Bridging TMD Frameworks: Intersections, Tensions, and Applications

09-13 March 2026

COLLINS ASYMMETRIES FOR PION-IN-JET PRODUCTION IN POLARIZED PP AND LP COLLISIONS

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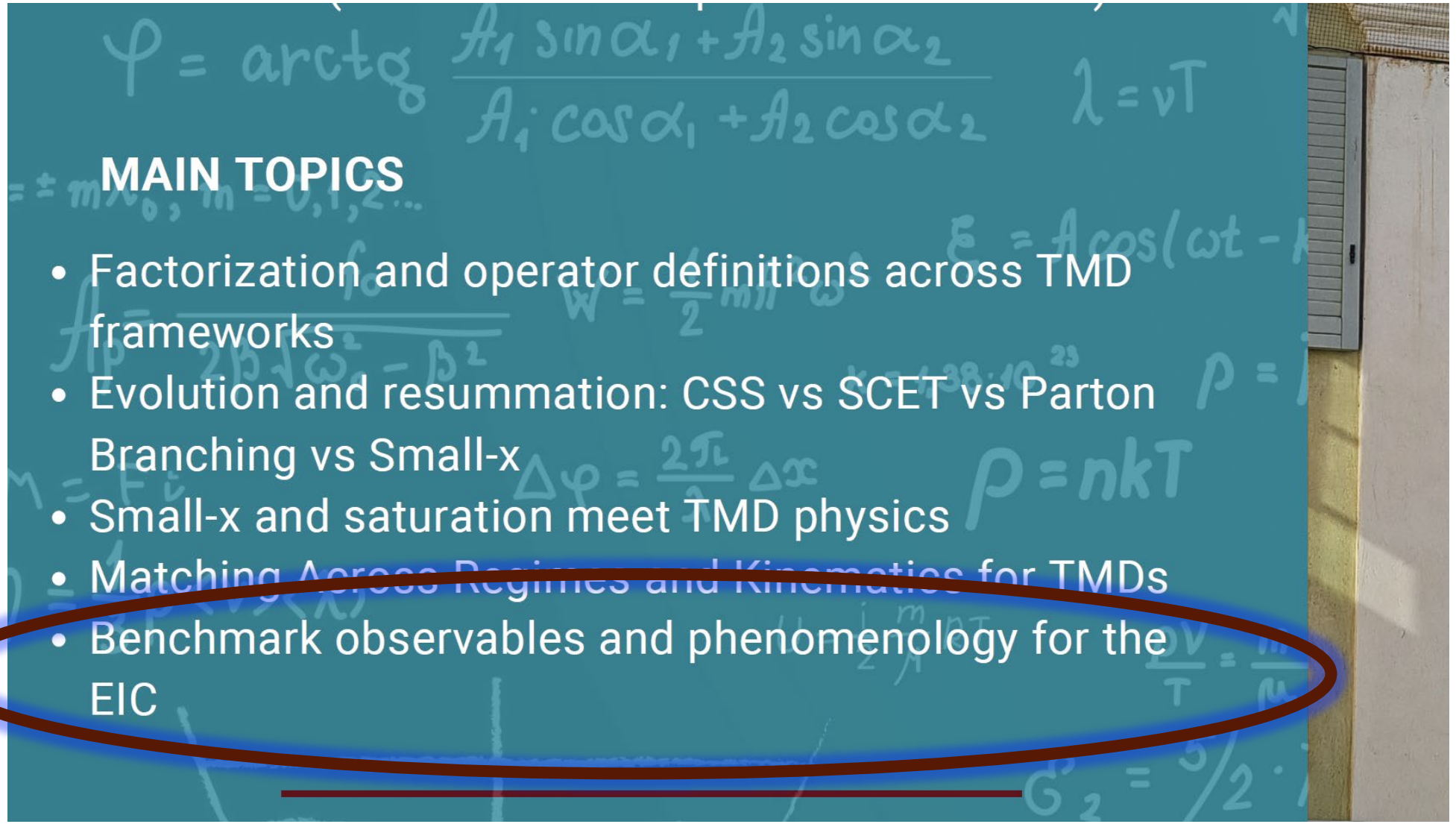
PLB 2025 & in progress

Bridging TMD Frameworks: Intersections, Tensions, and Applications

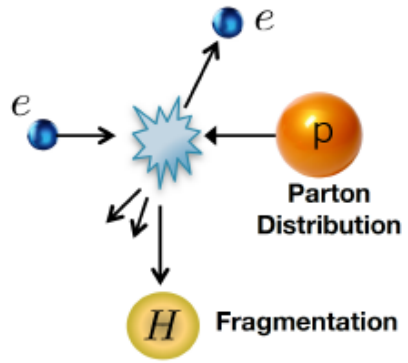


OUTLINE

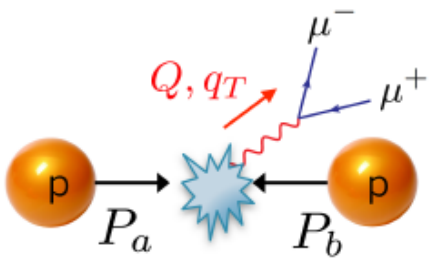
- hadron-in-jet in pp collisions
  - ✓Motivations
- Pion-in-jet production in pp collisions: Collins asymmetry
  - ✓Transversity and Collins FF from SIDIS and $e^+e^- \to h_1\ h_2 + X$
  - ✓Comparison with STAR data
  - ✓Universality, TMD factorization
- Pion-in-jet production in ℓp collisions: Collins asymmetry
  - ✓tool to constrain quark distributions
  - ✓Quasireal photon exchange contribution
  - ✓Estimates for EIC kinematics
- Concluding remarks

"STANDARD" TMD PROCESSESS

Semi-Inclusive DIS

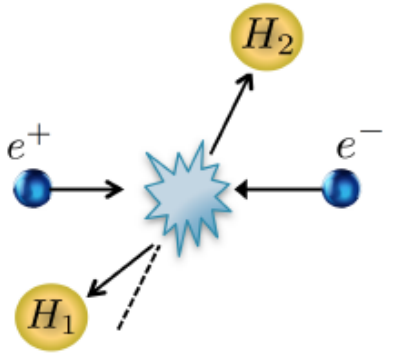


Drell-Yan



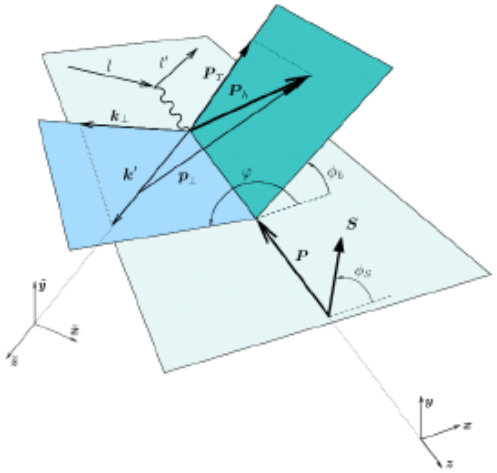
$q_T \ll Q$

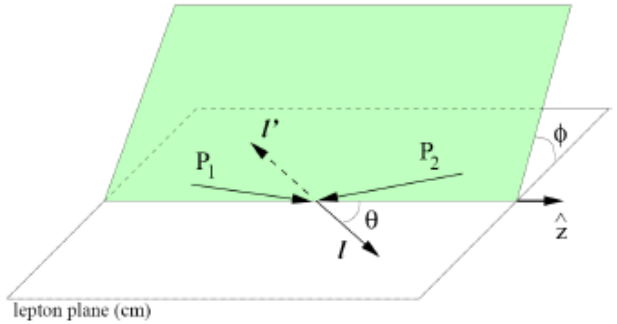
Dihadron in e+e-

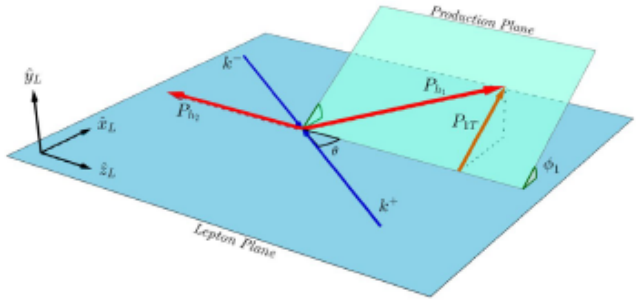


Two-scale processes:

$Q^2 \gg p_T^2 \geq \Lambda_{QCD}^2$

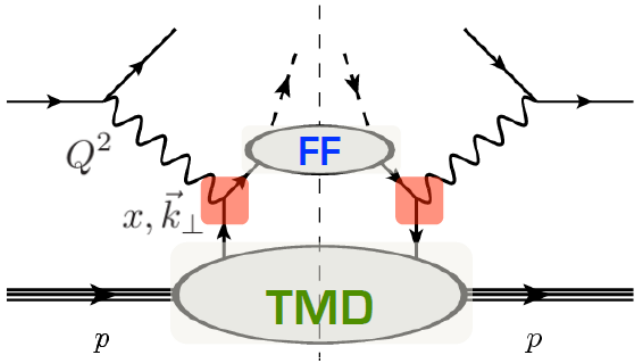






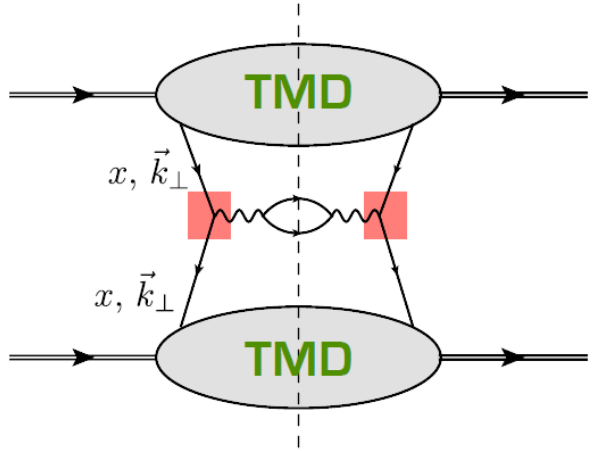
TMD FACTORIZATION

$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X$$



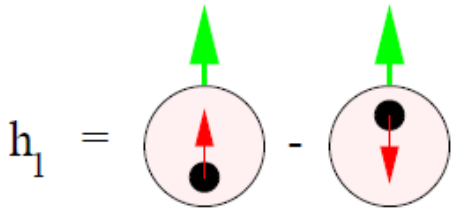
$$\mathrm{d}\sigma \sim \sum \mathrm{TMD}(x, \vec{k}_\perp) \otimes \mathrm{d}\hat{\sigma}_{hard} \otimes \mathrm{FF}(z, \vec{p}_\perp) + \mathcal{O}(\frac{P_T}{Q})$$

$$h(P_1) + h(P_2) \to \ell^+(l) + \ell^-(l')$$



$$\mathrm{d}\sigma \sim \sum \mathrm{TMD}(x, \vec{k}_\perp) \otimes \overline{\mathrm{TMD}}(x, \vec{k}_\perp) \otimes \mathrm{d}\hat{\sigma}_{hard}$$

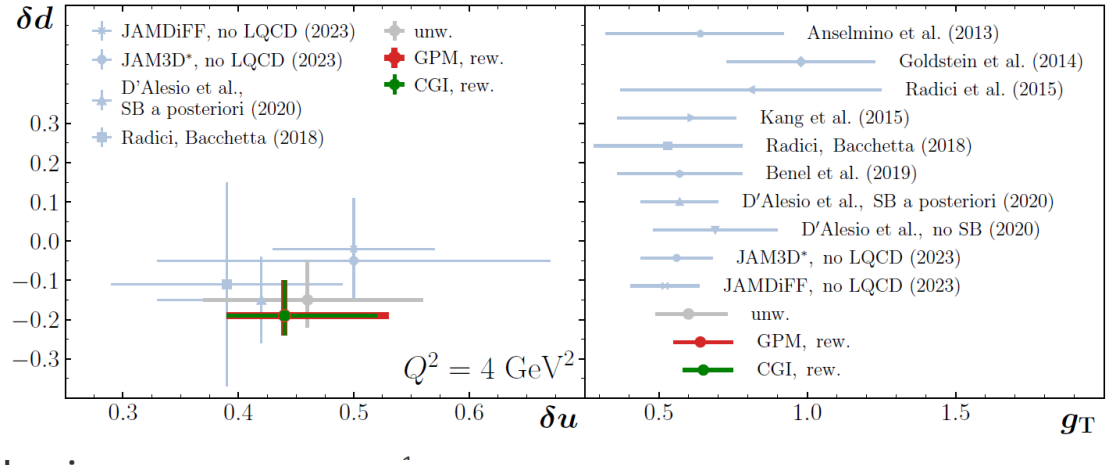
TRANSVERSITY



- collinear function for spin-1/2 hadrons at leading twist (+ f_1 and g_1)
- chiral-odd quantity ⇒ not accessible in inclusive DIS
- extracted in SIDIS together with Collins FF (TMD framework) or two-hadron production with dihadron FF (collinear pQCD)
- quarks contribute to nucleon tensor charge via the first Mellin moment of the non-singlet quark combination,

$$\delta q = \int_0^1 \left[h_1^q(x) - h_1^{\bar{q}}(x) \right] dx$$

$$g_T = \delta u - \delta d$$



a bridge between QCD phenomenology, lattice QCD and BSM physics

$$p^{\uparrow} p \rightarrow \text{jet} + h + X$$

Motivations

- Complementary [a bridge] to SIDIS and e^+e^- processes (TMD factorization):

- 2 ordered scales: p_{jT} (large) and intrinsic $p_{\perp h} = j_T$ (small) [$\perp$ w.r.t. jet]

- TMD effects (relevant) only in the fragmentation mechanism: **collinear initial state**

- Test of TMD factorization and universality of TMD FFs

- Several theo/exp studies for hadron-in-jet:

- Collins effect in pp collisions *[Yuan 2008; UD, Murgia, Pisano '11, '17; Kang et al. '17]*

  w/ or w/o TMD evolution: general agreement

- TMD-Jet-FFs and their relation to TMD FFs *[Kang et al. '20, '21, '23]*

- STAR data (pion-jet at different energies, p_T, binned) *[arXiv:2507.16355 [hep-ex]]*

$$p^{\uparrow} p \to \text{jet} + h + X$$

- Factorization scale and jet-cone radius R: proper initial scale $\mu = p_{\mathrm{jT}} R$
- Evolution up to $\mu = p_{\mathrm{jT}}$: resummation of single logs in jet param.
- At LO (no dependence on R) one can use $\mu = p_{\mathrm{jT}}$ *[Kang et al '17]*

- Backward going jet *[Boer, Bomhof, Hwang, Mulders '07]*
- to determine the partonic cm frame (different from the pp cm frame, depending on x_1, x_2)
- z and $p_\perp$ in TMDFF are not Lorentz boost invariant (pp vs e^+e^-)
- At high energies differences suppressed as $E_j/\sqrt{s}$: backward jet not relevant *[Boer '10]* …

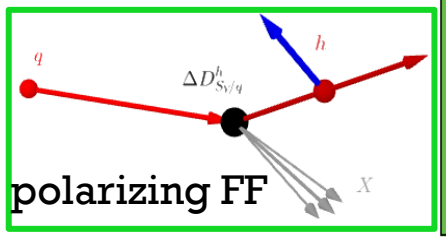
TMD FFs for spin ½ hadrons

Leading Quark TMDFFs

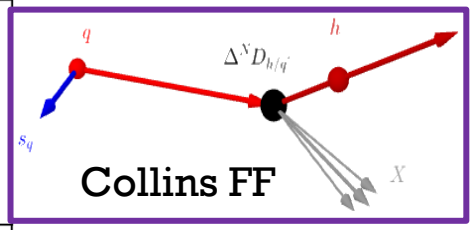
Hadron Spin Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		D_1 = Unpolarized		$H_1^\perp$ = Collins
Polarized Hadrons	L		G_1 = Helicity	$H_{1L}^\perp$ =
	T	$D_{1T}^\perp$ = Polarizing FF	$G_{1T}^\perp$ =	H_1 = Transversity $H_{1T}^\perp$ =

T-odd, chiral-even



polarizing FF



Collins FF

T-odd, chiral-odd

KINEMATIC CONFIGURATION

$$A^{\uparrow}(p_A)\, B(p_B) \to \mathrm{jet}(p_\mathrm{j}) + \pi(p_\pi) + X$$

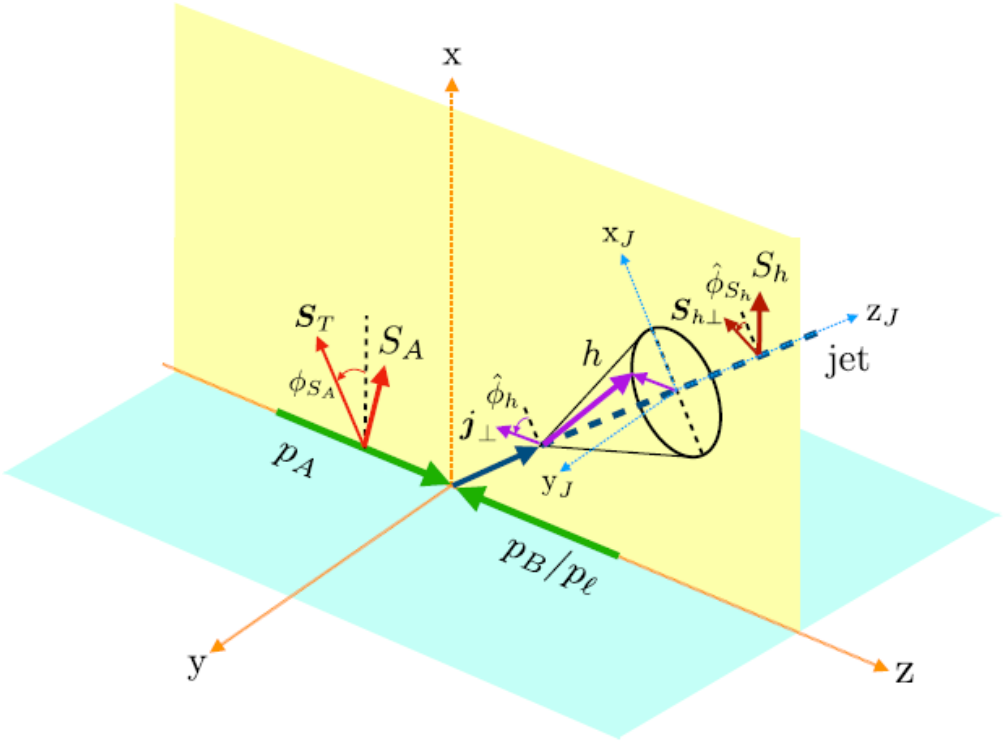


Figure from Kang, Lee, Zhao 2020

$$a\, b \to c\, d$$

$$p_a^\mu = x_a \frac{\sqrt{s}}{2}(1,0,0,1) \quad p_b^\mu = x_b \frac{\sqrt{s}}{2}(1,0,0,-1)$$

ab collinear

$$c \equiv \mathrm{jet} \quad \mathrm{LO}$$

$$p_c^\mu \equiv p_\mathrm{j}^\mu = p_{\mathrm{j}T}(\cosh\eta_\mathrm{j}, 1, 0, \sinh\eta_\mathrm{j})$$

AZIMUTHAL STRUCTURE

$$\frac{E_j\, d\sigma^{p(S,\phi_S)p\to \text{jet}\,\pi(\phi_\pi^H)\,X}}{d^3\boldsymbol{p}_j\, dz\, d^2\boldsymbol{p}_{\perp\pi}\, d\phi_S}$$

UD, Murgia, Pisano 2011

$$d\sigma(\phi_{S_A}, \phi_\pi^H) - d\sigma(\phi_{S_A} + \pi, \phi_\pi^H) \sim d\Delta\sigma_0 \sin\phi_{S_A}$$
$$+ d\Delta\sigma_1^- \sin(\phi_{S_A} - \phi_\pi^H) + d\Delta\sigma_1^+ \sin(\phi_{S_A} + \phi_\pi^H)$$
$$+ d\Delta\sigma_2^- \sin(\phi_{S_A} - 2\phi_\pi^H) + d\Delta\sigma_2^+ \sin(\phi_{S_A} + 2\phi_\pi^H),$$

Collins effect

$qq \to qq,$ $qg \to qg,$ $qg \to gq,$ $gq \to qg,$

$gq \to gq$ $gg \to q\bar{q},$ $q\bar{q} \to gg,$ $gg \to gg,$

COLLINS ASYMMETRY

$$\frac{E_j\, d\sigma^{p(S,\phi_S)p\to \text{jet}\,\pi(\phi_\pi^H)\,X}}{d^3\boldsymbol{p}_j\, dz\, d^2\boldsymbol{p}_{\perp\pi}\, d\phi_S}$$

$$A_N^{\sin(\phi_S-\phi_\pi^H)}(\boldsymbol{p}_j, z, p_{\perp\pi})$$

$$=2\,\frac{\int d\phi_S\, d\phi_\pi^H\, \sin(\phi_S-\phi_\pi^H)\,[\,d\sigma(\phi_S,\phi_\pi^H)-d\sigma(\phi_S+\pi,\phi_\pi^H)\,]}{\int d\phi_S\, d\phi_\pi^H\,[\,d\sigma(\phi_S,\phi_\pi^H)+d\sigma(\phi_S+\pi,\phi_\pi^H)\,]}$$

$$N[A_N^{\sin(\phi_S-\phi_\pi^H)}] \sim \sum_{a,b,c,d} h_1^a(x_a)\otimes f_1^b(x_b)\otimes \Delta\hat{\sigma}^{ab\to cd}\otimes H_1^{\perp\,c}(z,\boldsymbol{p}_{\perp\pi}^2)$$

Collinear initial state

PARAMETRIZATIONS

MSHT20nlo '21 DSSV '09 **FIT to SIDIS and e^+e^- data**

$$h_1^q(x, Q_0^2) = \mathcal{N}_q^T(x)\frac{1}{2}\left[f_{q/p}(x, Q_0^2) + g_{1L}^q(x, Q_0^2)\right]$$

valence

$$\mathcal{N}_q^T(x) = N_q^T x^\alpha (1-x)^\beta \frac{(\alpha+\beta)^{\alpha+\beta}}{\alpha^\alpha \beta^\beta}, \quad (q = u_v, d_v)$$

$$H_1^{\perp q}(z, p_\perp^2) = \mathcal{N}_q^C(z)\frac{z m_h}{M_C}\sqrt{2e}\, e^{-p_\perp^2/M_C^2} D_{h/q}(z, p_\perp^2)$$

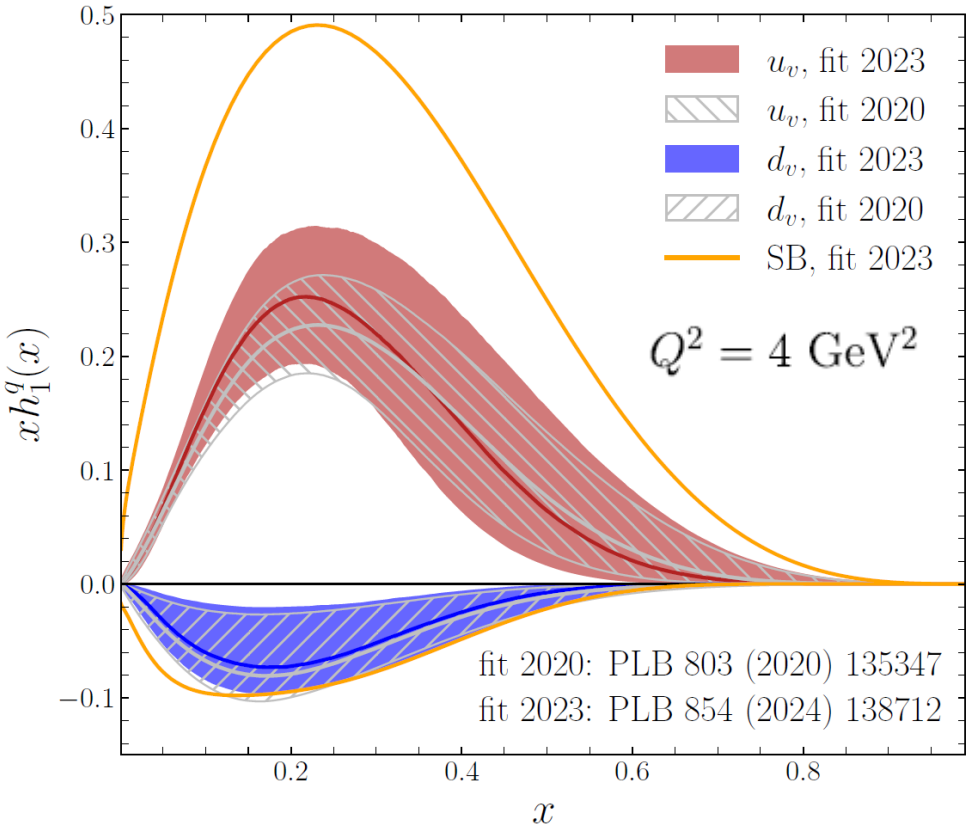
$$D_{h/q}(z, p_\perp^2) = D_{h/q}(z)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

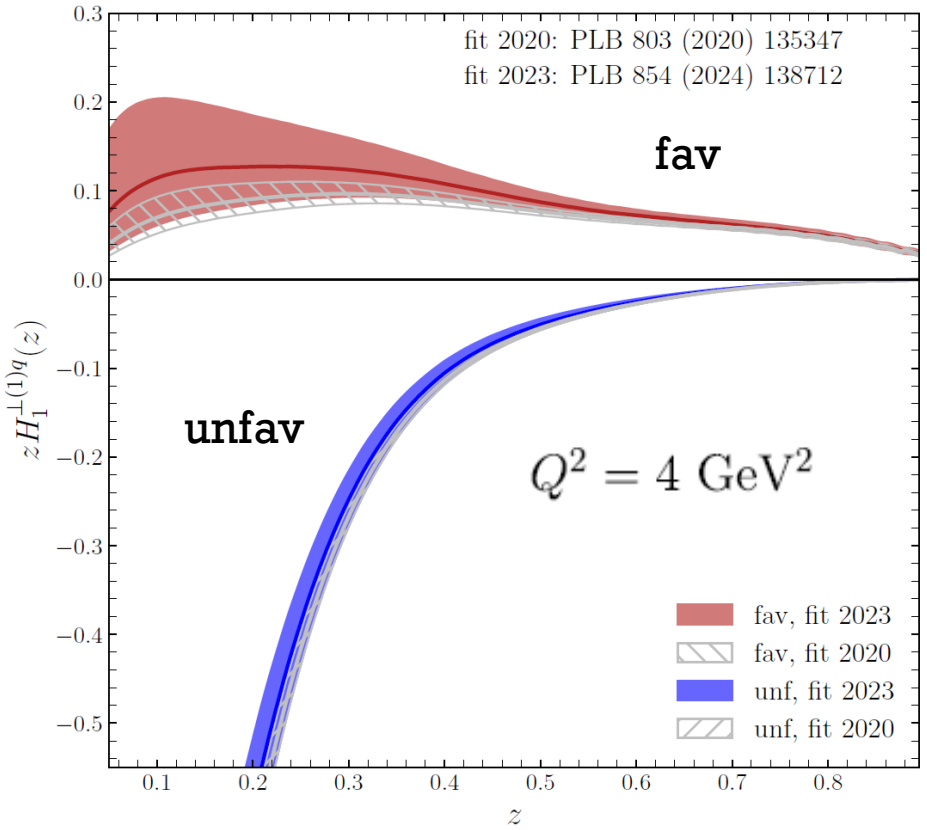
DEHSS '15

$$\mathcal{N}_{\rm fav}^C(z) = N_{\rm fav}^C z^\gamma, \quad \mathcal{N}_{\rm unf}^C(z) = N_{\rm unf}^C$$

Boglione, UD, Flore, Gonzalez-Hernandez, Murgia, Prokudin, PLB2024

TRANSVERSITY AND COLLINS FFS





PLB 2020 UD, Flore, Prokudin

PLB 2024 Boglione, UD, Flore, Gonzalez-Hernandez, Murgia, Prokudin

Proper use of the Soffer bound in a fit

$$|h_1^q(x,Q^2)| \leq \frac{1}{2}\left[f_{q/p}(x,Q^2) + g_{1L}^q(x,Q^2)\right]$$

RESULTS

PREDICTIONS

STAR KINEMATICS

- $\sqrt{s}$ = 200, 510 GeV

p_{jT}: [6:30], [8:60] GeV $x_T = 2p_{jT}/\sqrt{s}$ comparable ranges

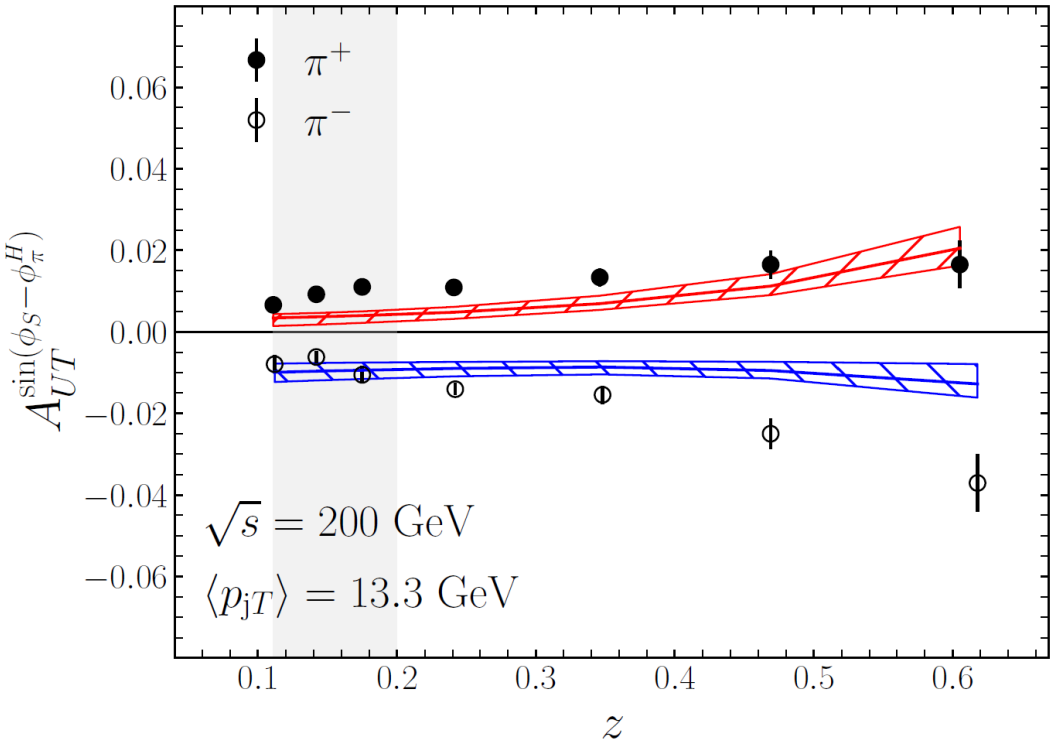
$0 \leq \eta_j \leq 0.9$ forward region

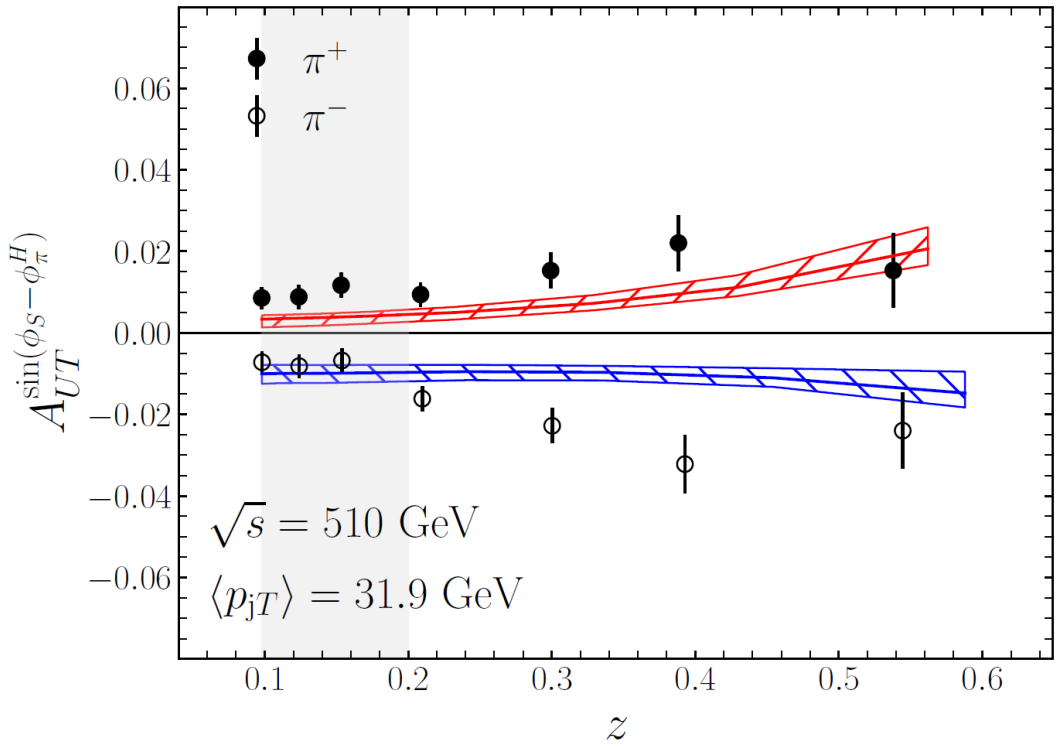
j_T:[0.1:1.25], [0.1:1.5] GeV pion transverse momentum w.r.t jet

z: $[0.1: 0.8]$

R: [0.05:0.6] jet-cone radius

RESULTS [PREDICTIONS VS. DATA] I

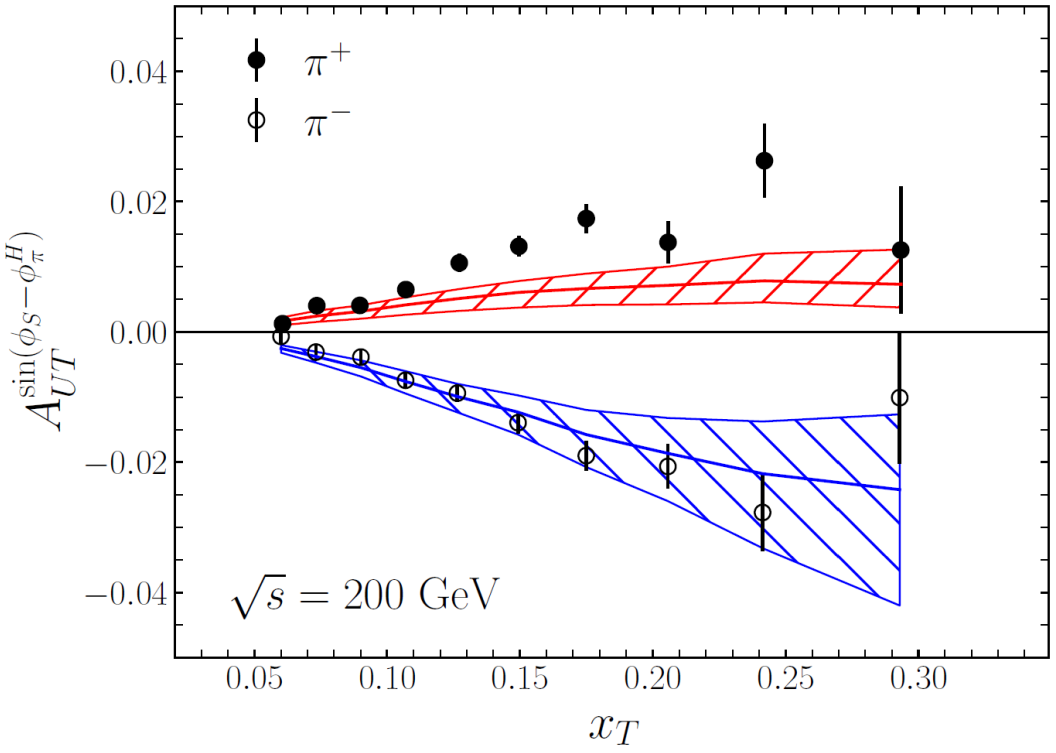


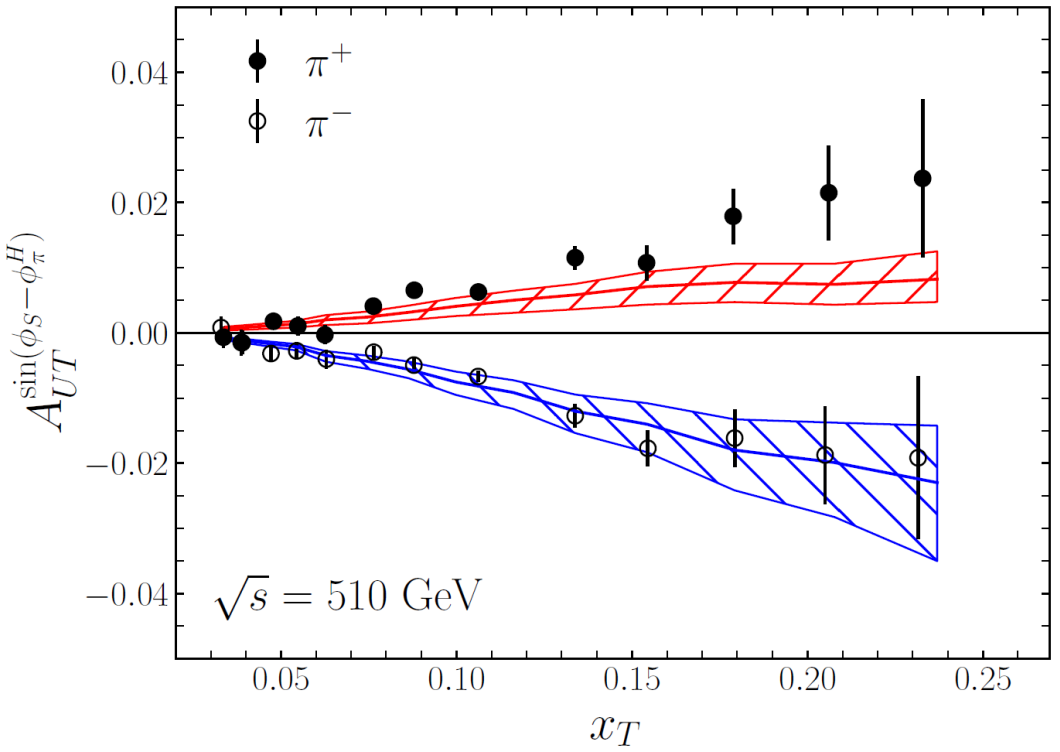


Collins azimuthal asymmetries **vs. z** at fixed p_{jT} integrated over η_j.
STAR data: PRD 2022 (200 GeV) and arXiv:2507.16355 [hep-ex] (510 GeV)

Central value: median of the compressed 10^6 MC sets; uncertainties at 2σ (CL).

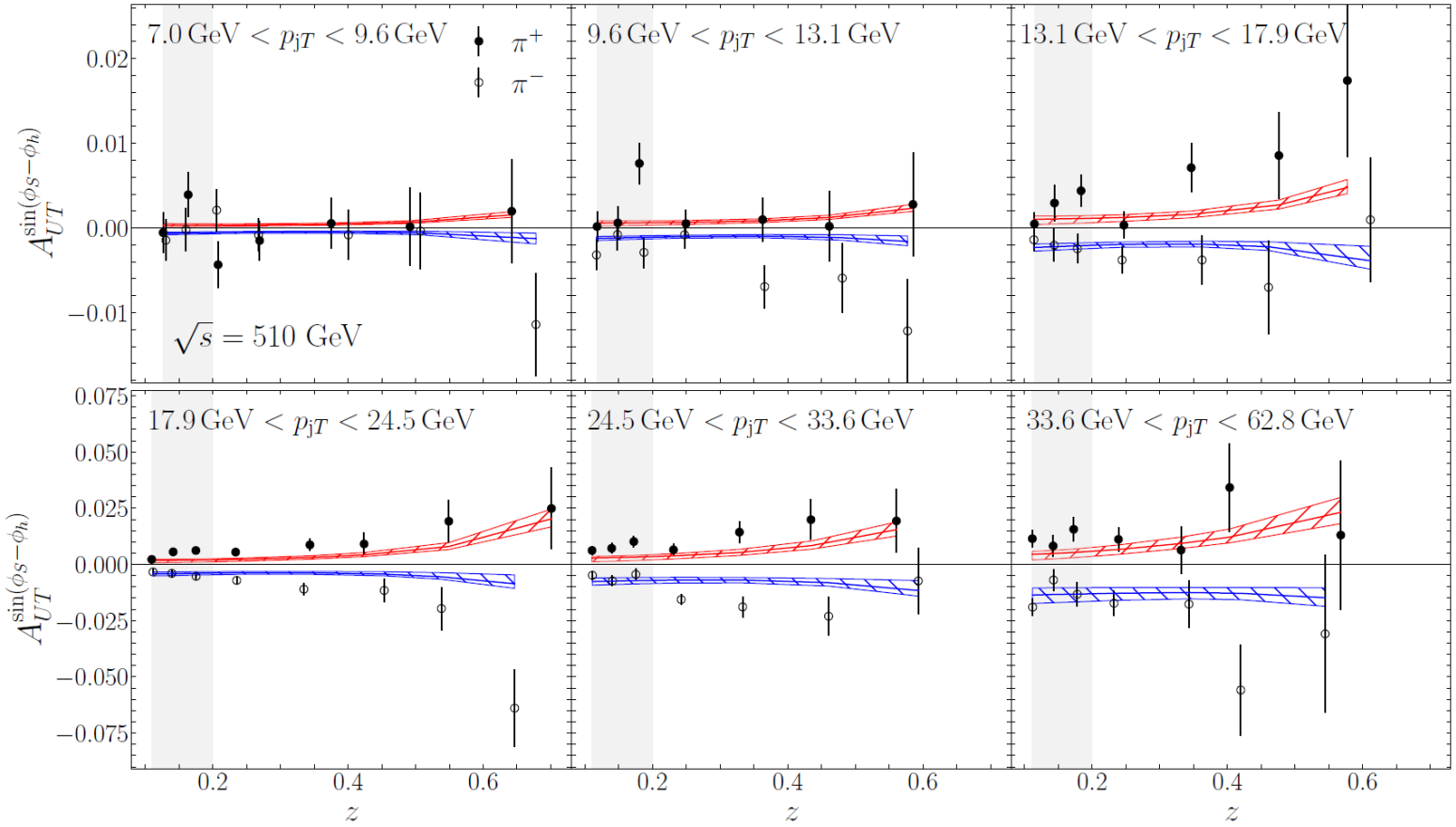
RESULTS [PREDICTIONS VS. DATA] II





Collins azimuthal asymmetries **vs.** $x_T = 2p_{jT}/\sqrt{s}$, integrated over z and over η_j.
STAR data: PRD 2022 (200 GeV) and arXiv:2507.16355 [hep-ex] (510 GeV)

RESULTS [PREDICTIONS VS. DATA] III



Collins azimuthal asymmetries **vs. z** integrated over η_j, for various **p_{jT} bins.**

STAR data: SPIN2023 **(510 GeV)**

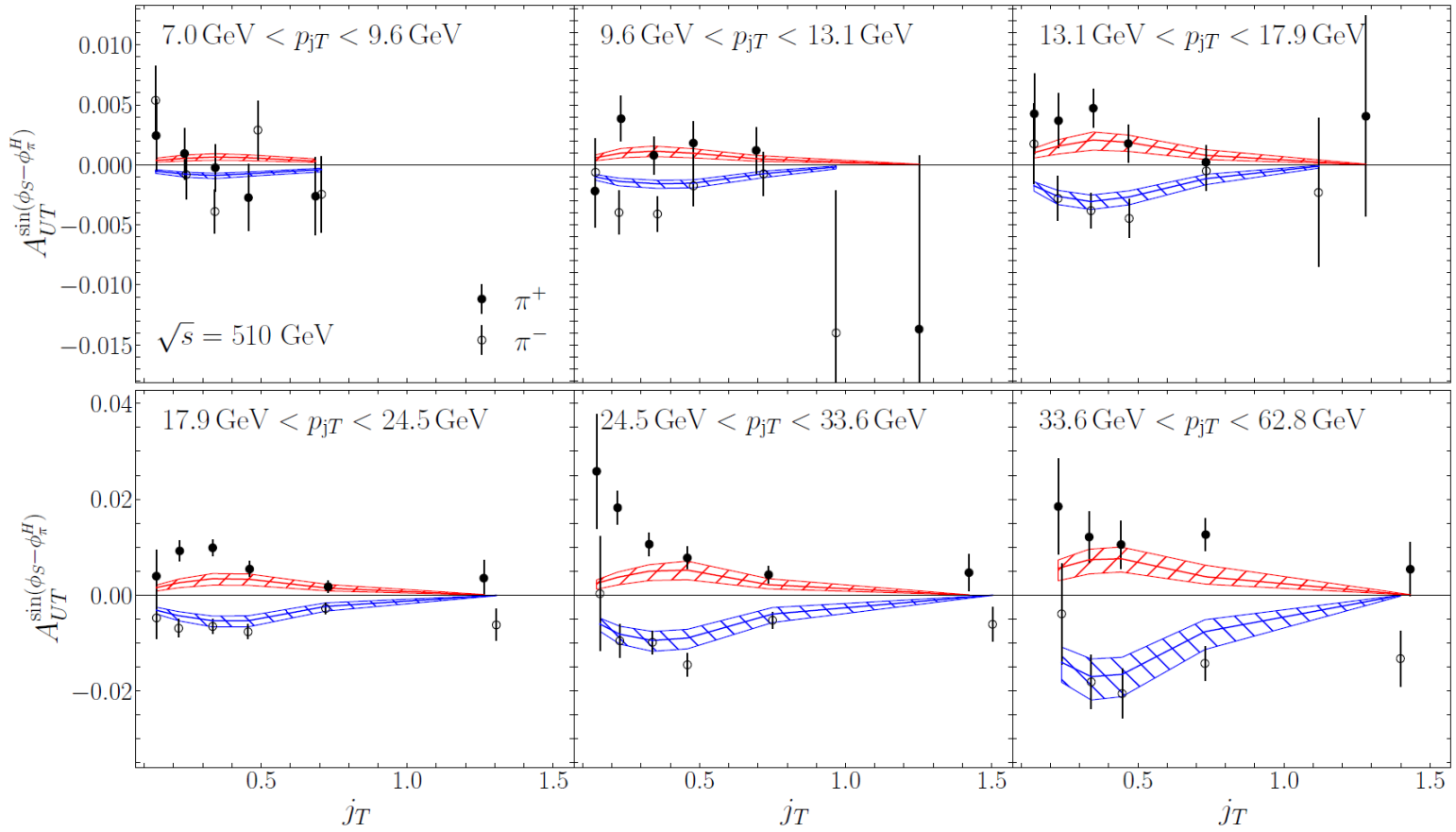
REMARKS I

- No significant energy dependence; good agreement data-theory
- No evidence of TMD-factorization breaking and mild evolution effect
- Discrepancies at moderate and large z: parametrization for the Collins FFs (unfavored FF same z dependence as the unpolarized TMD FF); extraction of the Collins FF limited to $z > 0.2$
- At fixed x_T, transversity probed at $x > x_T$, not covered by SIDIS data.

  access to transversity at large x.
- Importance of a combined fit, including this data set

RESULTS [PREDICTIONS VS. DATA] IV

(510 GeV)

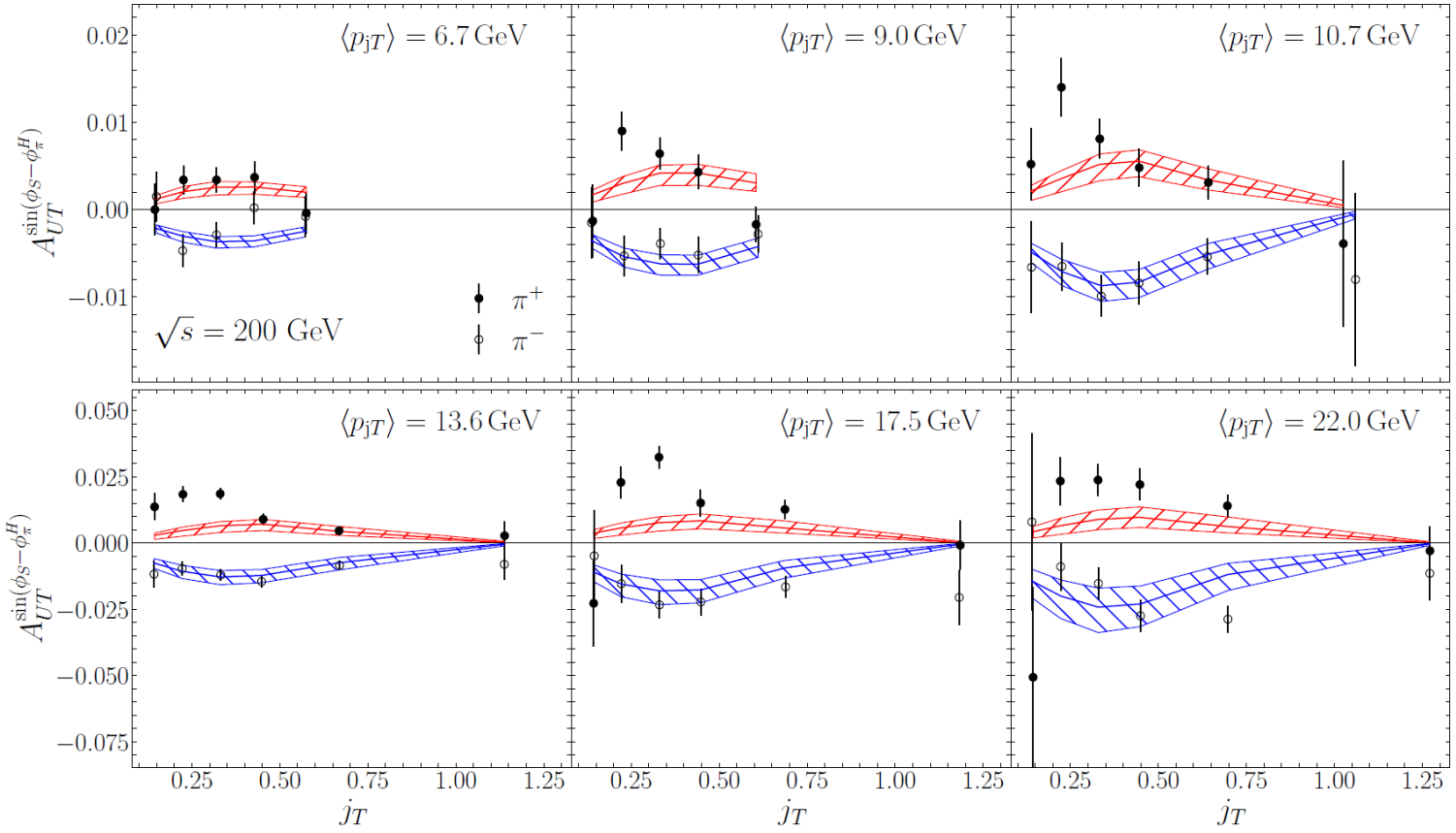


Collins azimuthal asymmetries **vs. j_T,** integrated over z and over η_j, for various **p_{jT} bins.**

STAR data: SPIN2023

RESULTS [PREDICTIONS VS. DATA] IV'

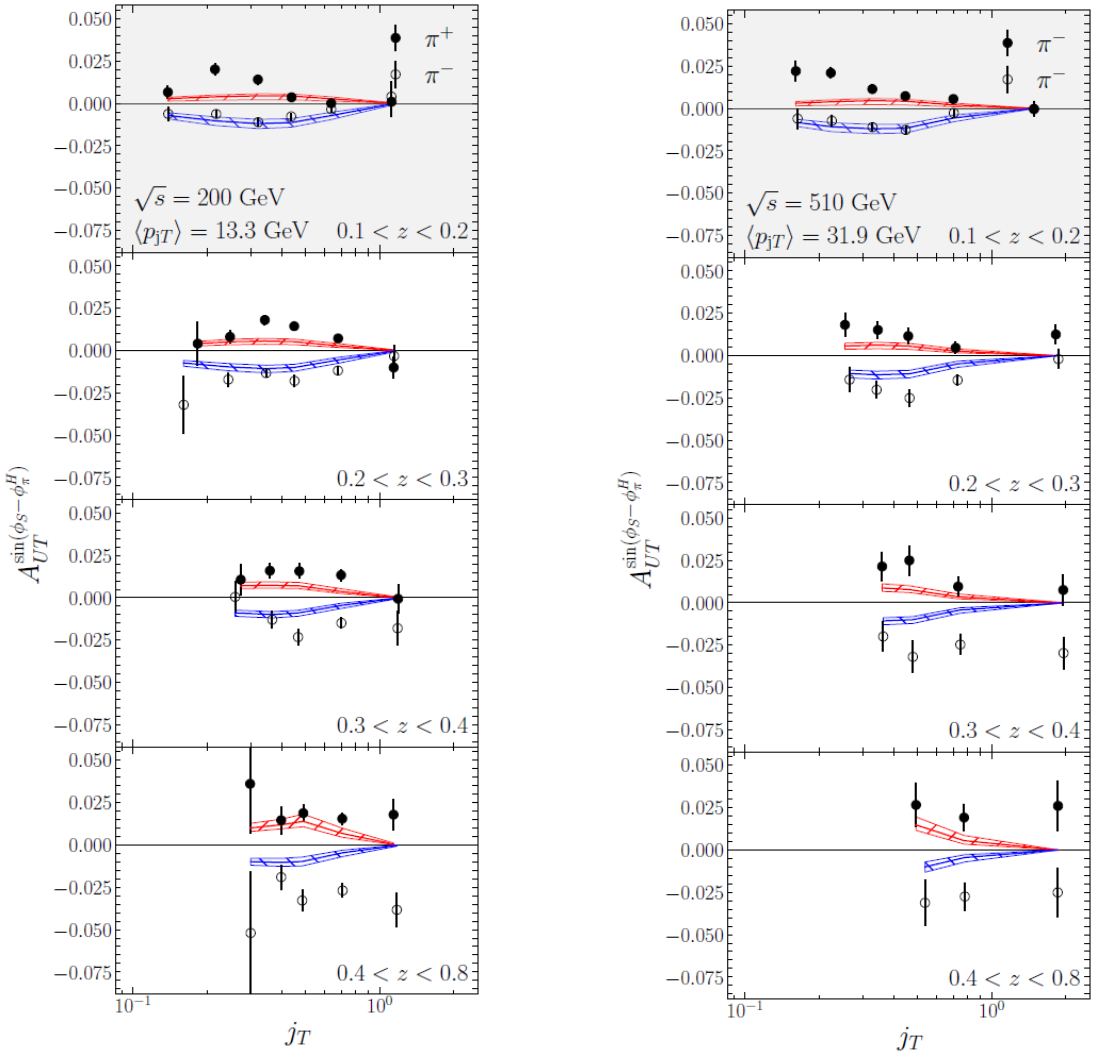
(200 GeV)



Collins azimuthal asymmetries **vs. j_T,** integrated over z and over η_j, for various **p_{jT} bins.**

STAR data: PRD 2022

RESULTS [PREDICTIONS VS. DATA] IV



Collins azimuthal asymmetries **vs. j_T,** integrated over η_j, for various ***z*** **bins.**
STAR data: PRD 2022 (200 GeV), SPIN2023 **(510 GeV)**

REMARKS II

- Good agreement (for a prediction).
- Challenging distributions, useful to learn about the intrinsic transverse momentum in the fragmentation mechanism and its correlation with z.
- Very/too simple parametrization of the j_T dependence for the TMD FFs, a Gaussian-like form, with no flavor and/or z dependences: to be reconsidered.
- larger z values: need of a more flexible parametrization.
- j_T behavior for different p_{jT} bins: mild evolution effects with the hard scale.
- more educated choice for the j_T dependence in TMD FFs and/or a more accurate treatment of the jet fragmentation mechanism.
- Importance of a combined fit, including this data set.

$$p^{\uparrow}(p)\,\ell(p_\ell) \to \mathrm{jet}(p_\mathrm{j})\,\pi(p_\pi) + X$$

Preliminary results

COLLINS ASYMMETRY

$$\frac{E_{\rm j}\, d\sigma^{p(S,\phi_S)\,\ell\to {\rm jet}\,\pi(\phi_\pi^H)\,X}}{d^3\boldsymbol{p}_{\rm j} dz d^2\boldsymbol{p}_{\perp\pi} d\phi_S}\,.$$

$$A_N^{\sin(\phi_S-\phi_\pi^H)}(\boldsymbol{p}_{\rm j}, z, p_{\perp\pi}) = 2\,\frac{\int d\phi_S\, d\phi_\pi^H\, \sin(\phi_S-\phi_\pi^H)\,[\,d\sigma(\phi_S,\phi_\pi^H) - d\sigma(\phi_S+\pi,\phi_\pi^H)\,]}{\int d\phi_S\, d\phi_\pi^H\,[\,d\sigma(\phi_S,\phi_\pi^H) + d\sigma(\phi_S+\pi,\phi_\pi^H)\,]}$$

Formally identical, but less channels involved w.r.t the *pp* case

$$p^{\uparrow}(p)\,\ell(p_\ell) \to \mathrm{jet}(p_\mathrm{j})\,\pi(p_\pi) + X$$

LO $\quad q\ell \to q\ell$

$$h_1^q(x) \otimes d\Delta\hat{\sigma}^{q\ell\to q\ell} \otimes H_1^{\perp q}(z, j_T)$$

$$d\Delta\hat{\sigma}^{q\ell\to q\ell} = -e_q^2 \frac{\hat{s}\hat{u}}{\hat{t}^2}$$

$$x = \frac{x_T e^{\eta_\mathrm{j}}}{2 - x_T e^{-\eta_\mathrm{j}}}$$

$$\hat{t} = -Q^2 = -x\sqrt{s}\, p_{\mathrm{j}T} e^{-\eta_\mathrm{j}}$$

Large $p_{\mathrm{j}T}$ implies large Q^2
(at not large positive η_j)

$$p^{\uparrow}(p)\,\ell(p_\ell) \to \mathrm{jet}(p_\mathrm{j})\,\pi(p_\pi) + X$$

But final lepton is not observed → also collinear…
quasireal photon exchange [WW approximation]

$$d\sigma(\phi_S, \phi_\pi^H) = d\sigma_{\mathrm{LO}}(\phi_S, \phi_\pi^H) + d\sigma_{\mathrm{WW}}(\phi_S, \phi_\pi^H)$$

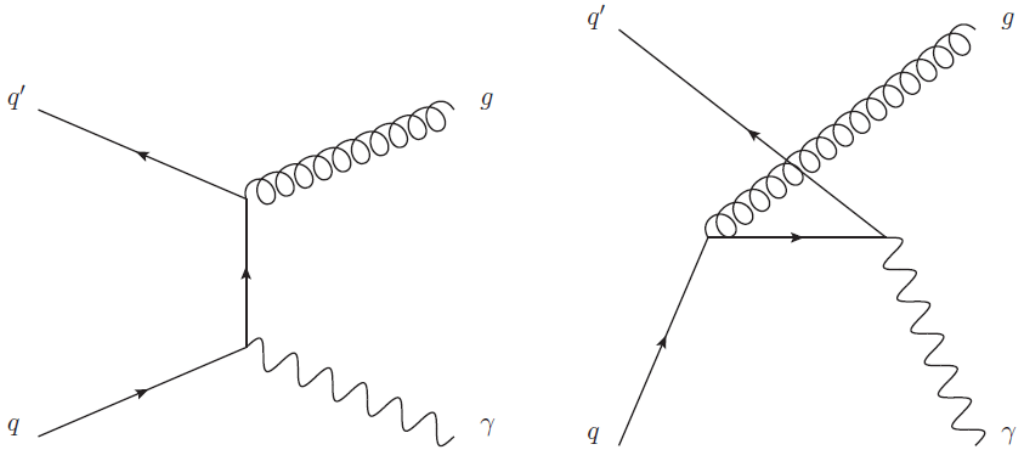
$$d\sigma_{\mathrm{WW}}(p\ell \to \mathrm{jet}\,\pi\,X) = \int dy\, f_{\gamma/\ell}(y)\, d\sigma(p\gamma \to \mathrm{jet}\,\pi\,X)$$

lepton as a source of quasi-real photons
$\ell \to \ell\gamma$ final lepton almost collinear

$$f_{\gamma/\ell}(y) = \frac{\alpha_{\mathrm{em}}}{2\pi}\left[\frac{1+(1-y)^2}{y}\ln\frac{Q^2_{\mathrm{max}}}{Q^2_{\mathrm{min}}(y)} + 2m_\ell^2 y\left(\frac{1}{Q^2_{\mathrm{max}}} - \frac{1}{Q^2_{\mathrm{min}}(y)}\right)\right]$$

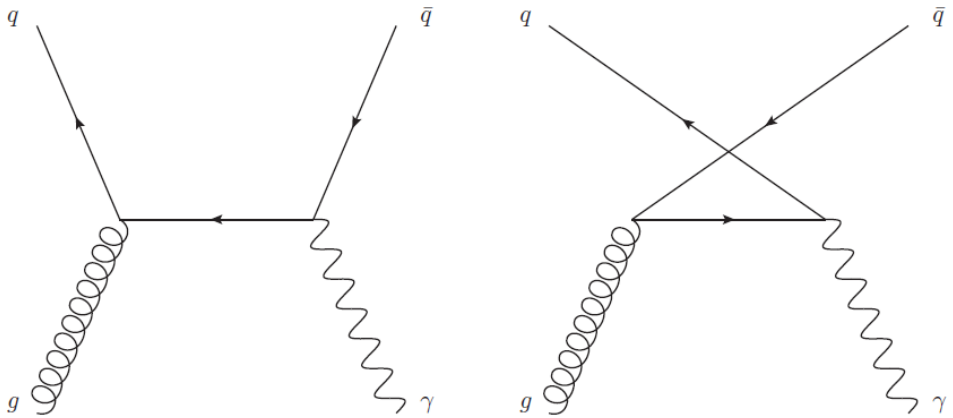
$$Q^2_{\mathrm{min}}(y) = m_\ell^2 y^2/(1-y) \qquad Q^2_{\mathrm{max}} = 1\mathrm{GeV}^2$$

$q\gamma \to qg$



$$d\hat{\sigma}^{q\gamma\to qg} = -\frac{4}{3}e_q^2\frac{\hat{s}^2+\hat{u}^2}{\hat{s}\hat{u}}$$

$g\gamma \to q\bar{q}$



$$d\hat{\sigma}^{g\gamma\to q\bar{q}} = e_q^2\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}$$

$\hat{u}$ dependence (absent in the LO term)

Numerator: only one channel $q\gamma \to qg$

$$f_{\gamma/\ell}(y)\otimes h_1^q(x)\otimes d\Delta\hat{\sigma}^{q\gamma\to qg}\otimes H_1^{\perp q}(z,j_T)$$

$$d\Delta\hat{\sigma}^{q\gamma\to qg} = \frac{8}{3}e_q^2$$

EIC KINEMATICS

$\sqrt{s}$ = 45, 105, 141 GeV

p_{jT}: [5:15], [10:30], [10:40] GeV

$|\eta_j| \leq 0.5$, 1, 1

$x_T = 2p_{jT}/\sqrt{s}$ comparable ranges

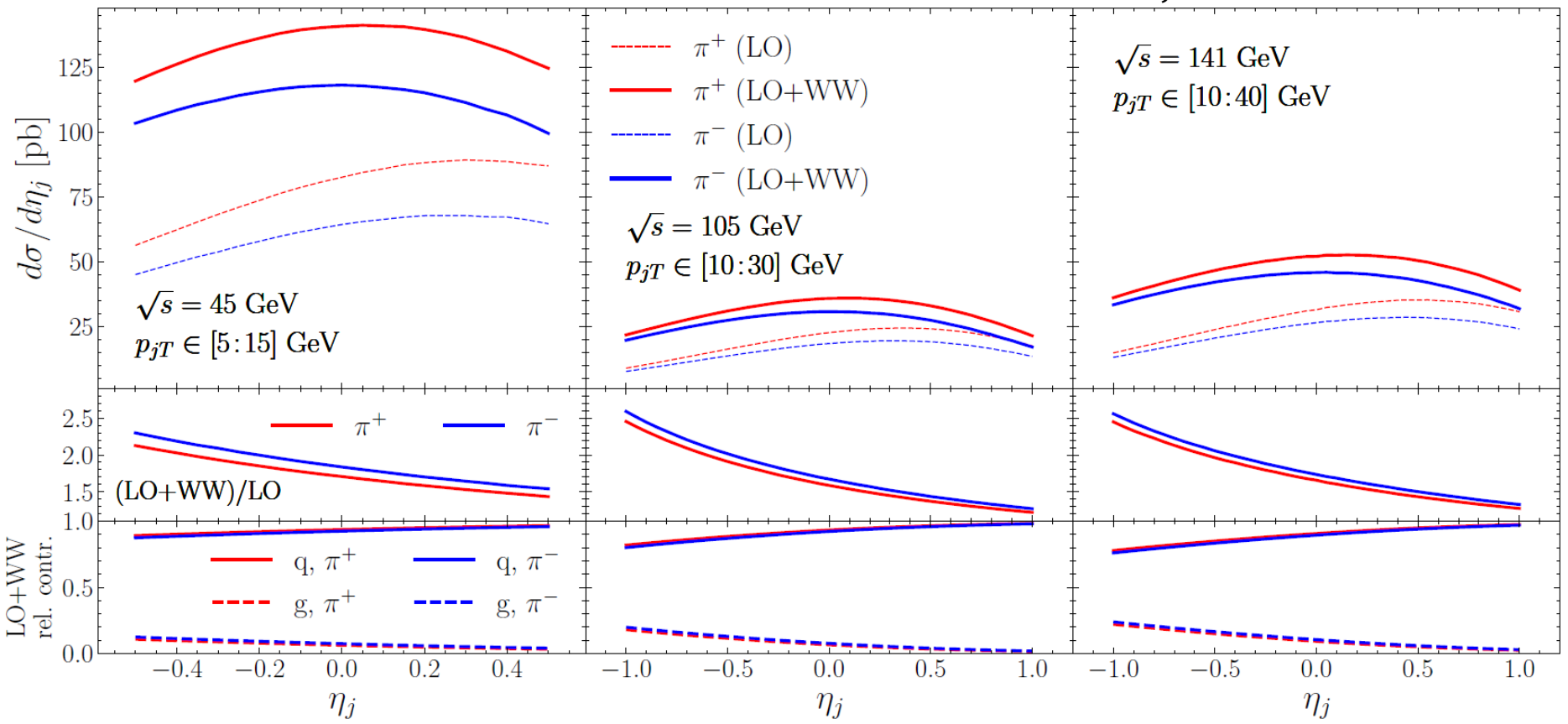
j_T:[0.1:1.5] GeV pion transverse momentum w.r.t jet

z: [0.1: 0.8]

R: [0.05:0.6] jet-cone radius

UNPOLARIZED CROSS SECTIONS

Unpol. cross sections as a function of η_j



Sizeable WW contr.

(LO+WW)/LO around 2-1.5

Quark dominance!!!

backward region: gluon-initiated channels, entering only via the WW term, plays some role

REMARKS I

- Marginal role played by gluons in the denominator: quark dominance

Numerator of the azimuthal asymmetry: only quark transversity (no gluon).

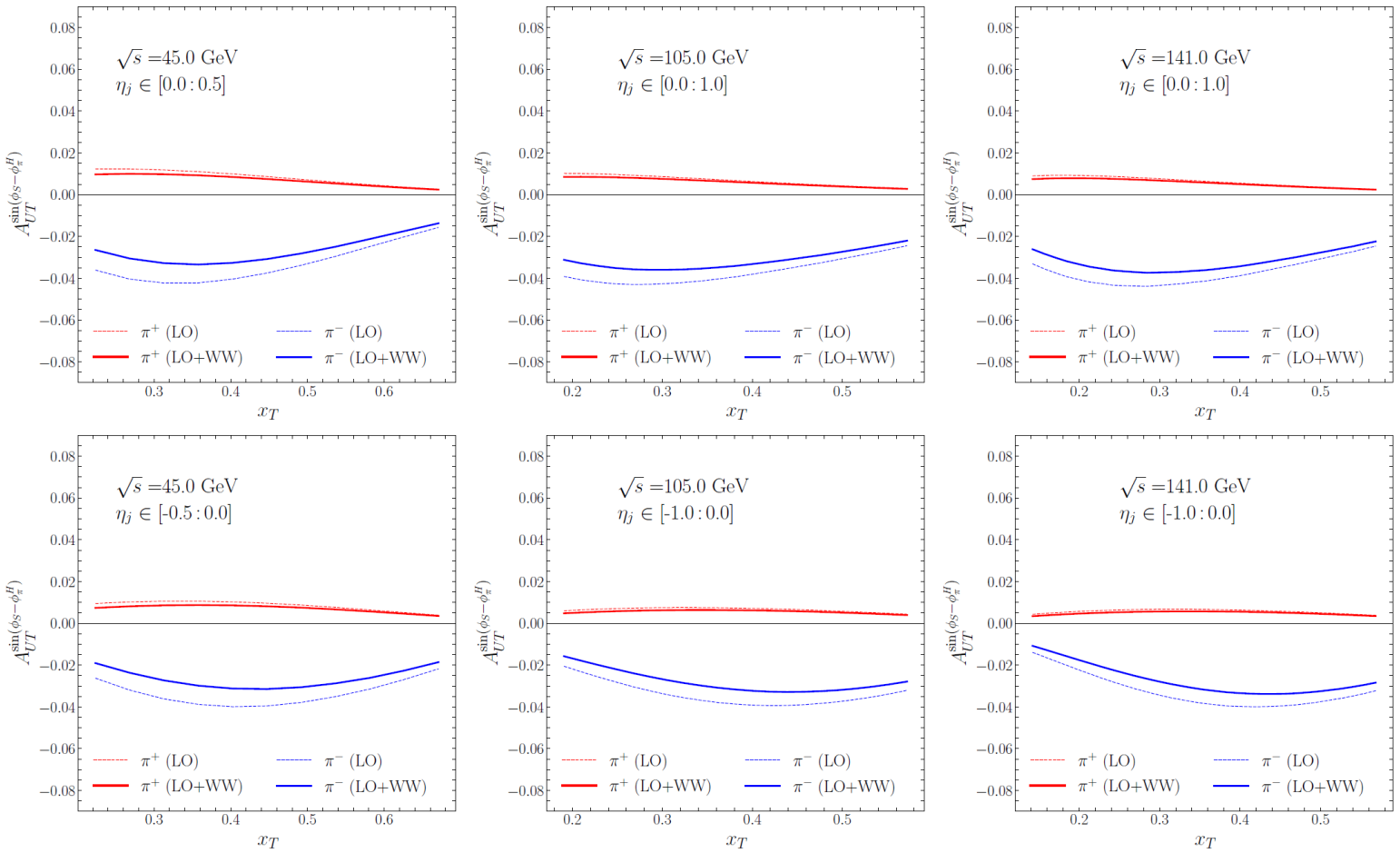
clear access to this distribution

In other words: expected sizeable asymmetries and/or more sensitive to h_1

- Important: azimuthal asymmetry in *pp* collisions,

denominator strongly affected/dominated by gluons: lower asymmetries

AZIMUTHAL ASYMMETRIES VS. x_T



- [LO+WW] results

  π^+ estimates around 1%,

  π^- larger values, in size:

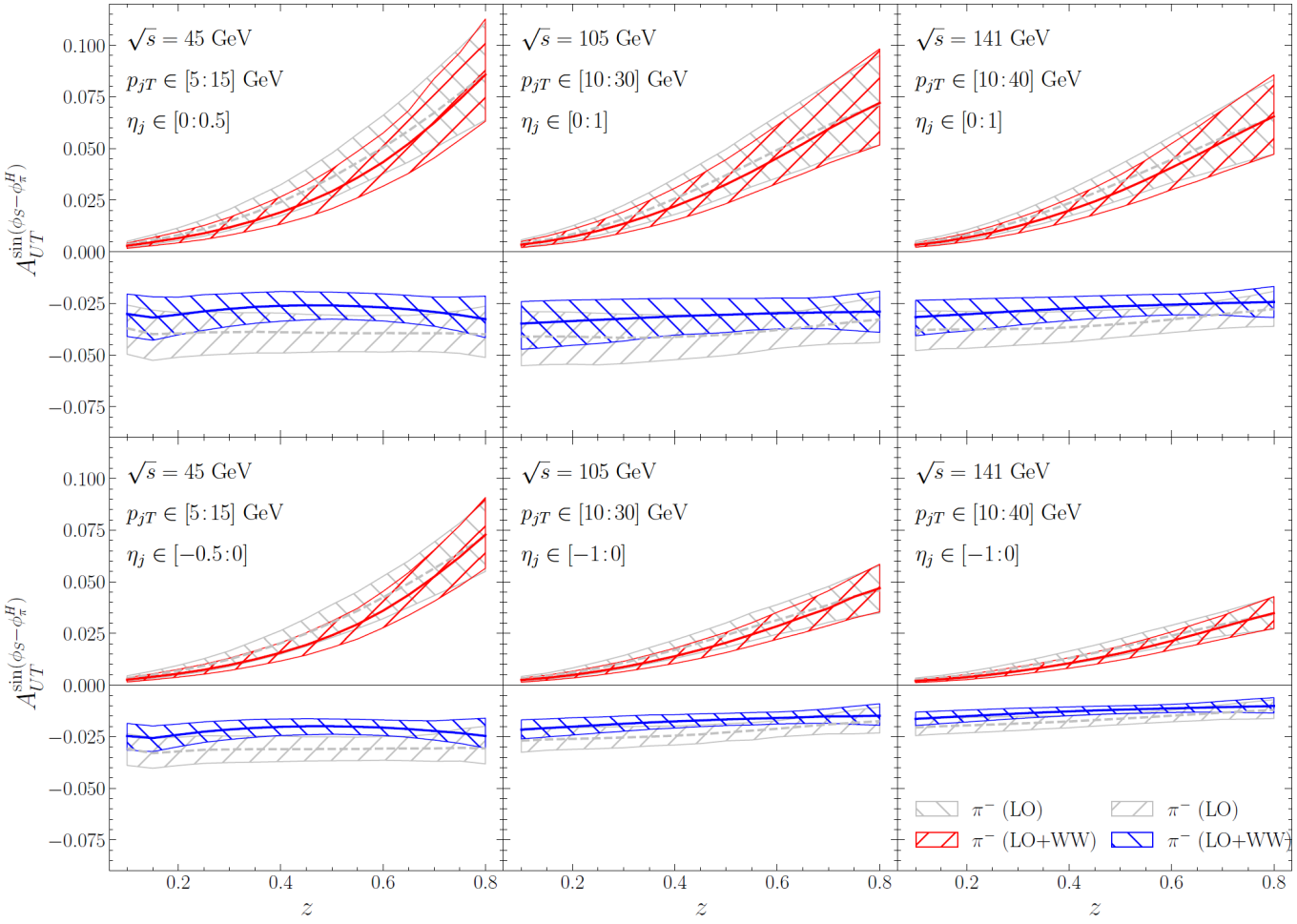
  3-4% forward region

  1-2% backward region.

- role of gluons in the backw. region.
- large size of the unfavored Collins FF, when integrated over z from the lowest value considered, coupled to transversity for up quarks: larger effect for π^- .
- Small x_T & backward region: low-x values, < 0.1 : access to transversity for sea quarks

$$x_{\min} = \frac{x_T e^{\eta_j}}{2 - x_T e^{-\eta_j}}$$

AZIMUTHAL ASYMMETRIES VS. *z*

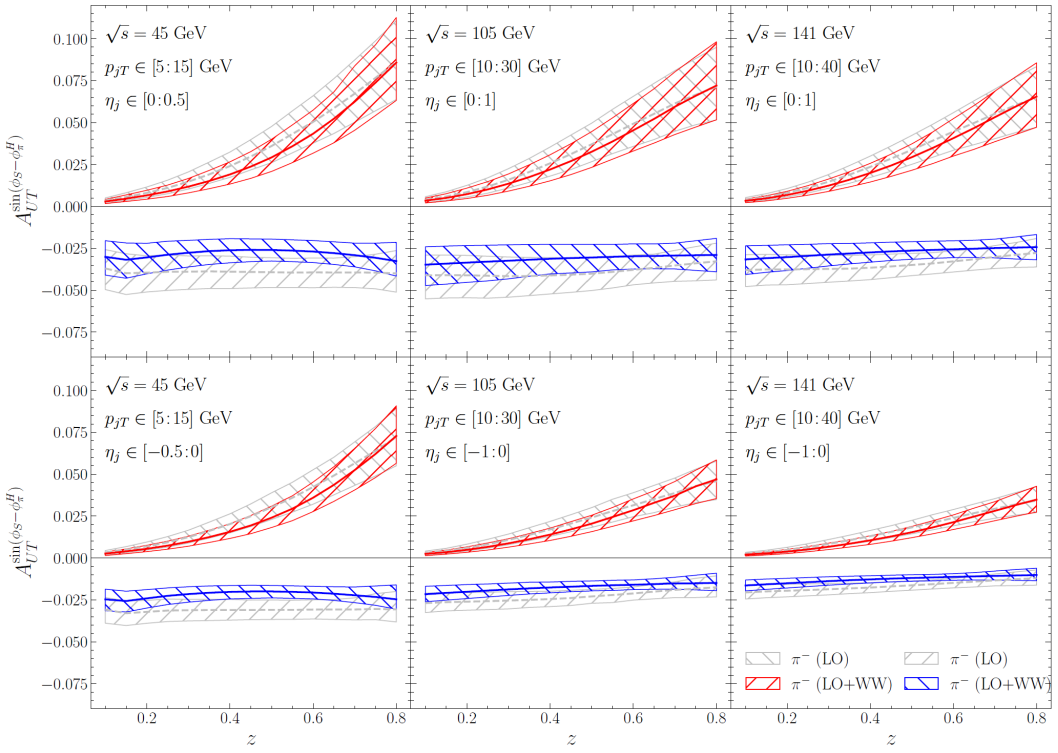


- π^+: increasing as a function of z around 8% in the forward region, around 4-5% in the backward region

- π^- : almost flat behavior around 3-4%

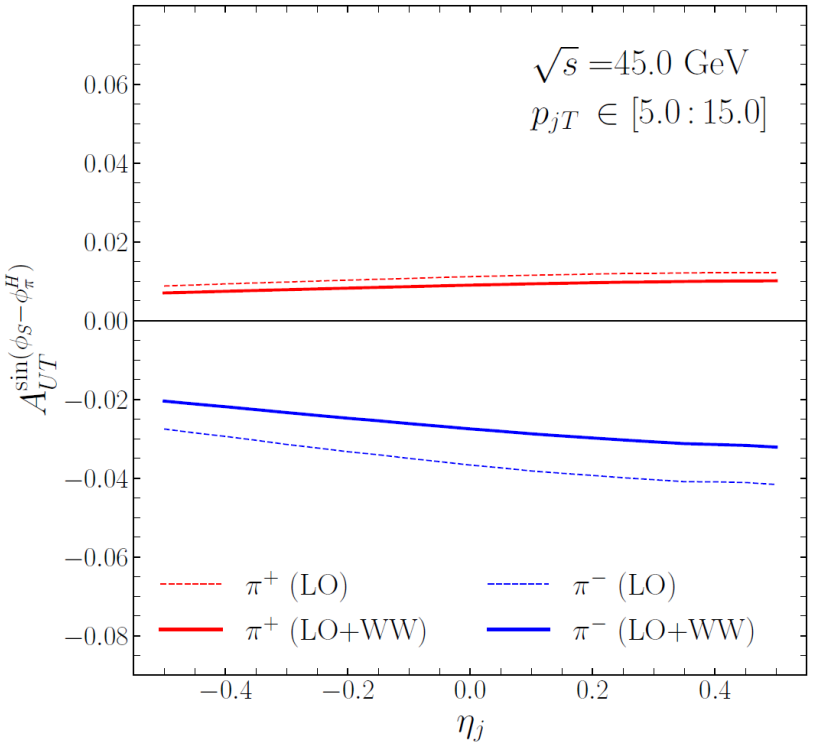
forward region: quark-initiated channel dominance (at lower energies also in the backward region).
This accounts for the differences between the forward and the backward regions at intermediate/large energies, where gluons start playing some role;

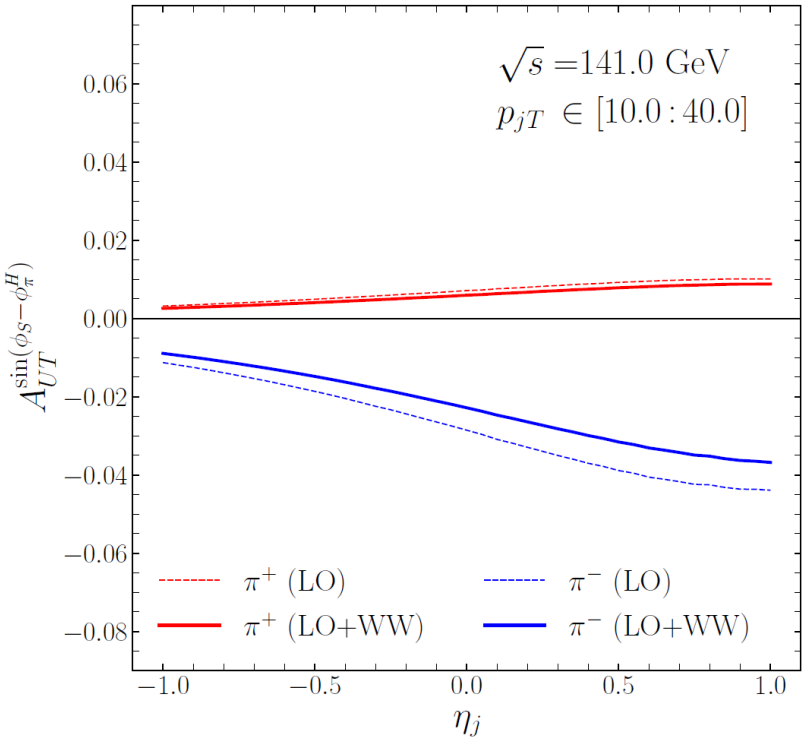
AZIMUTHAL ASYMMETRIES VS. *z*

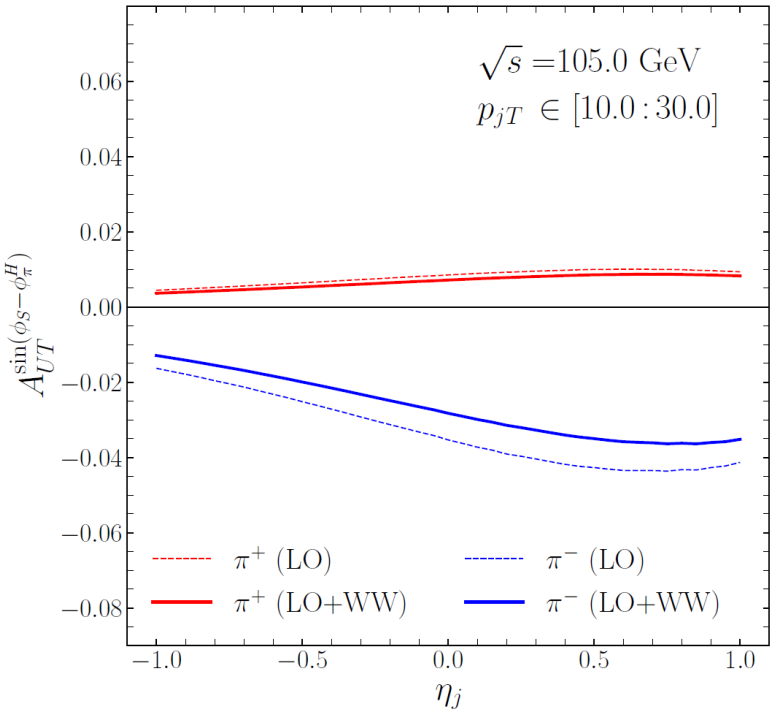


- (favored) unfavored Collins FFs at large *z* have (almost) exactly the same behavior as the corresponding unpolarized FFs, with the unfavored ones largely suppressed;
- dominance of the up-quark transversity for this process (charge factor).
- Large *z* - π^+ production: dominated by the transversity for the up quark times the favored Collins FF (both large), while for π^- the two terms entering the numerator involve always one suppressed factor.
- low-z: suppression of the favored Collins FF implies almost negligible π^+ azimuthal asymmetries.
- Azimuthal asymmetries in *pp* processes: gluons (in the denominator) play a non negligible role in all kinematic regions and no such up-quark dominance appears. This leads to much smaller absolute values (around 2%).

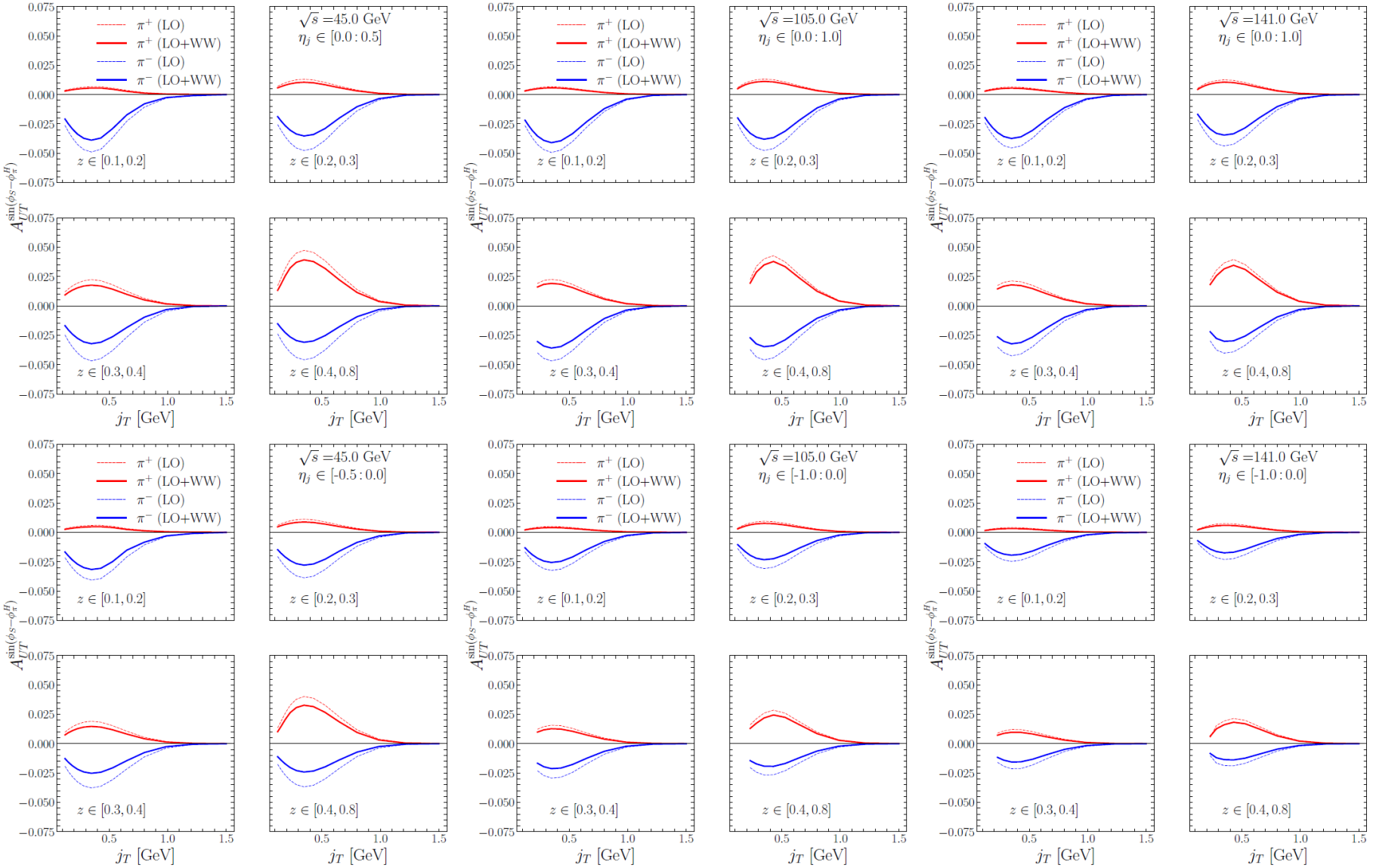
AZIMUTHAL ASYMMETRIES VS. η_J







Azimuthal asymmetries as a function of η_j



Azimuthal asymmetries as a function of j_T at fixed z bins

CONCLUDING REMARKS

- azimuthal asymmetries in h-jet in pp collisions: tool to "test" universality of TMD FFs and TMD factorization
- TMD scheme with TMD effects only in the fragmentation mechanism (advantages, direct access, two-scale processes)
- general agreement both for the Collins asymmetries (*and for the transverse Λ polarization*) in *pp* collisions
- azim. asym. for lepton-proton at EIC: sizable and testable, more sensitive to transversity valence and sea quarks, flavor decomposition of the Collins FFs
- Open theoretical issues in hadron-in-jet production:

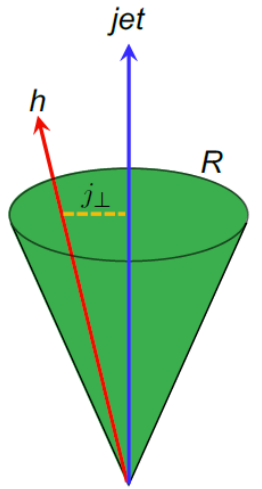
- Factorization breaking effects? TMD Factorization? Effective model?
- jet fragmentation mechanism beyond LO accuracy, jetTMDFFs, TMD FF parametrizations

THANKS for the ATTENTION

BACK-UP SLIDES

THE SEMI-INCLUSIVE TMD FRAGMENTING JET FUNCTION

$$\frac{d\sigma^{pp\to(\text{jet }h)X}}{dp_T d\eta dz_h d^2\boldsymbol{j}_\perp} = \sum_{a,b,c} f_a(x_a,\mu)\otimes f_b(x_b,\mu)\otimes H^c_{ab}(x_a,x_b,\eta,p_T/z,\mu)\otimes \mathcal{G}^h_c(z,z_h,\omega_J R,\boldsymbol{j}_\perp,\mu)$$



$$\mathcal{G}^h_c(z,z_h,\omega_J R,\boldsymbol{j}_\perp,\mu) = \mathcal{C}_{c\to i}(z,\omega_J R,\mu)\int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\,\boldsymbol{j}_\perp\cdot\boldsymbol{b}/z_h}\hat{\mathcal{D}}_{h/i}(z_h,\boldsymbol{b};\mu_J)$$

$$\mathcal{G}^h_c(z,z_h,\omega_J R,\boldsymbol{j}_\perp,\mu) = \mathcal{C}_{c\to i}(z,\omega_J R,\mu)\,\hat{\mathcal{D}}_{h/i}(z_h,\boldsymbol{j}_\perp;\mu_J)$$

$$\mu_J \sim \omega_J \tan(R/2) \to p_T R$$

$$\hat{\mathcal{D}}_{h/i}(z_h,\boldsymbol{j}_\perp;\mu_J) = \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\,\boldsymbol{j}_\perp\cdot\boldsymbol{b}/z_h}\hat{\mathcal{D}}_{h/i}(z_h,\boldsymbol{b};\mu_J)$$ TMD FF

$$\Lambda_{\rm QCD} \lesssim j_\perp \ll p_T R$$ TMD factorization

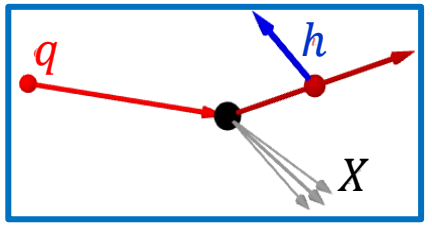
Kang, Liu, Ringer, Xing, JHEP 1711 (2017) 068

$$A(p_A)\, B(p_B) \to \mathrm{jet}(p_\mathrm{j})\, \Lambda^{\uparrow}(p_\Lambda)\, X$$

$$A(p_A)\,B(p_B) \to \mathrm{jet}(p_\mathrm{j})\,\Lambda^\uparrow(p_\Lambda)\,X$$

UD, Gamberg, Murgia, Zaccheddu '24

$$P_T^\Lambda(\boldsymbol{p}_\mathrm{j}, \xi, \boldsymbol{p}_{\perp\Lambda}) = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{d\Delta\sigma}{d\sigma_\mathrm{unp}}$$



- Polarization transverse w.r.t. the jet-Λ plane

$$\Delta D_{\Lambda^\uparrow/c}(\xi, p_{\perp\Lambda}) = \frac{p_{\perp\Lambda}}{\xi m_\Lambda} D_{1T}^{\perp c}(\xi, p_{\perp\Lambda})$$

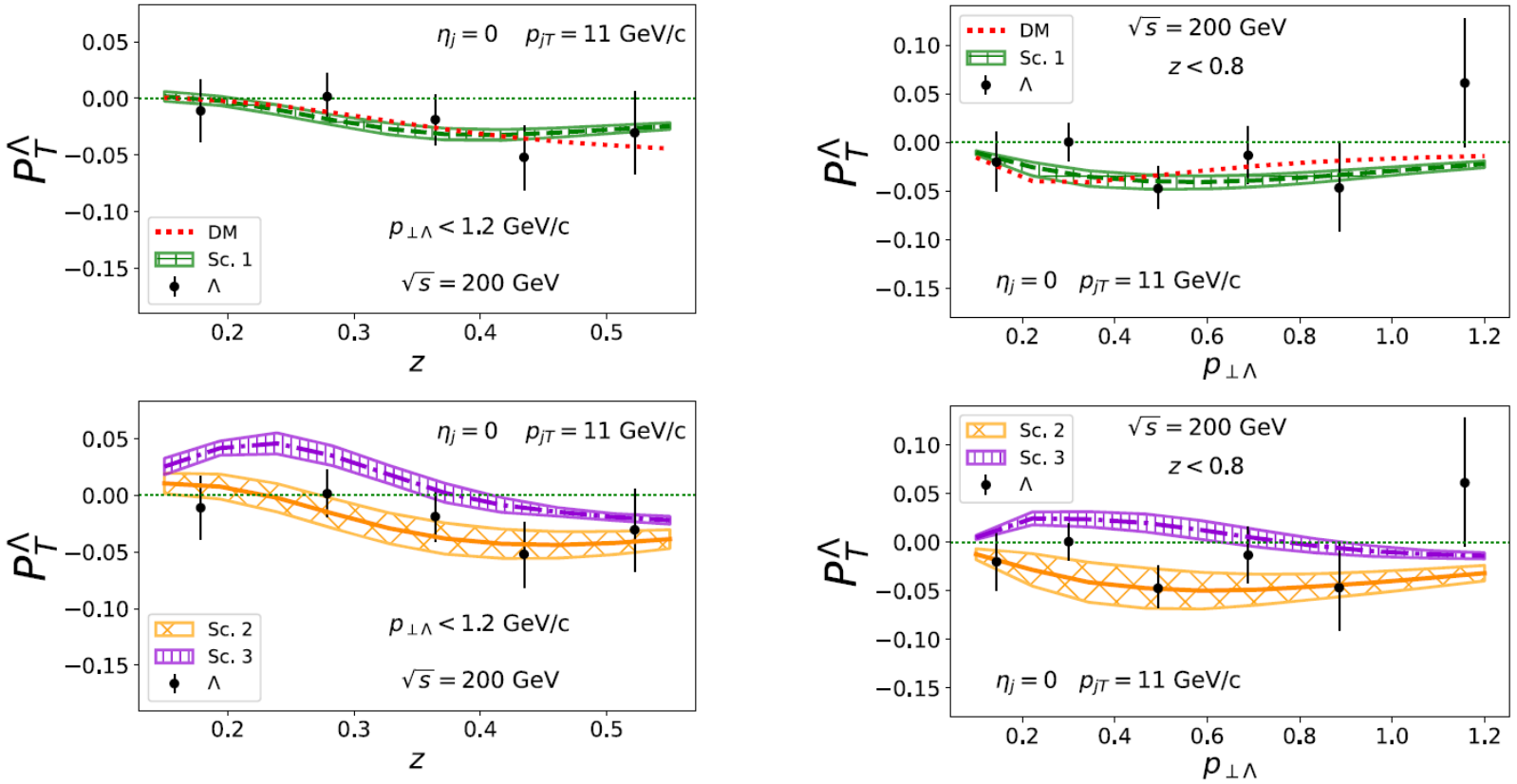
- Numerator involves the polarizing FF

- similar study/results *[Kang, Lee, Zhao '20]*

- Extraction within **CSS framework at NLL accuracy** from Belle $e^+e^- \to h_1\ h_2 + X$ data *[UD, Gamberg, Murgia, Zaccheddu '22, '23]*

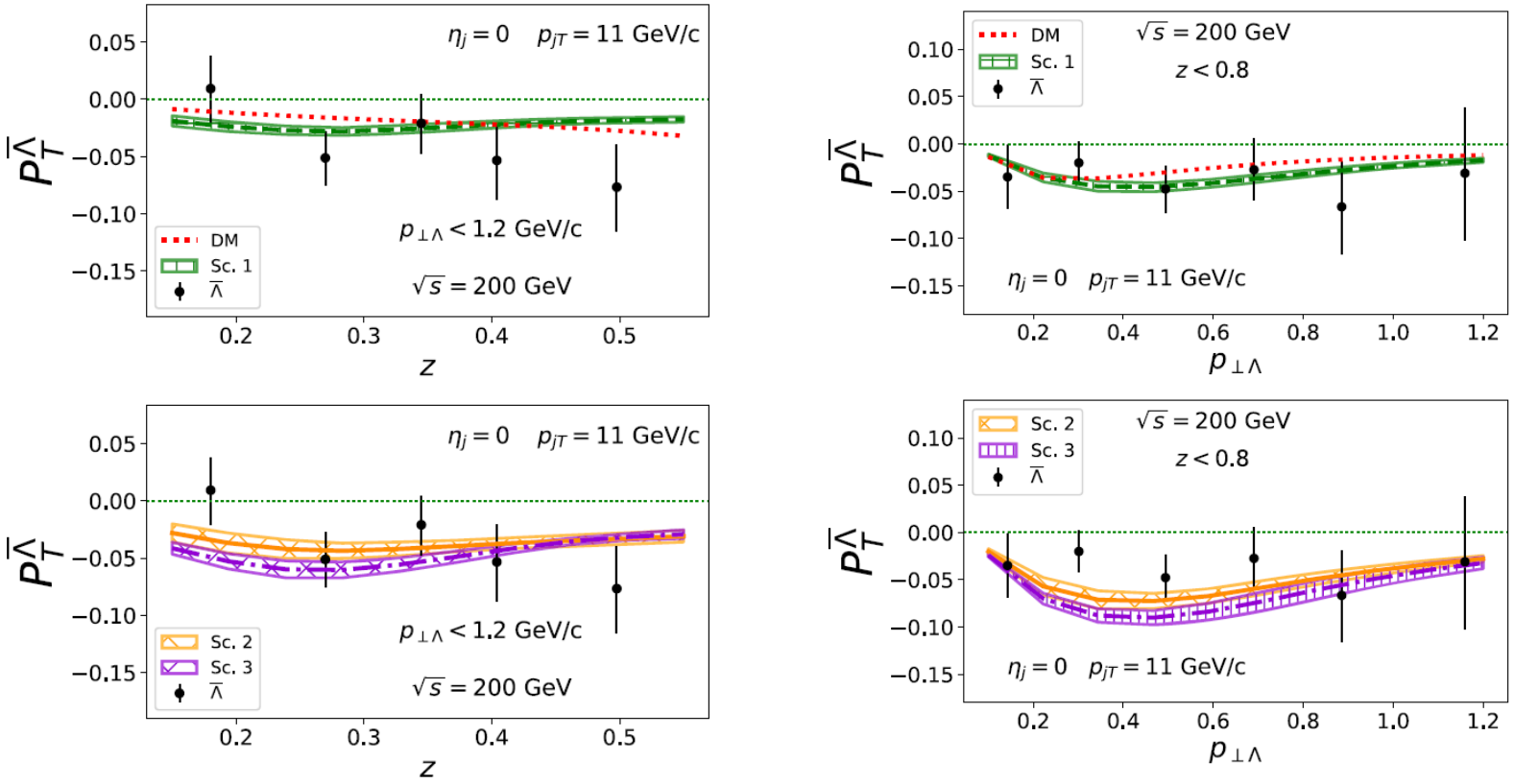
- STAR data *[Gao et al. @SPIN2023]*

RESULTS: Λ



Transverse Λ polarization vs. z (left) $p_{\perp\Lambda}$ (right) for different scenarios

RESULTS: $\overline{\Lambda}$



Transverse $\overline{\Lambda}$ polarization vs. z (left) $p_{\perp\Lambda}$ (right) for different scenarios

UNCERTAINTY BANDS

Compression procedure

We adopt as an indicator the Welch's t-statistic, defined as

$$t = \frac{\mu_a - \mu_{a'}}{\sqrt{\frac{\sigma_a^2}{N_{\text{set}}^a} + \frac{\sigma_{a'}^2}{N_{\text{set}}^{a'}}}},$$

which quantifies the difference of the arithmetical means of two samples with unequal variances and sizes. One has to verify that $|t|$ is such that the corresponding p-values are $\gtrsim 0.1$. Provided this condition holds, one can conclude that the sampled distribution and the original distribution are statistically equivalent. Notice that we sample directly in the parameter space.