Limit on Lorentz Invariance violation using the Gravitational Wave Interferometers

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Overview of GW Interferometers

The very low frequency "free spectral range" channel

Observation of tidal gradients

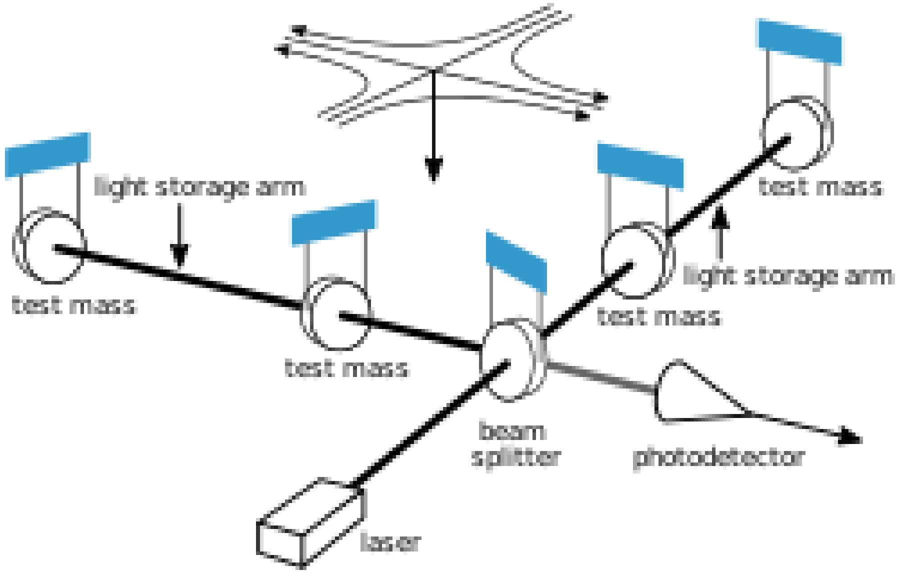
Limits on a Lorentz Invariance violating signal

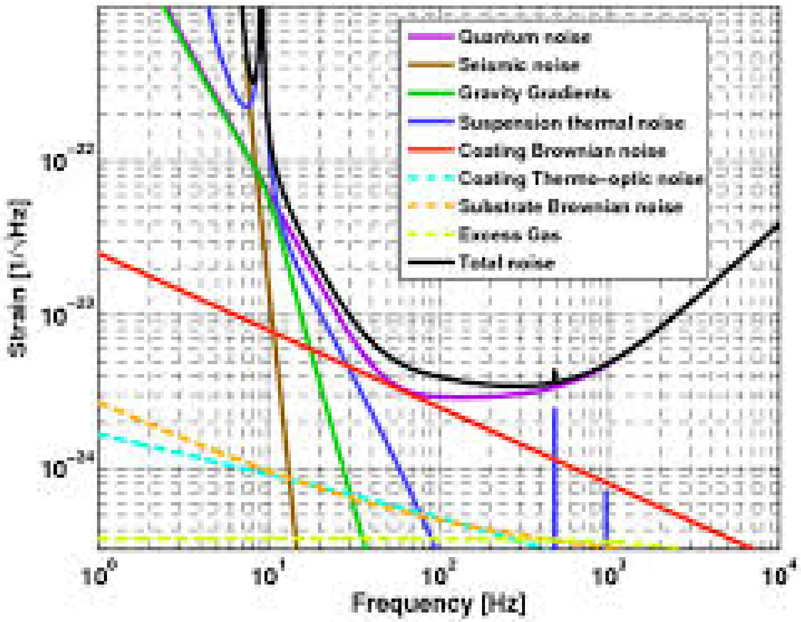
Twice yearly modulation and refractive index inconsistency

Overview of Gravitational Wave Interferometers

They are Michelson Interferometers with “free” suspended mirrors and Fabry–Perot cavities in the two orthogonal arms

They operate on a dark fringe, where they are maintained to $10^{-7}\ \lambda$, ($\lambda = 10^{-6}$ m) and record the signal at the dark port





The signal records the phase shift at the antisymmetric port

$$\frac{\delta\phi}{2\pi} = \frac{\delta\phi_1}{2\pi} - \frac{\delta\phi_2}{2\pi}$$

$$\frac{\delta\phi_i}{2\pi} = \left(\frac{\delta L_i}{L} + \frac{\delta f_i}{f} + \frac{\delta n_i}{n}\right)\frac{2L}{\lambda}$$ where i=1,2

The servo acts on δL and δf, but **not on δn**, to keep δϕ =0; but these actions are **recorded**. The integral of δϕ

$$\frac{1}{T}\int_{t-T/2}^{t+T/2}\frac{d\phi}{2\pi}dt = \frac{1}{T}\frac{2L}{\lambda}\int_{t-T/2}^{t+T/2}\frac{dn}{n}dt = \frac{2L}{\lambda}\Delta n(t)$$

because over the integration interval, δL and δf, average to zero when the interferometer is "locked".

The **arms** are held on resonance by adjusting the laser frequency and the arm length difference. They resonate when $f_0 = n_0 c/2L$ where f_0 is the carrier frequency and n_0 some large integer (~ 10^{10}). The "free spectral range" frequency, is $f_{fsr} = c/2L = 37.5$ kHz, and The arms will also resonate at sideband frequencies $f_{\pm 1} = f_0 \pm f_{fsr}$

If there is a **macroscopic** length difference ΔL, between the two arms, then when the carrier is **locked**, the $f_{\pm 1}$ sidebands are **off** the dark fringe by a phase shift

$$\Delta\phi_{\pm 1} /2\pi = \pm\, \Delta L/2L = \phi_{bias}$$

Typically, $\Delta L \approx 2$ cm, so that $\phi_{bias}^{(\text{single pass})} = 2.5\times 10^{-6}$

Thus the power at f_1 contains an **interference** term between the bias term A_{fsr} and any externally imposed signal A_ω

The demodulated amplitude in the fsr region is the sum of the amplitude due to the macroscopic arm-difference A_{fsr} and the audio amplitude A_{ω}. The **power**

$$P = |A_{fsr} + A_{\omega}|^2 = |A_{fsr}|^2 + 2|A_{fsr}||A_{\omega}|\cos(\omega t + \phi) + |A_{\omega}|^2$$

is modulated at the audio frequency ω, to a depth

$$M = \frac{P_{max} - P_{min}}{P_{max} + P_{min}} = 2\frac{|A_{fsr}||A_{\omega}|}{|A_{fsr}|^2 + |A_{\omega}|^2}$$

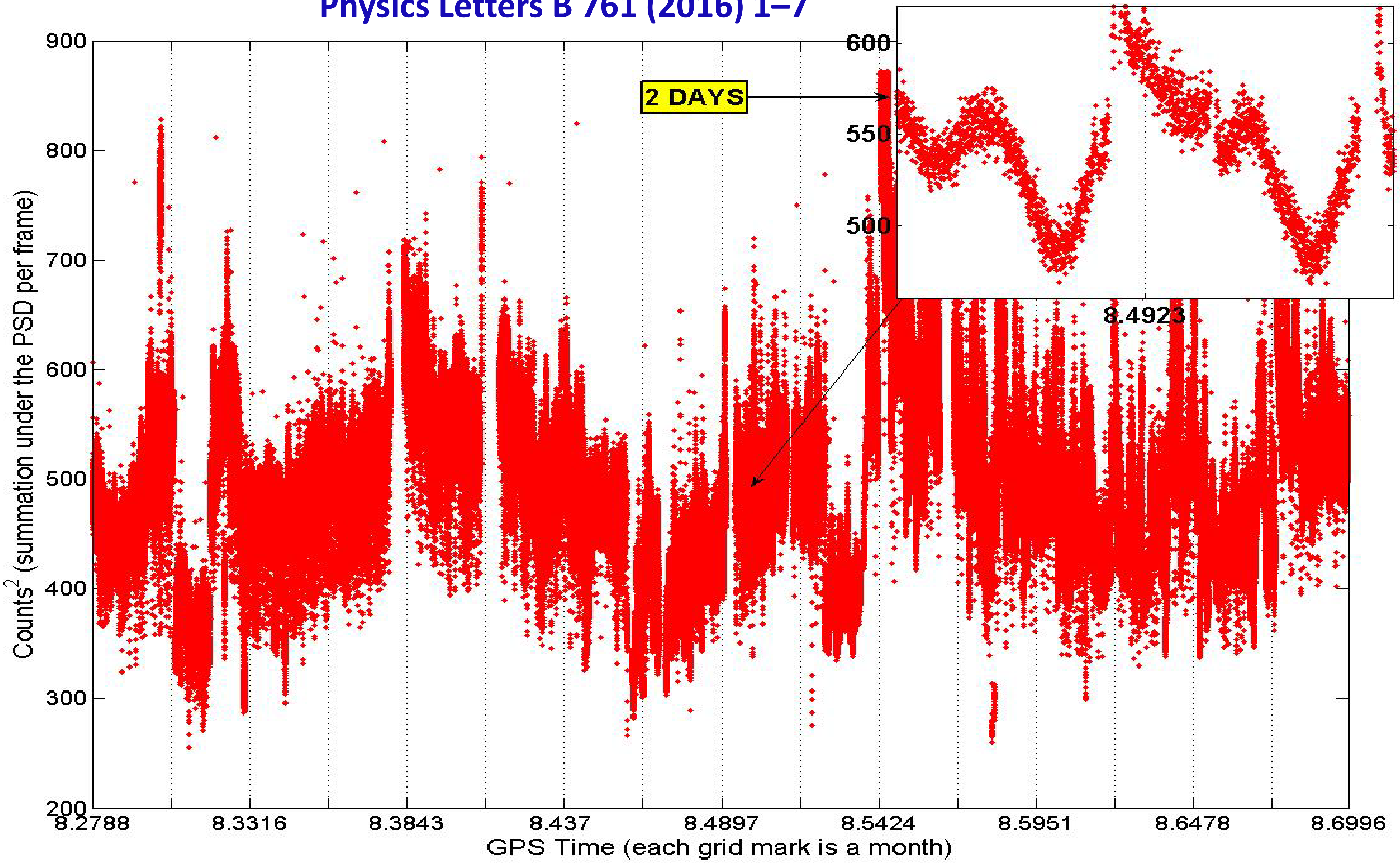
and when $A_{\omega} << A_{fsr}$ $M \approx 2\, A_{\omega} / A_{fsr}$.

The carrier is used to keep the interferometer in lock, (must be done rapidly), while the sideband at the fsr measures slow phase shifts.

The fsr frequency acts as a second Interferometer that operates in the “locked” optics.

Preliminary LIGO integrated fsr power from Apr. 06, 2006 - July 07, 2007

Physics Letters B 761 (2016) 1–7

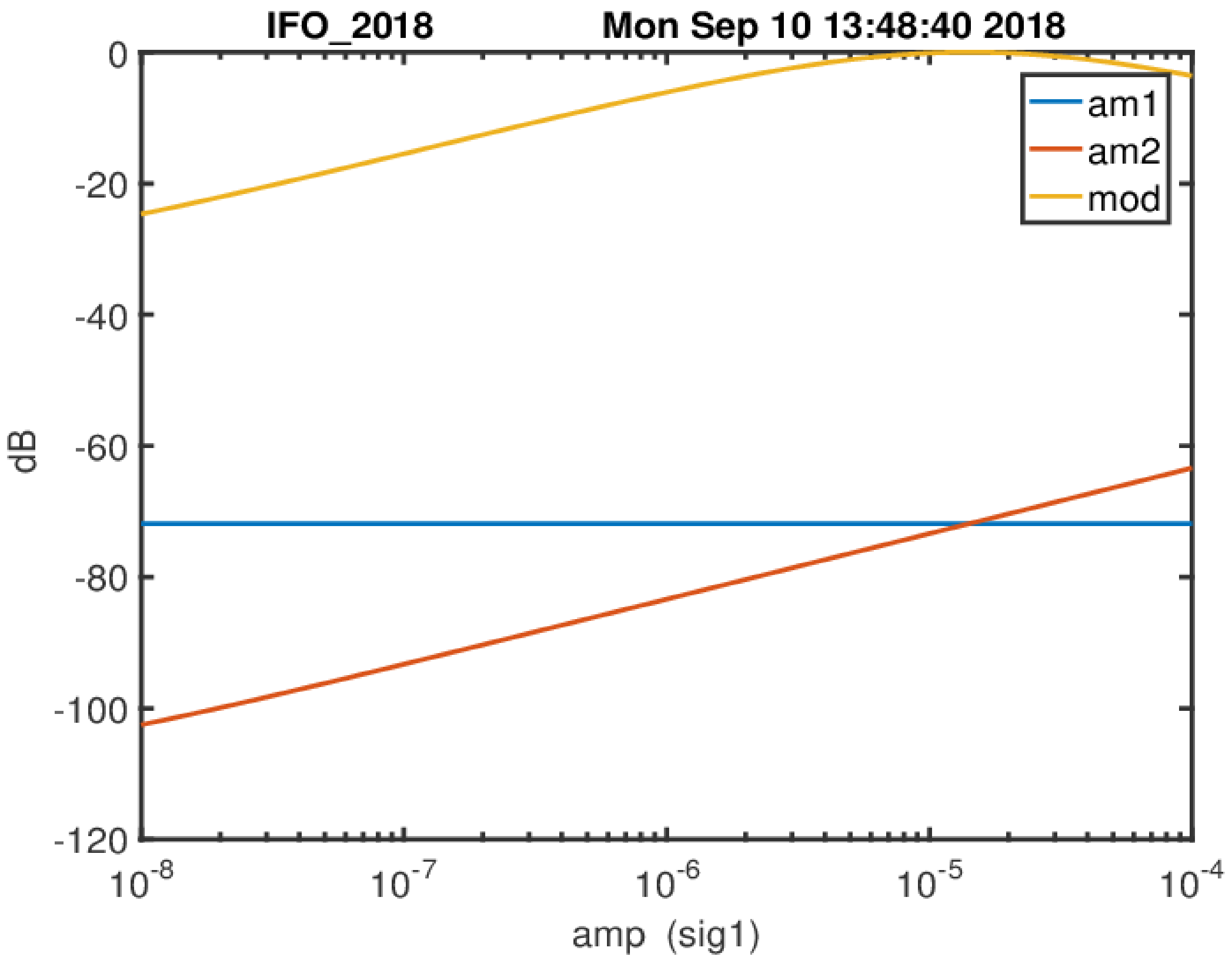


From observed modulation to phase shift at the dark port

In principle A_{fsr} can be calculated from the known ϕ_{bias} and the amplitude E_{+1} of the fsr sideband. The latter is not well known except that it is of order 10^{-7} of the carrier field. Knowing the modulation determines A_{ω}, but we must **propagate** the fields through the interferometer. We achieve this by modelling the interferometer (FINESSE) with macroscopic arm length difference $\Delta L = 2$ cm, and a single fsr sideband (no need to know the amplitude since it cancels in calculating the modulation) and plot the expected modulation as a function of the phase difference between the two arms. We find that

The observed modulation M = 0.10 = -20 db corresponds to $h = 1.25 \times 10^{-20}$, namely to a phase shift $\Delta\phi/2\pi = 2 \times 10^{-10}$

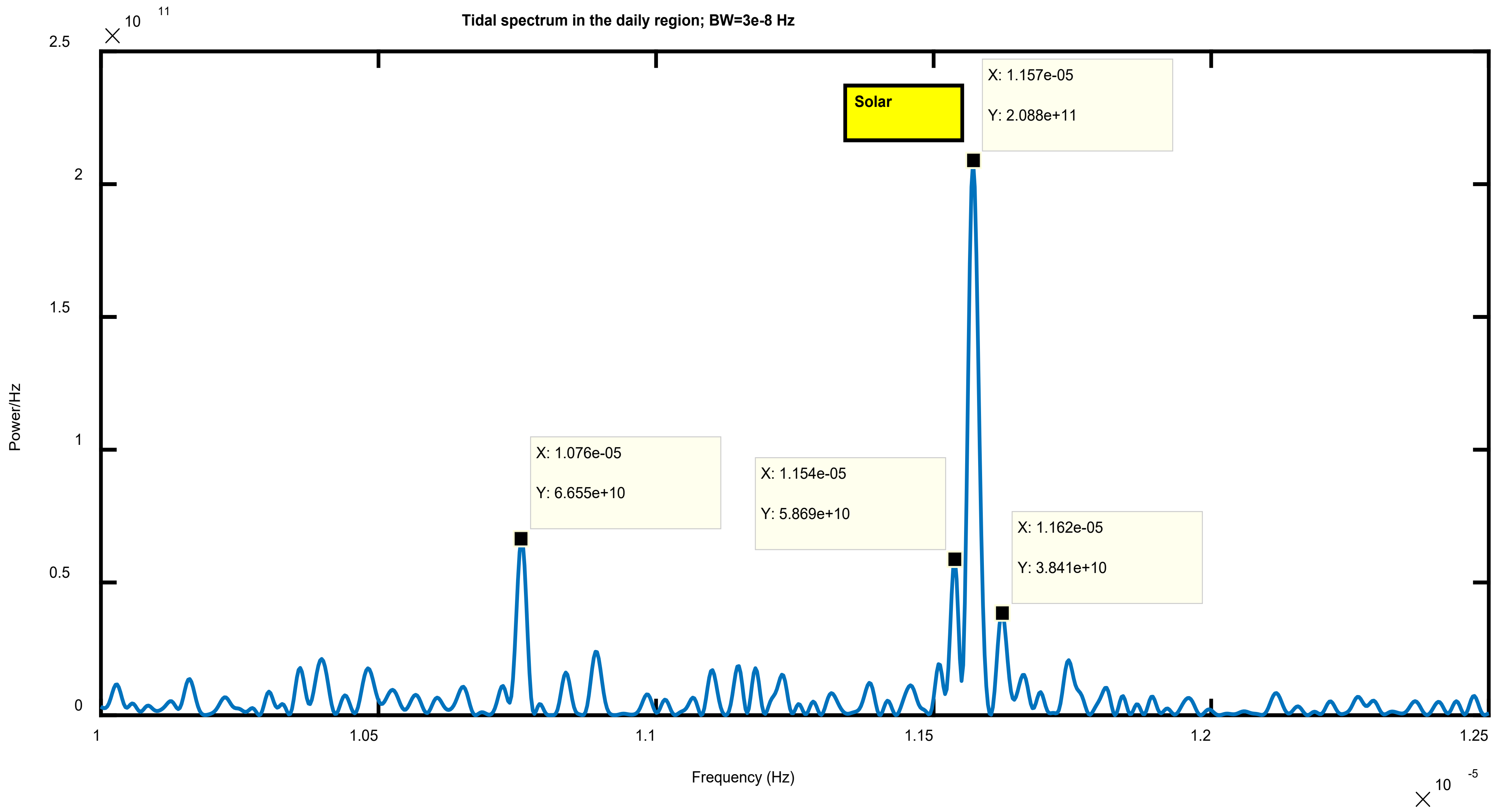
Modulation of the fsr power when propagating a signal (am2) through the IFO in the presence of an fsr sideband (am1), as modeled by FINESSE.



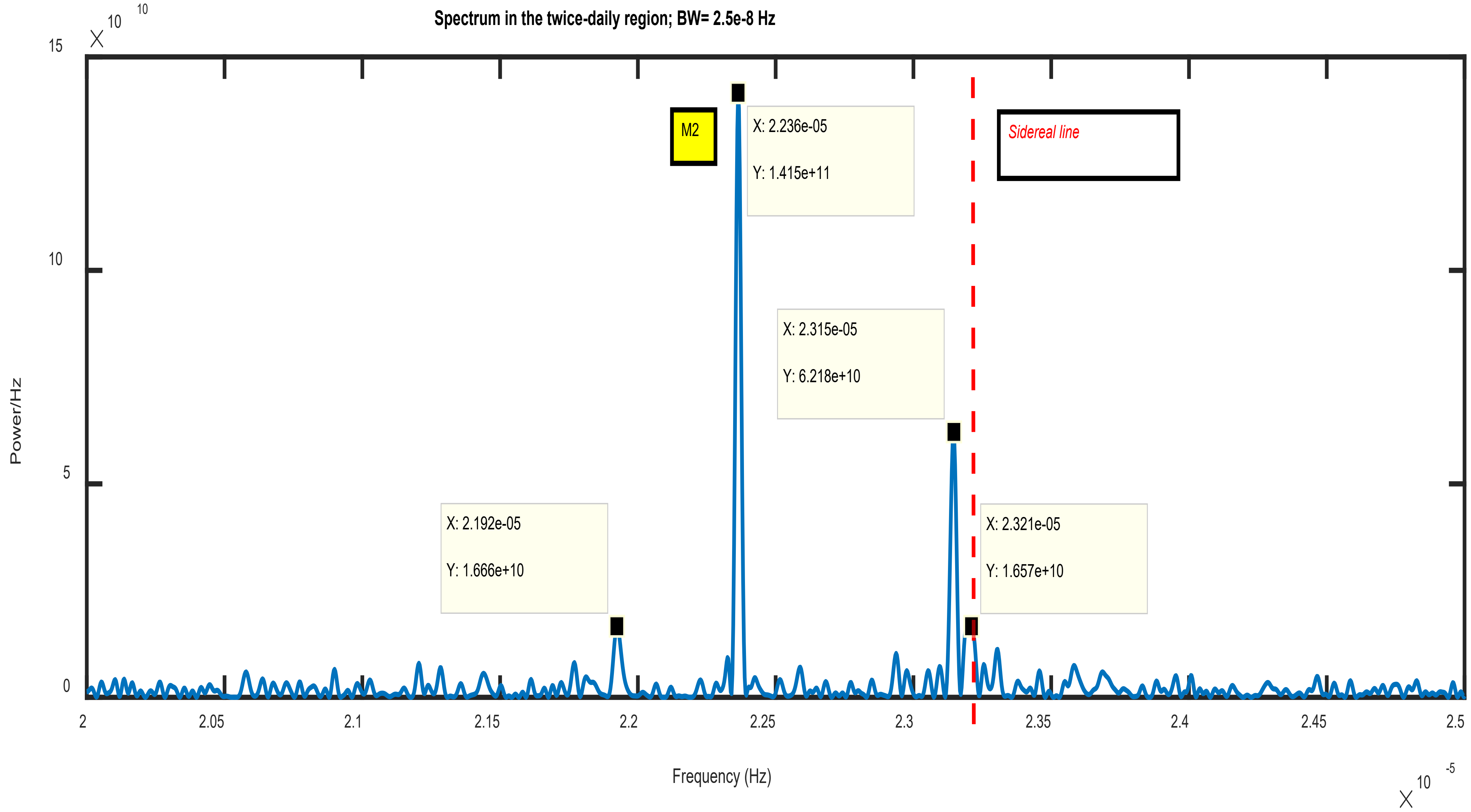
Blue A_{fsr}
Red A_{ω}
Yellow Modulation

$h = (\text{sig1}) \times (3.5 \times 10^{-13})$

Tidal Spectrum in Daily Region



Tidal Spectrum in Twice-Daily Region



Observed and known frequencies of tidal lines

Symbol	Measured	Predicted	L=lunar; S=solar
Long period			
Ss_a	6.536×10^{-8}	6.338×10^{-8}	S declinational
Diurnal			
O_1	1.07601×10^{-5}	1.07585×10^{-5}	L principal lunar wave
P_1	1.15384×10^{-5}	1.15424×10^{-5}	S solar principal wave
S_1	1.15741×10^{-5}	1.15741×10^{-5}	S elliptic wave of ${}^{s}K_1$
${}^{m}K_1, {}^{s}K_1$	1.16216×10^{-5}	1.16058×10^{-5}	L,S declinational waves
Twice-daily			
N_2	2.19240×10^{-5}	2.19442×10^{-5}	L major elliptic wave of M_2
M_2	2.23639×10^{-5}	2.23643×10^{-5}	L principal wave
S_2	2.31482×10^{-5}	2.31481×10^{-5}	S principal wave
${}^{m}K_2, {}^{s}K_2$	2.31957×10^{-5}	2.32115×10^{-5}	L,S declinational waves

The tidal gravity gradient

Since the fsr signal was acquired with **the IFO in lock** on a dark fringe, the **arm difference remained fixed**. However the tidal acceleration (force) has a horizontal component that varies harmonically at the tidal frequencies

$$d\Phi/dx = g_{horizontal} \approx 10^{-7}\ g = 10^{-6}\ m/s^2$$

Such a gradient modifies the frequency of the light propagating in the arms; we speak of a "red shift" between the two ends of the arm,

$$d\nu/\nu = \delta\Phi/c^2$$

and for the M2 tidal line, leads to a phase shift at the dark port (for a single traversal)

$$\frac{\delta\phi}{2\pi} = \frac{1}{\lambda_0} g_{hor} \frac{L^2}{c^2} \approx 2\times10^{-10}$$

Equivalent to strain $h = 1.25\times10^{-20}$ which implies a **modulation** of the fsr power with index M=0.10, in agreement with the data.

We can now use the M2 tidal gradient to calibrate all the lines in the fsr power spectrum

The Horizontal tidal gradients are given by

$$\mathrm{F}_{South} = C\left[\tfrac{3}{2}\sin 2\phi(\tfrac{1}{3} - \sin^2\delta) - \cos 2\phi \, \sin 2\delta \, \cos H + \tfrac{1}{2}\sin 2\phi \, \cos^2\delta \, \cos 2H\right]$$

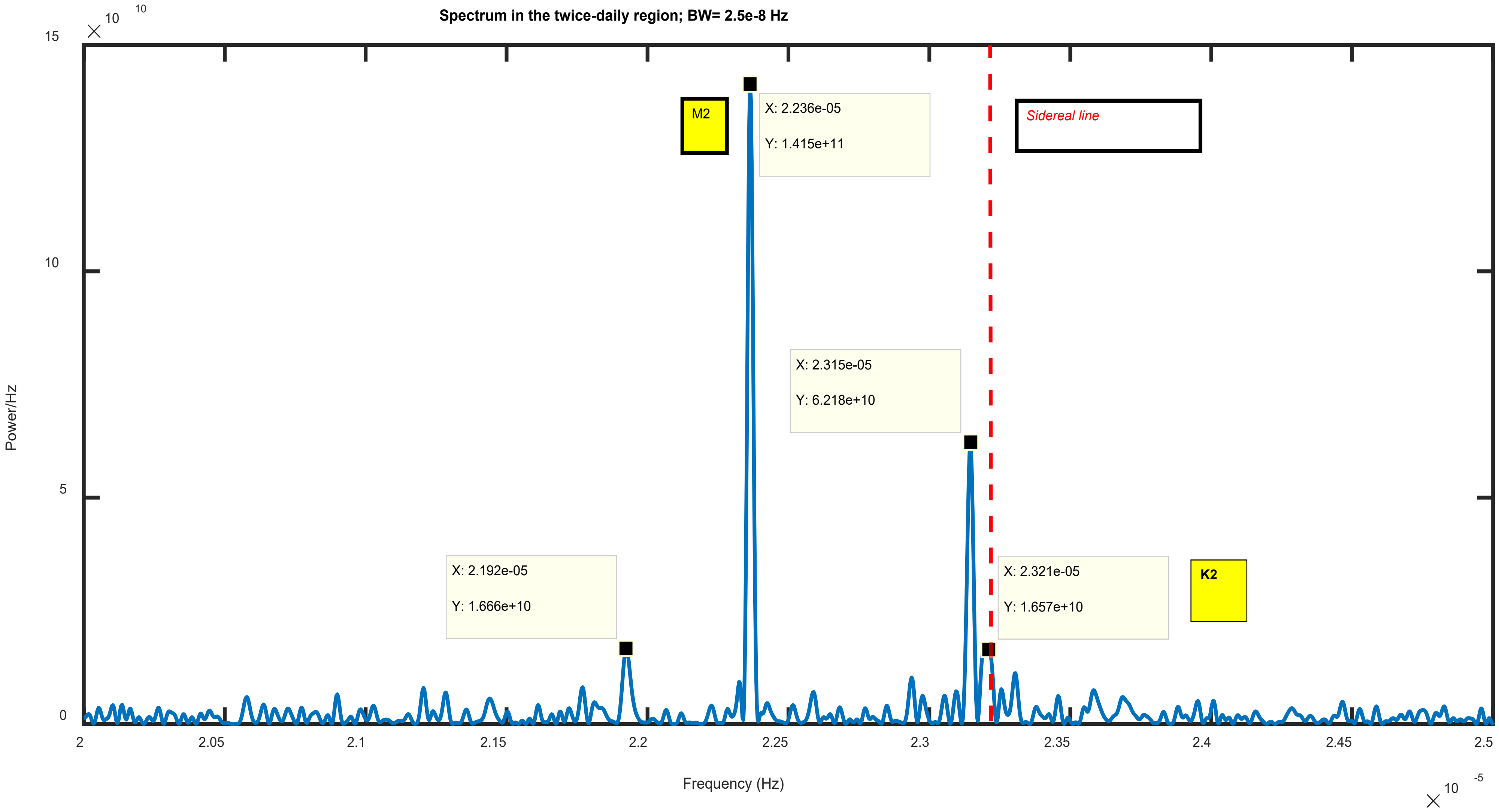
$$\mathrm{F}_{West} = C\left[\sin\phi \, \sin 2\delta \, \sin H + \cos\phi \, \cos^2\delta \, \sin 2H\right]$$

with the usual notation: H the hour angle and δ the declination of the perturbing body, and ϕ the latitude of the site,
We are interested in the terms that rotate at 2H, and for the M2 line at the Hanford site (setting $\langle\delta\rangle = 0$) the amplitudes are

$F_{South} \approx 0.7\times10^{-6}\ m/s^2$ $F_{West} \approx 10^{-6}\ m/s^2$

The amplitude of the M2 line, for multiple (≈ 100) traversals, corresponds to a phase shift, $\Delta\phi^{(multiple)}/2\pi \approx 1.2\times10^{-8}$

Spectrum in Twice Daily Region



Upper limit on a non-tidal phase shift at the twice daily sidereal frequency

Four tidal lines are observed in the twice daily region: N2, M2, S2 and K2. Their frequencies are exact (to 10^{-8} Hz) and their relative amplitudes agree with the known values (the measured power in the line is proportional to the tidal **amplitude** because it arises from interference) [1 gal = 10^{-2} m/s^2]

M2	Measured power P=3538 counts	known amplitude	91 μgal
K2	415 counts		11.5 μgal

Therefore the expected tidal power at K2 = 447 counts

Observed **non-tidal** power at K2 (twice daily sidereal) = 32 ± 34 counts

Normalizing to the known phase shift induced by the M2 tidal line

$\delta\phi^{(\text{single pass})}/2\pi = (1.1 \pm 1.2)\times 10^{-12}$ LV effect

Upper limit on Lorentz Invariance violation from the preliminary LIGO data

The upper limit on the phase shift implies a limit on dispersion

$$\delta n/n = (\delta\phi^{(\mathrm{single})}/2\pi)(\lambda/2L) = (1.4 \pm 1.5)\times 10^{-22}$$

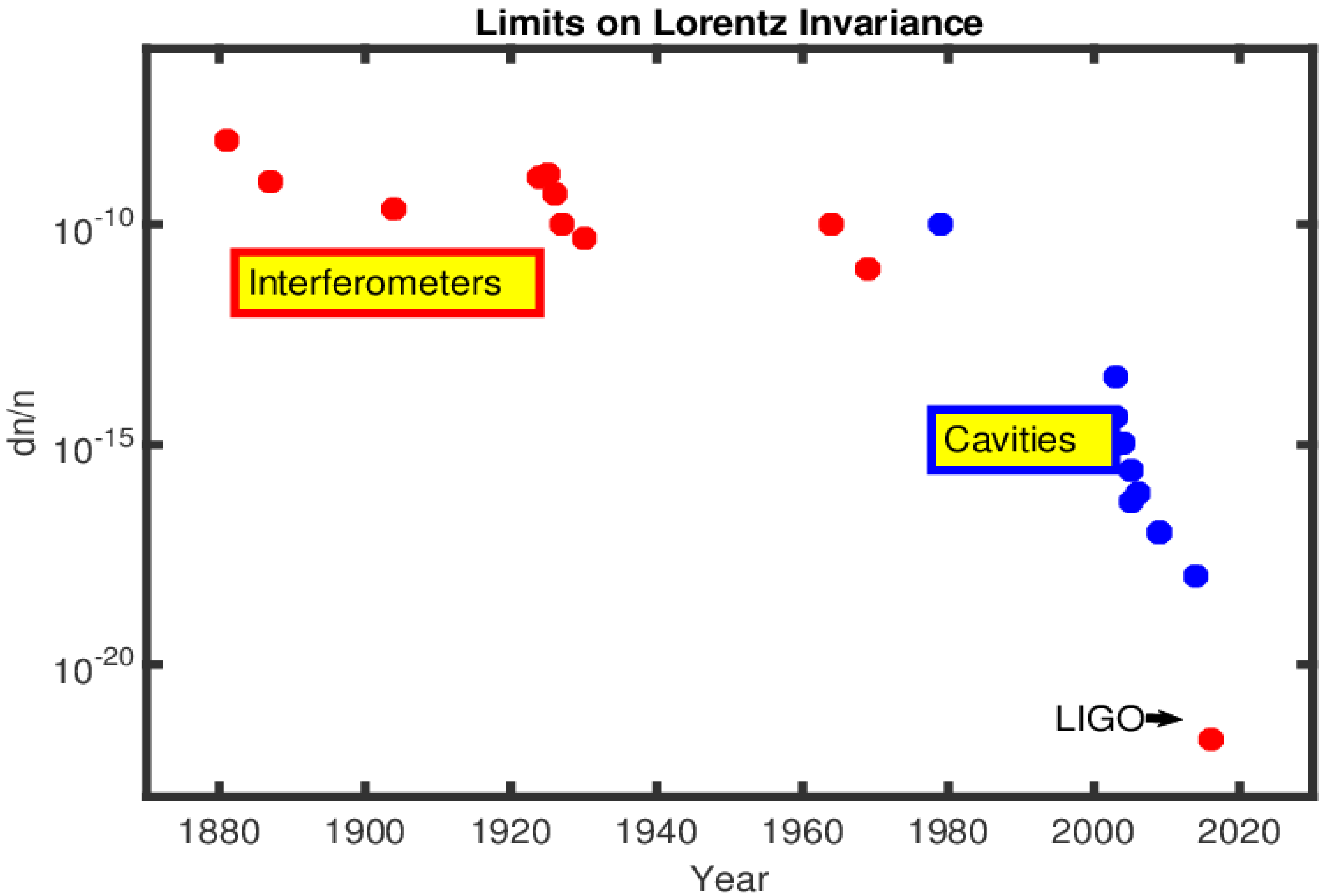
Compare to best limits

M. Nagel et al. Nature Com. 6:8174 (9/1/2015)

$\delta\nu/\nu < 10^{-18}$

Pruttivarasin et al. Nature 517, 592 Letter (2015)

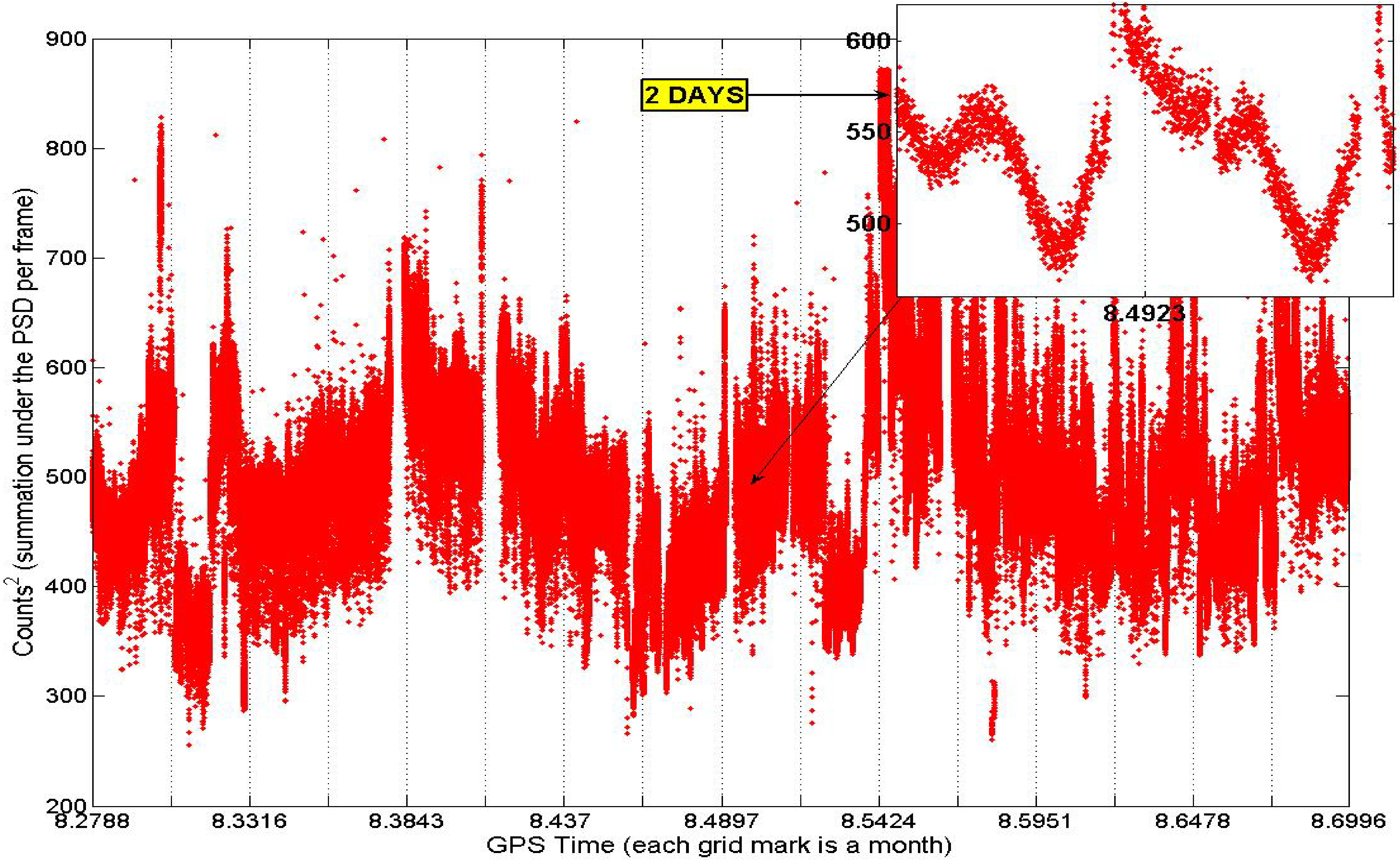
$\delta\nu/\nu < 10^{-18}$



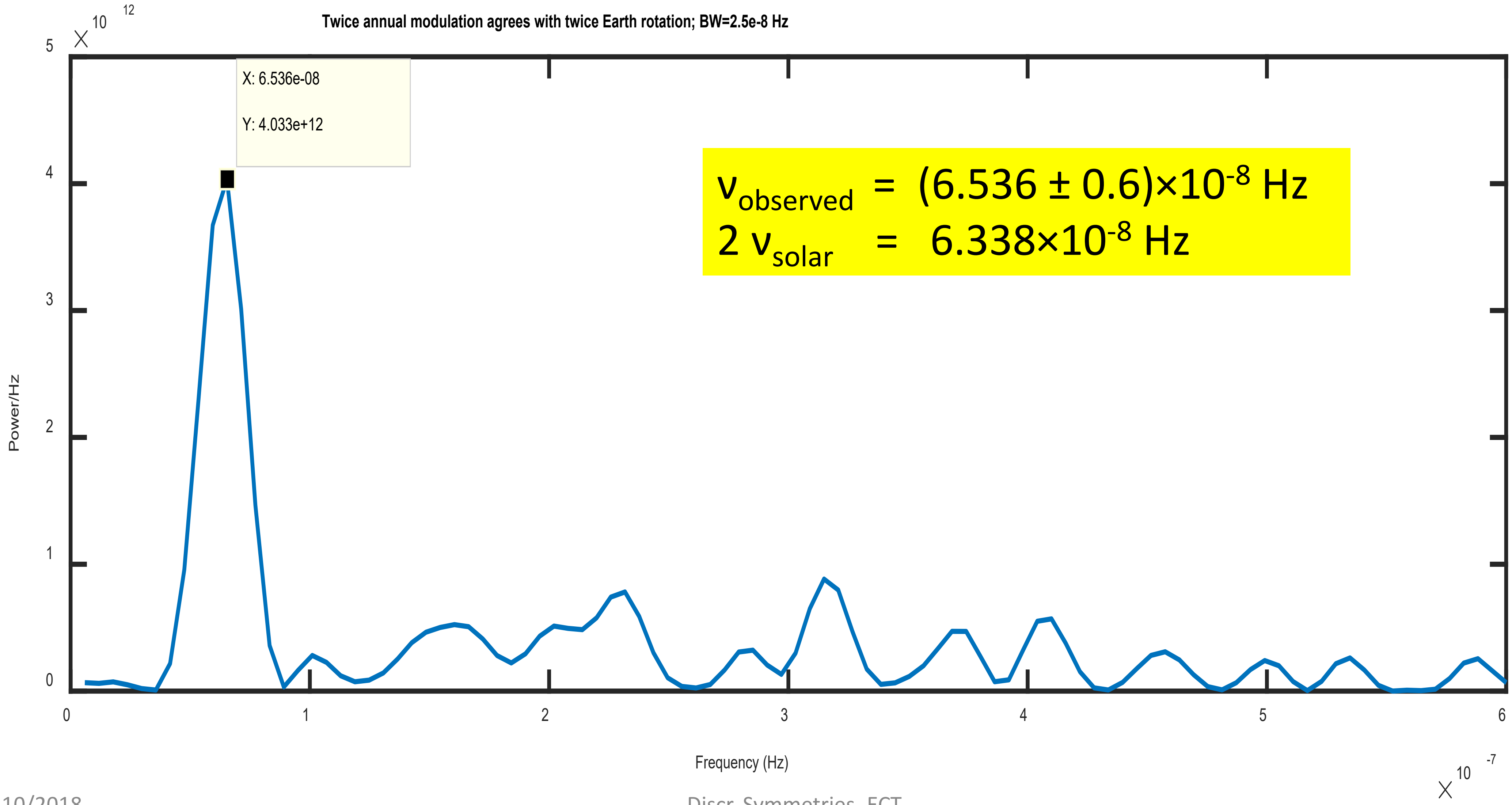
Limits on Lorentz Invariance
Interferometers
Cavities
LIGO
dn/n
10^{-10}
10^{-15}
10^{-20}
1880
1900
1920
1940
1960
1980
2000
2020
Year

The observed twice Yearly Modulation

Preliminary LIGO data of the integrated fsr power from Apr. 06, 2006 - July 07, 2007



The twice yearly modulation of the preliminary LIGO data is obvious, but not understood.



Standard Model Extension to include LV and CPT violating effects
V.A. Kostelecky and M. Mewes Phys. Rev. D66, 056005 (2002)

Introduce a modified Lagrangian for EM field; the 4-tensor k_F has 19 independent coefficients-

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} (k_F)_{\kappa\lambda\mu\nu} F^{\kappa\lambda} F^{\mu\nu}$$

By redefining the coefficients of k_F as 3x3 matrices we can write

$$\mathcal{L} = \frac{1}{2}[(1+\tilde{\kappa}_{\mathrm{tr}})\vec{E}^2 - (1-\tilde{\kappa}_{\mathrm{tr}})\vec{B}^2] + \frac{1}{2}\vec{E}\cdot(\tilde{\kappa}_{e+} + \tilde{\kappa}_{e-})\cdot\vec{E}$$

$$-\frac{1}{2}\vec{B}\cdot(\tilde{\kappa}_{e+} - \tilde{\kappa}_{e-})\cdot\vec{B} + \vec{E}\cdot(\tilde{\kappa}_{o+} + \tilde{\kappa}_{o-})\cdot\vec{B}. \qquad (11)$$

Why modulation at 2Ω appears

nation $\widetilde{\kappa}_{e-}^{JK}$, the antisymmetric combination $\widetilde{\kappa}_{o+}^{JK}$, and the trace component $\widetilde{\kappa}_{\rm tr}$ in the Sun-centered frame [25]. This gives

$$\begin{aligned}\Delta\bar{n} &= -\tfrac{1}{2}(\hat{l}_1^j\hat{l}_1^k - \hat{l}_2^j\hat{l}_2^k)\widetilde{\kappa}_{e-}^{jk}\\ &= -\tfrac{1}{2}(\hat{l}_1^j\hat{l}_1^k - \hat{l}_2^j\hat{l}_2^k)\big(\Lambda^j{}_J\Lambda^k{}_K\widetilde{\kappa}_{e-}^{JK} + \Lambda^j{}_T\Lambda^k{}_J\epsilon^{JKL}\widetilde{\kappa}_{o+}^{KL}\\ &\qquad - 2\Lambda^j{}_T\Lambda^k{}_T\widetilde{\kappa}_{\rm tr}\big).\end{aligned} \tag{12}$$

In this expression, the elements of the Lorentz transformation relating the Sun-centered frame and the laboratory frame can be taken as

$$\Lambda^0{}_T = 1,\quad \Lambda^0{}_J = -\beta^J,\quad \Lambda^j{}_T = -(R\cdot\vec{\beta})^j,\quad \Lambda^j{}_J = R^{jJ}, \tag{13}$$

where the matrix R^{jJ} rotating between the Sun-centered and laboratory frames is given by Eq. (C1) of Ref. [25], and β^J is given in terms of the orbital and laboratory boosts by Eq. (C2) of the same reference.

Results for dn/n, and SME Cartesian coefficients for different harmonics ω(rotation), Ω(orbital)

Harmonic	$\Delta\overline{n}/\overline{n}$
$\omega_\oplus$	$< 1.4 \times 10^{-20}$
$2\omega_\oplus$	$< 2.0 \times 10^{-22}$
$3\omega_\oplus$	$< 2.1 \times 10^{-22}$
$4\omega_\oplus$	$< 2.1 \times 10^{-22}$
$\Omega_\oplus$	$< 3.4 \times 10^{-20}$
$2\Omega_\oplus$	$(4.0 \pm 0.25) \times 10^{-19}$

Harmonic	Coefficient	Result
$\omega_\oplus$	$\lvert\tilde{\kappa}_{e-}^{XZ}\rvert$	$< 2.1 \times 10^{-20}$
	$\lvert\tilde{\kappa}_{e-}^{YZ}\rvert$	$< 2.1 \times 10^{-20}$
$2\omega_\oplus$	$\lvert\tilde{\kappa}_{e-}^{XY}\rvert$	$< 2.7 \times 10^{-22}$
	$\lvert\tilde{\kappa}_{e-}^{XX} - \tilde{\kappa}_{e-}^{YY}\rvert$	$< 5.5 \times 10^{-22}$
$\Omega_\oplus$	$\lvert\tilde{\kappa}_{\mathrm{tr}}\rvert$	$< 9.2 \times 10^{-10}$
	$\lvert\tilde{\kappa}_{o+}^{XY}\rvert$	$< 6.6 \times 10^{-15}$
	$\lvert\tilde{\kappa}_{o+}^{XZ}\rvert$	$< 5.7 \times 10^{-15}$
	$\lvert\tilde{\kappa}_{o+}^{YZ}\rvert$	$< 5.2 \times 10^{-15}$
2Ω	$\lvert\kappa_{tr}\rvert$	= $(3.1\pm0.2)\mathrm{x}10^{-9}$

*The coefficients are extracted from the data, (thanks to Alan Kostelecky and Matt Mewes), under the assumption that **only that particular coefficient differs from zero.***

The refractive index

From the modified Lagrangian (Eq.11)   Assuming that only $\kappa_{tr} \neq 0$

$$\frac{\partial L}{\partial \vec{E}} = \vec{D} = \epsilon \vec{D} = (1 + \kappa_{tr})\vec{E}$$

$$\frac{\partial L}{\partial \vec{B}} = \vec{H} = \mu \vec{B} = (1 - \kappa_{tr})\vec{B}$$

$$n = \sqrt{\epsilon\mu} = \sqrt{1 + \kappa_{tr}} / \sqrt{1 - \kappa_{tr}} \approx 1 + \kappa_{tr}$$

Thus $|n-1| \approx 10^{-9}$ which is much too large and excluded from the observation of **very high energy** gamma rays and **ultra high energy** Cosmic Rays

Limits on refractive index from ultra high energy cosmic rays and very high energy γ-rays

S. Coleman and S. Glashow (1999)

If $n>1$, a charged particle will emit Cherenkov radiation when $\beta n>1$ and loose energy. Therefore $n-1< 1/2\gamma^2$.
C.R. with $E \approx 200$ EeV $= 2\text{x}10^{20}$ eV have been observed. Assuming $M= 100$ GeV, $\gamma= 2\times10^9$ and **$n-1 < 10^{-19}$**

If $n<1$, photons propagate with phase velocity $c_p< c$ and are time-like, they will decay into massive particles, $\gamma\rightarrow e^+e^-$ and loose energy $1-n < ½(2m_e/E_\gamma)$
The observation of 60 TeV γ-rays implies **$n-1 > 1.4\text{x}10^{-16}$**

Conclusion on the observed twice annual modulation

The value of κ_{tr} extracted from the data is **incompatible** with the **photon sector** of the SME model.
[A. Kostelecky and M. Mewes Phys. Rev. **D 66** 56005 (2002)]

It could be due
(1) To Lorentz violation in another sector, or
(2) The SME model is not complete, or
(3) The data are incorrect.

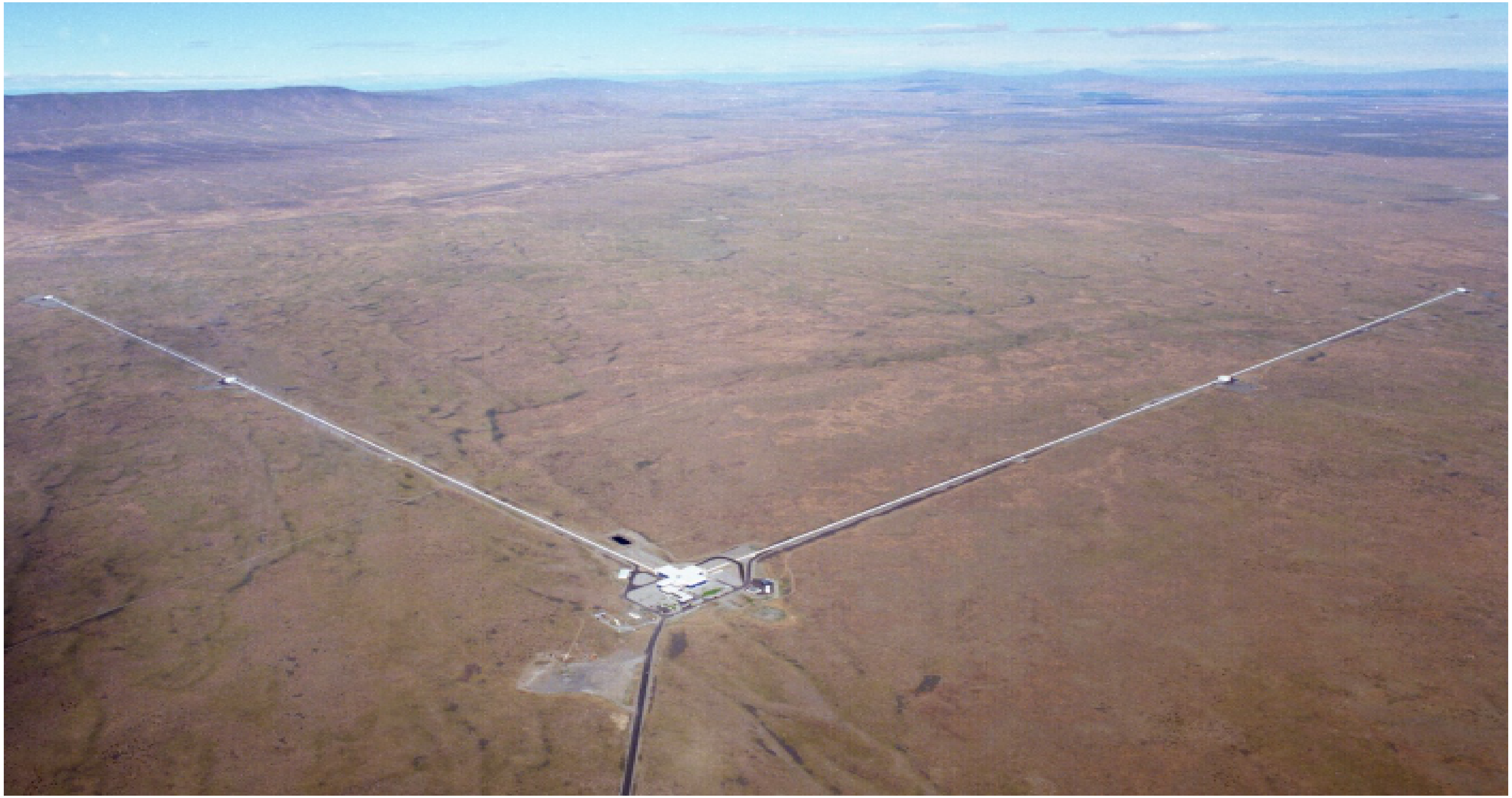
The observed twice yearly modulation can be explained phenomenologically by introducing a small dependence of the refractive index on the square of the velocity of the observer with respect to some inertial frame

$$dn/n = \alpha\beta^2 = \alpha|\beta|^2\cos^2\omega t = \alpha|\beta|^2(1/2)(1+\cos 2\omega t)$$

Obviously the dominant term will involve the Earth's orbital velocity, as compared to its rotational velocity, in agreement with the data.

This does not violate the Coleman-Glashow bounds

View of the Hanford LIGO Observatory



Acknowledgements

The data presented here were obtained during the LIGO S5 run by the dedicated efforts of the staff and operators of the LIGO Hanford Observatory and the members of the LSC.

Special thanks to D. Sigg who designed and installed the fsr channel and to W. Butler, C. Forrest, T. Fricke and S. Giampanis who analyzed the data. To A. Kostelecky and M. Mewes for interpreting the data in the SME model framework.

References: A. Melissinos (for the LSC) “The effect of the tides on the LIGO interferometers” 12th Marcel Grossman meeting. World Scientific, p.1719 (2012); arXiv:1001.0558v2[gr-qc] (2009) – **nine** years ago.

V. A. Kostelecky, A. Melissinos and M. Mewes Phys. Letters **B** 761, 1 (2016).

The Coleman Glashow argument

$$n = c/c_{ph} = \sqrt{(1+\kappa_{tr})/(1-\kappa_{tr})} \approx 1+\kappa_{tr}$$

Consider $n > 1$ then the threshold is when $n\beta = 1$

or $n^2\beta^2 = 1$ or $1/\gamma^2 \approx n^2 - 1 \approx 2(n-1) = 2\kappa_{tr}$

$$\kappa_{tr} = n - 1 < Mc^2/2E^2 = 10^{-19}$$

Consider $n < 1$ photon becomes time-like

$P = E/c_{ph} = E/(c/n) = nE/c$

But $(2mc^2)^2 = E^2 - c^2P^2 = E^2(1-n^2)$

or $E^2 > (2mc^2)^2/(1-n^2) \approx 2m^2c^4/(1-n)$

$$-\kappa_{tr} = (1-n) > 2m^2c^4/E^2 = 1.4 \times 10^{-16}$$

Expected phase shift from Galilean relativity

Light of wavelength λ, making a round trip in an arm of length L, moving with velocity β and at an angle θ with respect to an absolute frame acquires in Galilean relativity a phase shift

$$\frac{\phi}{2\pi} = \frac{cT}{\lambda} = \frac{2L}{\lambda}\frac{\sqrt{1+\beta^2\sin^2\theta}}{1-\beta^2\cos^2\theta} \approx \frac{2L}{\lambda}\left[1+\frac{\beta^2}{4}(3+\cos 2\theta)\right]$$

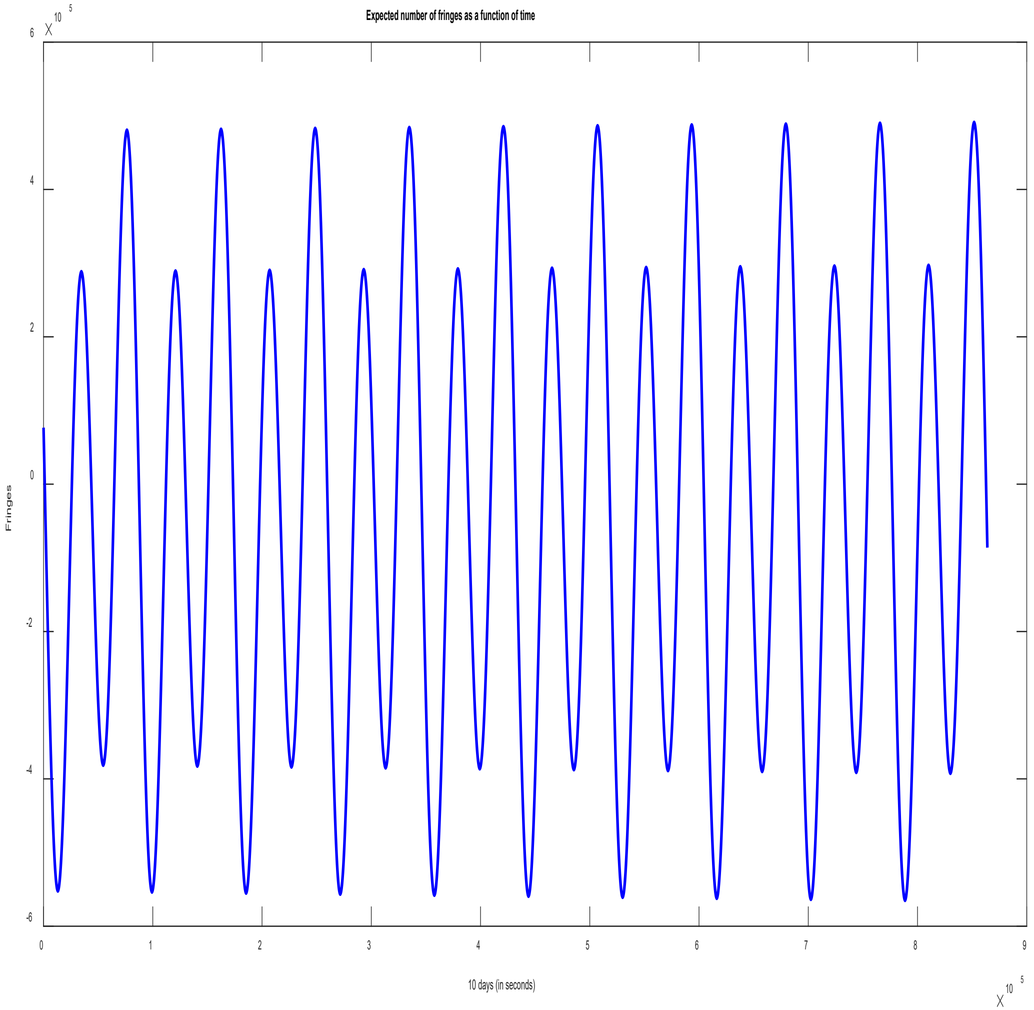
Thus the phase shift between two arms (of equal length) oriented at angles θ_A, θ_B is

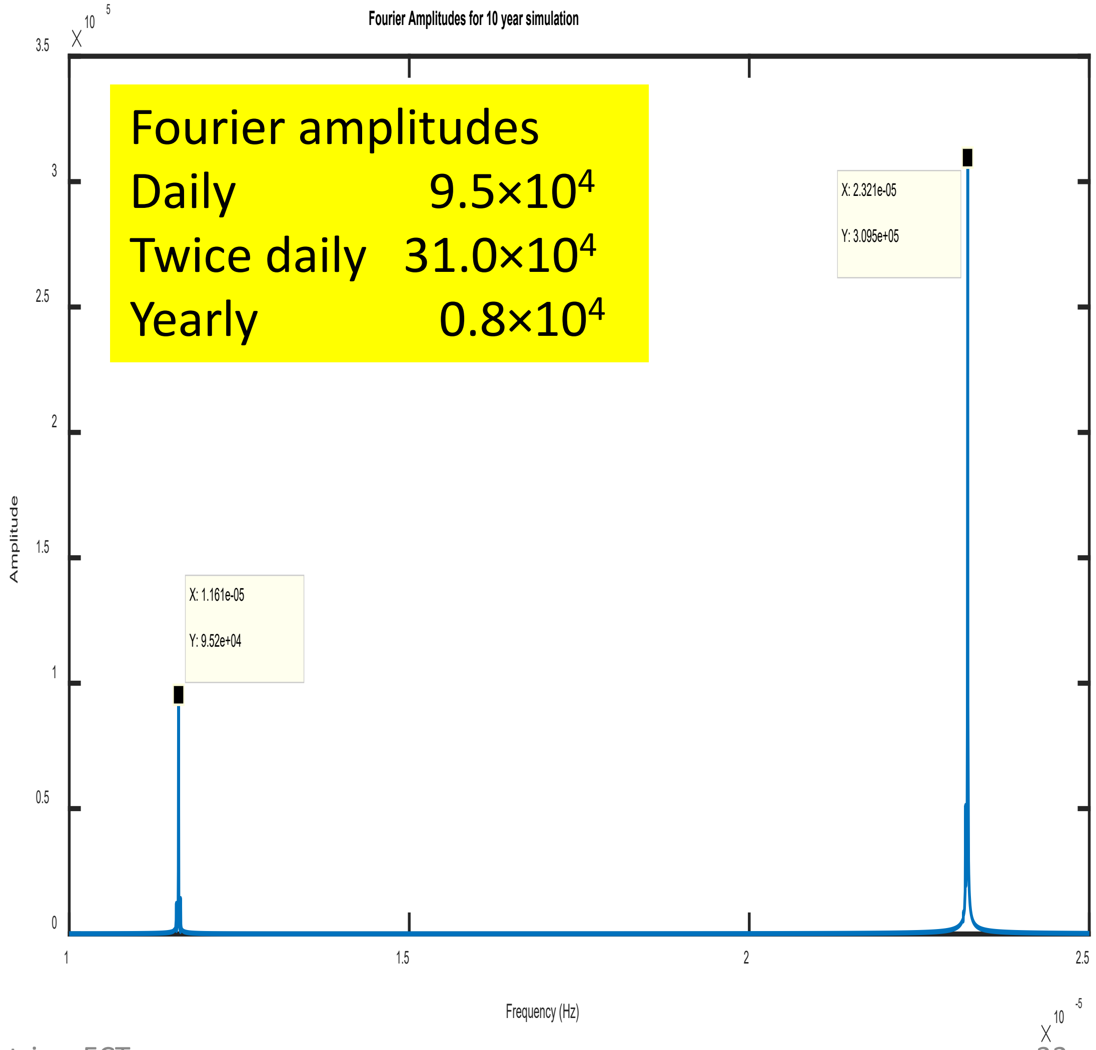
$$\frac{\Delta\phi}{2\pi} = \frac{L}{2\lambda}\beta^2[\cos 2\theta_A - \cos 2\theta_B]$$

The angles and velocity are evaluated in the SCCEF

$\beta_{daily(Hanford)} = 10^{-6}$ $\beta_{orbital} = 10^{-4}$

Expected Fringe shift ($\Delta\phi/2\pi$) for Galilean transformation at the Hanford LIGO Interferometer





Comparison of Michelson's and LIGO Interferometers

The "Figure of Merit" is length divided by Fringe resolution,

MM	$L = 11$ m	ΔFringe = 0.01	L/ΔFringe = 10^3
LIGO	$L = 4\times10^5$ m	ΔFringe = 2×10^{-10}	L/ΔFringe = $2\text{x}10^{15}$

For LIGO, given the Earth's velocity through the CMB frame, $\beta = 1.2\times10^{-3}$ the expected ΔFringe = 3.5×10^5, but we observe $< 2\times10^{-10}$, (multipass) thus in the RMS formalism

$$P_{MM} = (½ - \beta + \delta) < 6\times10^{-16}$$

Expressed as dispersion, the fringe shift, ΔFringe = $(\delta\nu)(2L/c)$, or

$$\delta\nu/\nu = (\lambda/2L)\times\Delta\text{Fringe} < 2.5\times10^{-22}$$

More about the SME coefficients

We analyze **only the twice daily** frequency

We do not separate the two quadratures but use the limit on the observed amplitude of the signal.
The dominant term is κ^{XY} , and the data imply the limit

$$\kappa^{XY} < 8\times10^{-23}$$

The twice yearly frequency is directly proportional to the trace of the "kappa" coefficients, leading to

$$\kappa_{tr} = (3.1\pm0.2)\times10^{-9} \quad !$$

SME coefficients

Normalizing the spectral power observed at the **twice yearly** frequency, to the power in the M2 tidal line, one finds

$$\delta n/n = (0.6 - 4.0)\times 10^{-19} \qquad \text{at} \quad 2\Omega_{orbital}$$

The twice yearly frequency is directly proportional to the trace of the "kappa" coefficients, leading to

$$\kappa_{tr} = (3.1 \pm 0.2)\times 10^{-9}$$

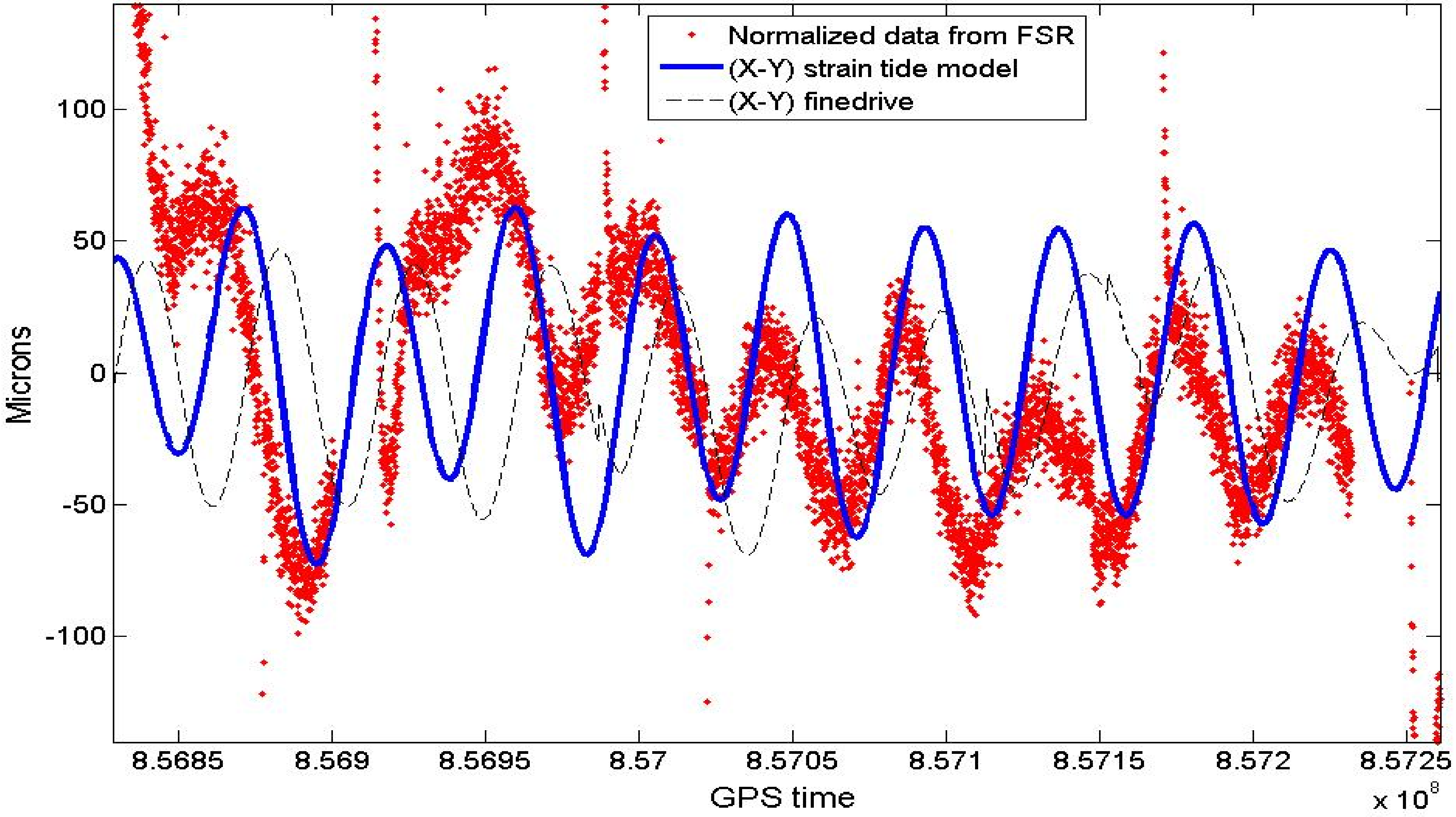
which is at the limit of existing measurements (arXive:0801.0287v9)

and could be instrumental.

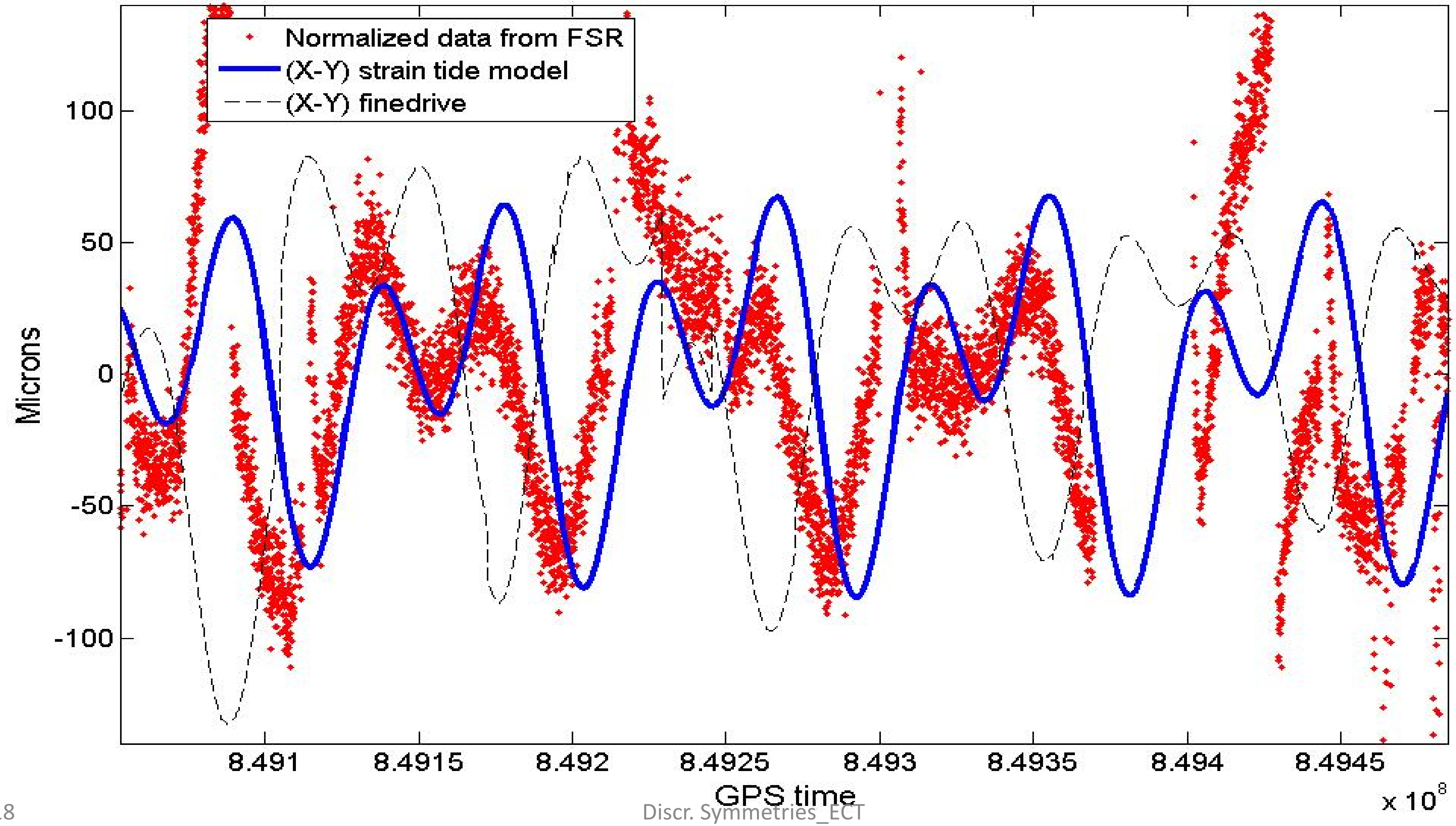
In contrast the absence of any signal at $\Omega_{orbital}$ sets a limit on

$$\kappa^{YZ}{}_{o+} < 10^{-17}$$

Predicted vs observed strain March 2 - 7, 2007



Predicted vs observed strain December 2 - 7,2006



Hand –waving evaluation of A_ω from modulation M

When $A_\omega \ll A_{fsr}$

$$M = \frac{P_{max} - P_{min}}{P_{max} + P_{min}} \approx 2\frac{A_\omega}{A_{fsr}} \qquad (1)$$

therefore $A_\omega \approx M A_{fsr}/2.$

But because of build up of the signal in the arms by a factor $B \approx 100$,
$A_\omega = (\Delta\phi_\omega/2\pi)BE_+$, whereas $A_{fsr} = (\Delta\phi_{bias}/2\pi)E_+$

$$(\Delta\phi_\omega)/2\pi = (M/2B)(\Delta\phi_{bias}/2\pi)$$

Numerically, $M = 0.1$, $B = 100$ $\Delta\phi_{bias}/2\pi = 2.5 \times 10^{-6}$

Leads to $\Delta\phi_\omega/2\pi \approx 10^{-9}$
Instead of the correct value $\Delta\phi_\omega/2\pi = 1.6 \times 10^{-10}$

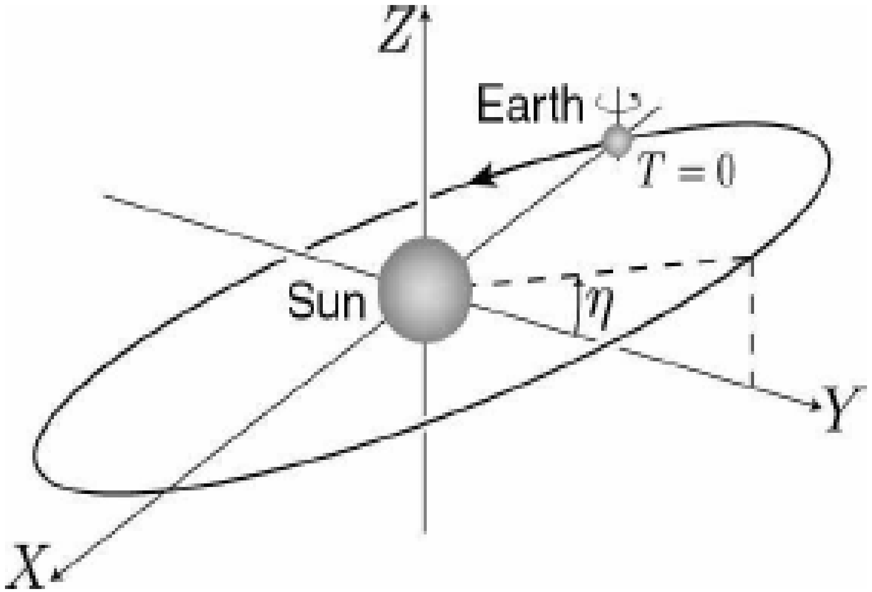
E_+ is the fsr field amplitude which cancels in the calculation since it contributes equally to $\Delta\phi_{bias}$ and to $\Delta\phi_\omega$

The Earth Tides

TABLE 3a. Principal Tidal Waves

Symbol	Argument number	Argument	Frequence	Amplitude	Origin (L, lunar; S, solar)
		Long period components			
M_o	055·555	0	0°, 000000	+ 50458	L constant flattening
S_o	055·555	0	0°, 000000	+ 23411	S constant flattening
S_a	056·554	$h - p_s$	0°, 041067	+ 1176	S elliptic wave
Ss_a	057·555	$2h$	0°, 082137	+ 7287	S declinational wave
M_m	065·455	$s - p$	0°, 544375	+ 8254	L elliptic wave
M_f	075·555	$2s$	1°, 098033	+ 15642	L declinational wave
		Diurnal components			
Q_1	135·655	$(\tau - s) - (s - p)$	13°, 398661	+ 7216	L elliptic wave of O_1
O_1	145·555	$\tau - s$	13°, 943036	+ 37689	L principal lunar wave
M_1	155·655	$(\tau + s) - (s - p)$	14°, 496694	− 2964	L elliptic wave of ${}^mK_1$
π_1	162·556	$(t - h) - (h - p_s)$	14°, 917865	+ 1029	S elliptic wave of P_1
P_1	163·555	$t - h$	14°, 958931	+ 17554	S solar principal wave
S_1	164·556	$(t + h) - (h - p_s)$	15°, 000002	− 423	S elliptic wave of ${}^sK_1$
${}^mK_1$	165·555	$\tau + s = \theta$	15°, 041069	− 36233	L declinational wave
${}^sK_1$	165·555	$t + h = \theta$	15°, 041069	− 16817	S declinational wave
ψ_1	166·554	$(t + h) + (h - p_s)$	15°, 082135	− 423	S elliptic wave of ${}^sK_1$
φ_1	167·555	$t + 3h$	15°, 123206	− 756	S declinational wave
J_1	175·455	$(\tau + s) + (s - p)$	15°, 585443	− 2964	L elliptic wave of ${}^mK_1$
OO_1	185·555	$\tau + 3s$	16°, 139102	− 1623	L declinational wave
		Semi-diurnal components			
$2N_2$	235·755	$2\tau - 2(s - p)$	27°, 895355	+ 2301	L elliptic wave of M_2
μ_2	237·555	$2\tau - 2(s - h)$	27°, 968208	+ 2777	L variation wave
N_2	245·655	$2\tau - (s - p)$	28°, 439730	+ 17387	L major elliptic wave of M_2
ν_2	247·455	$2\tau - (s - 2h + p)$	28°, 512583	+ 3303	L evection wave
M_2	255·555	2τ	28°, 984104	+ 90812	L principal wave
λ_2	263·655	$2\tau + (s - 2h + p)$	29°, 455625	− 670	L evection wave
L_2	265·455	$2\tau + (s - p)$	29°, 528479	− 2567	L minor elliptic wave of M_2
T_2	272·556	$2t - (h - p_s)$	29°, 958933	+ 2479	S major elliptic wave of S_2
S_2	273·555	$2t$	30°, 000000	+ 42286	S principal wave
R_2	274·554	$2t + (h - p_s)$	30°, 041067	− 354	S minor elliptic wave of S_2
${}^mK_2$	275·555	$2(\tau + s) = 2\theta$	30°, 082137	+ 7858	L declinational wave
${}^sK_2$	275·555	$2(t + h) = 2\theta$	30°, 082137	+ 3648	S declinational wave
		Ter-diurnal component			
M_3	355·555	3τ	43°, 476156	− 1188	L principal wave

Sun Centered Celestial Equatorial Frame used in the SME framework



Earth Centered Inertial frame

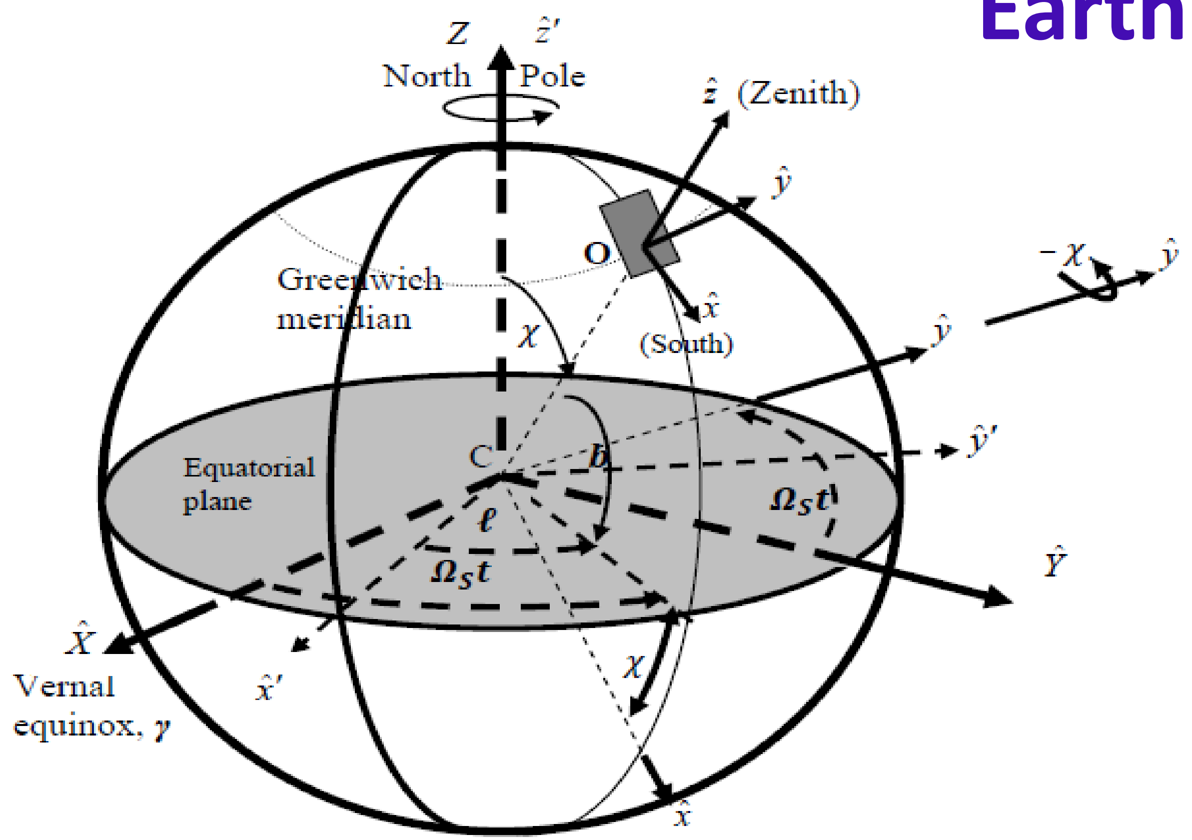
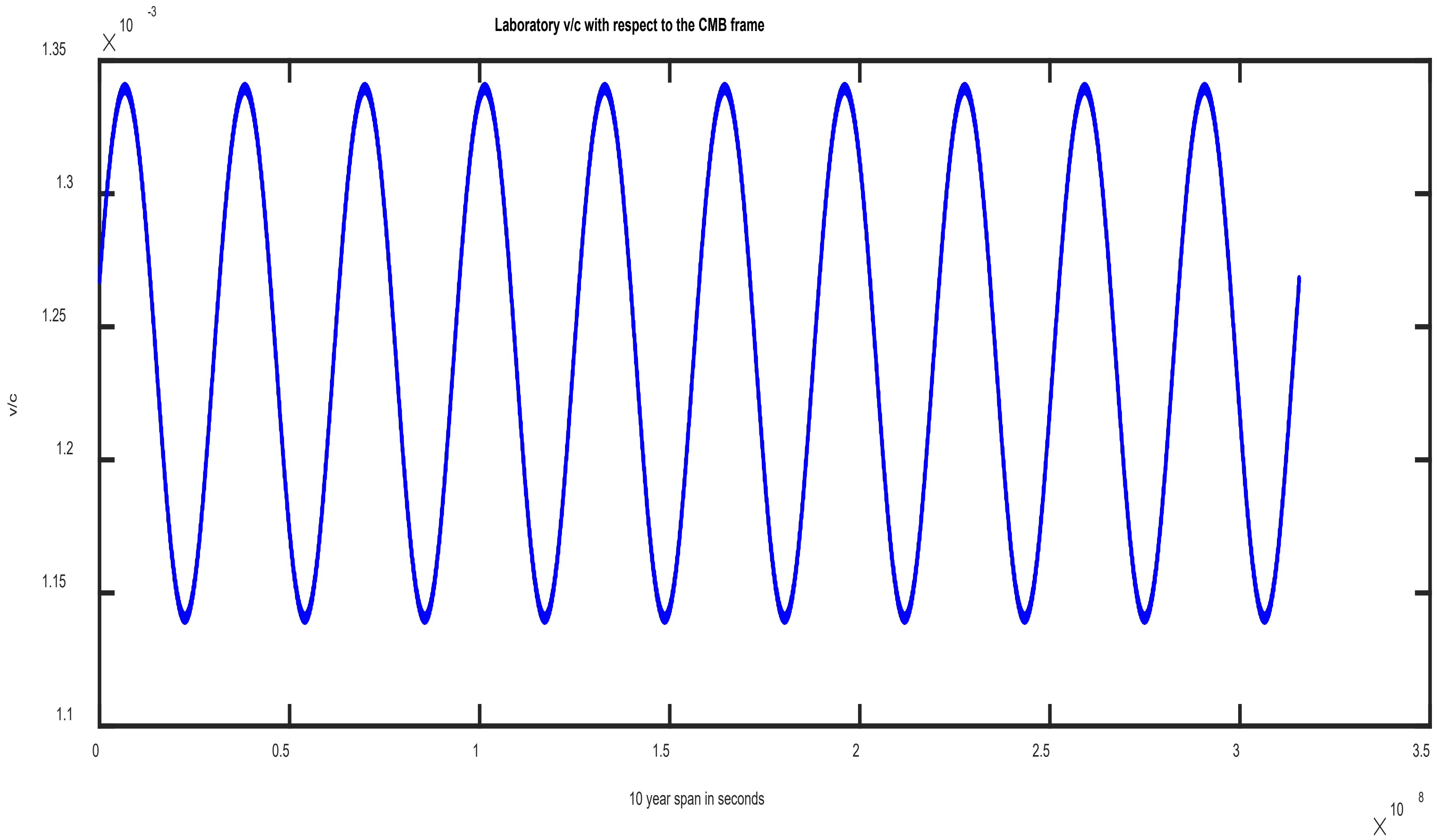


Figure 1. Schematic diagram of the Earth-centered inertial frame C(XYZ); the Earth-centered non-inertial frame C($x'y'z'$) embedded in and rotating with the Earth; and any arbitrary Earth based laboratory frame O(xyz) with (longitude, latitude) = (ℓ, b) and co-latitude, $\chi = \frac{\pi}{2} - b$ centered at C of the Earth's center and rotating with the Earth's axis with sidereal angular rotational velocity Ω_S. The time $t = 0$ starts on the first day of autumn 21st September (Autumnal Equinox). In order to derive the rotation matrices to make the transformation between C(XYZ) and O(xyz) frames, the rotation angles $-\chi$ and $-\Omega_S t$ with rotation axis $\hat{y}$ and $\hat{Z}$ respectively have been shown.

Earth velocity wrt to CMB frame for 10 years



(3) How the LOCK affects the Signal read-out.
The question is often brought up of how the signal can be read-out when the servo always returns the IFO to the dark fringe condition, especially for slow signals, that is disturbances at low or very low frequencies. Let's start with the gw signal, which through an imbalance in the phases of the light returning from the arms to the dark port generates a signal that (after demodulation) appears in the ASQ channel. This signal is sampled and recorded at 16 kHz, and after appropriate filtering is labeled DARM-ERR, and it is **integrated** and amplified to generate the DARM-CTRL signal which moves the mirrors back to the dark fringe condition. When the IFO is returned to the dark condition DARM-ERR becomes zero but DARM-CTRL is **NOT** affected since a null DARM-ERR does not add (nor subtract) from the previously calculated integral; note that this is also necessary **to keep** the mirrors on the dark fringe. DARM-CTRL changes (up or down) occur only when a signal (positive or negative) appears at the DARM-ERR output. Thus DARM-CTRL is a faithful record of all slow perturbations that have been applied to the IFO, and can be used to track slow or extra slow signals. Obviously it can be easily filtered against fast noise by averaging.

The next question is how this works for the fsr channel, namely how the corresponding integration is performed. Here the process is that every 8 second stretch of data is Fourier transformed (FFT'd) to provide the frequency spectrum (around the fsr) at time t_i These spectra contain amplitudes $A_i(\nu)$ typical of the phase shift introduced on the interferometer by the gravity gradient. In the meantime the servo returns the IFO to the dark condition by moving the mirrors, while the next frame contains amplitudes $A_j(\nu)$ typical of the value of the gradient at time t_j. It is the time series of the integrated **power** that is a measure of the effect of the time-varying gradient on the IFO. Mathematically, the FFT operation $A(\omega) = \int A(t)e^{i\omega t}dt$ is equivalent to an integration

(4) POWER in the first (37.52 kHz) sideband.
The enhancement at the fsr frequency observed in the power spectral density in Fig.1 (of LIGO-P1500157-v2), could be caused either by: (a) a differential strain oscillating at 37.52 kHz, or (b) a static arm difference which introduces a phase shift $\delta\phi/2\pi = \Delta L/L$ in the region of the sideband frequency $\nu_1 = \nu_0 \pm \nu_{fsr}$. For case(a) we use the interferometer calibration to find that the corresponding strain density is

$$h = 4.6 \times 10^{-23}/\sqrt{\mathrm{Hz}}. \tag{4}$$

We designate the carrier signal amplitude density by $A_0/\sqrt{\mathrm{Hz}}$ and similarly the fsr signal amplitude density by $A_1/\sqrt{\mathrm{Hz}}$. It then follows that

$$\frac{A_0}{\sqrt{\mathrm{Hz}}} = E_0 \frac{\Delta\phi_0}{\sqrt{\mathrm{Hz}}} = E_0 \frac{L}{\lambda} \frac{h}{\sqrt{\mathrm{Hz}}} \qquad \text{and} \qquad \frac{A_1}{\sqrt{\mathrm{Hz}}} = \frac{E_1}{\sqrt{\mathrm{Hz}}} \frac{\Delta L}{L}. \tag{5}$$

Here E_0 is the optical (electric) field at the carrier frequency, and $E_1/\sqrt{\mathrm{Hz}}$ is the density of the optical (electric) field in the fsr region. Setting $A_0/\sqrt{\mathrm{Hz}}$ equal to $A_1/\sqrt{\mathrm{Hz}}$, and using the numerical value of h given by Eq(4), and $\Delta L = 10^{-2}$ m, we find

$$E_1/\sqrt{\mathrm{Hz}} = 8 \times 10^{-8} E_0 \qquad \text{or correspondingly} \qquad E_1^2/\mathrm{Hz} = 6.4 \times 10^{-15} E_0^2$$

In terms of leakage from the carrier this corresponds to -284 dBc at 37 kHz, which seems reasonable.

Since the width of the E_1 field is ≈ 250 Hz, the power in the first sideband is $P_1 \approx 1.6 \times 10^{-12} P_0$. This is quite sufficient power to produce the signals observed in the fsr region.