Towards a short-range effective theory for deformed halo nuclei

Live-Palm Kubushishi
P.Capel, D.R.Phillips, H-W.Hammer

Department of Physics and Astronomy
Ohio University

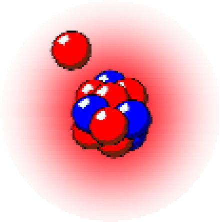
October 21, 2025 - Trento

Overview

1. Introduction/motivation
   - Halo Nuclei
   - Halo-EFT
   - Breakup reactions with halo nuclei
2. EFT-inspired calculations of ^{11}Be [potential model]
   - Bound states
   - Resonant states
3. A short-range effective theory for deformed halo nuclei?
   - ^{11}Be spectrum
   - ^{17}C spectrum
   - ^{31}Ne spectrum
4. Summary/perspectives

Halo nuclei

- Light, neutron-rich nuclei with large matter radius
- Low $\mathbf{S}_{\mathrm{n}}$ or $\mathbf{S}_{2\mathrm{n}}$: one or two loosely-bound neutrons
- Clusterised structure: neutrons can tunnel far from the core
  $\rightarrow$ halo-nucleus $\equiv$ compact core + valence neutron(s)



- **Our case study** : $^{11}\mathrm{Be} \equiv {}^{10}\mathrm{Be} + \mathrm{n}$
- Short-lived $\rightarrow$ studied via reactions (e.g. **breakup**)
  $\rightarrow$ need of an **effective few-body** model for reaction calculations
  $\rightarrow$ **Halo-EFT**

Halo-EFT description of ^{11}Be

- Halo-structure $\rightarrow$ separation of scales (in energy/distance)
  $\rightarrow$ small parameter $\eta = \sqrt{\frac{\mathrm{S_{1n}}}{\mathrm{E_{2^+}}}}$ or $\frac{\mathrm{R_{core}}}{\mathrm{R_{halo}}} \simeq 0.4 < 1$
  $\rightarrow$ expansion of the core-neutron Hamiltonian along η,
  i.e. reproducing the low-energy (viz. long distance) behaviour of the system
  [Bertulani, Hammer, van Kolck, NPA 712, 37 (2002)]
  Review: **[Hammer, Ji, Phillips, JPG 44, 103002 (2017)]**
- ^{11}Be $=^{10}$Be(0^+)+n **[core has no internal structure]**
  $\rightarrow$ **single-particle description**: $H(\mathbf{r}) = \mathrm{T}_{\mathbf{r}} + \mathrm{V_{cn}}(\mathbf{r})$
- Effective Gaussian potentials in each partial wave ℓj @NLO ($\ell \leqslant 1$):

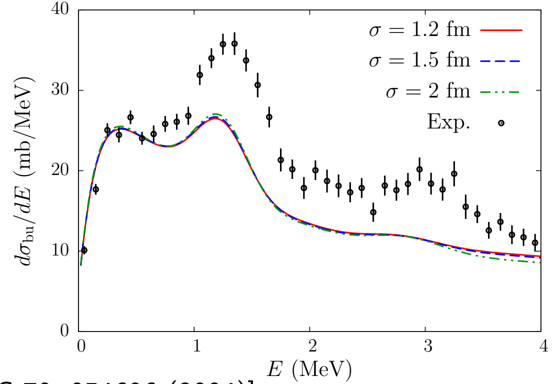
$$\mathrm{V_{cn}(r)} = \mathrm{V}^{(0)}_{\ell \mathrm{j}} \mathrm{e}^{-\frac{\mathrm{r}^2}{2\sigma^2}} + \mathrm{V}^{(2)}_{\ell \mathrm{j}} \mathrm{r}^2 \mathrm{e}^{-\frac{\mathrm{r}^2}{2\sigma^2}}$$

  $\mathrm{V}^{(0)}_{\ell \mathrm{j}}$ and $\mathrm{V}^{(2)}_{\ell \mathrm{j}}$ fitted to reproduce:
  $\rightarrow$ $\mathbf{S}_{\mathrm{n}}$ & asymptotic normalization coefficient (**ANC**) for bound states
  $\rightarrow$ effective range parameters for continuum states

  $\sigma :=$ cut-off $\rightarrow$ evaluates sensitivity to short-range physics

What is the problem ?

- Assumption: ^{10}Be remains in its 0^+ ground state still valid ?
  $\rightarrow$ Nuclear breakup: ^{11}Be+C $\rightarrow$ ^{10}Be+n+C



Exp: [Fukuda *et al.* PRC 70, 054606 (2004)]
Th.: [**L.-P.K** & P. Capel, PRC 111, 054618 (2025)]
$\Rightarrow$ Missing peaks @ $\frac{5}{2}^+$ and $\frac{3}{2}^+$ resonances $\rightarrow$ single-particle picture is not enough
$\Rightarrow$ Missing [^{10}Be(2^+)] degree of freedom **[Moro & Lay, PRL 109, 232502 (2012)]**

Core excitation within Halo-EFT

- **Extension of Halo-EFT to include core excitation**:

$$H(\mathbf{r},\xi) = \mathrm{T}_{\mathbf{r}} + \mathrm{V}_{\mathrm{cn}}(\mathbf{r},\xi) + \mathrm{h_c}(\xi)$$

$\mathrm{h_c}(\xi)$:= intrinsic Hamiltonian of the core with eigenstates $\chi^{\mathrm{c}}_{\mathrm{I}}(\xi)$

- **Halo-EFT particle-rotor model** [Bohr and Mottelson (1975)]:

$$\mathrm{V}_{\mathrm{cn}}(\mathbf{r},\xi) = \mathrm{V}_{\mathrm{cn}}(\mathrm{r}) + \beta\sigma \mathrm{Y}^0_2(\hat{\mathrm{r}}')\frac{\mathrm{d}}{\mathrm{d}\sigma}\mathrm{V}_{\mathrm{cn}}(\mathrm{r})$$

- Set of radial **coupled-channel** Schrödinger equations:

$$\left[\mathrm{T}^{\ell}_{\mathrm{r}} + \mathrm{V}_{\alpha\alpha}(\mathrm{r}) + \epsilon_\alpha - \mathrm{E}\right]\psi_\alpha(\mathrm{r}) = -\sum_{\alpha'\neq\alpha} \mathrm{V}_{\alpha\alpha'}(\mathrm{r})\psi_{\alpha'}(\mathrm{r})$$

with $\mathrm{V}_{\alpha\alpha'}(\mathrm{r}) = \langle \mathcal{Y}_\alpha(\hat{\mathrm{r}})\chi_\alpha(\xi)|\mathrm{V}_{\mathrm{cn}}(\mathbf{r},\xi)|\mathcal{Y}_{\alpha'}(\hat{\mathrm{r}})\chi_{\alpha'}(\xi)\rangle$, α=$\{\ell,\mathrm{s},\mathrm{j},\mathrm{I}\}$

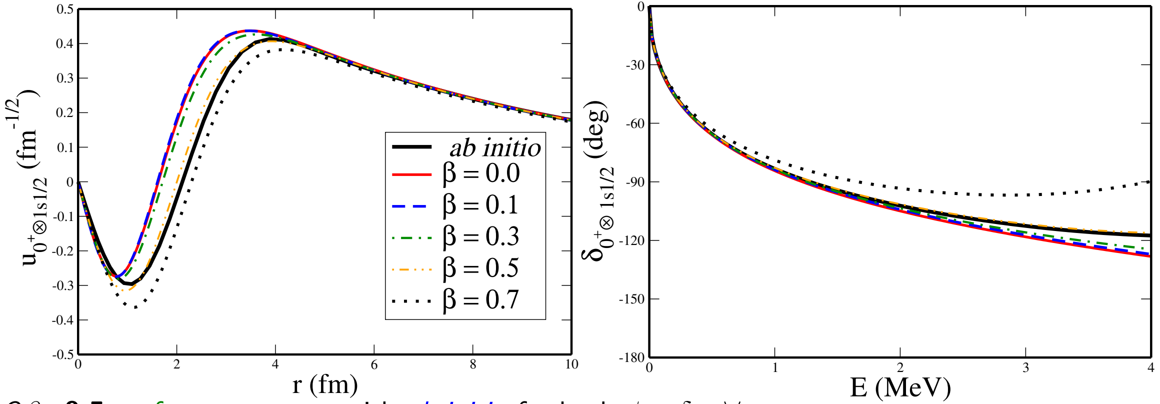
→ solved within the **R-Matrix method** on a Lagrange mesh
[D. Baye, Phys. Rep. 565 (2015) 1]

→ study impact of core excitation on: ψ_α, δ_α

Core excitation in ^{11}Be $\frac{1}{2}^+$ ground state

Compare to *ab initio* predictions [Calci et al., PRL 117, 242501 (2016)]

- $\Psi_{1/2^+} = \psi_{1s1/2}(\mathbf{r}) \otimes \chi_{0^+}^{10\text{Be}} + \psi_{0d5/2}(\mathbf{r}) \otimes \chi_{2^+}^{10\text{Be}} + \psi_{0d3/2}(\mathbf{r}) \otimes \chi_{2^+}^{10\text{Be}}$
- NLO potentials **fitted to** reproduce $\mathbf{S}_n$ and *ab initio* **ANC** for $\neq \beta$

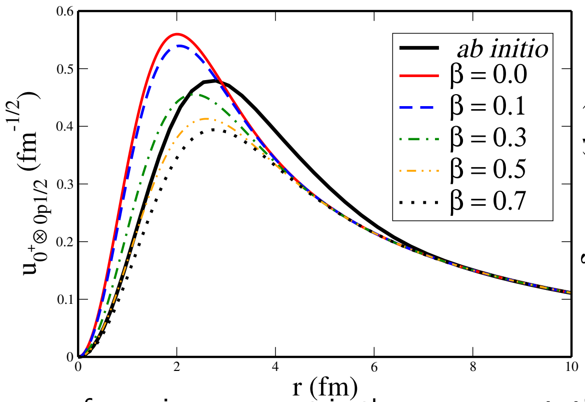


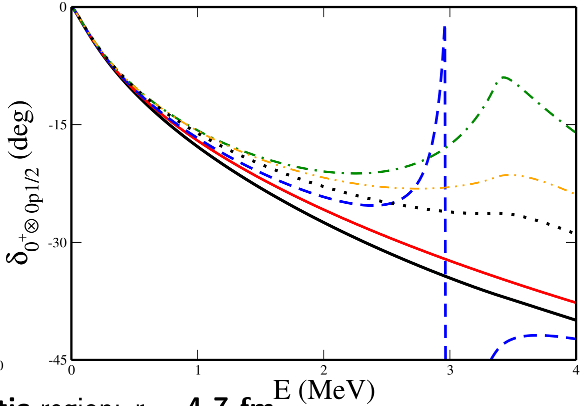
@β=**0.5** perfect agreement with *ab initio* for both ψ_α, δ_α, $\forall\sigma$

$\Rightarrow$ confirms the role of **core excitation** in structure of ^{11}Be g.s

Core excitation in ^{11}Be $\frac{1}{2}^-$ bound excited state

- $\Psi_{1/2^-} = \psi_{0p1/2}(\mathbf{r}) \otimes \chi_{0^+}^{^{10}\mathrm{Be}} + \psi_{0p3/2}(\mathbf{r}) \otimes \chi_{2^+}^{^{10}\mathrm{Be}} + \psi_{0f5/2}(\mathbf{r}) \otimes \chi_{2^+}^{^{10}\mathrm{Be}}$
- NLO potentials **fitted to** reproduce $\mathbf{S}_\mathrm{n}$ and *ab initio* **ANC** for $\neq \beta$





- wfs: no improvement in the **pre-asymptotic** region: r $\sim$ **4-7 fm**
- phase shifts: less good than without core excitation

$\Rightarrow$ No influence of **core excitation** on structure of ^{11}Be e.s.because **shell-model** state ?

Electric dipole strength: B(E1)

E1 transition from bound state to bound state: $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$

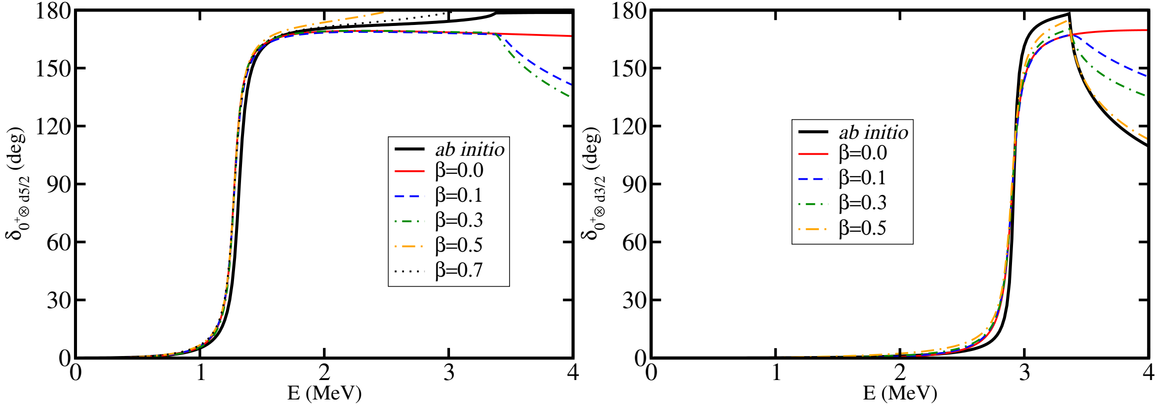
	B(E1) (e^2fm^2)	
σ (fm)	$\beta = 0.5$	$\beta = 0$
1.3	0.104	0.103
1.5	0.106	0.106
1.8	0.109	0.108
2.0	0.110	0.109
ab initio	0.117	
Experiments		
[PRC 28, 497]	0.116(12)	
[PLB 394, 11]	0.099(10)	
[PLB 650, 124]	0.105(12)	
[PLB 732, 210]	0.102(2)	

- **Core excitation** has **no influence** on B(E1)
- Good agreement with exp. data but lower than *ab initio*
- Ab initio **overestimates** exp. B(E1) $\rightarrow$ wrong **pre-asymptotic** region ?

Core excitation in low-energy resonances : $\frac{5}{2}^+$, $\frac{3}{2}^-$, $\frac{3}{2}^+$

Compare to *ab initio* predictions [Calci et al., PRL 117, 242501 (2016)]

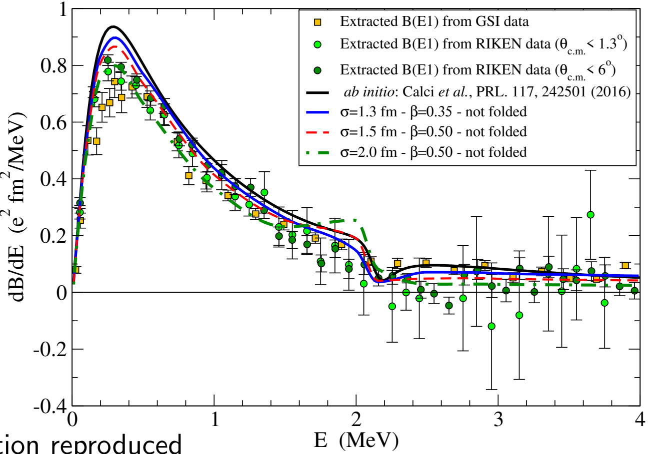
- NLO potentials **fitted to** reproduce exp. $\mathbf{E}_{\mathrm{res}}$ and Γ_{res} for $\neq \beta$



- Excellent agreement with *ab initio* results → probing **nature of resonances** $[\Gamma_{0^+}, \Gamma_{2^+}]$
- Direct access to scattering wfs, phase shifts → $\frac{\mathrm{dB(E1)}}{\mathrm{dE}}$, cross sections,...

dB(E1)/dE

E1 transition from $\frac{1}{2}^+$ bound state to the continuum with **final-state interactions**

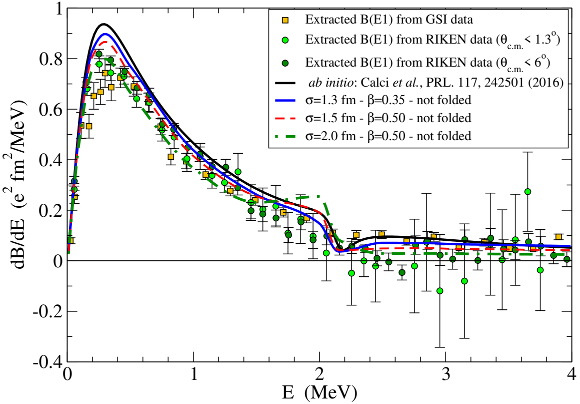


- *Ab initio* prediction reproduced
- **Good agreement** with exp. data reproduced but **overshoot at low** E (like *ab initio*)
- Significant σ-dependency because of $\frac{3}{2}^-$ phaseshift

Coulomb breakup & Equivalent Photon Method

Coulomb breakup: ^{11}Be+Pb $\rightarrow$ ^{10}Be+n+Pb @69AMeV $\rightarrow$ **E1**-dominated

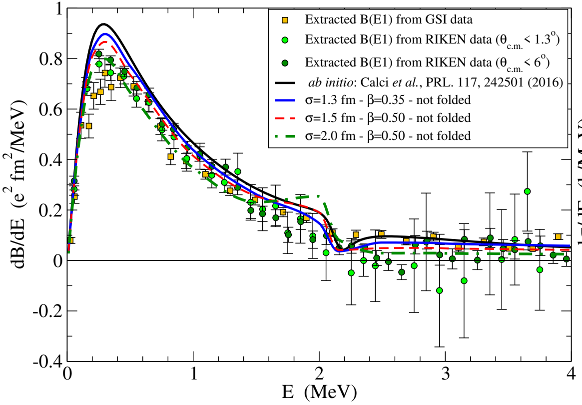
$$\frac{\mathrm{d}\sigma}{\mathrm{dE}} = \frac{16\pi^3}{9\hbar c}\mathrm{N_{E1}(E)}\frac{\mathrm{dB(E1)}}{\mathrm{dE}}$$

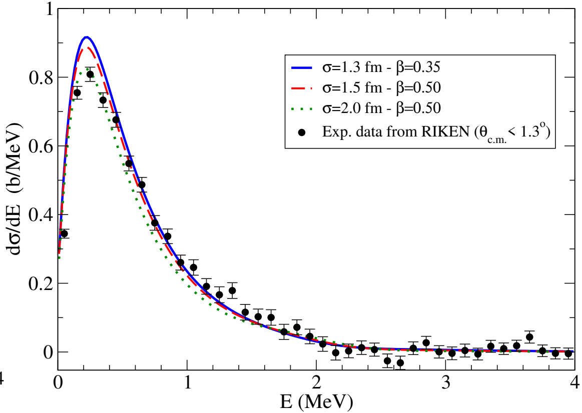


Coulomb breakup & Equivalent Photon Method

Coulomb breakup: ^{11}Be+Pb → ^{10}Be+n+Pb @69AMeV → **E1**-dominated

$$\frac{d\sigma}{dE} = \frac{16\pi^3}{9\hbar c} N_{E1}(E) \frac{dB(E1)}{dE}$$





→ B(E1) distribution overshoots reflected on cross-sections (**which are folded**)

Yes, it works, but is it really EFT?
Short-range effective theory for deformed halo nuclei?

A short-range effective theory for deformed halo nuclei?

Simple portable structure model for (breakup) reactions codes, including core deformation
$\rightarrow$ with **2 caveats**:

- Power counting?
- @NLO: non zero interactions in channels where $\ell \geq 1$ [mean field]

Idea: build $\mathrm{V_{eff}}$ as a series of **local contact** potentials **[Lepage, arXiv:nucl-th/9706029]**:

$$\mathrm{V_{eff}}(\mathrm{r}) = \mathrm{C_0}\,\delta_\sigma^{(3)}(\mathrm{r}) + \mathrm{C_2}\,\nabla^2\,\delta_\sigma^{(3)}(\mathrm{r}) + \mathrm{C_{2'}}\,\nabla\cdot\delta_\sigma^{(3)}(\mathrm{r})\,\nabla + \cdots + \mathrm{C_{2n+2}}\,\nabla^{\mathrm{n}}\,\delta_\sigma^{(3)}(\mathrm{r}) + \cdots$$

$\rightarrow$ each term:= $\mathrm{C_i}$ $\times$ operator **[$\mathrm{C_i}$:=coupling constants]**
$\rightarrow$ $\mathrm{C_i}$ properly tuned, operators respect symmetries

We want to describe **deformed** halo nuclei using:

- a **rotationally asymmetric** term generated solely by s-waves [**s-d coupling**]
- a power counting, i.e. hierarchy between different terms

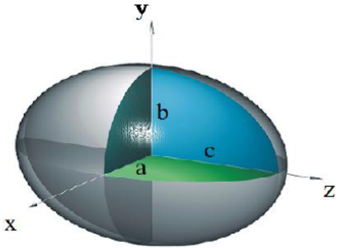
$\rightarrow$ Q: Can we reproduce the spectrum of deformed halo nuclei?

Geometry of deformed cores

Goal:=describe key features of the low-energy spectrum of (light) deformed one-neutron nuclei
Idea 1:= keep Halo-EFT and add deformation as subleading effect $\rightarrow$ perturbatively

Assumptions on the core:

- axially symmetric rigid rotor: $\hat{\mathrm{H}}_{\mathrm{core}} = \frac{\hat{\mathrm{I}}^2}{2\theta}$
  $\rightarrow$ rotational spectrum: 0^+g.s. (bandhead) and low-lying 2^+ excited state
- deformed ellipsoid along z-axis (symmetry axis) in intrinsic frame
  $\rightarrow$ stretching parameter ζ directly linked to β for small deformation



Operators and coupling constants (LECs)

From Halo-EFT: **[in momentum space]**

$$@\mathrm{LO} : \mathrm{V_{LO}} = \mathrm{C_0}$$

$$@\mathrm{NLO} : \mathrm{V_{NLO}} = \mathrm{C_0} + \mathrm{C_2}(\mathbf{p}^2 + \mathbf{p}'^2)$$

→ fine-tuned *s*-waves **[Kaplan, Savage, Wise (98)]**

Quadrupole operator:

$$@\mathrm{NNLO} : \mathrm{V_{sd}} = \mathrm{C_{sd}}\left[\mathrm{I}.\mathbf{q}.\mathrm{I}.\mathbf{q} - \frac{1}{3}(\mathrm{I}.\mathbf{q})^2\right] \qquad \text{with} \quad \mathbf{q} = \mathbf{p} - \mathbf{p'}$$

→ $\mathrm{C_{sd}}$:= LEC related to β

Hyperfine operator:

$$@\mathrm{LO} : \mathrm{V_{hf}} = \mathrm{C_{hf}}\ \mathrm{I}.\mathbf{j} \qquad \text{with} \quad \mathbf{j} = \boldsymbol{\ell} + \boldsymbol{s}$$

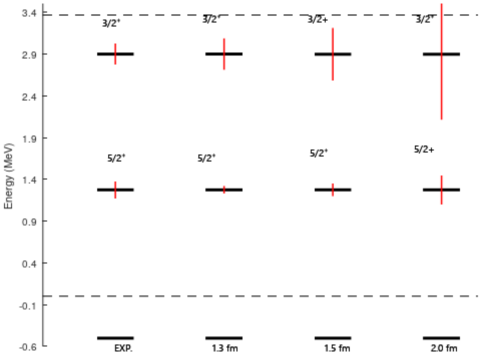
Core is a rigid rotor → $\hat{\mathrm{H}}_{\mathrm{core}} = \frac{\hat{\mathrm{I}}^2}{2\theta} \sim \mathbf{v}^2$ **N.B.** $\mathrm{I}=\mathrm{O}(1)$

→ higher order terms suppressed by powers of $\mathbf{v}^2$

Goal: tune $\mathrm{C_0}$, $\mathrm{C_2}$, $\mathrm{C_{sd}}$ and $\mathrm{C_{hf}}$ to reproduce low-energy spectrum of deformed halos

^{11}Be: positive parity states **[PRELIMINARY]**

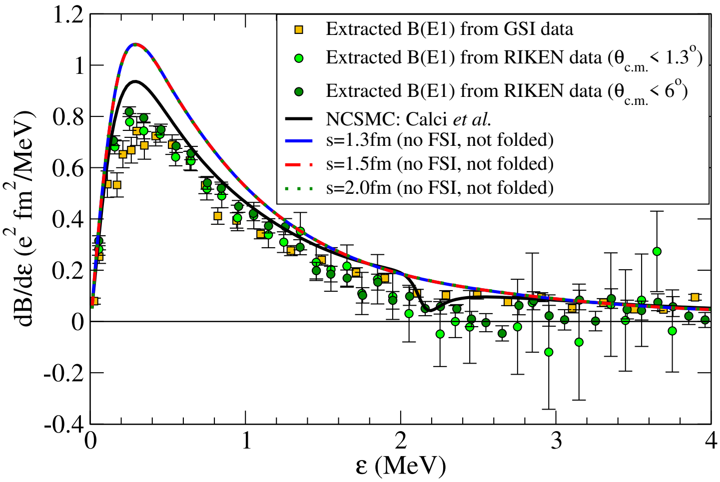
- $\frac{1}{2}^+$ g.s.; $\mathrm{S_{1n}}$=0.5 MeV; $\mathrm{E_{2^+}(^{10}Be)}$=3.368 MeV $\rightarrow$ $p_{\mathrm{rotor}} \gg p_{\mathrm{halo}}$
- Tune $\mathrm{C_0}$, $\mathrm{C_2}$, $\mathrm{C_{sd}}$ and $\mathrm{C_{hf}}$ against $\mathrm{S_{1n}}$, ANC, positions of resonances of ^{11}Be



- We reproduce the position of each state
- **Unprecised widths** for resonances $\rightarrow$ higher order effect

^{11}Be: dB(E1)/dE **[PRELIMINARY]**

Coulomb breakup: ^{11}Be+Pb $\rightarrow$ ^{10}Be+n+Pb @69AMeV $\rightarrow$ **E1**-dominated



Fair agreement with data but with **2 caveats** $\rightarrow$ no folding, no final-state interaction

What if $p_{halo} \sim p_{rotor}$?

Question: What about the case where $p_{halo} \sim p_{rotor}$?

$\rightarrow$ deformation (V_{sd}) enters @LO and we have:

$$\frac{p_{halo}{}^2}{2\mu} \sim \frac{I(I+1)}{2\theta}$$

$$\theta = \theta_{xx} = \theta_{yy} = \frac{A m_N}{5} R_{core}(1+\zeta^2) \quad \text{and} \quad \mu = \mu_0 m_N$$

$$\frac{p_{halo}{}^2}{2\mu_0} \sim \frac{I(I+1)}{\frac{A}{5} R_{core}^2 (1+\zeta^2)}$$

with different regimes:

$\zeta \gg 1$: **prolate** (:=elongation along z-axis); $\zeta \ll 1$: **oblate** (:=flattening); $\zeta = 1$: **spherical**

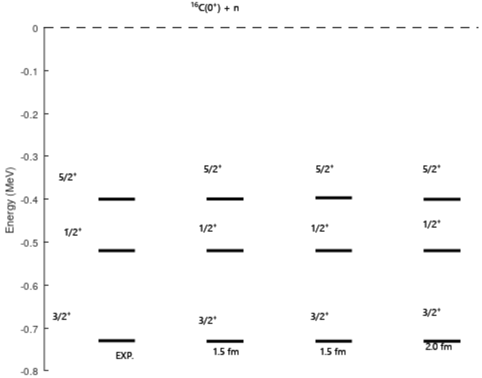
$\rightarrow$ relates geometry (moment of inertia), binding, nb of nucleons

$\rightarrow$ this scenario happens for heavier halos (larger nb of nucleons):

eg: ^{17}C, ^{19}C (**sd** shell), ^{31}Ne (**fp** shell)

^{17}C: halo excited states? [PRELIMINARY]

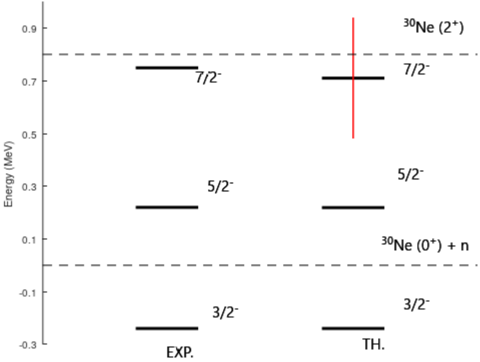
- $\frac{3}{2}^+$ g.s.; $\mathrm{S_{1n}}$=0.73 MeV; $\mathrm{E_{2^+}(^{16}C)}$=1.766 MeV
- $\frac{1}{2}^+$ e.s.; $\mathrm{S_{1n}}$=0.52 MeV $\rightarrow$ $p_{\mathrm{halo}} \sim p_{\mathrm{rotor}}$
- Tune $\mathrm{C_0}$, $\mathrm{C_{sd}}$ and $\mathrm{C_{hf}}$ against $\mathrm{S_{1n}}$ of the bound states



- Ok for position of each state **BUT** what about **transfer** data: $^{16}\mathrm{C(d,p)}$?

^{31}Ne: deformed p-wave halo [PRELIMINARY]

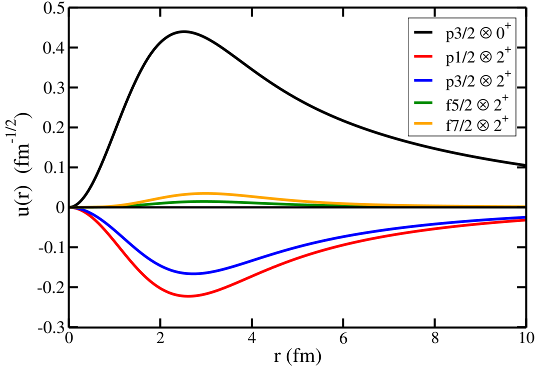
- $\frac{3}{2}^-$ g.s.; S_{1n}=0.24 MeV; $E_{2^+}(^{30}Ne)$=0.801 MeV $\rightarrow$ $p_{halo} \sim p_{rotor}$
- Tune C_0, C_{sd} and C_{hf} against S_{1n}, positions of the resonances



- We reproduce the position of each state
- **No scattering data** to compare to (no **exp. widths**)

^{31}Ne: $\frac{3}{2}^-$ ground state [PRELIMINARY]

Wave functions in each channel for β=0.56:



Other **models** available...but no scattering data:

Urata, et al. PRC 83, 041303(R) (2011); **Minomo, et al. PRL 108, 052503 (2012)**
Hong, Bertulani, Kruppa, PRC 96, 064603 (2017)

Outlook: E1-dissociation/Coulomb breakup **[Elkamhawy, Hammer JPG 50 02510 (2023)]**

Conclusion

We want to study reactions involving **one-neutron halo nuclei** :

- need of a **realistic few-body** model for reaction calculations
  $\rightarrow$ Halo-EFT

Our model of one-neutron halo nuclei [^{11}Be] provides:

- explicit inclusion of **core excitation within Halo-EFT**
- realistic description of both bound and low-lying resonant states in deformed halos [^{11}Be]
- portable structure model including deformation for reaction codes

[L.-P. Kubushishi and P. Capel, (2025), PRC 111 054618]
[L.-P. Kubushishi and P. Capel, (2025), arXiv:2406.10168]
[L.-P. Kubushishi and P. Capel, (2025), (in preparation)]

Outlook:

- same formalism to study structure and breakup of ^{17}C, ^{19}C (sd-shell), ^{37}Mg, ^{31}Ne
- short-range effective theory for deformed halo nuclei: ^{11}Be,^{17}C
  [L.-P. Kubushishi and D. R. Phillips, (2025), (in preparation)]
- include our model in reaction codes **(breakup, transfer,knock-out...)**

Thanks to my collaborators!

- Pierre Capel (JGU Mainz)
- Daniel R. Phillips (Ohio University)
- Hans-Werner Hammer (TU Darmstadt)