The Exotic Hybrid Meson: Charmonium at Finite Temperature

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INTRODUCTION

- The XYZs include manifestly exotic states that are not described by the quark-constituent model
- Despite > 20 years of theoretical study many remain unexplained (fully)
- HadSpec previously identified a hybrid supermultiplet in charmonium and bottomonium [JHEP 07 (2012) 126 and JHEP 02 (2021) 214]:
  - this includes the exotic 1^{-+} state, identified as a hybrid
  - all results at $T = 0$

  using distillation, large operator bases, GEVP and spin identification.

- Can temperature dependence shed light on the nature and structure of exotic mesons?
- Experimental results already in this direction
  - e.g. CMS: X(3872) [$\chi_{c1}(2P)$] seen in PbPb heavy ion collisions.

OPEN QUESTIONS

Distillation is a powerful tool for spectroscopy at T=0. Can it be effective at finite temperature?

Will a GEVP analysis work and/or help to shed light on patterns of melting and suppression?

Following the HadSpec approach consider an exploratory study:

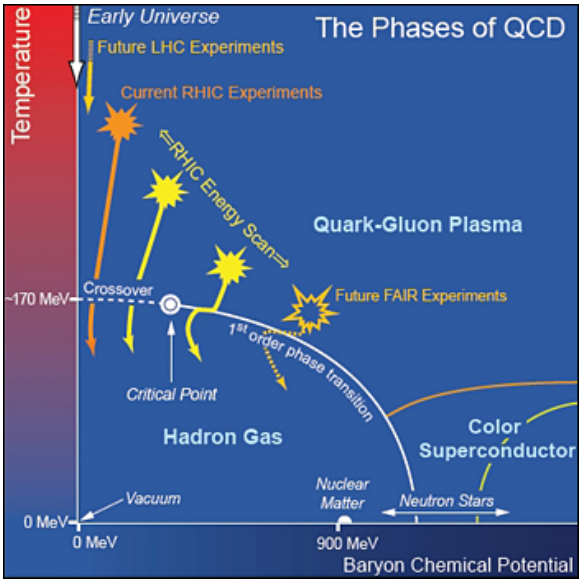
First (lattice) study of the charmonium exotic hybrid meson in medium (T>0).

First implementation of distillation at T>0.

Hybrid is readily accessible as a ground (low-lying) state in T_1^{-+} in charmonium (bottomonium) lattice calculations - straightforward spin/state identification.

Caveat: all results presented here are preliminary, with more details in 2512.11638 - Lattice 2025.

Setting the scene



Finite Temperature in LQCD

Euclidean time compactification: $\frac{1}{T} \longleftrightarrow a_t N_t$

To change temperature:

- fix a_t, vary N_t.
  Fine a_t helps ⇒ anisotropic lattices
  Approach taken here.
- fix N_t, vary a_t.
  Common approach but needs multiple gauge field ensembles.

FROM HADSPEC TO THE FASTSUM ENSEMBLES

- From HadSpec: the $T = 0 (N_t = 128)$ ensemble, used for excited and exotic charmonium spectroscopy JHEP 2012, 126 (2012); PRD 78 (2008) 054501
  - Symanzik-improved anisotropic tree-level tadpole-improved gauge action.
  - Anisotropic Wilson clover fermion action, stout-smeared spatial links, $N_f = 2 + 1$
  - $N_t \times 24^3$, $m_\pi \sim 385\mathrm{MeV}$
  - $\xi = a_s/a_t \sim 3.5; a_s \sim 0.12\mathrm{fm}$, $a_t^{-1} \sim 5.7\mathrm{GeV}$
- To FASTSUM "Gen 2" ensembles at $T > 0$

N_t	128	40	36	32	28	24	20	16
$T(MeV)$	44	141	156	176	201	235	281	352
T/T_c	0.243	0.777	0.864	0.972	1.11	1.30	1.56	1.94

THE HADSPEC RECIPE FOR (MESON) SPECTROSCOPY: FOLLOWED HERE

- Construct a basis of local and non-local operators $\bar{\Psi}(x)\Gamma D_i D_j \ldots \Psi(x)$ from distilled fields [PRD80 (2009) 054506].
  - Operators of definite J^{PC} subduced into relevant lattice irrep.
  - include up to 3 derivatives to sample the spatial structures well.

- Build a correlation matrix of two-point functions

$$C_{ij} = \langle 0|\mathcal{O}_i \mathcal{O}_j^\dagger|0\rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E_n} e^{-E_n t}$$

- Solve generalised eigenvalue problem $C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$
  - eigenvalues: $\lambda^{(n)}(t) \sim e^{-E_n t}\left[1 + O(e^{-\Delta E t})\right]$ yield principal correlators
  - eigenvectors: $Z_i^{(n)} = \sqrt{2E_n}e^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$ related to overlaps - helps identify continuum spin.

PITFALL 1: FINITE TEMPERATURE EFFECTS

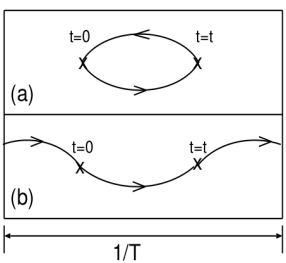
In euclidean lattice thermal field theory, (inverse) temperature is identified with the (finite) temporal extent.

Temporal correlations of zero-momentum meson operators $\bar{q}\Gamma q$ with $\{\Gamma, \gamma_0\} \neq 0$ include a non-negligble wraparound (scattering) contribution, regardless of irrep.

This single quark propagation exponentially suppressed at $T \sim 0$. Phys. Rev. D 75, 094502 (2007), Phys. Rev. D 84, 094504 (2011) , Phys. Rev. D 112, 014506 (2025)

$C_{(a)}(t) \propto \exp^{-m_q t} \times \exp^{-m_q t} = \exp^{-2m_q t}$

$C_{(b)}(t) \propto \exp^{-m_q t} \times \exp^{-m_q (L_t - t)} = \exp^{-m_q L_t}$, a constant ($L_t = 1/T$)



SOLUTION(S)

In the full HadSpec basis there are operators with and without constant contribution.

- ✓ Midpoint subtraction $\tilde{C}(t) = C(t) - C(N_t/2)$ or time derivative Phys. Rev. D 75, 094502 (2007)
- × But makes the GEVP noisier and numerically unstable for small N_t due to cancellations.

Use only operators with $\{\Gamma, \gamma_0\} = 0$.

- ✓ No constant term!
- × Fewer operators available for each lattice irrep Λ^{PC} and GEVP.

PITFALL 2: TEMPORAL BOUNDARY CONDITIONS

Given periodic temporal boundary conditions for gauge fields

If operators $\mathcal{O}_i$ and $\mathcal{O}_j$ have different time-reversal symmetries then

$$C_{ij}(t) = \left\langle \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0) \right\rangle \text{ satisfies } C(-t) = -C(t).$$

Having entries in the correlation matrix with different time-reversal symmetries is a problem for the GEVP eigenvalues with our BC.

Eur. Phys. J. C 78 (2018) 12, 1018

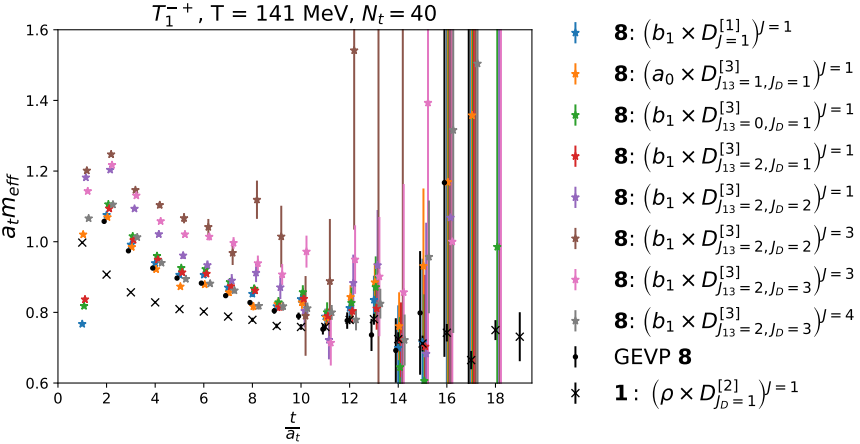
- ✓ Consider operator basis by time-reversal symmetry to ensure $C(-t) = C(t)$.
- × Fewer operators available per GEVP.

Still have $\approx 3-8$ operators per irrep in most cases.

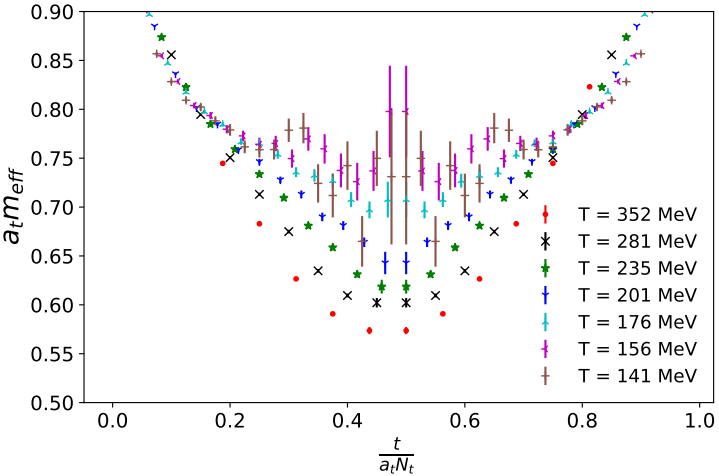
OPERATORS FOR THE HYBRID T_1^{-+}: STUDY AT LOW T

18 operators → **9** T-Even + **9** T-Odd. T-parity op groups perform equally well.

9 T-even. Comprising **1** with $\{\Gamma, \gamma_0\} = 0$ and **8** with $\{\Gamma, \gamma_0\} \neq 0$.
Only 1 "ideal" operator, do we lose a lot compared to the other 8?



A FIRST LOOK: TEMPERATURE DEPENDENCE

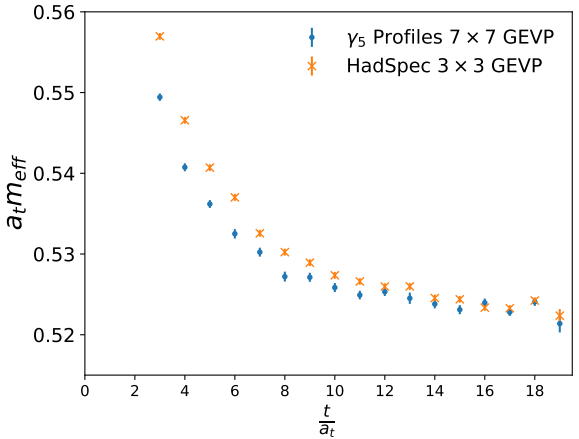


- At $T = 44$ MeV ($N_t = 128$) HadSpec result: $a_t m \approx 0.75$.
- Cosh effective mass formula gradually loses validity: **melting**.
- Note a large gap between 176 MeV $< T_c = 181$ MeV < 201 MeV.

BRIEF ASIDE - GETTING MORE FROM LESS

How to increase the operator basis size while respecting the two restrictions?

Distillation profiles. Phys. Rev. D 106, 034501 (2022)



- ✓ One Γ operator with 7 profiles outperforms 3 HadSpec operators with standard distillation.
- ✓ Combination HadSpec + Profiles is now proven to help substantially.
- ✓✓ Profiles preserve Γ - related symmetries.

EXPLORING T-DEPENDENCE WITH RECONSTRUCTED CORRELATORS

- Would like to compare correlators at different temperatures

$$G(\tau;T) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau,\omega;T)\,\rho(\omega;T),$$
$$K(\tau,\omega;T) = \frac{\cosh[\omega(\tau-1/2T)]}{\sinh(\omega/2T)}$$

- Two sources of temperature dependence in correlators:
  1. kinematical: kernel $K(\tau,\omega;T)$ — boring
  2. dynamical: spectral function $\rho(\omega;T)$ — interesting

- Avoid effect 1?

Reconstructed correlators

Avoiding effect 1 Phys. Rev. D **69**, 094507 (2004), JHEP **04** (2010), 099, Phys. Rev. D **86** (2012), 014509, arXiv:2209.14681

- Rewrite the kernel, for integer $m = T/T_0$ (where m typically an integer) - relating kernels at two temperatures:

$$\frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)} = \sum_{n=0}^{m-1} \frac{\cosh[\omega(\tau + n/T - 1/2T_0)]}{\sinh(\omega/2T_0)}$$

and so

$$K(t, \omega; T) = \sum_{n=0}^{m-1} K(t + a_t n N_t, \omega; T_0)$$

Reconstruction continued

- Use the new kernel to form a reconstructed correlator: at a higher temperature using a summation of the low/zero-temperature correlator.

$$G_{\mathrm{rec}}(t, T; T_0) = \sum_{n=0}^{m-1} G(t + n/T; T_0)$$

and the ratio

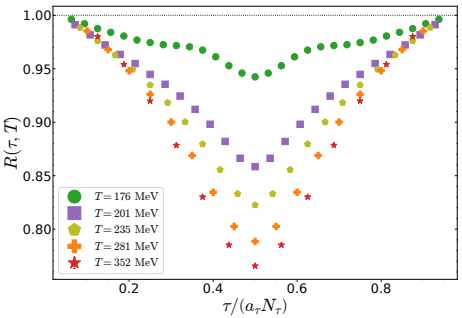
$$R(t, T) = G_{\mathrm{rec}}(t, T; T_0) \,/\, G(t, T)$$

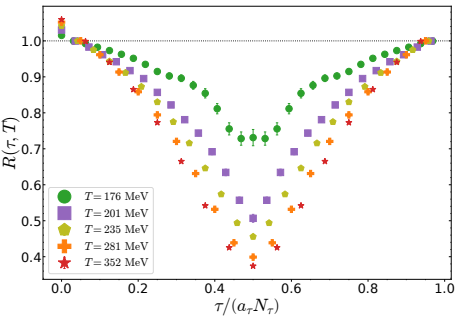
probes the change in $\rho(\omega)$ with temperature.

Removes the trivial (boring) temperature dependence of kernel

Allows study of the temperature-dependent change in the spectral function ρ

TEMPERATURE DEPENDENCE IN CORRELATORS





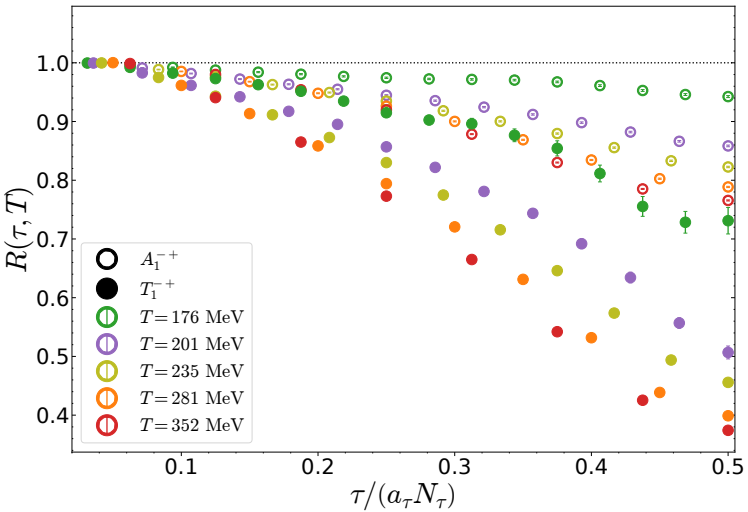
A_1^{-+} (pseudoscalar) ground state.

T_1^{-+} (hybrid) ground state

- Deviations from R=1 indicate changes to spectral function
- Stronger response seen in T_1^{-+}

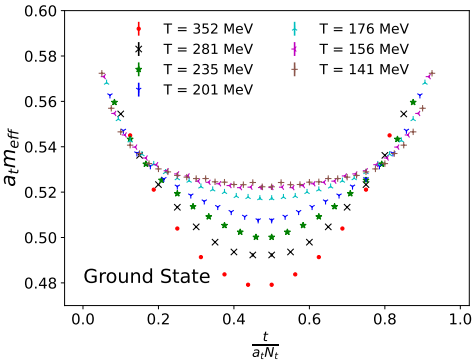
COMPARING TEMPERATURE DEPENDENCE OF THE A_1 AND T_1

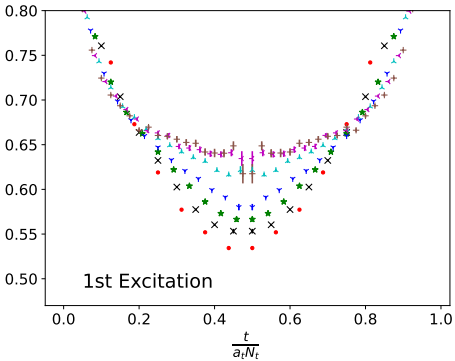
Compare the open (A_1) and closed (T_1) symbols for each colour (temperature)



- Changes for T_1^{-+} exotic hybrid more significant than for A_1^{-+} pseudoscalar.

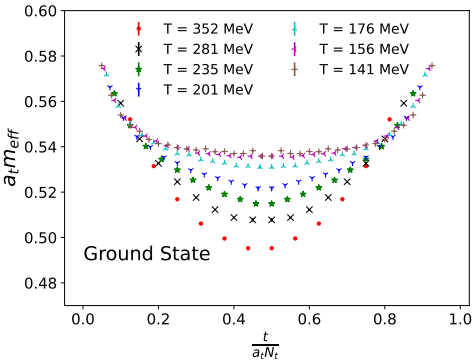
OTHER CHANNELS: PSEUDOSCALAR, A_1^{-+}

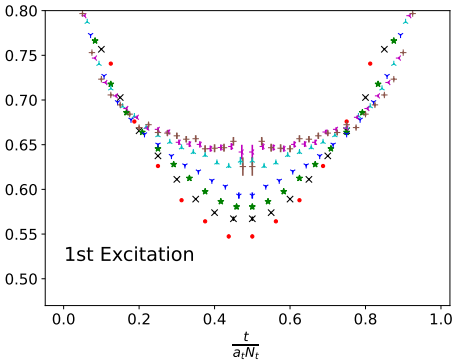




- 3×3 GEVP works well to resolve ground state and first excitation.
- Similar pattern to the 1^{-+} as temperature increases.

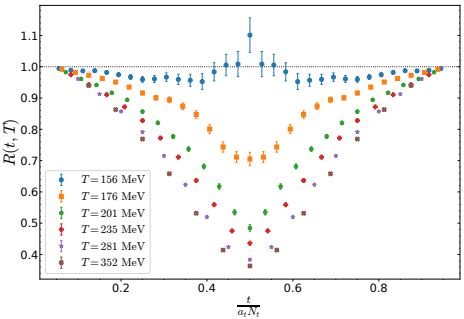
OTHER CHANNELS: VECTOR, T_1^{--}

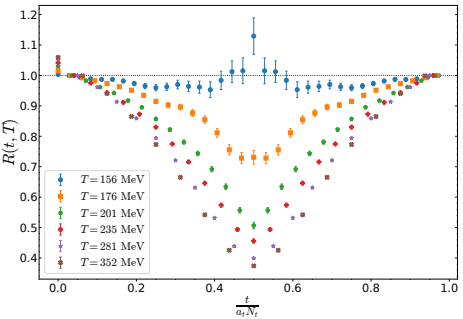




- 5 × 5 GEVP works well to resolve ground state and first excitation.
- Similar pattern to the 1^{-+} as temperature increases.

Back to the temperature dependence of the A_1^{-+} & T_1^{-+}





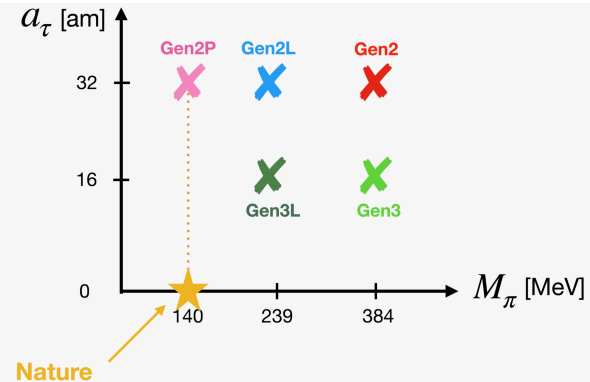
A_1^{-+} 2nd excited state.

T_1^{-+} ground state

- Observation: the ground state T_1 responds to temperature similarly to an excitation of the A_1

Next steps

Explore with other/new FASTSUM ensembles[1]



Focus on **Gen3**:

- ✓ Similar characteristics to Gen2 . . .
- ✓ . . . but **twice** as many temporal points at each temperature.
- ✓ Promises significant improvement in MEM/BR systematics.
- × Early days: charm mass parameter tuning in progress.

[1] Gen2P & Gen3L in preparation

CONCLUSIONS AND OUTLOOK

Methods:

- ✓ Distillation + HadSpec operator “machinery” adapt well to finite temperature...
- × ... at the cost of a reduced operator basis...
- ✓ Good operators remain fundamental!
- ✓ Distillation profiles can make up for this reduction - under investigation!
- ✓ A first application of Distillation in finite temperature setting.

At finite temperature:

- The exotic T_1^{-+} is more strongly affected by temperature than the A_1^{-+},
  - at a level comparable to the 3rd state in the A_1^{-+} channel.
- Spectral function methods e.g. MEM/BR planned to better understand patterns of melting and suppression.