TECHNISCHE UNIVERSITÄT DARMSTADT

CORRELATIONS IN RELATIVISTIC HEAVY ION COLLISIONS

Pan-American Few-Body Physics Boot Camp: Fostering Collaboration

22.10.2025

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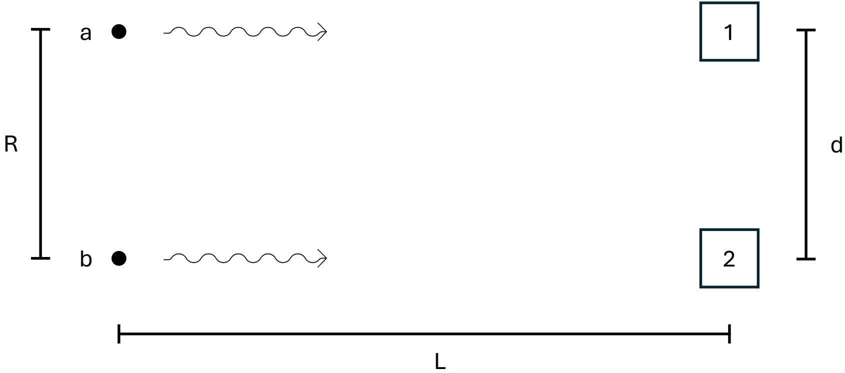
CONTENTS

HANBURY-BROWN-TWISS INTERFEROMETRY

Section 1

HANBURY-BROWN-TWISS INTERFEROMETRY



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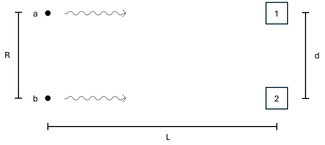

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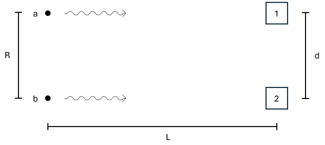

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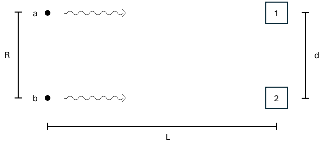

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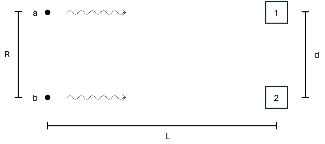

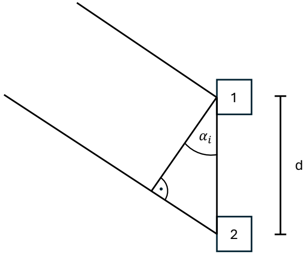

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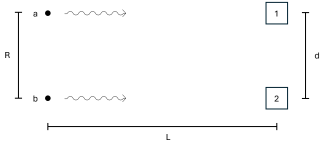

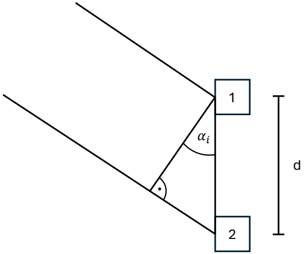

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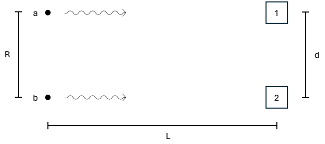

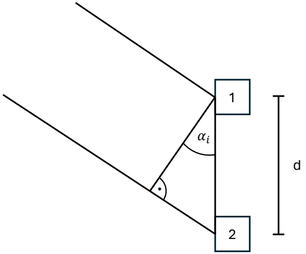

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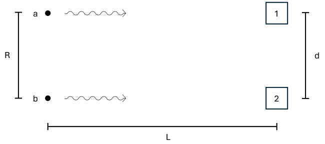

[model: Baym, arXiv preprint nucl-th/9804026, 1998]

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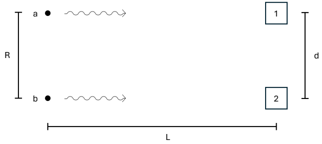

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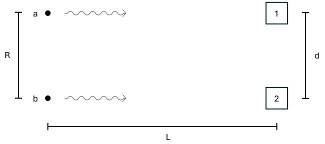

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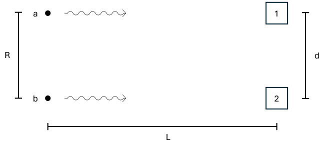

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$\Rightarrow$ if k and L are known we can extract R by differing d and measuring the correlated signal

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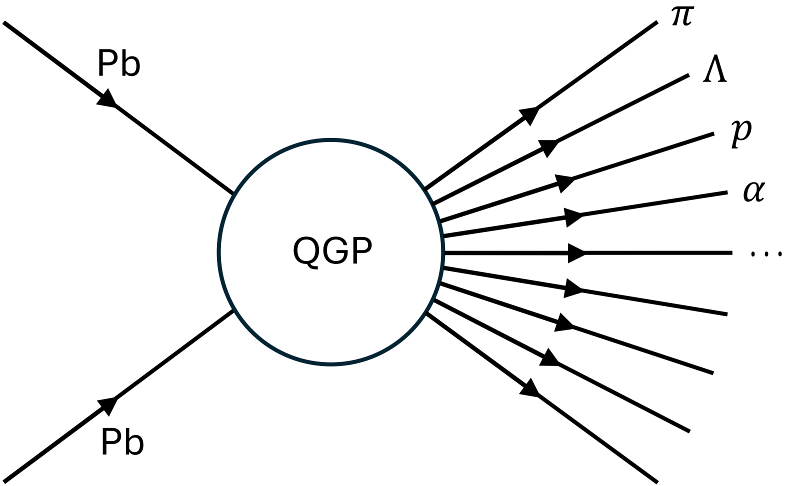
HEAVY ION COLLISIONS

Section 2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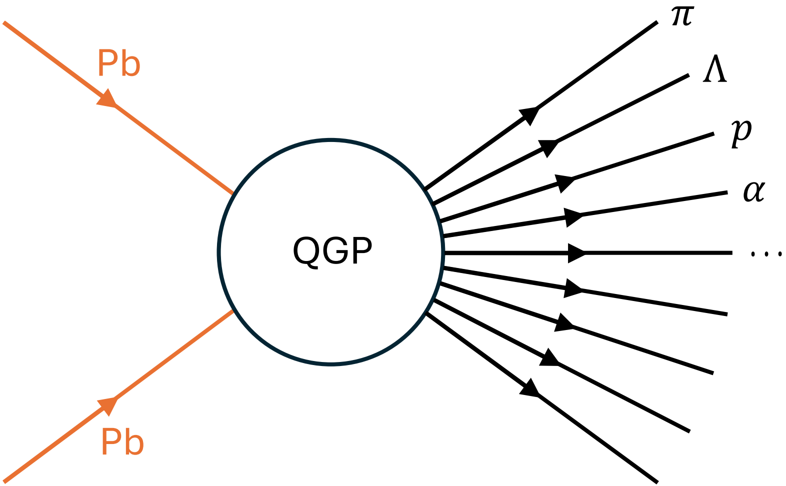

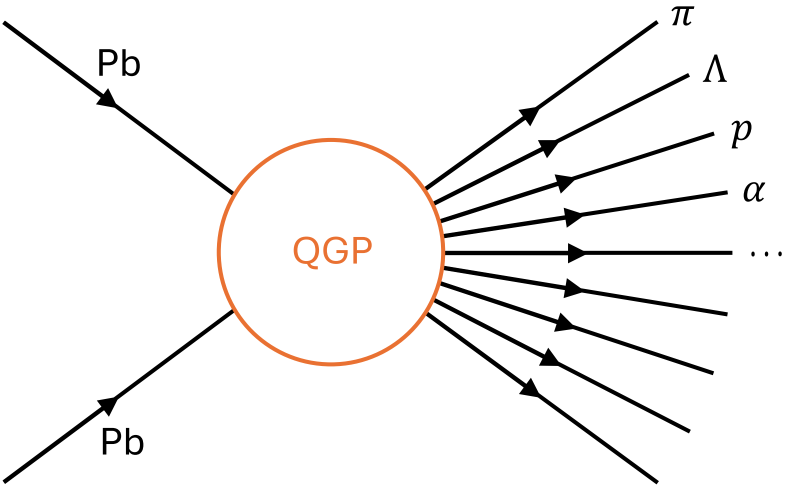
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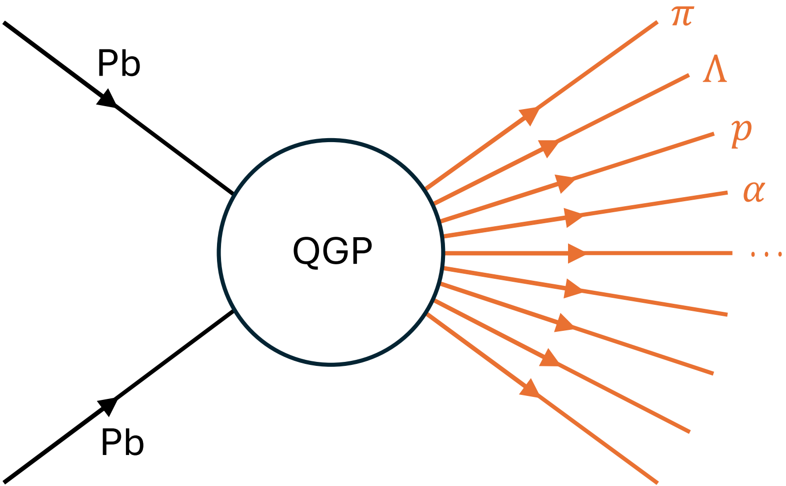
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DEFINITION OF THE CORRELATION FUNCTION

Section 3

TECHNISCHE UNIVERSITÄT DARMSTADT

DEFINITION OF THE CORRELATION FUNCTION

- Start from the *i*-particle distribution

$$N_i(\vec{p}_1, \vec{p}_2, ..., \vec{p}_i) = \left\langle a^\dagger_{\vec{p}_1}\ a^\dagger_{\vec{p}_2}\ ...\ a^\dagger_{\vec{p}_i}\ a_{\vec{p}_i}\ ...\ a_{\vec{p}_2}\ a_{\vec{p}_1} \right\rangle$$

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- Define the correlation function [Wiedemann Heinz, Physics Reports 319, 145–230 (1999)]

$$\boxed{C(\vec{p}_1, \vec{p}_2) = \frac{N_2(\vec{p}_1, \vec{p}_2)}{N_1(\vec{p}_1) N_1(\vec{p}_2)}}$$

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- If particles tend to turn up in pairs we observe correlations above unity for small relative momenta

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FINAL STATE INTERACTIONS

Section 4

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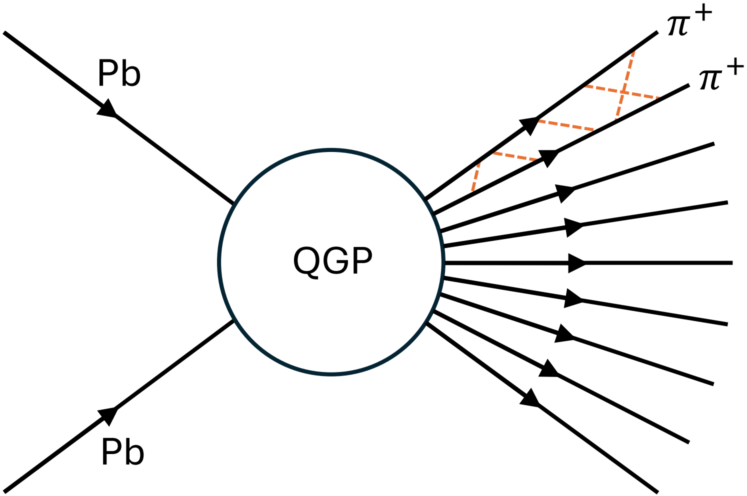

FINAL STATE INTERACTIONS

- Correlation function in presence of final state interactions

$$\boxed{C(\vec{q}) = \int \mathrm{d}^3 r \, S(\vec{r}) \left|\psi_{\vec{q}}(\vec{r})\right|^2}$$

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- model dependence of the source

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NUCLEON-NUCLEON CORRELATIONS

Section 5

NUCLEON-NUCLEON CORRELATIONS

- The absolute value squared of the *s*-wave function for NN scattering is given by

$$\left|\psi_{NN}(\vec{q},\vec{r})\right|^2 = \frac{1}{4}\left|\psi_{{}^1S_0}(\vec{q},\vec{r})\right|^2 + \frac{3}{4}\left|\psi_{{}^3S_1}(\vec{q},\vec{r})\right|^2$$

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$$\psi_{S_{0/1}}(\vec{q},\vec{r}) = e^{iqz} + f_q^{(0/1)}\frac{e^{iqr}}{r}$$

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- Approximating the scattering amplitude in *s*-wave approximation using ERE yields

$$f_q(\theta) = f_q = \frac{1}{-\frac{1}{a_0} + \frac{1}{2}r_0q^2 - iq}$$

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NUCLEON-NUCLEON CORRELATIONS

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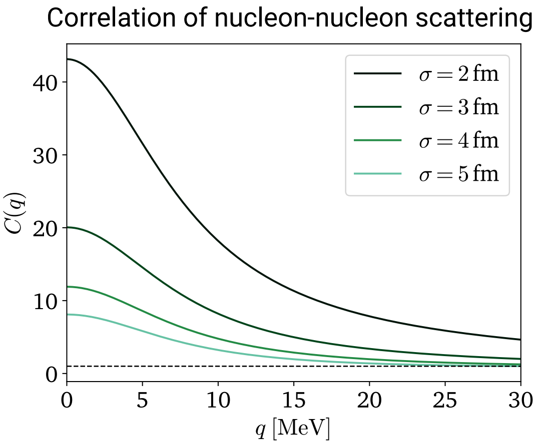
- With this we can calculate

$$C_{NN}(q) = 1 + \frac{e^{-2\sigma^2 q^2} \left|f_q\right|^2}{\sigma^2 q} \left[q + \mathrm{Erfi}\left(\sqrt{2}\sigma q\right) \left(-\frac{1}{a_0} + \frac{1}{2} r_0 q^2 \right) \right]$$



Parameters: $a_{^1S_0} = -23.71\,\mathrm{fm}$, $r_{^1S_0} = 2.70\,\mathrm{fm}$, $a_{^3S_1} = 5.42\,\mathrm{fm}$ and $r_{^3S_1} = 1.76\,\mathrm{fm}$

[V. A. Babenko and N. M. Petrov, Physics of Atomic Nuclei 70, 669–675 (2007)]

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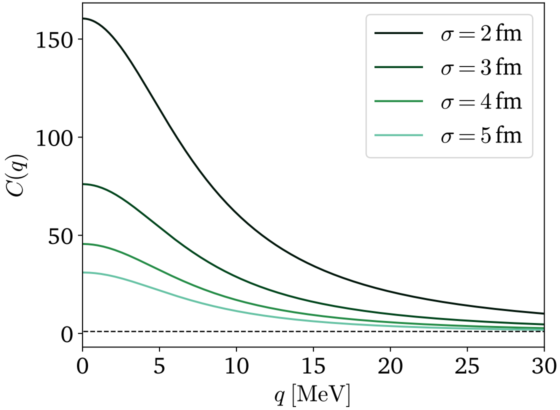
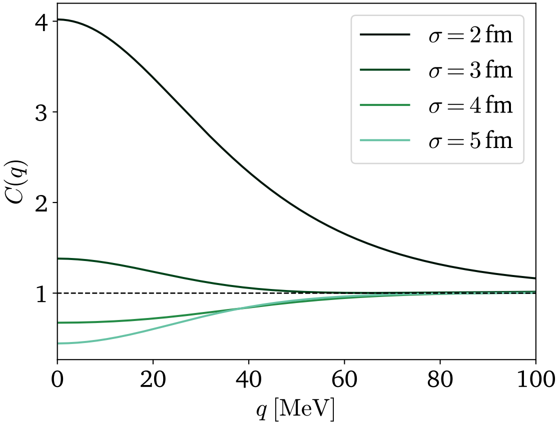
Separate correlations for $^1S_0$ and $^3S_1$ channels





Parameters: $a_{^1S_0} = -23.71$ fm, $r_{^1S_0} = 2.70$ fm, $a_{^3S_1} = 5.42$ fm and $r_{^3S_1} = 1.76$ fm

[V. A. Babenko and N. M. Petrov, Physics of Atomic Nuclei 70, 669–675 (2007)]

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NUCLEON-NUCLEON CORRELATIONS

- The limit $q \to 0$ can be calculated to be

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$\Rightarrow$ Discuss systems with Coulomb and square-well potentials

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TOY PROBLEMS: COULOMB AND SQUARE WELL

Section 6

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COULOMB SCATTERING PROBLEM

- The Schrödinger equation with attractive Coulomb potential and $\kappa = \alpha\, m/k$ is given by

$$\left(\nabla^2 + \frac{2\kappa k}{r} + k^2\right)\psi = 0$$

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- One can derive the scattering solution proportional to e^{ikz} for $r \to \infty$ to be
  [Landau and Lifshitz, Quantum mechanics: non-relativistic theory]

$$\psi\left(\vec{r}\right) = \mathrm{e}^{\pi\kappa/2}\Gamma(1 - i\kappa)\mathrm{e}^{ikr\cos\theta}\,{}_1F_1(i\kappa, 1, ikr(1 - \cos\theta))$$

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- The decomposition into partial waves is given by [A. Messiah, Quantum mechanics]

$$\psi(\vec{r}) = \sum_{l=0}^{\infty}(2l+1)i^l\mathrm{e}^{i\sigma_l}R^l(kr)P_l(\cos\theta)$$

with $\sigma_l = \arg(\Gamma(l + 1 - i\kappa))$ and $R^l(kr) = \frac{2^l\mathrm{e}^{\kappa\pi/2}|\Gamma(l+1-i\kappa)|}{(2l+1)!}\mathrm{e}^{ikr}(kr)^l\,{}_1F_1(l + 1 - i\kappa, 2l + 2, -2ikr)$

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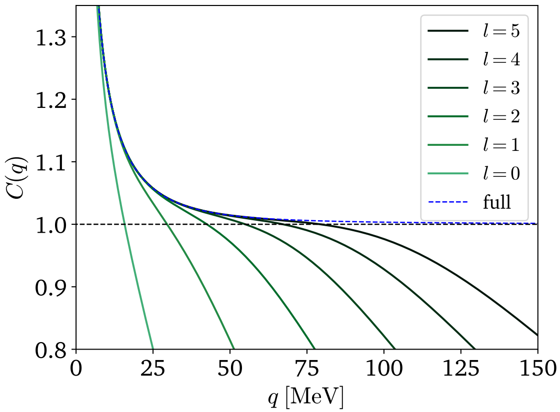
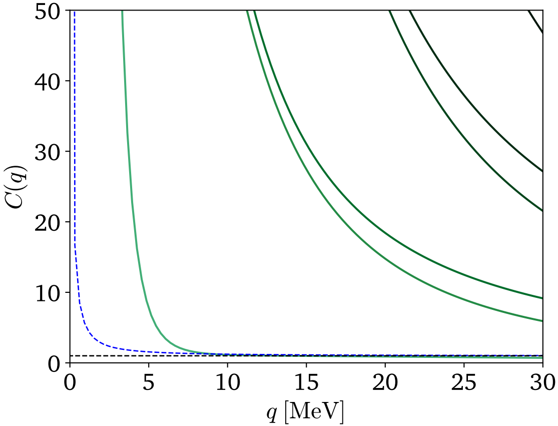
Coulomb correlation functions (pion-proton system), on the right for asymptotic partial waves





Parameters: $\sigma = \sqrt{2} \cdot 3\,\mathrm{fm}$

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SQUARE WELL

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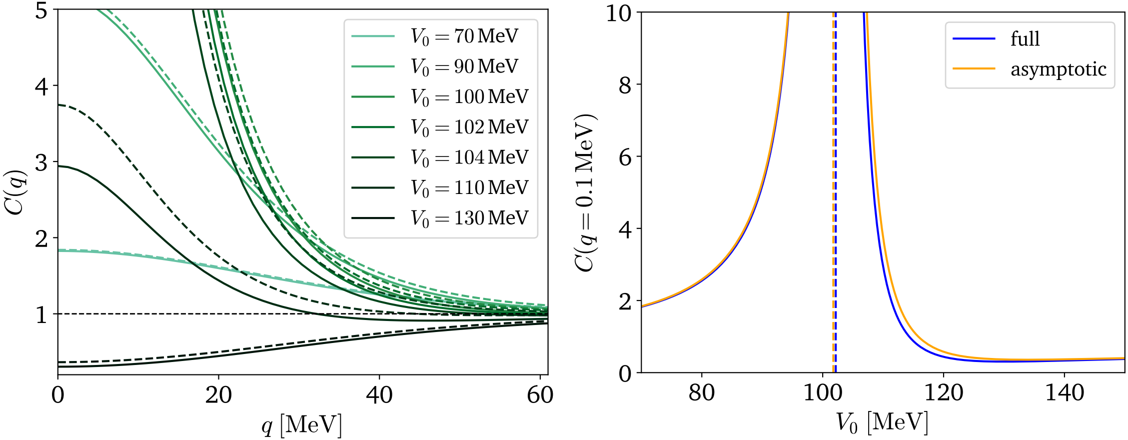
with the norm A, the modified wave number $k' = \sqrt{k^2 + 2V_0}$ (with $m = 1$) and

$$\cot \delta = (k \sin(kr) \sin(k'r) + k' \cos(kr) \cos(k'r)) \;/\; (k \cos kr \sin(k'r) - k' \sin(kr) \cos(k'r))$$

Square well correlation functions



Parameters: $\sigma = \sqrt{2} \cdot 3\,\text{fm}$, $b = 1\,\text{fm}$, $\mu = 938/2\,\text{MeV}$

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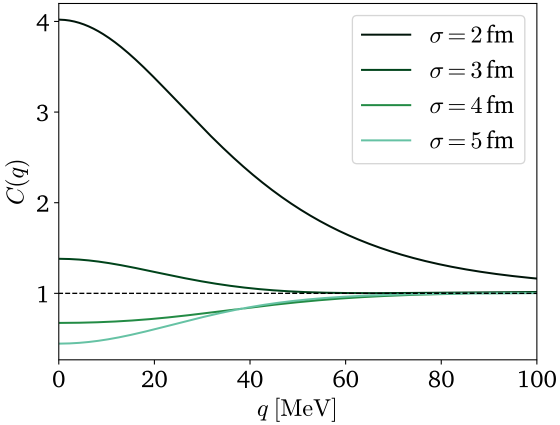
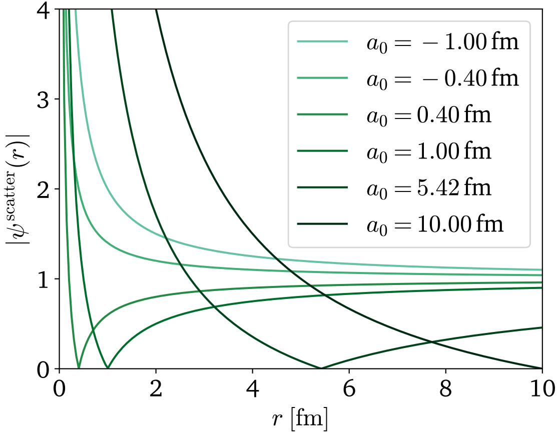
Nucleonic $^3S_1$ correlation and wave functions





Parameters: (left plot:) $\sigma = \sqrt{2} \cdot 3\,\mathrm{fm}$, $a_{^3S_1} = 5.42\,\mathrm{fm}$, (both plots:) $r_{^3S_1} = 1.76\,\mathrm{fm}$, (right plot:) $q = 0.1\,\mathrm{MeV}$

[V. A. Babenko and N. M. Petrov, Physics of Atomic Nuclei 70, 669–675 (2007)]

SUMMARY AND OUTLOOK

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- Looking forward we will combine the femtoscopic approach with an EFT calculation for the hypertriton

THANK YOU FOR YOUR ATTENTION!

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CORRELATION FUNCTION OF IDENTICAL BOSONS

- Define the source function (also Wigner Function) to be

$$S(x, K) = \langle N \rangle_P \int \mathrm{d}^4 r \, \mathrm{d}^4 p \, \mathrm{d}^4 y \; \rho(r, p) \, e^{iy(K-p)} J_0^* \left(x + \frac{1}{2} y \right) J_0 \left(x - \frac{1}{2} y \right)$$

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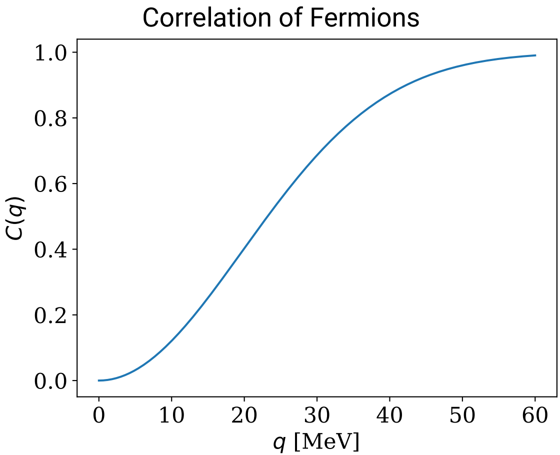
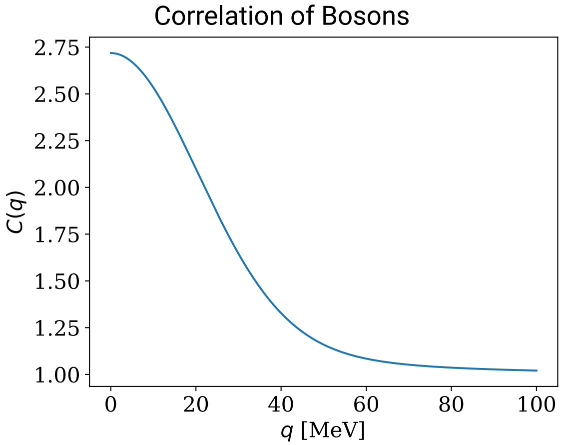
- With this we can calculate

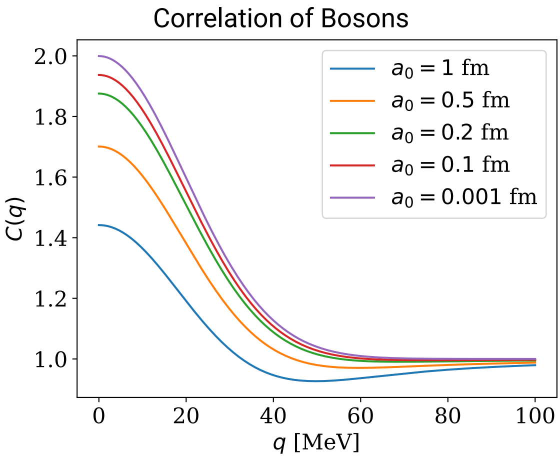
$$C\left(\vec{q},\vec{K}\right) = 1 + \frac{\left|\int \mathrm{d}^4 x \, S(x,K) e^{-iqx}\right|^2}{\left|\int \mathrm{d}^4 x \, S(x,K)\right|^2}$$



Parameters: $\sigma = 5\,\mathrm{fm}$



Parameters: $\sigma = 5\,\mathrm{fm}$, $a_0 = -1\,\mathrm{fm}$, $r_0 = 0\,\mathrm{fm}$



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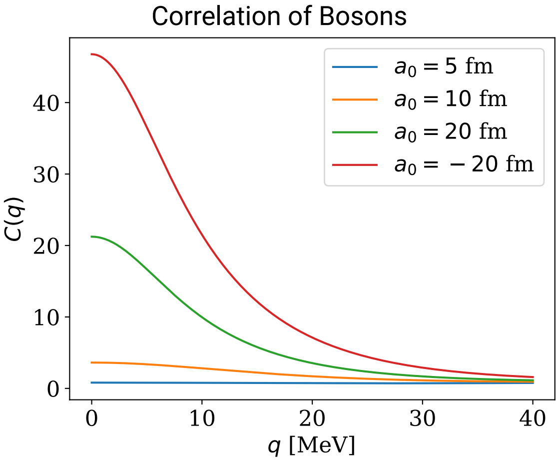

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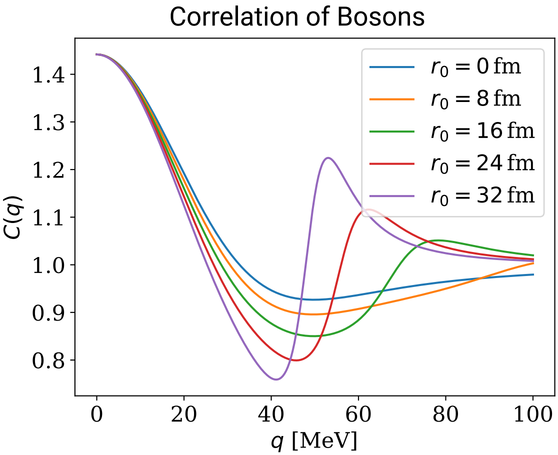

Parameters: $\sigma = 5\,\mathrm{fm}$ and $a_0 = 1\,\mathrm{fm}$

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RESULTS FOR GAUSSIAN MODEL

- Model the emission function by

$$S(x, K) = \delta(t)\,\frac{1}{\left(2\sigma\sqrt{\pi}\right)^3}\,\exp\left[-\frac{\vec{x}^2}{4\sigma^2}\right]$$

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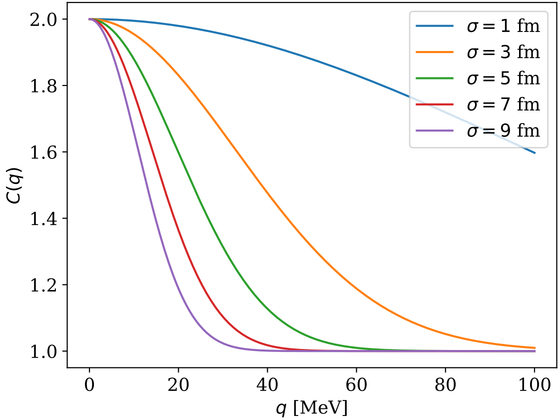
RESULTS FOR GAUSSIAN MODEL



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RESULTS: IDENTICAL FERMIONS AND BOSONS

- For identical particles we need to consider an explicitly (anti-) symmetrized wave function

$$\psi_{\text{asymptotic}}(\vec{q}, \vec{r}) = \frac{1}{\sqrt{2}} \left(e^{iqz} \pm e^{-iqz} + [f_q(\theta) \pm f_q(\pi - \theta)] \frac{e^{iqr}}{r} \right)$$

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- With this we obtain for fermions in S-wave approximation

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RESULT: IDENTICAL FERMIONS AND BOSONS

- With this we obtain for fermions in S-wave approximation

$$C_{\text{fermions}}(q) = 1 - e^{-2\sigma^2 q^2}$$

- While for Bosons

$$C_{\text{bosons}}(q) = 1 + e^{-2\sigma^2 q^2} + \frac{2e^{-2\sigma^2 q^2}\,|f_q|^2}{\sigma^2 q}\left[q + \mathrm{Erfi}\left(\sqrt{2}\sigma q\right)\left(-\frac{1}{a_0} + \frac{1}{2} r_0 q^2\right)\right]$$

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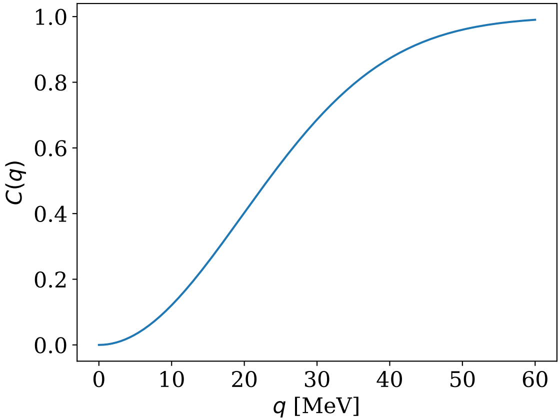
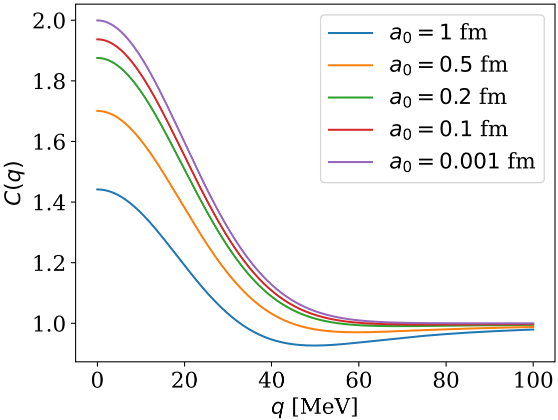
Correlation of Fermions



Parameters: $\sigma = 5\,\mathrm{fm}$

Correlation of Bosons



Parameters: $\sigma = 5\,\mathrm{fm}$, $r_0 = 0\,\mathrm{fm}$

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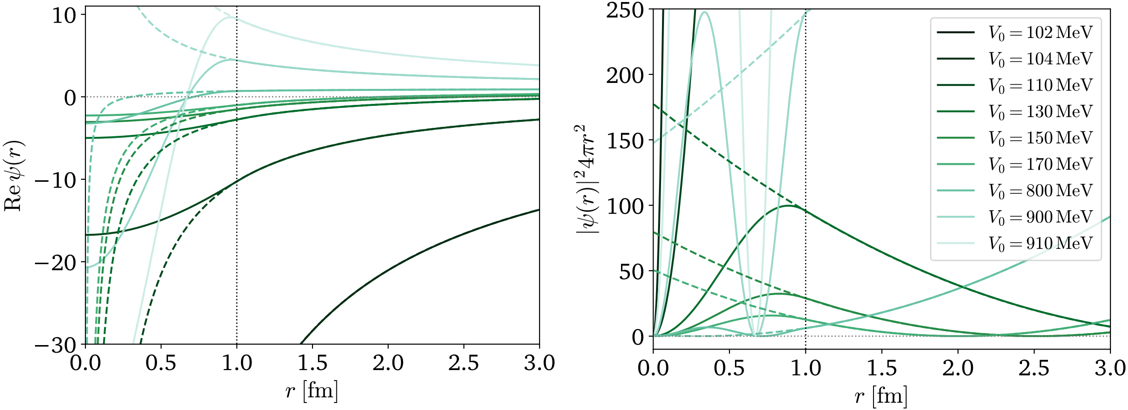
Square well wave functions



Parameters: $b = 1\,\mathrm{fm}$, $q = 0.1\,\mathrm{MeV}$