Beyond twist-2 TMDs: Generalized TMDs, NLP TMDs, quasiTMDs

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Generalities on TMDs

Definition

Core non-perturbative(*) objects for processes sensitive to 'transverse-momentum':

$$F(x, \boldsymbol{b}) \sim \int \frac{dz^-}{2\pi} e^{-ixz^-p^+} \langle p| \underbrace{\bar{\psi}(z, \boldsymbol{b}) W^\dagger}_{\overline{U}_1(z,\boldsymbol{b})} \Gamma \underbrace{W \bar{\psi}(0)}_{U_1(0)} |p\rangle$$

$W^\dagger W$ forms a staple-like gauge link between the fields. Examples of processes are:

- Drell-Yan $pp \to \gamma^*/Z + X \to \ell\bar{\ell} + X$ differential in transverse momentum of virtual boson;
- SIDIS $ep \to eh + X$ differential in transverse momentum of produced hadron;

General TMD definition

More in general, define widened class of TMD-like operators by composing U_N operators:

Semi-compact operators of definite geometrical twist

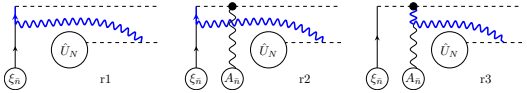
Very approximately: $U_N \sim U_1$ with extra field insertions (more on it later)

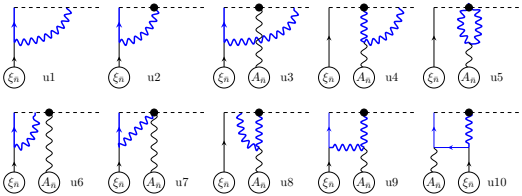
To directions to explore further classes of operators

- Combine $\bar{U}_N U_M$ for $N, M \geq 1$
- Combines multiple U_1 together

Generalities on TMDs
Perturbative structure

Renormalization and evolution





For $\overline{U}_N U_M$ evolution is a double-scale evolution: rapidity and UV scales:

$$\frac{d\log(F(x,\boldsymbol{b}))}{d\log\mu^2} = \frac{\gamma_F(\mu^2,\zeta)}{2} \qquad \frac{d\log(F(x,\boldsymbol{b}))}{d\log\zeta} = D(\mu^2,\boldsymbol{b})$$

The perturbative structure of TMDs

Perturbative structure for small field transverse separation:

$$F(x, \boldsymbol{b}) \to C(x, \log(\mu b)) \otimes f(x, \mu) + O(b^2)$$

TMD matching is OPE at vanishing $\boldsymbol{b}$.
It is not a 1-to-1 map with collinear distributions: on rhs tower increasing twist contributions.

TMD distributions f_1, g_{1L}, h_1, $h_{1T}^{\perp}$

Power	Collinear distributions					
1	$f(x)$					
b^2	$f(x)$	T_3	T_4			
b^4	$f(x)$	T_3	T_4	T_5	T_6	
b^6	$f(x)$	T_3	T_4	T_5	T_6	$\cdots$
	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

TMD distributions $f_{1T}^{\perp}$, g_{1T}, $h_1^{\perp}$, $h_{1L}^{\perp}$

Power	Collinear distributions					
1	$f(x)$	T_3				
b^2	$f(x)$	T_3	T_4	T_5		
b^4	$f(x)$	T_3	T_4	T_5	T_6	$\cdots$
b^6	$f(x)$	T_3	T_4	T_5	T_6	$\cdots$
	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

Why go beyond standard leading-power TMDs?

Several directions can give access to complementary information (compared to experiment) or new information altogether:

- Non-forward kinematics $\Rightarrow$ GTMDs
- power corrections $\Rightarrow$ NLP TMDs $\sim$ twist-three
- lattice-accessible equal-time correlators $\Rightarrow$ quasi-TMDs
- multi-parton hadron amplitudes $\Rightarrow$ baryon LFWFs

TMD factorization, richer operator content, new mixing patterns.

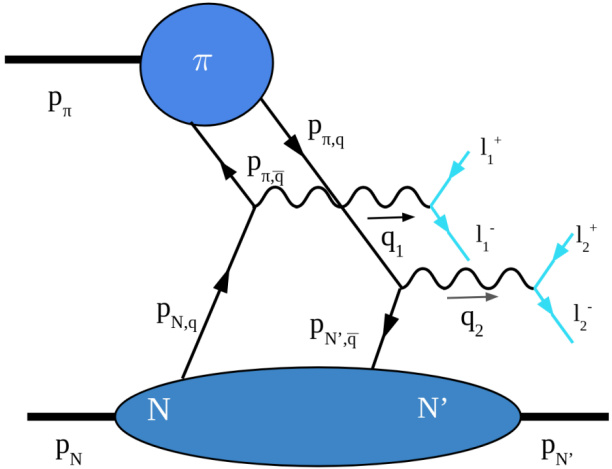
Leading-twist GTMDs: one-loop matching onto GPDs

GTMDs as the non-forward extension of TMDs

- GTMDs depend simultaneously on x, skewness ξ, transverse separation/momentum, and momentum transfer $\boldsymbol{\Delta}_T$.
- Can be thought as either:
  - transverse-momentum extension of Generalized Parton distributions
  - off-forward extension of TMDs
- For phenomenology one needs the analogue of the TMD small-b OPE: GTMDs matched onto GPDs.

$$F_i^{[Y]}(x,\xi,\boldsymbol{b},\boldsymbol{\Delta}_T;\mu,\zeta) = \big[C_{i/j}^{Y/\Gamma}\otimes f_j^{[\Gamma]}\big](x,\xi,\boldsymbol{\Delta}_T;\mu_b) + \mathcal{O}(b\Lambda_{\rm QCD})$$

staple-like Wilson lines, skewed external kinematics, richer structure.



Hard, exclusive reactions with small transverse momentum e.g. Double Drell-Yan in pion-proton where $q^2_{T,(1,2)} \ll q^2_{(1,2)}$

What is different in the GTMD matching problem?

- Matching is not single-helicity channel: different GPD polarizations contribute to one GTMD.
- The ERBL region ($\xi > x$) brings complex phases ($\Rightarrow$ process-dependent) mixing between T-even and T-odd pieces.
- The complete one-loop result is not just a direct adaptation of the forward one.

TMD beyond the forward case have a complex structure

Polarization mixing

i,Y \ j,Γ	q,U	g,U	q,L	g,L	q,T	g,T
q,U	$\bullet$	$\bullet$	0	0	0	$\bullet$
g,U	$\bullet$	$\bullet$	0	0	0	$\bullet$
q,L	0	0	$\bullet$	$\bullet$	0	$\bullet$
g,L	0	0	$\bullet$	$\bullet$	0	$\bullet$
q,T	0	0	0	0	$\bullet$	0
g,T	$\bullet$	$\bullet$	$\bullet$	$\bullet$	0	$\bullet$

Worth noting: for the mixed channels $(L/T), (T/L)$ the DGLAP part of the matching is the difference of the ERBL regions at $\pm\xi$, not the sum, because of the different symmetry properties.

Imaginary part in the evolution

From the UV renormalization constants of the GTMD operator

$$Z^\dagger(k_1^+)Z(k_2^+)$$

$k_{1,2}^+$ is the momentum of the left(right) field in the correlator:

$$k_1^+ = (1-\kappa)xp^+ \ (k_2^+ = (1+\kappa)xp^+)$$

$$Z(k^+) = 1 + \left(\frac{\alpha_s}{4\pi}\right)\frac{C_F}{\epsilon}\left(\frac{3}{2} + 2\log\left(\frac{\delta}{isk^+/p^+}\right)\right)$$

one has

$$Z^\dagger((1-\kappa)xp^+)Z((1+\kappa)xp^+) = (..) - \frac{\alpha_s}{4\pi}\left[\log(-is\mathsf{sign}(1-\kappa)) + \log(is\mathsf{sign}(1+\kappa))\right]$$

$s =$ direction of the staple-like gauge link.

$$-\log(-is\mathsf{sign}(1+\kappa)) - \log(is\mathsf{sign}(1-\kappa)) = -i\pi s\theta(\kappa-1)$$

The master formula

$$\mathbb{F}_i^{[Y]}(\mu_b, \mu_b^2) = \left[\mathcal{C}_{i/j}^{Y/\Gamma,e} \otimes f_j^{[\Gamma]}\right](\mu_b, \mu_b^2) + is\left[\mathcal{C}_{i/j}^{Y/\Gamma,o} \otimes f_j^{[\Gamma]}\right](\mu_b, \mu_b^2)$$

$$\begin{pmatrix} \mathbb{F}_i^{[Y],e}(\mu,\zeta) \\ \mathbb{F}_i^{[Y],o}(\mu,\zeta) \end{pmatrix} = R_i\left[(\mu,\zeta) \leftarrow (\mu_b, \mu_b^2)\right] \times \begin{pmatrix} \cos(\phi(\mu)) & -\sin(\phi(\mu)) \\ s\sin(\phi(\mu)) & s\cos(\phi(\mu)) \end{pmatrix} \left[\begin{pmatrix} \mathcal{C}_{i/j}^{Y/\Gamma,e} \\ \mathcal{C}_{i/j}^{Y/\Gamma,o} \end{pmatrix} \otimes f_j^{[\Gamma]}\right](\mu_b, \mu_b^2)$$

$\phi(\mu) \propto \theta(\kappa - 1)D(b, \mu)$ evolved Collins-Soper kernel

At leading power 32 GTMDs. Of them, 28 have NLO matching

A couple of examples

$$\mathcal{F}^{[Y]}_{i/H}(x,\xi,\boldsymbol{b},\boldsymbol{\Delta}_T) = \bar{u}_{\lambda'}(p')\frac{M^Y_i(x,\xi,\boldsymbol{b},\boldsymbol{\Delta}_T)}{2p^+(1-\xi^2)}u_\lambda(p)$$

$$M^U_i = \gamma^+\left(S^{0;+;i}_{1,1a} + \gamma_5\frac{i\varepsilon^{b\Delta_T}_T}{M|\boldsymbol{b}|}S^{0;+;i}_{1,1b}\right)+\cdots$$

$$\begin{aligned}
S^{0;+;q}_{1,1a} &= C^{U/U}_{q/q}\otimes\left[(1-\xi^2)H^q-\xi^2E^q\right]+C^{U/U}_{q/g}\otimes\left[(1-\xi^2)H^g-\xi^2E^g\right]\\
&\quad-\frac{2(\boldsymbol{b}\cdot\boldsymbol{\Delta}_T)^2-\boldsymbol{b}^2\boldsymbol{\Delta}_T^2}{4M^2\boldsymbol{b}^2}C^{U/T}_{q/g}\otimes\left[E^g_T-\xi\widetilde{E}^g_T+2\widetilde{H}^g_T\right]\\
S^{0;+;q}_{1,1b} &= \frac{(\boldsymbol{b}\cdot\boldsymbol{\Delta}_T)}{2M|\boldsymbol{b}|}C^{U/T}_{q/g}\otimes\left[\xi E^g_T-\widetilde{E}^g_T\right]
\end{aligned}$$

Numerical setup

1) Non-perturbative part: forward TMD of Pavia19

2) U and L GPDs from Goloskokov-Kroll (GK) model

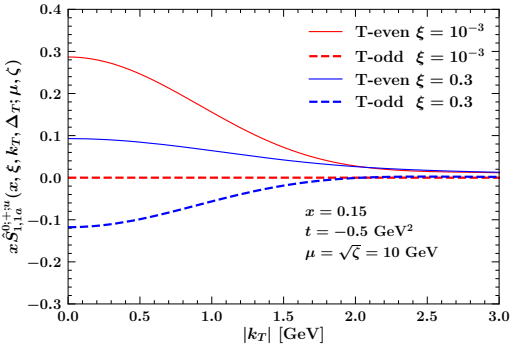
3) Linearly polarized gluon GPDs: arbitrary scaled down of U and L GPDs
$\widetilde{H}^g_T = c\widetilde{H}^g$, $E^g_T = cE^g$, $\widetilde{E}^g_T = c\widetilde{E}^g$, $c \sim 10^{-1}$

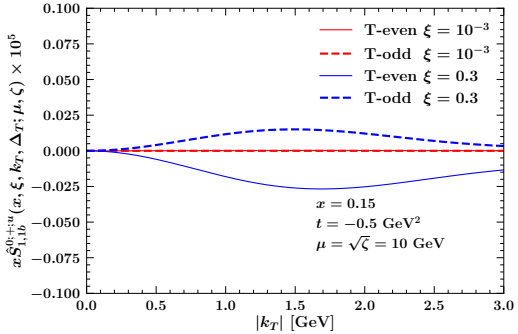
4) Kinematic variables: $M \sim 1$ GeV,

$$|t| \geq |t_{\text{min}}| = \frac{4M^2\xi^2}{1-\xi^2} \Leftrightarrow t = -0.5 \text{ GeV}^2 \quad \xi \lesssim 0.35$$

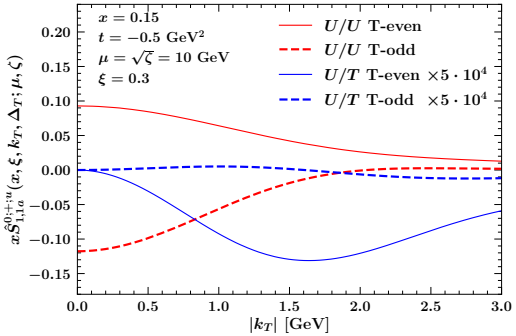
Not full phenomenology: proof of concept for estimating sizes of effects

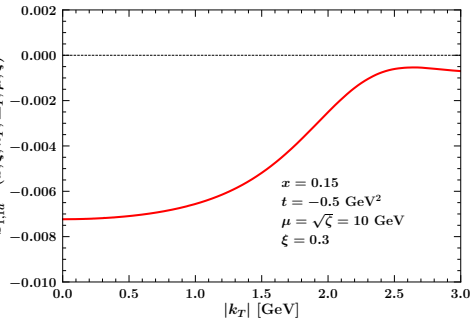
Some results I





Some results II





Next-to-leading power TMDs

Next-to-leading power TMDs
General definitions

Definitions

Recall $F(z, \boldsymbol{b}) \sim \langle \overline{U}_1(z, \boldsymbol{b}) U_1(0) \rangle$
Introduce extra gluon field via

$$U_2^\mu(\{z_1, z_2\}, \boldsymbol{b}) = \mathcal{W} g F^{\mu+}(z_2) \mathcal{W} \psi(z_1, \boldsymbol{b})$$

Construct NLP TMDs as

$$F^{[\Gamma]}_{\mu,21}(z_1, z_2, \boldsymbol{b}) \sim \langle p | \overline{U}_{\mu,2}(z_1, z_2, \boldsymbol{b}) \Gamma U_1(0) | p \rangle$$
$$F^{[\Gamma]}_{\mu,12}(z_1, z_2, \boldsymbol{b}) \sim \langle p | \overline{U}_1(z_1, \boldsymbol{b}) \Gamma U_{\mu,2}(z_2, 0) | p \rangle$$

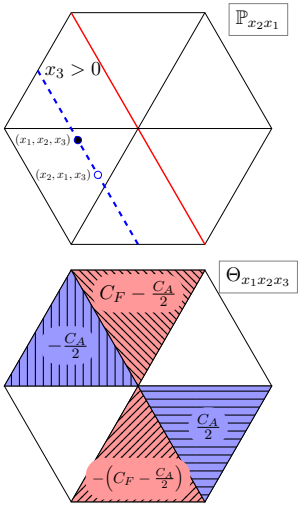
In momentum space: three fractions of momentum $x_{1,2,3}$, one conservation conditions $\delta(x_1 + x_2 + x_3)$.

UV evolution

Evolution is complex and longitudinal-momenta dependent

$$\frac{d}{d\log\mu^2}\Phi^{[\Gamma]}_{\mu,21} = \left(\frac{\Gamma_{\text{cusp}}}{2}\ln\left(\frac{\mu^2}{\zeta}\right) + \Upsilon_{x_1x_2x_3} + 2\pi i s\Theta_{x_1x_2x_3}\right)\Phi^{[\Gamma]}_{\mu,21} + \mathbb{P}^A_{x_2x_1}\otimes\Phi^{[\gamma^\nu\gamma^\mu\Gamma]}_{\nu,21} + \mathbb{P}^B_{x_2x_1}\otimes\Phi^{[\gamma^\mu\gamma^\nu\Gamma]}_{\nu,21}$$

$$\mu^2\frac{d}{d\mu^2}\Phi^{[\Gamma]}_{\mu,12} = \left(\frac{\Gamma_{\text{cusp}}}{2}\ln\left(\frac{\mu^2}{\zeta}\right) + \Upsilon_{x_3x_2x_1} + 2\pi i s\Theta_{x_3x_2x_1}\right)\Phi^{[\Gamma]}_{\mu,12} + \mathbb{P}^A_{x_2x_3}\otimes\Phi^{[\Gamma\gamma^\mu\gamma^\nu]}_{\nu,12} + \mathbb{P}^B_{x_2x_3}\otimes\Phi^{[\Gamma\gamma^\nu\gamma^\mu]}_{\nu,12}$$



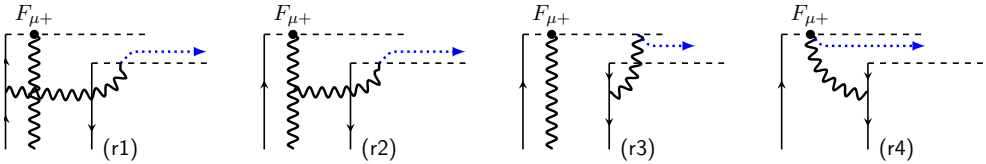
Definite T-parity combinations: evolution real, but mixes $\oplus$ and $\ominus$ distributions.

$$\Phi^{[\Gamma]}_{\mu,\oplus}(x_{123},b) = \frac{\Phi^{[\Gamma]}_{\mu,21}(x_{123},b) + \Phi^{[\Gamma]}_{\mu,12}(-x_{321},b)}{2}$$

$$\Phi^{[\Gamma]}_{\mu,\ominus}(x_1,x_2,x_3,b) = i\frac{\Phi^{[\Gamma]}_{\mu,21}(x_{123},b) - \Phi^{[\Gamma]}_{\mu,12}(-x_{321},b)}{2}$$

Correlator	T-even	T-odd
$\Phi^{[\Gamma]}_{11}$	$f_1,\ g_1,\ g_{1T},\ h_1,\ h^{\perp}_{1L},\ h^{\perp}_{1T}$	$f^{\perp}_{1T},\ h^{\perp}_1$
$\Phi^{[\gamma^+]}_{\mu\bullet}$	$f_{\oplus T},\ f^{\perp}_{\oplus L},\ f^{\perp}_{\oplus T},\ f^{\perp}_{\ominus}$	$f^{\perp}_{\oplus},\ f_{\ominus T},\ f^{\perp}_{\ominus L},\ f^{\perp}_{\ominus T}$
$\Phi^{[\gamma^+\gamma^5]}_{\mu\bullet}$	$g^{\perp}_{\oplus},\ g_{\ominus T},\ g^{\perp}_{\ominus L},\ g^{\perp}_{\ominus T}$	$g_{\oplus T},\ g^{\perp}_{\oplus L},\ g^{\perp}_{\oplus T},\ g^{\perp}_{\ominus}$
$\Phi^{[i\sigma^{\alpha+}\gamma^5]}_{\mu\bullet}$	$h_{\oplus},\ h^{\perp}_{\oplus},\ h_{\ominus L},\ h^{D\perp}_{\ominus T},$ $h^{S\perp}_{\ominus T},\ h^{A\perp}_{\ominus T},\ h^{T\perp}_{\ominus T},\ h^{\perp}_{\ominus L}$	$h_{\oplus L},\ h^{D\perp}_{\oplus T},\ h^{S\perp}_{\oplus T},\ h^{A\perp}_{\oplus T},$ $h^{T\perp}_{\oplus T},\ h^{\perp}_{\oplus L},\ h_{\ominus},\ h^{\perp}_{\ominus}$

Rapidity evolution



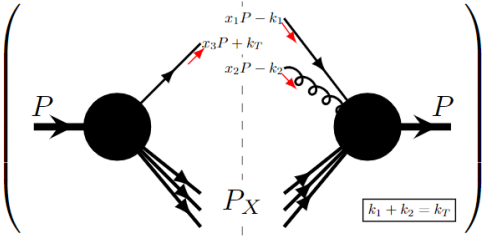
(r1,r2,r3) 'standard' rapidity divergence diagrams: $\Phi^{\mu}_{21,12} = R(b^2)\Phi^{\mu}_{21,12}$.

(r4) 'special' rapidity divergence: $\Phi^{\mu}_{21,12} = \partial^{\mu} D(b^2)\log(\delta)\Phi_{11}$.

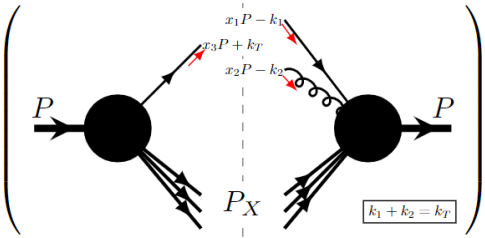
Additive divergences, must cancel in observables in-between terms

Physical picture

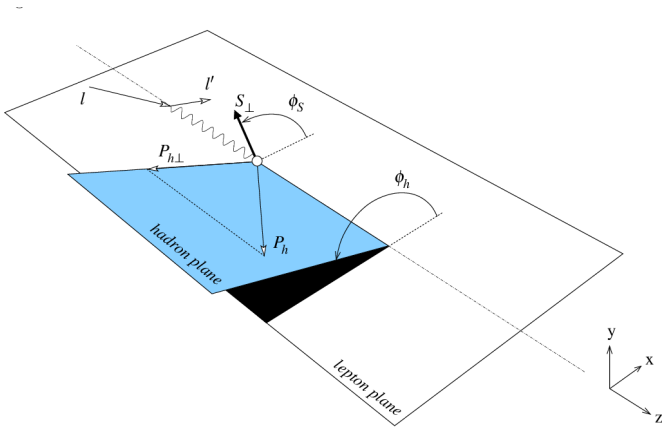
$$\Phi_{\oplus}(x_1 < 0, x_2 < 0, x_3 > 0, k_T) \propto \mathrm{Re}$$



$$\Phi_{\ominus}(x_1 < 0, x_2 < 0, x_3 > 0, k_T) \propto \mathrm{Im}$$



Physical observables



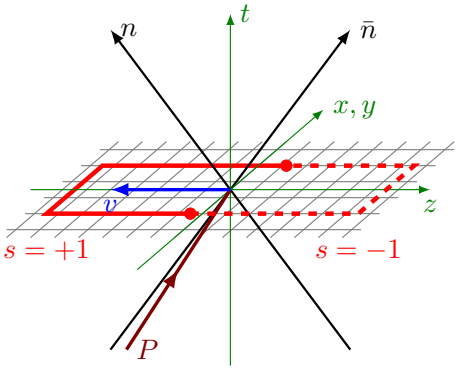
LP and NLP TMDs contributes to different angular modulations

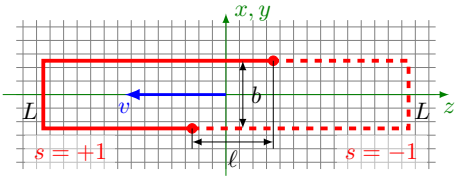
E.g.

$F_{UU}^{\cos 2\phi} \Rightarrow$ LP

$F_{UU}^{\cos \phi} \Rightarrow$ NLP

Lattice correlators





Next-to-leading power TMDs
Computational technique

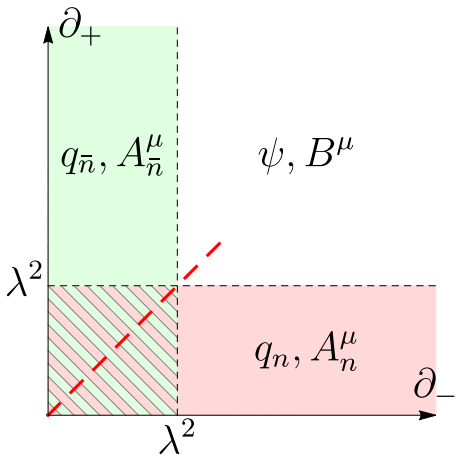
The background field method

Introducing copies of QCD fields like

$$\phi(y) \to \phi(y) + \phi_n(y) + \phi_{\bar{n}}(y)$$

with specific scaling properties

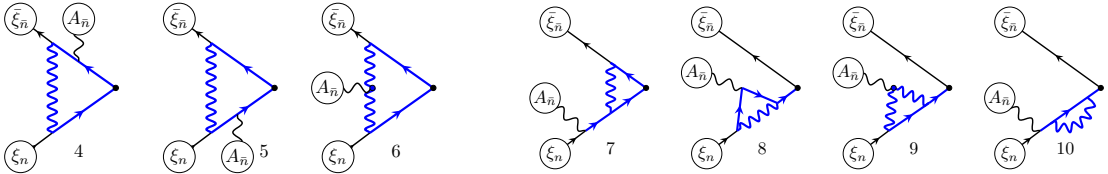
$$\{\partial^+, \partial^-, \partial_T\}\phi_{\bar{n}} \lesssim P^+\{1, \lambda^2, \lambda\}\phi_{\bar{n}}$$



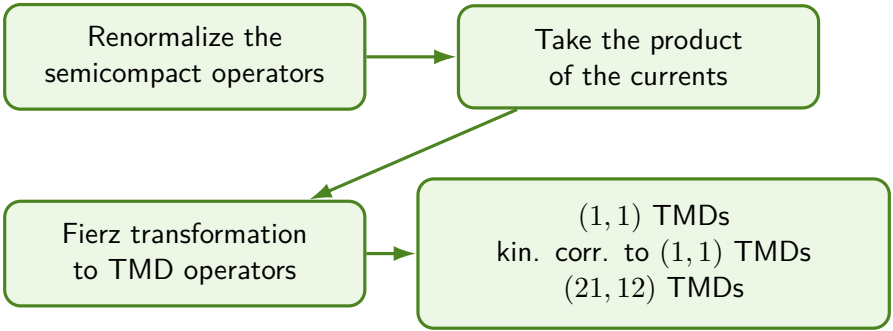
SIDIS Diagram examples

Power expansion happens at the level of the e.m. current of SIDIS

$$J^\mu[\psi+\psi_n+\psi_{\bar n},\cdots]=\bar\psi_{\bar n}\gamma^\mu\psi_n+\bar\psi\gamma^\mu\psi_n+\cdots$$



Workflow


Renormalize the semicompact operators
Take the product of the currents
Fierz transformation to TMD operators
$(1,1)$ TMDs
kin. corr. to $(1,1)$ TMDs
$(21,12)$ TMDs

Next-to-leading power TMDs
Results

Results

Coefficient functions for genuine NLP part is complex! Together with transformation to $\oplus, \ominus$ basis leads to real cross section.

$$\widetilde{W}^{\mu\nu}_{\mathsf{kNLP}}(y) = -i|C_1(\mu^2, Q^2)|^2 \sum_{n,m}$$

$$\frac{\bar{n}^\mu \mathrm{Tr}[\gamma^\rho \overline{\Gamma}^+_m \gamma^\nu \overline{\Gamma}^-_n] + \bar{n}^\nu \mathrm{Tr}[\gamma^\mu \overline{\Gamma}^+_m \gamma^\rho \overline{\Gamma}^-_n]}{q_-} \Phi^{[\Gamma^+_n]}_{11} \left(\frac{\partial}{\partial b^\rho} - \frac{\partial_\rho \mathscr{D}}{2} \ln\left(\frac{\zeta}{\bar{\zeta}}\right)\right) \Delta^{[\Gamma^-_m]}_{11} + \cdots$$

$$\widetilde{W}_{\mathsf{gNLP}} = i \sum_{n,m} \int_{-\infty}^{\infty} d\hat{u}_1 d\hat{u}_2 d\hat{u}_3 \, \delta(\hat{u}_1 + \hat{u}_2 + \hat{u}_3) \delta(\tilde{x} - \hat{u}_3)$$

$$T^{\mu\nu\rho}_-(\bar{n}, n) \left(\mathbb{C}_R(x, \hat{u}_2) \mathbf{\Phi}^{[\Gamma^+_n]}_{\rho,\oplus} \Delta^{[\Gamma^-_m]}_{11} + \pi \mathbb{C}_I(x, \hat{u}_2) \mathbf{\Phi}^{[\Gamma^+_n]}_{\rho,\ominus} \Delta^{[\Gamma^-_m]}_{11} \right) + \cdots$$

Lattice correlator

The same logic can be applied to equal-time correlators viewed as lattice 'observables'
The current is now the operator

$$J_v^i(y;L) = [\infty b + Lv, y + Lv][y + Lv, y]q^i(y)$$

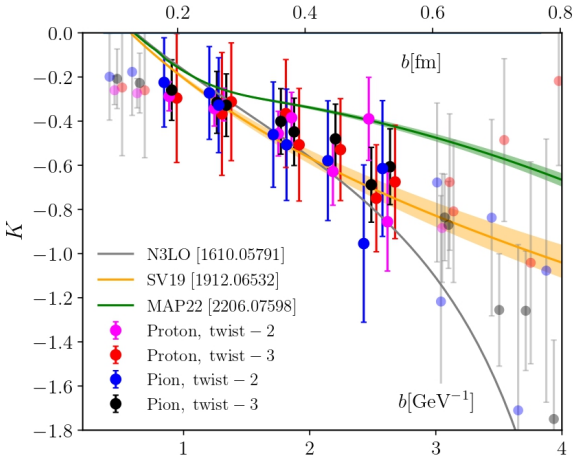
The background fields are the $\bar{n}$ field and a v-field, that scales

$$\{\partial_+, \partial_-, \partial_T\}q_v \lesssim P_+\{\lambda^2, \lambda^2, \lambda\}q_v$$

Full factorized form

$$
\begin{aligned}
\Omega^{[\Gamma]}(x,b,\mu) = {} & \Psi(b;\mu,\bar{\zeta})\mathbb{C}_{11}\Phi_{11}^{[\Gamma]}(x,b;\mu,\zeta) \\
& + \frac{i}{2xP_+}\mathbb{C}_{11}\Psi(b)\left(\partial_\mu - \frac{1}{2}[\partial_\mu \mathcal{D}(b,\mu)]\ln\left(\frac{\bar{\zeta}}{\zeta}\right)\right)\Phi_{11}^{\left[\gamma^\mu\gamma^+\Gamma+\Gamma\gamma^+\gamma^\mu\right]}(x,b;\mu,\zeta) \\
& + \frac{1}{2xP_+}\mathbb{C}_{11v}\boldsymbol{\Psi}_{\mu,21}^{(0)}(b;\mu,\bar{\zeta})\Phi_{11}^{\left[\gamma^\mu\gamma^+\Gamma+\Gamma\gamma^+\gamma^\mu\right]}(x,b;\mu,\zeta) \\
& + \frac{i}{2xP_+}\Psi(b;\mu,\bar{\zeta})\int_{-1}^{1}dx_2\Big[\mathbb{C}_R(x,x_2)\boldsymbol{\Phi}_{\mu,\oplus}^{\left[\gamma^\mu\gamma^+\Gamma-\Gamma\gamma^+\gamma^\mu\right]}(\tilde{x},b;\mu,\zeta) \\
& \qquad + s\pi\mathbb{C}_I(x,x_2)\boldsymbol{\Phi}_{\mu,\ominus}^{\left[\gamma^\mu\gamma^+\Gamma-\Gamma\gamma^+\gamma^\mu\right]}(\tilde{x},b;\mu,\zeta) \\
& \qquad - i\mathbb{C}_R(x,x_2)\boldsymbol{\Phi}_{\mu,\ominus}^{\left[\gamma^\mu\gamma^+\Gamma+\Gamma\gamma^+\gamma^\mu\right]}(\tilde{x},b;\mu,\zeta) \\
& \qquad + is\pi\mathbb{C}_I(x,x_2)\boldsymbol{\Phi}_{\mu,\oplus}^{\left[\gamma^\mu\gamma^+\Gamma+\Gamma\gamma^+\gamma^\mu\right]}(\tilde{x},b;\mu,\zeta)\Big]
\end{aligned}
$$

How to use it

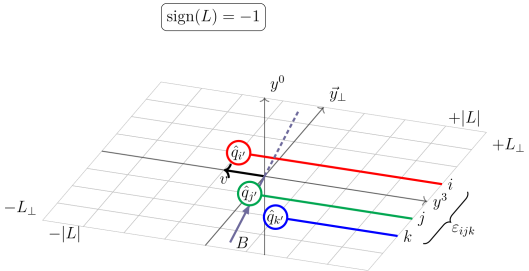


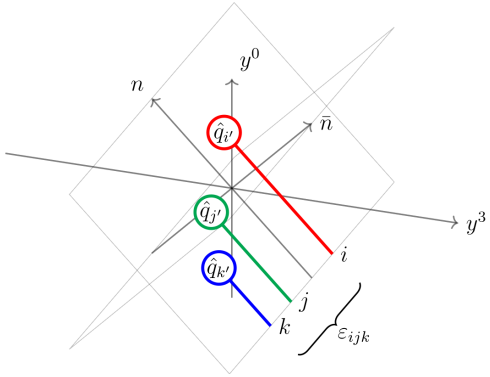
Construct ratios of qTMDs

$$\frac{\int dx \Omega^{[\Gamma]}(x, b, \mu; P_1)}{\int dx \Omega^{[\Gamma]}(x, b, \mu; P_2)}$$

is directly related to Collins-Soper kernel

Baryon (quasi-)LFWFs





$$\widetilde{\Omega}_v(y_1, y_2, y_3; P, S) = \langle 0|\varepsilon_{ijk} J_{v,i}(y_1)\, J_{v,j}(y_2)\, J_{v,k}(y_3)|P, S\rangle$$

Can re-use most of the results from qTMDs (same currents, different combinations).

Also in this case, complex coefficient function and complex evolution

$$C_{111;\overline{\mathsf{MS}}}(\{x_q\}) = \prod_{q=1}^{3} C_{1,\mathsf{fin};\overline{\mathsf{MS}}}(x_q) = \cdots + a_s C_F i\pi \mathsf{sign}(L) \sum_{q=1}^{3} \mathsf{sign}(x_q)$$

$$\Phi_{111,\mathsf{sub}}(\mu, \{\zeta_q\}) = \left(\prod_{q=1}^{3} \left(\frac{\zeta_q}{\zeta_{q,0}} \right)^{-\frac{1}{4}\sum_{w\neq q'} \mathscr{D}\left(b^2_{wq',\mu}\right)} \exp \int \frac{d\mu^2}{\mu^2} \gamma_{U1}\left(x_q, \frac{\zeta_q}{\mu^2}\right) \right) \Phi_{111,\mathsf{sub}}(\mu_0, \{\zeta_{q,0}\})$$

Conclusions

1. Generalized TMDs: complex evolution in ERBL and complex matching
2. Next-to-leading power factorization of SIDIS and quasi-TMDs
3. Small preview of baryon quasi-LFWFs

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