

A photograph of a grand, two-story courtyard with a central staircase and classical columns. The building has a light-colored facade with blue shutters and a balcony with a decorative iron railing. The courtyard is paved with cobblestones and has several potted plants.

Alpha clustering in light nuclei

Louis Heitz

Pan-American Few-Body physics Boot Camp : Fostering Collaboration
13-24 October 2025

Disclaimers

I. Work in progress

II. *The many-body problem... often turns into a body of many problems.*

1. Cluster phenomenology

2. « Ab Initio » methods

**3. Projected Generator
Coordinate method**

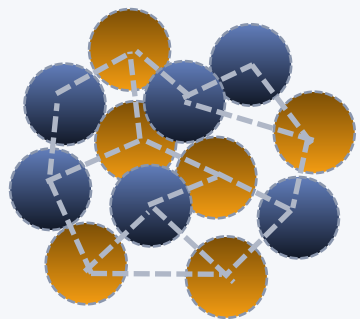
4. Preliminary results : ^{12}C

5. Conclusion

Alpha clustering

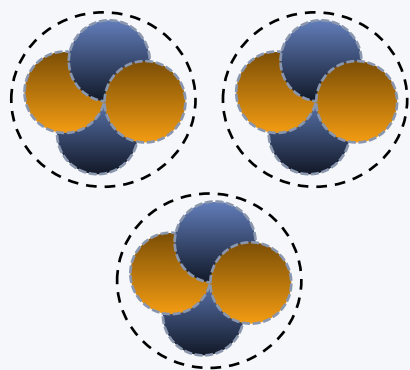
« Molecular substructure with α particles »

« Emergence of few-body physics »



6 protons
6 neutrons

^{12}C



3α ?

Heavy nuclei ($A \gtrsim 150$)
 α emitters from G.S.

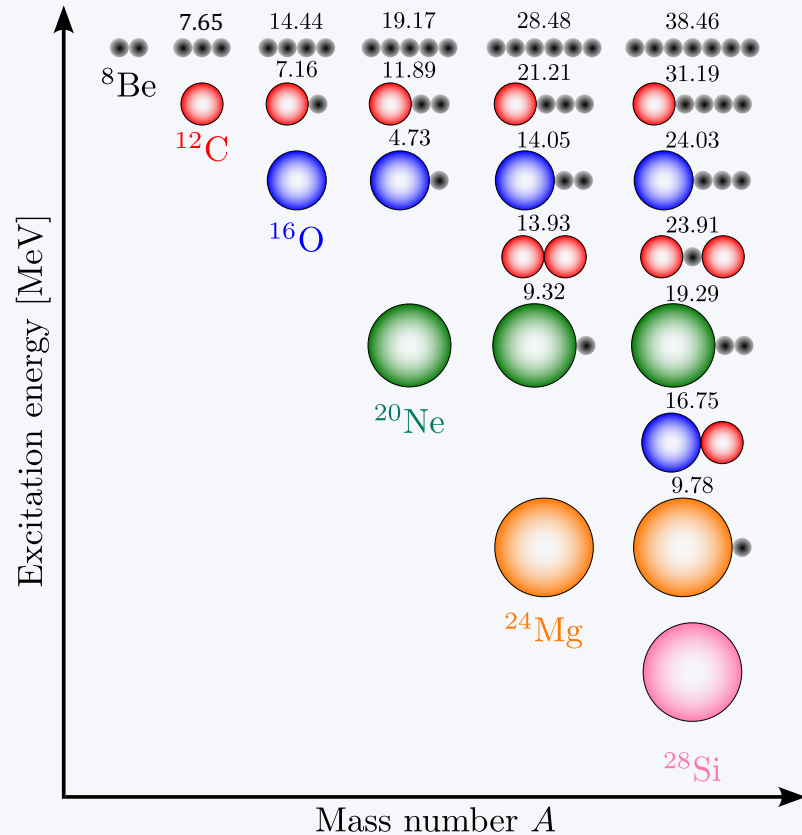
(Some) **Experimental evidence ?**

Light nuclei
Excited states

Decay modes

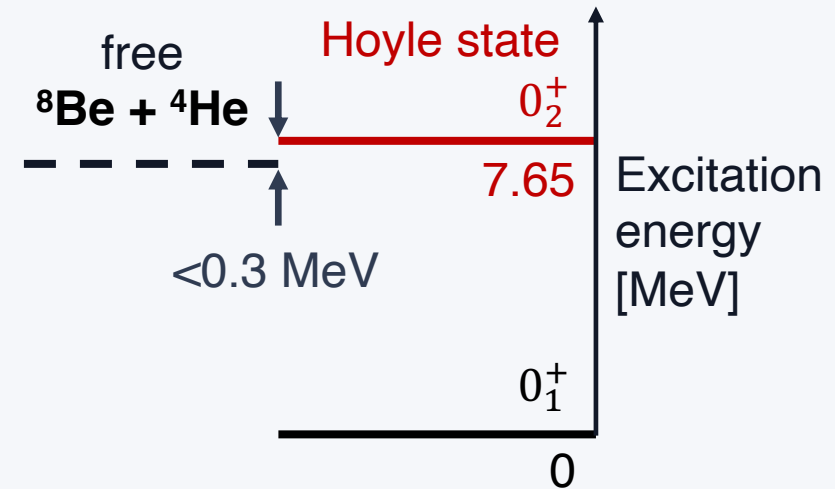


Light self-conjugate nuclei Even-Even, $N=Z$



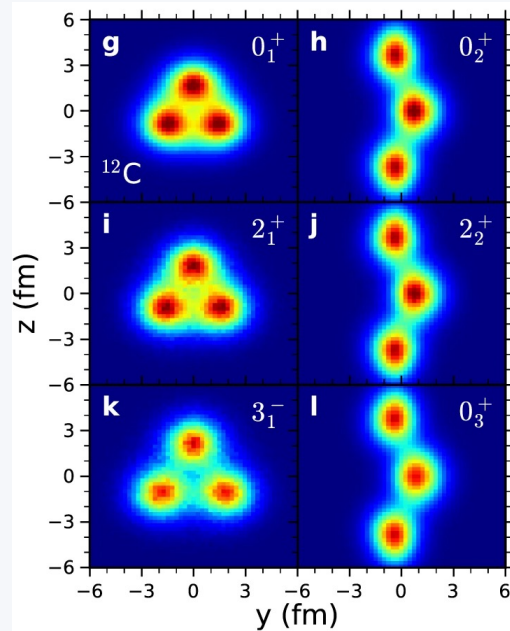
« Ikeda » diagram
Clustered excited states close to α threshold

Specific example : ^{12}C

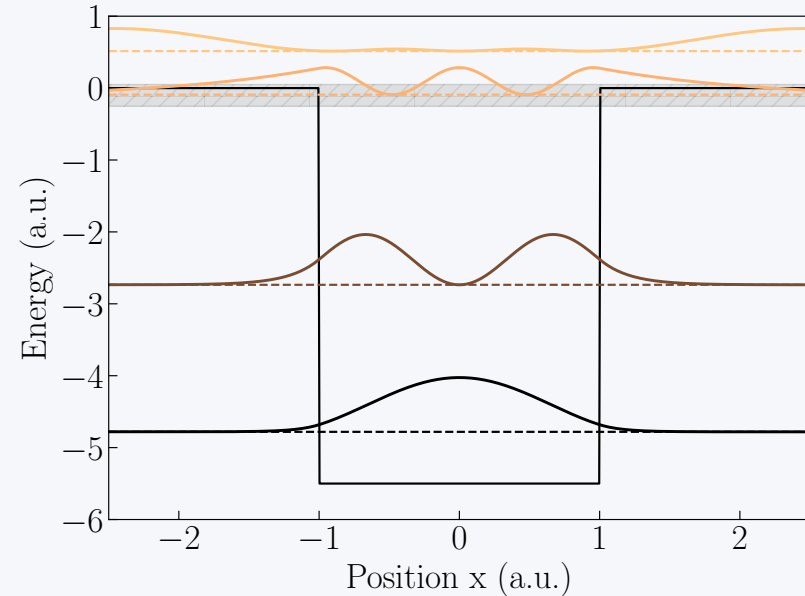


Theoretical challenges in describing clustered states

Shen et al., Nat. Commun. 14 (2023)



Best described via
Rotationnal symmetry breaking
« Collective » behaviour



Vinicity to continuum
Challenges Bound-state oriented
methods

Overview : theoretical methods

Alpha dofs [THSR, α GCM]

Nucleons dofs, pheno [AMD,(P)GCM]

Shell Model [NCSpSM, LSSM]

Lattice [NLEFT]

Group theory

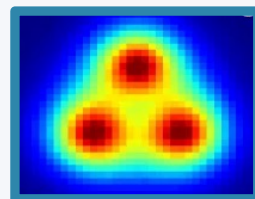
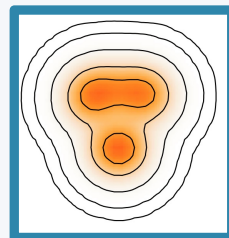
Freer et al., Rev. Mod. Phys. 90 (2018)

Note : $A=12$ accessible to NCSM,
but Hoyle state not captured

D'Alessio et al., Phys. Rev. C 102 (2020)

Choudhary et al., Phys. Rev. C 107 (2023)

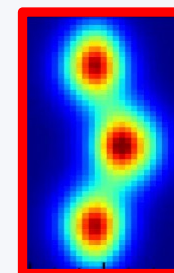
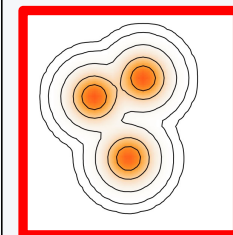
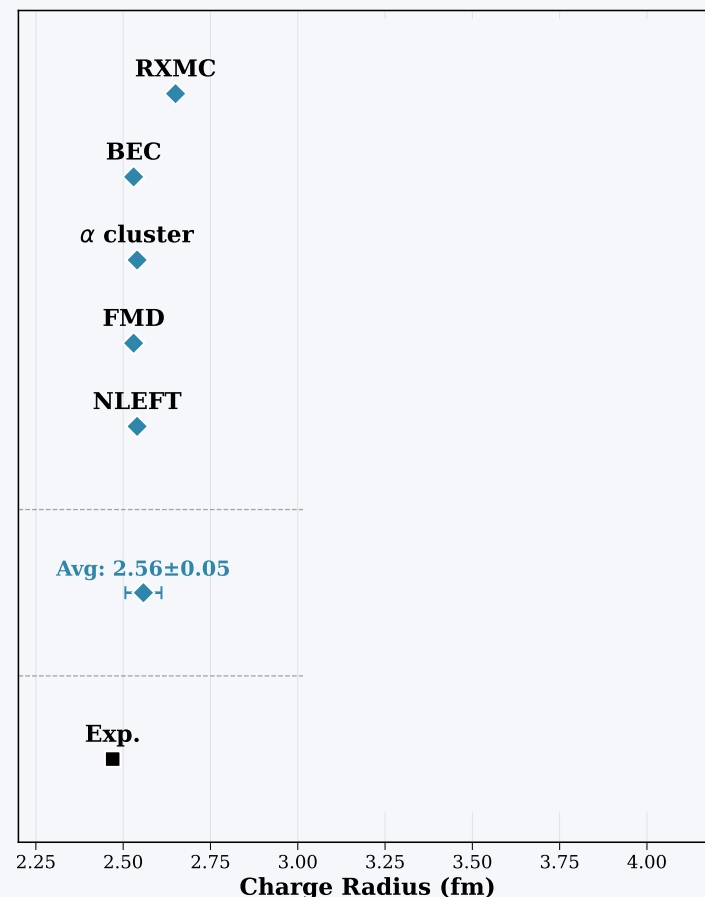
Nature of the Hoyle state still « elusive »



Example : charge radii

Ground state

Hoyle state



Pictures : most probable configurations

Goal of this study

Hoyle state via « Ab Initio » methods

A-body Schrödinger equation

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

« Chiral interactions »

Here : EM 1.8/2.0 & Hütter N3LO

$$V^{\text{LO}} + V^{\text{NLO}} + \dots$$

Ab Initio has, in particular, to be
« Systematically improvable »

Ekström *et al.*, Front. Phys **11** (2023)

Expansion methods to
alleviate exponential scaling

$$|\Psi\rangle \simeq |\Theta^{(0)}\rangle + |\Theta^{(1)}\rangle + \dots$$

Epelbaum *et al.*, Rev. Mod. Phys. **81** (2009)

Machleidt *et al.*, Phys. Rep. **503** (2011)

Drischler *et al.*, Prog. Part. Nuc. Phys. **121** (2021)

Hergert, Front. Phys. **8** (2020)

Ekström *et al.*, Front. Phys **11** (2023)

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Dr  schler *et al.*, Prog. Part. Nuc. Phys. **121** (2021)

Computational bottleneck : 3N forces

→ normal ordered to obtain 2N effective
(density-dependant) force

→ « Good » approximation up to $A \lesssim 100$

Roth *et al.*, Phys. Rev. Lett. **109** (2012)

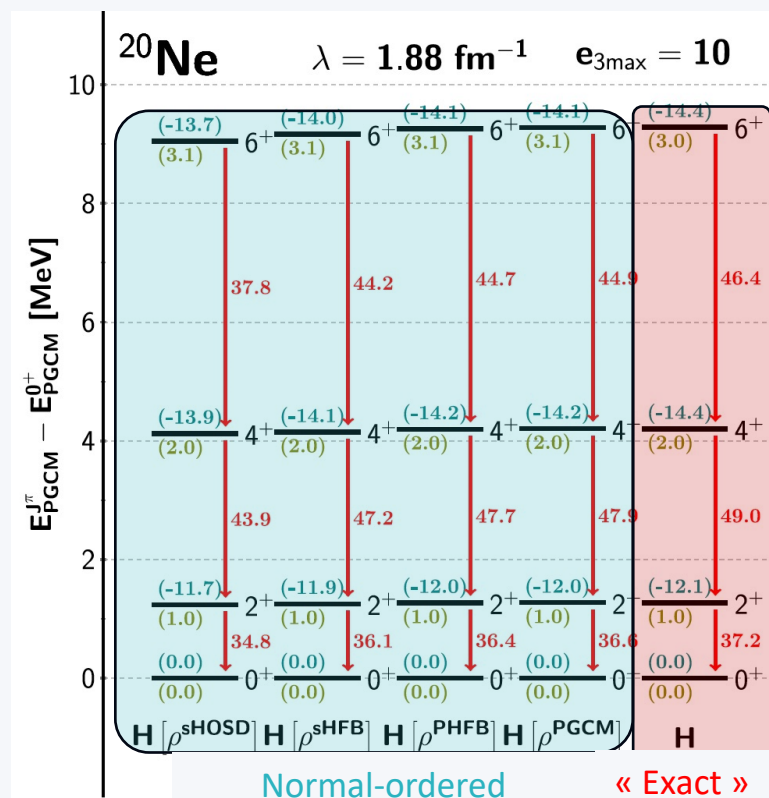
Note : interactions are SRG-evolved

Choice of interactions

EM: good pheno but inconsistencies

H  ther : more consistent but « bad » saturation

→ **Reflected in clusters physics ?**



Expansion methods to alleviate exponential scaling

$$|\Psi\rangle \simeq |\Theta^{(0)}\rangle + |\Theta^{(1)}\rangle + \dots$$

Curse of dimensionality

$$n = \dim(\mathcal{H}_1)$$

$$\dim(\mathcal{H}_A) = [\dim(\mathcal{H}_1)]^A \sim n^A$$

→ **Need polynomial methods**

1 (expansion) Many-body method

=

1 Reference

+ 1 Wave operator

Hamiltonian partitioning

$$H = H_0 + H_1$$

$$H_0 |\Theta_k^{(0)}\rangle = \epsilon_k^{(0)} |\Theta_k^{(0)}\rangle$$

$$|\Psi_k\rangle = \Omega_k |\Theta_k^{(0)}\rangle = |\Theta_k^{(0)}\rangle + |\Theta_k^{(1)}\rangle + |\Theta_k^{(2)}\rangle + \dots$$

« **Easy** » to solve
Cost ~ Mean-field
 $\mathcal{O}(n^4)$

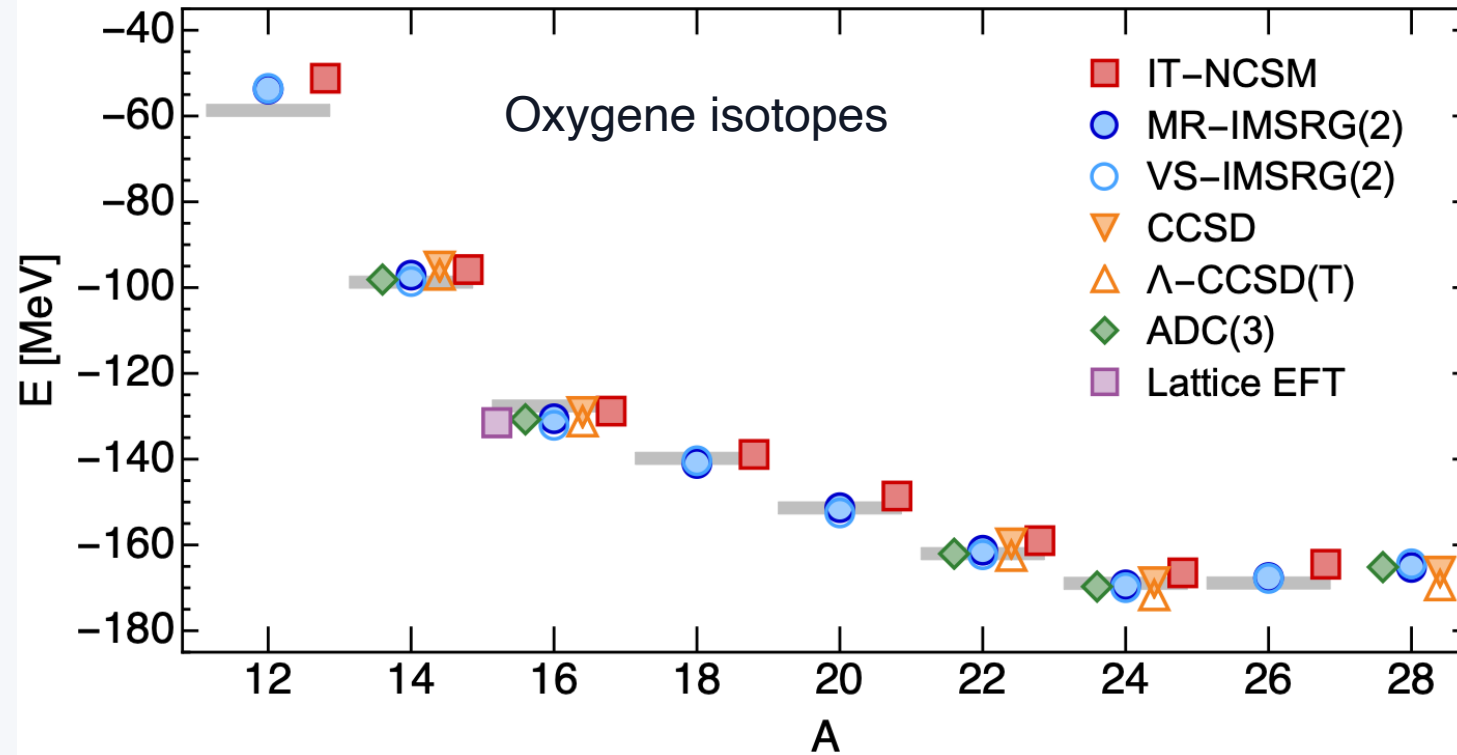
Add correlation on top
Via « Wave operator » Ω_k
Cost ~ $\mathcal{O}(n^{5\dots 7})$

Examples of « wave operators »

Coupled Cluster (cf Francesca's talk), IMSRG,
Green's function, Many body PT

Benchmark

≠ Many-body methods give
similar results



Hergert, Front. Phys. 8 (2020)

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Drischler *et al.*, Prog. Part. Nuc. Phys. **121** (2021)

Ab Initio has to be

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Ekström *et al.*, Front. Phys **11** (2023)

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Hergert, Front. Phys. **8** (2020)

Ekström *et al.*, Front. Phys **11** (2023)

Here, work at first order in **expansion method**, via
Projected Generator Coordinate Method («PGCM »)

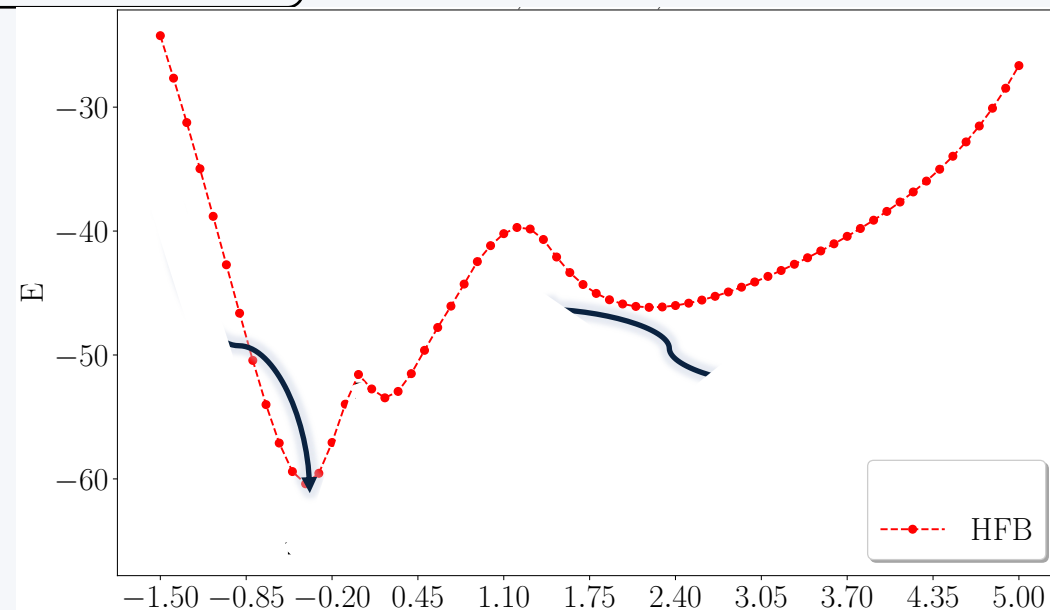
PGCM in a nutshell

1. Break symmetries : constrained Hartree-Fock-Bogoliubov (“**HFB**”)
2. Restore symmetries : use **projection** techniques (“**PHFB**”)
3. **Mix** PHFB states for fluctuations, via variational principle (“**GCM**”)

$$\delta \langle \Phi(q) | H - \lambda q | \Phi(q) \rangle = 0$$

$$| \Phi_K^\sigma(q) \rangle = P_K^\sigma | \Phi(q) \rangle$$

$$| \Psi_\epsilon^\sigma \rangle = \sum_{qK} f_{qK;\epsilon}^\sigma | \Phi_K^\sigma(q) \rangle$$



Collective coordinate(s) $q(= \beta_2)$

$= | \Theta^0 \rangle = \text{reference state for PT } (\Omega_k)$

My Goal : build «best » reference by choosing « wisely » set of $\mathbf{q}$

Note - $\mathcal{O}(n^4 \times N_{\text{proj}} \times N_{\text{gcm}}^2)$

PGCM : Strengths & Limitations



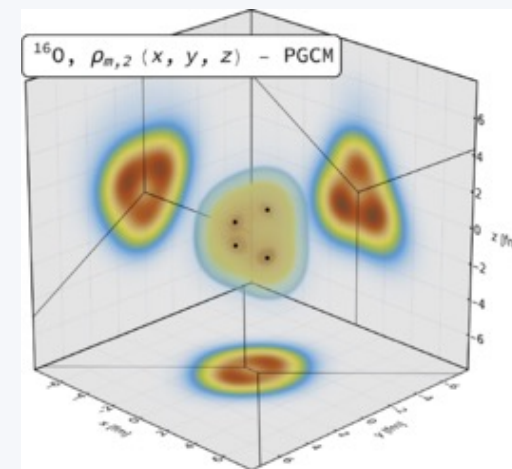
Mild scaling with mass number A
Efficiently capture collective behaviours
If $|\Theta^0\rangle$ is « good », PT is « good »

« Educated guess » for collective coordinates
PGCM (+ PT) numerically costly
Numerical convergence is « tricky »

Status - with chiral interactions

Efficient for spectroscopic quantities
Successfully used for nuclear deformation studies at RHIC
Benchmarked against other methods

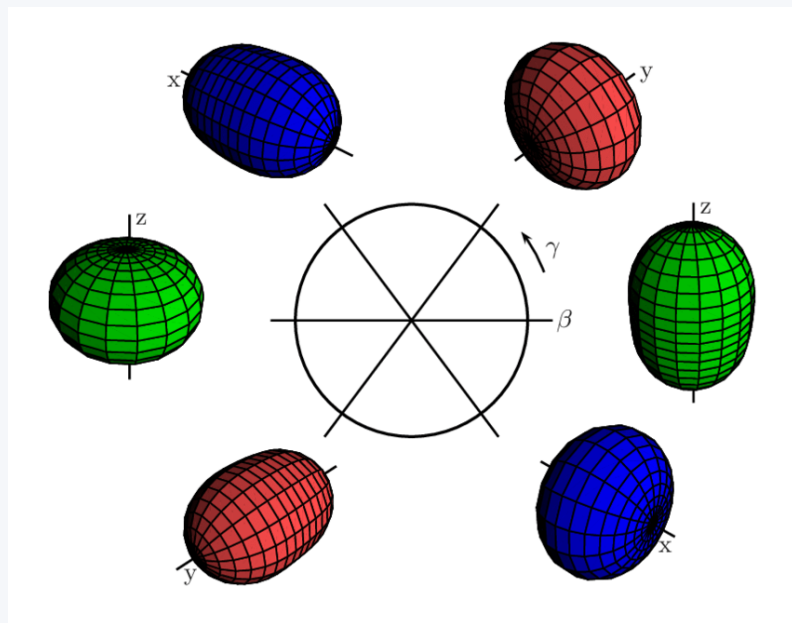
Giacalone et al., Phys. Rev. Lett. 135 (2025)
CMS Collaboration, arXiv:2510.02580 (2025)
ALICE Collaboration, arXiv:2509.06428 (2025)
Bofos et al., arXiv:2507.12037 (2025)



PGCM for ^{12}C

Chosen collective coordinates

$(\beta, \gamma) = \text{« nuclear shape »}$



All symmetries are broken

Π, N, Z, J, M

$\rightarrow N_{\text{proj}} = N_{\pi} N_N N_Z N_{\alpha} N_{\beta} N_{\gamma} \sim 10^6 \text{ factor}$

Number of states for GCM

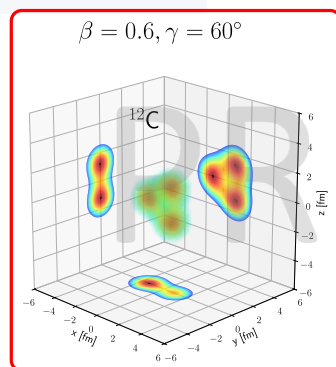
$N_{\text{gcm}} \sim 50-100 \rightarrow 10^{3..4} \text{ factor}$

Projected Energy surfaces (« PFHB »)

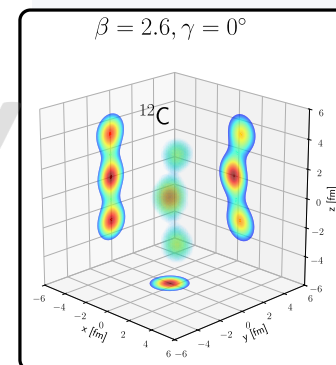
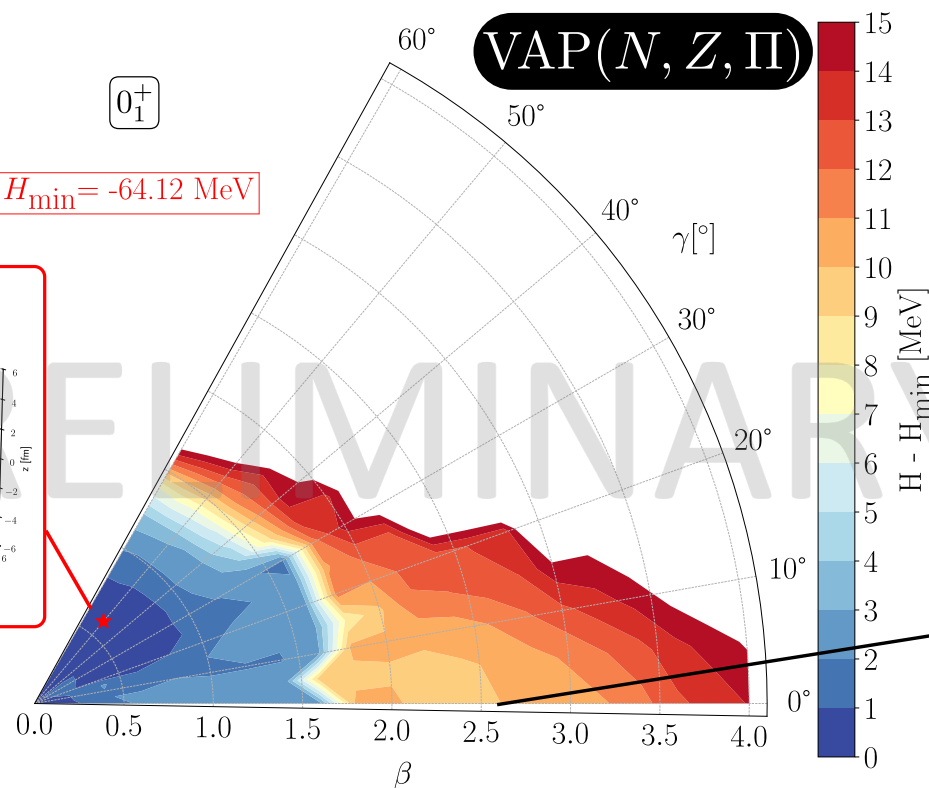
$$e_{\max} = 6, \hbar\omega = 10, e3_{\max} = 18, EM1.8/2.0$$

Absolute
minimum (GS) :
triangle shape

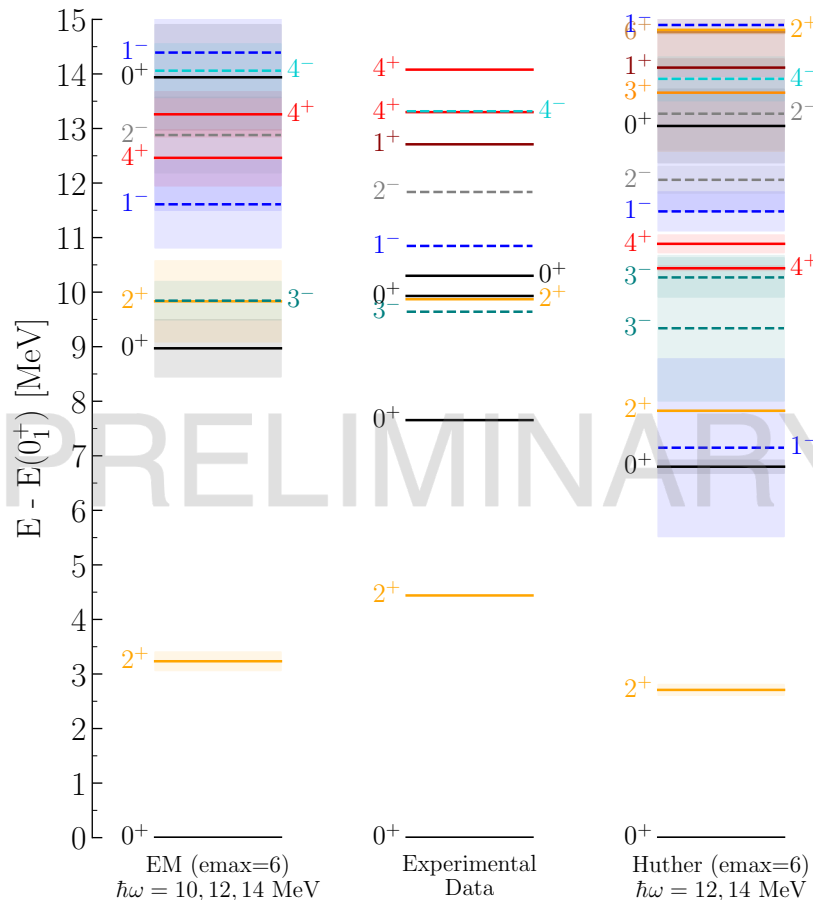
Second
minimum :
bent-arm



$$H_{\min} = -64.12 \text{ MeV}$$



* $e_{\max}$: HO basis size



Reasonable agreement

Low lying states are
qualitatively captured

Need to increase basis size
to ensure convergence

+

Add PT on top (another
work) for quantitative
statement

Note : « UQ » correspond to $\neq \hbar\omega$, not (at all) final

Conclusion & Outlook

This work [PGCM for ^{12}C]

In good shape, increase basis size for convergence

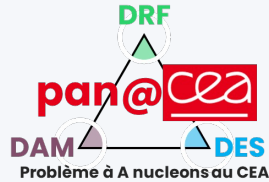
- Include continuum
- Add PT on top

Perspectives

Systematics in self-conjugate nuclei
Interplay between nucleonic and α dofs ?
Feedback on interaction ?
Interplay with reaction ?

Thank you !

Thesis supervisors



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