Percolation in Excitable Media: Insights into Signal Propagation in Heart-like Systems

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In collaboration with Dr. Sanchari Goswami

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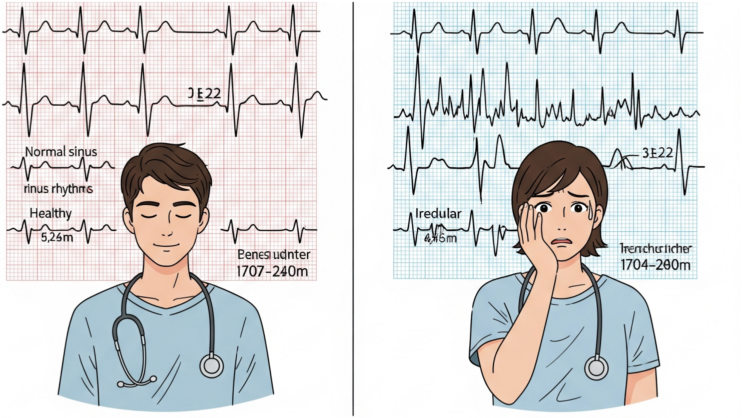
Motivation to Study Biological Cells



Modeling signal propagation as a non-equilibrium percolation process allows us to bridge biological complexity with tools from statistical mechanics. This helps understand arrhythmic transitions as critical phenomena.

What is Percolation?

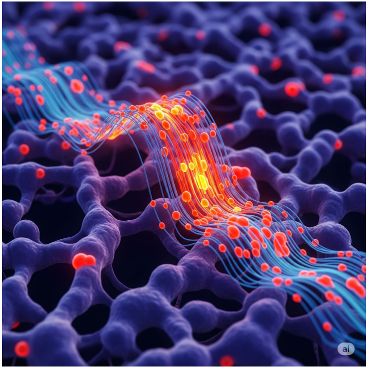
Percolation is a well-established framework in statistical mechanics. Here, we apply it to understand signal propagation through excitable biological tissue—a far-from-equilibrium, disordered system. The model captures phase-transition-like behavior: a sharp shift from signal failure to sustained propagation.

Two type of sites,

- Open: allows *something* propagation.
- Close: blocks *something* propagation (damaged or refractory tissue).

In our case *Something* is *Electrical Signal*

Connecting Statistical Mechanics and Biophysics



Excitable biological tissues, such as cardiac networks, operate far from equilibrium. We use percolation theory—a central tool in statistical mechanics—to model how localized activation can lead to system-wide transitions, much like energy barriers in chemical reactions.
This model, though simple, reveals key signatures of transition-like behavior, including signal failure and critical thresholds.

Types of Percolation

Directed

Only forward direction is allowed.

Isotropic

All directions are allowed.

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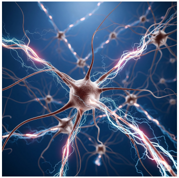

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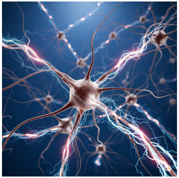
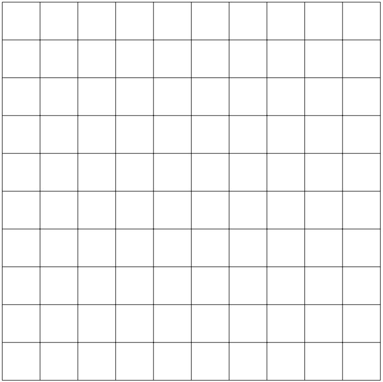



Example: Network Connectivity in Internet, Electrical signal through neurons etc.

Model Description

- Categorizing three types of cells,



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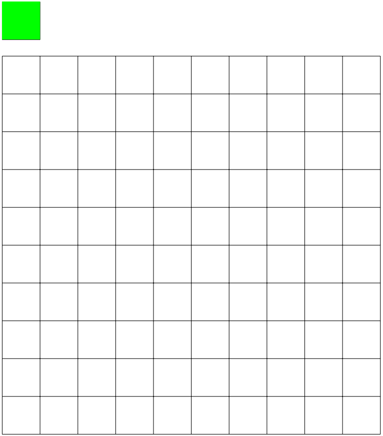

- Categorizing three types of cells,
  1. Active cell: contains Action Potential(AP).

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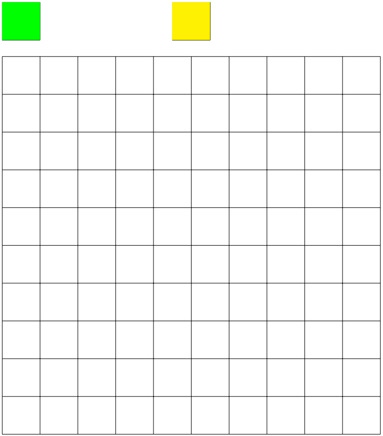

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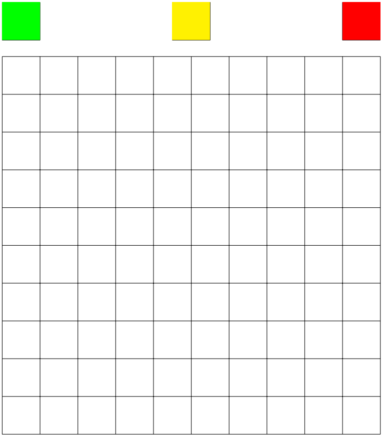

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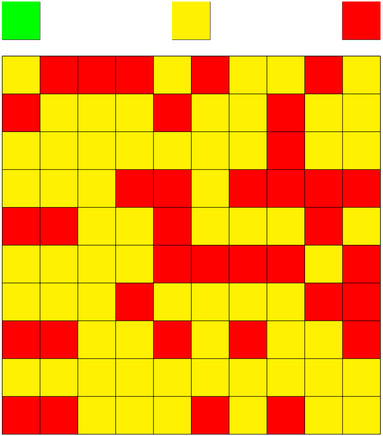

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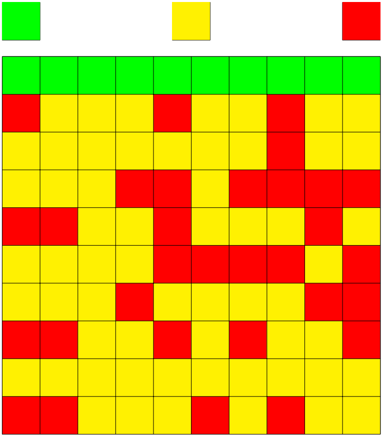

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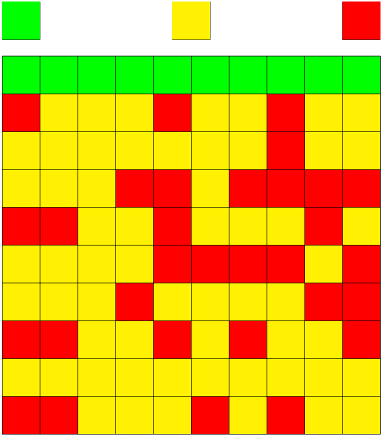

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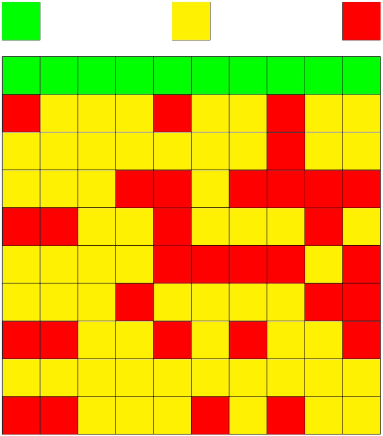

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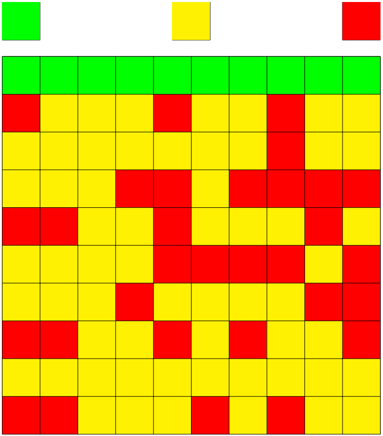

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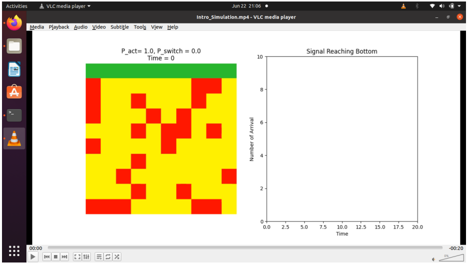
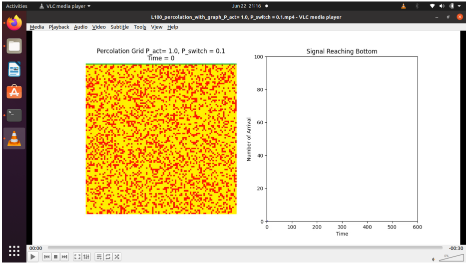
- Transformation rule:

Simulation

$$p_{act} = 1 \ \& \ p_{switch} = 0$$



$$p_{act} = 1 \ \& \ p_{switch} = 0.1$$



Study of No. of Arrival N_A

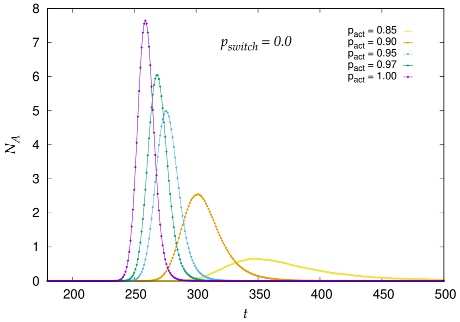


Figure 1: N_A versus t with $L = 200$, $p_{switch} = 0$ and $0.85 \leqslant p_{act} \leqslant 1$.

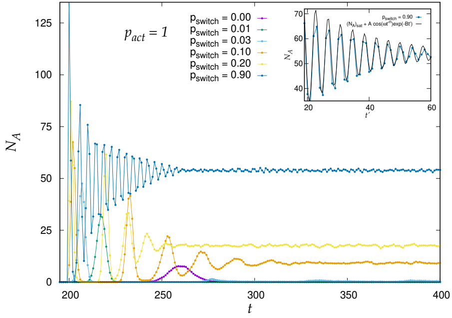


Figure 2: N_A versus t with $L = 200$, $p_{act} = 1$ and $0 \leqslant p_{switch} \leqslant 1$.

Fraction of Percolating Path P_N

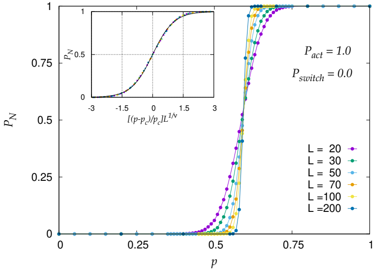


Figure 3: P_N versus p with various L and in the inset data collaps by $\frac{1+\tan \kappa x}{2}$ with $\nu = \frac{4}{3}$ and $\kappa = 0.96$.

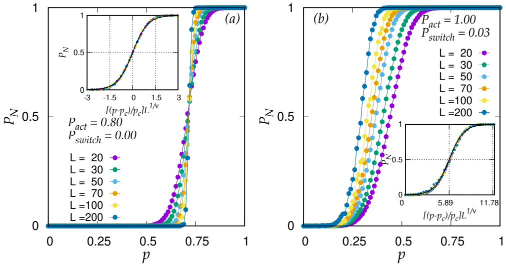
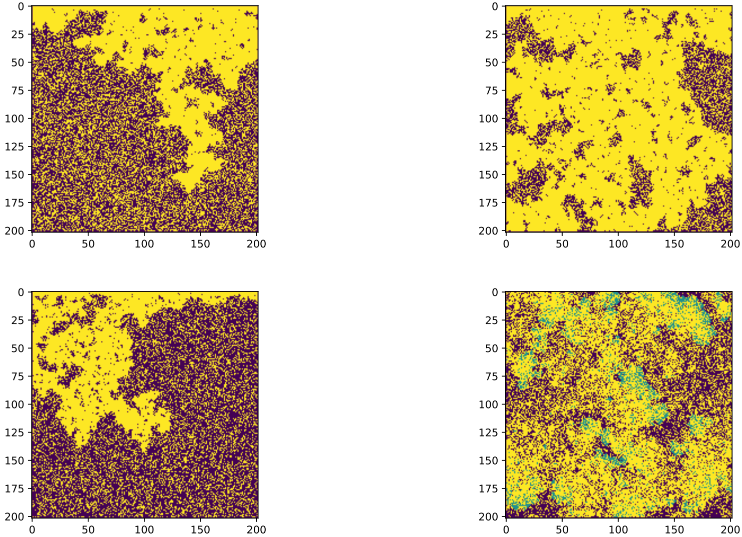


Figure 4: P_N versus p with various L, (a) $p_{act} = 0.8$ and $p_{switch} = 0$ data collaps has been shown $\frac{1+\tan \kappa x}{2}$ with $\nu = \frac{4}{3}$ and $\kappa = 1$, (b) $pact = 1$ and $p_{switch} = 0.03$ data collaps fitted by $\frac{1+\tan \kappa(x-\epsilon)}{2}$, $\nu = 3.7$, $\kappa = 0.52$ and $\epsilon = 5.89$.

Snapshots



What is Tortuosity?

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Results

What is Tortuosity?

$$\text{Tortuosity,} \quad \tau = \frac{\text{Total distance traveled}}{\text{Shortest distance}} = \frac{x_0}{L} \qquad (1)$$

Tortuosity τ

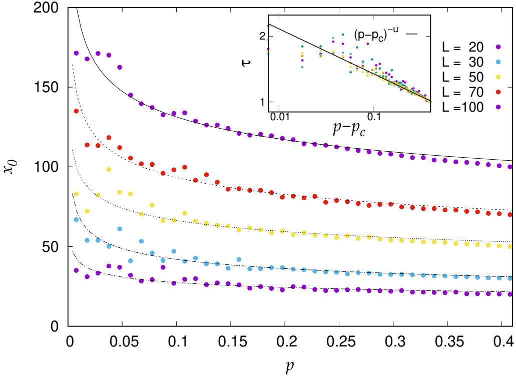


Figure 5: Average distance travelled x_0 vs $p - p_c$ with $p_{act} = 1.00$ and $p_{switch} = 0$. In inset Tortuosity τ vs $p - p_c$ has shown.

$$\tau \propto (p - p_c)^{-u} \tag{2}$$

Largest cluster size M

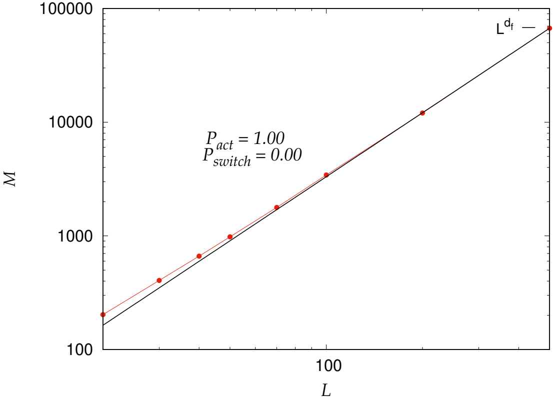


Figure 6: M vs L with $p_{act} = 1.00$ and $p_{switch} = 0.0$ here $d_f = 1.811 \pm 0.011$.

$$M \propto L^{d_f} \tag{3}$$

Cluster size distribution

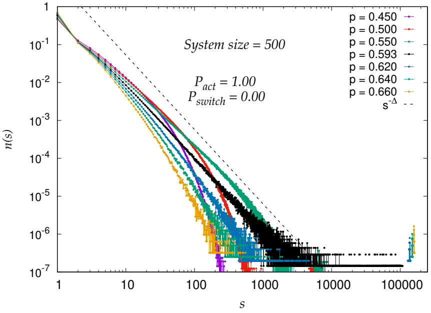


Figure 7: $n(s)$ versus s with $L = 500$, $p_{act} = 1.0$ and $p_{switch} = 0$ here $\Delta = 1.964 \pm 0.030$.

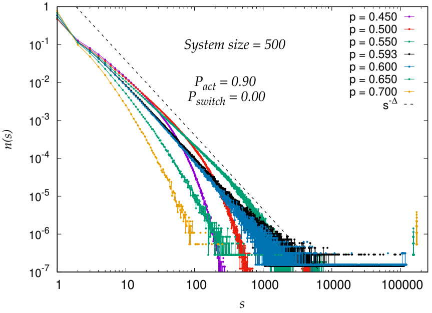


Figure 8: $n(s)$ versus s with $L = 500$, $p_{act} = 0.9$ and $p_{switch} = 0$ here $\Delta = 1.911 \pm 0.002$.

$$n_s \propto s^{-\Delta} \exp(-s/s_{max}) \tag{4}$$

Results

Weight of the Percolating Cluster,

$$P_\infty \sim (p - p_c)^\beta \tag{5}$$

Mean Cluster Size,

$$\langle s \rangle \sim (p - p_c)^{-\gamma} \tag{6}$$

Cutoff Cluster Size,

$$s_{max} \sim |p - p_c|^{-1/\sigma} \tag{7}$$

$$\sum_{0}^{\infty} n_s \sim (p - p_c)^{2-\alpha} \tag{8}$$

Critical Exponents

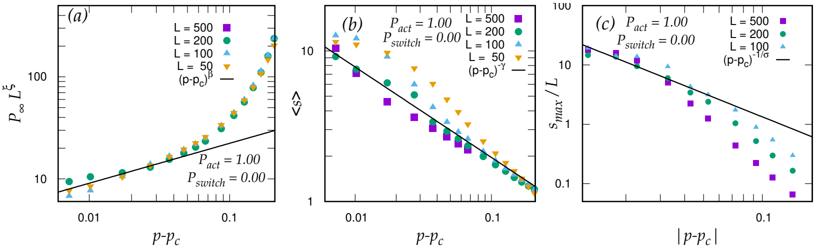


Figure 9: Study of critical exponents : (a) $P_\infty L^\xi$ vs $p - p_c$ with $\xi = 1.85$, which gives $\beta = 2.859$, (b) $\langle s \rangle$ vs $p - p_c$ with $\gamma = 0.600 \pm 0.021$ and (c) $\frac{s_{max}}{L}$ vs $|p - p_c|$ which gives $\sigma = 0.377$.

These exponents approximately satisfy the relation,

$$\beta + \gamma = \frac{1}{\sigma} \tag{9}$$

Critical Exponents

Model	Δ	α	β	γ	σ	ν
Heart Model $p_{act} = 1.0$ $p_{switch} = 0.0$	1.964	2.0004	2.859	0.600	0.377	4/3

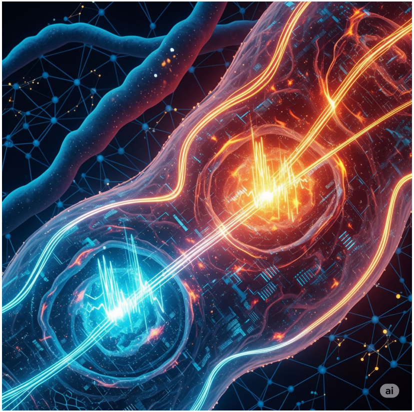
Table 1: Critical exponents.

Therefore,

$$\alpha + 2\beta + \gamma \geqslant 2 \tag{10}$$

Rushbrooke Inequality is satisfied.

Toward AI-Driven Analysis

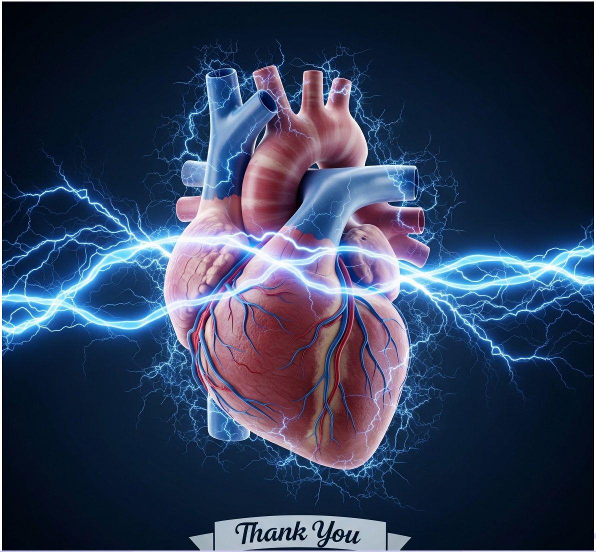


Percolation-generated data offers a foundation for machine learning applications in cardio physics helping identify thresholds for signal failure or predict arrhythmic risk from structural patterns.

Conclusion:

Our study illustrates how percolation theory can reveal critical thresholds in cardiac systems—providing a statistical mechanics lens on arrhythmogenic transitions.
Looking forward, the patterns and thresholds uncovered here could guide data-driven tools or machine learning models for arrhythmia prediction.

M.A.Molla, S.Goswami, "An insight of heart-like systems with percolation", Physics Letter A, **518**, 129695 (2024).


Thank You