Transverse Spin Effects at NLO Accuracy in Semi-Inclusive Deep Inelastic Scattering

Fragmentation effects in single and double spin asymmetries

Bridging TMD Frameworks: Intersections, Tensions, and Applications
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Summary

What am I going to talk about?

1. Motivation
2. Collinear twist-3 factorization in SIDIS
3. NLO analysis
4. Phenomenology and numerical estimates

1. Motivation

Motivation

The story begins almost 50 years ago...

Mid 70s: $p + p^{\uparrow} \to h$ collisions at Argonne [Klem et al., 1976] and FermiLab [Bunce et al., 1976]:

Single Spin Asymmetries (SSA)

$$A_{UT} = \frac{\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}}{\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}} = \frac{\sigma_{UT}}{\sigma_{UU}} \sim 40\%$$

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Much later, proposed DSA measurements

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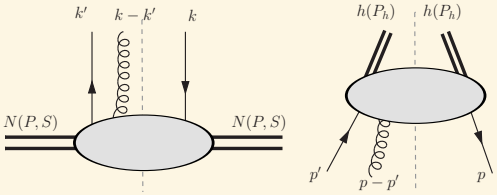
- ***Twist-3 observables***
- ***Probing quark-gluon-quark correlations within hadrons***

Motivation



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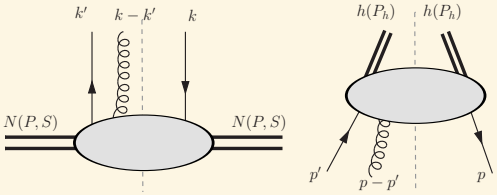

Easier case to probe quark-gluon-quark correlations: **lepton-nucleon collisions**

1. Single-inclusive hadron production
   $\ell + N^{\uparrow} \to h + X$
2. ***Semi-inclusive hadron production***
   $\ell(l) + N^{\uparrow}(P) \to \ell(l') + h(P_h) + X$

[Mulders & Tangerman, 1996] [Boer, Mulders, Pijlman, 2003]

[Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, 2007]

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz} = \frac{2\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T} + \varepsilon\,F_{UU,L} + S_\parallel \lambda_e \sqrt{1-\varepsilon^2}\,F_{LL} + |\boldsymbol{S}_\perp|\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + |\boldsymbol{S}_\perp|\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}\Big\},$$

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$$A_{UT}^{\sin\phi_S} \sim \frac{F_{UT}^{\sin\phi_S}}{F_{UU}}, \qquad A_{LT}^{\cos\phi_S} \sim \frac{F_{LT}^{\cos\phi_S}}{F_{UU}}$$

We focus on $P_{h\perp}$-integrated observables. Very few NLO studies for unintegrated [Kanazawa & Koike, 2013] or $P_{h\perp}$-weighted [Kang, Vitev, Xing, 2013] [Dai et. al., 2015] [Chen, Ma, Zhang, 2018] [Yoshida, 2016] analogous observables.

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Research questions:

- ***Does factorization hold at 1-loop level for these sub-leading twist observables? (Not obvious at all!)***
- ***If so, are NLO corrections important?***

2. Collinear twist-3 factorization in SIDIS

Parton distribution functions

First, PDFs: the "nucleon to parton" process

Organized in a ***quark-quark distribution correlator***

$$\Phi^q_{ij}(x) = \int \frac{\mathrm{d}\lambda}{2\pi}\, e^{i\lambda x}\, \langle N(P,S)|\bar{\psi}^q_j(0)\psi^q_i(\lambda n)|N(P,S)\rangle$$
$$= \frac{1}{2}\,(\not{P})_{ij}\, f^q_1(x) - \frac{1}{4}\,([\not{P},\not{S}]\gamma_5)_{ij}\, h^q_1(x) + \cdots$$

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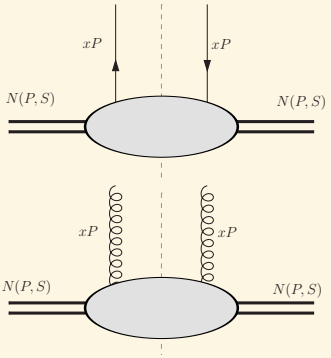
where:

- $f^q_1(x)$ usual unpolarized PDF
- $h^q_1(x)$ transversity PDF $\Rightarrow$ transversely polarized nucleons

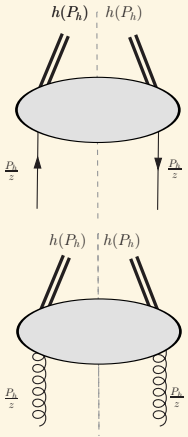


Twist-3 fragmentation functions classified as:

Intrinsic twist-3

$$\Delta^q_{ij}(z) = \sum_X \int \frac{\mathrm{d}\lambda}{2\pi}\, e^{-i\frac{\lambda}{z}}\, \langle 0|\psi^q_i(0)|h(P_h);X\rangle \times \langle h(P_h);X|\bar{\psi}^q_j(\lambda m)|0\rangle$$
$$= \frac{z^{2\epsilon}}{z}\,(\not{P}_h)_{ij}\, D_1^q(z) - \frac{z^{2\epsilon}}{z}\frac{iM_h}{2}\,([\not{P}_h, \not{n}])_{ij}\, H^q(z) + \frac{z^{2\epsilon}}{z} M_h\,(\mathbb{I})_{ij}\, E^q(z) + \cdots$$

Kinematical approximation: $p^\mu = \frac{P_h^\mu}{z}$

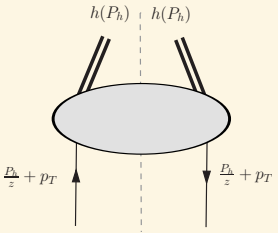


Twist-3 fragmentation functions classified as:

Kinematical twist-3

$$\Delta^{\rho}_{\partial,ij}(z) = \int \mathrm{d}^{d-2}p_T \, p_T^{\rho} \, \Delta_{ij}(z, p_T)$$
$$= \frac{z^{2\epsilon}}{z} \frac{iM_h}{2} \left([\not{P}_h, \gamma^{\rho}] - [\not{P}_h, \not{n}] P_h^{\rho} \right)_{ij} H_1^{\perp(1)}(z) + \cdots$$

Kinematical approximation: $p^{\mu} = \frac{P_h^{\mu}}{z} + p_T^{\mu}$
$\Rightarrow$ collinear expansion in p_T

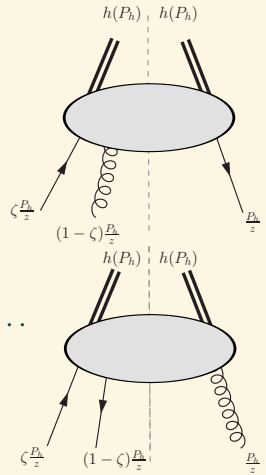


Twist-3 fragmentation functions classified as:

Dynamical twist-3

$$\Delta^{qg,\rho}_{F,ij}(z,\zeta) = \sum_X \int \frac{\mathrm{d}\lambda}{2\pi} \int \frac{\mathrm{d}\eta}{2\pi}\, e^{-i\frac{\lambda}{z}\zeta - i\frac{\eta}{z}(1-\zeta)}$$
$$\times\ \langle 0|ig_S\mu^\epsilon m_\sigma F^{\sigma\rho}(\eta m)\psi_i(0)|h(P_h);X\rangle$$
$$\times\ \langle h(P_h);X|\bar{\psi}_j(\lambda m)|0\rangle$$
$$= \frac{z^{2\epsilon}}{z}\frac{iM_h}{2}\left([\not{P}_h,\gamma^\rho] - [\not{P}_h,\not{n}]P_h^\rho\right)_{ij} i\left(\hat{H}^{qg}_{FU}(z,\zeta)\right)^* + \cdots$$

Kinematical approximation ($\zeta \equiv z/z'$):
$p^\mu = \frac{P_h^\mu}{z}, p'^\mu = \zeta\, p^\mu$



Equation of motion

Twist-3 functions are connected with each other via QCD EOM

$$\underbrace{(1-\epsilon)2z\int_0^1 \mathrm{d}\zeta \frac{\mathrm{Im}[\hat{H}^{qg}_{FU}(z,\zeta)]}{1-\zeta}}_{\equiv \tilde{H}^q(z)} = H^q(z) + 2z\, H_1^{\perp(1),q}(z)$$

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$$\underbrace{(1-\epsilon)(-2z)\int_0^1 \mathrm{d}\zeta \frac{\mathrm{Re}[\hat{H}^{qg}_{FU}(z,\zeta)]}{1-\zeta}}_{\equiv \tilde{E}^q(z)} = E^q(z) - z\frac{m_q}{M_h} D_1^q(z)$$

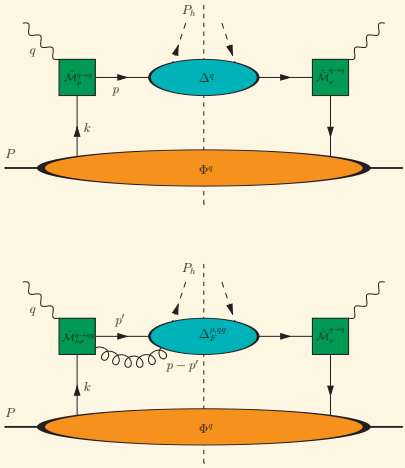
Fundamental even at LO!

SIDIS

$$\mathrm{d}\sigma^{\mathrm{SIDIS}} = \frac{1}{F}\mathrm{d}\Pi_{\mathrm{LIPS}}\frac{(4\pi)^2\alpha_{\mathrm{em}}^2}{Q^4}L_{\mu\nu}W^{\mu\nu}$$

$$x_B = \frac{Q^2}{2P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot l}, \quad z_h = \frac{P\cdot P_h}{P\cdot q}$$

- Setup in PP_h collinear frame, then Lorentz transform to Breit frame
- Gluons in light-cone gauge $m\cdot A = 0$



3. NLO analysis

How do the diagrams look like?

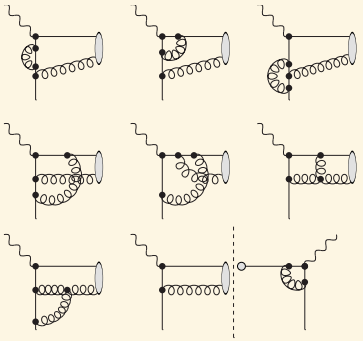


Figure: Dynamical twist-3 virtual graphs.

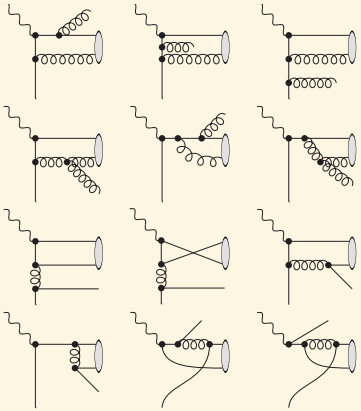


Figure: Dynamical twist-3 real graphs.

Consistency checks throughout the calculation

1. Gauge invariance in the sense of κ:

$$\sum_{\lambda=\pm 1} \epsilon^{\mu}_{\lambda}(r)\, \epsilon^{\nu *}_{\lambda}(r) = -g^{\mu\nu} + \kappa\, \frac{r^{\mu} m^{\nu} + r^{\nu} m^{\mu}}{r \cdot m}$$

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$\Rightarrow$ ***all require the use of the EOM relations (in d dimensions)!***

NLO analysis

We are left with $1/\epsilon$ poles... renormalization of chiral-odd twist-3 FFs?

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Known, independent calculation in the literature *[Ma & Zhang, 2017]*

$$\mu\frac{\partial\hat{e}(z,\mu)}{\partial\mu}=\frac{z\alpha_s}{\pi}\int\frac{d\xi_1 d\xi_2}{\xi_1\xi_2}\Big\{\mathrm{Re}\hat{E}_F(x_1,x_2)\Big[-C_F\frac{2\xi_1\xi_2}{(1-\xi_2)_+(\xi_2-\xi_1)}+\frac{N_c}{2}\frac{\xi_1}{\xi_2-\xi_1}+\frac{1}{2N_c}$$
$$-\frac{\xi_1\delta(\xi_2-1)}{2(1-\xi_1)}\Big(C_F+\frac{\ln\xi_1}{N_c(1-\xi_1)}\Big)\Big]+\frac{C_F}{N_c}\mathrm{Re}\hat{E}_G(x_1,x_2)\frac{1-\xi_2}{1-\xi_2+\xi_1}\Big\},$$

$$\mu\frac{\partial\hat{e}_\partial(z,\mu)}{\partial\mu}=\frac{\alpha_s C_F}{\pi}\Big[\frac{3}{2}\hat{e}_\partial(z)+2\int_z^1\frac{d\xi}{\xi}\frac{1}{(1-\xi)_+}\hat{e}_\partial(x)\Big]+\frac{\alpha_s}{\pi}\int\frac{d\xi_1 d\xi_2}{\xi_1\xi_2}\Big\{\mathrm{Im}\hat{E}_F(x_1,x_2)$$
$$\Big[\frac{N_c\xi_2}{2(\xi_2-\xi_1)^2}\Big(-\xi_1-\frac{\xi_2(1-\xi_2)}{1-\xi_1}\Big)-C_F\Big(\xi_2+\frac{\xi_2}{\xi_1-\xi_2}\Big)\Big]-\frac{C_F}{N_c}\mathrm{Im}\hat{E}_G(x_1,x_2)\frac{(1-\xi_2)^2}{1-\xi_2+\xi_1}\Big]\Big\},$$

$$\mu\frac{\partial\hat{e}_I(z,\mu)}{\partial\mu}=\frac{\alpha_s}{\pi}\Big\{3zC_F\hat{e}_\partial(z)+4zC_F\int_z^1\frac{d\xi}{\xi}\frac{1}{(1-\xi)_+}\hat{e}_\partial(x)\Big\}+\frac{z\alpha_s}{\pi}\int\frac{d\xi_1 d\xi_2}{\xi_1\xi_2}\Big\{\mathrm{Im}\hat{E}_F(x_1,x_2)$$
$$\Big[C_F\frac{2\xi_1(\xi_2^2-\xi_1)}{(1-\xi_2)_+(\xi_2-\xi_1)^2}-\frac{1}{2N_c}\frac{\xi_2(\xi_1+\xi_2)}{(\xi_2-\xi_1)^2}+\frac{\xi_1\delta(\xi_2-1)}{2(1-\xi_1)}\Big(C_F+\frac{\ln\xi_1}{N_c(1-\xi_1)}\Big)$$
$$-C_F\frac{\xi_1}{\xi_2-\xi_1}\Big]-\frac{C_F}{N_c}\mathrm{Im}\hat{E}_G(x_1,x_2)\frac{1-\xi_2}{1-\xi_2+\xi_1}\Big\},$$

Figure: Evolution of $E(z)$, $H_1^{\perp(1)}(z)$ and $H(z)$ in [Ma & Zhang, 2017]

NLO analysis

Factorization holds

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B\mathrm{d}y\mathrm{d}z_h} \sim F_{UU,T} + \varepsilon F_{UU,L} + |\boldsymbol{S}_\perp|\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S\, F_{UT}^{\sin\phi_S} + |\boldsymbol{S}_\perp|\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S\, F_{LT}^{\cos\phi_S}$$

Unpolarized structure functions at NLO (known, e.g. [de Florian, Stratmann, Vogelsang, 1997])

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$$F_{UU,T}(x_B,z_h,\mu^2) = x_B\sum_{q,\bar{q}} e_q^2\Big\{ f_1^q(x_B,\mu^2)D_1^q(z_h,\mu^2)$$

$$+\frac{\alpha_S(\mu^2)}{2\pi}\left[f_1^q\otimes\mathcal{C}_1^{q\to q}\otimes D_1^q + f_1^g\otimes\mathcal{C}_1^{g\to q}\otimes D_1^q + f_1^q\otimes\mathcal{C}_1^{q\to g}\otimes D_1^g\right](x_B,z_h,\mu^2)\Big\}$$

$$F_{UU,L}(x_B,z_h,\mu^2) = x_B\frac{\alpha_S(\mu^2)}{2\pi}\sum_{q,\bar{q}} e_q^2\Big[f_1^q\otimes\mathcal{C}_L^{q\to q}\otimes D_1^q$$

$$+ f_1^g\otimes\mathcal{C}_L^{g\to q}\otimes D_1^q + f_1^q\otimes\mathcal{C}_L^{q\to g}\otimes D_1^g\Big](x_B,z_h,\mu^2)$$

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$$F_{UT}^{\sin\phi_S}(x_B, z_h, \mu^2) = -x_B \frac{4M_h}{Q}\sum_{q,\bar{q}} e_q^2 \Bigg\{ h_1^q(x_B,\mu^2)\frac{\tilde{H}^q(z_h,\mu^2)}{2z_h}$$

$$+ \frac{\alpha_S(\mu^2)}{2\pi}\int_{x_B}^1 \frac{\mathrm{d}w}{w}\int_{z_h}^1 \frac{\mathrm{d}v}{v}\int_0^1 \mathrm{d}\zeta\; h_1^q\left(\frac{x_B}{w},\mu^2\right)$$

$$\times \left[\mathcal{C}_{UT}^{q\to qg}(w,v,\zeta,\mu^2)\frac{\mathrm{Im}\,\hat{H}_{FU}^{qg}\left(\frac{z_h}{v},\zeta,\mu^2\right)}{1-\zeta} + \mathcal{C}_{UT}^{q\to\bar{q}q}(w,v,\zeta,\mu^2)\,\mathrm{Im}\,\hat{H}_{FU}^{\bar{q}q}\left(\frac{z_h}{v},\zeta,\mu^2\right)\right]\Bigg\}$$

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Polarized structure functions at NLO

$$F_{LT}^{\cos\phi_S}(x_B,z_h,\mu^2) = +x_B\frac{4M_h}{Q}\sum_{q,\bar{q}} e_q^2\Bigg\{-x_B\frac{M}{M_h}\,g_T^q(x_B,\mu^2)D_1^q(z_h,\mu^2) + \mathcal{O}(\alpha_S)$$

$$- h_1^q(x_B,\mu^2)\frac{\tilde{E}^q(z_h,\mu^2)}{2z_h} + \frac{\alpha_S(\mu^2)}{2\pi}\int_{x_B}^1\frac{\mathrm{d}w}{w}\int_{z_h}^1\frac{\mathrm{d}v}{v}\int_0^1\mathrm{d}\zeta\; h_1^q\left(\frac{x_B}{w},\mu^2\right)$$

$$\times\left[\mathcal{C}_{LT}^{q\to qg}(w,v,\zeta,\mu^2)\frac{\mathrm{Re}\,\hat{H}_{FU}^{qg}\left(\frac{z_h}{v},\zeta,\mu^2\right)}{1-\zeta} + \mathcal{C}_{LT}^{q\to\bar{q}q}(w,v,\zeta,\mu^2)\,\mathrm{Re}\,\hat{H}_{FU}^{\bar{q}q}\left(\frac{z_h}{v},\zeta,\mu^2\right)\right]\Bigg\}$$

4. Phenomenology and numerical estimates

Case study: $A_{UT}^{\sin\phi_S}$ *asymmetry*

"One of the more striking results of this analysis is the observation of large sub-leading twist $2\langle \sin\phi_S/\sqrt{2\varepsilon(1+\varepsilon)}\rangle^h_{U\perp}$ *Fourier amplitudes. In particular, they provide the largest twist-3 signal in this measurement. They surprise also with a large kinematic dependence [. . .]"*

– *[HERMES collaboration, 2020]*

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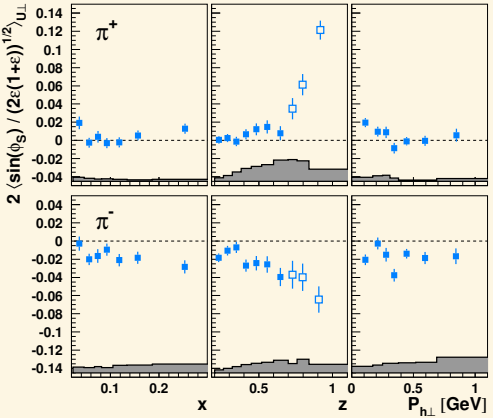
"One of the more striking results of this analysis is the observation of large sub-leading twist $2\langle \sin\phi_S/\sqrt{2\varepsilon(1+\varepsilon)}\rangle^h_{U\perp}$ *Fourier amplitudes. In particular, they provide the largest twist-3 signal in this measurement. They surprise also with a large kinematic dependence [...]"*

– *[HERMES collaboration, 2020]*



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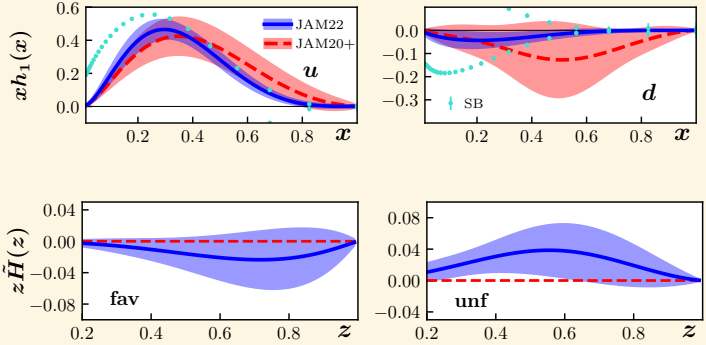
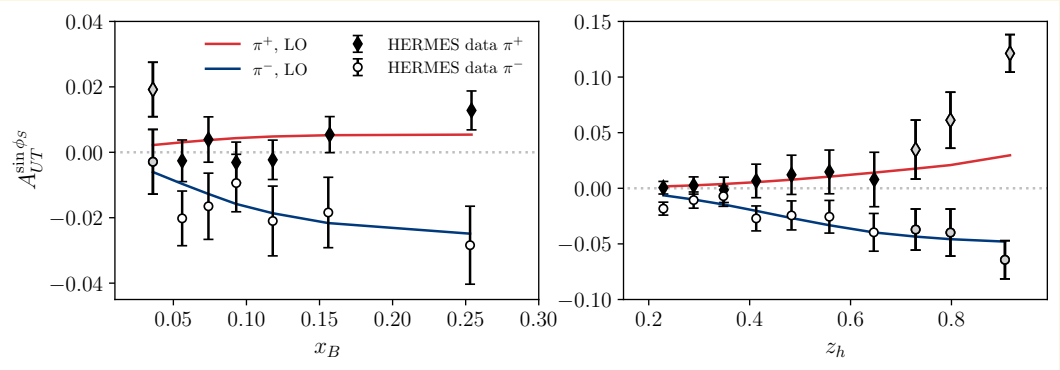


Figure: Transversity PDF $h_1(x)$ and twist-3 FF $\tilde{H}(z)$ from [Gamberg et al., 2022]

Phenomenology and numerical estimates

Asymmetry projections: π^- channel significantly $\neq 0$



Phenomenology and numerical estimates

For NLO curves, we need input models for $\mathrm{Im}\,\hat{H}^{qg}_{FU}(z,\zeta)$ ***and*** $\mathrm{Im}\,\hat{H}^{\bar{q}q}_{FU}(z,\zeta)$

We parametrize such that

- EOM relation is satisfied $\frac{\tilde{H}^q(z)}{2z} = \int_0^1 \mathrm{d}\zeta\, \frac{\mathrm{Im}\,\hat{H}^{qg}_{FU}(z,\zeta)}{1-\zeta}$

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$$\mathrm{Im}\, \hat{H}^{qg}_{FU}(z,\zeta) = \frac{\tilde{H}^q(z)}{2z}\, \mathcal{F}^q(\zeta; 1, a_q, b_q, \gamma_q, \tilde{a}_q, \tilde{b}_q)$$

$$\mathrm{Im}\, \hat{H}^{\bar{q}q}_{FU}(z,\zeta) = \frac{\tilde{H}^q(z)}{2z}\, \mathcal{F}(\zeta; \bar{N}_q, c_q, c_q, \bar{\gamma}_q, \tilde{c}_q, \tilde{c}_q)$$

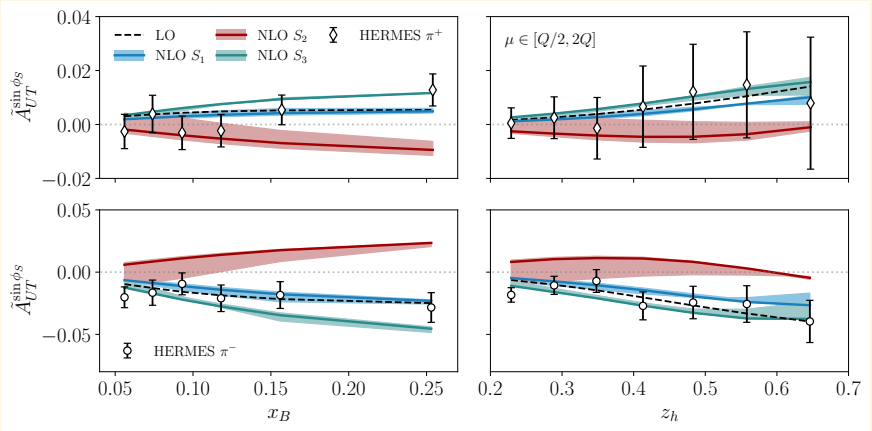
with

$$\mathcal{F}^q(\zeta; N_q, a_q, b_q, \gamma_q, \tilde{a}_q, \tilde{b}_q) = \frac{N_q\, \zeta^{a_q}(1-\zeta)^{b_q}(1+\gamma_q\, \zeta^{\tilde{a}_q}(1-\zeta)^{\tilde{b}_q})}{\mathbb{B}(a_q+2, b_q+1) + \gamma_q\, \mathbb{B}(a_q+\tilde{a}_q+2, b_q+\tilde{b}_q+1)}$$

Phenomenology and numerical estimates

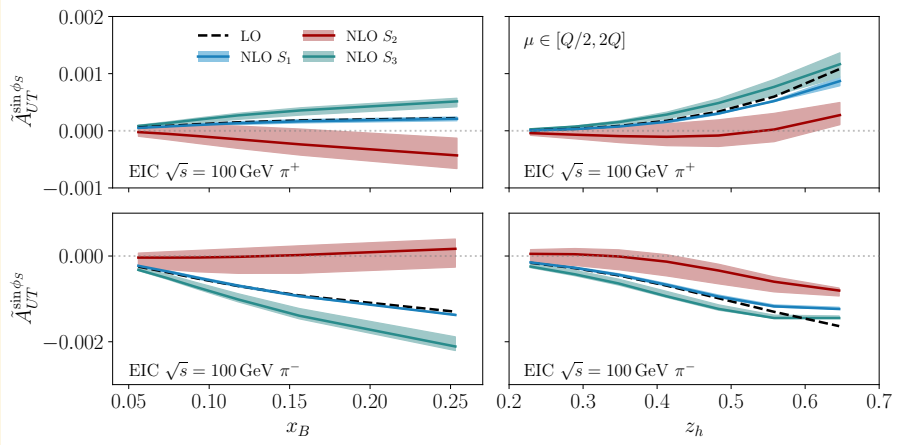
We study 3 model scenarios S_k (ignoring twist-3 FF evolution)

- S_1: similar to LO!
- S_2: corrections so large that SSA changes sign!
- S_3: enhance LO effect!



Phenomenology and numerical estimates

Same $\{x_B, y, z_h\}$ ***at future EIC***



5. Conclusions

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3 main ideas of this work

1. Showed that factorization holds for the $P_{h\perp}$-integrated SIDIS sub-leading twist cross section at NLO in QCD (non-trivial result!)
2. Showcased the importance of NLO corrections to the numerators of SAs, crucial for phenomenology
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Future directions

- Numerically implement evolution equations for chiral-odd twist-3 FFs
- Extraction of $\hat{H}_{FU}^{qg}(z,\zeta)$ and $\hat{H}_{FU}^{\bar{q}q}(z,\zeta)$ from data
- Extension to single-inclusive hadron production in lepton-nucleon collisions

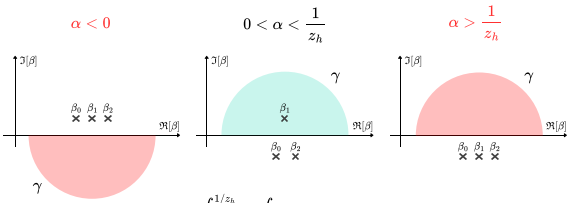
Thank you!

Further details: preprint out in the upcoming days

6. Extra

Contour integration on the light-cone

Decomposing loop momentum as $\mathrm{d}^d r = \mathrm{d}\alpha\, \mathrm{d}\beta\, \mathrm{d}^{d-2} r_\perp$



Scenarios

						qg				
	a_u	a_d	b_u	b_d	$\tilde{a}_u$	$\tilde{a}_d$	$\tilde{b}_u$	$\tilde{b}_d$	γ_u	γ_d
S_1	1.51	1.25	1.86	1.89	0.36	0.95	0.17	0.33	-1.54	-2.49
S_2	1.55	1.85	1.36	1.65	0.51	0.18	0.01	0.16	-1.47	-3.32
S_3	2.46	2.49	1.32	1.91	0.29	0.39	0.55	0.72	-3.64	-1.97

				$\bar{q}q$				
	$\bar{N}_u$	$\bar{N}_d$	c_u	c_d	$\tilde{c}_u$	$\tilde{c}_d$	$\bar{\gamma}_u$	$\bar{\gamma}_d$
S_1	1.14	1.06	2.30	1.48	0.31	0.76	-1.57	-1.88
S_2	1.58	0.61	1.38	1.26	0.15	0.69	-2.24	-3.44
S_3	1.49	2.81	2.05	1.76	0.93	0.13	-2.26	-2.59

Table: Model parameters for the scenarios S_1, S_2 and S_3.