

Studies of the isospin symmetry in the hyperon-nucleon interactions

Raffaele Del Grande*



Istituto Nazionale di Fisica Nucleare
LABORATORI NAZIONALI DI FRASCATI

Discrete symmetries in particle, nuclear and atomic physics and implications for our Universe



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*raffaele.delgrande@lnf.infn.it

Isospin symmetry

Proposed by Heisenberg after the discovery of neutron in 1932:

“The proton and the neutron can be regarded as two states of a single particle” (nucleon)

Motivation:

$$\begin{aligned} m_p &= 938.28 \text{ MeV}/c^2 \\ m_n &= 939.57 \text{ MeV}/c^2 \end{aligned}$$

The mass equivalence can be viewed as an energy degeneracy of the underlying interactions.

$$E = m \cdot c^2$$

→ **approximate symmetry of the strong interaction**
(originally believed to be exact).

→ u and d quarks have **similar masses** $m_u \sim m_d$, interchanging them works okay $u \leftrightarrow d$; interchanging **other quarks doesn't work as well**.



Werner Karl Heisenberg

Isospin properties

Isospin has exactly the same properties as spin:

$$\begin{aligned} [T_1, T_2] &= i T_3 & [T_2, T_3] &= i T_1 & [T_3, T_1] &= i T_2 \\ [T^2, T_i] &= 0 & T^2 &= T_1^2 + T_2^2 + T_3^2 \end{aligned}$$

- Eigenstates analogous to ordinary angular momentum

$$T^2 |I, I_3\rangle = I(I+1) |I, I_3\rangle \quad T_i |I, I_3\rangle = I_i |I, I_3\rangle$$

- T_1, T_2 and T_3 are non-commuting operators \Rightarrow cannot know observables I_1, I_2, I_3 simultaneously
- Isospin ladder operators for I_3 :

$$T_- = T_1 - i T_2 \quad (u \rightarrow d) \quad T_+ = T_1 + i T_2 \quad (d \rightarrow u)$$

$$T_- |I, I_3\rangle = \sqrt{I(I+1) - I_3(I_3 - 1)} |I, I_3 - 1\rangle \quad T_+ |I, I_3\rangle = \sqrt{I(I+1) - I_3(I_3 + 1)} |I, I_3 + 1\rangle$$

- Combination of isospin:

$$|I^{(1)}, I_3^{(1)}\rangle |I^{(2)}, I_3^{(2)}\rangle \rightarrow |I, I_3\rangle$$

$$I_3 = I_3^{(1)} + I_3^{(2)} \text{ additive}$$

$$I = |I^{(1)} - I^{(2)}|, |I^{(1)} - I^{(2)}| + 1, \dots, |I^{(1)} + I^{(2)}|$$

Clebsch-Gordan coefficients

Iso-multiplets

Isospin multiplets can be defined in the same way as for angular momentum and spin.

Using the notation $|I, I_3\rangle$ we have:

- Iso-doublet

$$\begin{aligned} m_p &= 938.28 \text{ MeV}/c^2 & |p\rangle &= |\tfrac{1}{2}, \tfrac{1}{2}\rangle \\ m_n &= 939.57 \text{ MeV}/c^2 & |n\rangle &= |\tfrac{1}{2}, -\tfrac{1}{2}\rangle \end{aligned}$$

- Iso-triplet

$$\begin{aligned} m_{\pi^+} &= 139.570 \text{ MeV}/c^2 & |\pi^+\rangle &= |1, +1\rangle \\ m_{\pi^0} &= 134.977 \text{ MeV}/c^2 & |\pi^0\rangle &= |1, 0\rangle \\ m_{\pi^-} &= 139.570 \text{ MeV}/c^2 & |\pi^-\rangle &= |1, -1\rangle \end{aligned}$$

- Iso-quadruplet

$$\begin{aligned} m_\Delta &= 1232 \text{ MeV}/c^2 & |\Delta^{++}\rangle &= |\tfrac{3}{2}, \tfrac{3}{2}\rangle & |\Delta^+\rangle &= |\tfrac{3}{2}, \tfrac{1}{2}\rangle \\ & & |\Delta^0\rangle &= |\tfrac{3}{2}, -\tfrac{1}{2}\rangle & |\Delta^-\rangle &= |\tfrac{3}{2}, -\tfrac{3}{2}\rangle \end{aligned}$$

Heisenberg postulates:

- p and n are up/down states of $I=\frac{1}{2}$
- pions are +1,0,-1 states of $I=1$
- strong interaction is invariant under isospin rotations
- isospin symmetry is violated by electromagnetic and weak interactions (e.g. β decay)

NN scattering Cross Sections

Isospin conservation is used to estimate the ratio between cross sections.

Example: Nucleon-Nucleon scattering

$$\begin{aligned}
 1) \quad p + p &\rightarrow d + \pi^+ \\
 2) \quad p + n &\rightarrow d + \pi^0
 \end{aligned}
 \Rightarrow \frac{\sigma(pp \rightarrow d\pi^+)}{\sigma(pn \rightarrow d\pi^0)} \sim \frac{|T_{pp \rightarrow d\pi^+}|^2}{|T_{pn \rightarrow d\pi^0}|^2}$$

$\rightarrow T_{fin, in} = \langle \psi_{fin} | V | \psi_{in} \rangle$ is the amplitude for the transition $in \rightarrow fin$
 \rightarrow The ratio of the phase spaces is 1.

Reaction 1):

initial state $|p\rangle = |\frac{1}{2}, \frac{1}{2}\rangle \quad \times \quad |p\rangle = |\frac{1}{2}, \frac{1}{2}\rangle \rightarrow |pp\rangle = |1, 1\rangle$

final state $|d\rangle = |0, 0\rangle \quad \times \quad |\pi^+\rangle = |1, 1\rangle \rightarrow |d\pi^+\rangle = |1, 1\rangle$

Reaction 2):

initial state $|p\rangle = |\frac{1}{2}, \frac{1}{2}\rangle \quad \times \quad |n\rangle = |\frac{1}{2}, -\frac{1}{2}\rangle$
 $\rightarrow |pn\rangle = 1/\sqrt{2} [|1, 0\rangle + |0, 0\rangle]$

final state $|d\rangle = |0, 0\rangle \quad \times \quad |\pi^0\rangle = |1, 0\rangle \rightarrow |d\pi^0\rangle = |1, 0\rangle$

Clebsch-Gordan coefficients

$$1/2 \times 1/2$$

		1		
	+1	1	0	
+1/2 + 1/2	1	0	0	
+1/2 - 1/2	1/2	1/2	1	
-1/2 + 1/2	1/2	-1/2	-1	
	-1/2 - 1/2		1	

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- 2) $p + n \rightarrow d + \pi^0$ \rightarrow The ratio of the phase spaces is 1.

$$T_{pp \rightarrow d\pi^+} = \langle d\pi^+ | V | pp \rangle = \langle 1, 1 | V | 1, 1 \rangle = T_{I=1}$$

$$T_{pn \rightarrow d\pi^0} = \langle d\pi^0 | V | pn \rangle = 1/\sqrt{2} \langle 1, 1 | V [| 1, 1 \rangle + | 0, 0 \rangle] = 1/\sqrt{2} T_{I=1}$$

$$\frac{\sigma(pp \rightarrow d\pi^+)}{\sigma(pn \rightarrow d\pi^0)} = 2$$

Energy of incident nucleon, Mev	Reaction	Angular distribution	Total cross section 10^{-27} cm^2	Reference
580 600	$p+p \rightarrow \pi^+ + d$ $n+p \rightarrow \pi^0 + d$	$(0.216 \pm 0.033) + \cos^2 \theta^*$ $(0.220 \pm 0.022) + \cos^2 \theta^*$	3.10 ± 0.24 1.5 ± 0.3	[²] Present experiment [⁵]
660 610	$p+p \rightarrow \pi^+ + d$ $p+p \rightarrow \pi^+ + d$	$(0.23 \pm 0.03) + \cos^2 \theta^*$	3.1 ± 0.2 3.15 ± 0.22	

[V. B. Flagin et al., Sov. Phys. JETP 35(8), 592 (1959)]

K⁻N and K⁻ multi-N absorptions

TABLE III. Branching ratios for K⁻ absorption at rest.

Reaction	Events/(stopping K ⁻) (%)
K ⁻ He ⁴ → Σ ⁺ π ⁻ H ³	9.3±2.3
→ Σ ⁺ π ⁻ dn	1.9±0.7
→ Σ ⁺ π ⁻ pnn	1.6±0.6
→ Σ ⁺ π ⁰ nnn	3.2±1.0
→ Σ ⁺ nnn	1.0±0.4
Total Σ ⁺ = (17.0±2.7)%	
K ⁻ He ⁴ → Σ ⁻ π ⁺ H ³	4.2±1.2
→ Σ ⁻ π ⁺ dn	1.6±0.6
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Total = Λ + Σ = (100-τ ⁺)%	

[P. A. Katz et al., Phys. Rev. D1 (1970) 1267-1276]

The first measurement of BRs for K⁻N and K⁻ multi-N absorptions were performed in bubble chamber experiments.

K⁻ "N" → Y π 1NA pionic process

K⁻ "NN" → Y N

2NA

K⁻ "NNN" → Y (NN)

3NA

non-pionic

K⁻ "NNNN" → Y (NNN)

4NA

global non-pionic = 2NA + 3NA + 4NA + ...

"NN" = bound nucleons

(NN), (NNN) = residual nucleons bound or unbound

Hard to distinguish experimentally:

Λ π⁰ and Σ⁰ π⁰

Λ (no π) and Σ⁰ (no π)

because Σ⁰ → Λ γ and π⁰ → γ γ decay via EM interaction.

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$$K^- \text{ "N" } \rightarrow Y \pi$$

1NA pionic process

Solution:

Isospin symmetry is used to determine the $\Lambda \pi^0$ and $\Sigma^0 \pi^0$ branching ratios.

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$$\text{Total} = \Lambda + \Sigma = (100 - \pi^+)^{\circ}\%$$

Hard to distinguish experimentally:

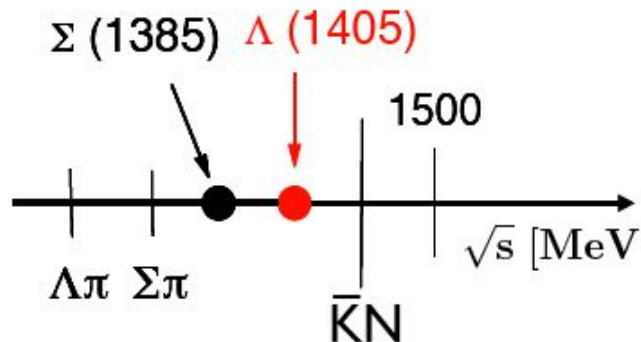
$$\Lambda \pi^0 \text{ and } \Sigma^0 \pi^0$$

$$\Lambda (\text{no } \pi) \text{ and } \Sigma^0 (\text{no } \pi)$$

because $\Sigma^0 \rightarrow \Lambda \gamma$ and $\pi^0 \rightarrow \gamma \gamma$ decay via EM interaction.

[P. A. Katz et al., Phys. Rev. D1 (1970) 1267-1276]

The controversial nature of the $\Lambda(1405)$



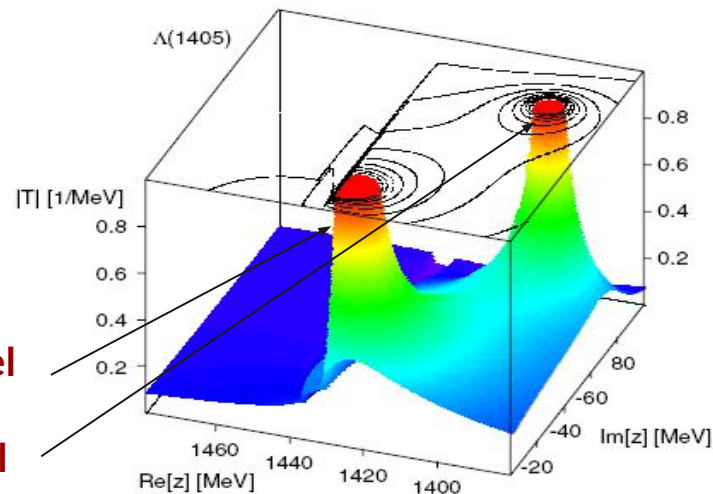
The $\Lambda(1405)$ ($I=0$) state does not fit with the simple three quarks model (uds) and it is commonly accepted that it is, at least partially, a $\bar{K}N$ bound state.

$$\Lambda(1405) \rightarrow (\Sigma \pi)^0 \text{ (BR=100\%)}$$

- Chiral SU(3) coupled channel dynamics:** the state is given by the superpositions of two poles of the $\bar{K}N$ scattering amplitude.

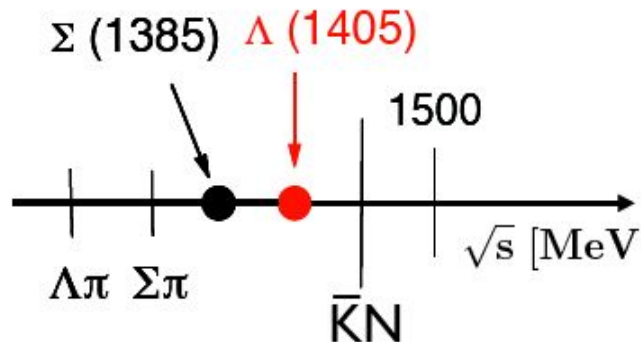
$M = 1425 \text{ MeV} \rightarrow$ mainly coupled to the $\bar{K}N$ channel

$M = 1380 \text{ MeV} \rightarrow$ mainly coupled to the $\Sigma\pi$ channel



- Phenomenological potentials models:** the $\Lambda(1405)$ is a pure $\bar{K}N$ bound state with mass $M=1405$ MeV, binding energy $BE = 27$ MeV and width $\Gamma=50$ MeV.

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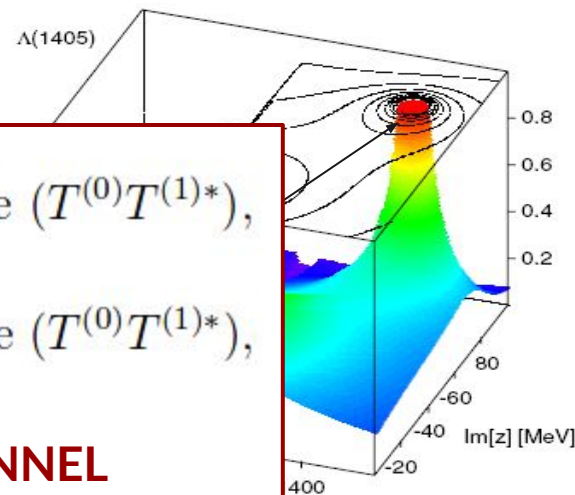
$M = 1425 \text{ MeV}$

$M = 1380 \text{ MeV}$

$$\frac{d\sigma(\pi^+\Sigma^-)}{dM_I} \propto \frac{1}{3}|T^{(0)}|^2 + \frac{1}{2}|T^{(1)}|^2 + \frac{2}{\sqrt{6}}\text{Re}(T^{(0)}T^{(1)*}),$$

$$\frac{d\sigma(\pi^-\Sigma^+)}{dM_I} \propto \frac{1}{3}|T^{(0)}|^2 + \frac{1}{2}|T^{(1)}|^2 - \frac{2}{\sqrt{6}}\text{Re}(T^{(0)}T^{(1)*}),$$

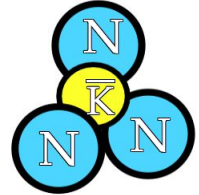
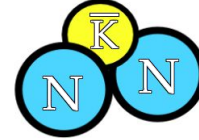
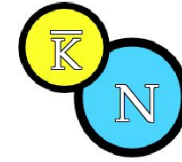
$$\frac{d\sigma(\pi^0\Sigma^0)}{dM_I} \propto \frac{1}{3}|T^{(0)}|^2, \quad \text{GOLDEN CHANNEL}$$



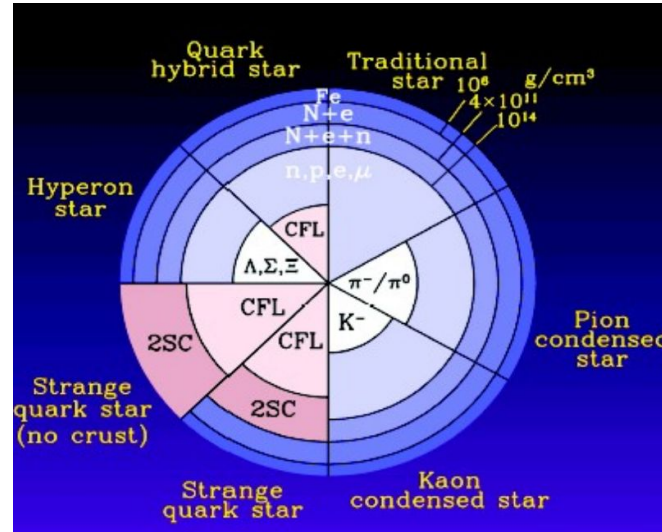
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Possible existence of kaonic bound state

The existence of kaonic bound state with multi-nucleons was suggested by S. Wycech (1986) and by Y. Akaishi, T. Yamazaki (2002) due to the **strongly attractive $\bar{K}N$ interaction in $I=0$ channel.**



Role of strangeness in
dense baryonic matter,
kaon condensation?
Strange quark matter?
Hyperons in Neutron Stars?



Determination of $\Sigma^0\pi^0$ and $\Lambda\pi^0$ BRs

Pionic processes $K^- \text{ "N"} \rightarrow Y \pi$

Final state:

$$|\Sigma^+\pi^-\rangle = 1/\sqrt{6} |2,0\rangle + 1/\sqrt{2} |1,0\rangle + 1/\sqrt{3} |0,0\rangle$$

$$|\Sigma^+\pi^0\rangle = 1/\sqrt{2} |2,1\rangle + 1/\sqrt{2} |1,1\rangle$$

$$|\Sigma^-\pi^+\rangle = 1/\sqrt{2} |2,0\rangle - 1/\sqrt{2} |1,0\rangle + 1/\sqrt{3} |0,0\rangle$$

$$|\Sigma^-\pi^0\rangle = 1/\sqrt{2} |2,-1\rangle - 1/\sqrt{2} |1,-1\rangle$$

$$|\Sigma^0\pi^-\rangle = 1/\sqrt{2} |2,-1\rangle + 1/\sqrt{2} |1,-1\rangle$$

$$|\Sigma^0\pi^0\rangle = \sqrt{2}/\sqrt{3} |2,0\rangle - 1/\sqrt{3} |0,0\rangle$$

$$|\Sigma^0\pi^+\rangle = 1/\sqrt{2} |2,1\rangle - 1/\sqrt{2} |1,1\rangle$$

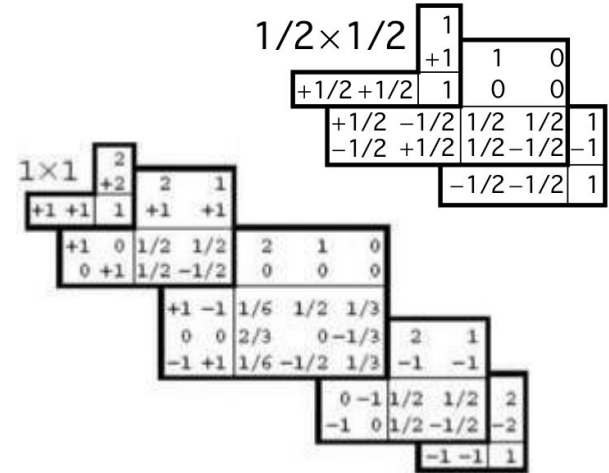
Initial state:

$$|K^-N\rangle = 1/2 |0,0\rangle + 1/2 |1,0\rangle + 1/\sqrt{2} |1,-1\rangle$$

$$|\Lambda\pi^-\rangle = |1,-1\rangle$$

$$|\Lambda\pi^0\rangle = |1,0\rangle$$

$$|\Lambda\pi^+\rangle = |1,+1\rangle$$



Isospin symmetry in S=-1 sector

Λ π final states

$$T_{K^-N \rightarrow \Lambda \pi^0} = \langle \Lambda \pi^0 | V | K^- N \rangle = 1/2 T_{I=1}$$

$$T_{K^-N \rightarrow \Lambda \pi^-} = \langle \Lambda \pi^- | V | K^- N \rangle = 1/\sqrt{2} T_{I=1}$$

$$T_{K^-N \rightarrow \Lambda \pi^+} = \langle \Lambda \pi^+ | V | K^- N \rangle = 0$$

$$\frac{BR(\Lambda \pi^0)}{BR(\Lambda \pi^-)} = \frac{1}{2}$$

Σ π final states

$$T_{K^-N \rightarrow \Sigma^+ \pi^-} = \langle \Sigma^+ \pi^- | V | K^- N \rangle = 1/(2\sqrt{2}) T_{I=1} + 1/(2\sqrt{3}) T_{I=0}$$

$$T_{K^-N \rightarrow \Sigma^+ \pi^0} = \langle \Sigma^+ \pi^0 | V | K^- N \rangle = 0$$

$$T_{K^-N \rightarrow \Sigma^- \pi^+} = \langle \Sigma^- \pi^+ | V | K^- N \rangle = 1/(2\sqrt{3}) T_{I=0} - 1/(2\sqrt{2}) T_{I=1}$$

$$T_{K^-N \rightarrow \Sigma^- \pi^0} = \langle \Sigma^- \pi^0 | V | K^- N \rangle = -1/2 T_{I=1}$$

$$\frac{BR(\Sigma^- \pi^0)}{BR(\Sigma^0 \pi^-)} = 1$$

$$BR(\Sigma^0 \pi^0) = \frac{1}{2} [BR(\Sigma^+ \pi^-) + BR(\Sigma^- \pi^+) - BR(\Sigma^- \pi^0)]$$

$$T_{K^-N \rightarrow \Sigma^0 \pi^+} = \langle \Sigma^0 \pi^+ | V | K^- N \rangle = 0$$

$$T_{K^-N \rightarrow \Sigma^0 \pi^0} = \langle \Sigma^0 \pi^0 | V | K^- N \rangle = -1/(2\sqrt{3}) T_{I=0}$$

$$T_{K^-N \rightarrow \Sigma^0 \pi^-} = \langle \Sigma^0 \pi^- | V | K^- N \rangle = 1/2 T_{I=1}$$

$$BR(\Sigma^0) = \frac{1}{2} [BR(\Sigma^+) + BR(\Sigma^-)]$$

Isospin symmetry in S=-1 sector

K^- ^4He absorptions at-rest:

[P. A. Katz et al., Phys. Rev. D1 (1970) 1267-1276]

$$\text{BR}(\Sigma^+\pi^-) = (32.0 \pm 6.3) \%$$

$$\text{BR}(\Sigma^-\pi^+) = (9.3 \pm 1.8) \%$$

$$\text{BR}(\Sigma^-\pi^0) = (3.9 \pm 1.0) \%$$

$$\text{BR}(\Sigma^0\pi^-) = (4.3 \pm 2.5) \%$$

$$\text{BR}(\Sigma^0\pi^0) = (18.7 \pm 3.3) \%$$

$$\text{BR}(\Lambda\pi^-) = (9.3 \pm 6.1) \%$$

$$\text{BR}(\Lambda\pi^0) = (4.7 \pm 3.1) \%$$

$$\text{BR}(\Sigma^+(\text{no } \pi)) = (2.5 \pm 1.0) \%$$

$$\text{BR}(\Sigma^-(\text{no } \pi)) = (4.6 \pm 1.2) \%$$

$$\text{BR}(\Sigma^0(\text{no } \pi)) = (3.6 \pm 0.8) \%$$

$$\text{BR}(\Lambda(\text{no } \pi)) = (5.6 \pm 2.6) \%$$

K^- ^{12}C absorptions at-rest:

[C. Vander Velde-Wilquet et al., Nuovo Cim. A39 (1977) 538-547]

$$\text{BR}(\Sigma^+\pi^-) = (29.4 \pm 1.0 \pm 5.0) \%$$

$$\text{BR}(\Sigma^-\pi^+) = (13.1 \pm 0.4 \pm 0.9) \%$$

$$\text{BR}(\Sigma^-\pi^0) = (2.6 \pm 0.6 \pm 1.0) \%$$

$$\text{BR}(\Sigma^0\pi^-) = (2.6 \pm 0.6 \pm 1.0) \%$$

$$\text{BR}(\Sigma^0\pi^0) = (20.0 \pm 0.7 \pm 3.0) \%$$

$$\text{BR}(\Lambda\pi^-) = (6.8 \pm 0.3 \pm 2.0) \%$$

$$\text{BR}(\Lambda\pi^0) = (3.4 \pm 0.2 \pm 1.0) \%$$

measured

from isospin relations

Isospin symmetry in S=-1 sector

K^- ^4He absorptions at-rest:

[P. A. Katz et al., Phys. Rev. D1 (1970) 1267-1276]

$$\text{BR}(\Sigma^+\pi^-) = (32.0 \pm 6.3) \%$$

$$\begin{aligned} \text{BR}(\Sigma^-\pi^+) &= (9.3 \pm 6.1) \% \\ \text{BR}(\Sigma^-\pi^0) &= (4.7 \pm 3.1) \% \end{aligned} \quad R = \frac{\text{BR}(\Lambda\pi^0)}{\text{BR}(\Sigma^0\pi^0)} = 0.25 \pm 0.17$$

$$\text{BR}(\Sigma^0\pi^-) = (4.5 \pm 2.5) \%$$

$$\text{BR}(\Sigma^0\pi^0) = (18.7 \pm 3.3) \%$$

$$\text{BR}(\Lambda\pi^-) = (9.3 \pm 6.1) \%$$

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$$\text{BR}(\Sigma^+(\text{no } \pi)) = (2.5 \pm 1.0) \%$$

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$$\text{BR}(\Sigma^0(\text{no } \pi)) = (3.6 \pm 0.8) \%$$

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K^- ^{12}C absorptions at-rest:

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$$\text{BR}(\Sigma^0\pi^-) = (2.6 \pm 0.6 \pm 1.0) \%$$

$$\text{BR}(\Sigma^0\pi^0) = (20.0 \pm 0.7 \pm 3.0) \%$$

$$\text{BR}(\Lambda\pi^-) = (6.8 \pm 0.3 \pm 2.0) \%$$

$$\text{BR}(\Lambda\pi^0) = (3.4 \pm 0.2 \pm 1.0) \%$$

measured

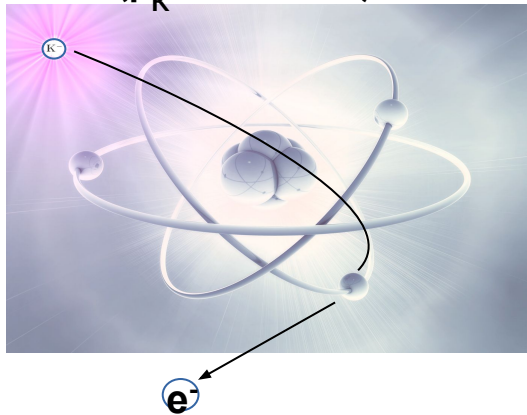
from isospin relations

K^- multi-N absorptions measurement

- First comprehensive study of K^- multi-nucleon absorptions is performed by the AMADEUS collaboration at DAΦNE collider in Frascati.
- Analyzed data sample contains K^- absorptions events of ^{12}C nuclei at-rest and in-flight ($p_K \sim 100 \text{ MeV}/c$) and was collected by the KLOE collaboration during the 2004-2005 data taking.

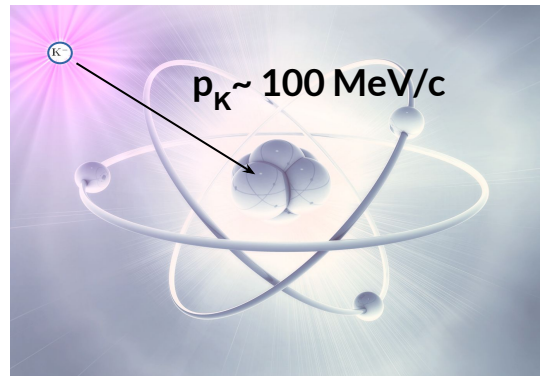
AT-REST

K^- absorbed from atomic orbitals
($p_K \sim 0 \text{ MeV}/c$)



IN-FLIGHT

($p_K \sim 100 \text{ MeV}/c$)



- Λ and proton in the final state are reconstructed. Also in this case it is not possible to disentangle the direct Λ production from the Σ^0 production followed by $\Sigma^0 \rightarrow \Lambda \gamma$.

Monte Carlo simulations

Processes to be simulated in order to interpret the experimental spectra:

K^- multi-nucleon absorption processes in

Λ p channel
 $(K^- + {}^{12}\text{C} \rightarrow \Lambda + p + R)$

- **2NA quasi-free (QF):** $K^- \text{ "pp" } \rightarrow \Lambda p$
- **2NA with final state interaction (FSI):**
 $K^- \text{ "NN" } \rightarrow \Lambda N$
 $\Lambda (N) + \text{"N"} \rightarrow \Lambda (N) + N'$
- **3NA:** $K^- \text{ "ppn" } \rightarrow \Lambda p n$
- **4NA:** $K^- \text{ "ppnn" } \rightarrow \Lambda p n n$

Σ^0 p channel
 $(K^- + {}^{12}\text{C} \rightarrow \Sigma^0 + p + R \quad \Sigma^0 \rightarrow \Lambda + \gamma)$

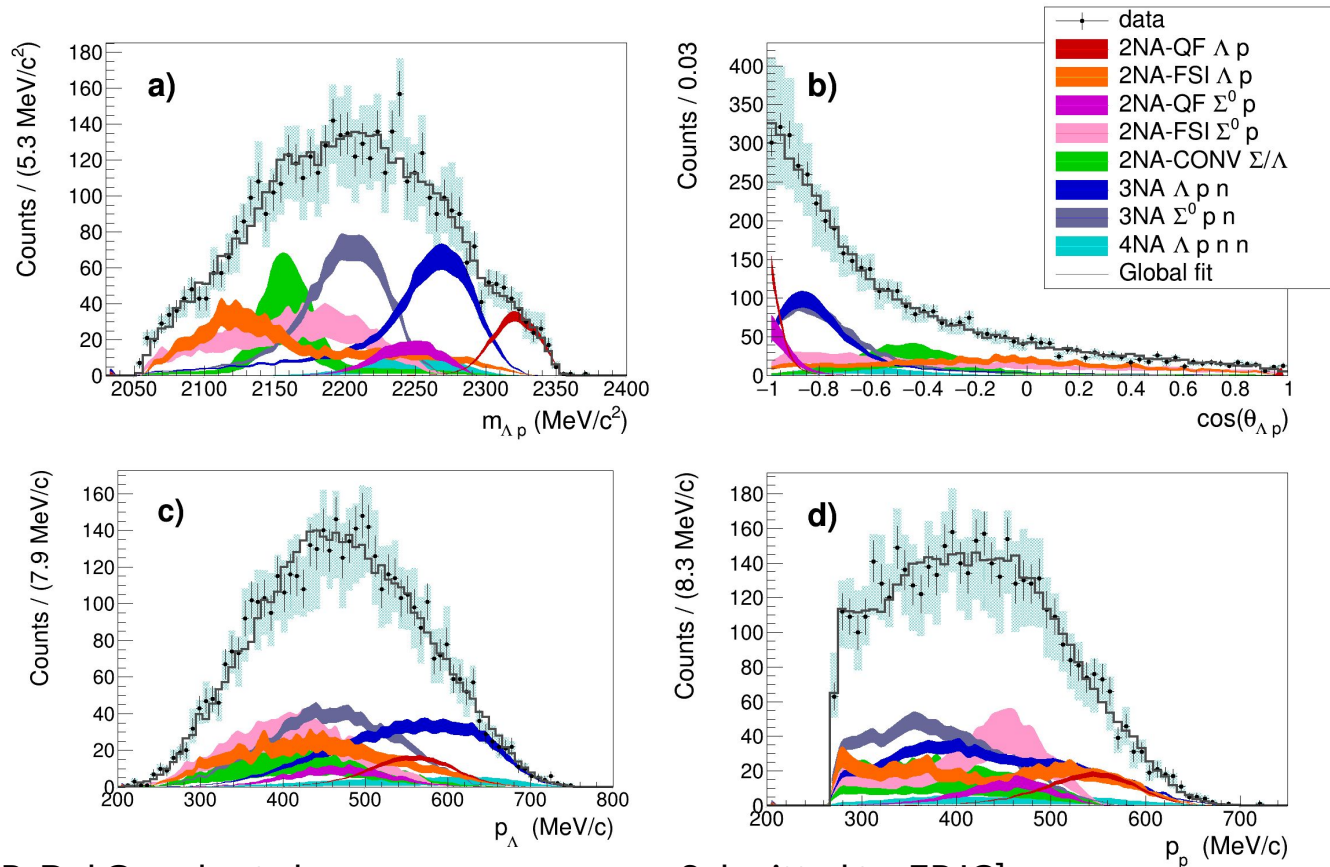
- **2NA quasi-free (QF):** $K^- \text{ "pp" } \rightarrow \Sigma^0 p$
- **2NA with final state interaction (FSI):**
 $K^- \text{ "NN" } \rightarrow \Sigma^0 N$
 $\Sigma^0 (N) + \text{"N"} \rightarrow \Sigma^0 (N) + N'$
- **3NA:** $K^- \text{ "ppn" } \rightarrow \Sigma^0 p n$
- **4NA:** $K^- \text{ "ppnn" } \rightarrow \Sigma^0 p n n$

- **2NA with Σ/Λ conversion:** $K^- \text{ "NN" } \rightarrow \Sigma N$
 $\Sigma \text{ "N" } \rightarrow \Lambda N'$

→ Single nucleon absorption included as a contribution to the systematic errors

[R. Del Grande et al., e-Print: [arXiv:1809.07212](https://arxiv.org/abs/1809.07212), Submitted to: EPJC]

K⁻ multi-N absorptions measurement



Simultaneous fit of:

- Λp invariant mass
- angular correlation
- proton momentum
- Λ momentum

Reduced chi-square:

$$\chi^2/\text{ndf} = 0.94$$

[R. Del Grande et al., e-Print: [arXiv:1809.07212](https://arxiv.org/abs/1809.07212), Submitted to: EPJC]

K⁻ multi-N absorptions measurement

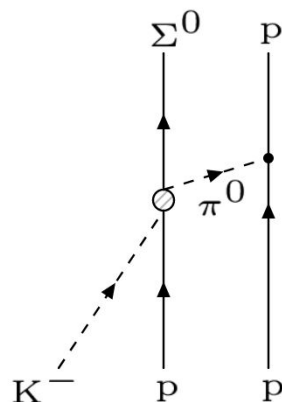
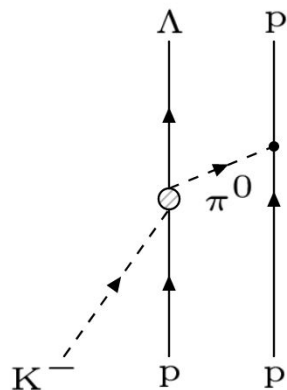
K⁻ absorption at-rest

K⁻ absorption in-flight

Process	Branching Ratio (%)	σ (mb)	@	p_K (MeV/c)
2NA-QF Λp	0.25 ± 0.02 (stat.) $^{+0.01}_{-0.02}$ (syst.)	2.8 ± 0.3 (stat.) $^{+0.1}_{-0.2}$ (syst.)	@	128 ± 29
2NA-FSI Λp	6.2 ± 1.4 (stat.) $^{+0.5}_{-0.6}$ (syst.)	69 ± 15 (stat.) ± 6 (syst.)	@	128 ± 29
2NA-QF $\Sigma^0 p$	0.35 ± 0.09 (stat.) $^{+0.13}_{-0.06}$ (syst.)	3.9 ± 1.0 (stat.) $^{+1.4}_{-0.7}$ (syst.)	@	128 ± 29
2NA-FSI $\Sigma^0 p$	7.2 ± 2.2 (stat.) $^{+4.2}_{-5.4}$ (syst.)	80 ± 25 (stat.) $^{+46}_{-60}$ (syst.)	@	128 ± 29
3NA $\Lambda p n$	1.4 ± 0.2 (stat.) $^{+0.1}_{-0.2}$ (syst.)	15 ± 2 (stat.) ± 2 (syst.)	@	117 ± 23
3NA $\Sigma^0 p n$	3.7 ± 0.4 (stat.) $^{+0.2}_{-0.4}$ (syst.)	41 ± 4 (stat.) $^{+2}_{-5}$ (syst.)	@	117 ± 23
4NA $\Lambda p n n$	0.13 ± 0.09 (stat.) $^{+0.08}_{-0.07}$ (syst.)	-		
2NA-CONV Σ/Λ	2.1 ± 1.2 (stat.) $^{+0.9}_{-0.5}$ (syst.)	-		

[R. Del Grande et al., e-Print: [arXiv:1809.07212](https://arxiv.org/abs/1809.07212), Submitted to: EPJC]

K⁻ multi-N absorptions measurement



According to the pion exchange model [E. Oset, H. Toki, Phys. Rev. C74 (2006) 015207] the K⁻ absorption on 2 nucleons occurs through the exchange of a virtual π:

$$\mathcal{R} = \frac{\text{BR}(K^- \text{ "pp" } \rightarrow \Lambda p)}{\text{BR}(K^- \text{ "pp" } \rightarrow \Sigma^0 p)} = \frac{\text{BR}(K^- \text{ "p" } \rightarrow \Lambda \pi^0)}{\text{BR}(K^- \text{ "p" } \rightarrow \Sigma^0 \pi^0)}$$

Using the BRs of K⁻2NA in Λp and Σ⁰p channels we measure:

$$\mathcal{R} = 0.7 \pm 0.2(\text{stat.})_{-0.3}^{+0.2}(\text{syst.})$$

Using BR(Λπ⁰) and BR(Σ⁰π⁰) measured by [C. Vander Velde-Wilquet et al., Nuovo Cim. A39 (1977) 538–547]:

$$R = 0.17 \pm 0.01(\text{stat.}) \pm 0.06(\text{syst.})$$

[R. Del Grande et al., e-Print: [arXiv:1809.07212](https://arxiv.org/abs/1809.07212), Submitted to: EPJC]

Isospin symmetry violation

The s-wave ($L=0$) meson-nucleon interaction in the $S=-1$ sector is studied in Ref. [E. Oset and A. Ramos, Nuclear Physics A 635 (1998) 99-120] by means of **coupled-channel Lippmann-Schwinger equation**:

$$T = V + VGT$$

using:

- Lower order Chiral Lagrangians \Rightarrow the potential V_{ij} has the following form:

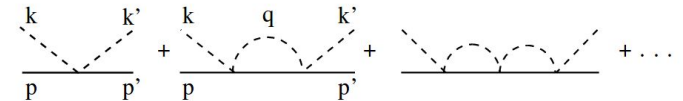
$$V_{ij} = -C_{ij} \frac{1}{4f^2} (k^0 + k'^0)$$

- a free parameter q_{max} as a cutoff in the integral:

$$V_{il} G_l T_{lj} = i \int \frac{d^4 q}{(2\pi)^4} \frac{M_l}{E_l(\vec{q})} \frac{V_{il}(k, q) T_{lj}(q, k')}{k^0 + p^0 - q^0 - E_l(\vec{q}) + i\epsilon} \frac{1}{q^2 - m_l^2 + i\epsilon}$$

M_l, E_l = intermediate baryon
 m_l = intermediate meson

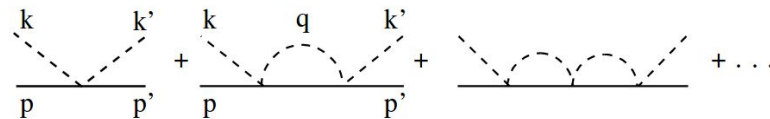
to accounts for higher order contributions in an effective way.



Isospin symmetry violation

- Couplings of K^-p to the eight channels

$$K^-p, K^0n, \pi^0\Lambda, \pi^0\Sigma^0, \pi^+\Sigma^-, \pi^-\Sigma^+, \eta\Lambda, \eta\Sigma^0$$



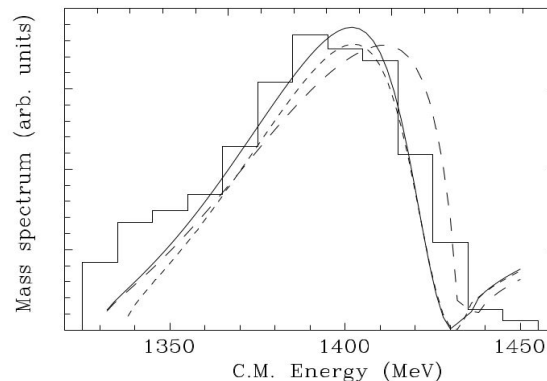
are considered. The couplings to $K^+\Xi^-$ and $K^0\Xi^0$ are demonstrated to be negligible.

Fitting procedure: → the coupling constant f and q_{max} are the free parameters;
 → the fit is performed to get the best reproduction of the threshold parameter (γ , R_c and R_n) and the $\Lambda(1405) \rightarrow \Sigma\pi$ invariant mass spectrum.

$$\gamma = \frac{\Gamma(K^-p \rightarrow \pi^+\Sigma^-)}{\Gamma(K^-p \rightarrow \pi^-\Sigma^+)} = 2.36 \pm 0.04$$

$$R_c = \frac{\Gamma(K^-p \rightarrow \text{charged particles})}{\Gamma(K^-p \rightarrow \text{all})} = 0.664 \pm 0.011$$

$$R_n = \frac{\Gamma(K^-p \rightarrow \pi^0\Lambda)}{\Gamma(K^-p \rightarrow \text{all neutral states})} = 0.189 \pm 0.015$$



Isospin symmetry violation

1. Amplitudes can be constructed using isospin formalism for which average masses are used for the isospin multiplets:

$\pi (\pi^+, \pi^0, \pi^-)$, $K (K^0, K^-)$, $K (K^+, K^0)$, $N (p, n)$ and $\Sigma (\Sigma^+, \Sigma^0, \Sigma^-)$

The coupled-channel equations are solved in the isospin basis for $I=0$ and $I=1$ cases and the amplitude for the physical channels are then constructed.

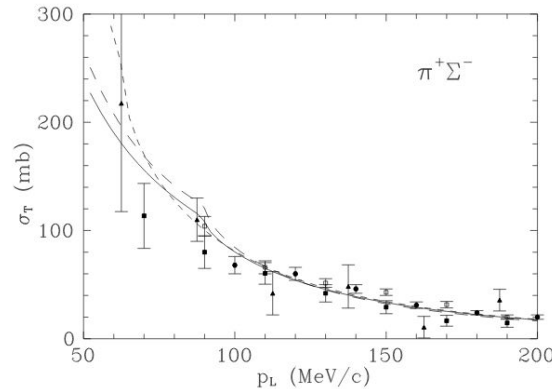
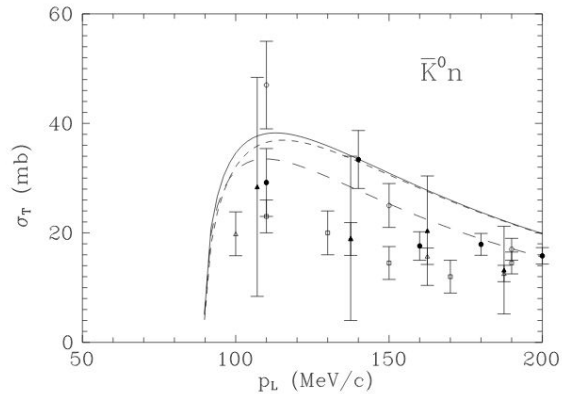
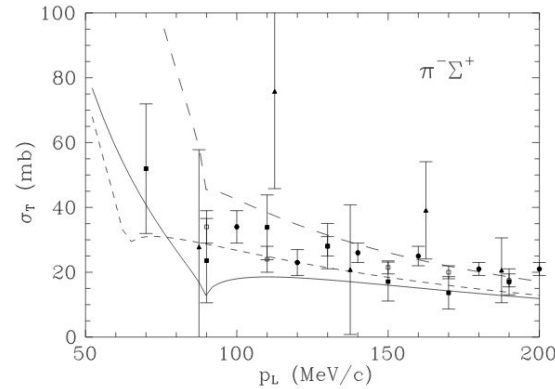
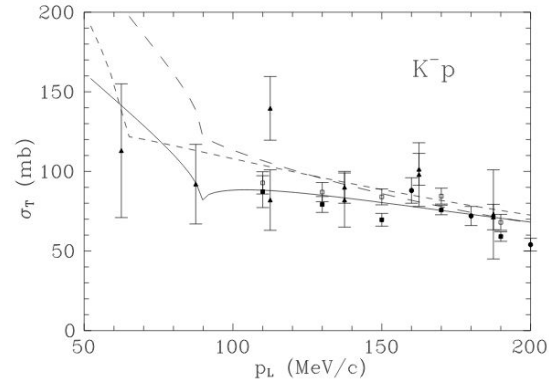
2. Alternatively the physical states using the physical masses are used.

This allow to investigate the isospin violation effects

[E. Oset and A. Ramos, Nuclear Physics A 635 (1998) 99-120]

Isospin symmetry violation

[E. Oset and A. Ramos, Nuclear Physics A 635 (1998) 99-120]



$K^- p \rightarrow K^- p, K^0 n, \pi^- \Sigma^+, \pi^+ \Sigma^-$

Cross section measurements are not used in the fit.

$$\sigma_{ij} = \frac{1}{4\pi} \frac{MM'}{s} \frac{k'}{k} |T_{ij}|^2$$

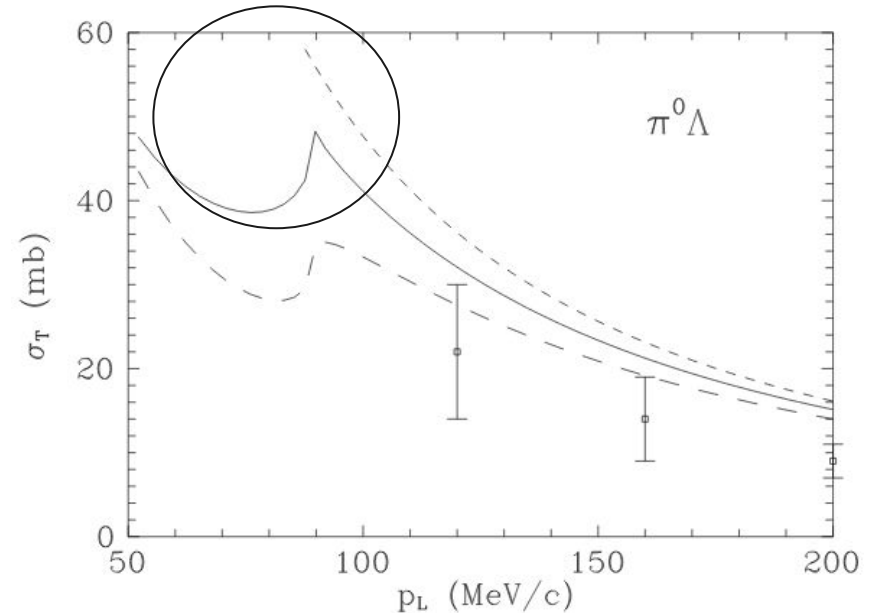
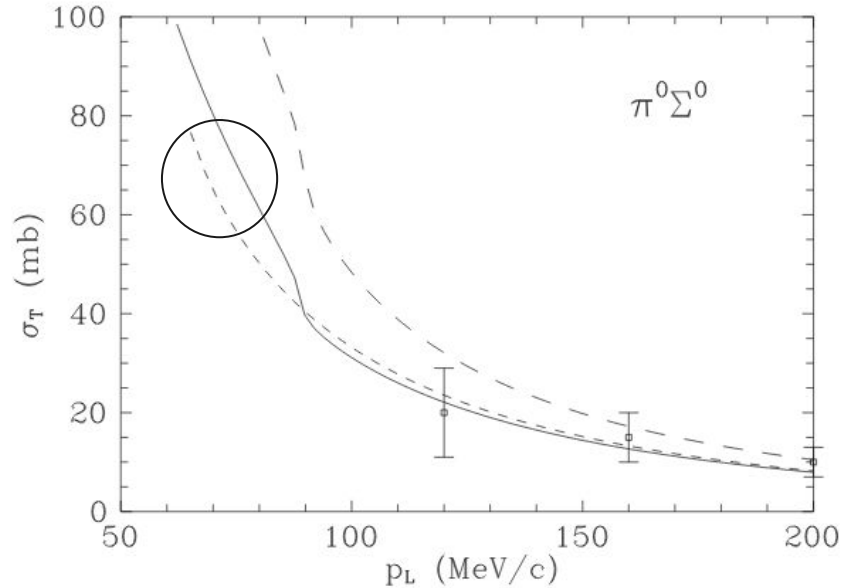
— physical masses

- - - isospin

— no $\eta \Lambda, \eta \Sigma^0$

Isospin symmetry violation

$$K^-p \rightarrow \pi^0 \Sigma^0, \pi^0 \Lambda$$



Isospin symmetry violation effect close to the K^-p threshold.

Summary and Outlooks

1. Isospin is not an exact symmetry of the strong interaction but Isospin invariance works very well when is used to estimate the cross sections and branching ratios for NN scattering
2. Isospin symmetry is used to determine the $BR(\Sigma^0\pi^0)$ and $BR(\Lambda^0\pi^0)$

$$\begin{aligned} \text{K- 4He: } BR(\Sigma^0\pi^0) &= (18.7 \pm 3.3) \% \\ BR(\Lambda\pi^0) &= (4.7 \pm 3.1) \% \\ R &= 0.25 \pm 0.17 \end{aligned}$$

$$\begin{aligned} \text{K- 12C: } BR(\Sigma^0\pi^0) &= (20.0 \pm 0.7 \pm 3.0) \% \\ BR(\Lambda\pi^0) &= (3.4 \pm 0.2 \pm 1.0) \% \\ R &= 0.17 \pm 0.01 (\text{stat.}) \pm 0.06 (\text{syst.}) \end{aligned}$$

3. Measuring the K^- 2NA in Λp and $\Sigma^0 p$ channels using ^{12}C target:
$$\mathcal{R} = 0.7 \pm 0.2(\text{stat.})^{+0.2}_{-0.3}(\text{syst.})$$
4. Isospin symmetry violation close to K^-p threshold is suggested in Ref. [E. Oset and A. Ramos, Nuclear Physics A 635 (1998) 99-120]
5. Lack of experimental data for $K^-p \rightarrow \Lambda\pi^0$ and $K^-p \rightarrow \Sigma^0\pi^0$ cross sections close to the threshold \rightarrow New data can used to test the models

Conclusions

Thank you for your attention