Precision charmonium spectroscopy on CLS ensembles

Gunnar Bali, Sara Collins, Wolfgang Söldner, **Sebastian Spiegel**

Universität Regensburg



Glueballs and Charmonium, ECT* Trento

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Motivation and scope

- Precision study of stable charmonia to set baseline for investigations of XYZ states.
- Compute low lying charmonium masses and decay constants from isoQCD quark-line connected correlation functions.
- Benchmark for studies including electromagnetic and/or annihilation effects.

I will show results obtained on 57 CLS gauge ensembles with $2.0\,\text{GeV} \lesssim a^{-1} \lesssim 5.1\,\text{GeV}$.

Cut-off effects: $\sim a^2 m_c^2$ for m_c.
Smaller for fine structure splittings $a^2(m_c v)^2 \sim 0.3\, a^2 m_c^2$
Even smaller for spin-independent splittings and $M_{\overline{\psi}} - 2M_{\overline{D}}$: $a^2(m_c v^2)^2 \sim 0.1\, a^2 m_c^2$.

For the moment being, we only study $J = 0$ and $J = 1$ charmonia
(smeared interpolating operators without derivatives).

What we neglect

- No c quark in the sea.
  Heavy flavours will only affect short distance quantities.
  Binding energies and momenta of a few 100 MeV $< m_c$.
  Sea quark effects are suppressed in any case. The heavier the sea quark, the more.

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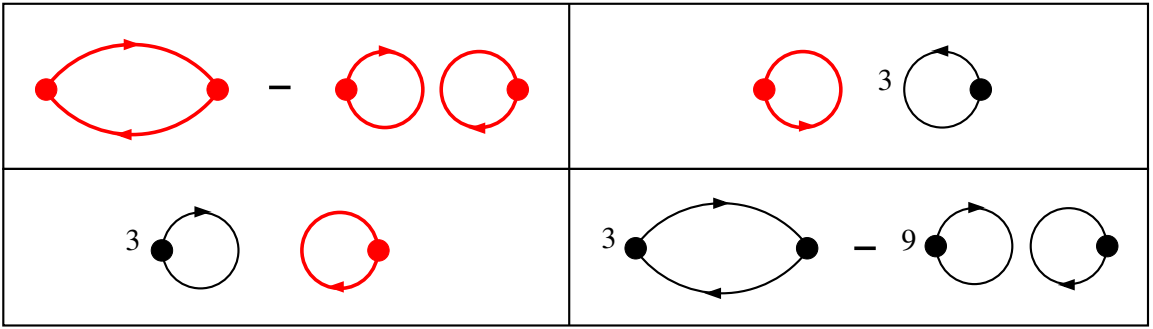
  Impact of q_c can, e.g., be modelled in potential models. Will increase hyperfine splittings.
- No quark-line disconnected contributions.
  Axial anomaly leads to heavier η_c and reduces 1S-splitting.
  Main effect: charmonia become resonances with decay widths.

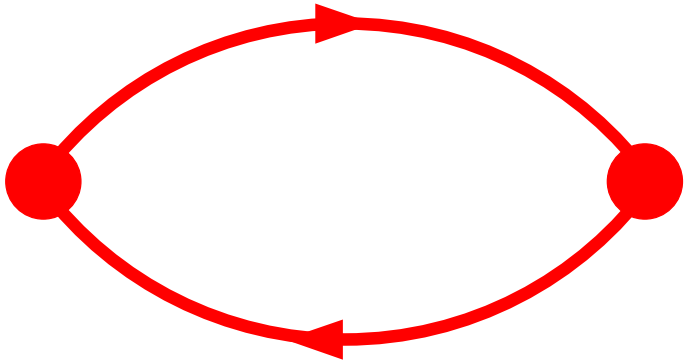
Correlation matrix for Wick contractions for $\bar{c}\Gamma c$

Simplified: no glueball-like interpolators, $m_u = m_d = m_s$.



state	$n^{2S+1}L_J$	J^{PC}	Γ	state	$n^{2S+1}L_J$	J^{PC}	Γ
η_c	1^1S_0	0^{-+}	γ_5	h_c	1^1P_1	1^{+-}	σ_{jk}
J/ψ	1^3S_0	1^{--}	γ_j	exotic	—	0^{--}	$\gamma_0\gamma_5$
χ_{c1}	1^3P_1	1^{++}	$\gamma_j\gamma_5$	exotic	—	0^{+-}	γ_0
χ_{c0}	1^3P_0	0^{++}	$\mathbb{1}$	exotic	—	1^{-+}	σ_{0j}

Wick contractions for $\bar{c}_1 \Gamma c_2$

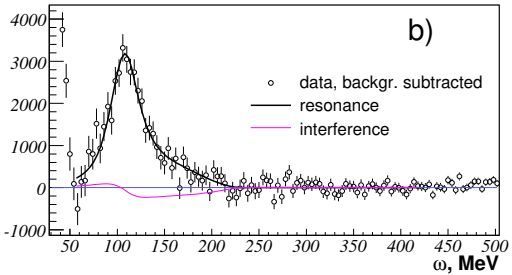


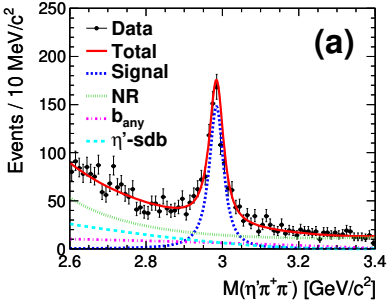
C is no good quantum number.

Only $\exists\, J^P$!

state	$n^{2S+1}L_J$	J^P	Γ
η_c	1^1S_0	0^-	γ_5,$\gamma_0\gamma_5$
J/ψ	1^3S_0	1^-	γ_j,σ_{0j}
χ_{c1}, h_c	1^1P_1	1^+	$\gamma_j\gamma_5$,σ_{jk}
χ_{c0}	1^3P_0	0^+	$\mathbb{1}$,γ_0

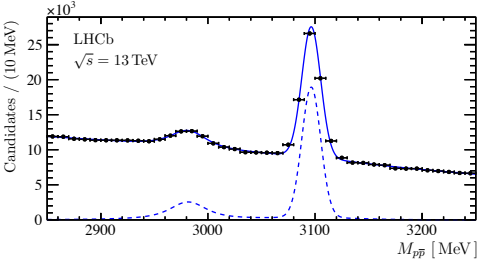
Experimental results for the $1S$ hyperfine splitting

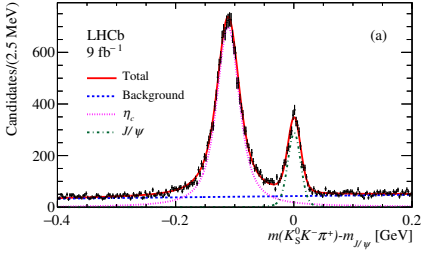




[KEDR, 1406.7644]

[Belli, 1805.03044]

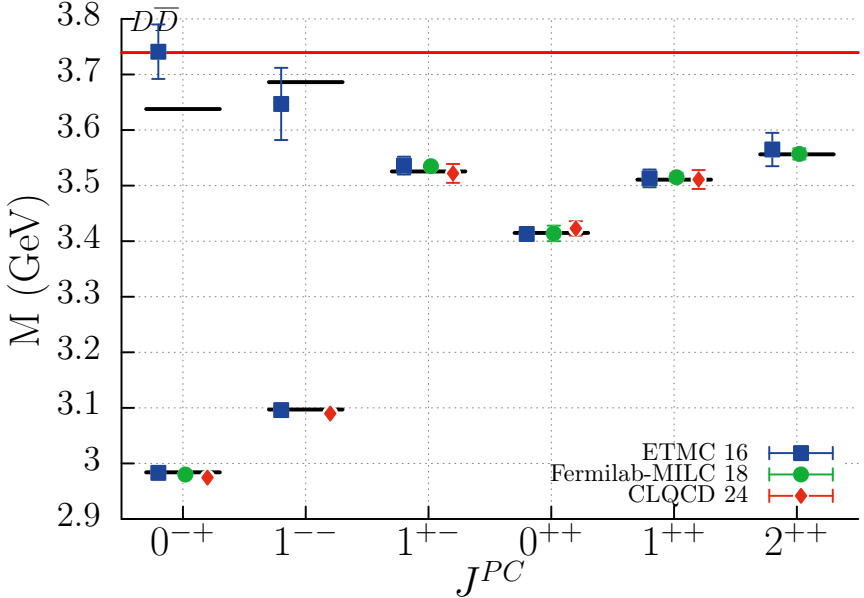




[LHCb, 1911.03326]

[LHCb, 2304.14891]

Continuum limit lattice results



Coordinated Lattice Simulations (CLS)

CLS groups at

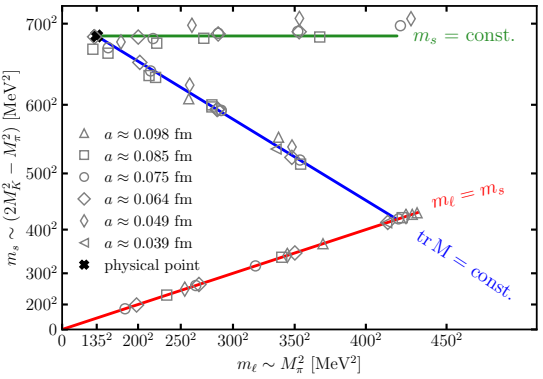
- HU Berlin
- Bern
- Bochum
- CERN
- Darmstadt
- Mainz
- UA Madrid
- Milano Bicocca
- Münster
- Odense/CP3 Origins
- Regensburg (RQCD)
- Roma I + II
- Wuppertal
- DESY/Zeuthen

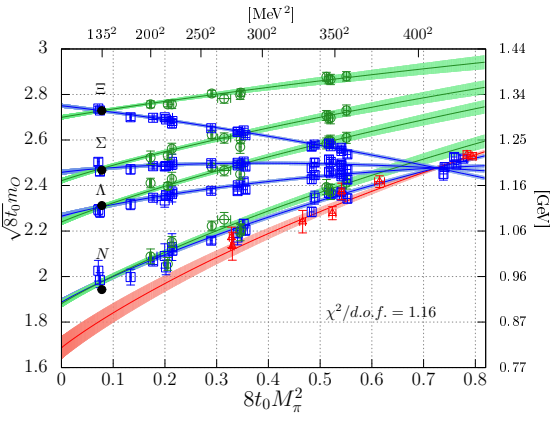
Coordinated generation of gauge ensembles using openQCD
`https://luscher.web.cern.ch/luscher/openQCD/`
[M Lüscher, S Schaefer, 1206.2809].

$N_f = 2 + 1$ flavours of non-perturbatively order-a improved Wilson fermions on tree-level Symanzik improved glue.

Keep it simple and local: no smeared action etc.

CLS ensembles used: m_ℓ-m_s plane





$\Xi \sim ss\ell$, $\Sigma, \Lambda \sim s\ell\ell$, $N \sim \ell\ell\ell$.

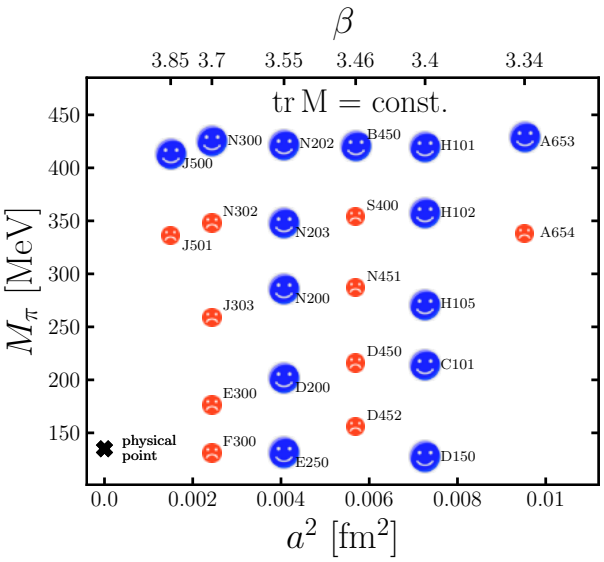
Three trajectories: can correct for mis-tuning of the masses.
Excellent control of the quark mass dependence.

$2m_\ell + m_s = \text{const.}$: investigate SU(3) flavour breaking (flavour averaged quantities roughly constant).

Approaching the physical point along this line involves M_K ↗ as M_π ↘.

Analysis status at Lattice 2024

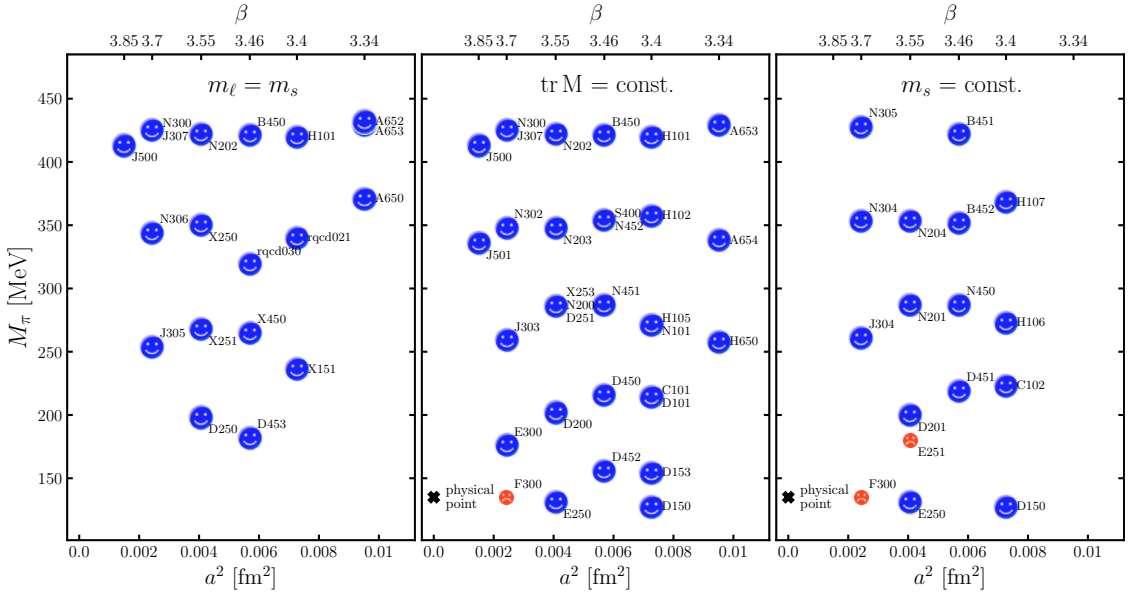
6 lattice spacings: $a = (0.098 \searrow 0.039)$ fm, $LM_\pi \gtrsim 4$ + smaller, $M_\pi = (420 \searrow 130)$ MeV.



$a < 0.06$ fm: open boundary conditions in time.

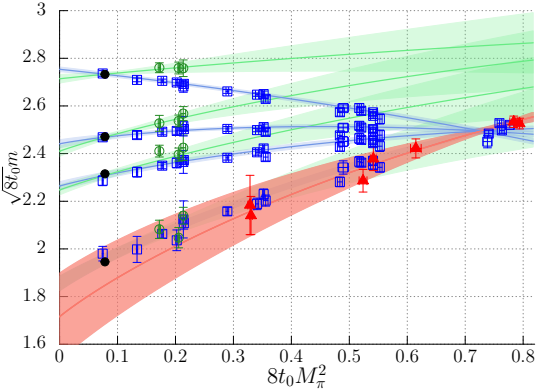
$a > 0.06$ fm: ensembles with either periodic or open boundary conditions.

Ensembles used in this analysis

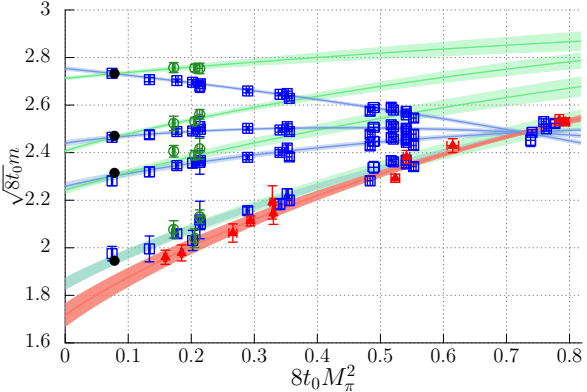


Best octet baryon spectrum fit

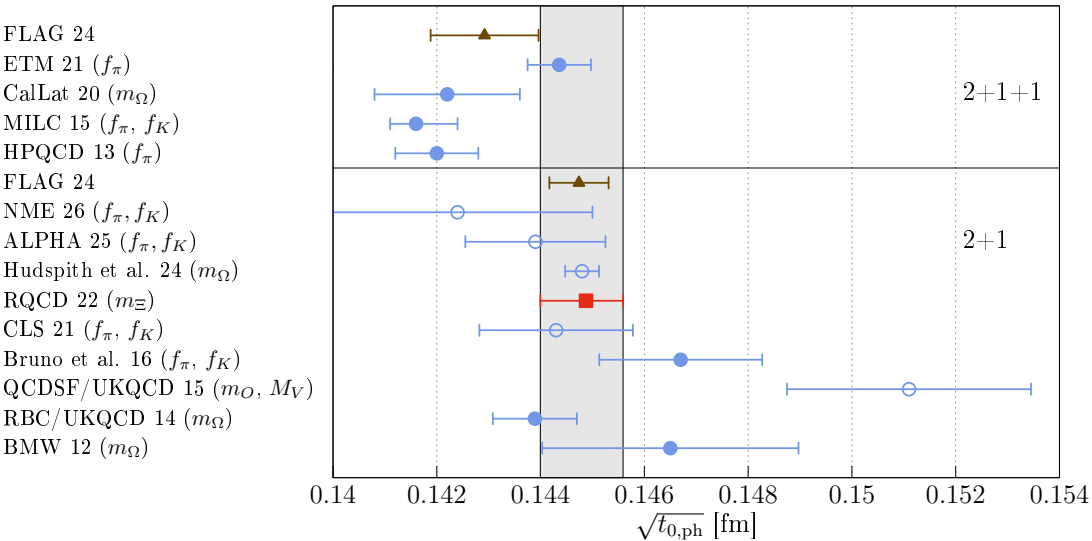
[RQCD, 2211.03744]



With additional $m_\ell = m_s$ ensembles



Scale setting



Scale set via $t_{0,\mathrm{ph}}^{-1/2} = 1.362(8)\,\mathrm{GeV}$ [RQCD, 2211.03744] from $m_\Xi = 1316.9\,\mathrm{MeV} \curvearrowright 1317.7\,\mathrm{MeV}$.

Definition of the lattice spacing a and of the physical point

One can very precisely compute the purely gluonic quantity t_0/a^2 at a lattice spacing $a(g^2)$.

For $a = 0$ (continuum limit):

$$t_0(M_\pi, M_K) = t_{0,\mathrm{ch}} + c\overline{M}^2 + \mathcal{O}(M^4), \quad \overline{M}^2 = \frac{1}{3}\left(2M_K^2 + M_\pi^2\right), \quad \delta M^2 = 2(M_K^2 - M_\pi^2).$$

t_0^* is the value of t_0 at the $M_\pi = M_K$ point where $12 t_0 \overline{M}^2 = 1.110$.

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To leading non-trivial order in ChPT charmonium masses M_ψ and the flavour-averaged D-meson mass $M_{\overline{D}} = \frac{1}{3}(2M_D + M_{D_s}) = 1899.4\,\mathrm{MeV}$ are independent of δM^2.

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$$\frac{M_\pi}{m_\Xi} = \frac{135\,\mathrm{MeV}}{1317.7\,\mathrm{MeV}}, \qquad \frac{M_K}{m_\Xi} = \frac{494\,\mathrm{MeV}}{1317.7\,\mathrm{MeV}}.$$

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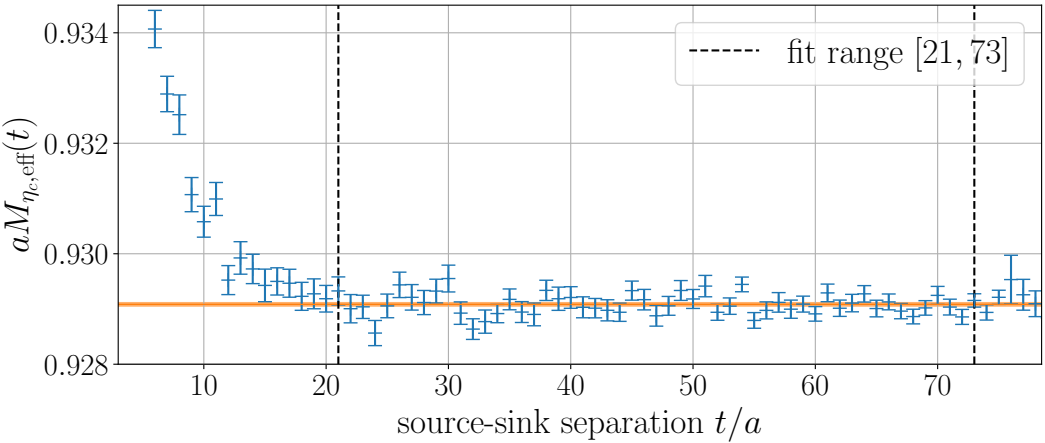
$t_{0,\mathrm{ph}}$ in GeV^{-2} is obtained from $t_{0,\mathrm{ph}} m_\Xi^2$ at the $a = 0$ physical point.

Ratio $t_0^*/t_{0,\mathrm{ph}} \approx 1$ at $a = 0$ is then used to assign a GeV^{-2} value to t_0^*.

Example for a mass determination



$a \approx 0.064\,\mathrm{fm}$, $M_\pi \approx 131\,\mathrm{MeV}$, $V = 192 \cdot 96^3 a^4$, $\qquad aM(t) = \operatorname{arccosh}\left[\frac{C(t+a)+C(t-a)}{2C(t)}\right]$

Extrapolation strategy

Quantities in lattice units: $\mathbf{M}$, $\mathbf{t}_0$ etc. Then for $m_q \to m_{q,\mathrm{ph}}$, $L \to \infty$, $a \to 0$:

$$\mathbf{M}_\psi(\{m_q\}, L, a)\sqrt{\mathbf{t}_0(\{m_q\}, a)} \longrightarrow M_\psi\sqrt{t_{0,\mathrm{ph}}}, \quad \mathbf{t_0}(\{m_q\}, a)a^2 \longrightarrow t_{0,\mathrm{ph}} \quad \text{etc.}$$

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Ideally, simulate at fixed values of the $O(a)$ improved coupling $g^2 = g_0^2[1 + b_g(g_0^2)a\overline{m}]$.

Instead, we simulate at fixed $\beta = 6/g_0^2$ values, thereby changing the lattice spacing by an $O(a)$ effect between points with different $\mathrm{Tr}\, M = 2m_\ell + m_s$.

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$$\mathbf{M}_\psi = \frac{1}{\sqrt{\mathbf{t}_0}}\left\{A_0 + c_c\mathbf{M}_{\overline{D}}\sqrt{\mathbf{t}_0} + \bar{c}\,\overline{\mathbf{M}}^2\mathbf{t}_0 + c_{c2}\mathbf{M}_{\overline{D}}^2\mathbf{t}_0 + \frac{c_a}{\mathbf{t}_0^*} + \frac{c_{ac}}{\mathbf{t}_0^*}\mathbf{M}_{\overline{D}}\sqrt{\mathbf{t}_0} + \ldots\right\}.$$

Extrapolation strategy 2

Fit according to

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The charmonium mass is then obtained from the dimensionless fit parameters A_0, c_c, $\bar{c}$ and c_{c2}, setting t_0, $M_{\overline{D}}$, M_π and M_K to their physical values.

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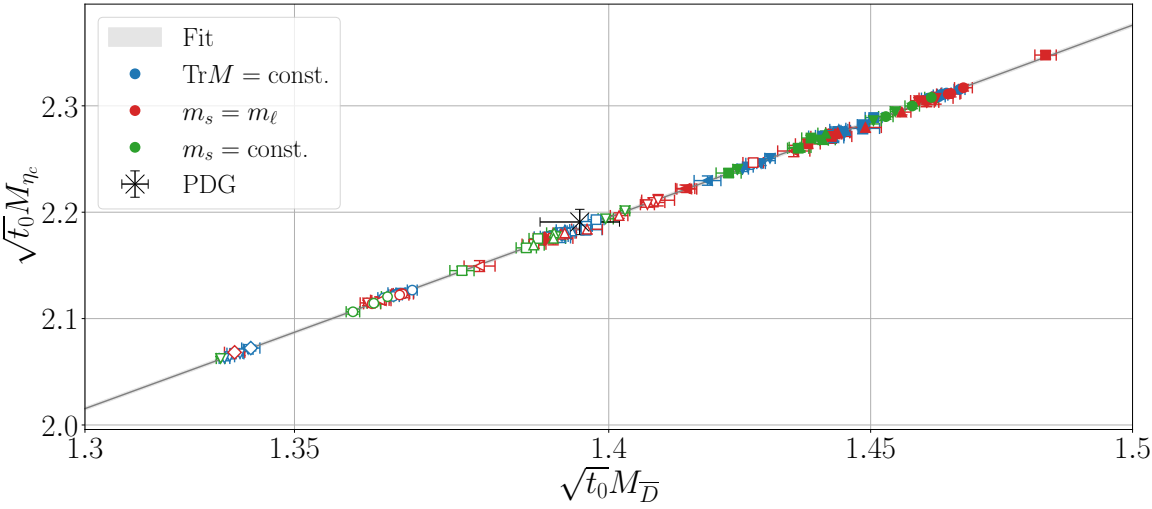
Two charm quark masses on each ensemble, mostly bracketing the physical one.

Charm quark mass interpolation

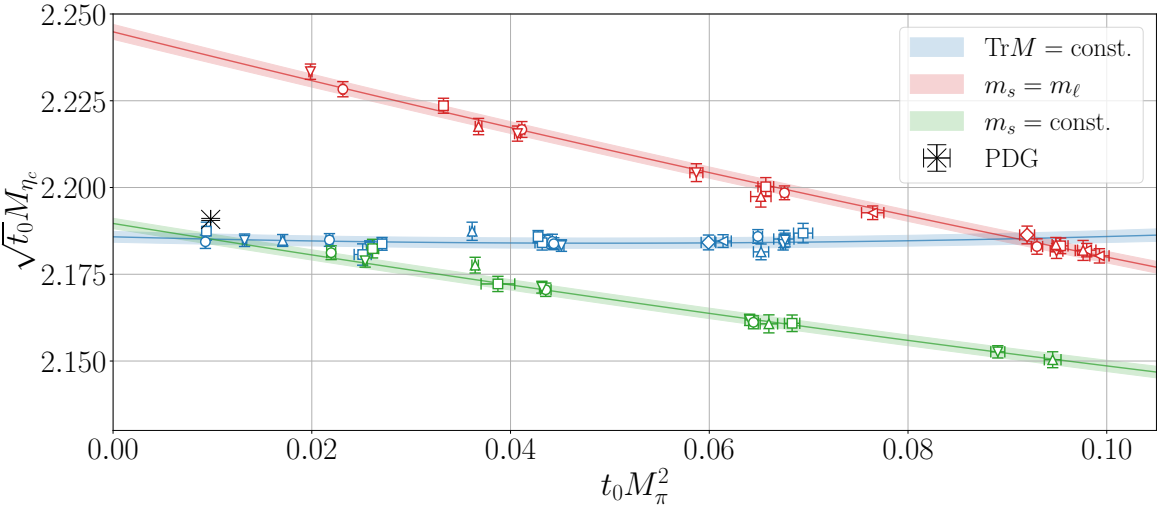


Smaller mass: open, Larger mass: solid.

Fit details

- On each ensemble, correlations between $\mathbf{M}_\pi$, $\mathbf{M}_K$ and the $\mathbf{M}_{\overline{D}}$ and $\mathbf{M}_\psi$ for the two charm quark masses are taken into account: 6×6 covariance matrices (5×5 for $m_s = m_\ell$).
- "x-errors" are included.
- $\mathbf{t}_0$ enters for each ensemble and $\mathbf{t}_0^*$ for each β-value (priors with errors).
- $t_{0,\mathrm{ph}}^{-1/2} = 1.362(8)\,\mathrm{GeV}$ is used to define the physical point and convert into GeV.
- Cuts: $M_\pi < 450\,\mathrm{MeV}$, $L > 2.3\,\mathrm{fm}$, $LM_\pi > 3.5$.
- 6–15 fit parameters, 114 data points.
  Best fit in terms of χ^2/d.o.f. has 12 (10) parameters for M_{η_c} ($M_{J/\psi} - M_{\eta_c}$).
- AIC model average with weights $w_i \propto \exp\left(-\frac{1}{2}(\chi_i^2 + 2k_i)\right)$ for model i with k_i parameters.

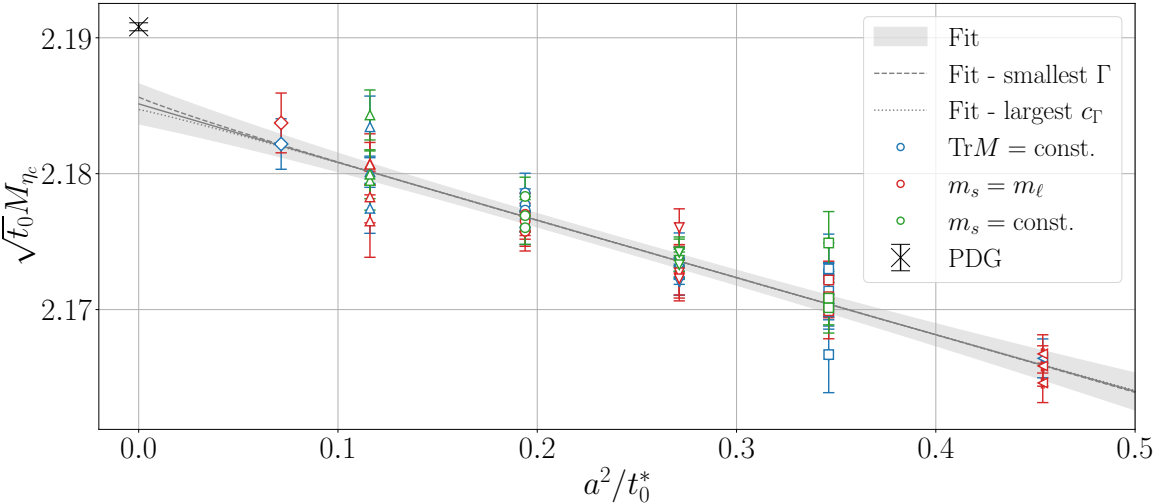
Pion mass dependence of the η_c mass (fit with largest AIC weight)



Error smaller than that of $\sqrt{t_{0,\mathrm{ph}}}$ because charm quark mass set via $\mathbf{M}_{\overline{D}} = M_{\overline{D}}\sqrt{t_{0,\mathrm{ph}}/\mathbf{t}_0}$.

↷ cancellations: only the difference $M_{\eta_c} - 2M_{\overline{D}}$ approximately scales with $\sqrt{t_{0,\mathrm{ph}}}$.

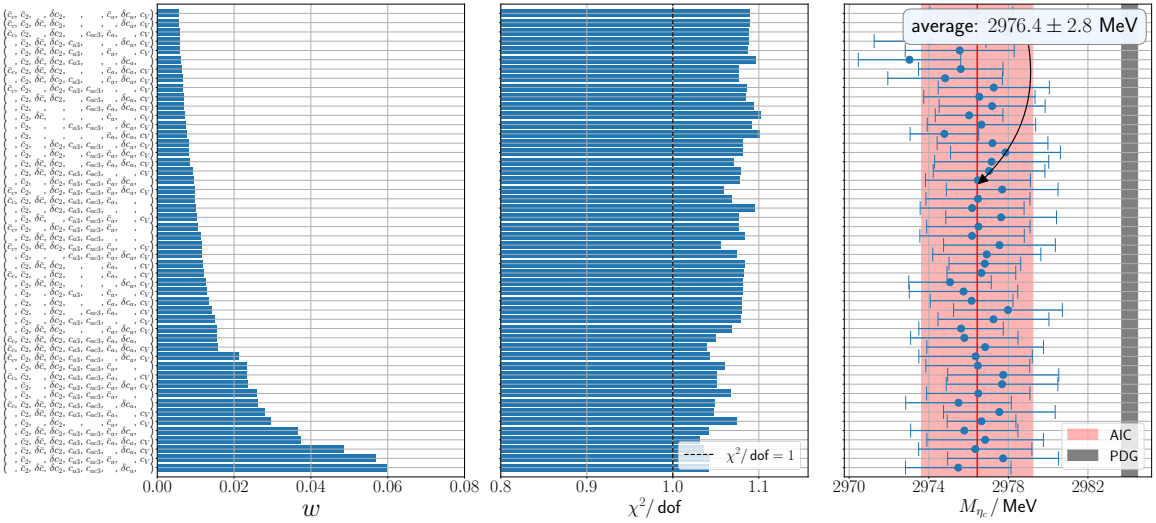
The η_c mass: continuum limit (fit with largest AIC weight)



Additional models: $a^2 \mapsto g^{2\Gamma} a^2$ with either smallest Γ or term with dominant c_Γ coefficient.

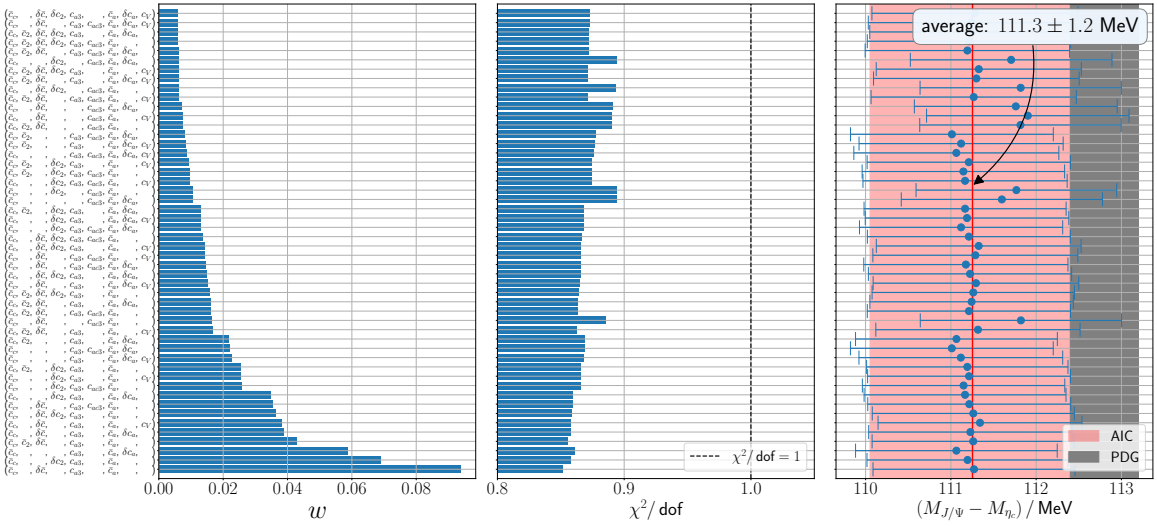
[N Husung, P Marquard, R Sommer, 2111.02347]

Model averaging for the η_c mass

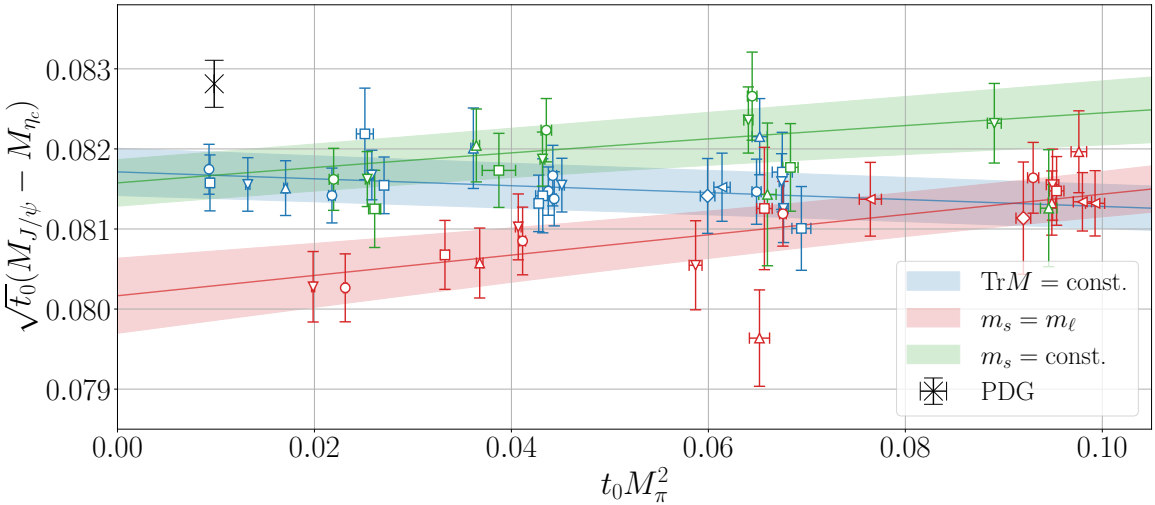


Also $M_{J/\psi}$ smaller than experiment for the isoQCD charm mass definition $M_{\overline{D}} = 1899.4\,\mathrm{MeV}$.

Model averaging for the $1S$ splitting

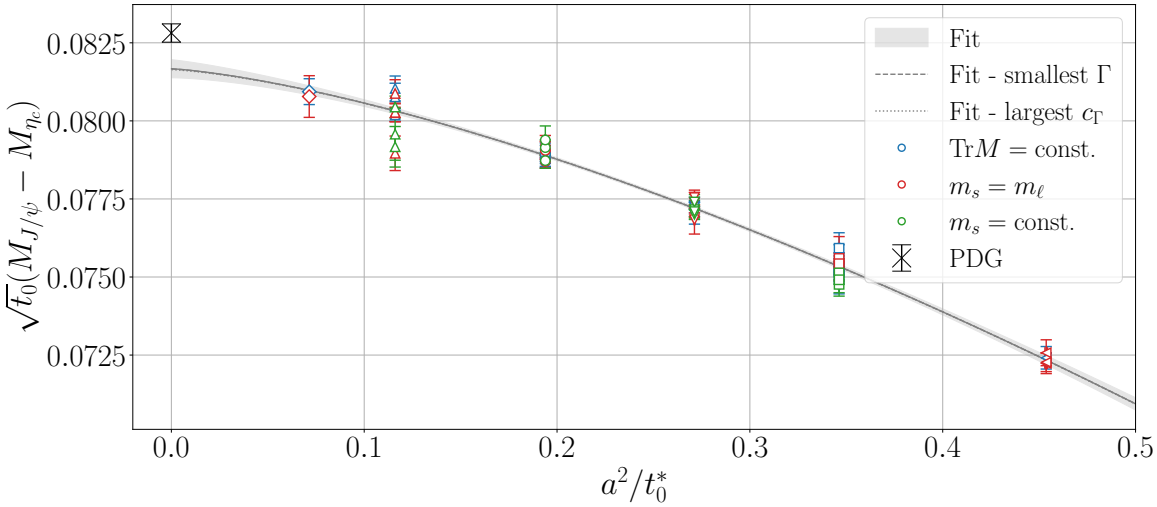


The $1S$ fine structure splitting: pion mass dependence



Fits with largest weights shown. Uncertainty of physical point definition ($t_{0,\mathrm{ph}}$) not included!

The 1S fine structure splitting: continuum limit



Fits with largest weights shown. Uncertainty of physical point definition ($t_{0,\mathrm{ph}}$) not included!

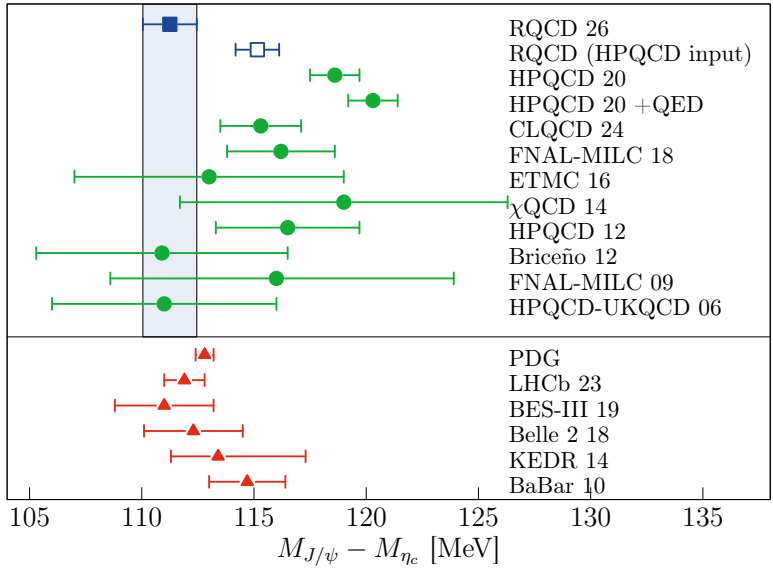
The fine structure splitting in isoQCD. Connected two-point function only.

HPQCD input:
$t_{0,\mathrm{ph}}^{-1/2} = 1.362(8)\,\mathrm{GeV}$
$\curvearrowright 1.389(8)\,\mathrm{GeV}$,
$M_{\overline{D}} = 1.8994\,\mathrm{GeV}$
$\curvearrowright M_{J/\psi} = 3.0969\,\mathrm{GeV}$.

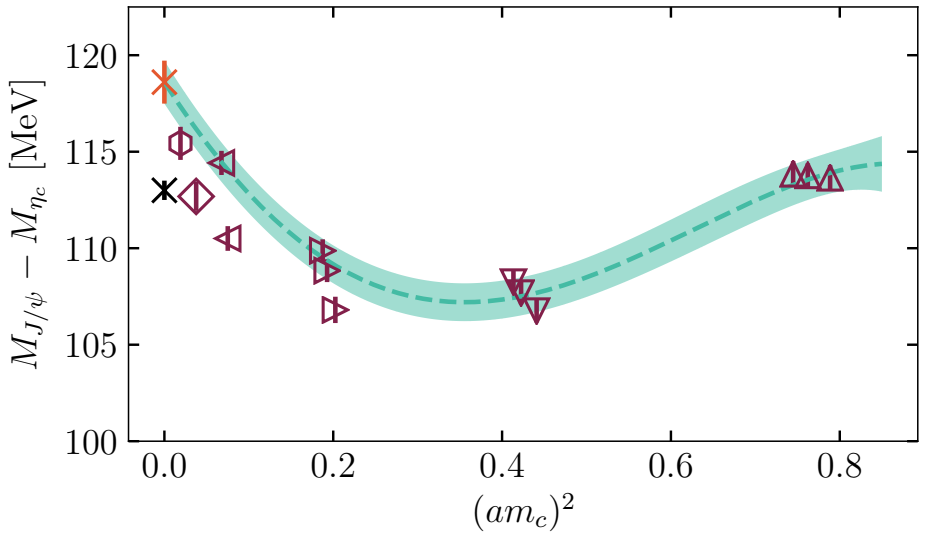
Potential models and [HPQCD, 2005.01845]:
$\lesssim 2\,\mathrm{MeV}$ increase due to QED.

[S Cali, F Knechtli, T Korzec, 1905.12971]:
$\sim 2\%$ increase of $M_{J/\psi} - M_{\eta_c}$, including 2 charm sea quarks.

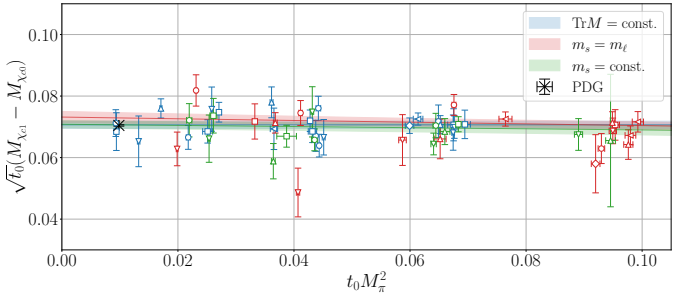
However, $\Gamma_{\eta_c} \approx 30\,\mathrm{MeV}$.

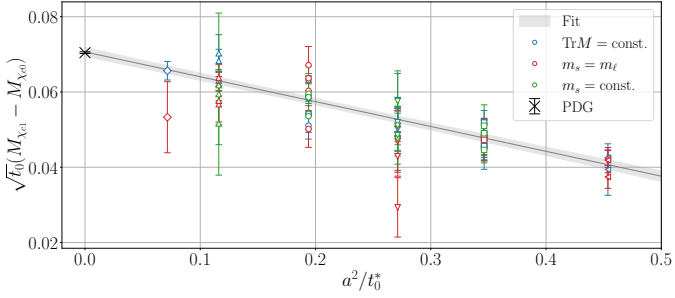


HPQCD continuum limit



The $\chi_{c1} - \chi_{c0}$ splitting



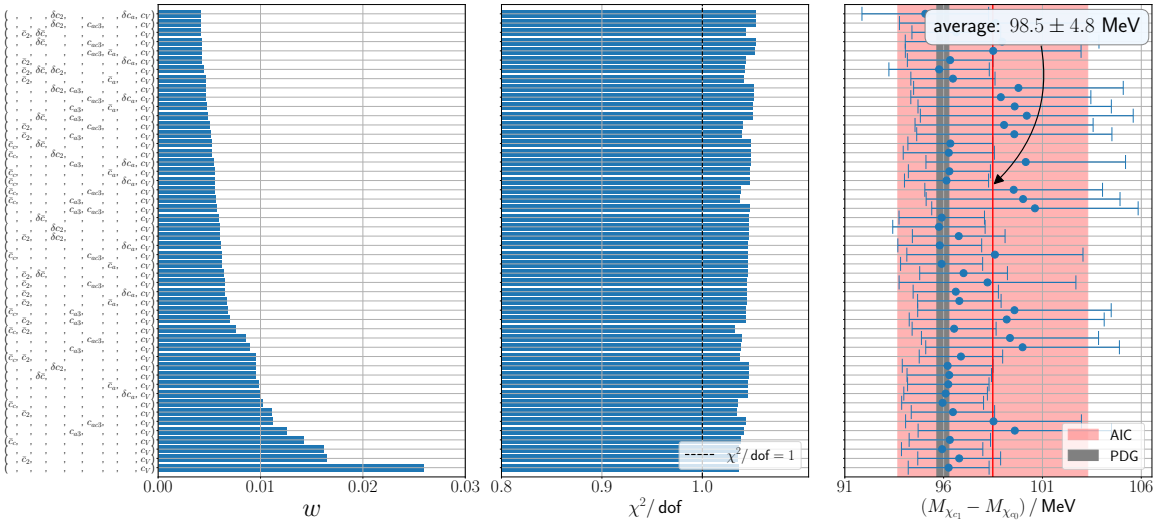


Fit with highest weight shown.

Experiment (PDG): 96.0(3.0) MeV, but $\Gamma_{\chi_{c0}} = 10.5(8)$ MeV.

Our result: 98.5(4.8) MeV.

Model averaging for the $\chi_{c1} - \chi_{c0}$ splitting



Summary and outlook

- From potential models and [HPQCD, 2005.01845], one would expect an increase of the $1S$ fine structure splitting by up to 2 MeV, due to electromagnetism.
- Given the decay width $\Gamma_{\eta_c} = 30.5(5)$ MeV, the agreement with experiment may be surprising.
  $M_{\eta_c} = 2976.4(2.8)$ MeV vs. 2984.1(4) MeV (PDG) (including EM, the difference is $2\,\sigma$).
- Not straightforward to include annihilation diagrams: the axial anomaly may decrease the $1S$ splitting but many decay channels and mixing with other flavour-singlet 0^{-+} states.
- Sea quark effects are very significant (as expected): the quenched splitting of 77(2)(6) MeV [QCD-TARO, hep-lat/0307004] becomes 111.3(1.2) MeV in isoQCD.
- $M_{\chi_{c1}} - M_{\chi_{c0}} = 98.5(4.8)$ MeV vs. 96.0(3) MeV (PDG).

Future plans

- Finalize the analysis and also compute the decay constants.
- Improve precision of scale setting.
  Optimize matching of isoQCD to QCD+QED in the charm sector.