Multi-canonical Methods and Lattice Field Theory

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Introduction

Lattice field theory simulations use Monte Carlo methods to evaluate

$$\langle O \rangle = \frac{1}{Z(\omega)} \int d[\phi] O(\phi) e^{-S(\phi,\omega)}, \qquad Z(\omega) := \int d[\phi] e^{-S(\phi,\omega)}$$

Parameters ω may include

- Inverse gauge coupling $\beta = 1/g^2$ $\leftrightarrow$ (Classical) Inverse temperature $\beta = 1/T$
- Coupling constants of interactions $\lambda, \lambda', \ldots$ and masses terms $m_u, m_d, \ldots$
- Boundary conditions, defects

Multi-canonical methods *aim to evaluate or relate multiple choices of* ω*.*

USING THIS TERM VERY GENERALLY ...

Overview

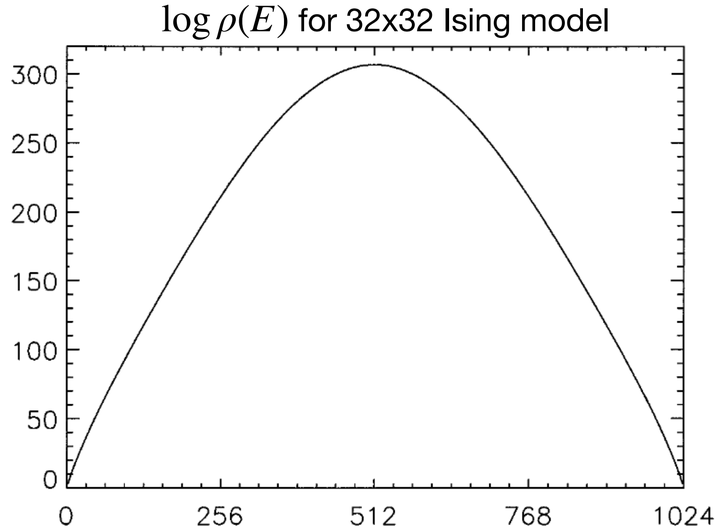
This workshop covers several related topics:

- Methods estimating/using the density of states
- Parallel tempering methods
- Transport methods (normalizing flows, diffusion models, non-equilibrium)
- Applications (where these methods are being or can be used)

Density-of-states methods

Often the action can be written as $\beta S(\phi)$

- **Canonical ensemble** for (classical) inverse temperature β and energy $E = S(\phi)$



Beale, PRL76 (1996) 78

Idea: Rewrite path integral as a **one-dimensional integral** $Z(\beta) = \int dE\, e^{-\beta E} \rho(E)$

using density of states $\rho(E) := \int d[\phi]\, \delta(S(\phi) - E)$

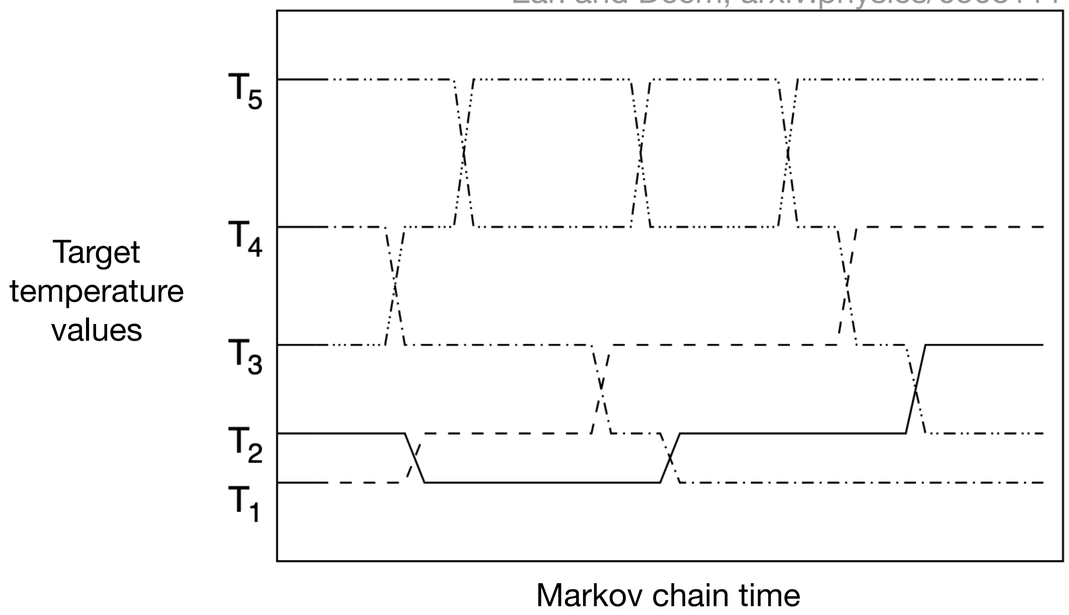
- Histogram methods, Wang-Landau, log-linear relaxation (LLR), nested sampling
- RESULT: $Z(\beta)$ easily evaluated for multiple β (multi-canonical evaluation)

Parallel tempering

Idea: **Parallel Markov chains** to evaluate at multiple parameters concurrently.

- Allowing swaps between chains can help with mixing times
- Obtain (correlated) estimates for all choices of parameters from one run

Earl and Deem, arxiv:physics/0508111

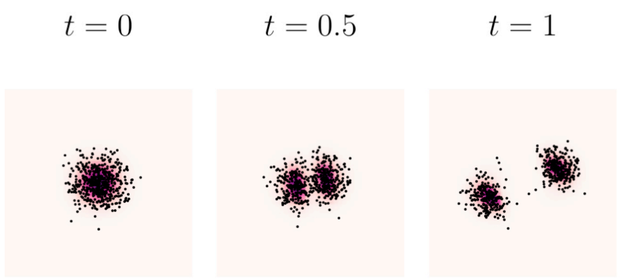


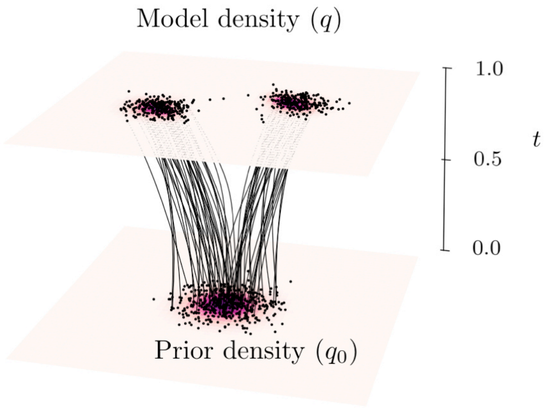
Measure transport

1. Start from samples ξ from an initial probability measure

2. Apply a **transformation** $f : \xi \mapsto \phi$

   - Deterministic transformation squishes/stretches/folds the density of points
   - Stochastic transformation mixes/diffuses the density of points
   - RESULT: we have a new probability measure over samples ϕ





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Funding —

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- INFN

Support —

- Director Ubirajara van Kolck
- ECT* admins, particularly Barbara Gazzoli

Enjoy Trento!

Highlights:

- Piazza del Duomo
- Christmas market
- Hiking/skiing in the Trentino area

A few notes:

- Free transit card available from your hotel
- Conference dinner Tuesday @ 8pm
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