From Non-Local OPE to High-energy OPE

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Bridging TMD frameworks:
Intersections, Tensions, and Applications

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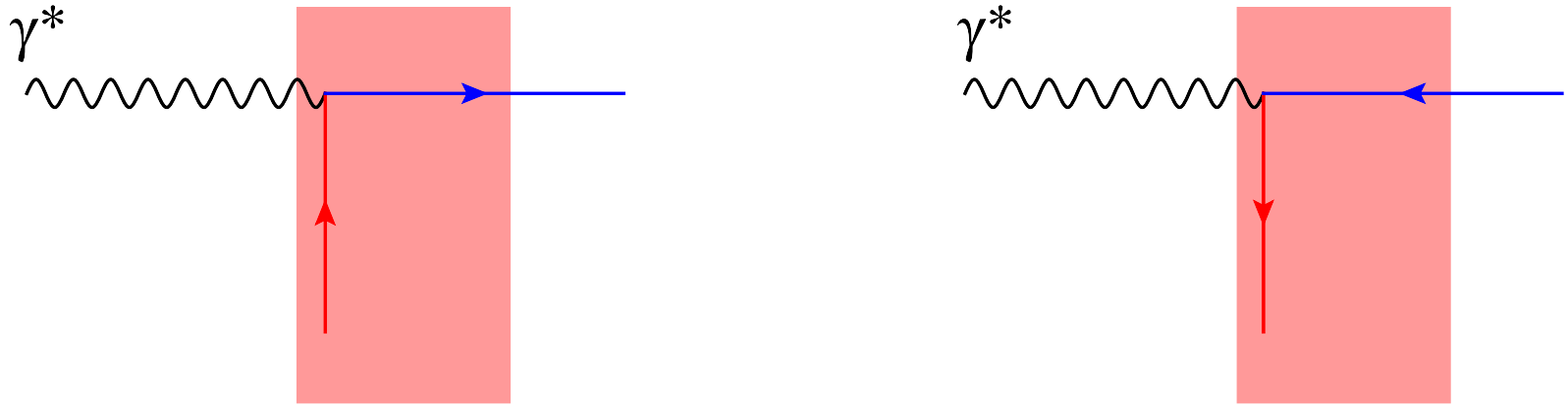
Motivation

- We want a unified operator description of DIS/SIDIS across different kinematic limits.
- We want to connect the light-cone OPE with the high-energy OPE.
- This bridge may clarifies how the small-x structure of DIS emerges from the general non-local expansion.



(a) (b)

Figure: Diagrams contributing to the transition amplitude $\gamma^*(q) \to q(k)$ in the right panel, and $\gamma^*(q) \to \bar{q}(k)$ in the left panel. The blue fermionic lines are the quantum fields, while the red ones are the classical fields.

Same diagrams where considered by: Altinoluk, Beuf, Mulani (2025)

Notation

$n_1 \cdot n_2 = 1 \qquad n_1^2 = n_2^2 = 0$

$$\mathrm{P}\exp\left\{ig\int_{y^+}^{x^+} dw^+ A^-(w^+ n_1 + x_\perp)\right\} = [x^+ n_1 + x_\perp, y^+ n_1 + x_\perp]$$
$$= [x^+, y^+]_x$$

infinite Wilson line

$$[\infty n_1 + x_\perp, -\infty n_1 + x_\perp] = [\infty n_1, -\infty n_1]_x$$

semi-infinite Wilson line

$$[\infty n_1 + x_\perp, x^+ + x_\perp] = [\infty n_1, x^+]_x \qquad [x^+ + x_\perp, -\infty n_1 + x_\perp] = [x^+, -\infty n_1]_x$$

High-energy boost

$$
\begin{aligned}
A^-(x^-,x^+,x_\perp) &\to \lambda A^-(\lambda^{-1}x^-,\lambda x^+,x_\perp)\,,\\
A^+(x^-,x^+,x_\perp) &\to \lambda^{-1} A^+(\lambda^{-1}x^-,\lambda x^+,x_\perp)\,,\\
A_\perp(x^-,x^+,x_\perp) &\to A_\perp(\lambda^{-1}x^-,\lambda x^+,x_\perp)\,.
\end{aligned}
$$

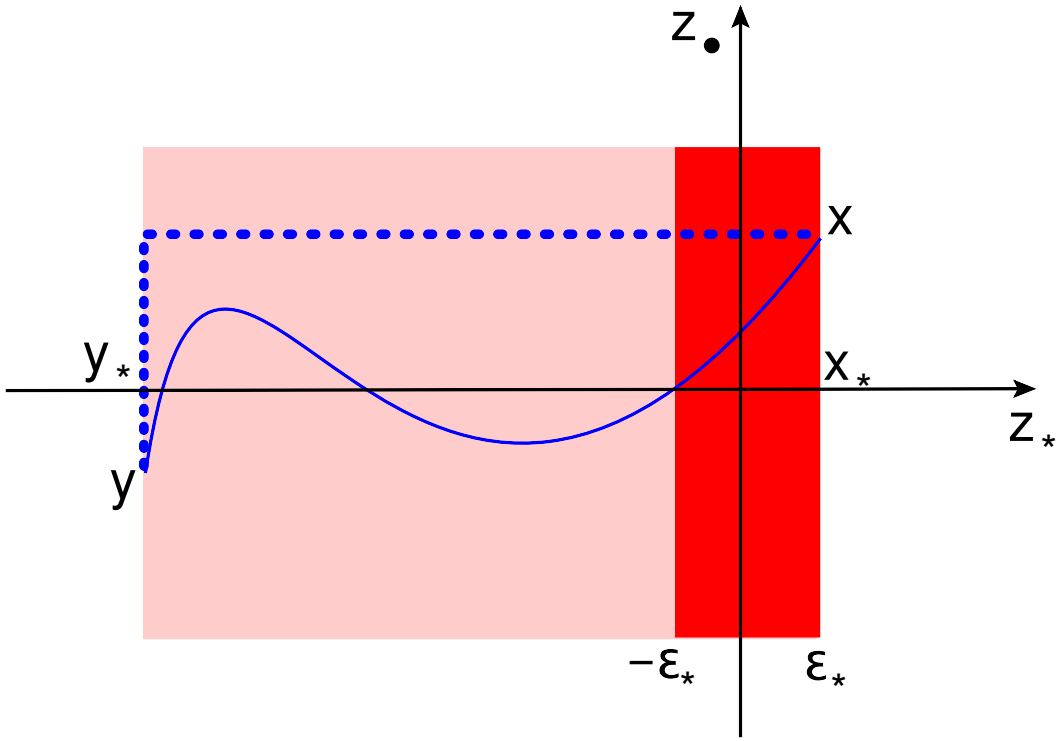
Consequently, the field strength gets is rescaled as follow

$$
\begin{aligned}
F_i{}^-(x^-,x^+,x_\perp) &\to \lambda F_i{}^-(\lambda^{-1}x^-,\lambda x^+,x_\perp)\,,\\
F_i{}^+(x^-,x^+,x_\perp) &\to \lambda^{-1} F_i{}^-(\lambda^{-1}x^-,\lambda x^+,x_\perp)\,,\\
F_i{}^+(x^-,x^+,x_\perp) &\to F^{-+}(\lambda^{-1}x^-,\lambda x^+,x_\perp)\,,\\
F_{ij}(x^-,x^+,x_\perp) &\to F_{ij}(\lambda^{-1}x^-,\lambda x^+,x_\perp)\,.
\end{aligned}
$$

and the spinor fields as

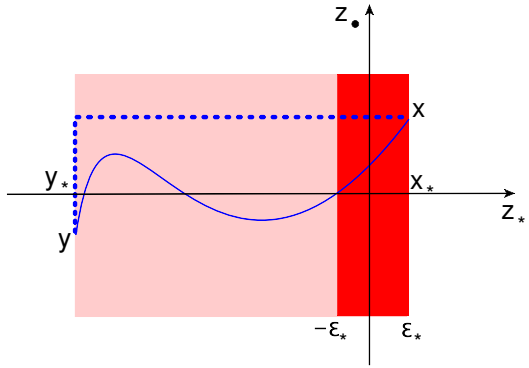
$$
\bar\psi t^a \not{n}_1 \psi \to \lambda \bar\psi t^a \not{n}_1 \psi\,, \quad \bar\psi t^a \gamma_\nu^\perp \psi \to \bar\psi t^a \gamma_\nu^\perp \psi\,, \quad \bar\psi t^a \not{n}_2 \psi \to \lambda^{-1} \bar\psi t^a \not{n}_2 \psi\,.
$$

Quark propagator with one point in the shock-wave



C.A.G. (2019)

Quark propagator with one point in the shock-wave



$$\langle x|\frac{i}{\not{P}+i\epsilon}|y\rangle$$

$$= \frac{1}{2}\Bigg[\theta(x^+-y^+)\Big([x^+,-\infty]_x \not{k}\not{n}_2 - g\int_{-\infty}^{x^+} dz^+ \gamma^i \not{n}_2 [x^+,z^+]_x F_i{}^-(z^+,x_\perp)[z^+,-\infty]_x\Big)$$

$$+\theta(y^+-x^+)\Big([x^+,+\infty]\not{k}\not{n}_2 + g\int_{x^+}^{+\infty} dz^+ \gamma^i \not{n}_2 [x^+,z^+]_x F_i{}^-(z^+,x_\perp)[z^+,n_1\infty]_x\Big)\Bigg]$$

$$\times\int \bar{d}^4k \frac{i\not{k}}{k^+(k^2+i\epsilon)} e^{-ik\cdot(x-y)}$$

C.A.G. (2019)

quark TMD

transverse polarization

$$\not{n}_1 = \gamma^-$$

$$\frac{1}{2\pi\delta(0)} \int \bar{d}^4k \bar{\delta}(k^2) \left| \langle q(k) | \gamma_T^*(q) \rangle_{\text{Fig. 1}a} \right|^2$$

$$= \frac{1}{2P^-} \sum_f e^2 e_f^2 \int d^2x d^2y \bar{d}^2k \, e^{i(q-k,x-y)_\perp} \int dx^+ dy^+ \, e^{-i\left(q^- - \frac{k_\perp^2}{2q^+}\right)(x^+ - y^+)}$$

$$\times \bar{\psi}(y^+, y_\perp)[y^+, \infty p_1]_y \not{n}_1 [+\infty n_1, x^+]_x \psi(x^+, x_\perp) + O(\lambda^{-2})$$

quark TMD

transverse polarization $\qquad \not{n}_1 = \gamma^-$

$$\frac{1}{2\pi\delta(0)} \int \bar{d}^4k \delta^-(k^2) \left| \langle q(k)|\gamma_T^*(q)\rangle_{\text{Fig. 1}a} \right|^2$$

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$$\times \bar{\psi}(y^+, y_\perp)[y^+, \infty p_1]_y \not{n}_1 [+\infty n_1, x^+]_x \psi(x^+, x_\perp) + O(\lambda^{-2})$$

using translation invariance, we define, up to a constant, the quark TMD

$$q_1(x_B, k_\perp) = \int d\Delta^+ d^2r \, e^{-i(k,r)_\perp} e^{i\left(x_B + \frac{k_\perp^2}{s}\right)P^-\Delta^+}$$

$$\times \langle N(P)|\bar{\psi}(0^+, 0_\perp)\,[0, \infty n_1]\,\gamma^-\,[\infty n_1, \Delta^+]_r\,\psi(\Delta^+, r_\perp)|N(P)\rangle$$

$$\frac{d^3\sigma^{\gamma_T^* \to q}}{dk^+ d^2k} = \frac{\alpha_{\text{em}} e_f^2}{s} \delta(q^+ - k^+) q_1(x_B, k_\perp)$$

inclusive DIS: naive quark pdf

$$\frac{1}{2\pi\delta(0)}\int \bar{d}^4k\delta^-(k^2)\left|\langle q(k)|\gamma_T^*(q)\rangle_{\text{Fig. }1a}\right|^2$$

$$=\frac{1}{2P^-}\sum_f e^2e_f^2\int d^2xd^2y\bar{d}^2k\,e^{i(q-k,x-y)_\perp}\int dx^+dy^+\,e^{-i\left(q^- -\frac{k_\perp^2}{2q^+}\right)(x^+-y^+)}$$

$$\times\bar{\psi}(y^+,y_\perp)[y^+,\infty p_1]_y\not{n}_1[+\infty n_1,x^+]_x\psi(x^+,x_\perp)+O(\lambda^{-2})$$

inclusive DIS: naive quark pdf

$$\frac{1}{2\pi\delta(0)}\int \bar{d}^4k\bar{\delta}(k^2)\left|\langle q(k)|\gamma_T^*(q)\rangle_{\text{Fig.}1a}\right|^2$$

$$=\frac{1}{2P^-}\sum_f e^2e_f^2\int d^2xd^2y\bar{d}^2k\,e^{i(q-k,x-y)_\perp}\int dx^+dy^+\left(e^{-i\left(q^--\frac{k_\perp^2}{2q^+}\right)(x^+-y^+)}\to 1\right)$$

$$\times\bar{\psi}(y^+,y_\perp)[y^+,\infty p_1]_y\not{n}_1[+\infty n_1,x^+]_x\psi(x^+,x_\perp)+O(\lambda^{-2})$$

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$$\times\bar{\psi}(y^+,y_\perp)[y^+,\infty p_1]_y\not{n}_1[+\infty n_1,x^+]_x\psi(x^+,x_\perp)+O(\lambda^{-2})$$

To get inclusive DIS $\longrightarrow$ complete the integration over the phase space

$$\frac{1}{2\pi\delta(0)}\int \bar{d}^4k\bar{\delta}(k^2)\theta(k^+)\left(\left|\langle q(k)|\gamma_T^*(q)\rangle_{\text{Fig. }1a}\right|^2+\left|\langle \bar{q}(k)|\gamma_T^*(q)\rangle_{\text{Fig. }1b}\right|^2\right)$$
$$=\frac{1}{P^-}\sum_f e^2e_f^2\int d^2xdx^+dy^+\,\langle\bar{\psi}(y^+,x_\perp)\not{n}_1[y^+,x^+]_x\psi(x^+,x_\perp)\rangle$$

$$q_f(x_B=0)=\frac{1}{4\pi}\int dx^+\langle N(P)|\bar{\psi}_f(0^+,0_\perp)\gamma^-[0^+,x^+]\psi_f(x^+,0_\perp)|N(P)\rangle$$

quark pdf at $x_B \neq 0$

$$\frac{1}{2\pi\delta(0)} \int \bar{d}^4k \bar{\delta}(k^2) \left| \langle q(k) | \gamma_T^*(q) \rangle_{\text{Fig. 1}a} \right|^2$$

$$= \frac{1}{2P^-} \sum_f e^2 e_f^2 \int d^2x d^2y \bar{d}^2k \, e^{i(q-k,x-y)_\perp} \int dx^+ dy^+ \, e^{-i\left(q^- - \frac{k_\perp^2}{2q^+}\right)(x^+ - y^+)}$$

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$$\times\bar{\psi}(y^+,y_\perp)[y^+,\infty p_1]_y \not{n}_1[+\infty n_1,x^+]_x\psi(x^+,x_\perp) + O(\lambda^{-2})$$

we complete the phase-space integration without making any small-x_B approximation in the integrand

$$\frac{1}{2\pi\delta(0)}\int \bar{d}^4k\delta^-(k^2)\theta(k^+)\left|\langle q(k)|\gamma_T^*(q)\rangle_{\text{Fig. 1}a}\right|^2$$

$$= \frac{1}{2P^-}\sum_f e^2e_f^2\int d^2xd^2y\, e^{i(q,\Delta)_\perp}\int dx^+dy^+\,\frac{is}{4\pi P^-\Delta^+}\, e^{-iq^-\Delta^+ - i\frac{\Delta_\perp^2 s}{4P^-\Delta^+}}$$

$$\times\bar{\psi}(y^+,y_\perp)[y^+,+\infty n_1]_y \not{n}_1[+\infty n_1,x^+]_x\psi(x^+,x_\perp)\text{sign}(\Delta^+) + O(\lambda^{-2})$$

$\Delta = x - y$

Let us define the operators

$$Q_{1,f}(x_\perp, x_B) \equiv g^2 \int_{-\infty}^{+\infty} dx^+ \int_{-\infty}^{x^+} dy^+ \, e^{ix_B P^- \Delta^+} \bar{\psi}_f(y^+, x_\perp)[y^+, x^+]_x \not{n}_1 \psi_f(x^+, x_\perp) \,,$$

and its adjoint conjugated

$$\bar{Q}_{1,f}(x_\perp, x_B) \equiv g^2 \int_{-\infty}^{+\infty} dy^+ \int_{-\infty}^{y^+} dx^+ \, e^{ix_B P^- \Delta^+} \, \bar{\psi}_f(y^+, x_\perp)[y^+, x^+]_x \not{n}_1 \psi_f(x^+, x_\perp) \,.$$

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Using translation invariance, we define

$$q_f(x_B) \equiv \frac{1}{4\pi} \int_0^{+\infty} dx^+ e^{ix_B P^- x^+} \langle N(P)| \hat{\bar{\psi}}_f(0^+, 0) \gamma^- [0^+, x^+] \hat{\psi}_f(x^+, 0_\perp) |N(P)\rangle$$

$$\bar{q}_f(x_B) \equiv -\frac{1}{4\pi} \int_0^{+\infty} dx^+ e^{-ix_B P^- x^+} \langle N(P)| \hat{\bar{\psi}}_f(x^+, 0_\perp) \gamma^- [x^+, 0^+] \hat{\psi}_f(0^+, 0_\perp) |N(P)\rangle$$

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Using translation invariance

$$q_f(x_B) + \bar{q}_f(x_B) = \frac{1}{4\pi}\int_{-\infty}^{+\infty} dx^+ e^{ix_B P^- x^+} \langle N(P)|\hat{\bar{\psi}}_f(0^+,0)\gamma^-[0^+,x^+]\hat{\psi}_f(x^+,0_\perp)|N(P)\rangle$$

Thus, transverse cross-section, is

$$\sigma_T^{\gamma^* P} = \frac{4\pi}{s}\alpha_{\rm em}\sum_f e_f^2\Big(q_f(x_B) + \bar{q}_f(x_B)\Big) + O(\lambda^{-2})$$

From non-local OPE to High-energy OPE

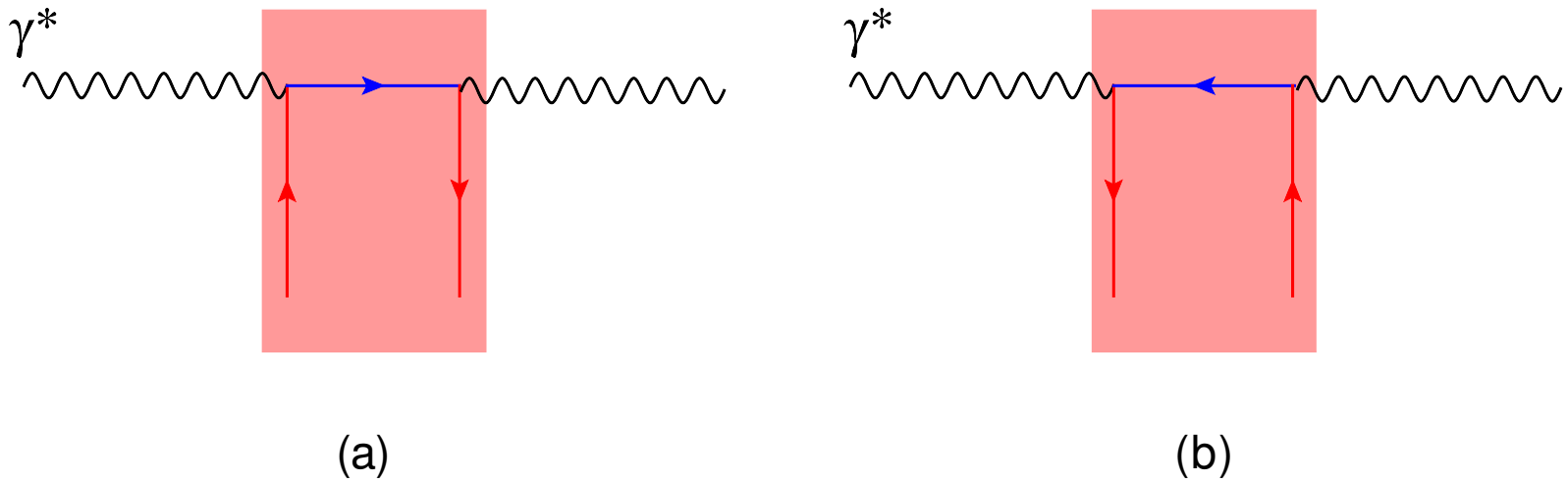

Figure: Diagrams contributing to the calculation of $T_{\mu\nu}$

$$T_{\mu\nu} = i \int d^4x\, e^{iq\cdot x} \langle N(P)|\mathrm{T}\{j_\mu(x) j_\nu(0)\}|N(P)\rangle$$

The non-local OPE

- a gauge-invariant generating formalism for towers of local operators,
- a natural derivation of evolution equations in coordinate space,
- a unified language for forward and non-forward parton distributions,
- conceptual precursor of the modern Wilson-line formalism used in collinear, TMD, and high-energy QCD.

Non-local OPE

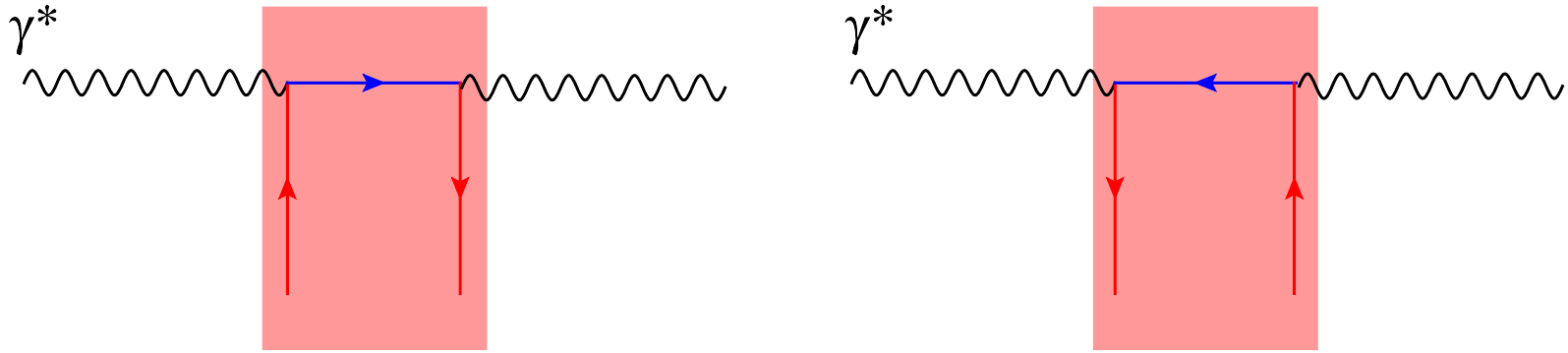

Figure: Diagrams contributing to the calculation of $T_{\mu\nu}$

$$T_{\mu\nu} = \lim_{\epsilon^+ \to 0} i \int d^4x d^4y \, e^{iq\cdot(x-y)} \frac{i}{2\pi^2[(x-y)^2 - i\epsilon]^2}$$
$$\times \Big(\langle N(P)|\mathrm{T}\{\bar{\psi}_c(x)\gamma_\mu(\not{x}-\not{y})[x,y]\gamma_\nu\psi_c(y)\}|N(P+\epsilon^+ n_1)\rangle$$
$$- \langle N(P)|\mathrm{T}\{\bar{\psi}_c(y)\gamma_\nu(\not{x}-\not{y})[y,x]\gamma_\mu\psi_c(x)\}|N(P+\epsilon^+ n_1)\rangle \Big)$$

Non-local OPE

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$$- \langle N(P)|\mathrm{T}\{\bar{\psi}_c(y)\gamma_\nu(\not{x} - \not{y})[y,x]\gamma_\mu \psi_c(x)\}|N(P+\epsilon^+ n_1)\rangle \Big)$$

$$\langle \mathrm{T}\psi(x)\bar{\psi}(y)\rangle = \frac{i(\not{x} - \not{y})}{2\pi^2[(x-y)^2 - i\epsilon]^2}[x,y]$$

with the straight gauge link between two points x^μ and y^μ given by

$$[x,y] = \mathrm{P}\exp\left\{ ig \int_y^x dz^\mu \, A_\mu(z) \right\}$$

From non-local OPE to high-energy OPE

Consider transverse polarization $\Delta = x - y$

$$\frac{1}{2}\sum_{\lambda=\pm 1}\varepsilon^i_\lambda \varepsilon^{*j}_\lambda T_{ij} = -\lim_{\epsilon^+\to 0}\frac{\sum_f e_f^2}{4\pi^2}\Delta^+\int dx^+dx^-d^2xdy^+dy^-d^2y\,\frac{e^{iq^+\Delta^- + iq^-\Delta^+ - i(q,\Delta)_\perp}}{\left[2\Delta^-\Delta^+ - \Delta_\perp^2 - i\epsilon\right]^2}$$
$$\times\Big(\langle N(P)|\hat{\bar{\psi}}(x^+,x_\perp)\not{n}_1[x^+,y^+]_x\hat{\psi}(y^+,y_\perp)|N(P+\epsilon^+n_1)\rangle$$
$$-\langle N(P)|\hat{\bar{\psi}}(y^+,y_\perp)\not{n}_1[y^+,x^+]_y\hat{\psi}(x^+,x_\perp)|N(P+\epsilon^+n_1)\rangle + O(\lambda^{-2})$$

From non-local OPE to high-energy OPE

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$q^- = -\frac{Q^2}{2q^+} = -x_BP^-$ at $q_\perp = 0$, take the residue integrating over Δ^-

$$\frac{1}{2}\sum_{\lambda=\pm1}\varepsilon^i_\lambda\varepsilon^{*j}_\lambda T_{ij} = \lim_{\epsilon^+\to0}\frac{1}{4\pi}\frac{\sum_f e_f^2}{\Delta^+}\int dx^+dy^+\theta(x^+-y^+)d^2xd^2y\,e^{i\frac{s\Delta_\perp^2}{4P^-\Delta^+}-i\frac{Q^2}{2q^+}\Delta^+}$$
$$\times\Big(\langle N(P)|\hat{\bar\psi}(x^+,x_\perp)\not{n}_1[x^+,y^+]_x\hat\psi(y^+,y_\perp)|N(P)\rangle$$
$$-\langle N(P)|\hat{\bar\psi}(y^+,y_\perp)\not{n}_1[y^+,x^+]_y\hat\psi(x^+,x_\perp)|N(P+\epsilon^+n_1)\rangle\Big) + O(\lambda^{-2})$$

High-energy limit : $\frac{i\alpha_q s}{4\pi P^- \Delta^+} \exp\left\{-i\frac{s\Delta_\perp^2}{4P^-\Delta^+}\right\} = \delta^{(2)}(\Delta_\perp)$

$$\frac{1}{2}\sum_{\lambda=\pm 1} \varepsilon^i_\lambda(q)\varepsilon^j_\lambda(q)\mathrm{Im}T_{ij} = \frac{2\pi}{s}\sum_f e_f^2\left(q_f + \bar{q}_f\right) + O(\lambda^{-2})$$

$$q_f(x_B) \equiv \frac{1}{4\pi}\int_0^{+\infty} dx^+ e^{ix_B P^- x^+} \langle N(P)|\hat{\bar{\psi}}_f(0^+,0)\gamma^-[0^+,x^+]\hat{\psi}_f(x^+,0_\perp)|N(P)\rangle$$

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$$\varepsilon^\mu \varepsilon^{*\nu}\frac{1}{\pi}\mathrm{Im}\, T_{\mu\nu} = \quad \varepsilon^\mu \varepsilon^{*\nu} W_{\mu\nu}$$

$$\frac{1}{2}\sum_{\lambda=\pm 1} \varepsilon^i_\lambda(q)\varepsilon^j_\lambda(q) W_{ij} = \frac{1}{2\pi}\frac{1}{\alpha_{\mathrm{em}}}\left[\frac{4\pi\alpha_{\mathrm{em}}}{s}\sum_f e_f^2(q_f + \bar{q}_f)\right]$$

$$\frac{1}{2}\sum_{\lambda=\pm 1} \varepsilon^i_\lambda(q)\varepsilon^j_\lambda(q)\frac{1}{\pi}\mathrm{Im}\, T_{ij} = \frac{1}{\pi}\left[\frac{2\pi}{s}\sum_f e_f^2\left(q_f + \bar{q}_f\right)\right]$$

Conclusions

- From non-local OPE to Local OPE
  - It isolates the operators that survive in the Regge limit.
  - It explains how Wilson lines arise from the Balitsky–Braun light-ray operators.

$\Rightarrow$ Book-keeping procedure to relate higher twist corrections to sub-eikonal corrections

We need a formalism

- to unifies the light-cone and shock-wave descriptions of high-energy scattering.

- the provides a common framework for quark distributions, TMDs, and dipole observables at the varius kinematic regimes that will be achivable at the EIC