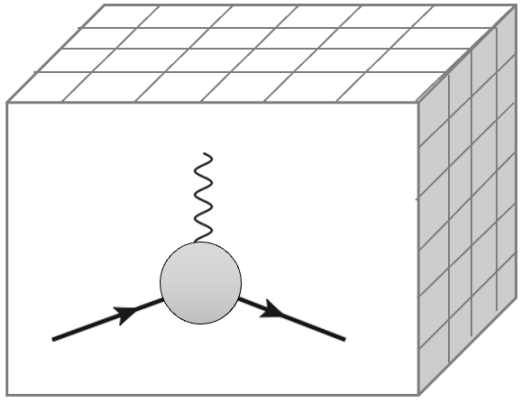
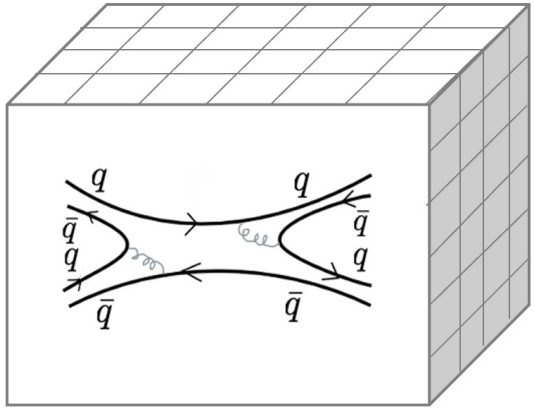
Charmoniumlike and other exotic hadrons from lattice





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on sabbatical semester (2026): Rome Tor Vergata / Berlin DESY / University of Regensburg

Glueballs and Charmonium, ECT Trento, March 2026

Aims

Hadron with certain quantum numbers

Does it exist ?

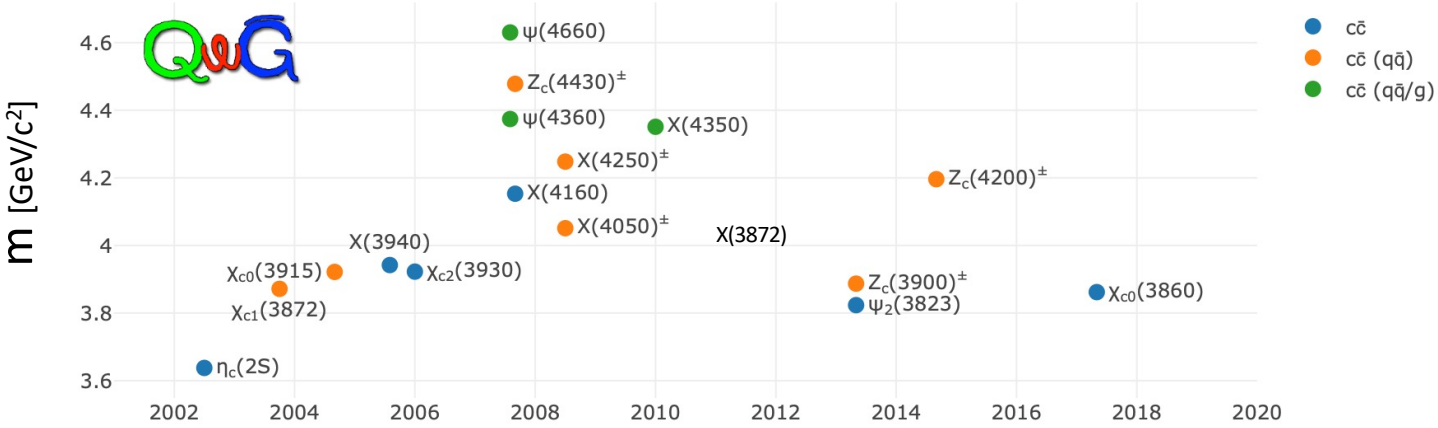
Mass ?

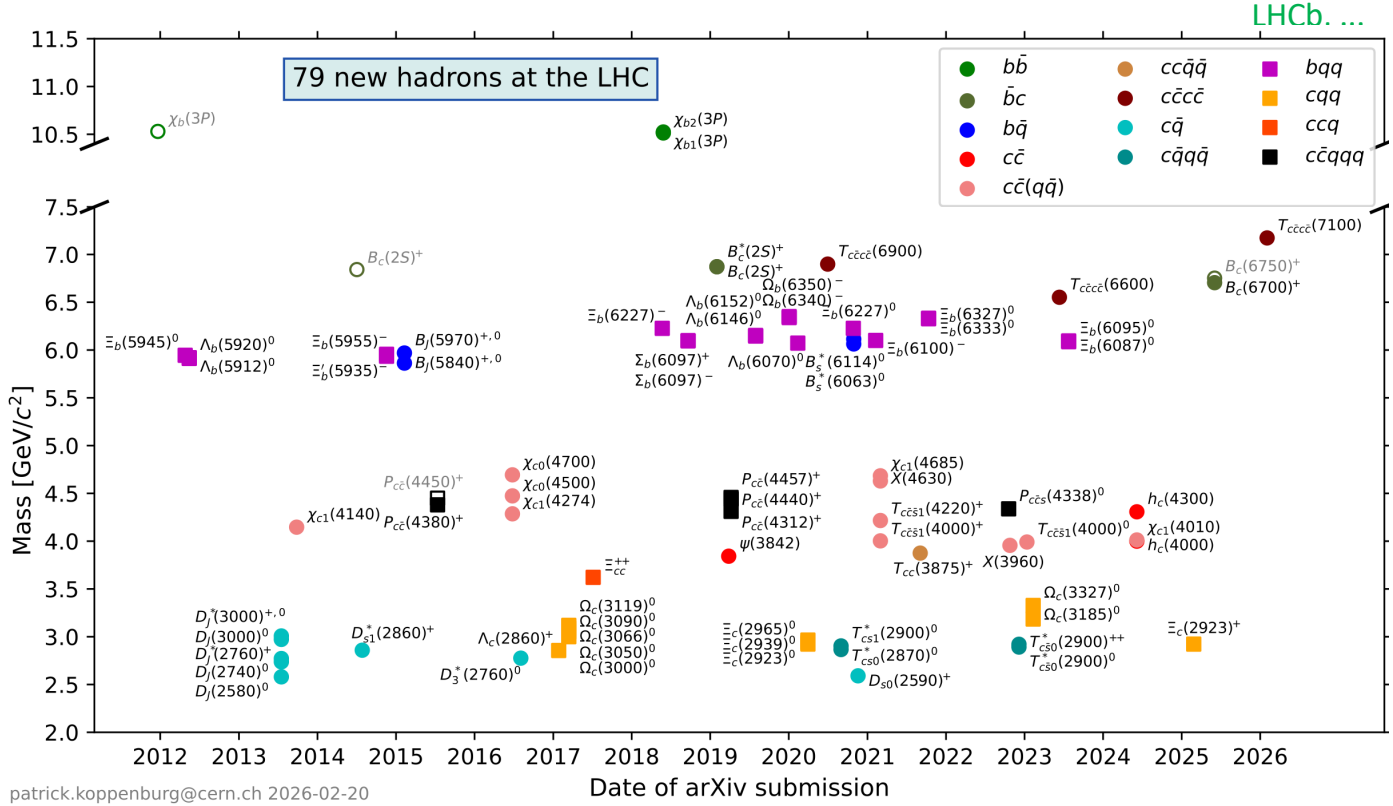
Width ?

Binding energy ?

Structure ?

Binding mechanism ?



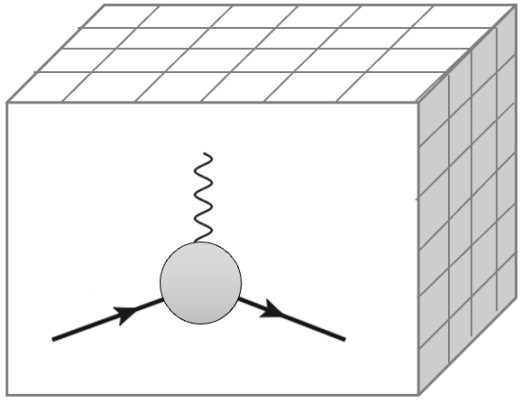


Outline

ab-intio theoretical study with lattice QCD

Structure of strongly stable hadrons:

- $bb\bar{u}\bar{d}$ probed by EM current
- quenched glueball probed by $T^{\mu\nu}$
- charmonia probed by charge densities

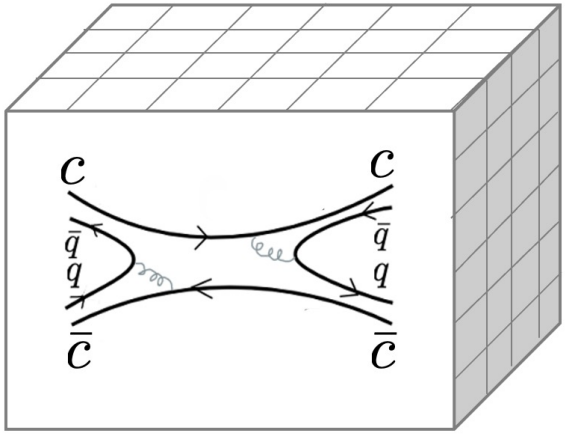


recent (2025)

Spectropscopy via scattering

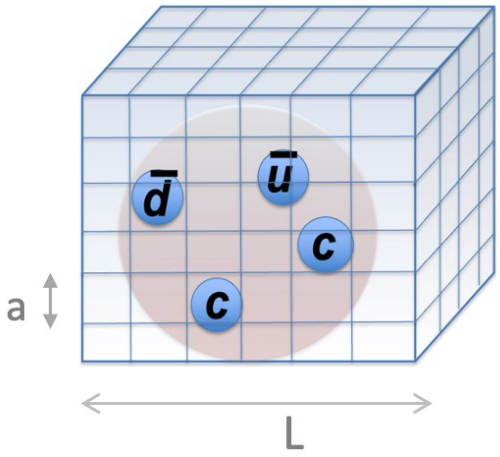
Charmoniumlike resonances and states near threshold

- $J^{PC}=1^{++}$, 1^{--} , 0^{++}, 2^{++} , isospin 0



old (2013-2020)

Quantum ChromoDynamics on lattice

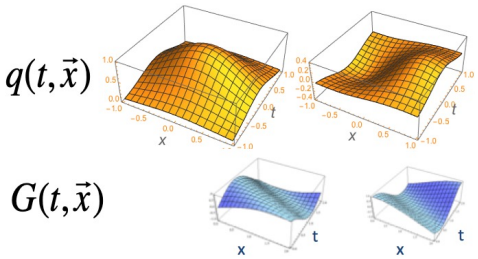


- numerical path integral in discretized finite Eucledian space-time $t_M = -it$
- input: g_s , m_q ; output: physics observables & their dependence on m_q , N_c

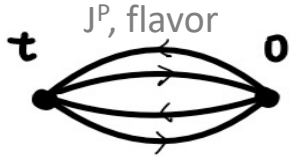
caution: several results with not be at physical $m_{u/d}$

$$\mathcal{L}_{QCD} = \frac{1}{4} G_a^{\mu\nu} G_a^{\mu\nu} + \bar{q} i \gamma_\mu (\partial^\mu + i g_s G_a^\mu T^a) q - m_q \bar{q} q$$

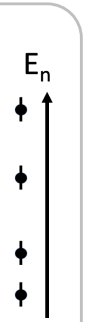
$$S_{QCD} = \int d^4x \; \mathcal{L}_{QCD}$$

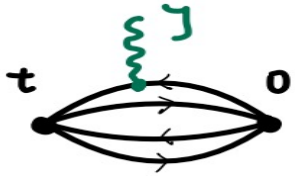


$$\langle C \rangle \propto \int \mathcal{D}G\mathcal{D}q\mathcal{D}\bar{q} \; C \; e^{-S_E/\hbar}$$



$$\langle 0|\mathcal{O}_f(t)\mathcal{O}_i^\dagger(0)|0\rangle = \sum_n A_n \; e^{-E_n t}$$

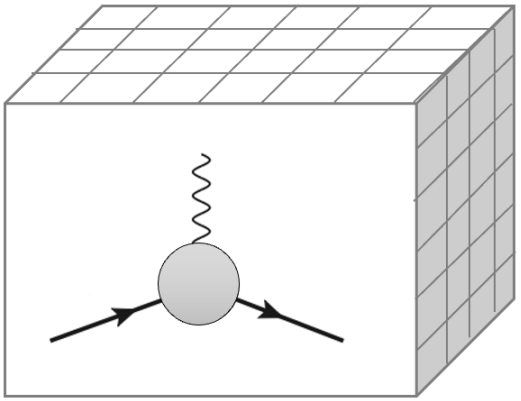


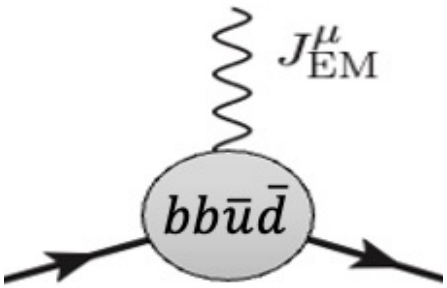


$$\langle 0|\mathcal{O}_f(t) \; J \; \mathcal{O}_i^\dagger(0)|0\rangle$$

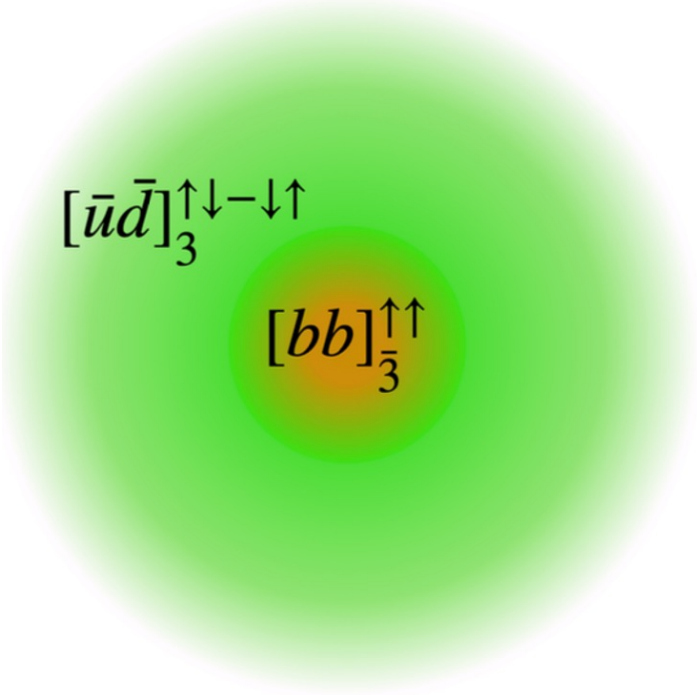
$$\langle H_f|J|H_i\rangle$$

Structure of strongly stable hadrons





Structure of strongly stable tetraquark







Ivan Vujmilovic, Sara Collins, Luka Leskovec, SP [arxiv: 2510.17549]

CLS ensembles, Nf=2+1: $m_{\pi}\approx290$ MeV, $a\approx0.064$ fm, $N_L=48$

Doubly bottom tetraquark

$T_{bb}: \; bb\bar{u}\bar{d}\,, \; J^P = 1^+, \; I = 0$

Mass:

strongly stable
bound state

previous lattice studies:

determined the mass,

but not the structure

$b\bar{u}$ $b\bar{d}$

$m_B + m_{B^*}$

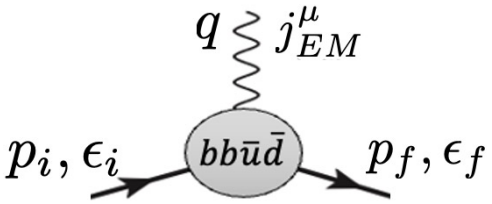
64 MeV

$bb\bar{u}\bar{d}$

$$m_{T_{bb}} - (m_B + m_{B^*}) = -64(10)\ \text{MeV}$$

Form factors

$$\langle T_{bb}(p_f, \epsilon_f)|j^\mu_{EM}(q)|T_{bb}(p_i, \epsilon_i)\rangle : \; F_C(q^2), \; F_M(q^2), \; F_{\mathcal{Q}}(q^2)$$



$$\hat{j}^\mu_{EM,cont} = \sum_q e_q \bar{q}\gamma^\mu q$$

$$q^2 = (p_f - p_i)^2 \equiv -Q^2 < 0$$

$$Q^2 = 0: \; F_C(0) = -1, \; F_M(0) = 2m \cdot \mu, \; F_{\mathcal{Q}}(0) = m^2 \cdot \mathcal{Q}$$

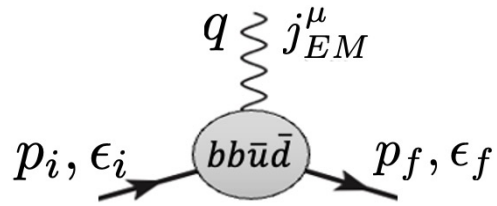
$$\hat{j}^\mu_{u/d} = Z^V_{u/d} \cdot \left(\frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d\right) \qquad \hat{j}^\mu_b = Z^V_b \cdot \left(-\frac{1}{3}\bar{b}\gamma^\mu b\right)$$

$Z^V_{u/d}$ and Z^V_b fixed from $F^b_C(Q^2 = 0) \equiv -\frac{2}{3}, \quad F^{\bar{u}}_C(0) \equiv -\frac{2}{3}.$

Doubly bottom tetraquark

$T_{bb}: \; bb\bar{u}\bar{d}\,, \; J^P = 1^+, \; I = 0$

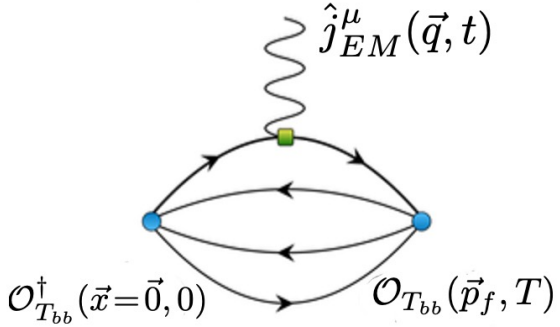
Form factors



$\langle T_{bb}(p_f,\epsilon_f)|j^\mu_{EM}(q)|T_{bb}(p_i,\epsilon_i)\rangle$: $F_C(q^2),\; F_M(q^2),\; F_{\mathcal{Q}}(q^2)$

$$q^2 = (p_f - p_i)^2 \equiv -Q^2 < 0$$

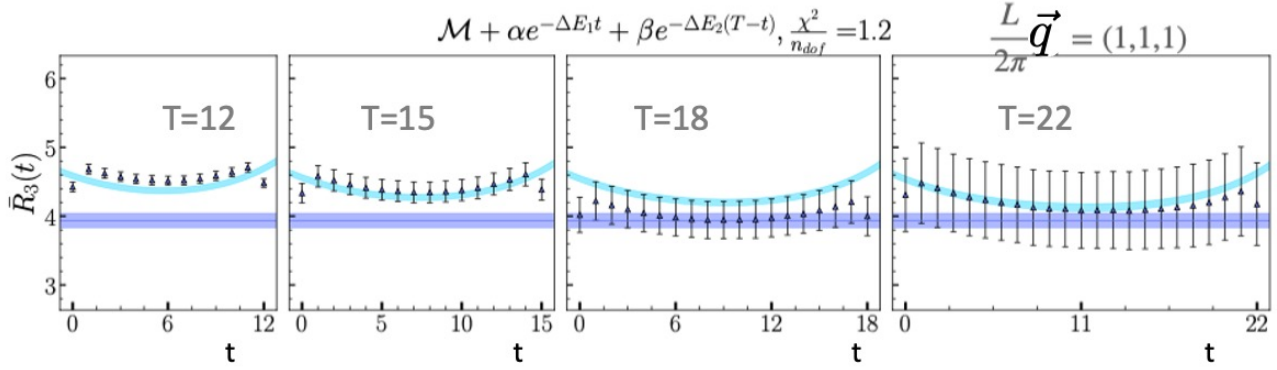
Form factors from lattice



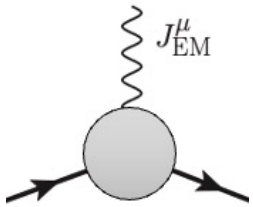
$$\sum |h_f\rangle\langle h_f| \quad \sum |h_i\rangle\langle h_i|$$

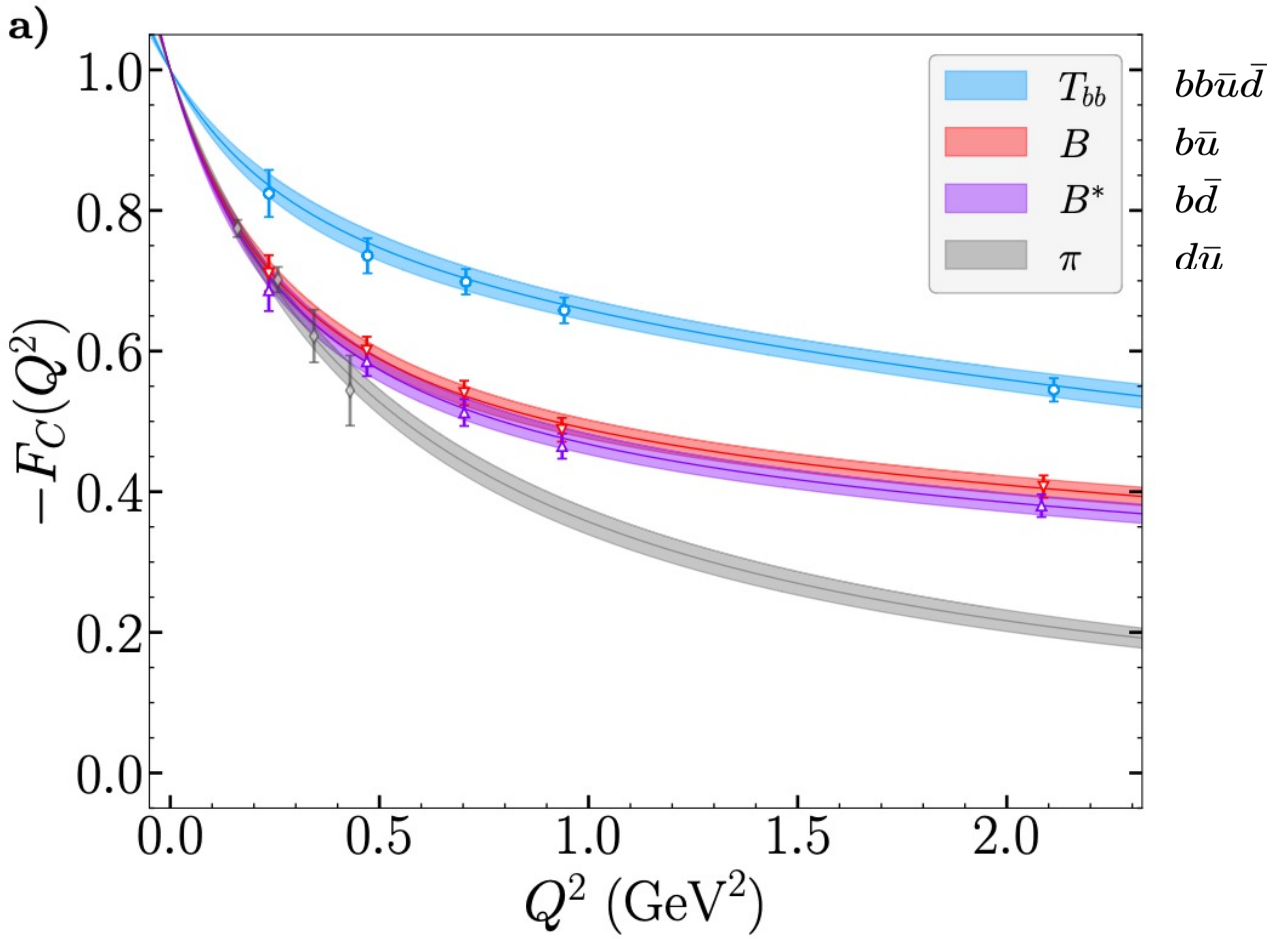
$$c_{3pt} = \langle\Omega|\mathcal{O}_{T_{bb}}(\vec{p}_f,T)\; j^\mu_{EM}(\vec{q},t)\; \mathcal{O}^\dagger_{T_{bb}}(0)|\Omega\rangle =$$

$$= \langle\Omega|\mathcal{O}_{T_{bb}}|T_{bb}\rangle \; \langle T_{bb}|j^\mu_{EM}(\vec{q})\;|T_{bb}\rangle \; \langle T_{bb}|\mathcal{O}^\dagger_{T_{bb}}|\Omega\rangle \; e^{-E_f(T-t)} e^{-E_i t} + \ldots.$$



Results: charge form factors of $T_{bb}, B^{(*)}, \pi$



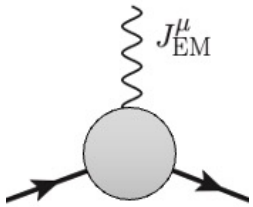


$$z(Q^2, t_+, t_0) = \frac{\sqrt{t_+ + Q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ + Q^2} + \sqrt{t_+ - t_0}}$$

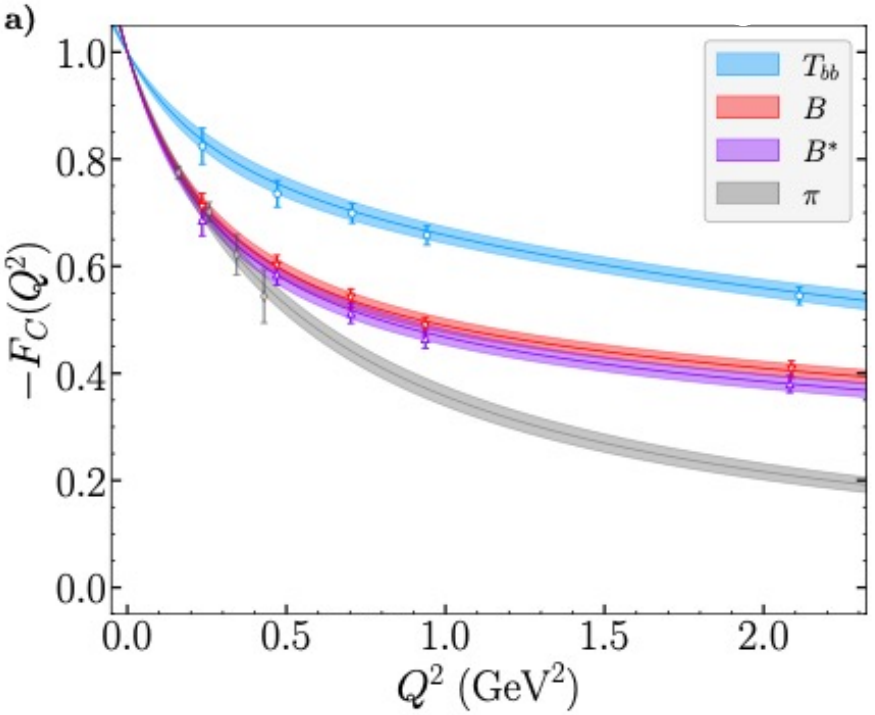
h	m_r (GeV)	t_+ (GeV^2)
T_{bb}	m_ω	$(3m_\pi)^2$
B, B^*	m_ρ	$(2m_\pi)^2$
π	m_ρ	$(2m_\pi)^2$

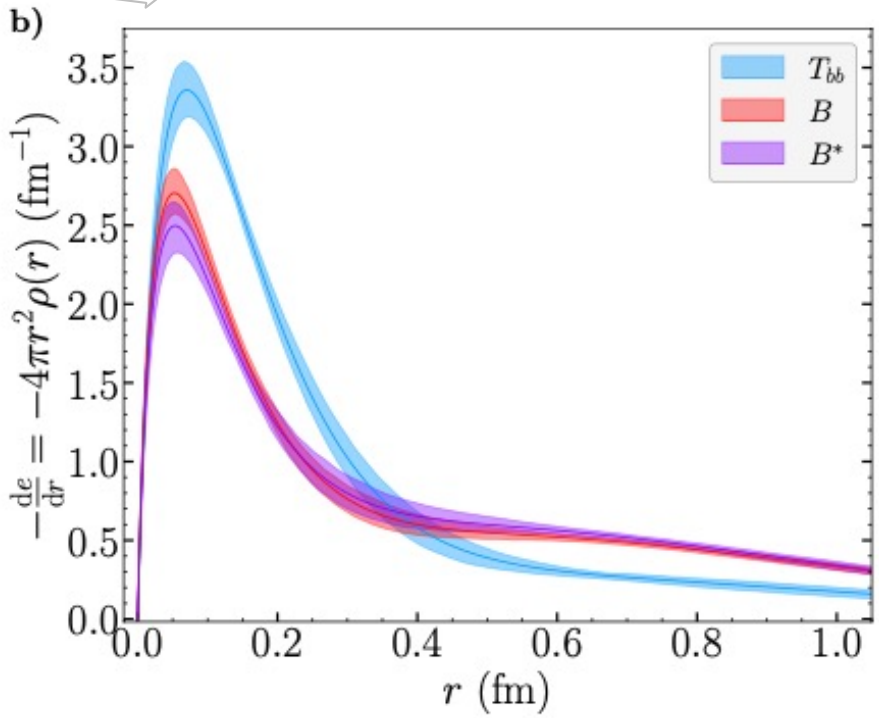
$$F(Q^2) = \frac{1}{1 + \frac{Q^2}{m_r^2}} \sum_{n=0}^{\infty} a_n z^n(Q^2, t_+, t_0)$$

Results: charge distributions of $T_{bb}, B^{(*)}, \pi$

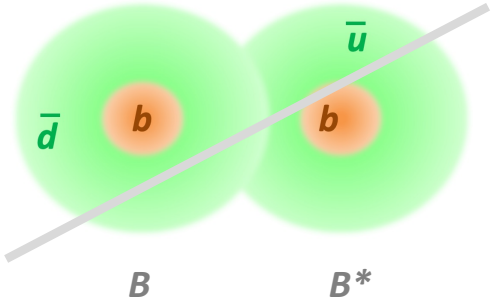


$$\rho(r) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\ e^{i\vec{q}\cdot\vec{r}} F_C(|\vec{q}|^2)$$





$$\langle r_C^2 \rangle = \int r^2\ [\rho\ 4\pi r^2] dr = 6\frac{dF_C(0)}{dQ^2}$$

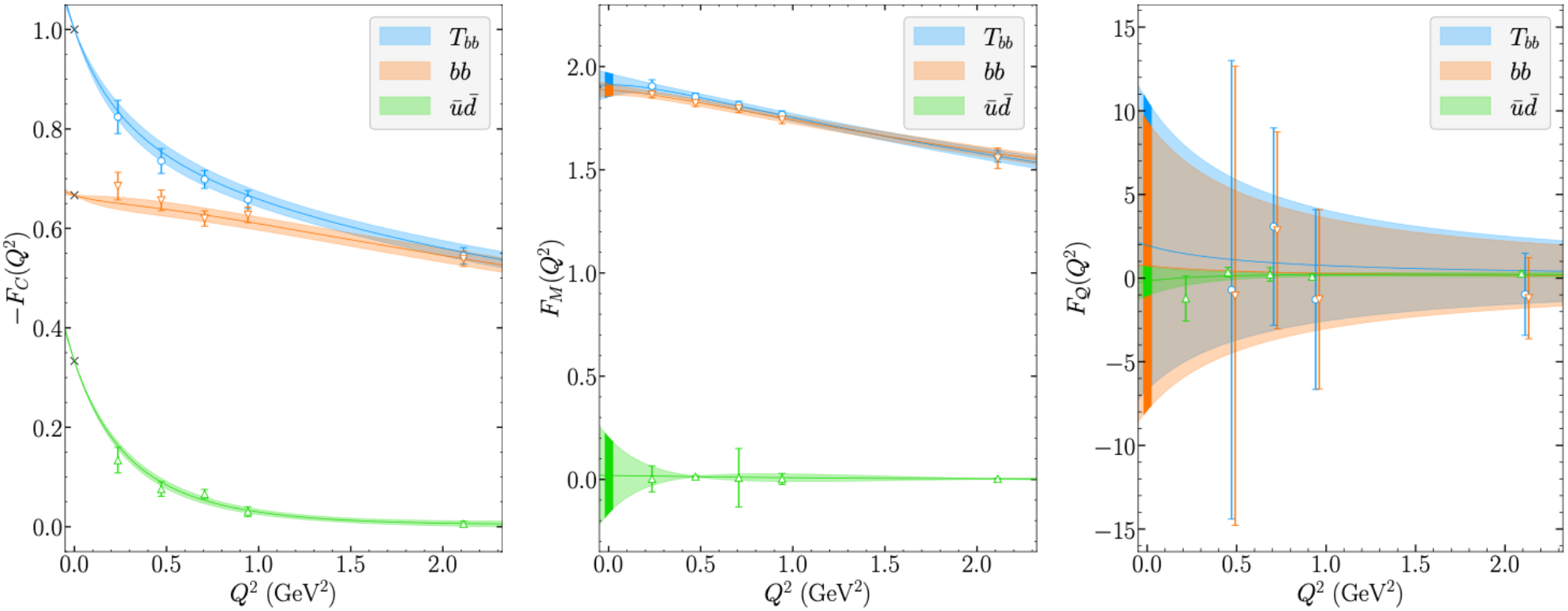


$$r_C^{T_{bb}} \ll r_C^B + r_C^{B^*}$$

- disfavors $B - B^*$ structure

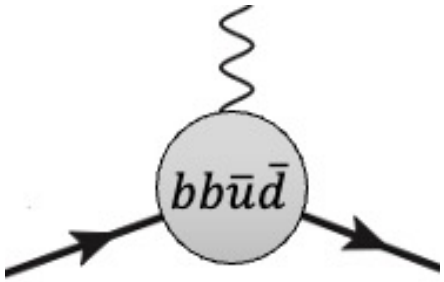
h	$\sqrt{\langle r_C^2 \rangle}$ (fm)
T_{bb}	0.499(31)
B	0.692(21)
B^*	0.698(23)
π	0.652(20)

Results: T_{bb} form factors

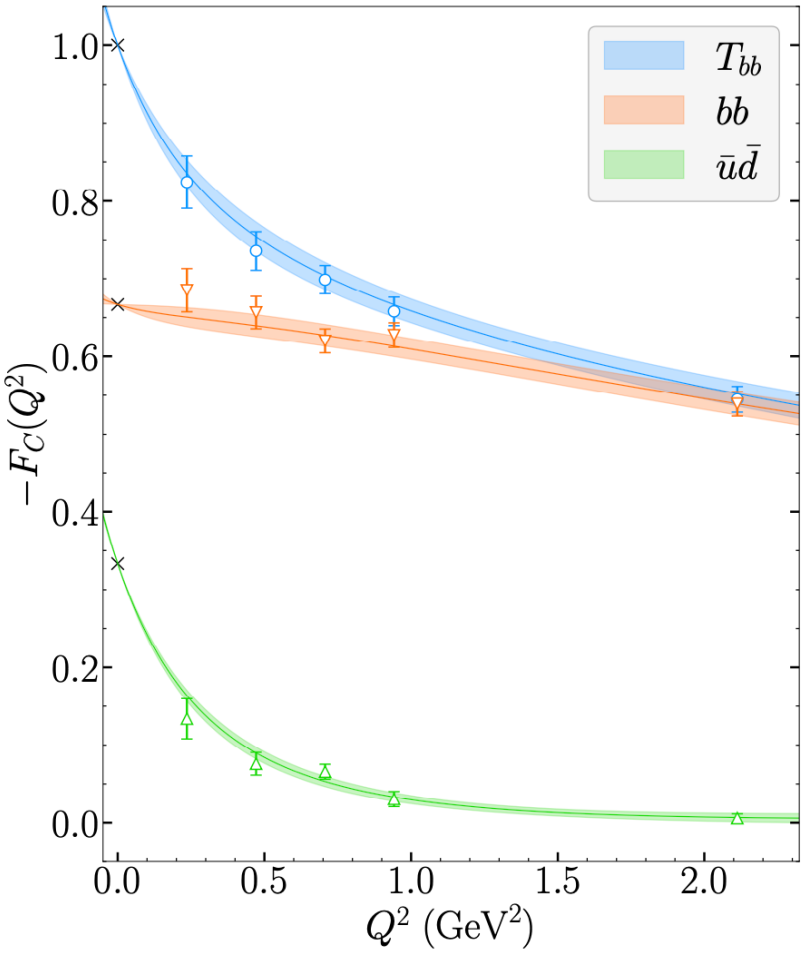


$$\hat{\jmath}^{\mu}_{u/d} = \tfrac{2}{3}\bar{u}\gamma^{\mu}u - \tfrac{1}{3}\bar{d}\gamma^{\mu}d \qquad \hat{\jmath}^{\mu}_{b} = -\tfrac{1}{3}\bar{b}\gamma^{\mu}b$$

$$\hat{\jmath}^{\mu}_{EM} = \hat{\jmath}^{\mu}_{u/d} + \hat{\jmath}^{\mu}_{b}$$

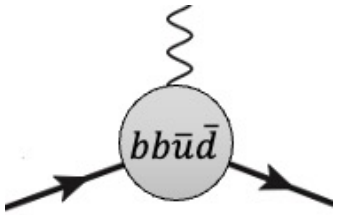


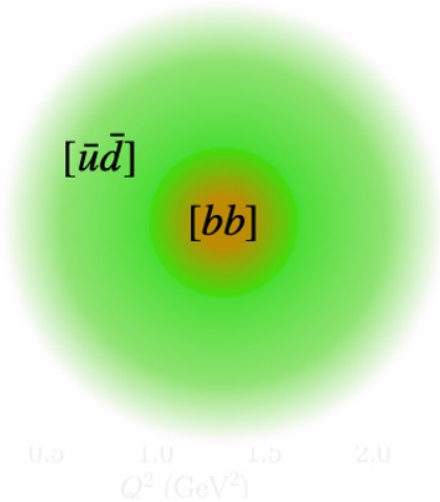
Charge form factor and radius



$$\sqrt{\langle r_C^2 \rangle} = \sqrt{6 \cdot \frac{\mathrm{d}F_C}{\mathrm{d}Q^2}(0)}$$

	T_{bb}	$[bb]$	$[\bar{u}\bar{d}]$
$\sqrt{\langle r_C^2 \rangle}$ (fm)	0.499(31)	0.174(59)	0.511(14)



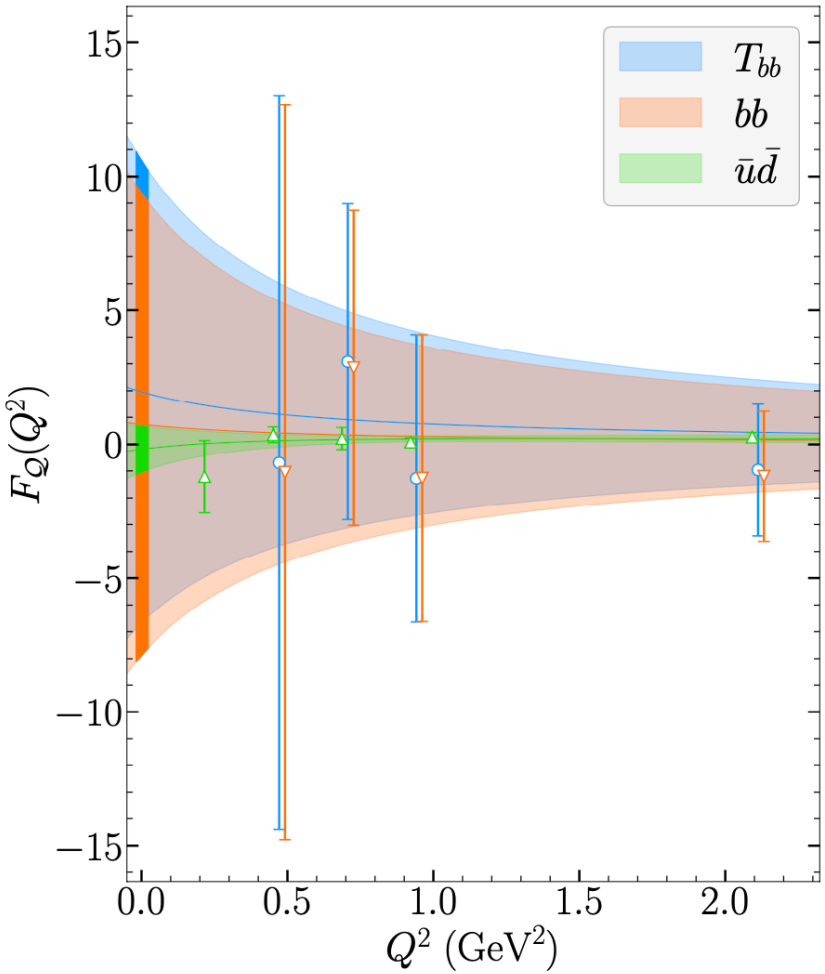


- $r_C^{bb} << r_C^{ud}$
- strongly favors diquark antidiquark

$$\{[bb]_{\bar{c}}^{l_{bb},s_{bb}}[\bar{u}\bar{d}]_c^{I=0,l_{\bar{u}\bar{d}},s_{\bar{u}\bar{d}}}\}_{l_{12}}^{J^P=1^+}$$

Electric quadropole form factor of T_{bb}

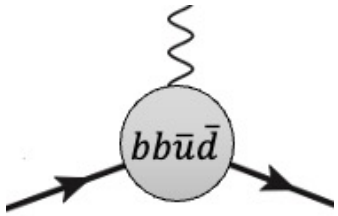
$$F_{\mathcal{Q}}(0) = m^2 \cdot \mathcal{Q}$$



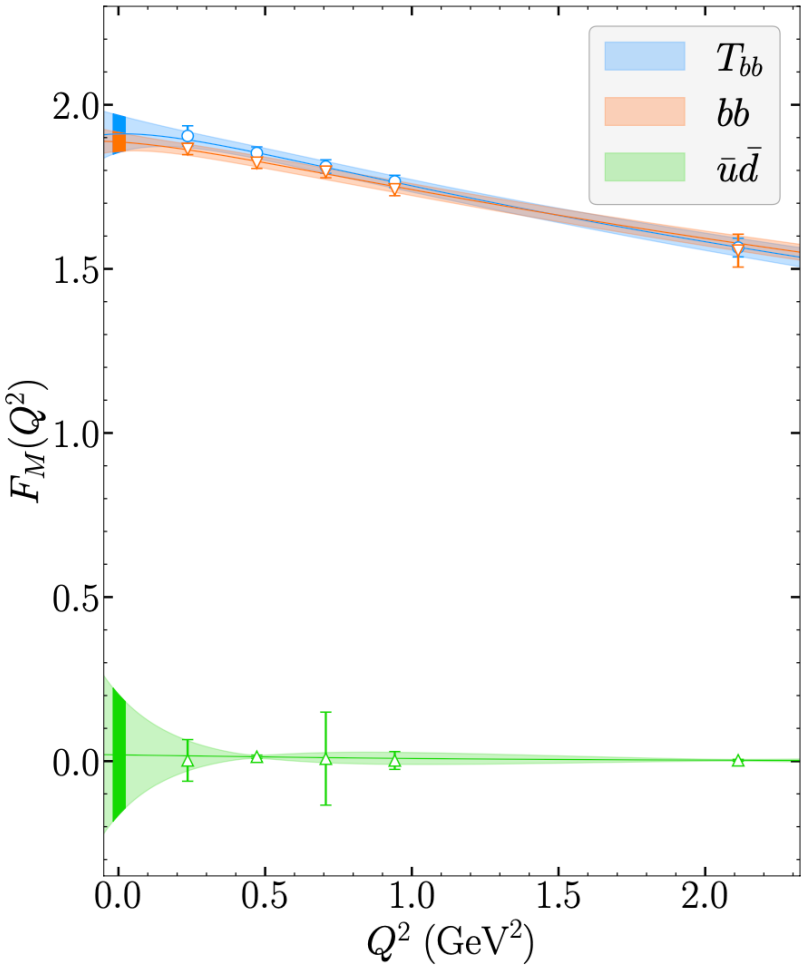
- all three components consistent with zero
- spherical distributions
- orbital angular momenta zero

$$l_{bb} = l_{\bar{u}\bar{d}} = l_{12} = 0$$

- as expected for the ground state



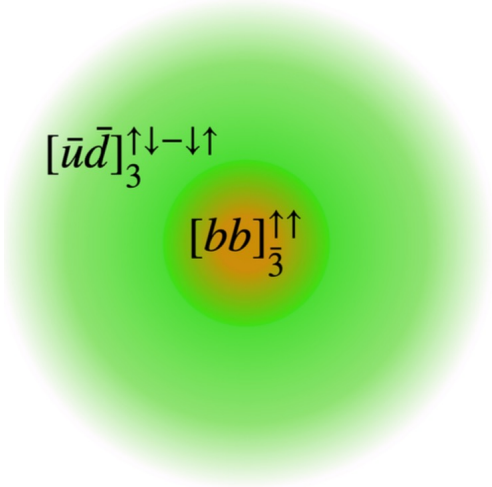
Magnetic form factor and magnetic moment of T_{bb}

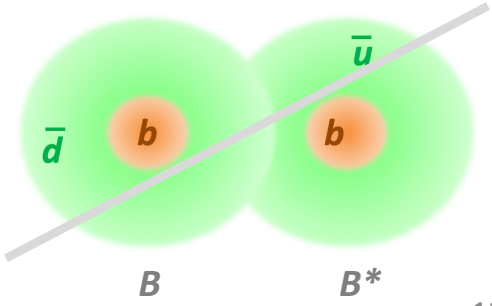


$$\mu_{T_{bb}} = \langle T_{bb}| \sum_{q=u,d,b} \frac{e_q}{2m_q}(\hat{l}_q + g_q \hat{s}_q)|T_{bb}\rangle = \frac{F_M(0)}{2m_{T_{bb}}}$$

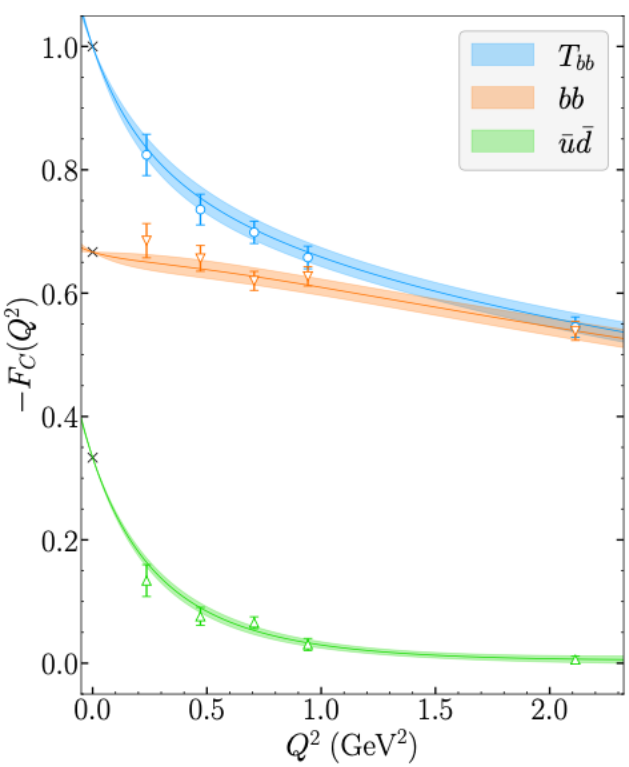
	T_{bb}	$[bb]$	$[\bar{u}\bar{d}]$
$2m_{T_{bb}} \cdot \mu$	1.912(57)	1.887(29)	0.02(18)

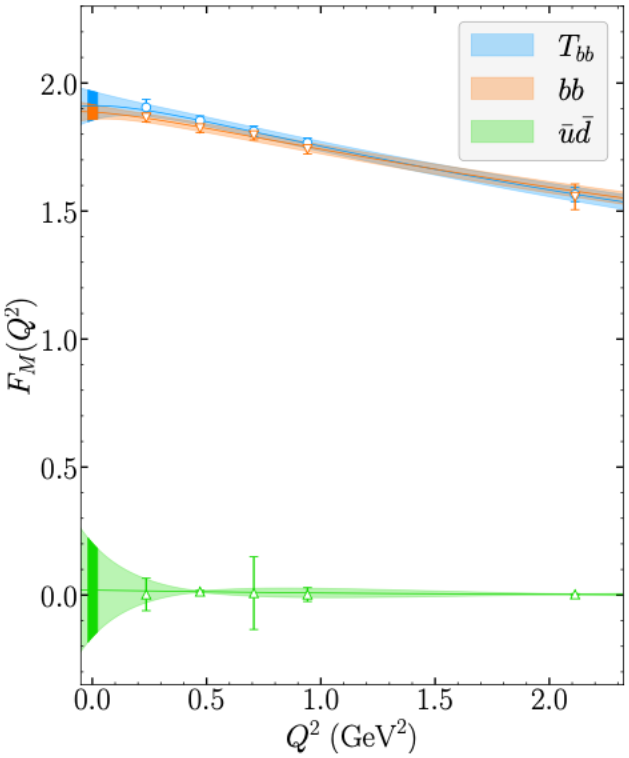
$$s_{\bar{u}\bar{d}} = 0, \quad s_{bb} = 1$$

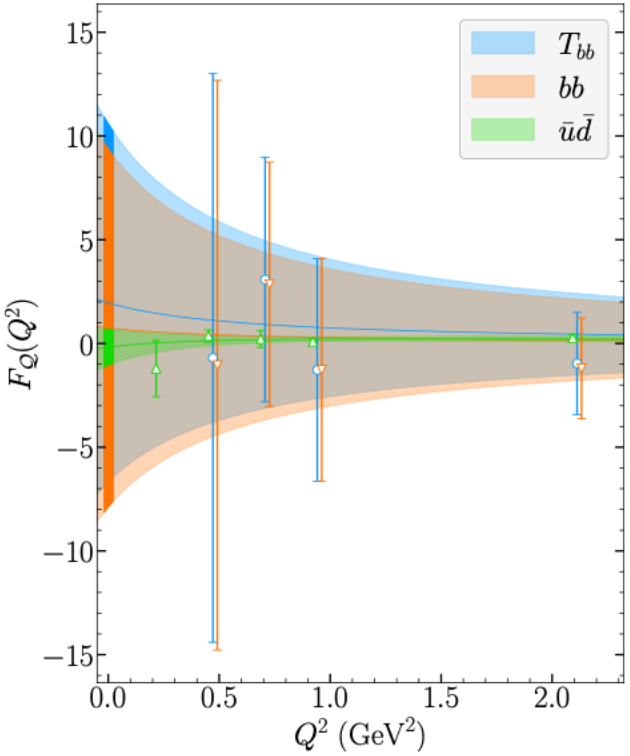




Results: T_{bb} form factors $bb\bar{u}\bar{d}$







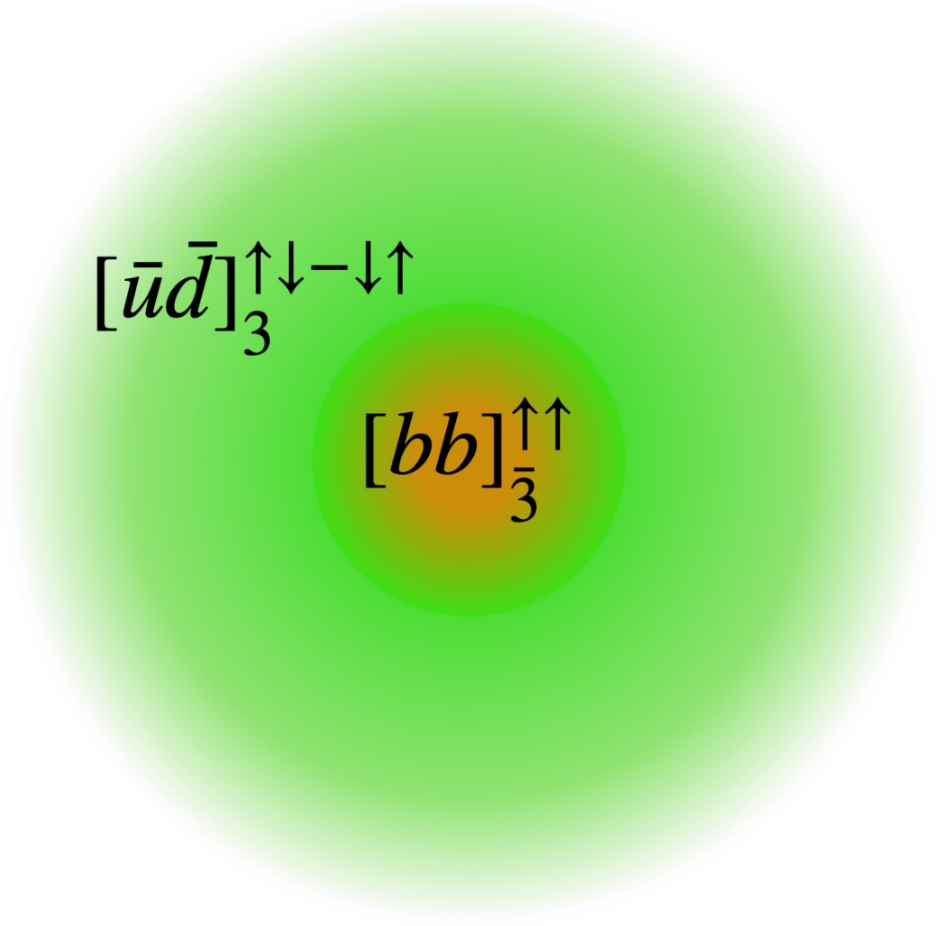
$$\{[bb]^{l_{bb}=0,s_{bb}=1}_{\bar{c}}[\bar{u}\bar{d}]^{I=0,l_{\bar{u}\bar{d}}=0,s_{\bar{u}\bar{d}}=0}_{c}\}^{J^P=1^+}_{l_{12}=0}$$

$c = 3, 6$?

Pauli principle

$c = 3$

Summary on structure of T_{bb}



$$\left\{[bb]_{\bar{3}}^{l_{bb}=0,s_{bb}=1}[\bar{u}\bar{d}]_{3}^{I=0,l_{\bar{u}\bar{d}}=0,s_{\bar{u}\bar{d}}=0}\right\}_{l_{12}=0}^{J^P=1^+}$$

spectral function methods:

Joshua Hoffer, Gernot Eichmann, Christian Fischer: arxiv:2409.05779, PRD: E_b=110(40) MeV



Structure of a glueball

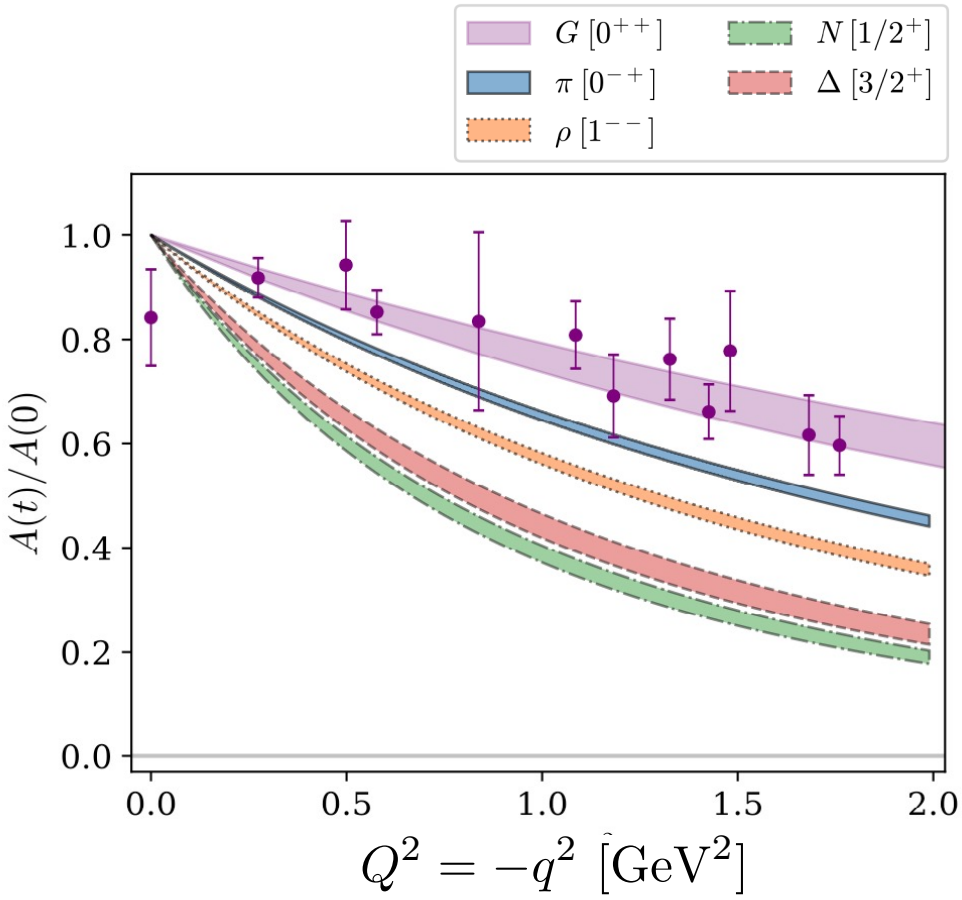
Energy-momentum form factor of scalar glueball

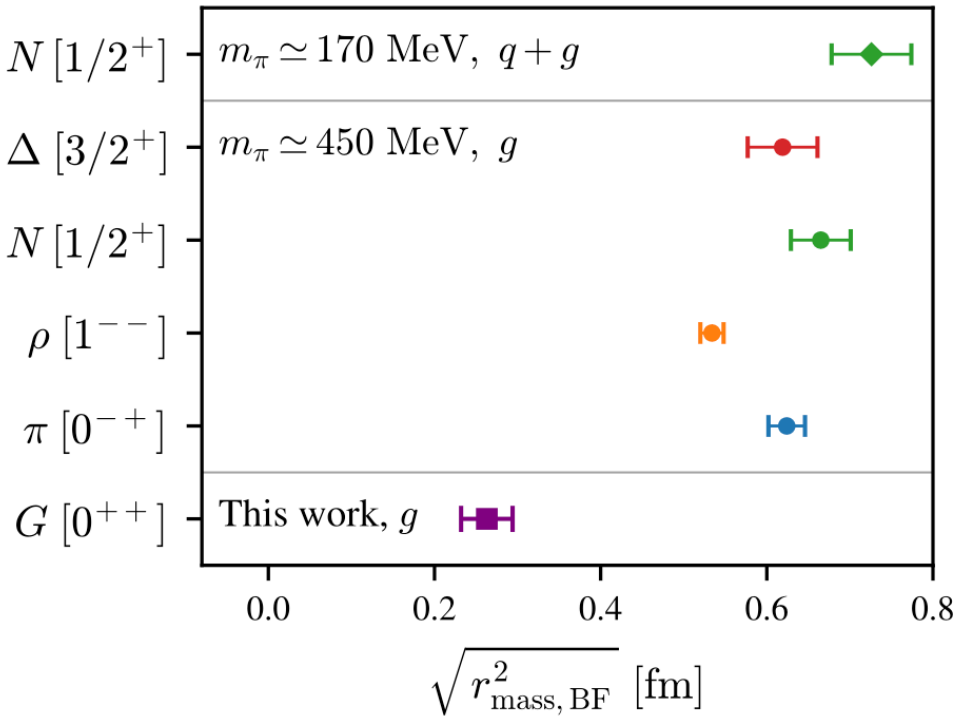
(gravitational) in quenched pure–glue theory



$$\langle G[0^{++}](p')| T^{\mu\nu} |G[0^{++}](p)\rangle$$

energy momentum tensor





Abbott, Hackett, Pefkou, Romero-Lopez, Shanahan: 2508.21821, PRL

Glueball is small !

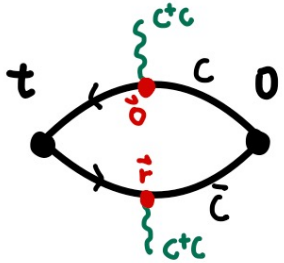
Charmonia and charmoniumlike states

$\bar{c}c$ $\bar{c}c\bar{q}q$ I=0

Portraits of charmonia $\bar{c}c$

The observed spatial distributions for 1P charmonia exhibit important relativistic effects that can be understood in the Dirac theory

$$C(\vec{r}) \propto \langle \mathcal{O}\ c^\dagger c(\vec{0})\ c^\dagger c(\vec{r})\ \mathcal{O}^\dagger \rangle$$



Portraits of $\bar{c}Gc$ charmonium hybrids

neglecting strong their decays

Hybrid c G c with $J^{PC}=1^{-+}$
this work: c c found in l=0

example: χ_{c0}

non-relativistic quark model:

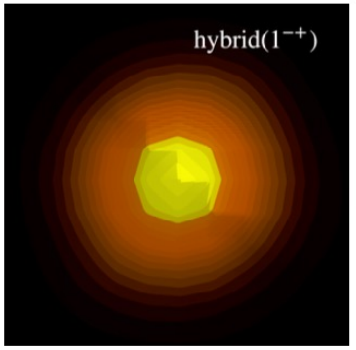
incompatible since it predicts l=1

$$|\phi(0)| = 0$$

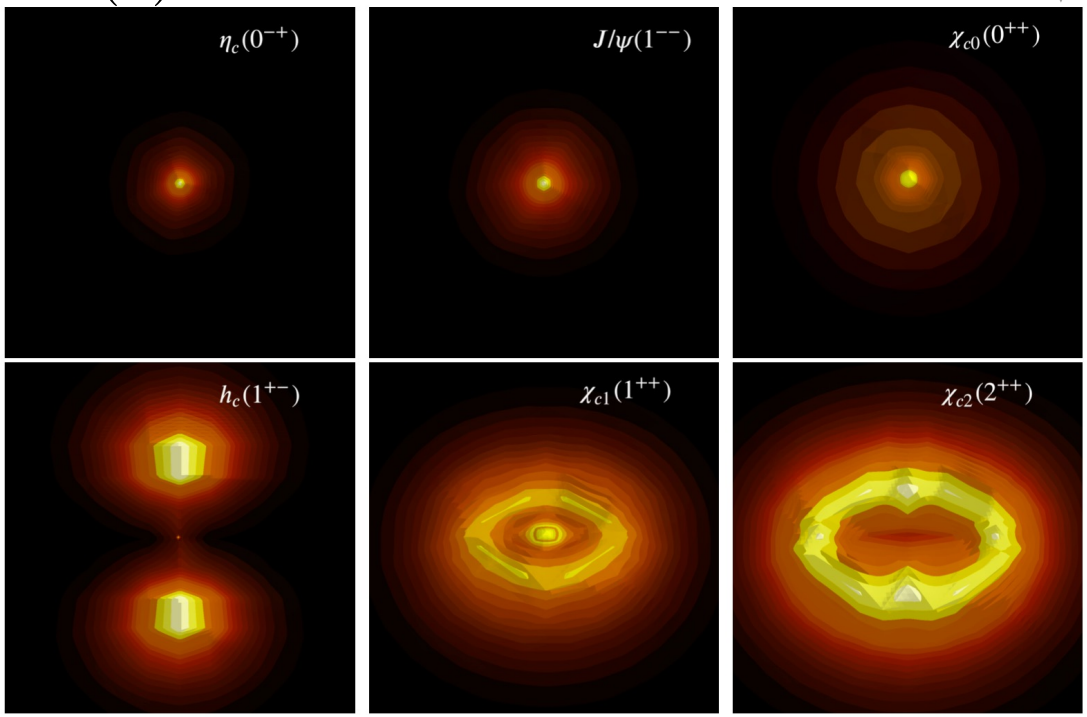
relativistic model:

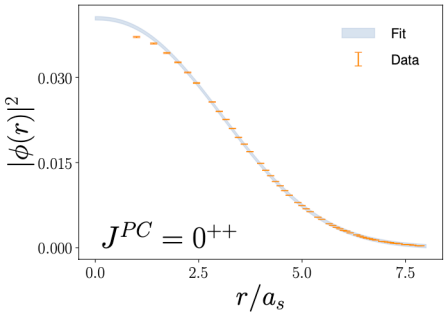
compatible, lower-comp. with l ' = 0 dominates near origin

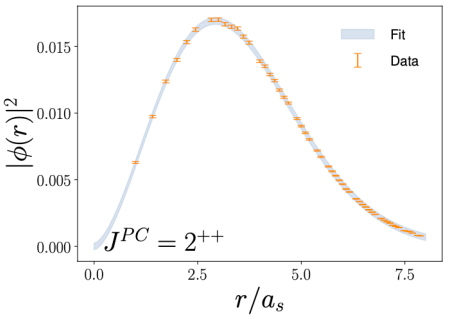
$$|\phi(0)| \neq 0 \qquad \begin{pmatrix} F_{n,\kappa}(r)\chi^{(l)}_{j,m}(\theta,\phi) \\ iG_{n,\kappa}(r)\chi^{(l')}_{j,m}(\theta,\phi) \end{pmatrix}$$



$C(\vec{r})$

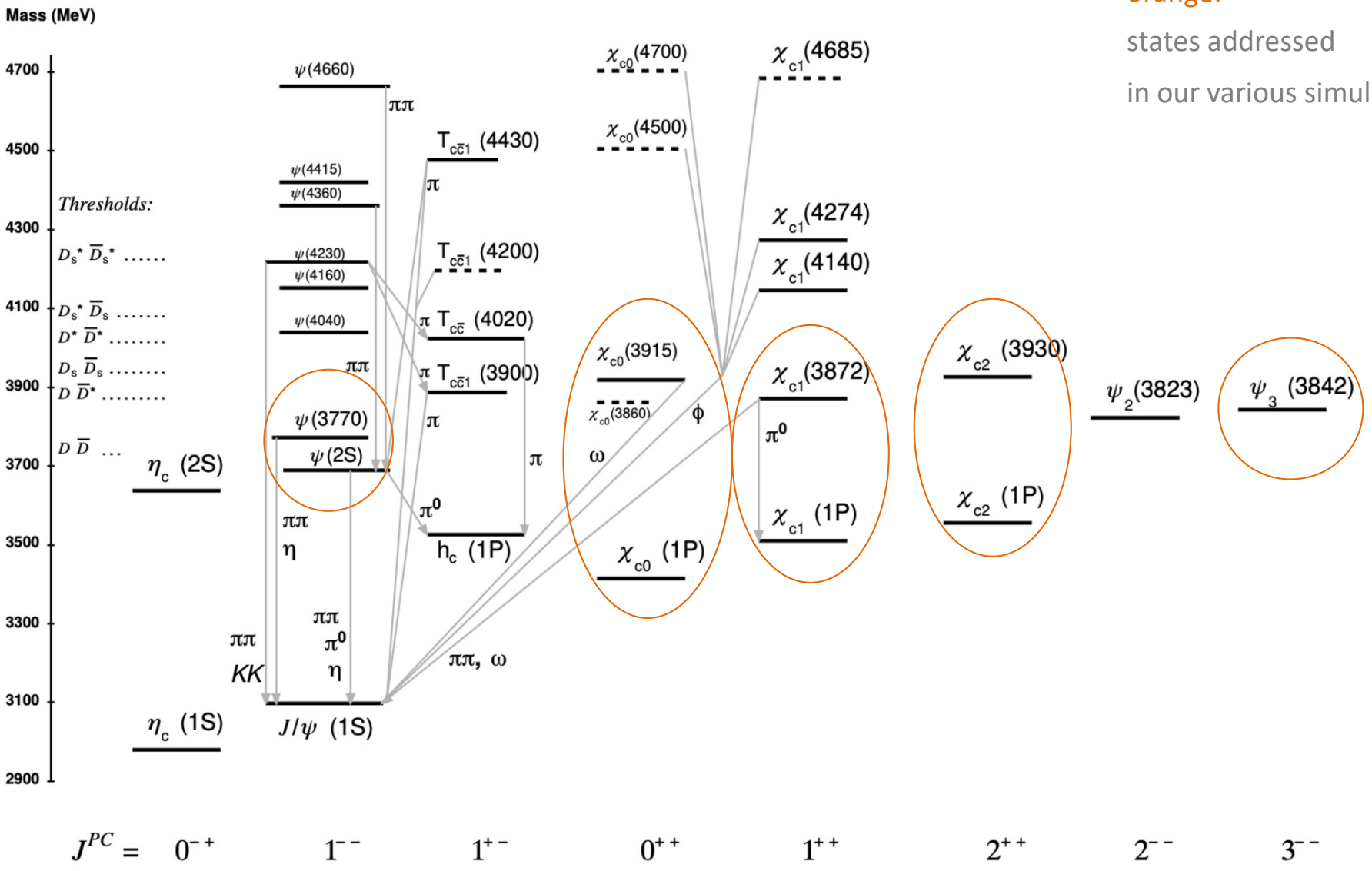






Q. Liu et al, 2505.21193

Charmonia and charmoniumlike states, PDG 2024



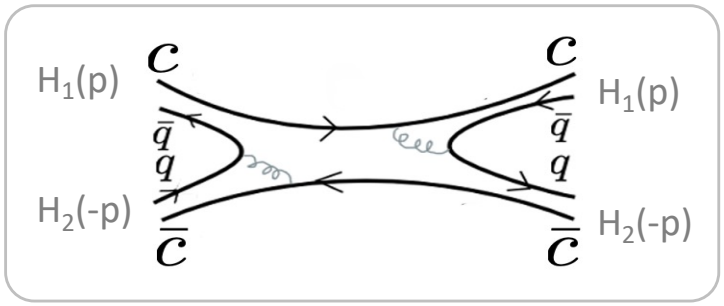
orange:

states addressed

in our various simulations

Charmoniumlike states from one-channel scattering

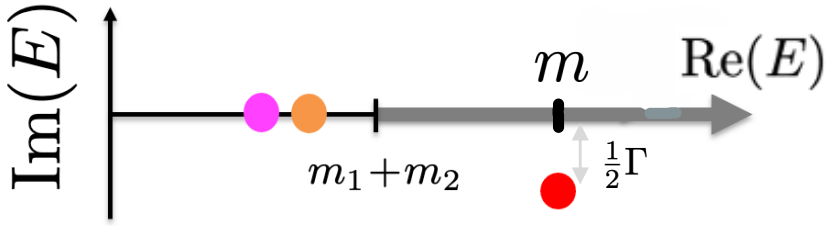
scattering amplitude T(E) for given partial wave l



$$S(E) = e^{2i\delta(E)} = 1 + i\,\frac{p}{4\pi E}\,T(E)$$

$$\rightarrow \boxed{T(E) = \frac{8\pi E}{p\cot\delta - ip}}$$

$$E = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2}$$



Virtual st. p = - i |p|, sheet II

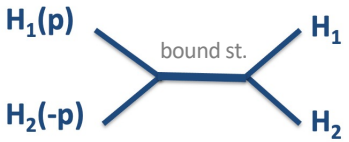
Bound st. p = i |p|, sheet I

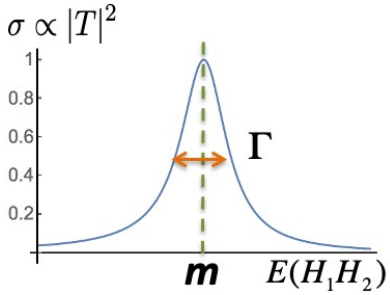
Resonance sheet II

$p^2<0$

$$T(E) \propto \frac{1}{E^2 - m^2}$$

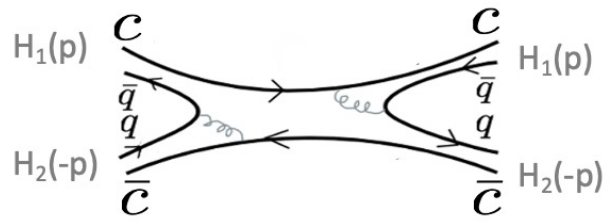
$$T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$



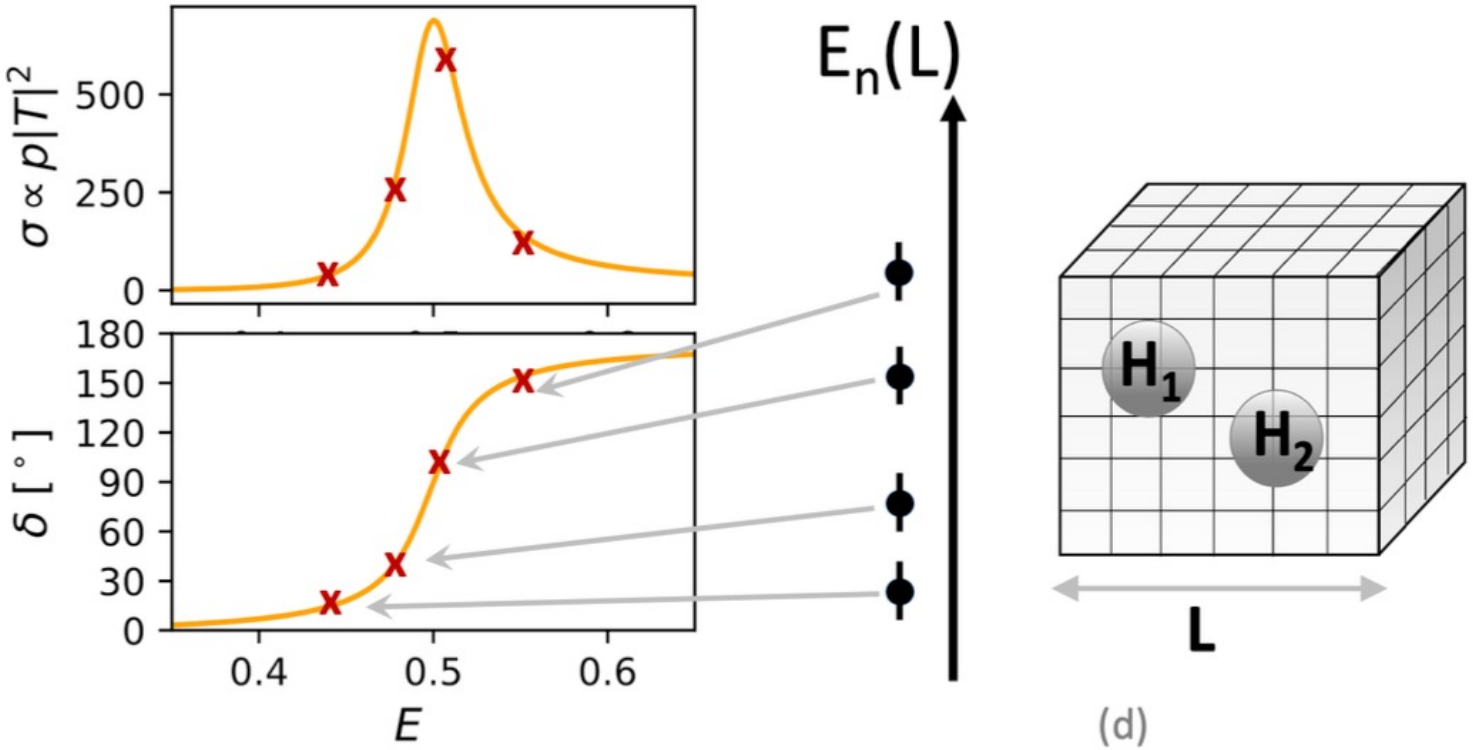


Strategy to determine E_n, δ(E), T(E)

$$T(E) \propto \frac{1}{p \cot \delta - ip}$$



$$\langle \Omega | \mathcal{O}_f(t)\ \mathcal{O}_i(0) | \Omega \rangle = \sum_n \langle \Omega | \mathcal{O}_f | n \rangle\ e^{-E_n t}\ \langle n | \mathcal{O}_i | \Omega \rangle$$



at infinite volume δ(E), T(E) ← Luscher 1991 — E(L) energies from lattice with spatial extent L

Charmonia and charmoniumlike states from CLS ensmbles

CLS ensembles, Nf=2+1: $m_\pi \approx 280$ MeV, $m_K \approx 467$ MeV

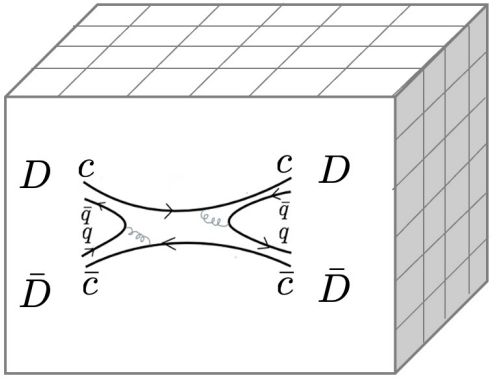
two values of m_c : $m_D \approx 1762$ MeV, 1927 MeV ($m_D^{exp}=1867$ MeV)

$a \approx 0.086$ fm, $N_L=24$ & 32

Wick contractions with distillation

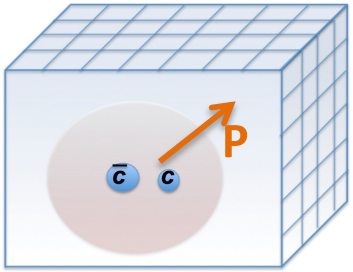
Collins, Padmanath, Mohler, Piemonte, SP
1905.03506 PRD: $\mathbf{J^P=1^-, 3^-}$
2011.02541 JHEP: $\mathbf{J^P=0^+, 2^+}$

Charmonia with $J^{PC}=1^{--}, 3^{--}$



Operators

$$\mathcal{O}^{\bar{c}c} = \left(\bar{c}\Gamma c\right)_{\vec{P}}$$

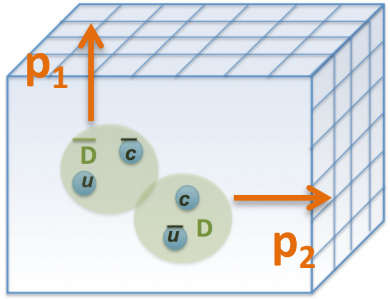


$$\mathcal{O}^{\bar{D}D} = \left(\bar{c}\Gamma_1 q\right)_{\vec{p}_1}\left(\bar{q}\Gamma_2 c\right)_{\vec{p}_2}$$
$$= \bar{D}(\vec{p}_1)D(\vec{p}_2)$$

$\vec{P} = \vec{p}_1 + \vec{p}_2$ P: 0

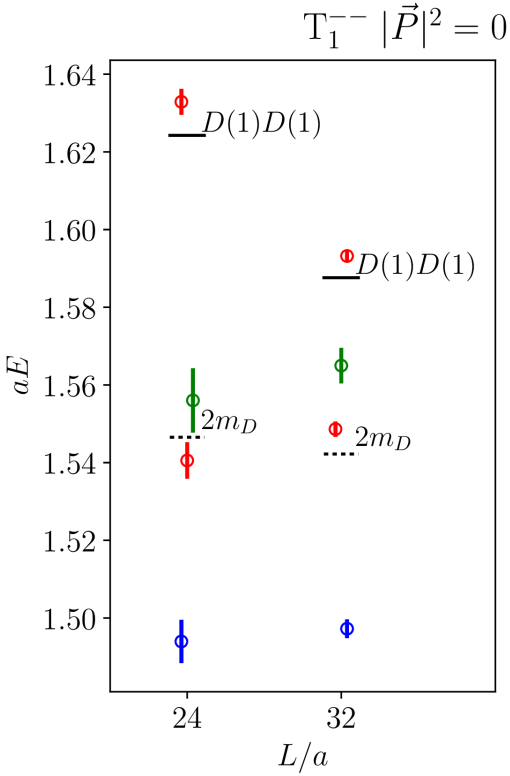
(0,0,1) $2\pi/N_L$

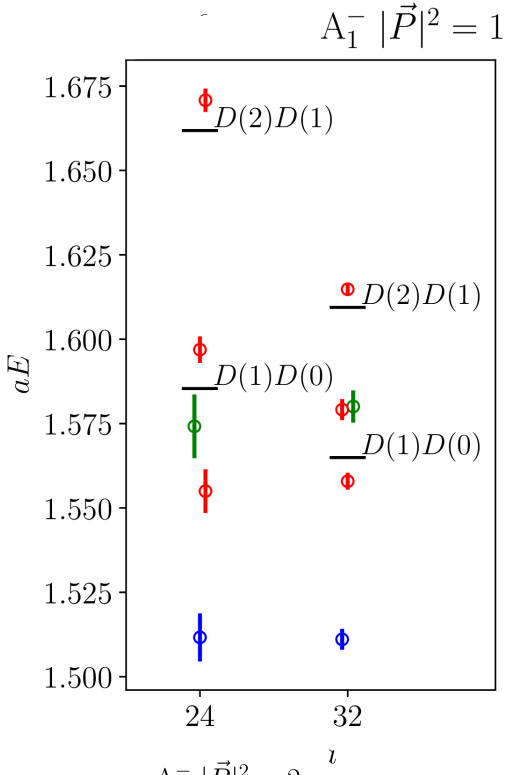
(1,1,0) $2\pi/N_L$



Eigen-energies E_n

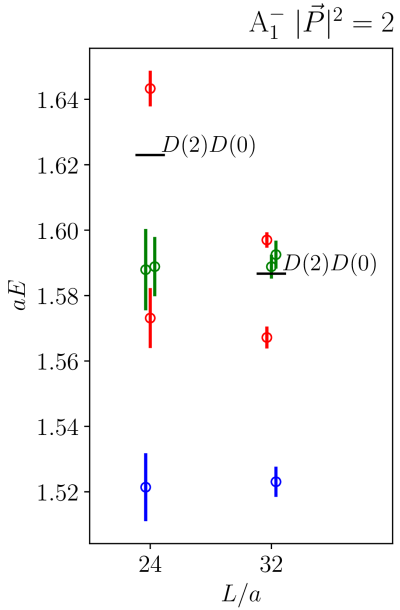
$m_D \approx 1762$ MeV





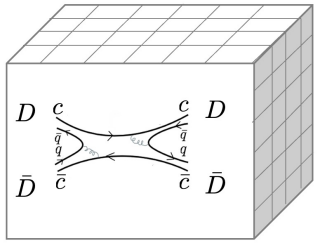
green: 3-

blue: $\psi(2S)$

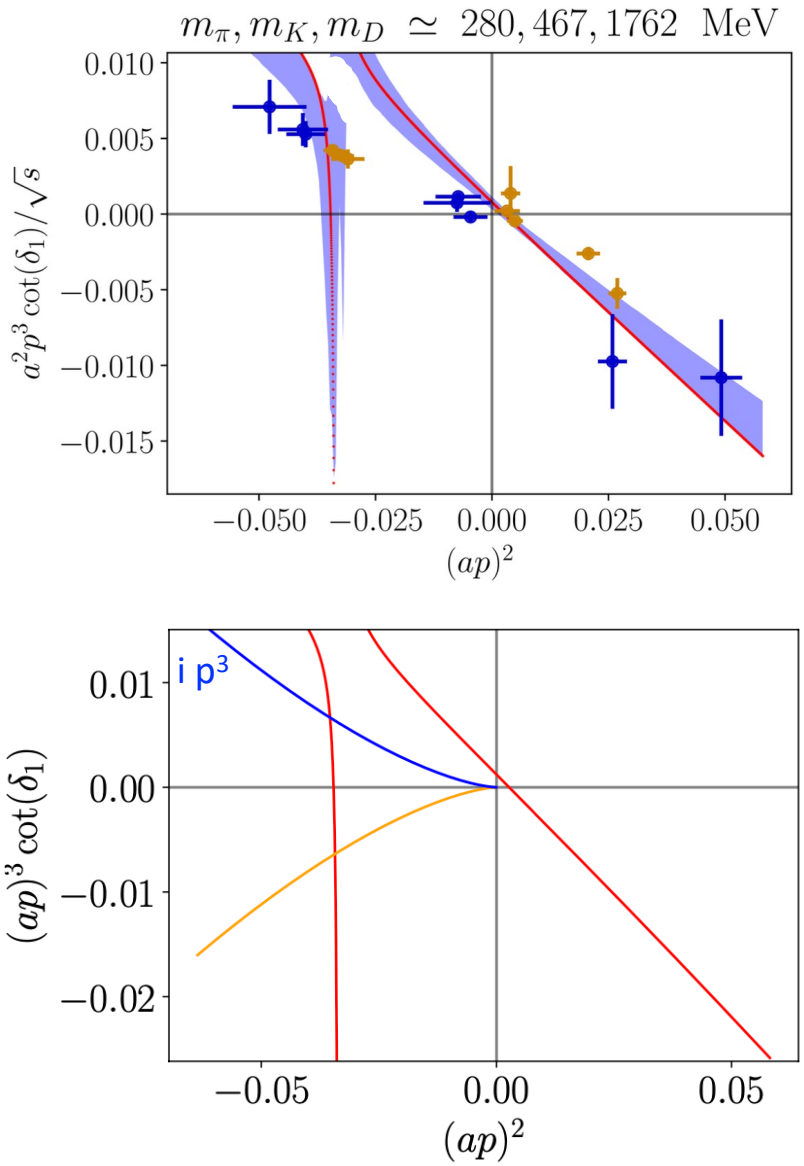


Charmonia with $J^P=1^-$

$$\frac{p^3\cot(\delta_1)}{\sqrt{s}} = \left(\frac{G_1^2}{m_1^2-s} + \frac{G_2^2}{m_2^2-s}\right)^{-1}$$



$$T \propto \frac{1}{p^3\cot\delta - ip^3}$$



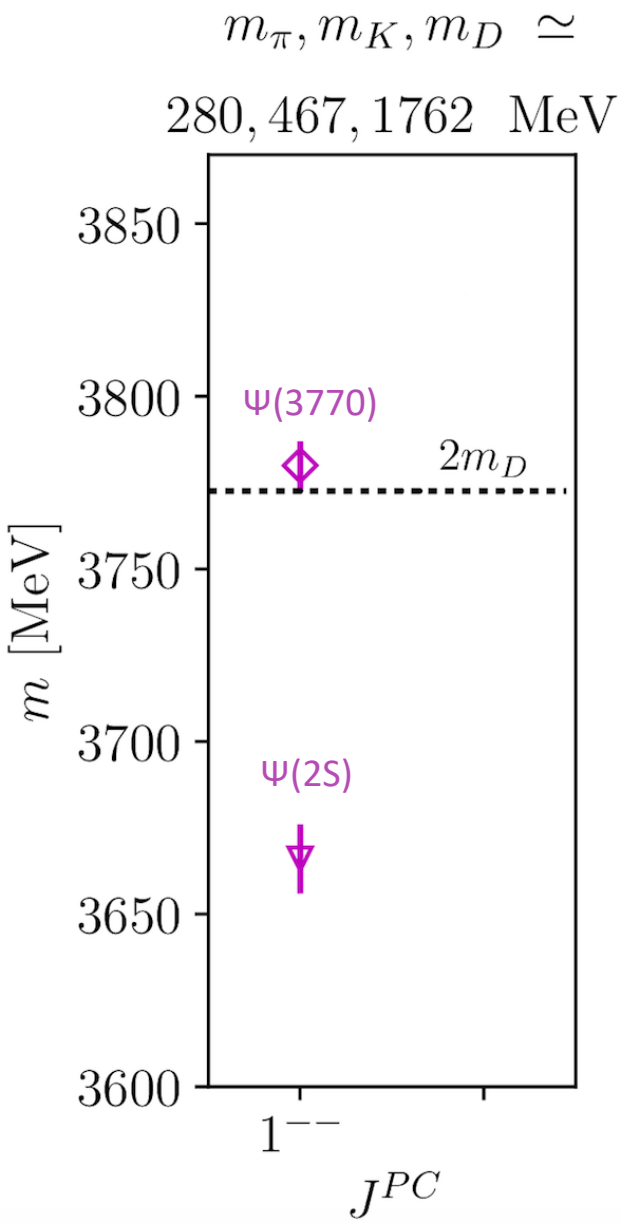
resonance Ψ(3770)

$$\frac{p^3\cot\delta_1}{\sqrt{s}}|_{s\simeq m^2} = \frac{6\pi}{g^2}(m^2-s)$$

$$\Gamma = \frac{g^2\ p^3}{6\pi\ s}$$

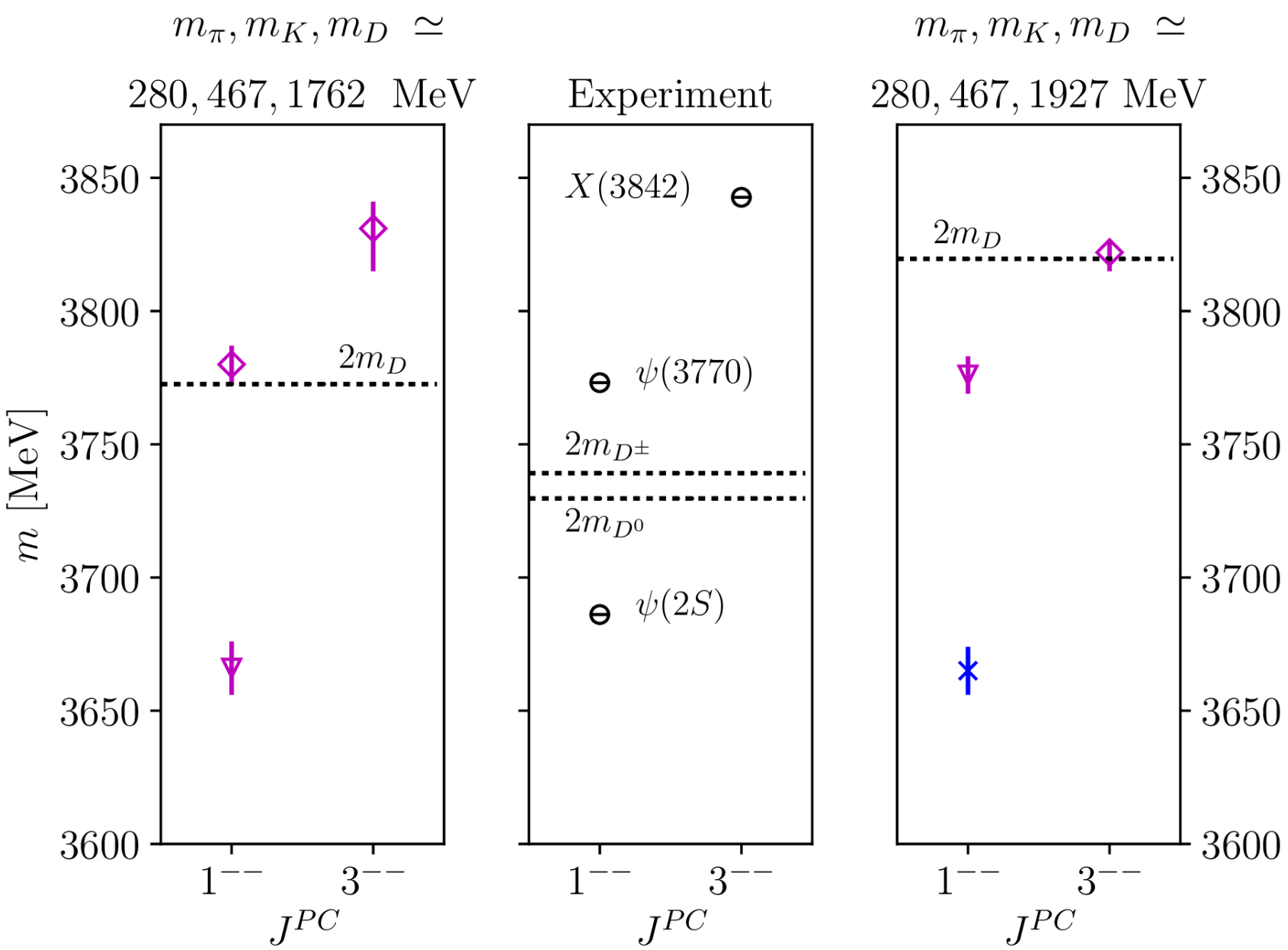
	g
lat	16 ± 2
exp	19 ± 1

bound state Ψ(2S)



$$m = m^{\rm lat} - M_{\rm av}^{\rm lat} + M_{\rm av}^{\rm exp} \qquad M_{\rm av} = \frac{1}{4}(m_{\eta_c} + 3m_{J/\psi})$$

Masses of charmonium states with $J^P=1^-, 3^-$

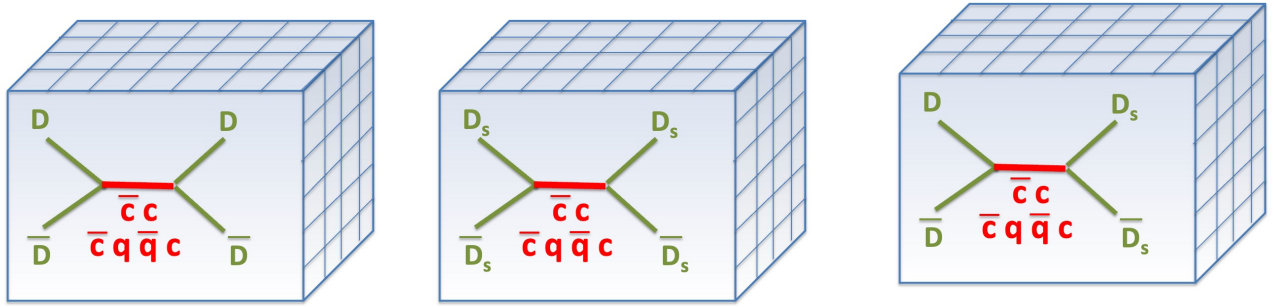


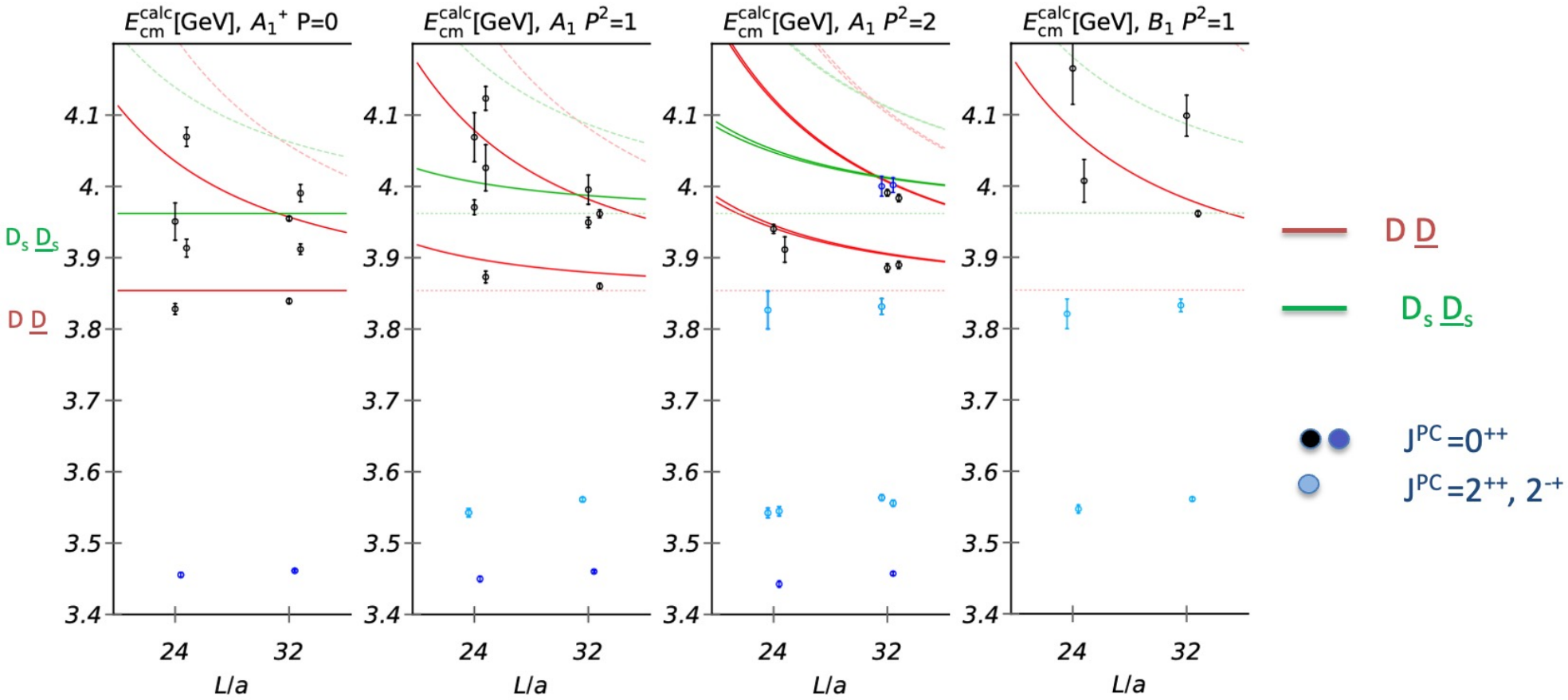
Charmoniumlike states with $J^P=0^+,2^+$

two coupled channels $D\bar{D} - D_s\bar{D}_s$

main approx: omitting lowerlying $\eta_c\eta$ channel

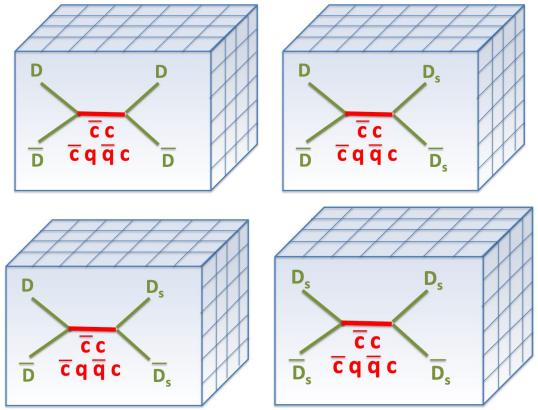
$\mathcal{O}: \ \bar{c}c, \bar{D}D, \bar{D}_sD_s, J/\psi\omega$



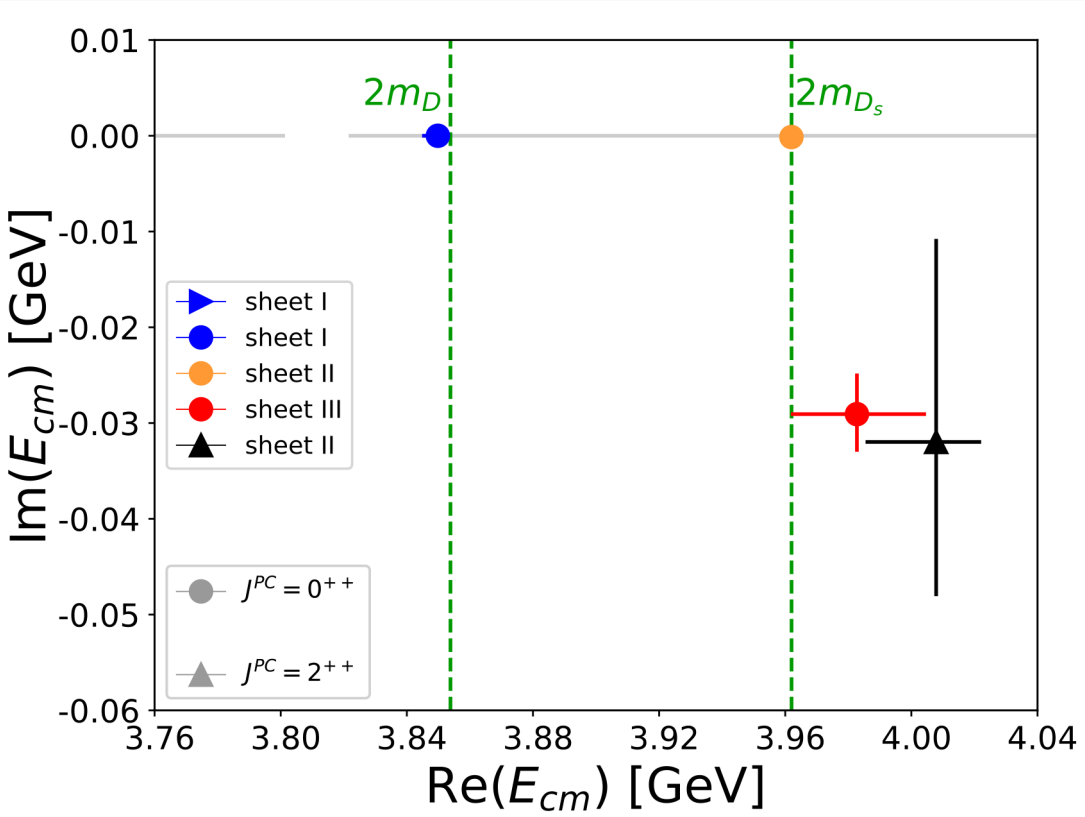


Charmoniumlike states: poles in the scattering amplitude

$$T(E) = \begin{pmatrix} T_{aa}(E) & T_{ab}(E) \\ T_{ab}(E) & T_{bb}(E) \end{pmatrix}$$



$m_D \approx 1927$ MeV

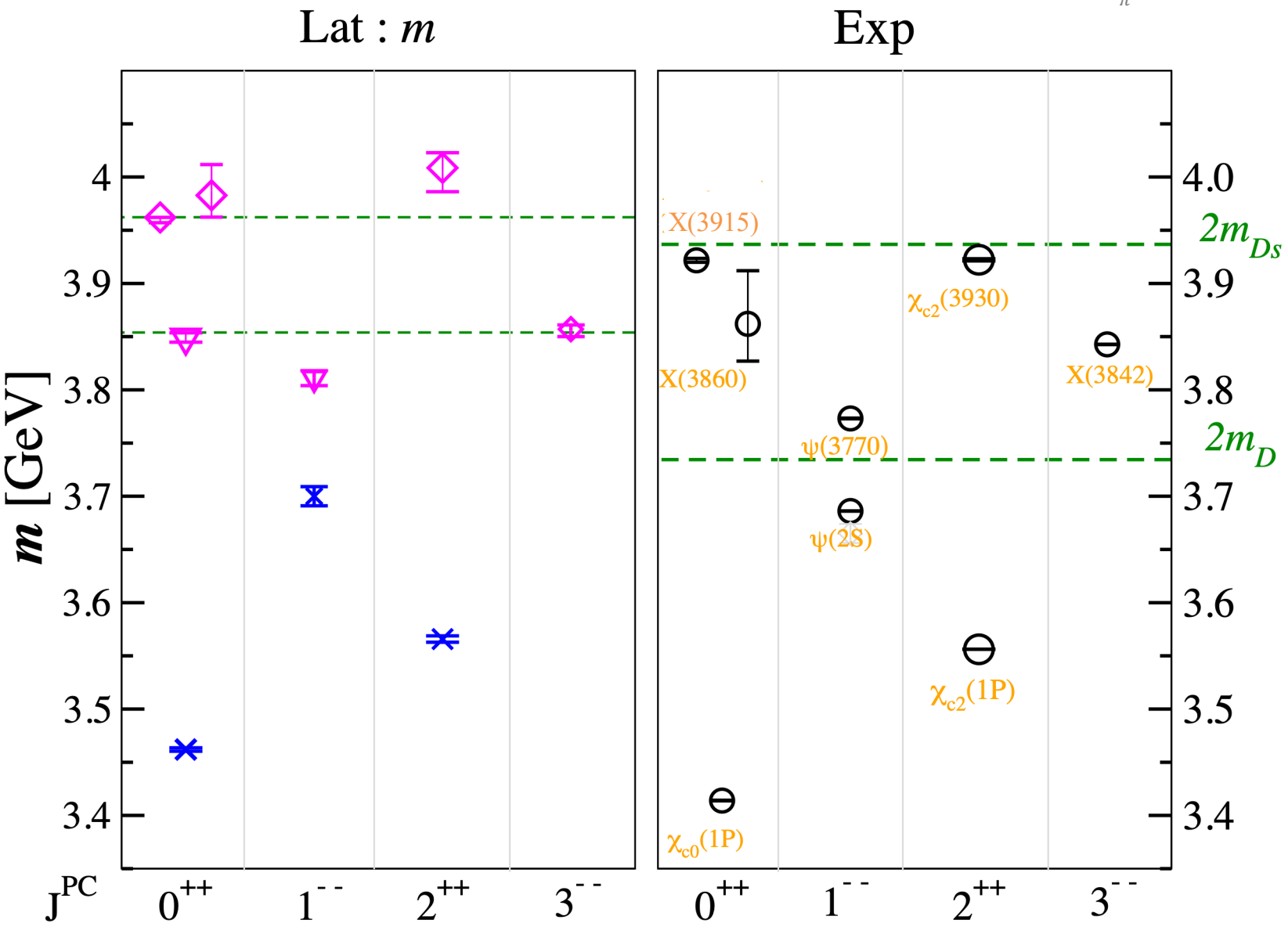


reanalysis finds similar poles
Shi, Albaladejo, Du, Guo, Nieves 2410.19563

Charmonia and charmoniumlike states: masses

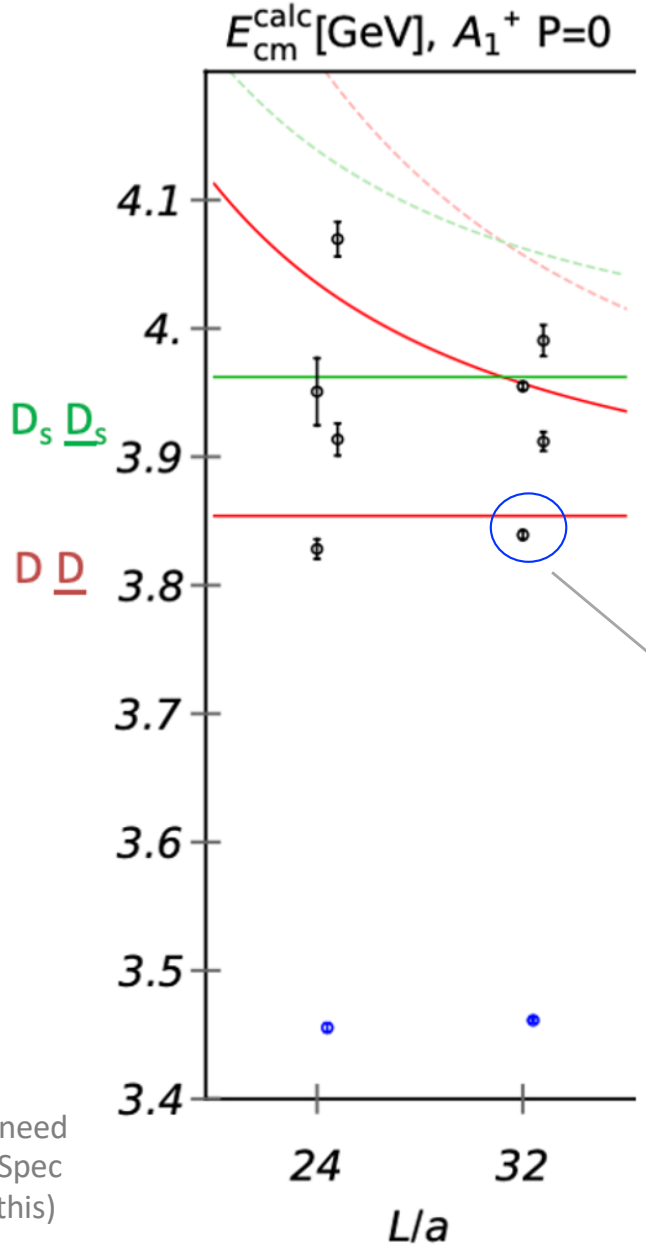
$m_D \approx 1927$ MeV

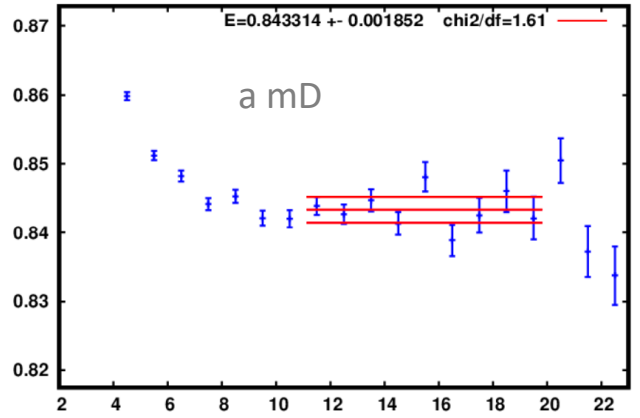
$m_\pi \approx 280$ MeV

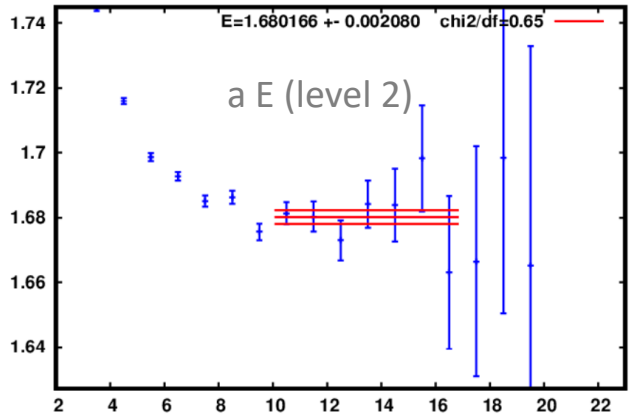


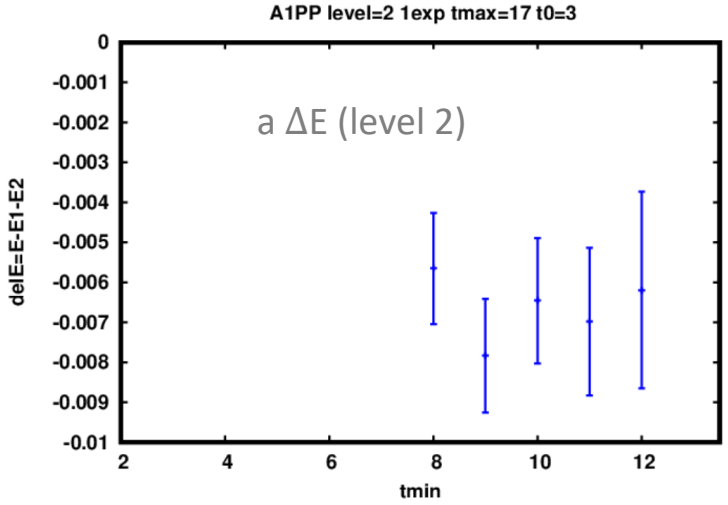
Charmoniumlike states with $J^P=0^+$

$m_D \approx 1927$ MeV









disagreement:

- in physics conclusions
- ΔE of D$\underline{\text{D}}$ threshold level

- our 2011.02541: ΔE<0

- HadSpec 2602.09862: ΔE≈0

(also earlier 2309.14070 PRL with mpi~390 MeV)

Christopher Thomas will likely talk about this channel

Reasons ??

- lowerlying $\eta_c\eta$ $\sim 3.54\ GeV$ would need to be included from our side (HadSpec finds negligible coupling of D$\underline{\text{D}}$ to this)
- both groups use just one lat. spacing
- ??

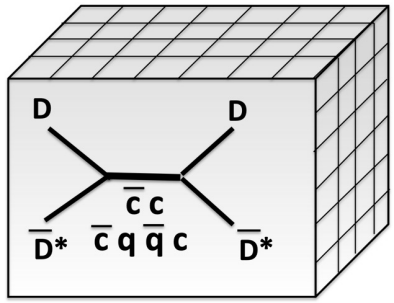
Charmoniumlike X(3872)= $\chi_{c1}(3872)$

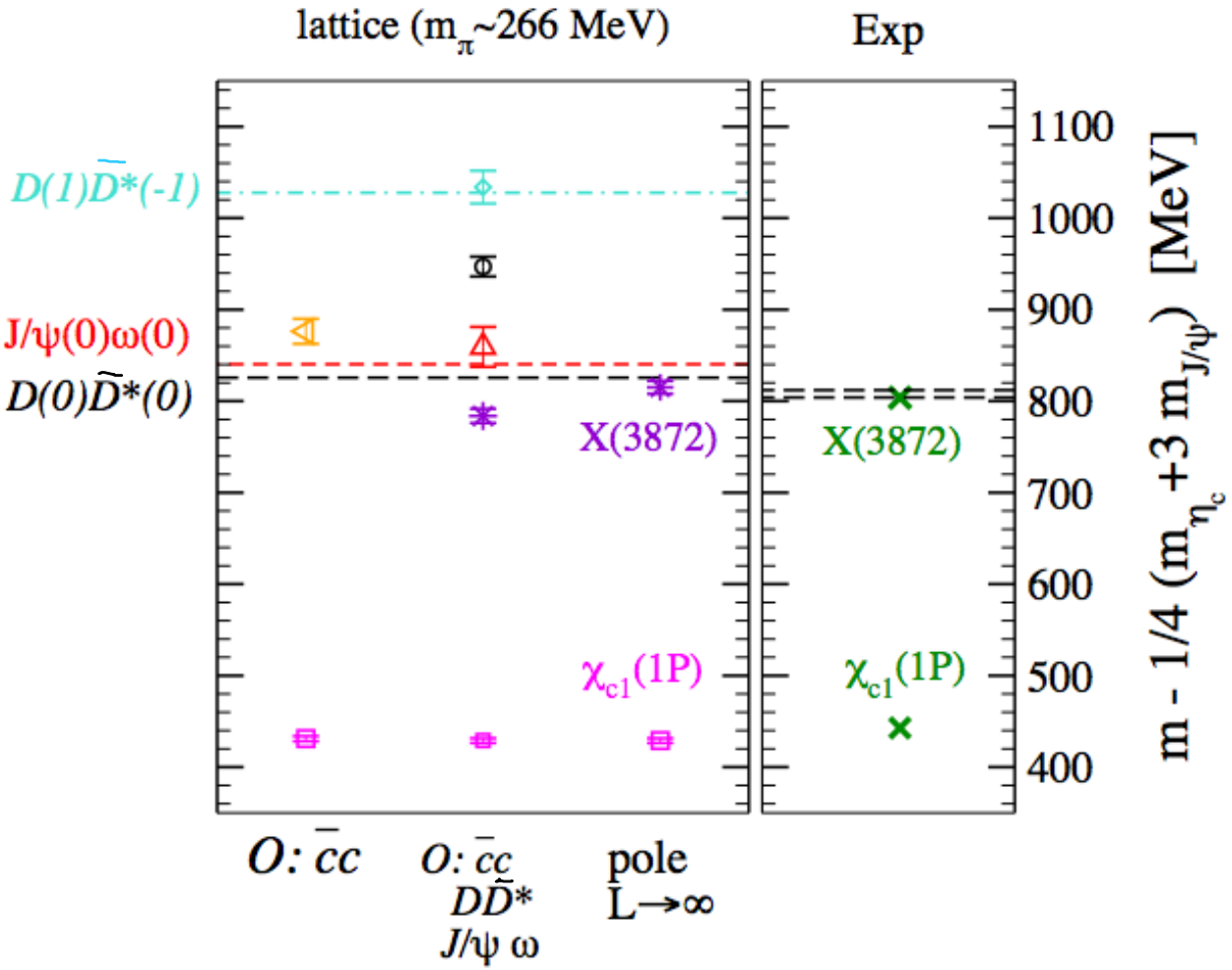
$m_\pi \approx 266$ MeV, $L \approx 2$ fm, $N_f = 2$

SP, Leskovec : 1307.5172, PRL 2013: old !

Charmoniumlike X(3872), $J^{PC}=1^{++}$

SP, Leskovec : 1307.5172, PRL 2013





Subsequent work

[Padmanath,Lang,SP: 1503.03257, PRD 2015]

[cu][cu] did not affect eigen-energies

A number of simplifications:

- $J/\psi\ \pi\pi$ omitted
- DD* d-wave omitted
- just one small volume
- no moving frames
- charm-annihillation omitted
  (as omitted in majority of charmoniumlike studies)
- ...

still the only published lattice simulation of X(3872) using scattering to date ?!

charmonium-like states with $J^P=1^{++}$, I=0

$\bar{c}c + \bar{c}c\bar{q}q$

$\chi_{c1}(4010) \;:\; \bar{c}c\ (2P)$

$D\bar{D}^*\ th.$

$\chi_{c1}(3872) \;:\; D\bar{D}^*$

$\chi_{c1}(1P) \;:\; \bar{c}c\ (1P)$

reanalysis of lattice data by SP, Leskovec 1307.5172 PRL 2013

by Shi, Albaladejo, Du, Guo, Nieves 2410.19563

finds three states in the 1++ channel (shown on the left),

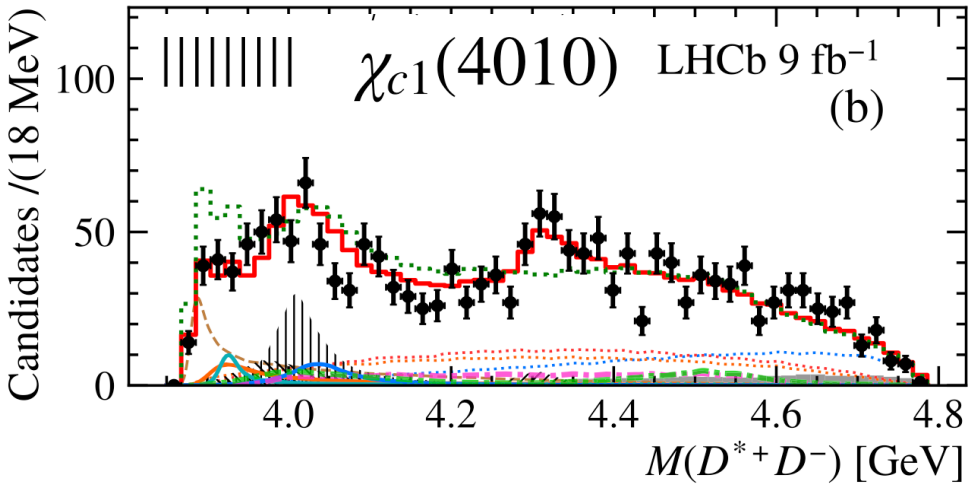
roghly consistent with experimental masses;

caution, many disclaimers:

not many energy levels to analyze and fits are underconstrained

conclusions on χ_{c1} (4010) from lattice are not solid

χ_{c1} (3872) claimed also in lattice simulation 2402.14541;
extraction of χ_{c1} (4010) is not solid in my opinion

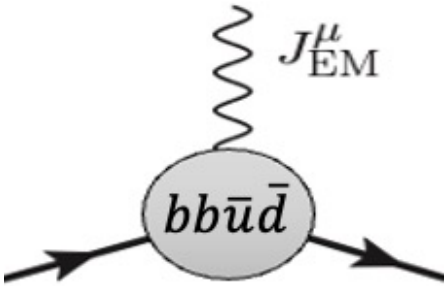


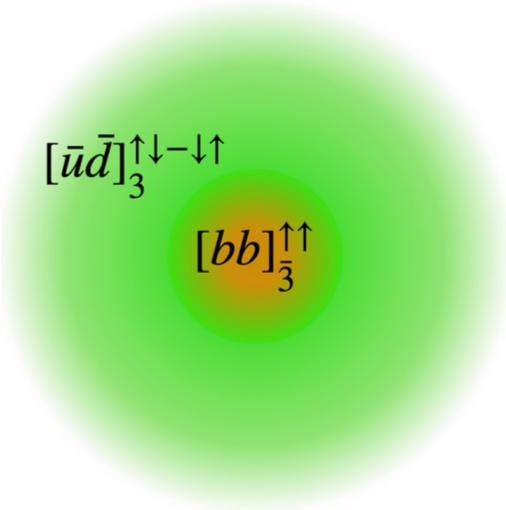
Conclusions

Structure

- $bb\bar{u}\bar{d}$ has

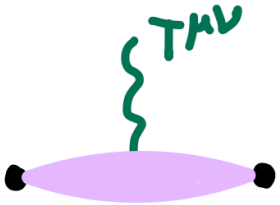
  has diquark-antidiquark structure





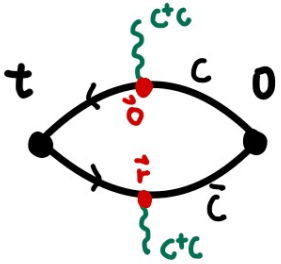
- **quenched scalar glueball**

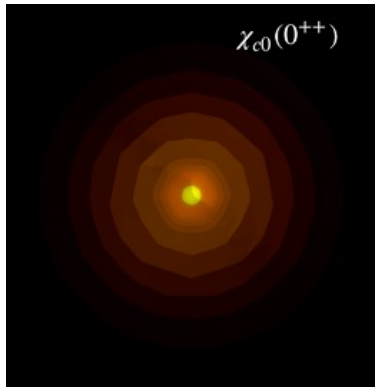
  is small



- **portraits of charmonia**

  exhibit significant relativistic effects



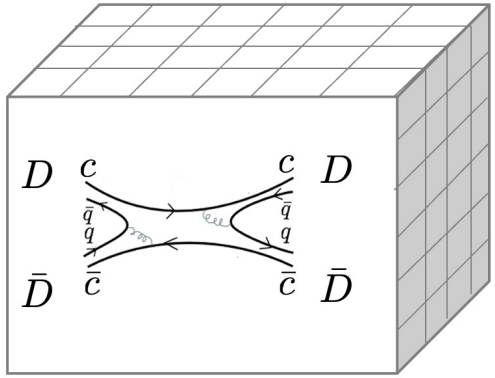


Conclusions cont.

Spectroscopy from scattering

charmonia and charmoniumlike states

$\bar{c}c$ $\bar{c}c\bar{q}q$



1^{--}, 2^{++}, 3^{--} : conventional

0^{++} : disagreement of our and Hadspec results

1^{++}: found signature of X(3872) in 2013

plenty of room for improvement

backup

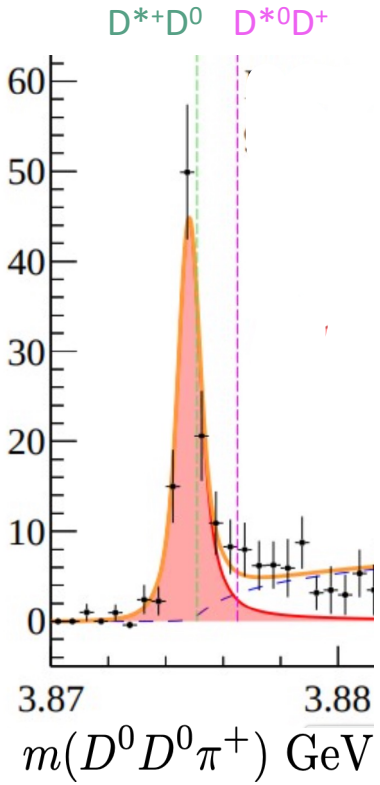
T_{cc} $ccd\bar{u}$ I=0, $J^P=1^+$

experiment

$$D^* \to D\pi$$

$$m_{\pi^0} \simeq 135 \text{ MeV}$$
$$m_{D^{*+}} - m_{D^+} \simeq 140 \text{ MeV}$$

$$\delta m_{pole} = -0.36 \pm 0.04 \text{ MeV}$$

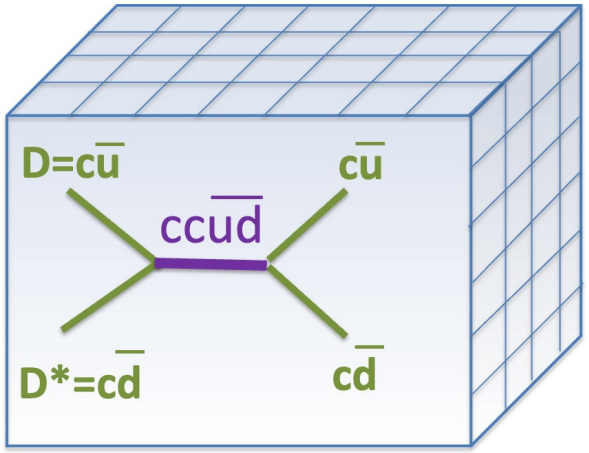


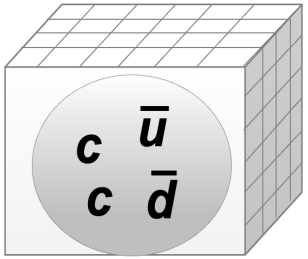
LHCb 2021 Nature Physics

lattice

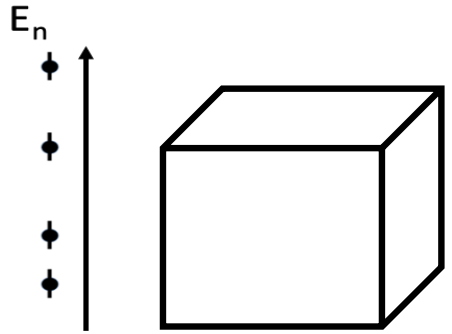
$$D^* \not\to D\pi$$

$$m_u = m_d > m_{u,d}^{ph}$$





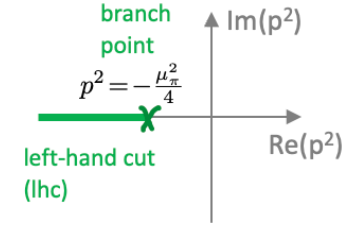
T_{cc}: incorporating the "left-hand cut"



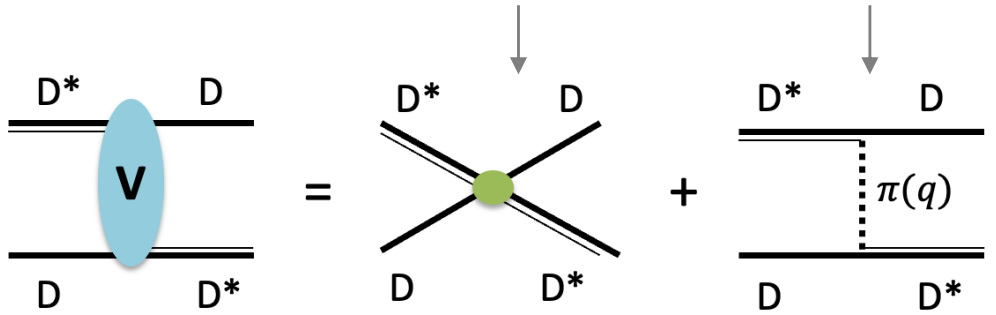
$$\hat{H}|\psi\rangle = E_n|\psi\rangle$$

$$\hat{H} = \hat{W}_{kin} + \hat{V}$$

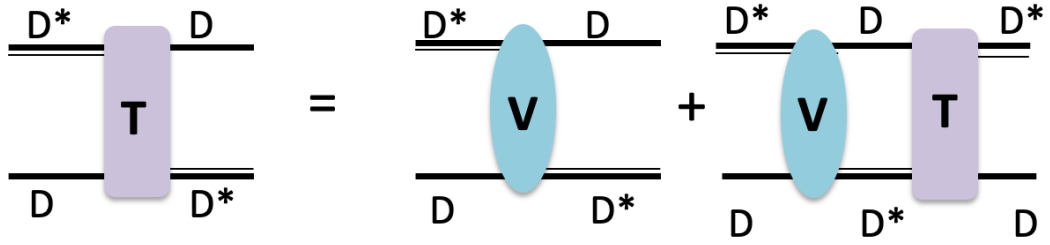
$$\mu_\pi^2 = m_\pi^2 - (m_{D^*} - m_D)^2$$



$$2c_0 + 2c_2(p^2 + p'^2) \qquad \frac{g_c}{2f_\pi}\vec{q}$$



Bethe-Salpeter equation



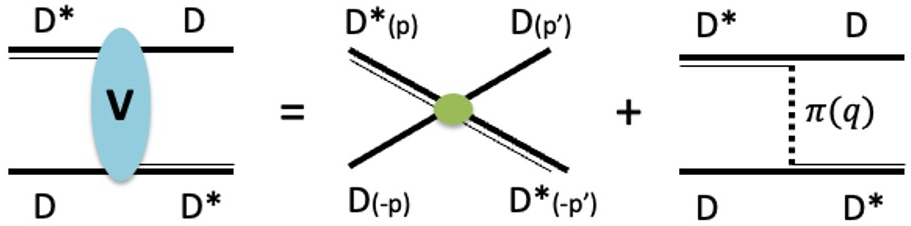
plane wave method: Meng, Baru, Epelbaum et al., 2312.01930, PRD
Collins, Nefediev, Padmanath , SP, 2402.14715, PRD
Meng, Pacheco, Baru, Epelbaum, Padmanath, SP, 2411.06266, PRD 25

T_{cc} : interpretation

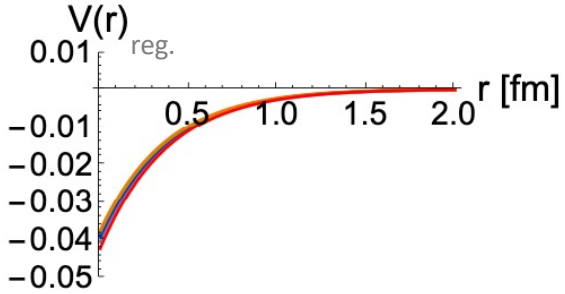
m_c

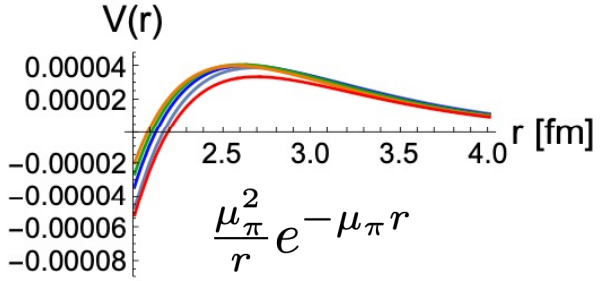
$m_\pi \simeq 280$ MeV

V(q)

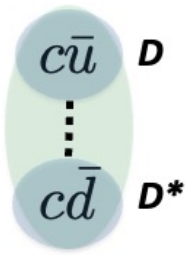


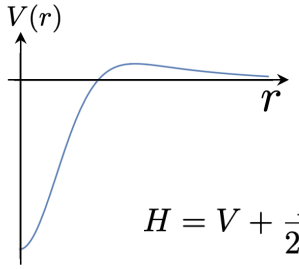
FT $\quad V_{\rm reg}(r) = \int \frac{d^3q}{(2\pi)^3} e^{i\boldsymbol{q}\boldsymbol{r}} \left(V_\pi^S(\boldsymbol{q}) + V_{\rm CT} \right) f_{\rm reg}(\boldsymbol{q}^2)$





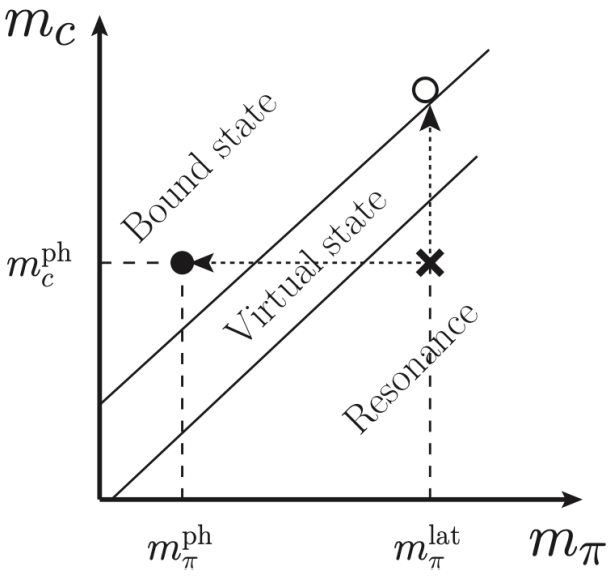
$\mu_\pi^2 = m_\pi^2 - (m_{D^*} - m_D)^2 > 0$





$H = V + \frac{p^2}{2m_r}$

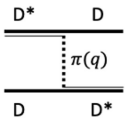
V almost independent on m_c



T_{cc} : pole trajectory $ccd\bar{u}$

Collins, Nefediev, Padmanath , SP, 2402.14715, PRD

analysis incorporates analytic properties related to

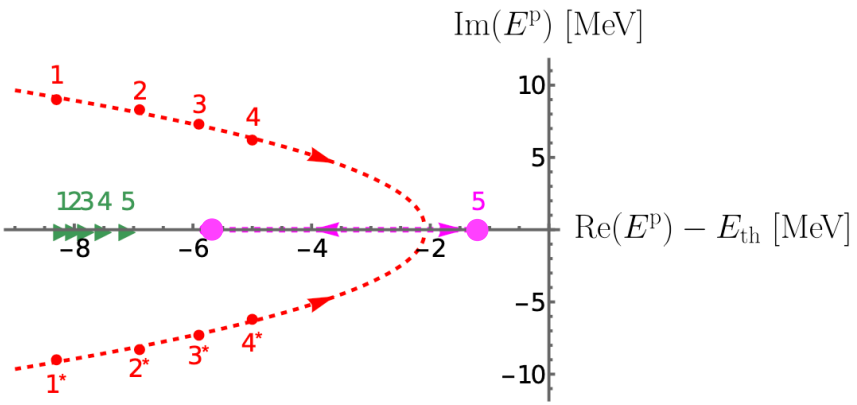




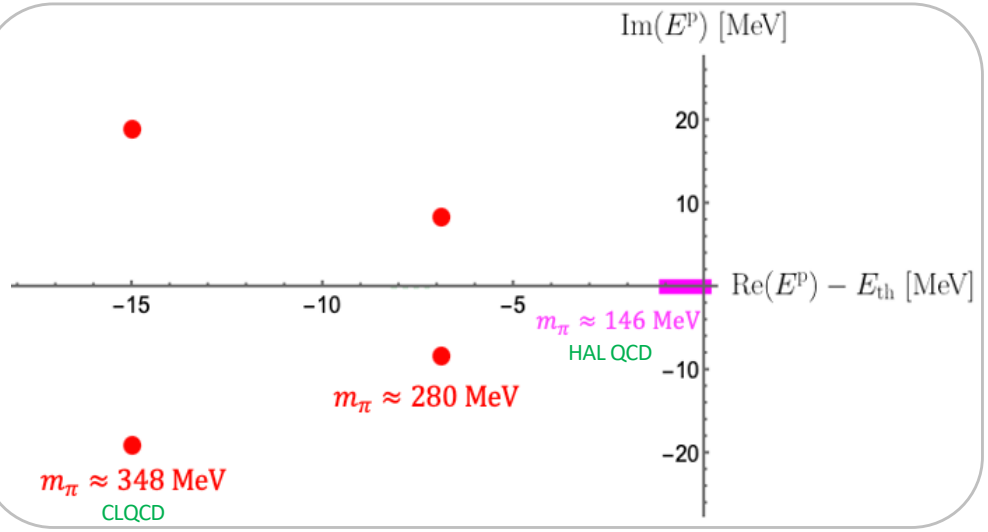
$m_\pi \simeq 280$ MeV

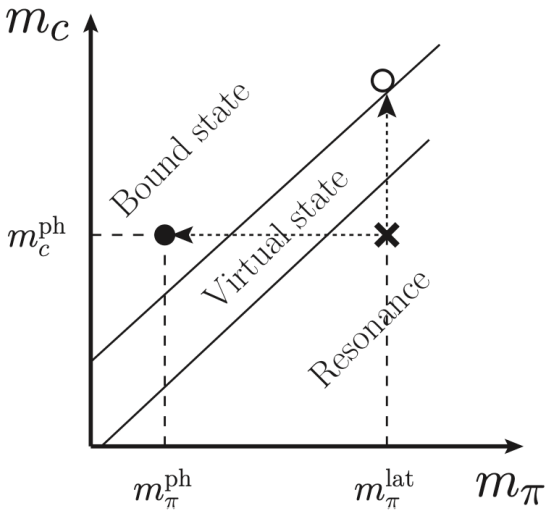


m_c^{ph}



1: $m_D \approx 1.7$ GeV 5: $m_D \approx 2.4$ GeV arrow: increasing m_c



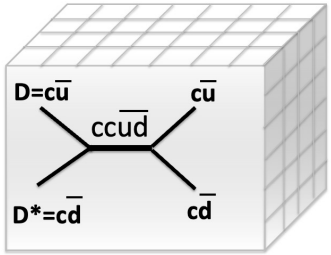


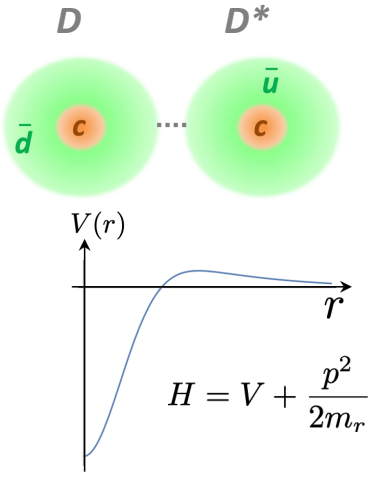
"phase" diagram

resonance pole

virtual state pole

left hand cut





V almost independent on m_c

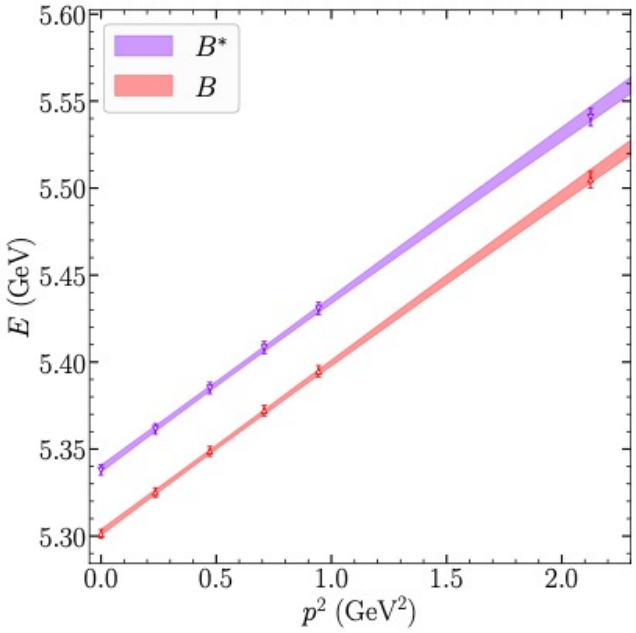
attraction increases with decreasing $m_{u/d}$

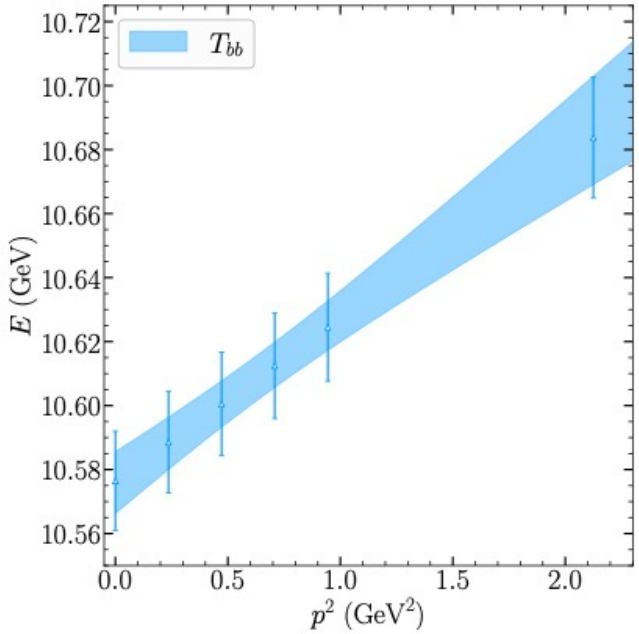
Lattice setup

- single $N_f = 2 + 1$ gauge ensemble, generated by CLS
- $V = N_L^3 \times N_T = 40^3 \times 128$
- $a \approx 0.06379(37)$ fm, $L \approx 2.56$ fm
- u/d quarks: isospin symmetry; isotropic, $\mathrm{O}(a)$ improved Wilson action → pion mass $m_\pi \approx 290$ MeV
- b quarks: anisotropic relativistic heavy quark action (RHQ)

h	m_h (GeV)	c
B	5.3020(17)	1.020(12)
B^*	5.3387(20)	1.021(14)
T_{bb}	10.5765(98)	1.04(11)

B, B^* lattice masses well within 1% of PDG values





$$m_{T_{bb}} - (m_B + m_{B^*}) = -64(10)\ \mathrm{MeV}$$

$$E = \sqrt{m^2 + (c\vec{p})^2}$$

Chen, Phys. Rev. D 64, 034509 (2001), arXiv:hep-lat/0006019

El-Khadra, Kronfeld, MacKenzie, hep-lat/9604004, Fermilab approach

$$\mathcal{S}_{WC}^{Q} = a^4 \sum_x \bar{Q}(x) \left(m_Q + \gamma_t \nabla_t - \frac{1}{2}\Delta_t + \nu \sum_{i=1}^{3} \left(\gamma_i \nabla_i - \frac{1}{2}\Delta_i \right) - \frac{c_E}{2} \sum_{i=1}^{3} \sigma_{it} F_{it} - \frac{c_B}{2} \sum_{i<j} \sigma_{ij} F_{ij} \right) Q(x)$$

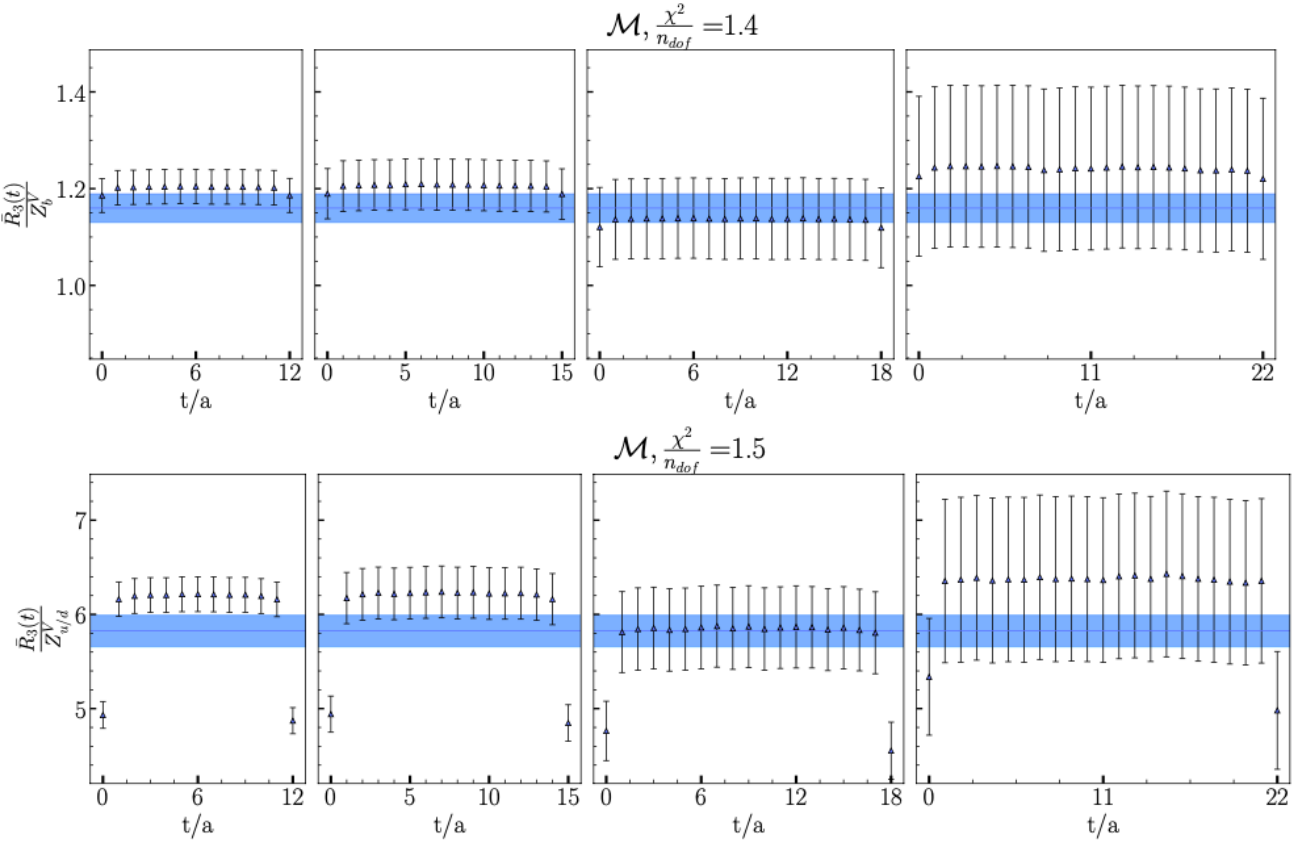
Backup: vector current renormalization

$$\hat{j}^{\mu}_{EM} = Z^{V}_{u/d}\left(\frac{2}{3}\bar{u}\gamma^{\mu}u - \frac{1}{3}\bar{d}\gamma^{\mu}d\right) - Z^{V}_{b}\cdot\frac{1}{3}\bar{b}\gamma^{\mu}b$$

- renormalization conditions:
  - $F_C^{b,T_{bb}} \equiv -\frac{2}{3}$
  - $F_C^{\bar{u},T_{bb}} \equiv -\frac{2}{3}$

  } at Q^2=0:

- $Z_b^V = 3.97(11), \quad Z_{u/d}^V = 0.725(16)$



Fits to the form factors

[Boyd et al.,hep-ph/9412324,hep-ph/9702300], [Bourrely et al.,0807.2722]

$$F(Q^2) = \frac{1}{1+\frac{Q^2}{M_r^2}} \sum_n a_n z^n(Q^2; t_{cut}, t_0), \qquad z(Q^2; t_{cut}, t_0) = \frac{\sqrt{t_{cut}+Q^2} - \sqrt{t_{cut}-t_0}}{\sqrt{t_{cut}+Q^2} + \sqrt{t_{cut}-t_0}},$$

$$t = q^2 = -Q^2.$$

	M_r	t_{cut}
T_{bb}	M_ω	$(3m_\pi)^2$
$B^{(*)}, \pi$	M_ρ	$(2m_\pi)^2$

Tuneable parameter t_0 mostly set to 0.

