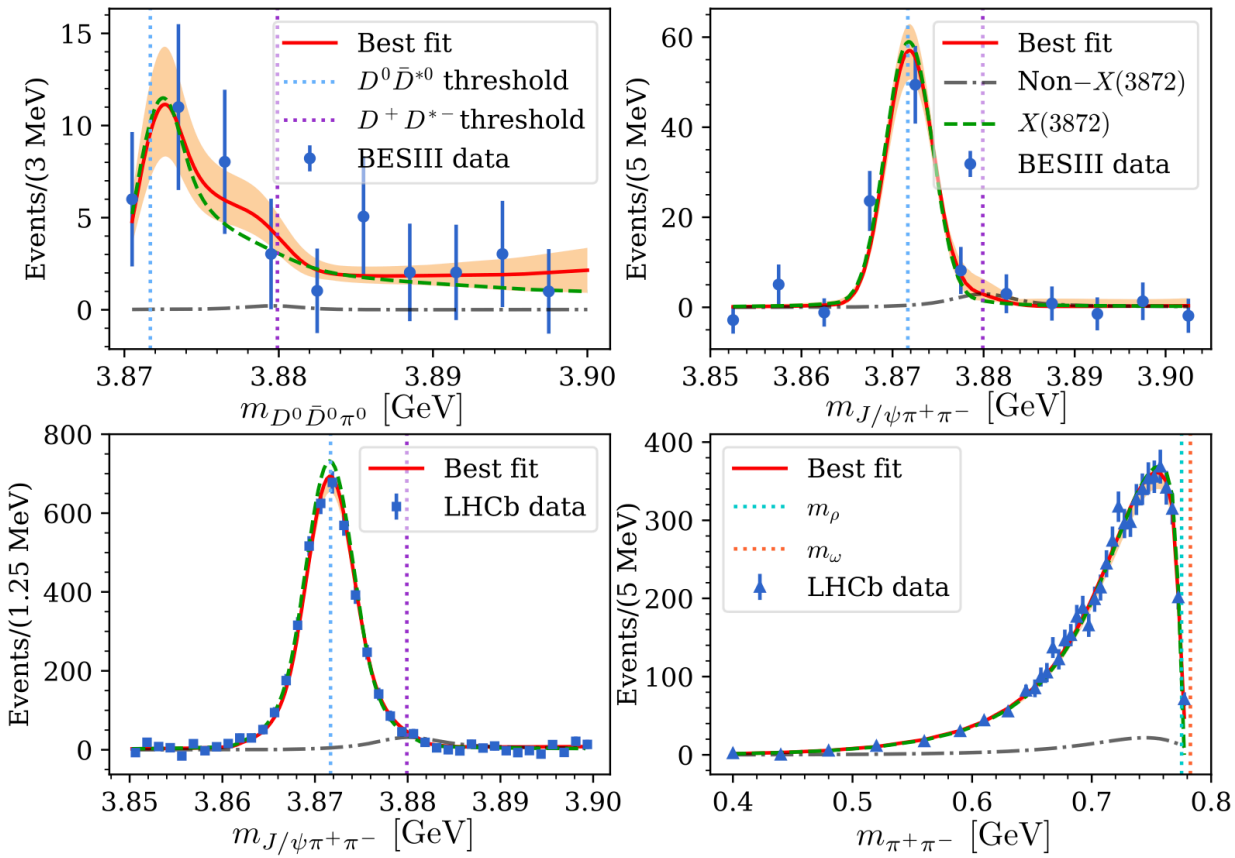
  $E_W = \left(3.1 \pm 0.7 + i\,1.3^{+1.9}_{-0.6}\right)$ MeV

  relative to D^+D^{*-} threshold

  on hidden sheet (R_{12})

  pred.: Zhang et al. JHEP08(2024)130

  lattice support: Sadl et al. PRD111(2025)054513



analysis gives compositeness of $X(3872)$=0.97(2)

using Li et al. PRD105(2022)L071502

W_{c1} effect amplified through interference with X

Possible Future measurements

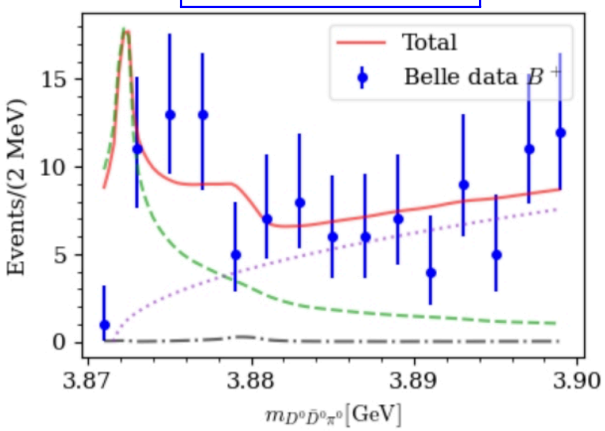
W_{c1} signal could be enhanced by switching production source:

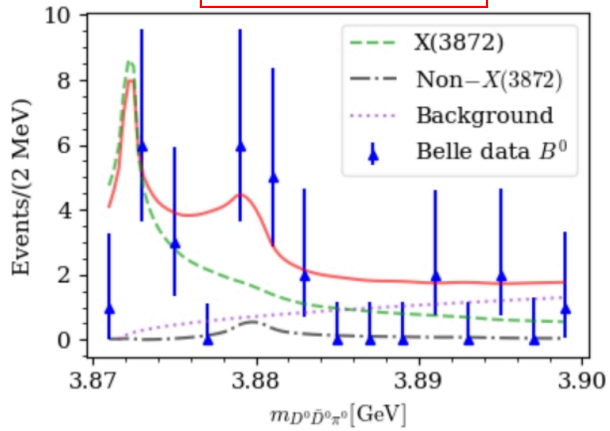
For example $\Gamma(B^+ \to K^+ D^0 \bar{D}^{*0}) > \Gamma(B^+ \to K^+ D^+ D^{*-})$ $(P_\pm/P_0 = 0.5)$

but $\Gamma(B^0 \to K^0 D^0 \bar{D}^{*0}) < \Gamma(B^0 \to K^0 D^+ D^{*-})$ $(P_\pm/P_0 = 2)$

$B^\pm$ decays

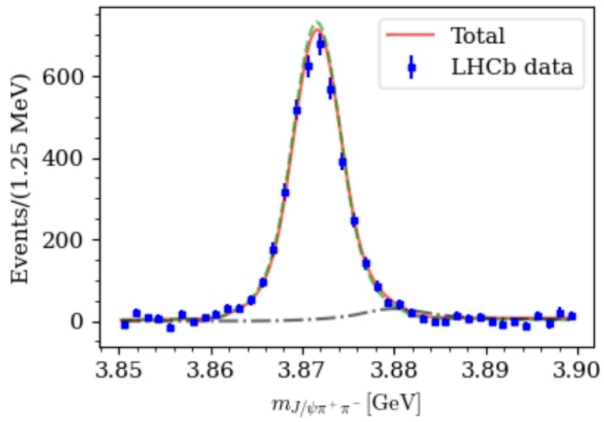
B^0 decays

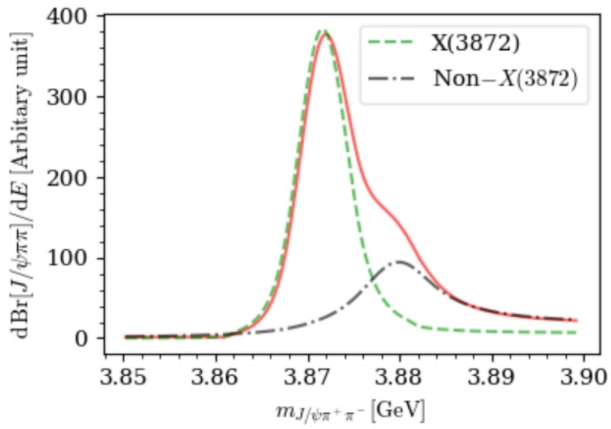




$B \to K[D^0 \bar{D}^{*0}]$

Belle, PRD107(2023)112011





$B \to K[J/\psi\pi^+\pi^-]$

LHCb, JHEP08(2020)123

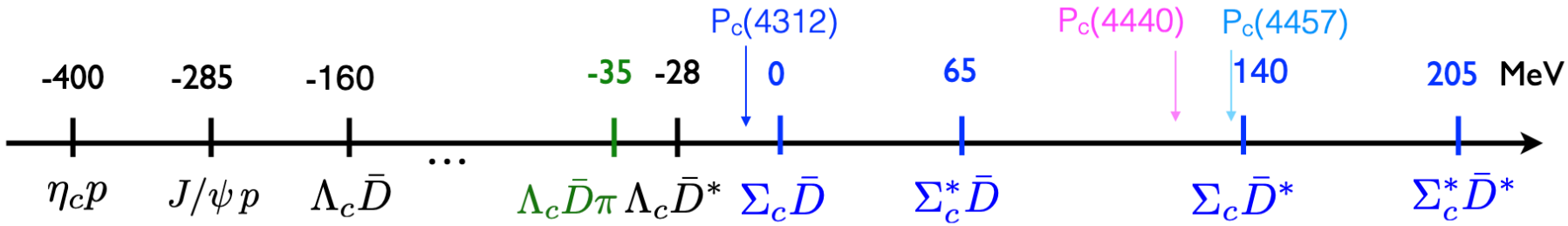
Example III: Pentaquarks as Molecules

Predicted in phenomenological analysis — J. J. Wu, R. Molina, E. Oset and B. S. Zou, PRL 105(2010), 232001

Various pion less EFT studies — Nieves, Oset, Geng, Valderrama, ...

We again apply pion full EFT & fit to data — M. L. Du et al., PRL 124 (2020) 7, 072001

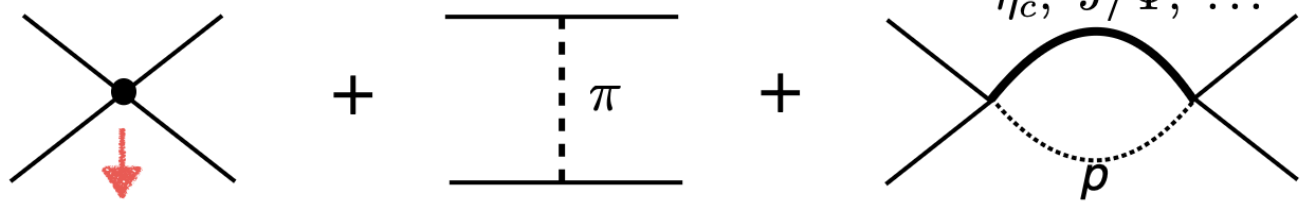
Potentially relevant thresholds:



Systematic study of pertinent channels — Idea:

- Formation of states in $\Sigma_c^{(*)} \bar{D}^{(*)}$ channels (elastic channels)
- Aim at fitting experimental data for $\Lambda_b \to KpJ/\psi$
- Predictions for $\Lambda_b \to K\Sigma_c^{(*)} \bar{D}^{(*)}$ and $\Lambda_b \to K\eta_c p$
- Is there room for $\Lambda_b \to K\Lambda_c \bar{D}^{(*)}$? (Predicted by many models)

Formalism

$V_{\rm LO}^{\rm eff}$ = 

HQSS: 2 S-S wave LECs at O(Q^0)
1 S-D wave LEC at O(Q^2)

Long range: OPE

Imaginary part
from inelastic channels

Total number of parameters without/with the $\Lambda_c\bar{D}^{(*)}$ channels

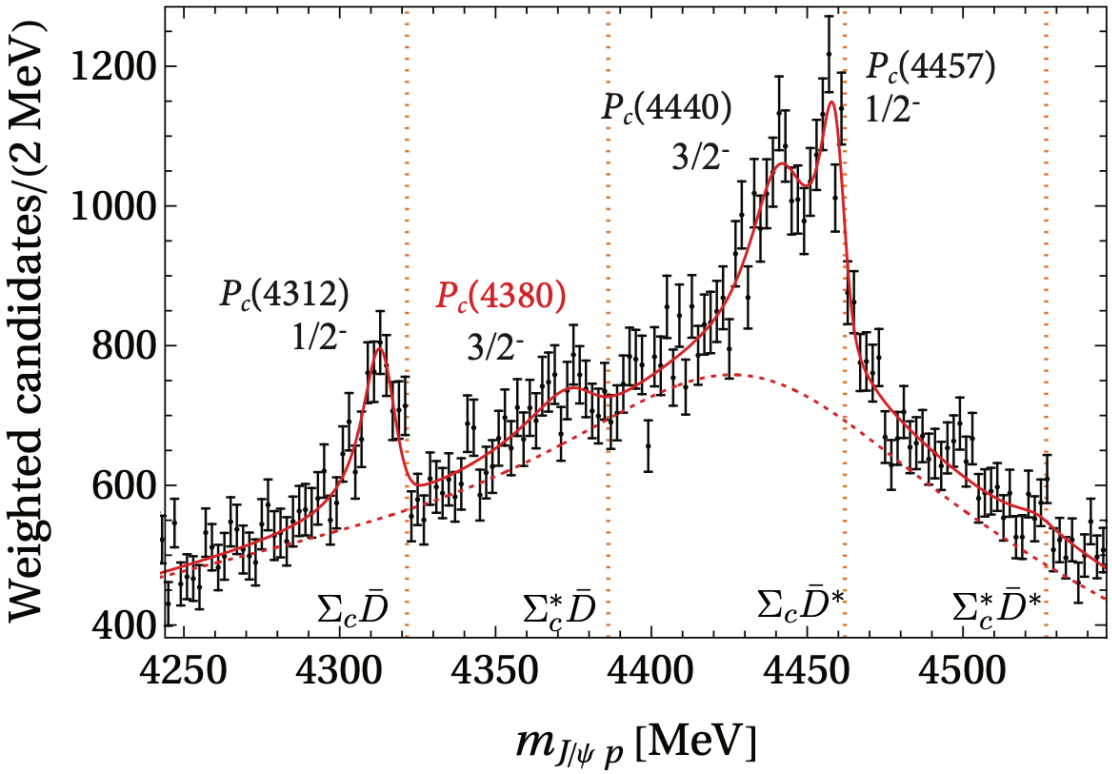
- elastic channels: $\Sigma_c^{(*)}$ has $j_{\rm light} = 1$; $D^{(*)}$ has $j_{\rm light} = 1/2$
  $\Longrightarrow$ 2 counter terms with $j_{\rm light} = 1/2$ & $j_{\rm light} = 3/2$, respectively
- $J/\psi p$ and $\eta_c p$: transitions in S and D wave
  $\Longrightarrow$ 2 parameters for $\Sigma_c^{(*)}\bar{D}^{(*)} \to p(J/\psi/\eta_c)$
- $\Lambda_c^{(*)}$ has $j_{\rm light} = 0$; $D^{(*)}$ has $j_{\rm light} = 1/2$
  $\Longrightarrow$ 1 parameter for $\Lambda_c\bar{D}^{(*)} \to \Sigma_c^{(*)}\bar{D}^{(*)}$
- 1 (2) parameters for the promoted $S-D$ counter terms

$\Longrightarrow$ 5 parameters (7 parameters) control the scattering,
but the 2 first are most important

- .... and 7 parameters to control the production

Results for $\Lambda_b \to KpJ/\psi$

data: LHCb, PRL122 (2019)222001



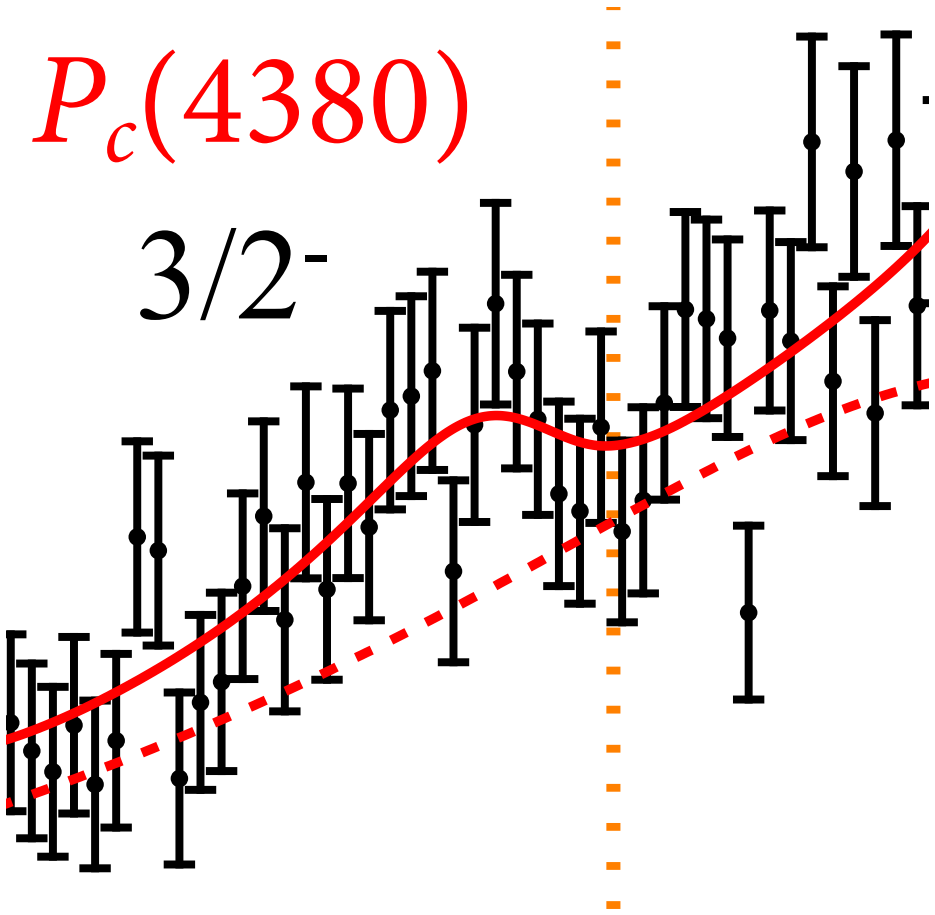
- Data described well ($\chi^2/dof \approx 1$)
- Without OPE two good fits (different order of the $\Sigma\bar{D}^*$ states)
- With OPE unique fit
- Observe indications for narrow $P_c(4380)$

$\Longrightarrow$ Fit provides 1 $\Sigma_c\bar{D}$, 1 $\Sigma_c^*\bar{D}$, 2 $\Sigma_c\bar{D}^*$ and 3 $\Sigma_c^*\bar{D}^*$ molecules (compared to 10 for the diquark picture with different J^P)

$\Longrightarrow$ basically 2 parameters fixed by fitting 3 states

$\Longrightarrow$ Not all show up in the data (importance of production vector?)

Zooming in ....



Spin symmetric couplings give additional states

Source coupling of $\Sigma_c^* D$ channel different from 0 by 1.7 σ

Its confirmation would provide support for molecular picture!

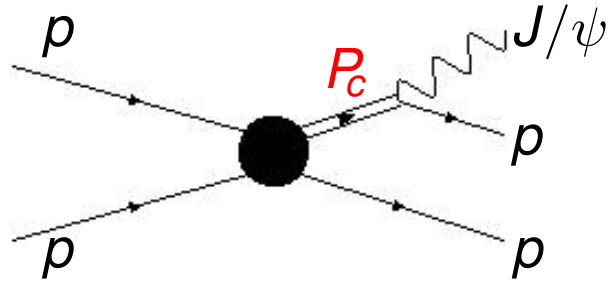
three states still missing!!

The amount of spin symmetry violation is specific to the structure

$\Longrightarrow$ Molecules follow the thresholds!

M. Cleven et al., PRD92(2015)014005

Opportunities at FAIR

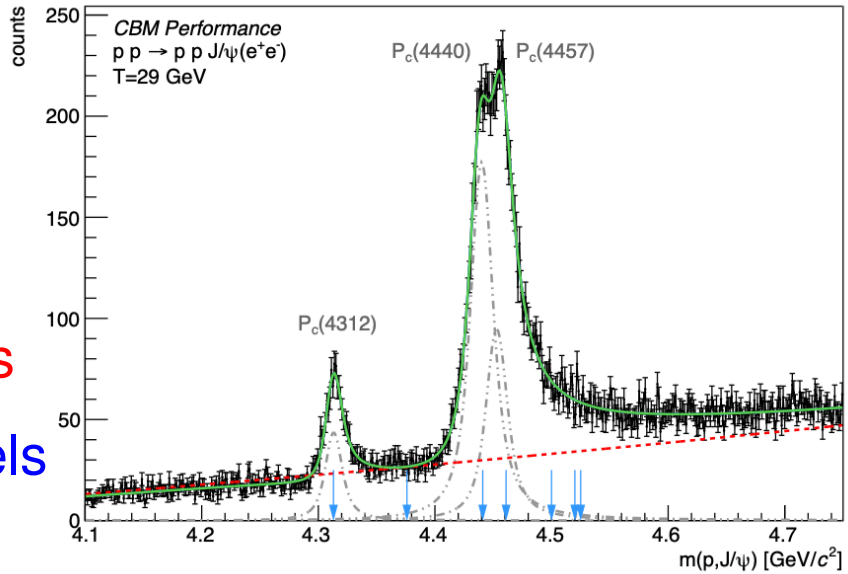


Production in *pp* collisions with SIS100 (proton momenta: 5-30 GeV/*c*):

- reaction fully exclusive
- allows for Dalitz-plot analysis
- no kinematic phase-space limitation
- production consistent with heavy quark spin symmetry

  $\Longrightarrow$ $\Sigma_c \bar{D}^{(*)}$ & $\Sigma_c^* \bar{D}^{(*)}$ at similar rates
- completely different crossed channels compared to $\Lambda_b \to J/\psi p K$
- far future perspective (?):
  Double polarised measurement allows for determination of parity

Simulation for CBM:



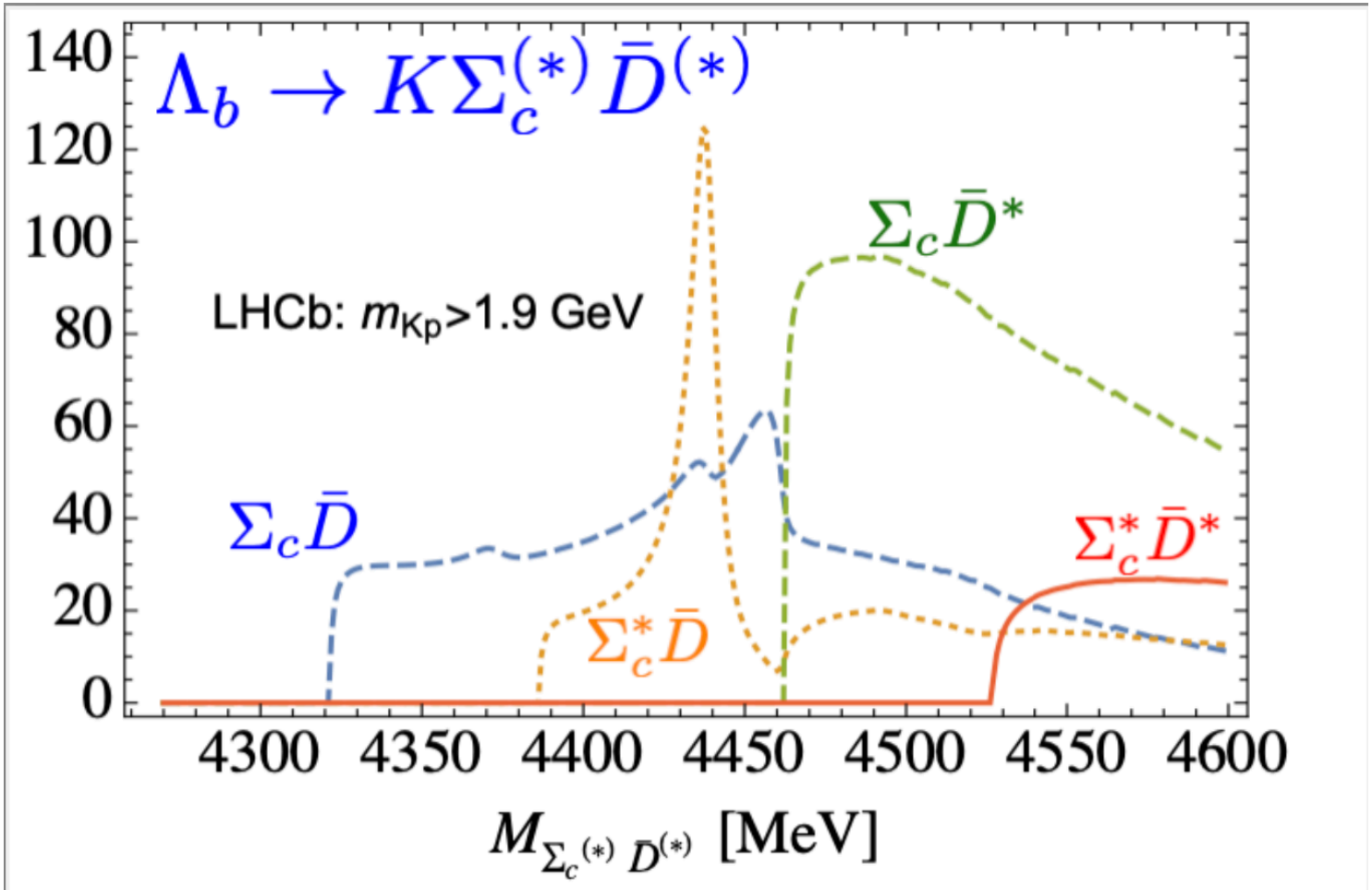
Messchendorp *et al.*, arXiv:2512.15986 [hep-ex]

C.H. et al. PLB590(2004)39

Signatures in elastic channels

Typical spectrum

Du et al., JHEP 08 (2021) 157

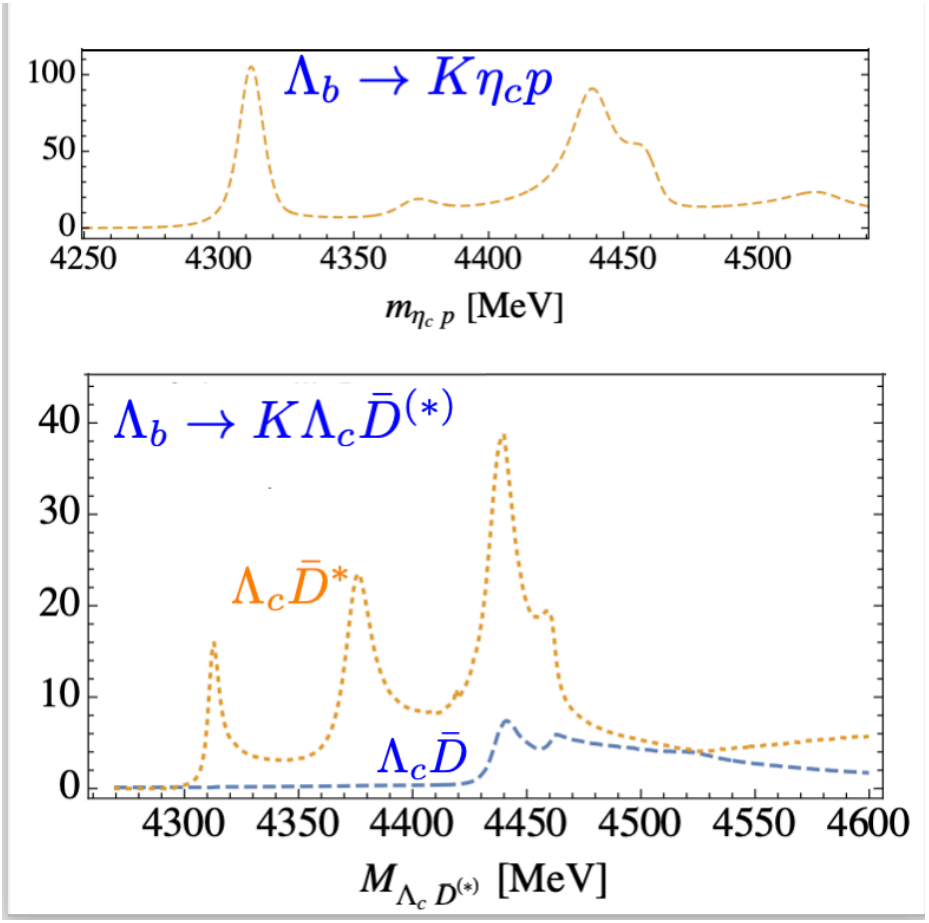


strong threshold enhancement - unavoidable for hadronic molecules

Signatures in inelastic channels

Du et al., JHEP 08 (2021) 157

- For $\Sigma_c^{(*)}\bar{D}^{(*)} \to \Lambda_c\bar{D}^{(*)}$ OPE + two add. para. (S-S and S-D)



- $\eta_c p$ channel similar to $J/\psi p$ (spin symmetry!)
- $\Lambda_c\bar{D}^{(*)}$ final states not constrained yet
- Shown: Some typical spectrum
- Data even compatible with very weak signal
- $(\Lambda_c\bar{D}\to\Sigma_c\bar{D})_{\mathrm{S-wave}} = 0$
  $(\Lambda_c\bar{D}\to\Sigma_c^*\bar{D})_{\mathrm{DS-wave}} = 0$

Summary and Perspectives

- The recent and future data have the potential to allow us to identify the prominent components in exotic states

to-do for experiment

- Continue great performance! Especially needed:
  - data for different quantum numbers and
  - data for line shapes in different systems

to-do for theory

- Provide more predictions for the different scenarios in EFTs as well as lattice QCD
- Go beyond most simple approaches
  e.g. study interplay of regular quarkonia with exotics
  - potentially significant mixing

    Kalashnikova et al., PRD80(2009)074004; Takizawa et al., PTEP(2013)093D01; Ortega et al., JPG 40(2013)065107; Coito et al., EPJC73(2013)2351; Cincioglu et al., EPJC76(2016)576
  - negligible mixing

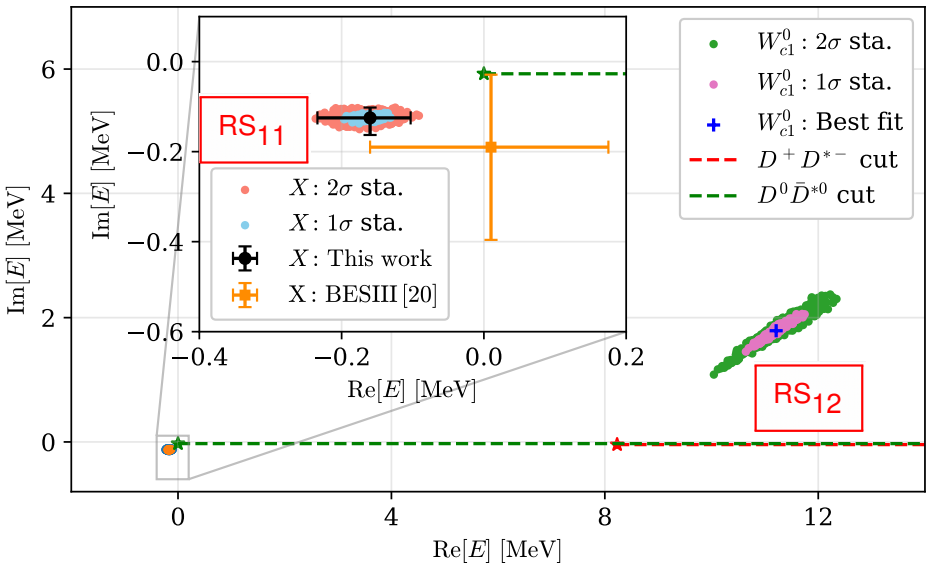
    van Beveren et al., PLB641(2006)265; Hammer et al., EPJA52(2016)330, C.H. et al., PRD106(2022)114003

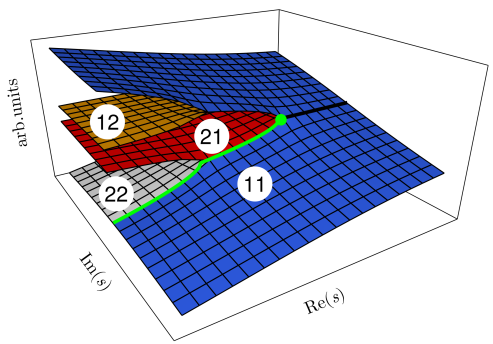
  $\Longrightarrow$ Exploit further BOEFT and Dyson-Schwinger approaches

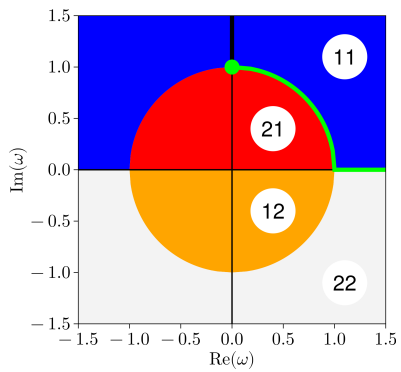
Thanks a lot for your attention

BACK-UP SLIDES

For the experts







conformal map: $k_{1/2} = \sqrt{\frac{\mu_{1/2}\Delta}{2}}\left(\omega \pm \frac{1}{\omega}\right)$

$$E = \frac{k_1^2}{2\mu_1} = \frac{k_2^2}{2\mu_2} + \Delta = \frac{\Delta}{4}\left(\omega^2 + \frac{1}{\omega^2} + 2\right)$$

Zhang, Guo, PLB 863 (2025) 139387, C.H. [arXiv:2508.20694 [hep-ph]]

Pole trajectories X vs. W
(pure contact int.; particle basis)

$$V^{\rm ct} = \frac{1}{2}\begin{pmatrix} C_{0X}+C_{1X} & C_{0X}-C_{1X} \\ C_{0X}-C_{1X} & C_{0X}+C_{1X} \end{pmatrix}$$

- start: Isospin limit & $C_{0X}=C_{1X}$
  → channels decouple
  → Poles on RS_{21} and RS_{12}
- $M_{D^{(*)+}} \neq M_{D^{(*)0}}$ & $C_{0X}\neq C_{1X}$:
  → thresholds move apart
  → states couple and repel each other
  - one near neutral thres. gets bound ($X(3872)$)
  - one near charged thres. gets width & moves up on RS_{12}

  $\Longrightarrow$ threshold cusp (W_{c1})