Transverse Momentum Moments

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MINISTERIO DE CIENCIA E INNOVACIÓN
Financiado por la Unión Europea NextGenerationEU
Plan de Recuperación, Transformación y Resiliencia
AGENCIA ESTATAL DE INVESTIGACIÓN

Outline

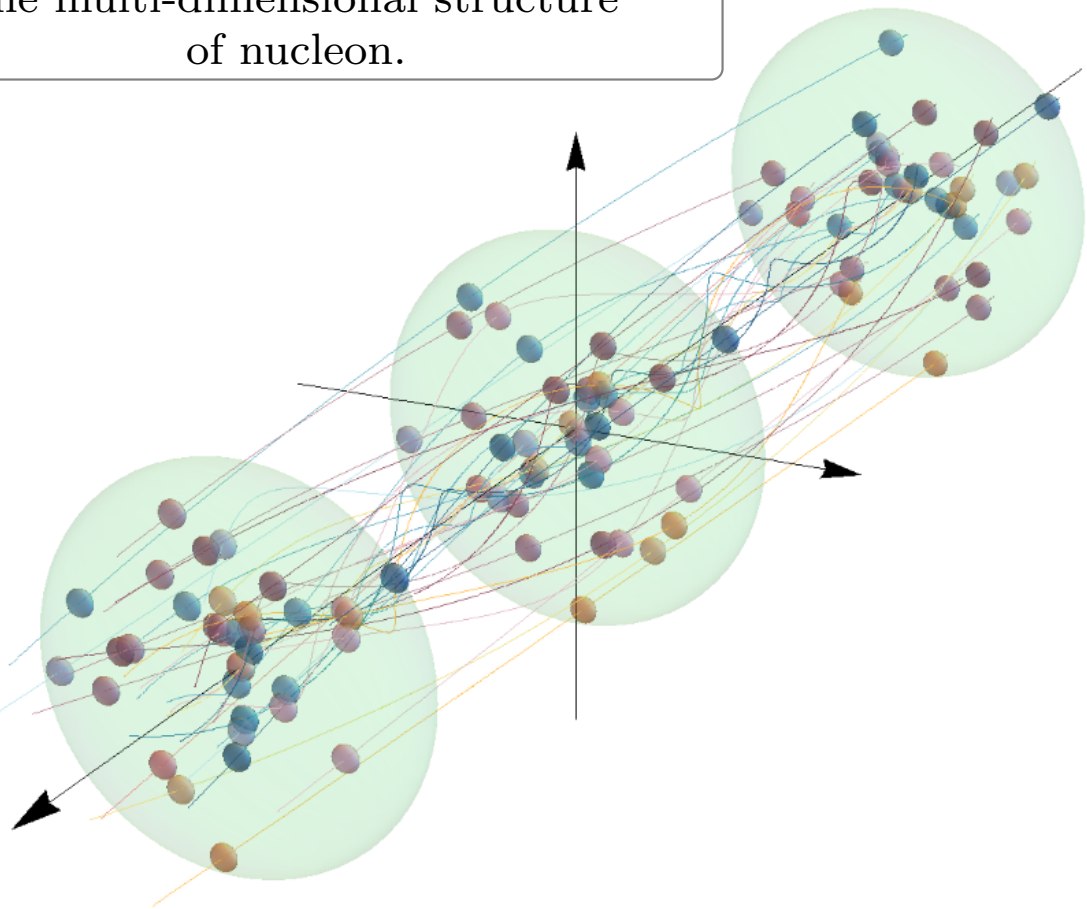
Transverse Momentum Moments (TMM)
from 3D to 1D

$$\int d^2\mathbf{k}_T F(x, k_T)? =? f(x)$$

- OPE vs. integration
- Scheme dependence (TMD-scheme vs. $\overline{\text{MS}}$-scheme)
- What have we learned?

Hadron is a 3D object

Nucleon tomography aims to explore the multi-dimensional structure of nucleon.



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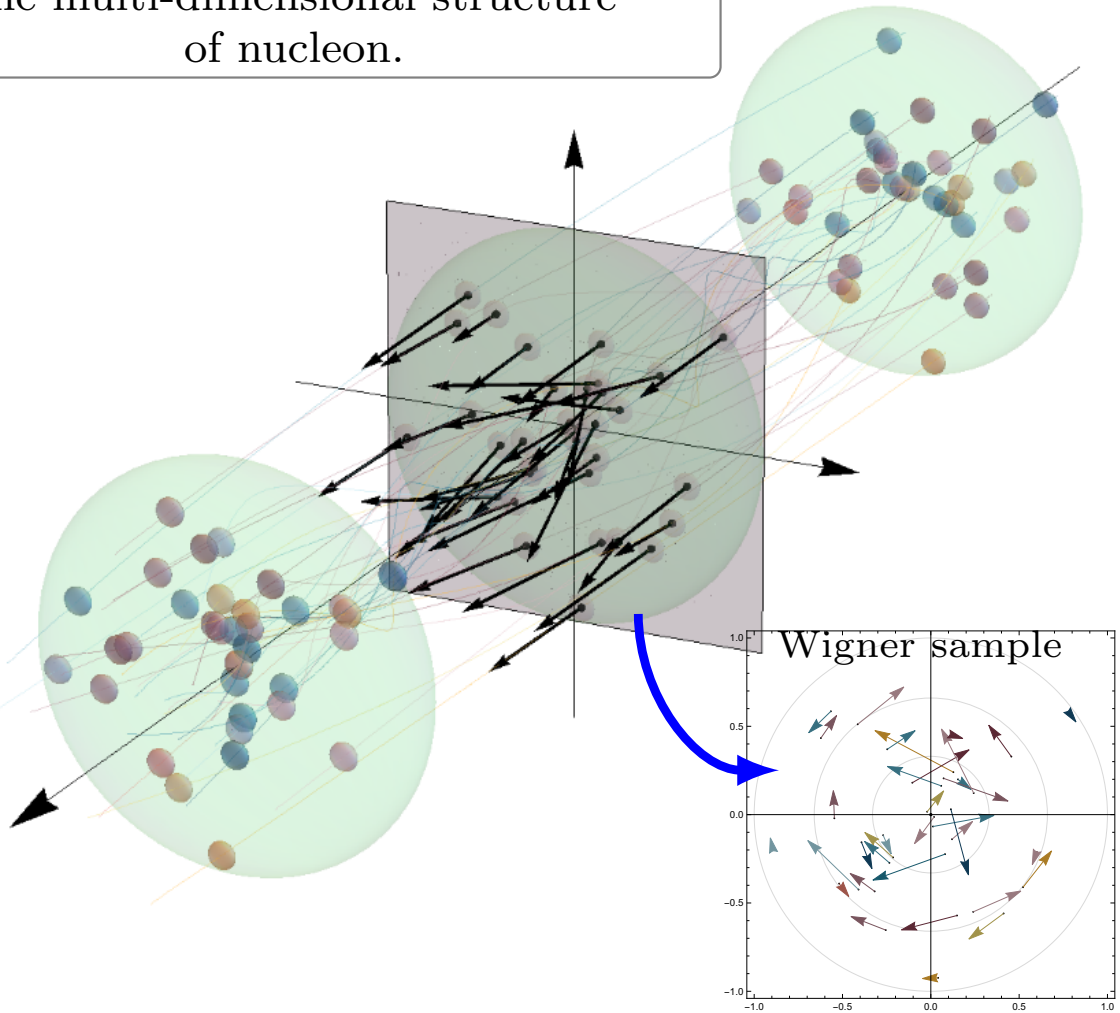

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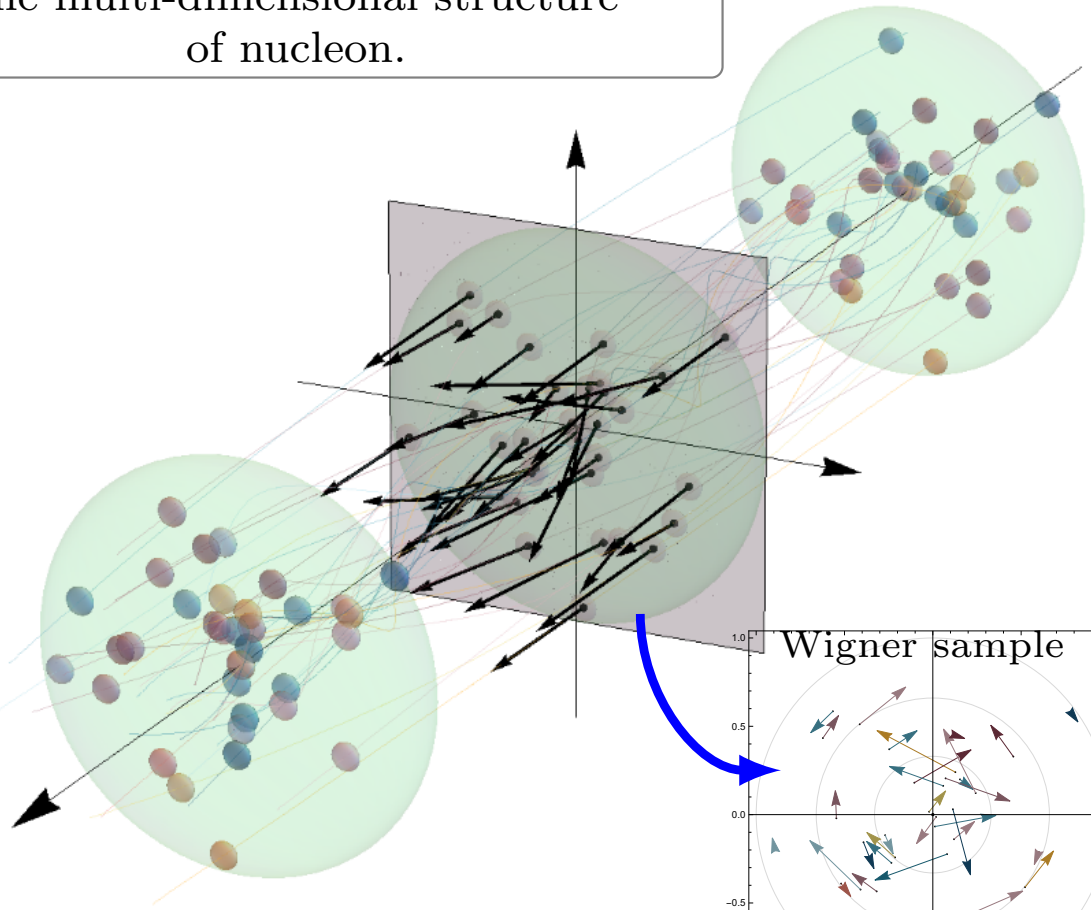
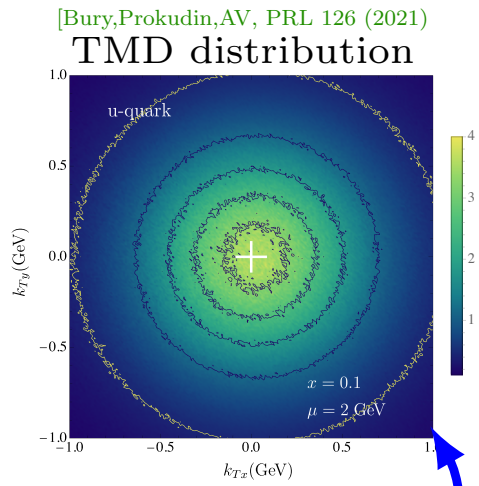
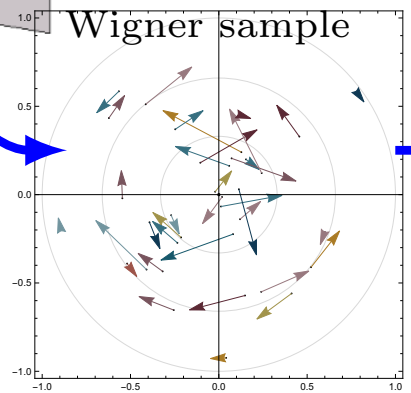
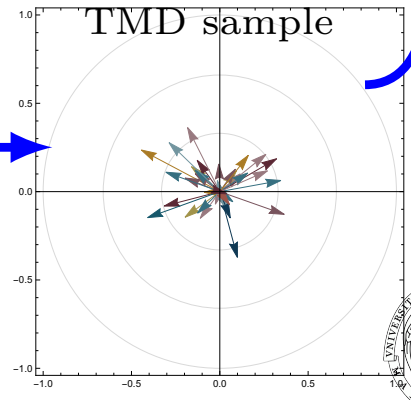


[Bury,Prokudin,AV, PRL 126 (2021)







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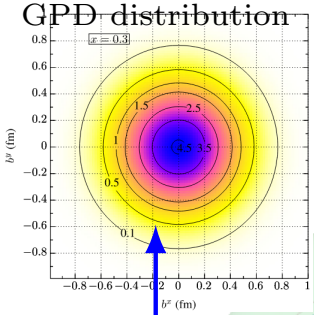
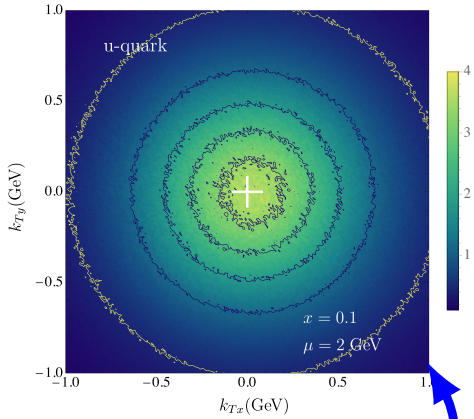
[Hashamipour,et al, PRD 102 (2020)]

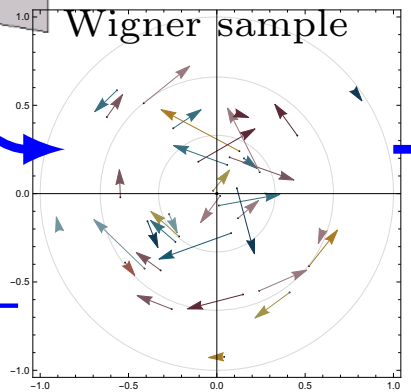
GPD distribution

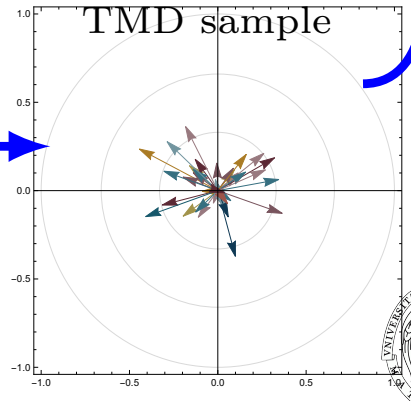


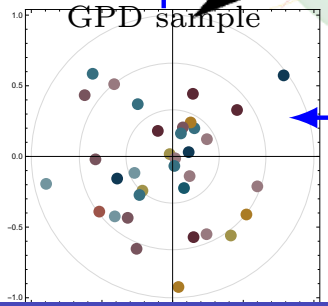
[Bury,Prokudin,AV, PRL 126 (2021)]

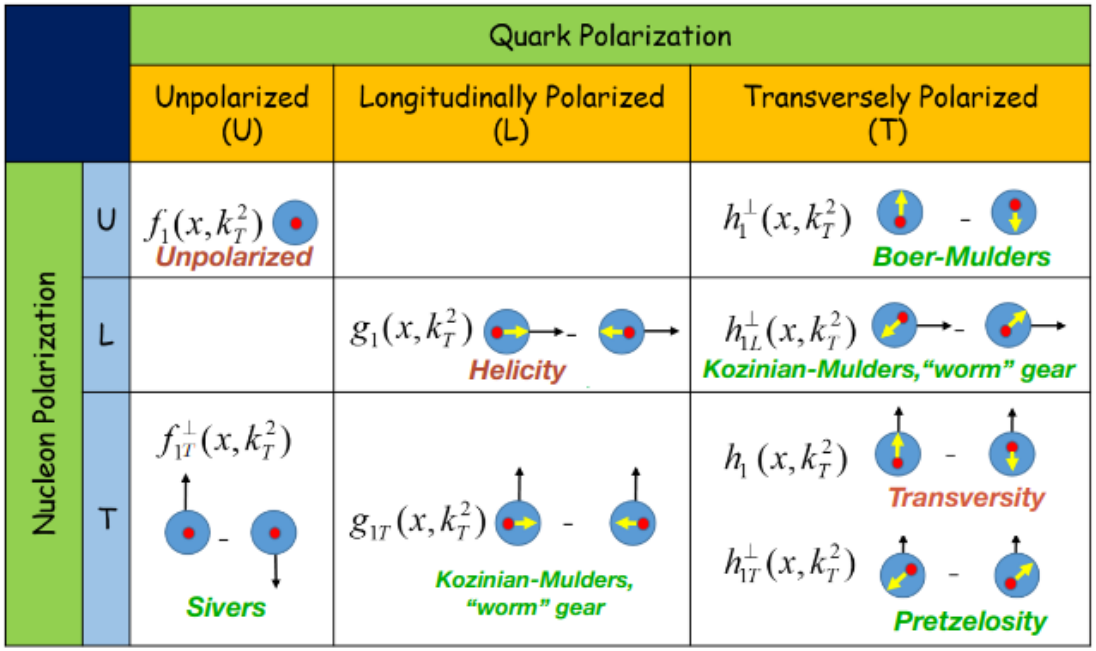
TMD distribution







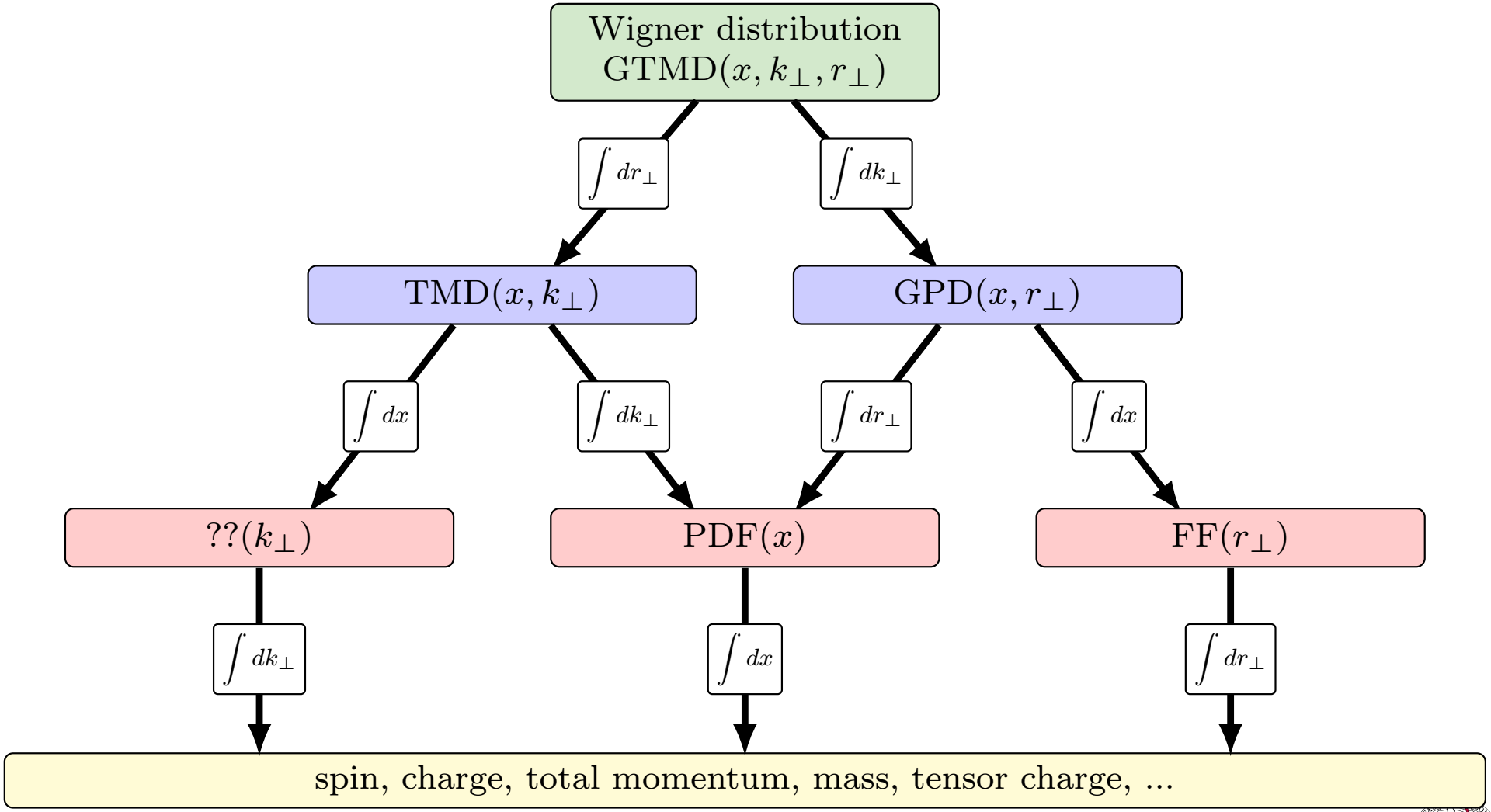


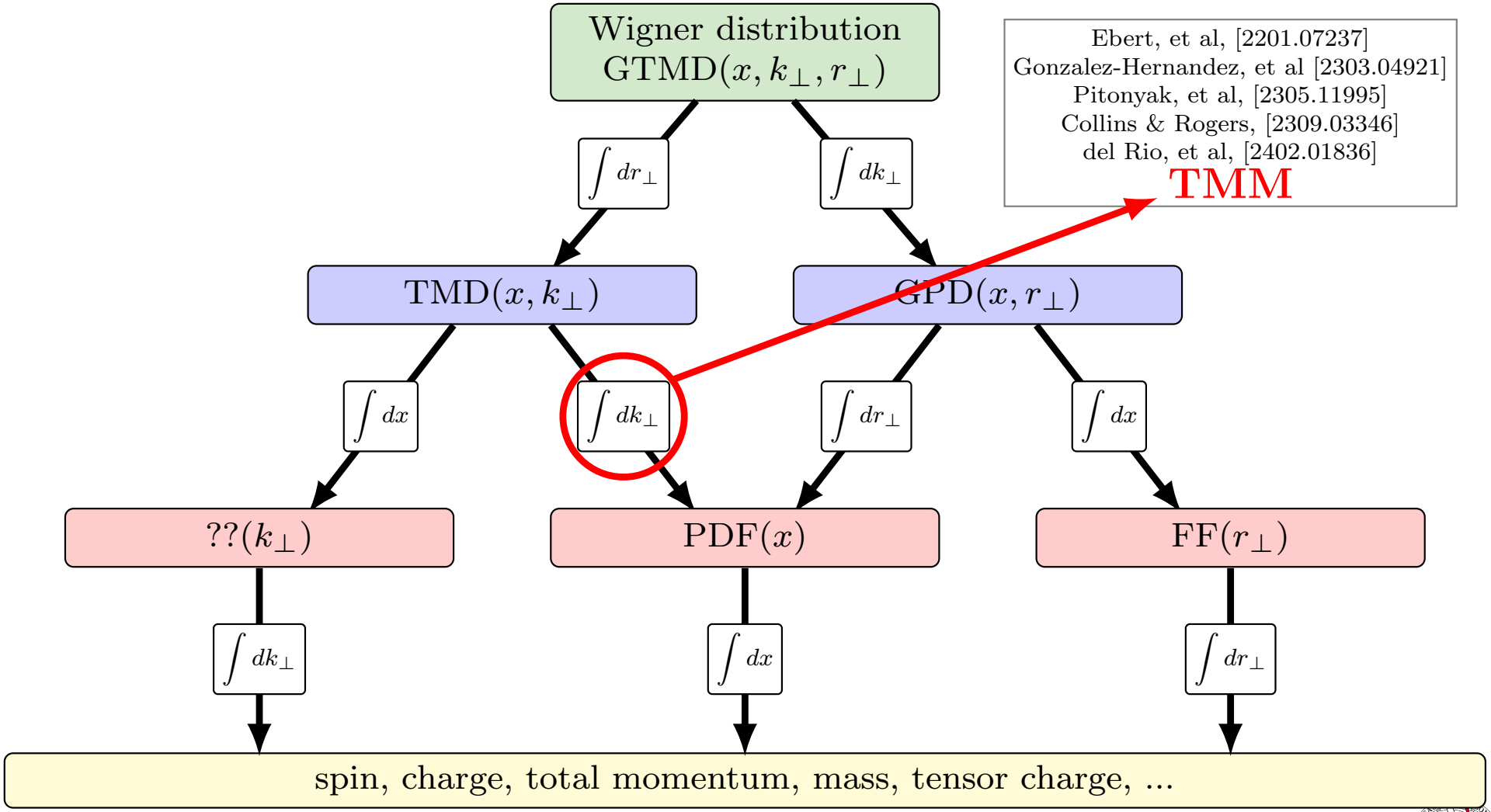


$$F(x,k_T) = \int \frac{d\lambda d^2b}{(2\pi)^3} e^{-ix\lambda p_+ - i(bk)_T} \langle p,s|\bar{q}(\lambda n + b)[\text{staple Wilson line}]\gamma^+ q(0)|p,s\rangle$$

Interpretation as parton densities is "naive"

- TMDs are not positive definite (not theoretically, nor practically)
- Double-scale evolution complicates a lot


Wigner distribution
GTMD$(x, k_\perp, r_\perp)$
$\int dr_\perp$
$\int dk_\perp$
TMD$(x, k_\perp)$
GPD$(x, r_\perp)$
$\int dx$
$\int dk_\perp$
$\int dr_\perp$
$\int dx$
$??(k_\perp)$
PDF(x)
FF$(r_\perp)$
$\int dk_\perp$
$\int dx$
$\int dr_\perp$
spin, charge, total momentum, mass, tensor charge, ...


Wigner distribution
GTMD$(x, k_\perp, r_\perp)$
$\int dr_\perp$
$\int dk_\perp$
Ebert, et al, [2201.07237]
Gonzalez-Hernandez, et al [2303.04921]
Pitonyak, et al, [2305.11995]
Collins & Rogers, [2309.03346]
del Rio, et al, [2402.01836]
TMM
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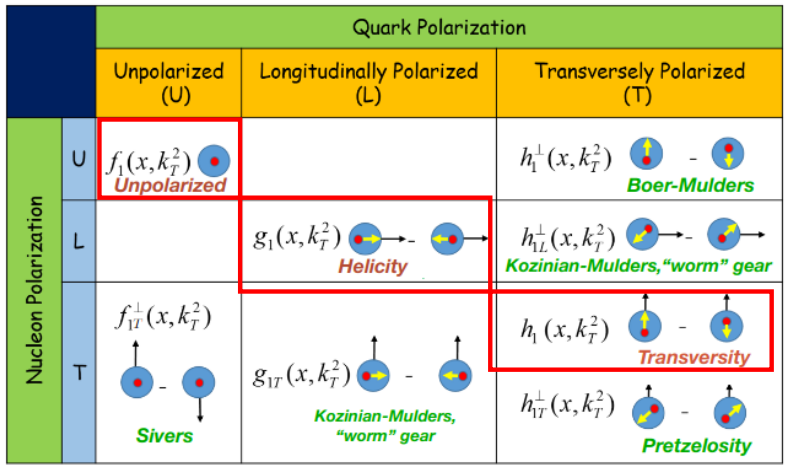
TMDs are 3D
The question is how can we related them to 1D (collinear) distributions

Very Naively:

$$\int d^2 k_T F(x, k_T) = f(x)$$



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Very Naively:

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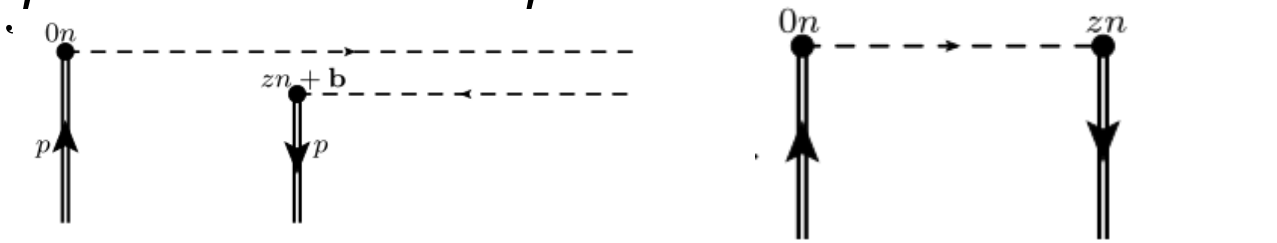
$$F(x,k_T) = \int \frac{d^2b}{(2\pi)^2} e^{i(\mathbf{b}\mathbf{k}_T)} \tilde{F}(x,b)$$

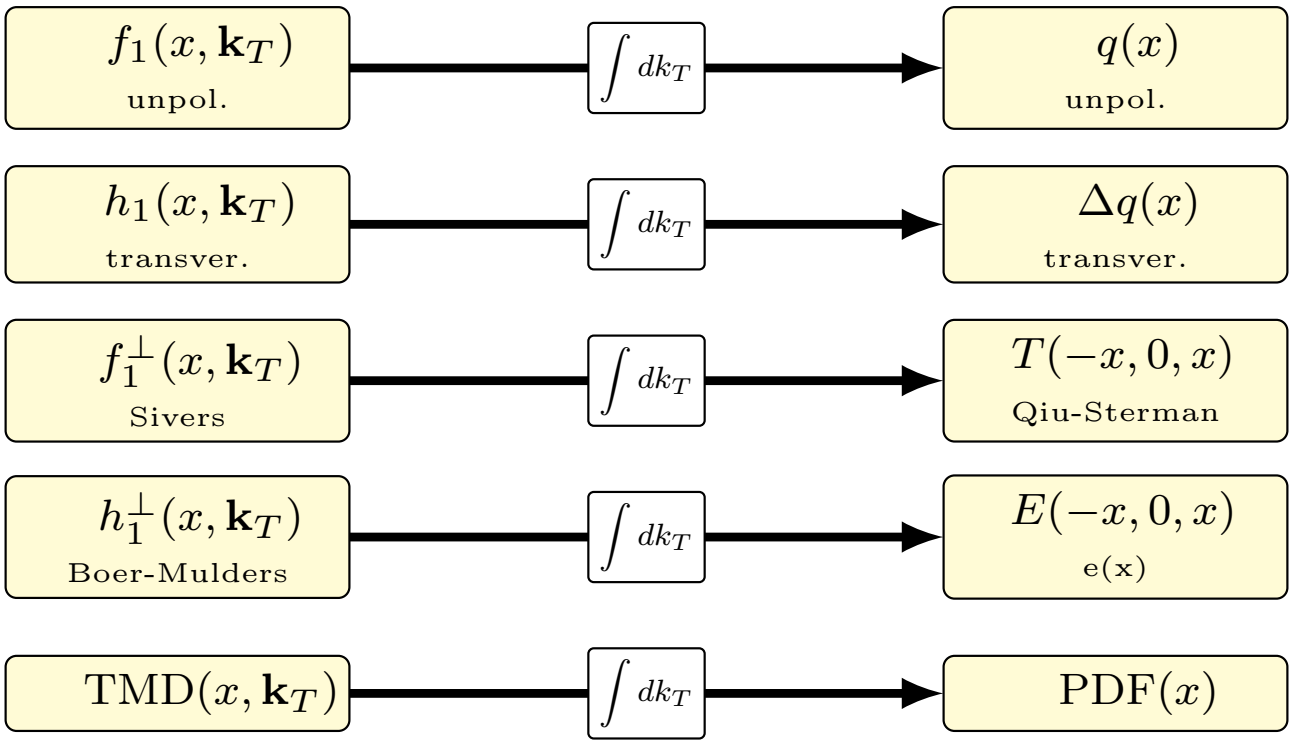
$$\downarrow \qquad \downarrow$$

$$\int d^2\mathbf{k}_T F(x,k_T) = \int d^2b \delta^{(2)}(b)\tilde{F}(x,b) = \tilde{F}(x,\mathbf{0})$$



Collinear distribution from TMDs

Naively



Small-b operator product expansion

$$F(x,k_T) = \int \frac{d^2b}{(2\pi)^2} e^{i(\mathbf{b}\mathbf{k}_T)} \tilde{F}(x,b)$$

$$\int d^2\mathbf{k}_T F(x,k_T) = \int d^2b \delta^{(2)}(b)\tilde{F}(x,b) = \tilde{F}(x,\mathbf{0})$$

- In small-b the operator changes its type. So, one should perform OPE.
- Different operators $\rightarrow$ different renormalization $\rightarrow$ logarithms of b

$$\tilde{F}(x,\mathbf{0}) \quad = \quad \int d^2\mathbf{k}_T F(x,k_T) \textbf{ is indefined}$$

Collinear distribution from TMDs

Properly

$f_1(x, \mathbf{k}_T, \mu, \zeta)$ unpol.	OPE →	$C(x, k_\perp; \mu, \zeta; \mu') \otimes$ N^{3}LO	$q(x, \mu')$ unpol.
$h_1(x, \mathbf{k}_T, \mu, \zeta)$ transver.	OPE →	$C(x, k_\perp; \mu, \zeta; \mu') \otimes$ N^{2}LO	$\Delta q(x, \mu')$ transver.
$f_1^\perp(x, \mathbf{k}_T, \mu, \zeta)$ Sivers	OPE →	$C(x, k_\perp; \mu, \zeta; \mu') \otimes$ NLO	$T(-x, 0, x, \mu')$ Qiu-Sterman
$h_1^\perp(x, \mathbf{k}_T, \mu, \zeta)$ Boer-Mulders	OPE →	$C(x, k_\perp; \mu, \zeta; \mu') \otimes$ NLO	$E(-x, 0, x, \mu')$ e(x)
TMD$(x, \mathbf{k}_T, \mu, \zeta)$	OPE →	$C(x, k_\perp; \mu, \zeta; \mu') \otimes$	PDF(x, μ')

1. Coefficient function $\sim \ln^n(\mathbf{k}^2)/\mathbf{k}^2$

2. Three scales: (μ, ζ) in TMD, μ' in collinear PDF

This is also reflected in the data

Presently we know all (quark) TMDs to some degree of precision

Name	Function	Twist of leading matching	Twist-2 distributions in matching	Twist-3 distributions in matching	Order of leading power coef.function
unpolarized	$f_1(x,b)$	tw-2	$f_1(x)$	–	N^{3}LO (α_s^3)
Sivers	$f_{1T}^{\perp}(x,b)$	tw-3	–	$T(-x,0,x)$	NLO (α_s^1)
helicity	$g_{1L}(x,b)$	tw-2	$g_1(x)$	$\mathcal{T}_g(x)$	NLO (α_s^1)
worm-gear T	$g_{1T}(x,b)$	tw-2/3	$g_1(x)$	$\mathcal{T}_g(x)$	NLO (α_s)
transversity	$h_1(x,b)$	tw-2	$h_1(x)$	$\mathcal{T}_h(x)$	NNLO (α_s^2)
Boer-Mulders	$h_1^{\perp}(x,b)$	tw-3	–	$\delta T_\epsilon(-x,0,x)$	NLO (α_s)
worm-gear L	$h_{1L}^{\perp}(x,b)$	tw-2/3	$h_1(x)$	$\mathcal{T}_h(x)$	NLO (α_s)
pretzelosity	$h_{1T}^{\perp}$	tw-3/4	–	$\mathcal{T}_h(x)$	LO (α_s^0)

We know how to insert "collinear" into TMD
but how extract "collinear" from TMD?

Why would on need it?

- Only unpolarized PDF is well-known, many other PDFs are less-known or unreachable
- TMD can set further constraints on PDF (joined TMD+PDF fits ?)
- How to determine higher moments?

The main problem is the evolution.
TMD distribution obeys CSS-evolution

$$\frac{d}{d\ln\mu}F(x,b;\mu,\zeta)=\gamma_F(\mu,\zeta)F(x,b;\mu,\zeta),\qquad \frac{d}{d\ln\zeta}F(x,b;\mu,\zeta)=\mathcal{D}(b,\mu)F(x,b;\mu,\zeta)$$

PDF obeys DGLAP-evolution

$$\frac{d}{d\ln\mu}f(x,\mu)=\int_x^1\frac{dy}{y}P(y,\mu)f(\frac{x}{y},\mu)$$

$$\int_{-\infty}^{\infty} d^2\mathbf{k}_T F(x, \mathbf{k}_T; \mu, \zeta) = \text{UV divergent}$$

$$\sim \frac{1}{\mathbf{k}_T^2}$$

$$\int_{-\infty}^{\infty} d^2\mathbf{k}_T \theta(|\mathbf{k}_T| < \mu') F(x, \mathbf{k}_T; \mu, \zeta) = \tilde{f}(x, \mu, \zeta, \mu')$$

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power correction
$$\sim \frac{\langle \mathbf{k}_T^2 \rangle}{\mu'^2}$$

$$\frac{d}{d \ln \mu'} \tilde{f}(x, \mu, \zeta, \mu') = (\text{DGLAP} + (\mu, \zeta)) \tilde{f}(x, \mu, \zeta, \mu') + \mathcal{O}(\mu'^{-2})$$

proven at all-orders of PT
[O. del Rio, et al, 2402.01836]

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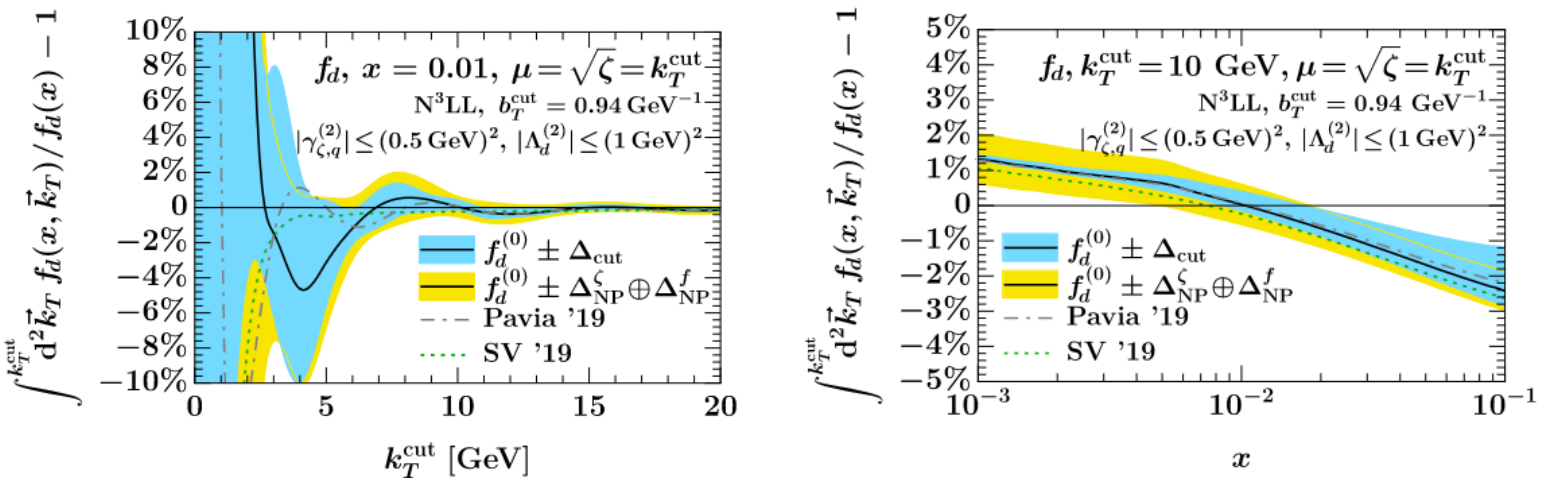
what to do with this?
1. $\zeta = \mu^2 = \mu'^2$
2. ζ-prescription

proven at all-orders of PT
[O. del Rio, et al, 2402.01836]

Collinear distribution from TMDs

$$\int^{\mu} d^2\mathbf{k}_T f_1(x, \mathbf{k}_T; \mu, \mu^2) = \mu \int_0^{\infty} db J_1(\mu b) f(x, b; \mu, \mu^2) =\simeq q(x, \mu)$$

[Ebert, et al 2201.07237]
[Gonzalez-Hernandez, et al, 2205.05750]



One can restore (tw2) collinear PDF up to few %. **Can we do better?**

$$\frac{d}{d\ln\mu'}\tilde{f}(x,\mu=\sqrt{\zeta}=\mu') = (\text{DGLAP} + \underbrace{\text{finite terms}}_{\sim\alpha_s^2\beta_0 C_1+\ldots})\tilde{f}(x,\mu=\sqrt{\zeta}=\mu') + \mathcal{O}(\mu'^{-2})$$

I.e. it is not the $\overline{\text{MS}}$-scheme but some other scheme (TMD-scheme) situation simular to lattice-schemes, Larin scheme, etc.

To match it to $\overline{\text{MS}}$ one should compute the scheme-transition coefficient.

- for optimal TMD

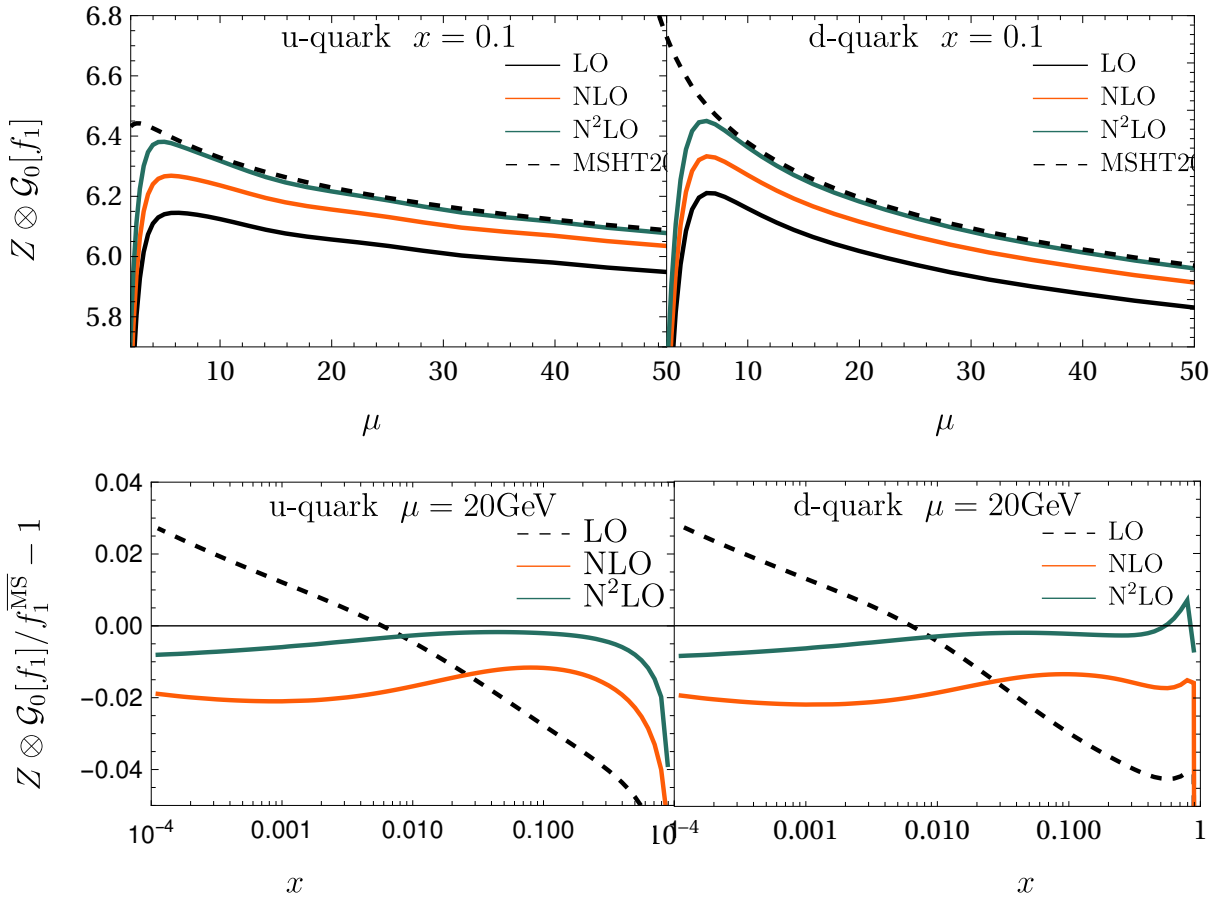
$$Z^{\overline{\text{MS}}/\text{TMD}} = \mathbf{1} - \alpha_s C_1 - \alpha_s^2 (C_2 - C_1 \otimes C_1)$$
$$-\alpha_s^3 \left[C_3 + C_1\otimes C_1\otimes C_1 - C_1\otimes C_2 - C_2\otimes C_1 + \frac{2\zeta_3}{3} P_1\otimes(P_1-\beta_0\cdot\mathbf{1})\otimes(P_1-2\beta_0\cdot\mathbf{1})\right] + \mathcal{O}(\alpha_s^4),$$

- for general TMD

$$Z^{\overline{\text{MS}}/\text{TMD2}} = \mathbf{1} - \alpha_s \overline{C}_1 - \alpha_s^2 \left[\overline{C}_2 - \overline{C}_1\otimes\overline{C}_1 - \zeta_3\Gamma_0\left(P_1 + \left(\frac{\gamma_1}{2} - \frac{2\beta_0}{3}\right)\cdot\mathbf{1}\right)\right] + \mathcal{O}(\alpha_s^3),$$

Collinear distribution from TMDs

$$\int^{\mu} d^2\mathbf{k}_T f_1(x,\mathbf{k}_T;\mu,\mu^2) = \mu \int_0^{\infty} dbJ_1(\mu b) f(x,b;\mu,\mu^2) = Z^{\mathrm{TMD}/\overline{\mathrm{MS}}}(\mu) \otimes q(x,\mu)$$



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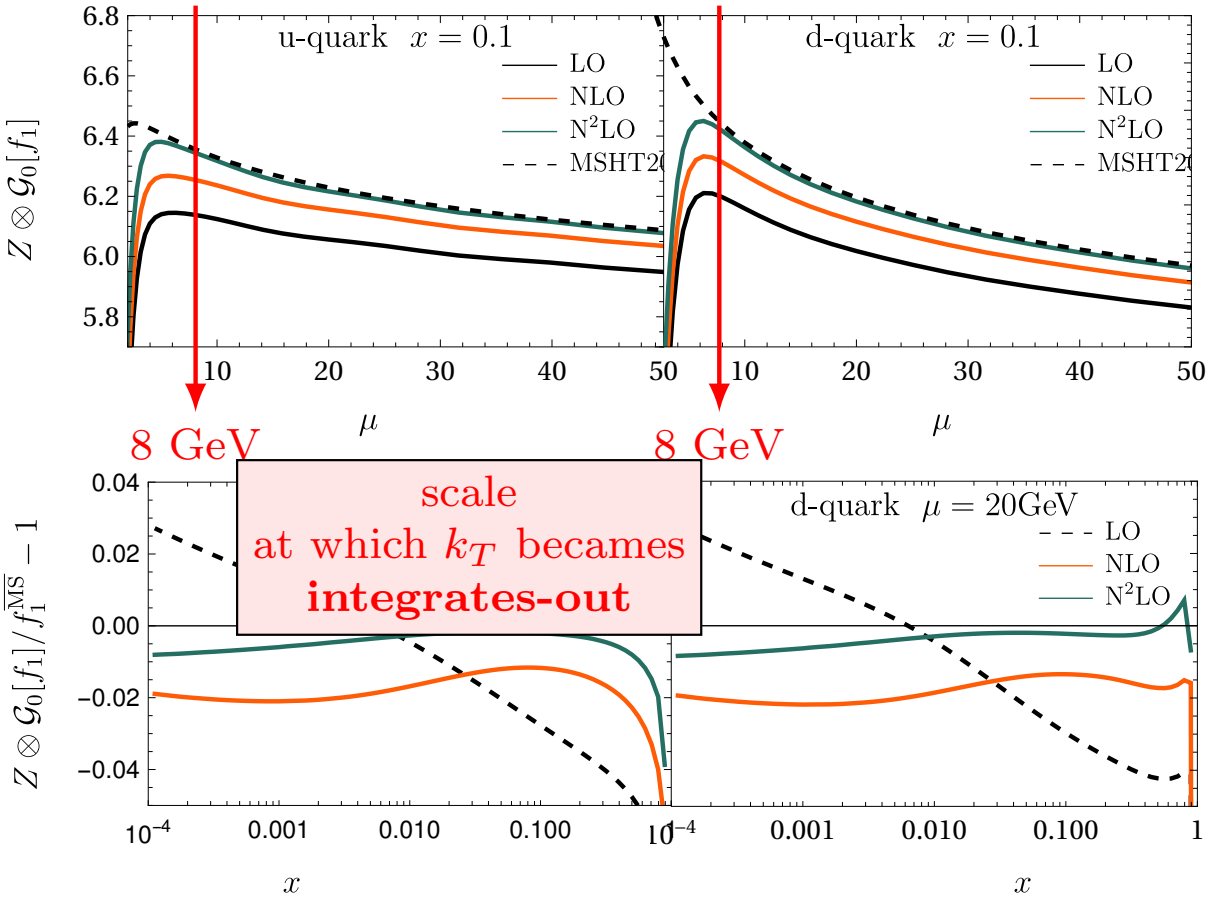

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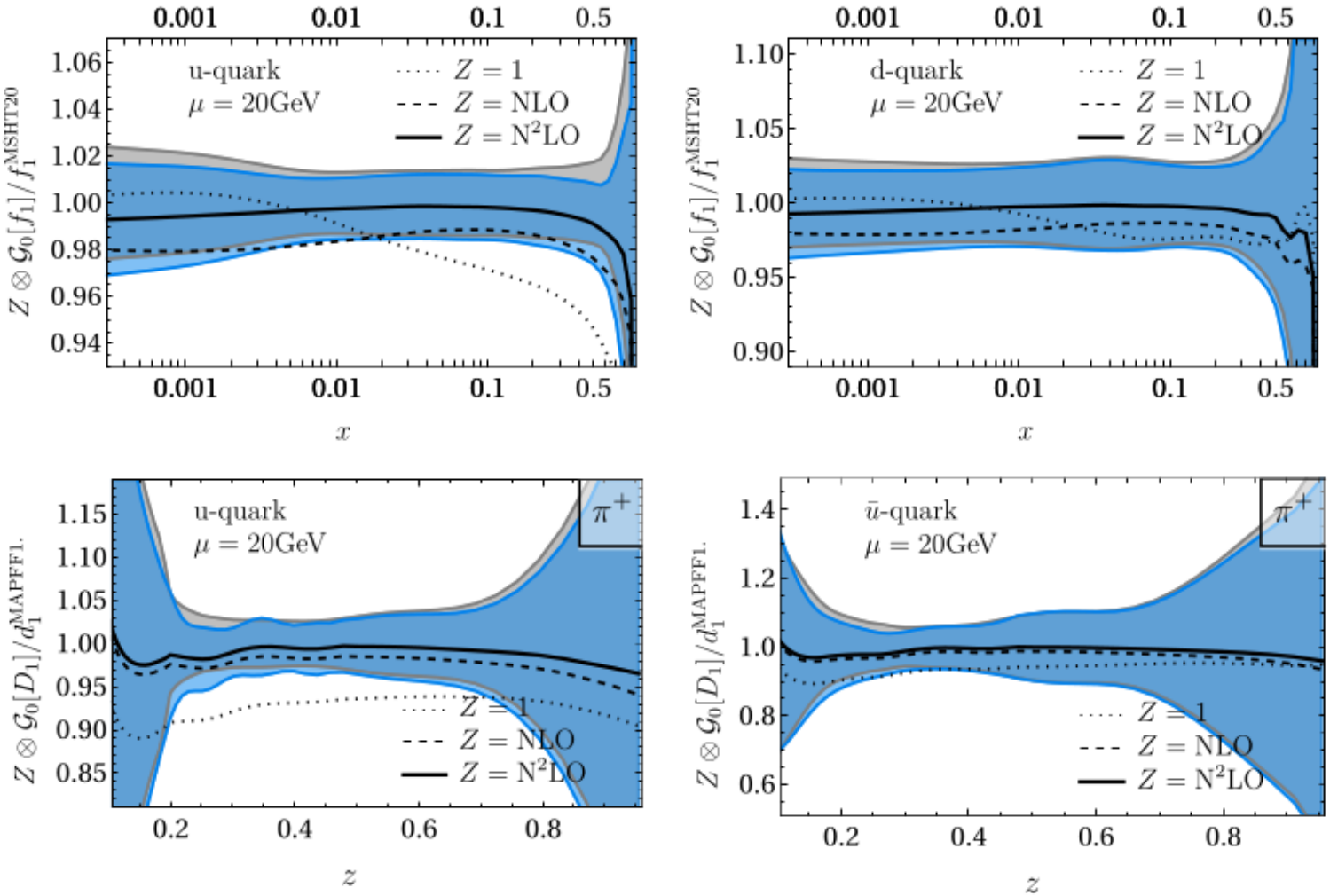

What about higher Transverse Momentum Moments (TMM)?

$$\int d^2\mathbf{k}_T \mathbf{k}_T^{\mu_1} ... \mathbf{k}_T^{\mu_n} f_1(x, \mathbf{k}_T) \simeq \int e^{-ix\lambda p+} d\lambda \langle p, s|\bar{q}[\lambda n, \infty n] \overleftrightarrow{\mathcal{D}_T^{\mu_1}} ... \overleftrightarrow{\mathcal{D}_T^{\mu_n}} \gamma^+ [\infty n, 0] q(0)|p, s\rangle$$

twist-n matrix element

0'th TMM $\Rightarrow$ twist-2
1'th TMM $\Rightarrow$ twist-3
2'th TMM $\Rightarrow$ twist-4

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$$\int^{\mu} d^2\mathbf{k}_T \mathbf{k}_T^{\mu_1} ... \mathbf{k}_T^{\mu_n} f_1(x, \mathbf{k}_T) = M_{\mu_1 ... \mu_n}(x, \mu) + \mathcal{O}\left(\frac{1}{\mu^2}\right)$$

Twist-n matrix element in TMD scheme
How to to pass to $\overline{\text{MS}}$-scheme?

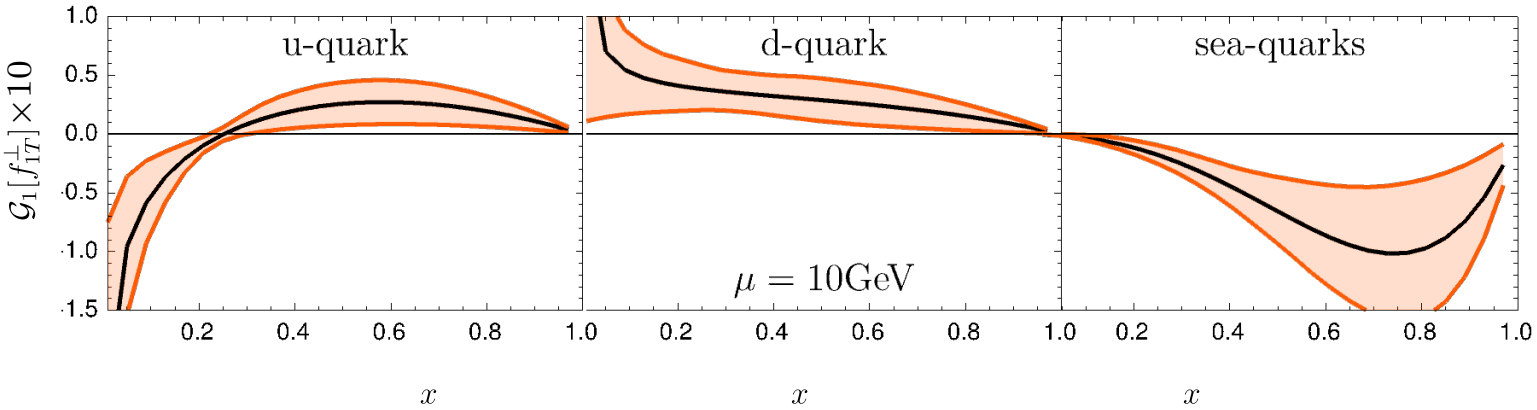
1'st TMM$\Longrightarrow$ tw3 distributions

$$\mathcal{G}_1[F](x,\mu) \quad = \quad \int db(\mu b) J_2(\mu b) F(x,b)$$

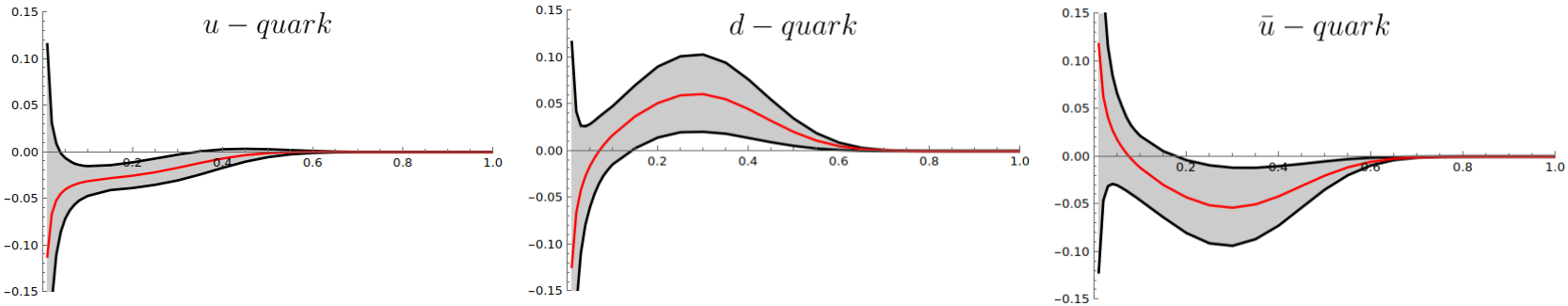
$$\begin{aligned}
\mathcal{G}_1[f_{1T}^{\perp}](x,\mu) &= \pm\frac{\pi}{2} T^{(\mathrm{TMD})}(-x,0,x;\mu) + \mathcal{O}(\mu^{-2}),\\
\mathcal{G}_1[g_{1T}^{\perp}](x,\mu) &= \frac{x}{2}\int_x^1 \frac{dy}{y}\Delta q^{(\mathrm{TMD})}(y,\mu) + x\int_{-1}^1 dy_1 dy_2 dy_3 \delta(y_1+y_2+y_3)\int_0^1 d\alpha\delta(x-\alpha y_3)\Big[\\
&\qquad \frac{\Delta T^{(\mathrm{TMD})}(y_{123};\mu)}{y_2^2} + \frac{T^{(\mathrm{TMD})}(y_{123};\mu) - \Delta T^{(\mathrm{TMD})}(y_{123};\mu)}{2y_2 y_3}\Big] + \mathcal{O}(\mu^{-2}),\\
\mathcal{G}_1[h_{1L}^{\perp}](x,\mu) &= -\frac{x^2}{2}\int_x^1 \frac{dy}{y}\delta q^{(\mathrm{TMD})}(y,\mu)\\
&\quad -x\int_{-1}^1 dy_1 dy_2 dy_3 \delta(y_1+y_2+y_3)\int_0^1 d\alpha\alpha\delta(x-\alpha y_3) H^{(\mathrm{TMD})}(y_{123};\mu)\frac{y_3-y_2}{y_2^2 y_3} + \mathcal{O}(\mu^{-2}),\\
\mathcal{G}_1[h_1^{\perp}](x,\mu) &= \mp\frac{\pi}{2} E^{(\mathrm{TMD})}(-x,0,x;\mu) + \mathcal{O}(\mu^{-2}),
\end{aligned}$$

TMD(2)-scheme is a kind of MS-scheme, but not $\overline{\mathrm{MS}}$-scheme.
The scheme matching factor cannot be computed because it connects
$(x_1,x_2,x_3) \leftrightarrow x$.
but up to NLO the results are the same (we do not know NLO).

$T(-x, 0, x)$ from Sivers function
[Bury,Prokudin, AV, 2103.03270]

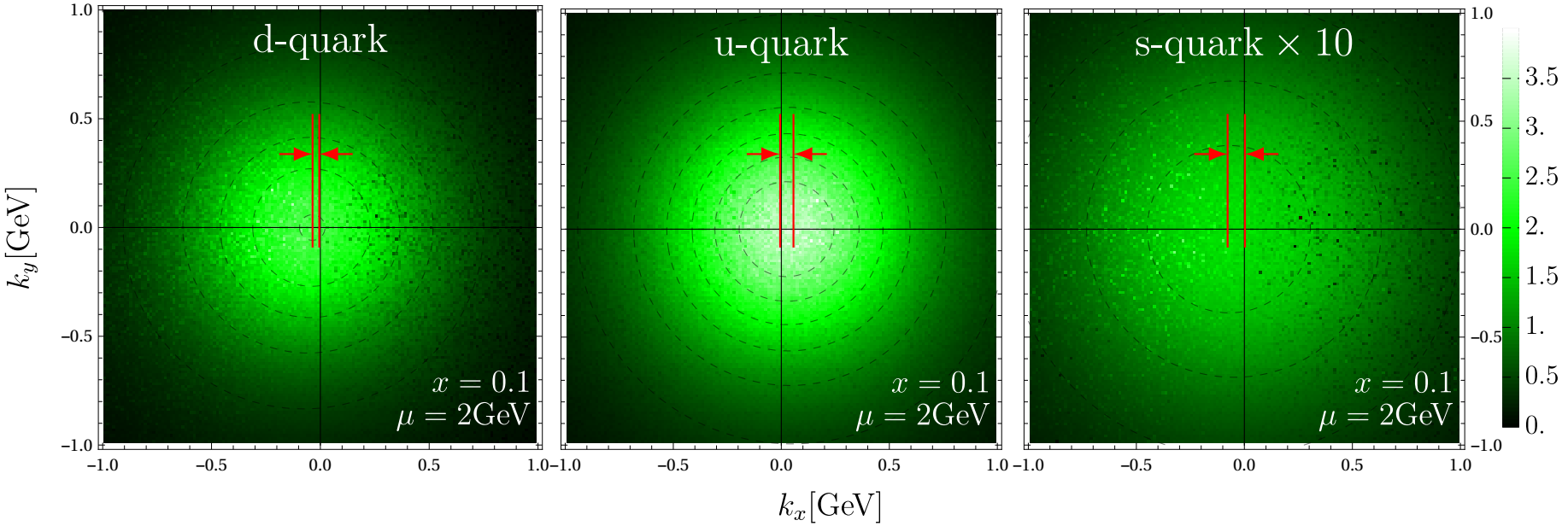


$T(-x, 0, x)$ from the direct fit of $T(x_1, x_2, x_3)$
[AV, Portela, Rodini, 2511.04294]



Avarage transverse momentum of quarks

$$\langle \mathbf{k}_\mu \rangle = -\pi \epsilon_{\mu\nu} s_T^\nu M \int_0^1 dx T(-x,0,x)$$ [O.del Rio, A.Prokudin, AV, 2402.01836]



$\langle k_x \rangle_u = 9.5^{+7.0}_{-7.1}$MeV, $\langle k_x \rangle_d = -18.8^{+18.4}_{-17.7}$MeV,

$\langle k_x \rangle_s = -5.0^{+6.4}_{-7.4}$MeV, $\langle k_x \rangle_c = 2.3^{+3.2}_{-3.3}$MeV,

$\langle k_x \rangle_{\bar{u}} = -2.5^{+6.3}_{-6.4}$MeV, $\langle k_x \rangle_{\bar{d}} = 16.2^{+17.9}_{-19.0}$MeV,

$\langle k_x \rangle_{\bar{s}} = 2.2^{+7.2}_{-7.2}$MeV, $\langle k_x \rangle_{\bar{c}} = -0.7^{+3.5}_{-3.8}$MeV,

$\langle k_x \rangle_g = -36.9^{+36.4}_{-39.8}$MeV,

Budkard sum rule

[M.Budkardt, hep-ph/0311013]

$$\sum_{f=q,\bar{q},g} \langle k_x \rangle_f = -33.^{+34.}_{-32.}\text{MeV}$$

2'th TMM $\Longrightarrow$ twist-4

$$\int^{\mu} d^2\mathbf{k}_T \mathbf{k}_T^2 f_1(x,\mathbf{k}_T) = \int_0^{\infty} db\, \mu^3 \frac{J_1(\mu b) - J_3(\mu b)}{2} f_1(x,b)$$

$$\simeq \int e^{-ix\lambda p_+} d\lambda \langle p,s|\bar{q}[\lambda n,\infty n] \overleftrightarrow{\mathcal{D}}^2 \gamma^+ [\infty n, 0] q(0)|p,s\rangle$$

TMD-scheme is not MS-scheme, it has power divergence

$$\int^{\mu} d^2\mathbf{k}_T \mathbf{k}_T^2 f_1(x,\mathbf{k}_T) \simeq \mu^2 ... + ...$$

To pass to a MS-scheme (not $\overline{\text{MS}}$-scheme) one needs to subtract the divergence

2'th TMM $\Longrightarrow$ twist-4

$$\int^{\mu} d^2\mathbf{k}_T \mathbf{k}_T^2 f_1(x,\mathbf{k}_T) = \int_0^{\infty} db\, \mu^3 \frac{J_1(\mu b) - J_3(\mu b)}{2} f_1(x,b)$$

$$\simeq \int e^{-ix\lambda p_+} d\lambda \langle p,s|\bar{q}[\lambda n, \infty n] \overleftrightarrow{\mathcal{D}}^2 \gamma^+ [\infty n, 0] q(0)|p,s\rangle$$

$$\int^{\mu} d^2\mathbf{k}_T \mathbf{k}_T^2 f_1(x,\mathbf{k}_T;\mu,\mu^2) - \underbrace{\mu^2 \mathrm{AS}}_{\text{analytic}} = \langle \bar{q} D^2 q \rangle_{\overline{\mathrm{MS}}}$$

$$\begin{aligned}
\mathrm{AS}[\mathcal{G}_{1,0}[F]](x,\mu) \;=\; & \Big\{ \alpha_s P_1 + \alpha_s^2 \left[P_2 + (C_1 - P_1) \otimes (P_1 - \beta_0 \mathbf{1}) \right] \\
& + \alpha_s^3 \Big[P_3 + (C_1 - P_1) \otimes (P_2 - \beta_1 \mathbf{1}) + (C_2 - P_2) \otimes (P_1 - 2\beta_0 \mathbf{1}) \\
& - (C_1 - P_1) \otimes (P_1 - \beta_0 \mathbf{1}) \otimes (P_1 - 2\beta_0 \mathbf{1}) \Big] + \mathcal{O}\left(\alpha_s^4\right) \Big\} \otimes f(x,\mu),
\end{aligned}$$

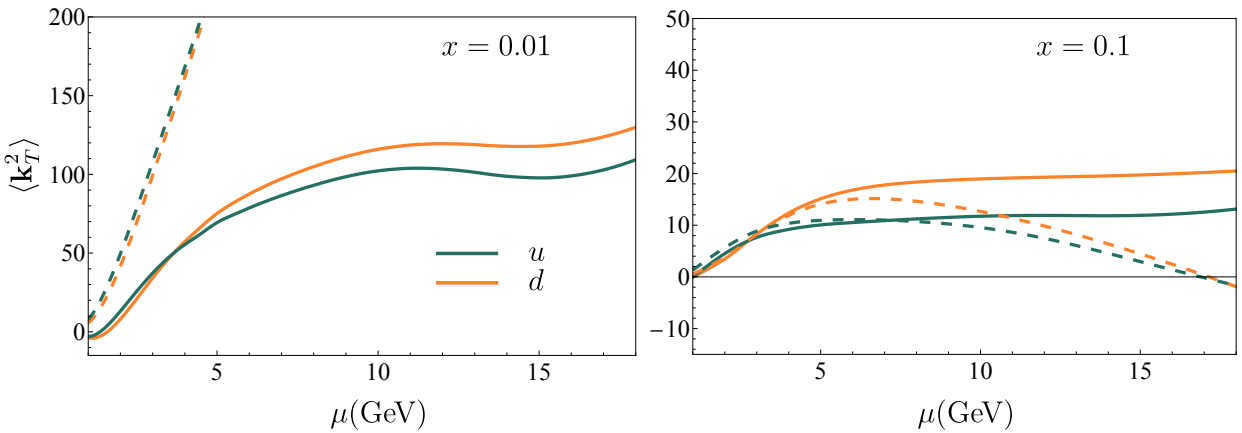
(for general case see (89) in [2402.01836])

2'th TMM $\Longrightarrow$ twist-4

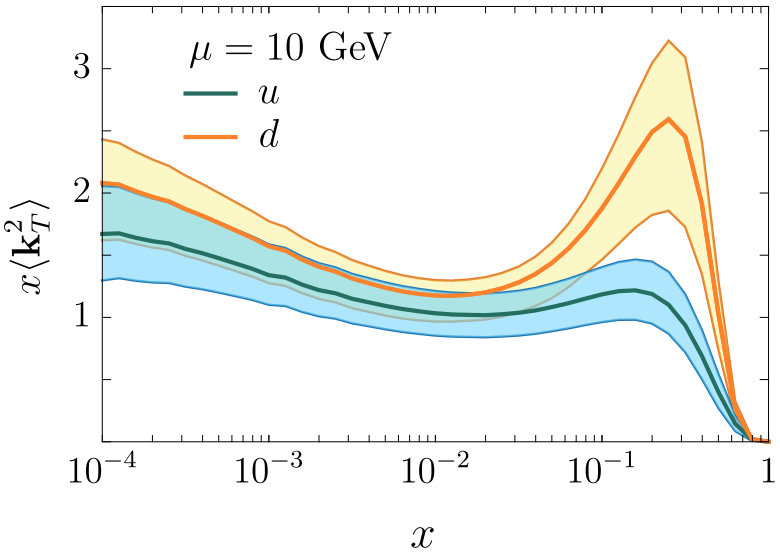
$$\int^{\mu} d^2\mathbf{k}_T \mathbf{k}_T^2 f_1(x,\mathbf{k}_T) = \int_0^{\infty} db\, \mu^3 \frac{J_1(\mu b) - J_3(\mu b)}{2} f_1(x,b)$$

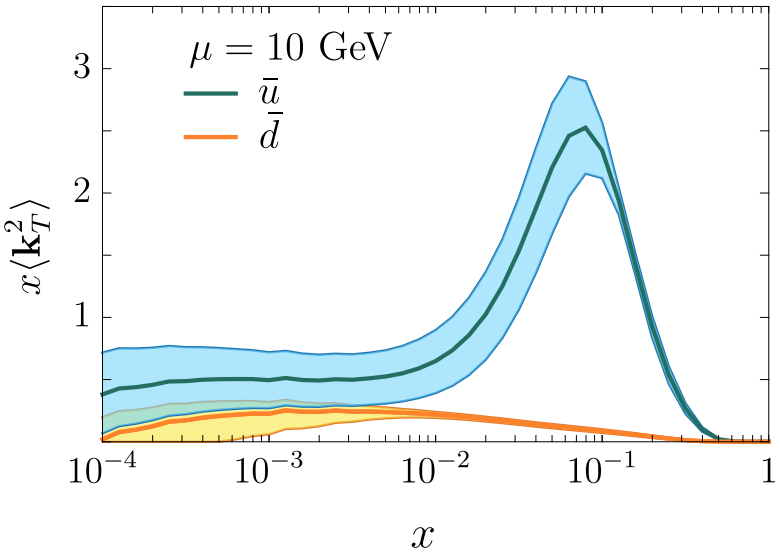
$$\simeq \int e^{-ix\lambda p_+} d\lambda \langle p,s|\bar{q}[\lambda n,\infty n] \overleftrightarrow{\mathcal{D}}^2 \gamma^+ [\infty n, 0] q(0)|p,s\rangle$$

$$\int^{\mu} d^2\mathbf{k}_T \mathbf{k}_T^2 f_1(x,\mathbf{k}_T;\mu,\mu^2) - \underbrace{\mu^2 \mathrm{AS}}_{\text{analytic}} = \langle \bar{q} D^2 q \rangle_{\overline{\mathrm{MS}}}$$



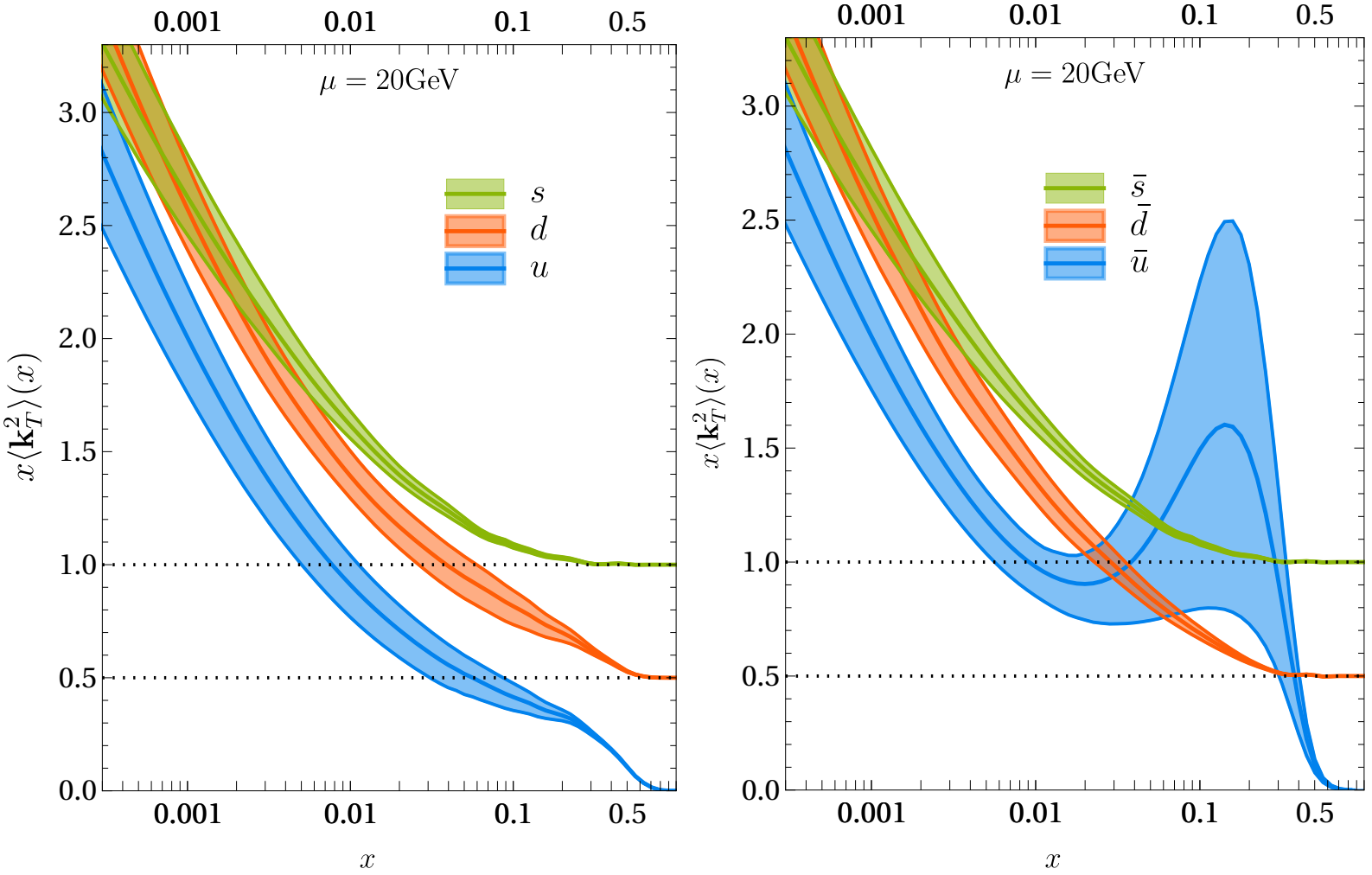
ART23





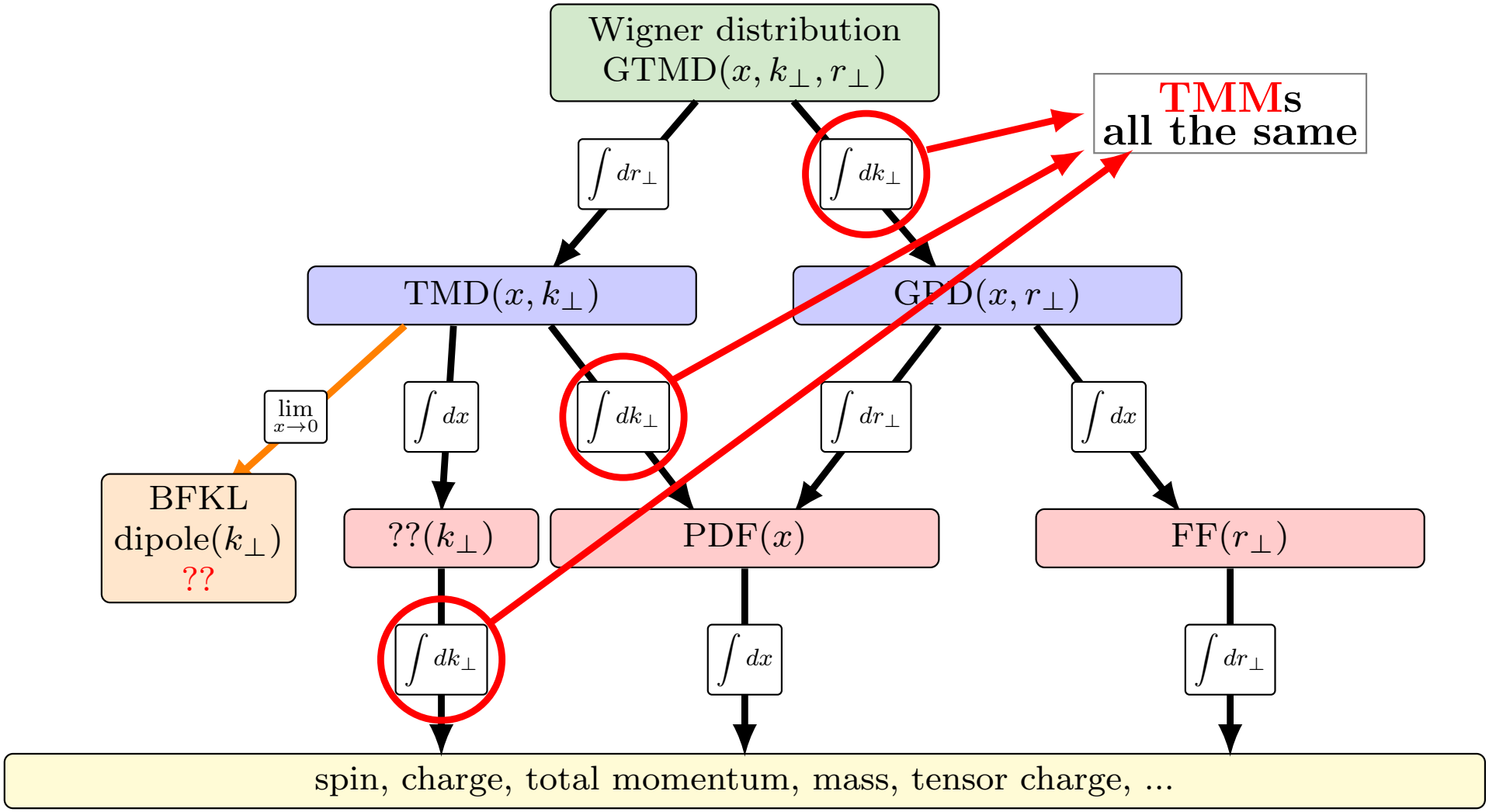
Interesting question: what is $\langle k_T^2 \rangle(\mu) = \int_0^1 dx \langle k_T^2 \rangle(x, \mu)$
(could be interpreted as "avarage transverse momentum" carried by all quarks)
It is ∞, since "number of partons" $\int_0^1 dx f_1(x, \mu) \to \infty$.
However, "parton-number" sum rule for valence quark $\int_0^1 dx (f_q(x) - f_{\bar{q}}(x))$ =finite.
So, one could expect $\langle k_T^2 \rangle_{val.}$ also be finite.
one should match asymptotics of q and $\bar{q}$ at $x \to 0$.

ART25

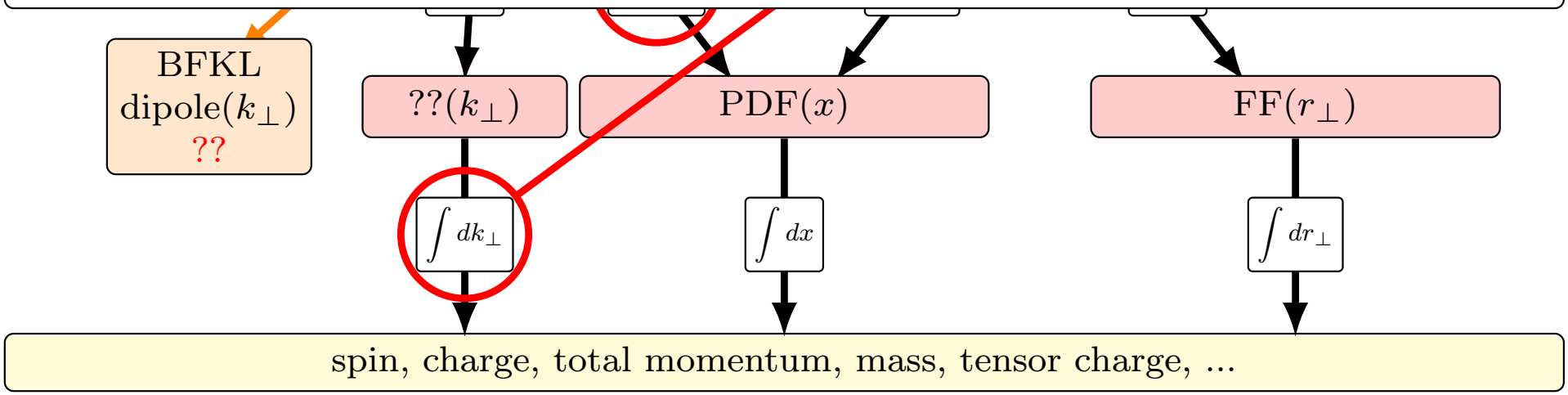


$\langle \mathbf{k}^2 \rangle_{\text{val.d}} = 1.0^{+0.4}_{-0.5}\text{GeV}^2$, $\langle \mathbf{k}^2 \rangle_{\text{val.u}} = -4.7^{+3.4}_{-3.5}\text{GeV}^2$

Work in progress


Wigner distribution
GTMD$(x, k_\perp, r_\perp)$
TMDs
all the same
$\int dr_\perp$
$\int dk_\perp$
TMD$(x, k_\perp)$
GPD$(x, r_\perp)$
$\lim_{x\to 0}$
$\int dx$
$\int dk_\perp$
$\int dr_\perp$
$\int dx$
BFKL
dipole$(k_\perp)$
??
??$(k_\perp)$
PDF(x)
FF$(r_\perp)$
$\int dk_\perp$
$\int dx$
$\int dr_\perp$
spin, charge, total momentum, mass, tensor charge, ...

Wigner distribution

Conclusions

- Simple and clear expressions how to get TMMs from TMDs
- 0'th TMMs = PDF (in $\overline{\text{MS}}$-scheme)
- 1'th TMMs = PDF of tw3 (in MS-scheme)
- 2'th TMMs = PDF of tw4 (can be turned to MS-scheme)
- 3+'th TMMs cannot be turned to MS-scheme simply
- Good and straightforward way to compare "integrated" elements, and interpretation