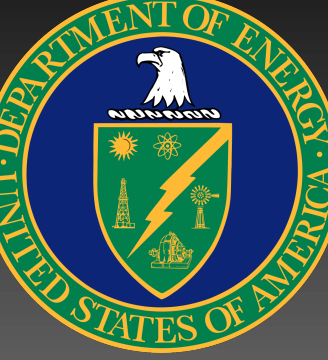




Bayesian Inference of QGP properties from the RHIC Beam Energy Scan Program

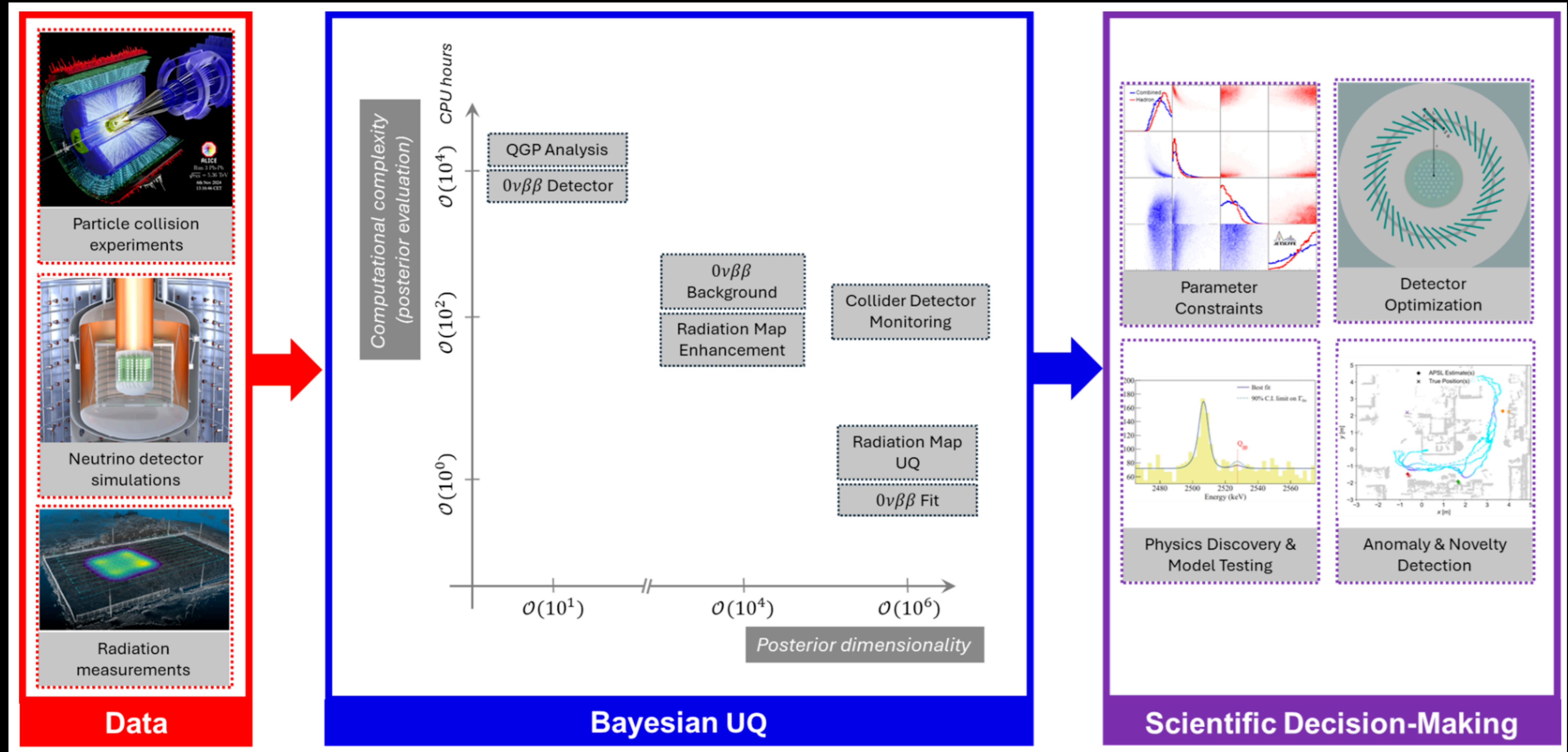
Chun Shen (Wayne State University)

In collaboration with Syed Afrid Jahan, Hendrik Roch, Björn Schenke, and Wenbin Zhao

C. Shen, B. Schenke and W. Zhao, Phys. Rev. Lett. 132, 072301 (2024)
H. Roch, S. A. Jahan and C. Shen, Phys. Rev. C 110, 044904 (2024)
S. A. Jahan, H. Roch and C. Shen, Phys. Rev. C 110, 054905 (2024)
S. A. Jahan, H. Roch and C. Shen, arXiv:2507.11394 [nucl-th]

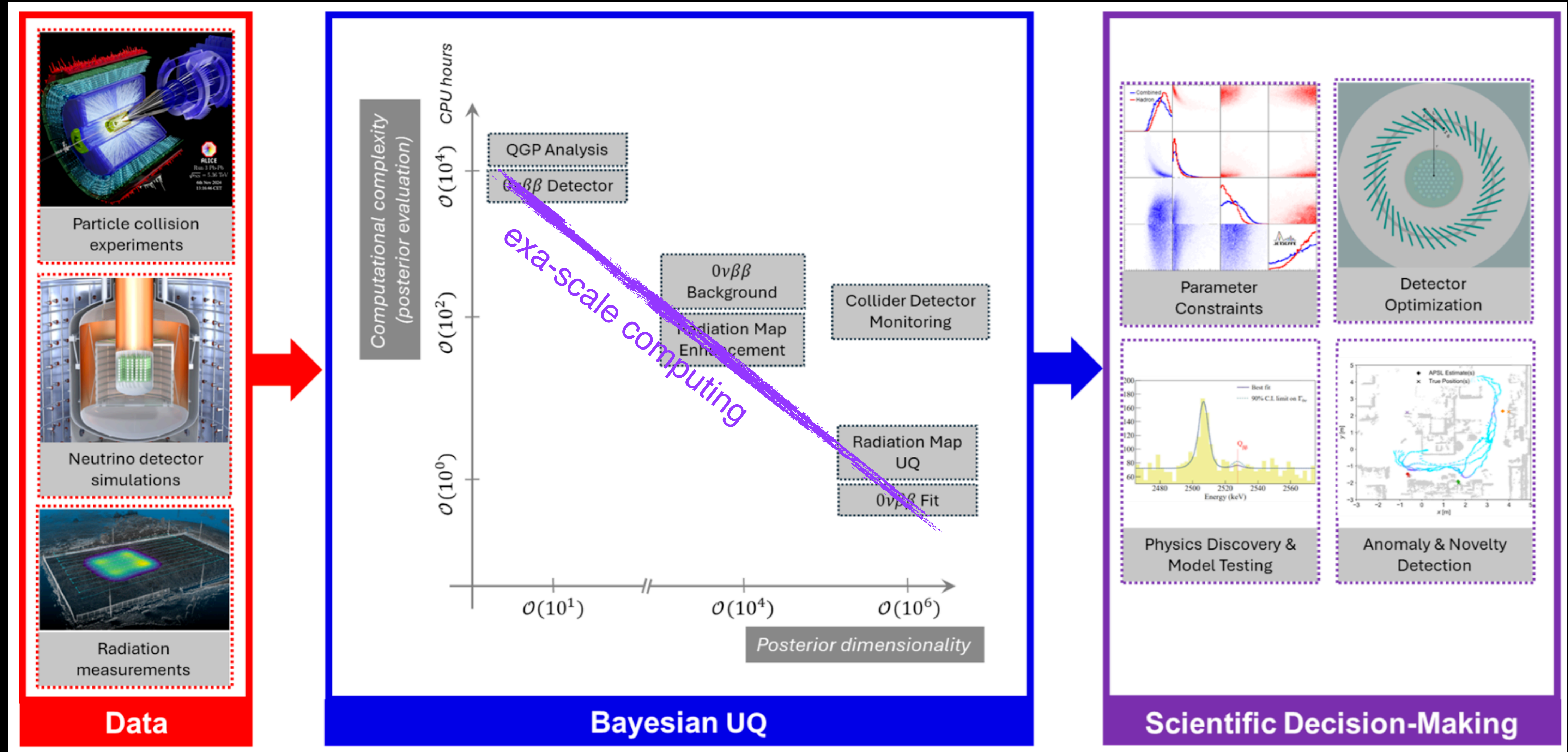
**ISNET 11, ECT*, Trento
November 17, 2025**

COLLABORATIVE EFFORT ON BAYESIAN-ENABLED NP



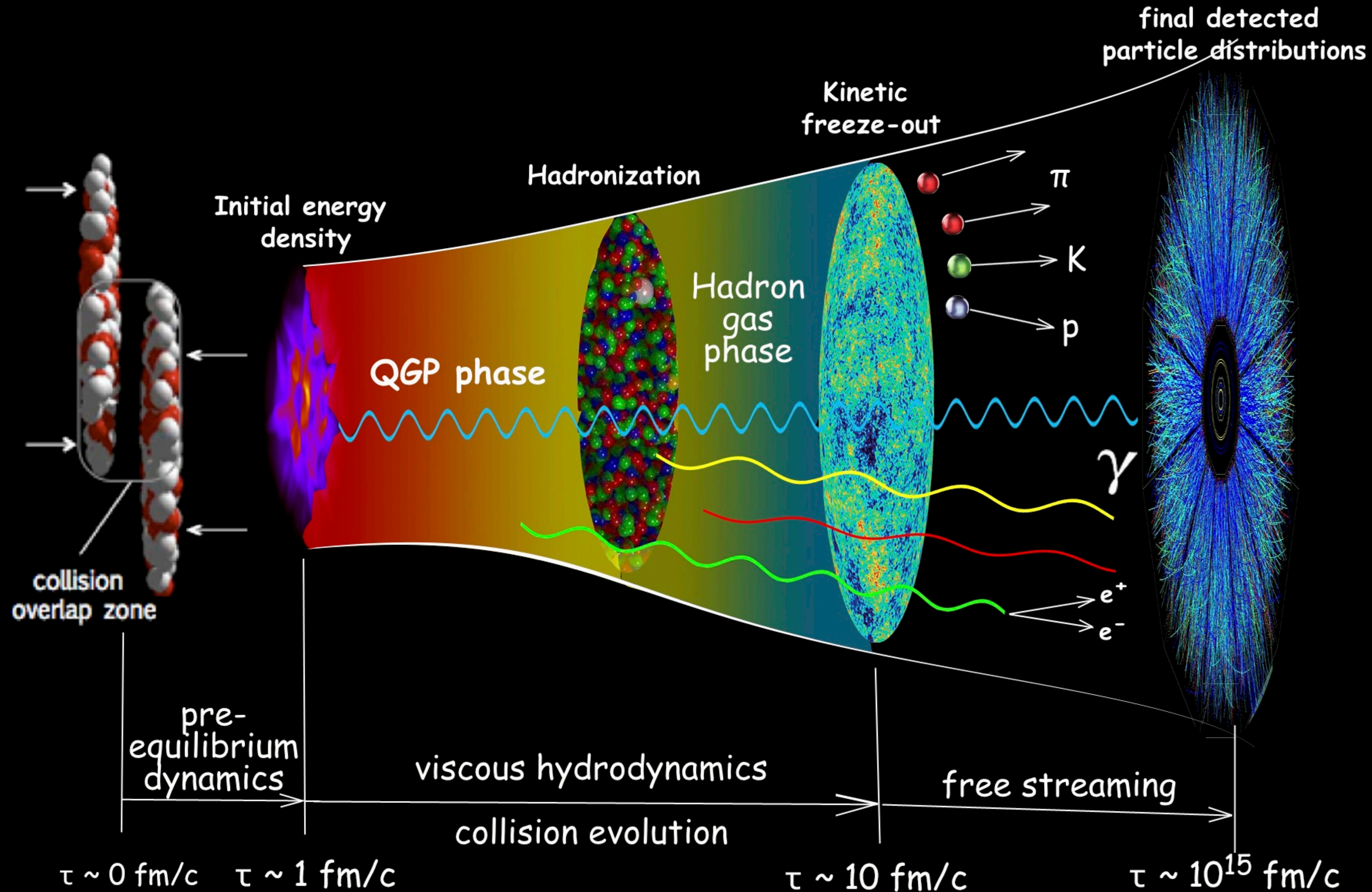
Develop common tools for Bayesian inference with expensive models at high dimensions

COLLABORATIVE EFFORT ON BAYESIAN-ENABLED NP



Develop common tools for Bayesian inference with expensive models at high dimensions

RELATIVISTIC HEAVY-ION COLLISIONS

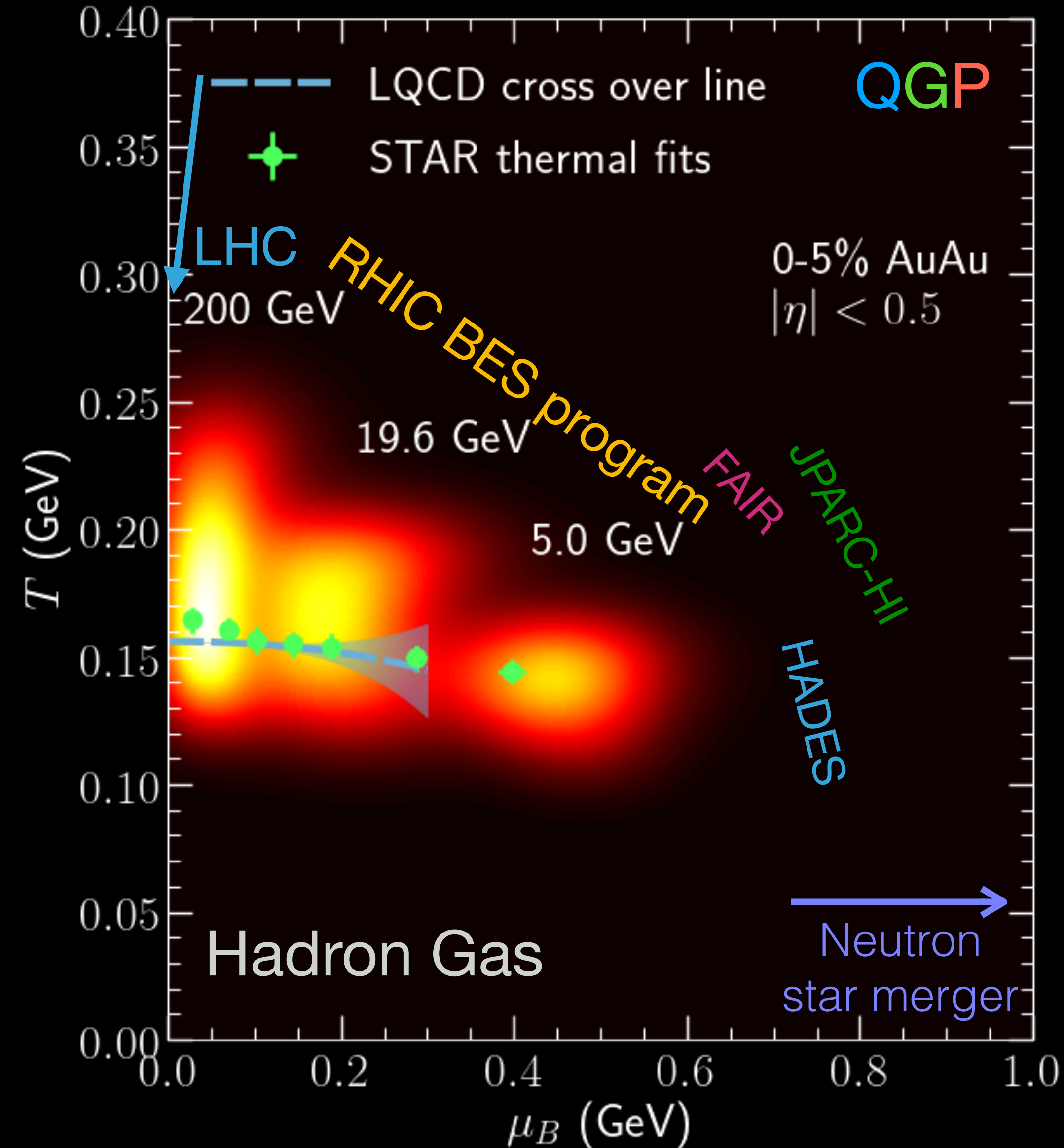
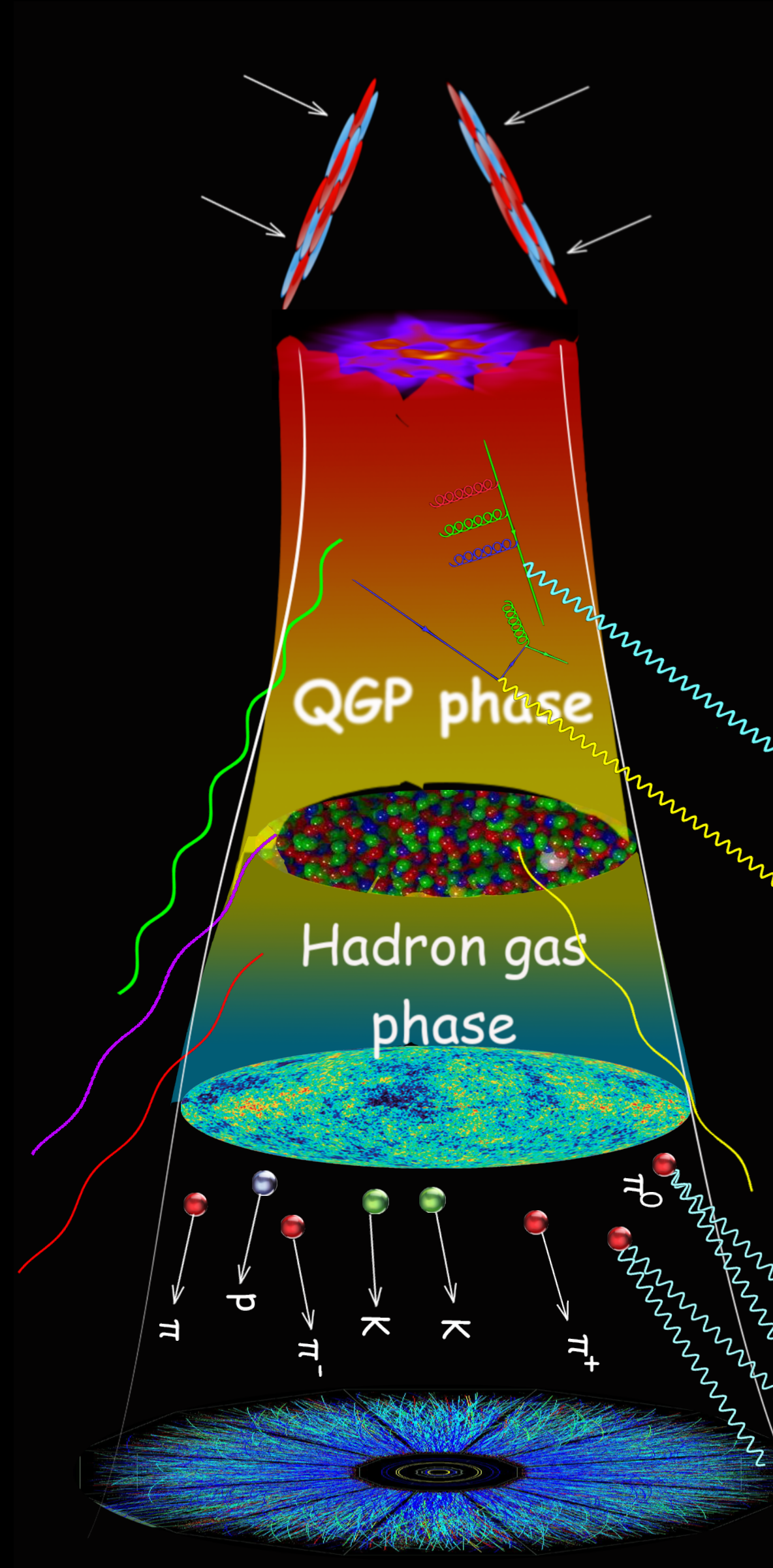


Relativistic heavy-ion collisions are tiny and have ultra-fast dynamics

It serves as a multi-purpose laboratory for many-body QCD

Physics insights reply on phenomenological theory-to-model comparisons

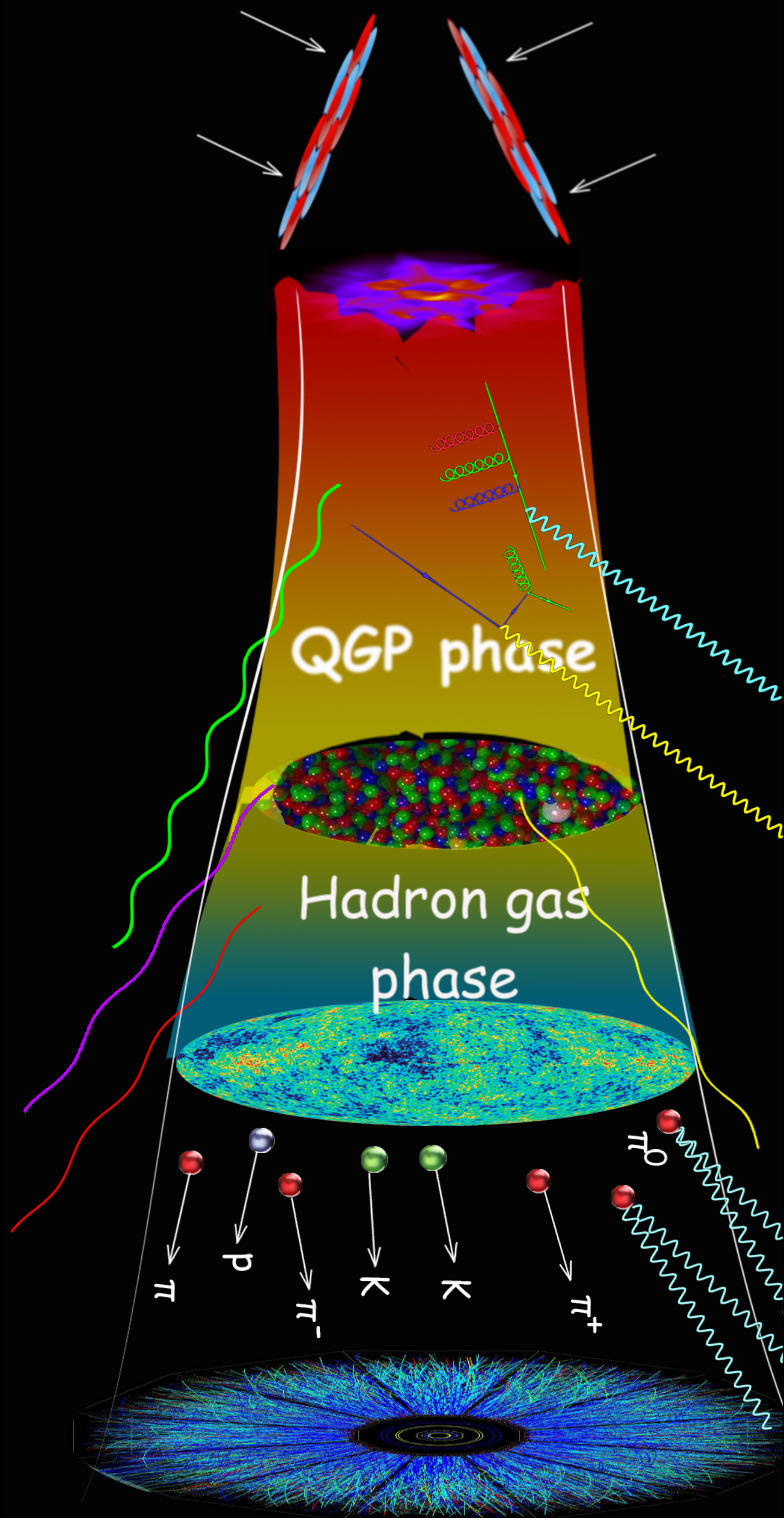
PROBING THE NUCLEAR MATTER PHASE DIAGRAM



- Search for a critical point & 1st order phase transition
 $c_s^2(T, \{\mu_q\})$
- How do the QGP transport properties change with baryon doping?
 $(\eta/s)(T, \{\mu_q\}), (\zeta/s)(T, \{\mu_q\})$
- Access to new transport phenomena
Charge diffusion

DEFINING THE QUARK-GLUON PLASMA

Which **properties of hot QCD matter** can we determine from relativistic heavy ion data (LHC, RHIC, and future FAIR/NICA/JPAC)?



Equation of State $T^{\mu\nu} \Longleftrightarrow e, P, s$
 $c_s^2 = \partial P / \partial e|_{s/n}$

Shear and bulk viscosities
 $\eta/s(T, \mu_B), \zeta/s(T, \mu_B)$

Charge diffusion D_B, D_Q, D_S

Electromagnetic emissivity

Energy-momentum transport
 $\hat{q}, \hat{e}, \hat{e}_2, \dots$

Spectra, collective flow, femtoscopy

Anisotropic flow v_n
Flow correlations

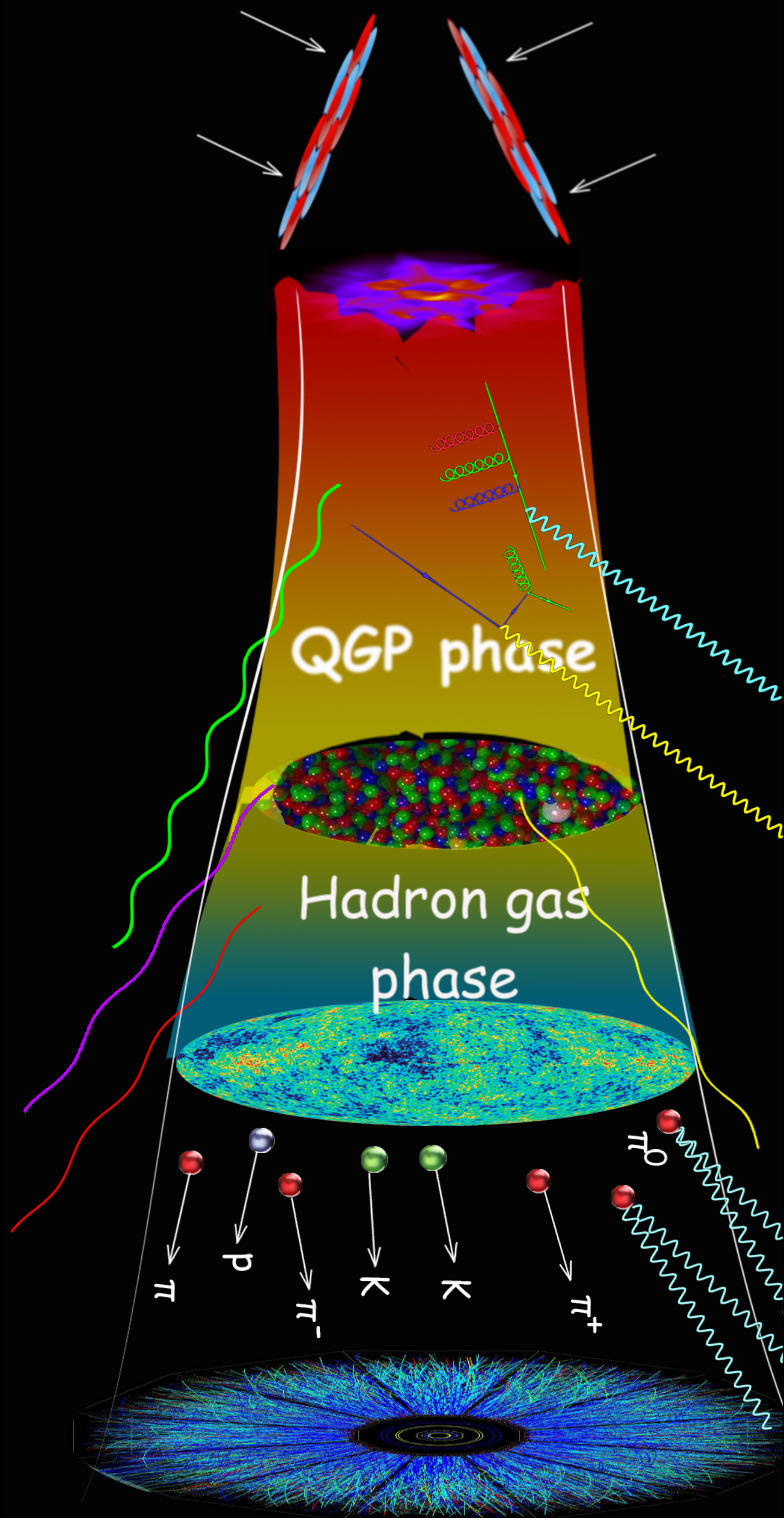
Balance functions

Photons and dileptons

Jets and heavy-quarks

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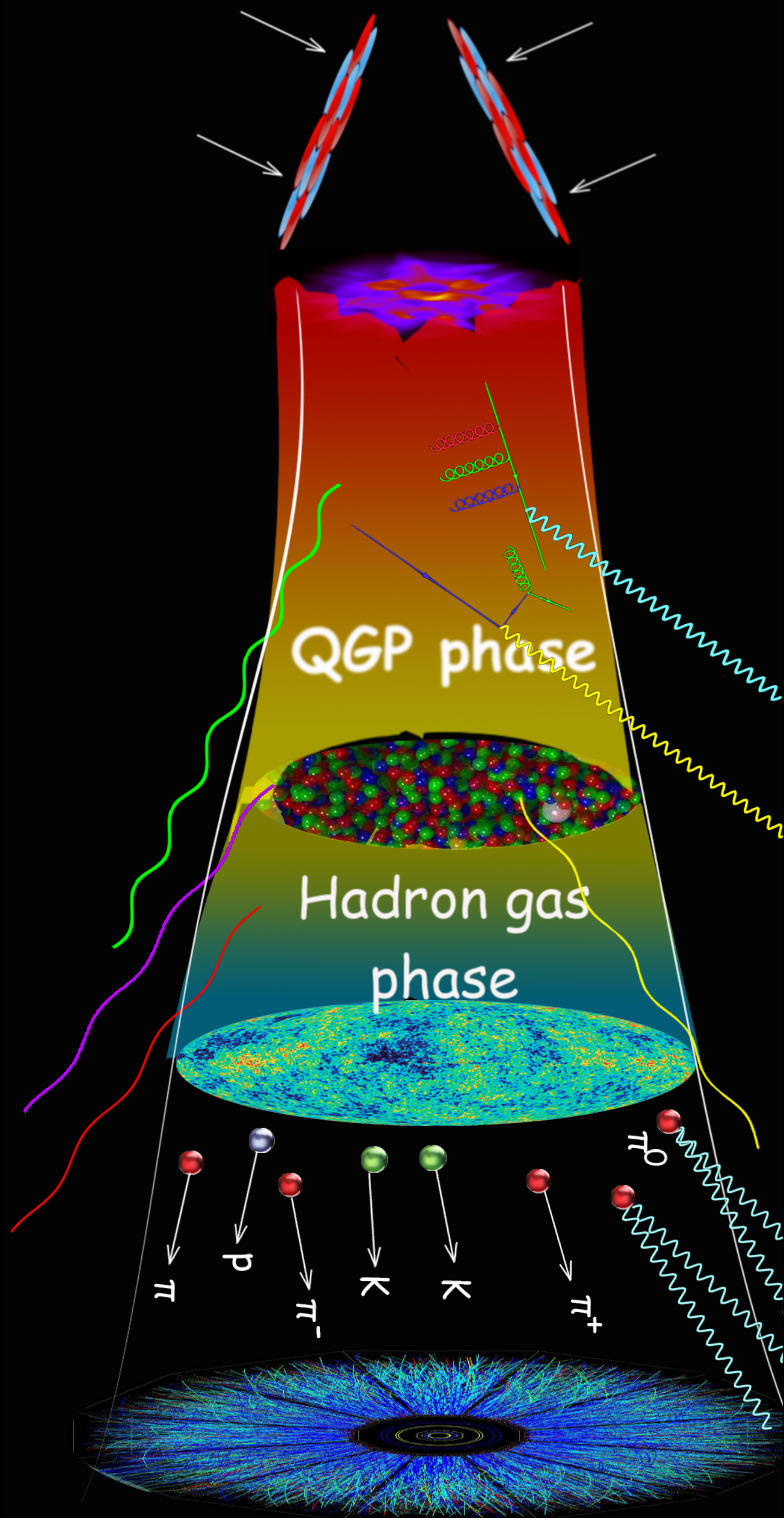
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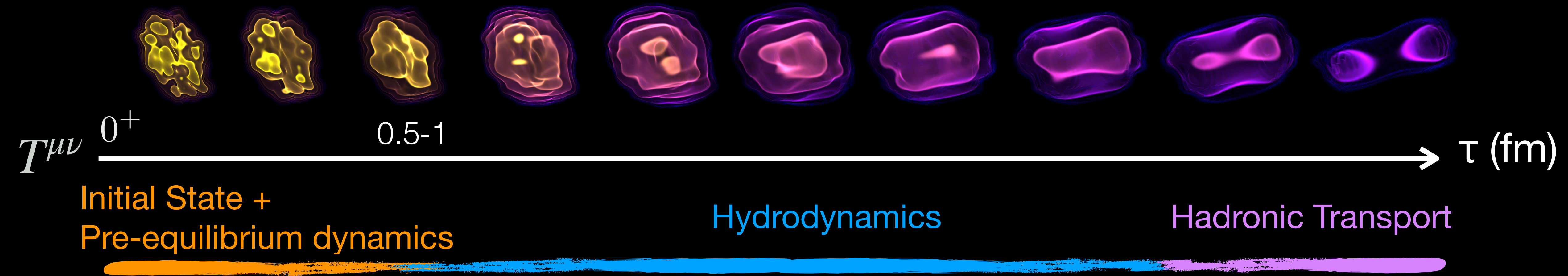
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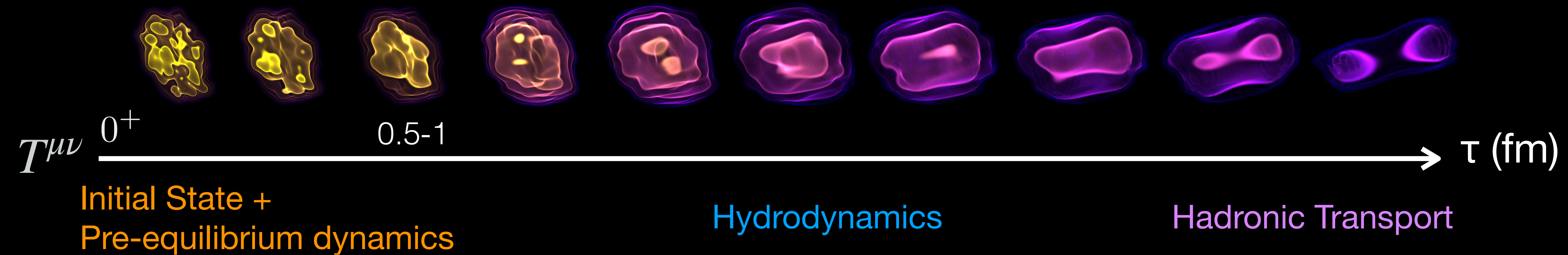
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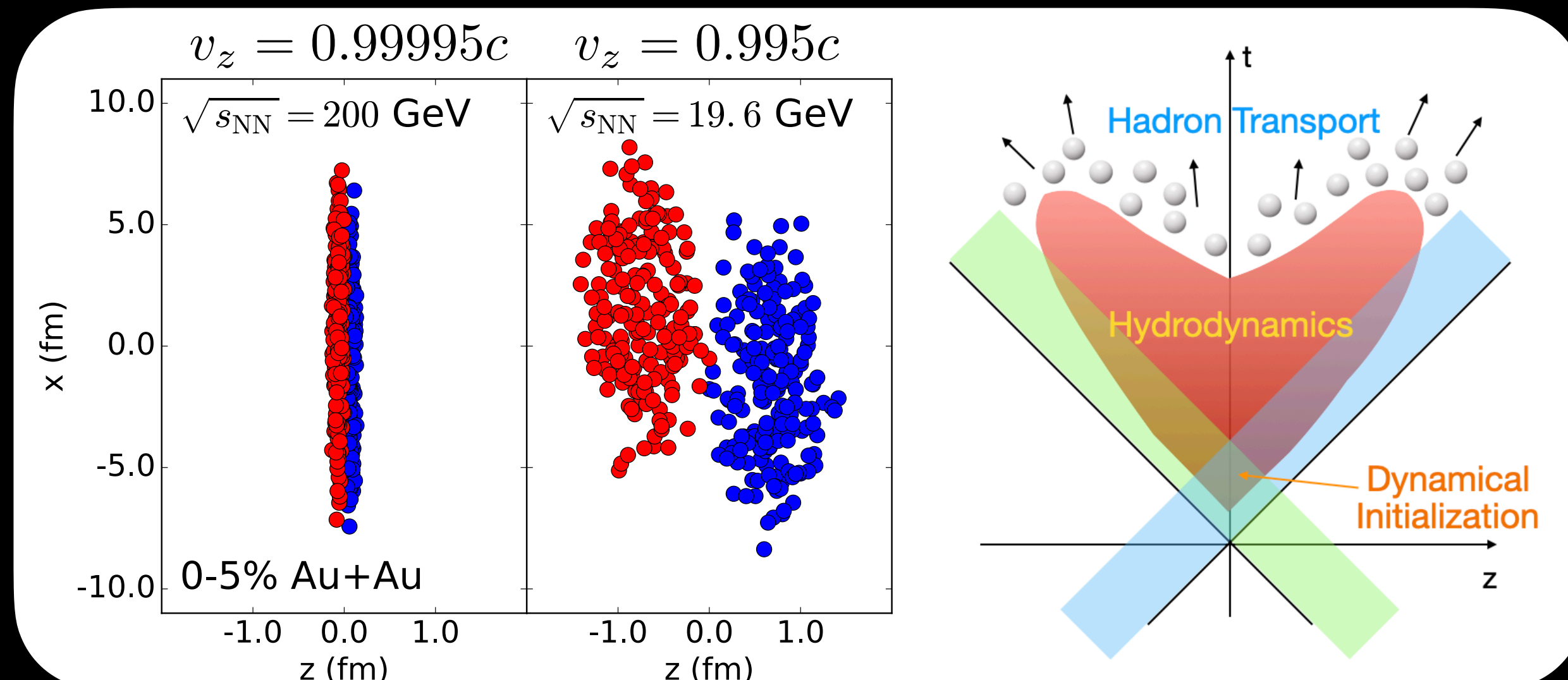
THE MULTI-STAGE THEORETICAL FRAMEWORK



THE MULTI-STAGE THEORETICAL FRAMEWORK



RHIC BES energies



$$\partial_\mu T^{\mu\nu} = J^\nu_{\text{source}}$$

$$\partial_\mu J^\mu = \rho_{\text{source}}$$

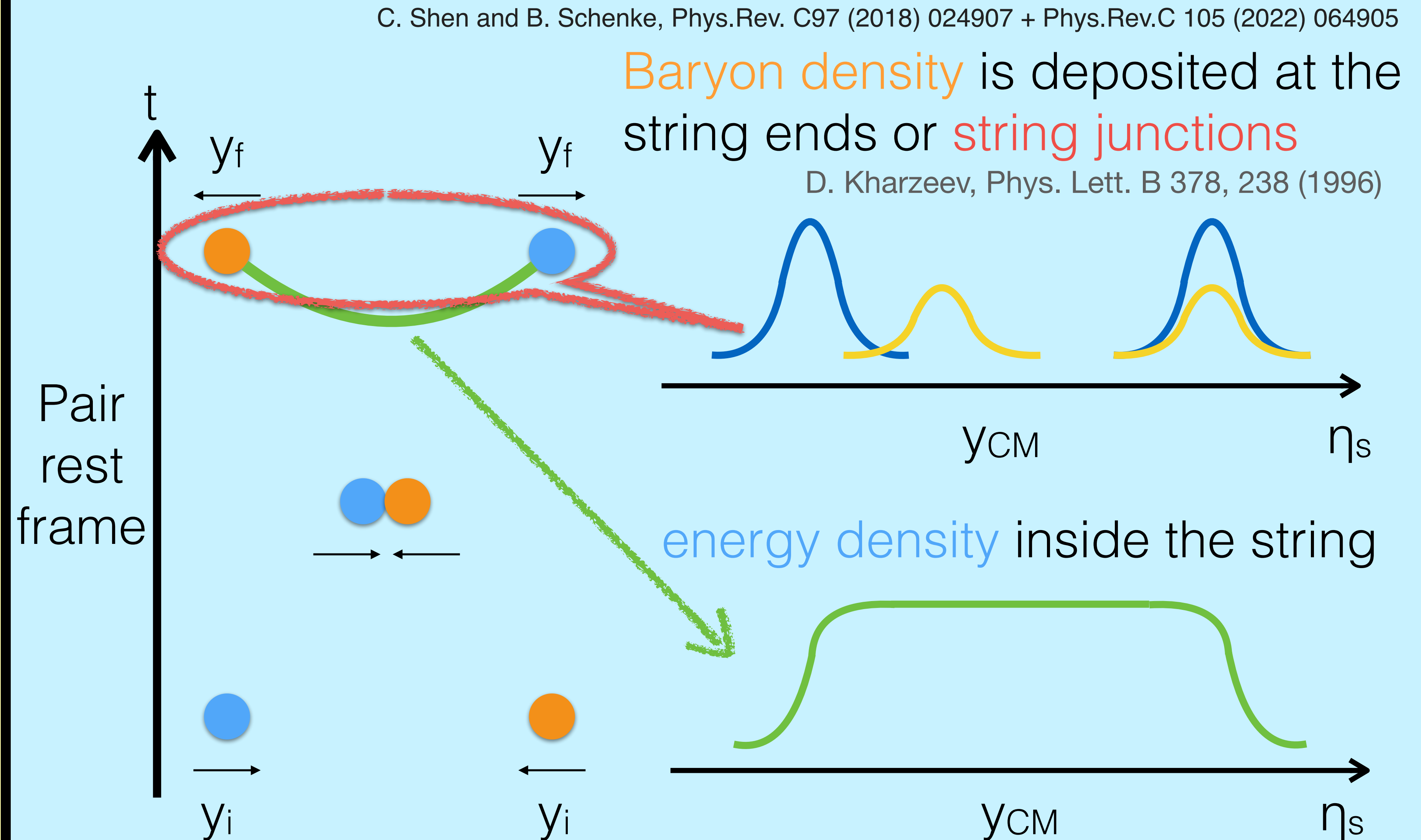
3D MC-GLAUBER MODEL WITH STRING DECELERATION

- Transverse collision geometry is determined by MC-Glauber model
- 3 valence quarks are sampled from PDF with

$$\sum_i x_i \leq 1$$

- Incoming quarks are decelerated with a string tension σ ,

$$dp^z/dt = -\sigma$$



Imposed conservation for energy, momentum, and net baryon density

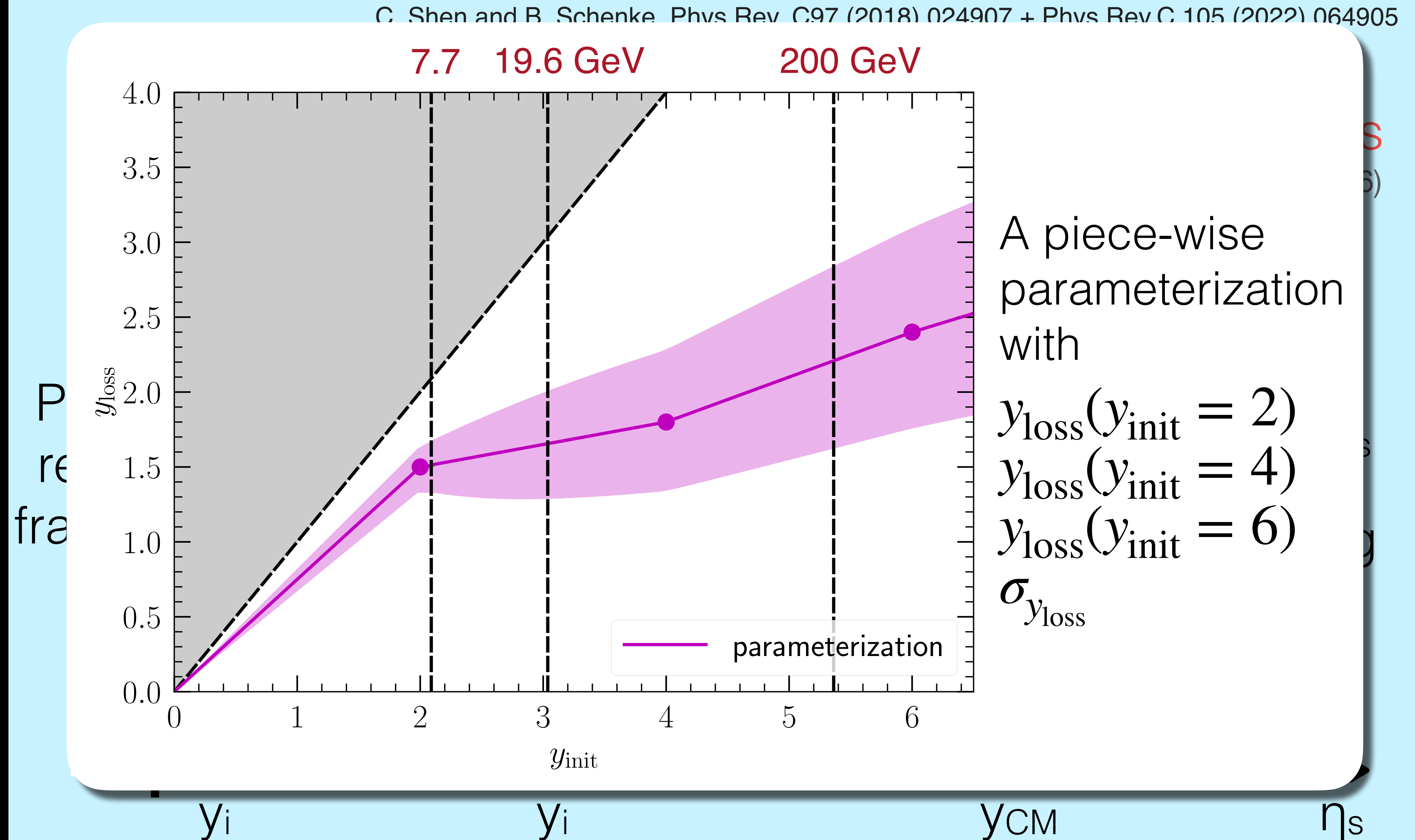
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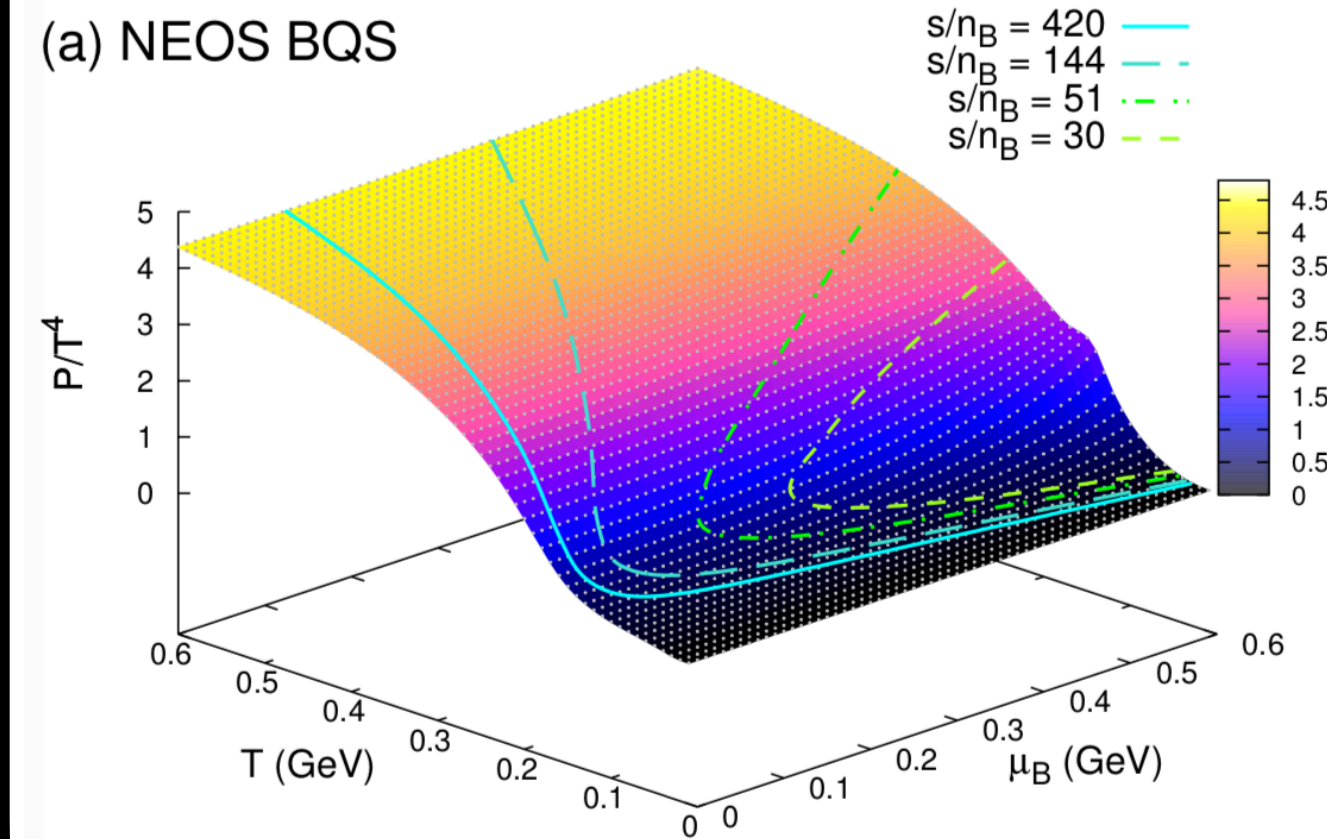
Imposed conservation for energy, momentum, and net baryon density

3D HYDRODYNAMICS WITH FINITE BARYON CURRENT

$$\begin{aligned}\partial_\mu T^{\mu\nu} &= J^\nu_{\text{source}} \\ \partial_\mu J^\mu &= \rho_{\text{source}}\end{aligned} +$$

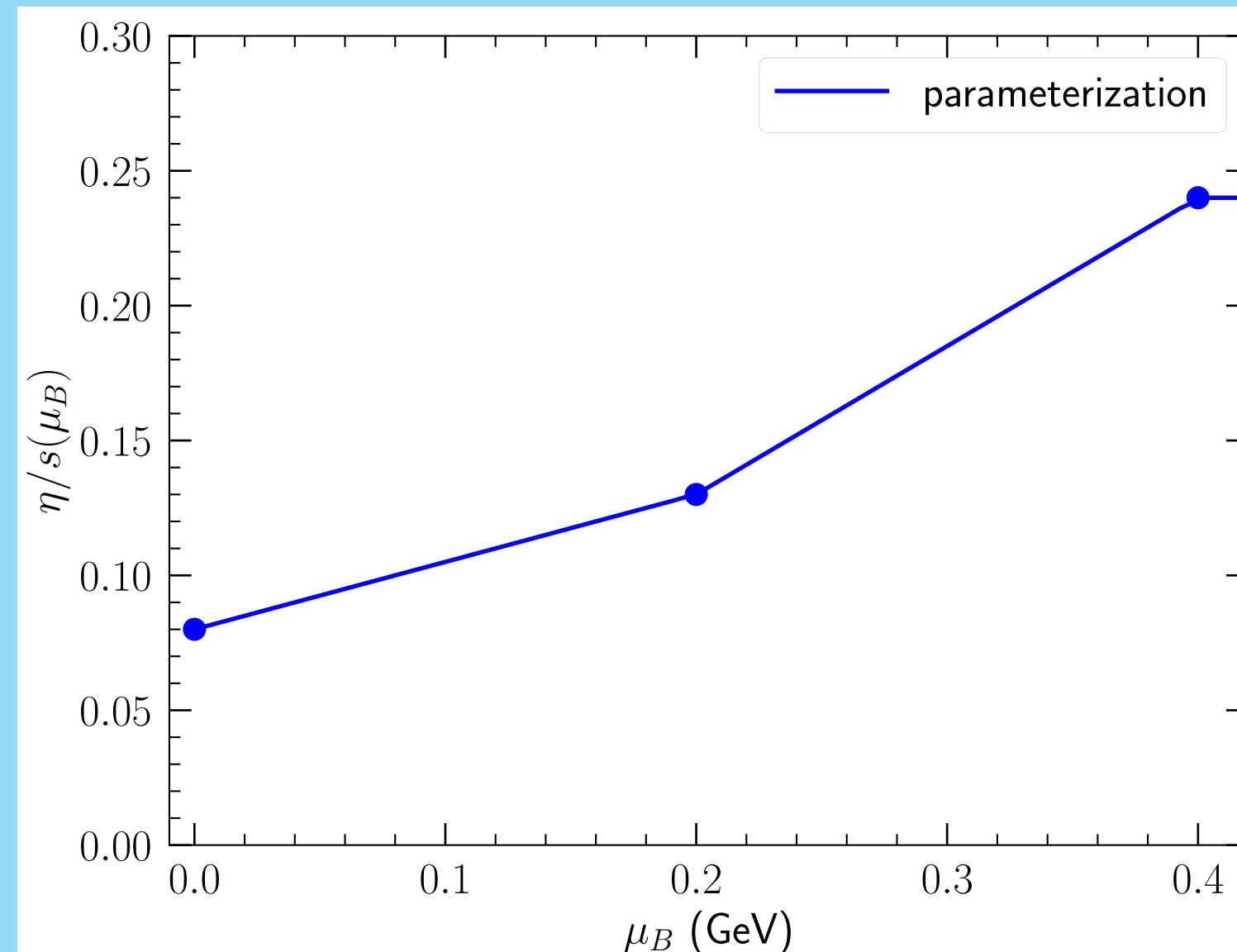
$$n_s = 0 \quad n_Q = 0.4n_B$$

(a) NEOS BQS

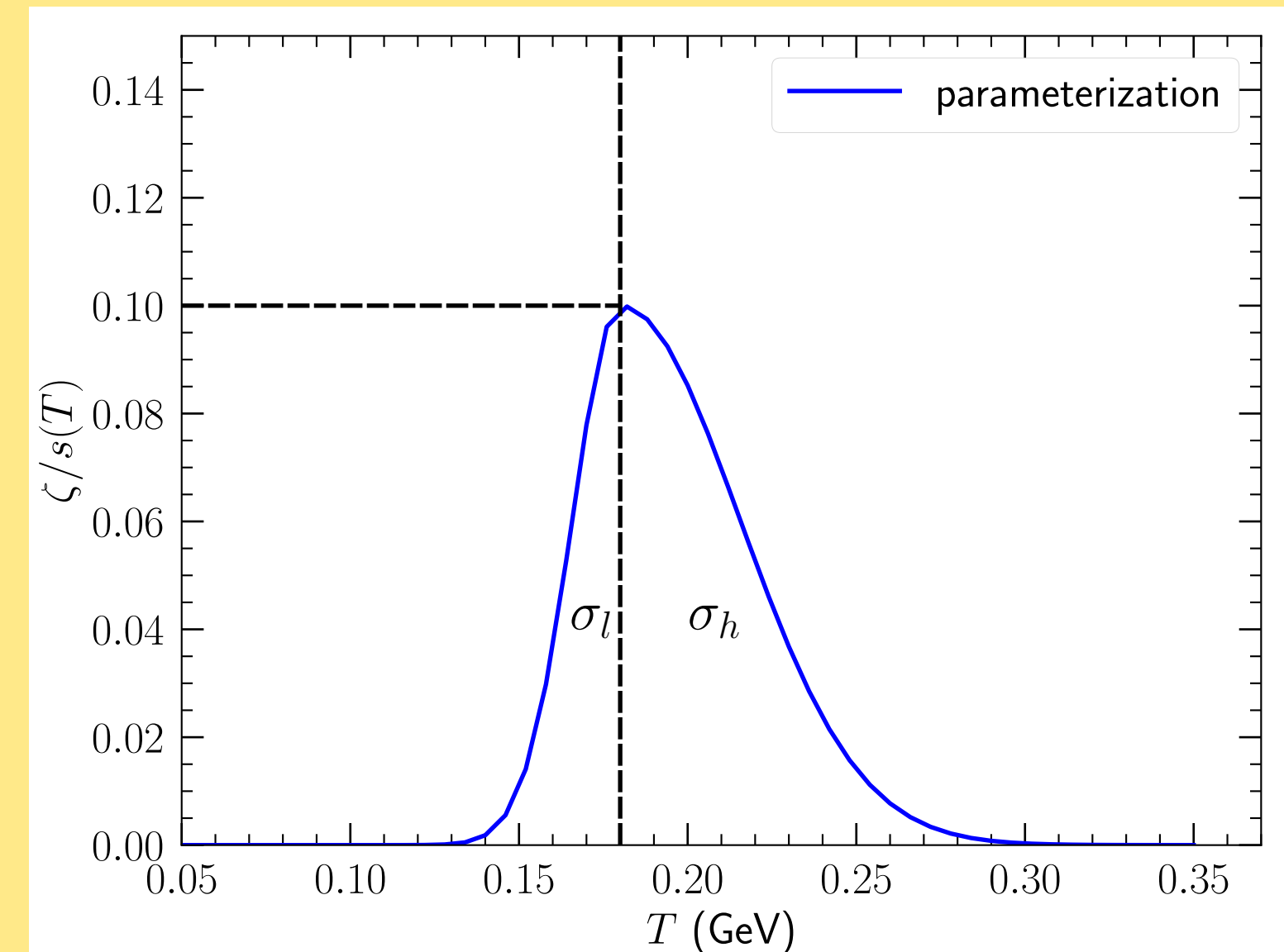


A. Monnai, B. Schenke and C. Shen, Phys. Rev. C100, 024907 (2019)

$\eta/s(\mu_B)$ has a piece-wise parameterization



$\zeta/s(T)$ is parameterized with an two-piece asymmetric Gaussian



THE INVERSE PROBLEM: BAYESIAN INFERENCE

A 20-dimensional model parameter space

Parameter	Prior	Parameter	Prior
B_G (GeV ⁻²)	[1, 25]	$\alpha_{\text{string tilt}}$	[0, 1]
$\alpha_{\text{shadowing}}$	[0, 1]	α_{preFlow}	[0, 2]
$y_{\text{loss},2}$	[0, 2]	η_0	[0.001, 0.3]
$y_{\text{loss},4}$	[1, 3]	η_2	[0.001, 0.3]
$y_{\text{loss},6}$	[1, 4]	η_4	[0.001, 0.3]
$\sigma_{y_{\text{loss}}}$	[0.1, 0.8]	ζ_{max}	[0, 0.2]
α_{Rem}	[0, 1]	$T_{\zeta,0}$ (GeV)	[0.15, 0.25]
λ_B	[0, 1]	$\sigma_{\zeta,+}$ (GeV)	[0.01, 0.15]
σ_x^{string} (fm)	[0.1, 0.8]	$\sigma_{\zeta,-}$ (GeV)	[0.005, 0.1]
$\sigma_\eta^{\text{string}}$	[0.1, 1]	e_{sw} (GeV/fm ³)	[0.15, 0.5]

$P(y_{\text{exp}} | \theta)$
Likelihood

Posterior
 $P(\theta | y_{\text{exp}})$

Bayes Theorem:

$$P(\theta | y_{\text{exp}}) = \frac{P(y_{\text{exp}} | \theta)P(\theta)}{P(y_{\text{exp}})}$$

- Expensive models require training emulators

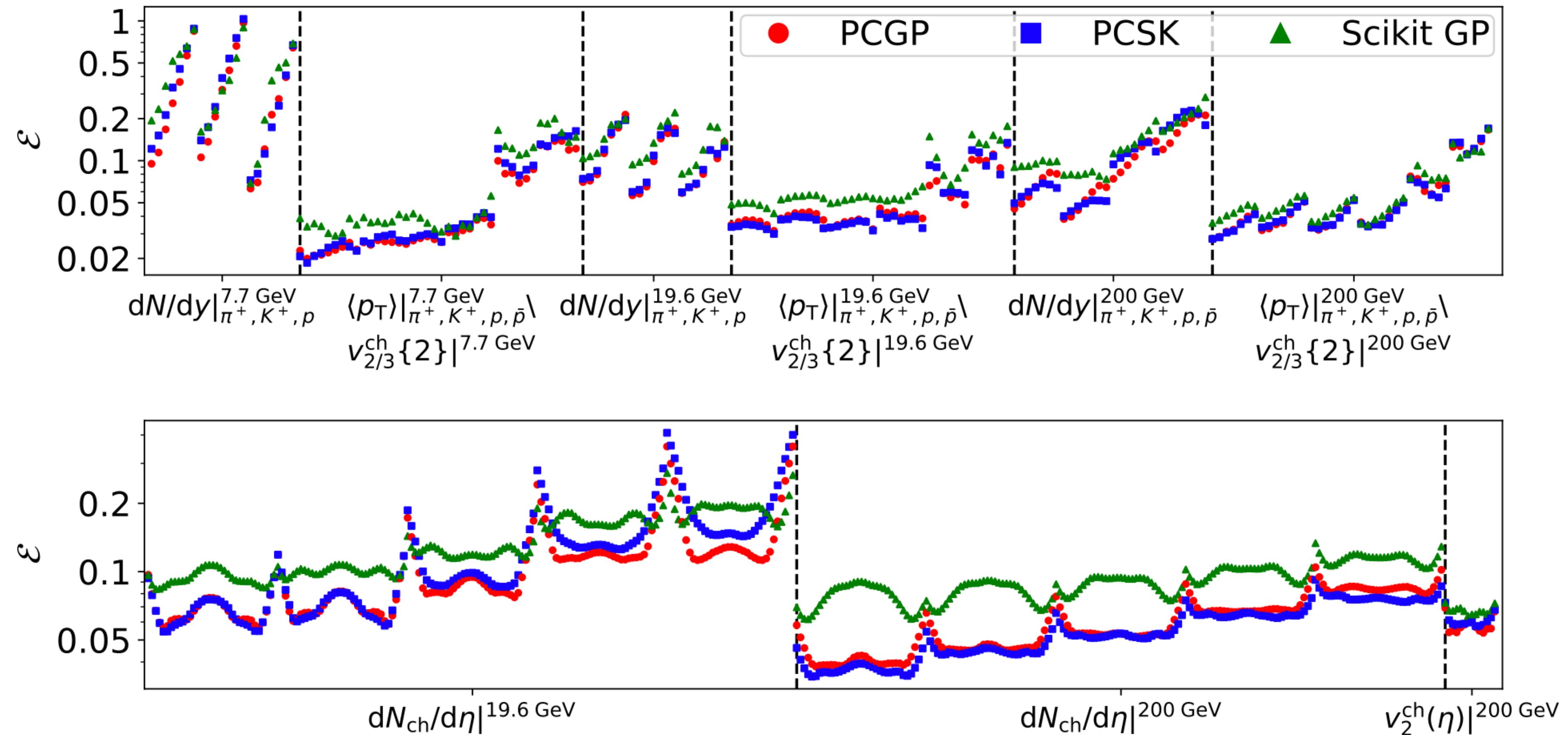
~600 experimental data points

Au+Au	STAR midrapidity data vs. centrality	PHOBOS rapidity distribution
200 GeV	$dN/dy(\pi^+, K^+, p, \bar{p})$ $\langle p_T \rangle(\pi^+, K^+, p, \bar{p})$ $v_2^{\text{ch}}\{2\}, v_3^{\text{ch}}\{2\}$	$dN^{\text{ch}}/d\eta$ $v_2(\eta)$
19.6 GeV	$dN/dy(\pi^+, K^+, p)$ $\langle p_T \rangle(\pi^+, K^+, p, \bar{p})$ $v_2^{\text{ch}}\{2\}, v_3^{\text{ch}}\{2\}$	$dN^{\text{ch}}/d\eta$
7.7 GeV	$dN/dy(\pi^+, K^+, p)$ $\langle p_T \rangle(\pi^+, K^+, p, \bar{p})$ $v_2^{\text{ch}}\{2\}, v_3^{\text{ch}}\{2\}$	

EVALUATE THE QUALITY OF MODEL EMULATION

H. Roch, S. A. Jahan and C. Shen, Phys. Rev. C 110, 044904 (2024)

$$\mathcal{E} = \sqrt{\left\langle \left(\frac{\text{prediction} - \text{truth}}{\text{truth}} \right)^2 \right\rangle}$$



- With the same amount of training data, the PCGP and PCSK emulators are more accurate than the standard GP from the scikit-learn package

EVALUATE THE QUALITY OF MODEL EMULATION

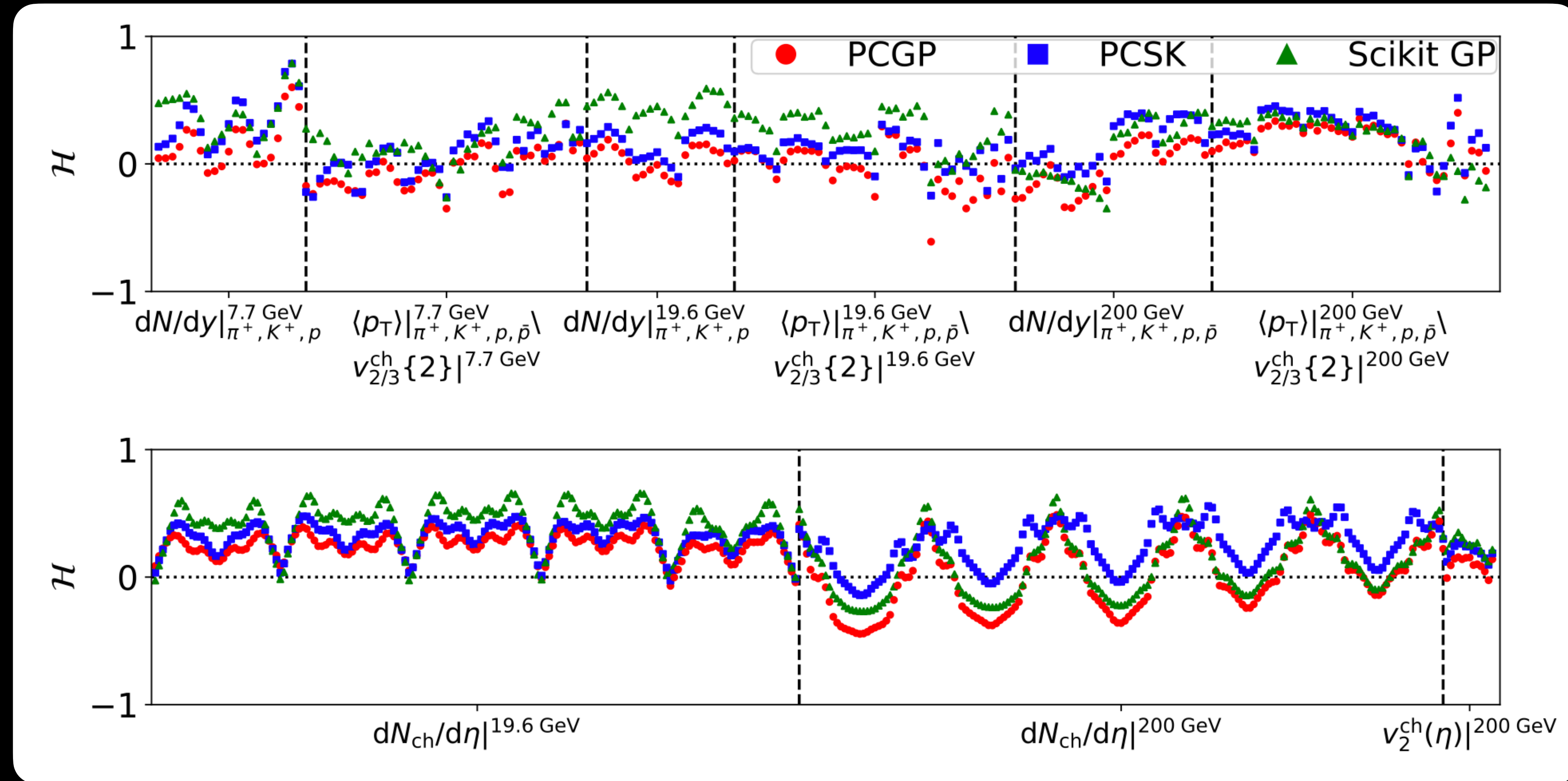
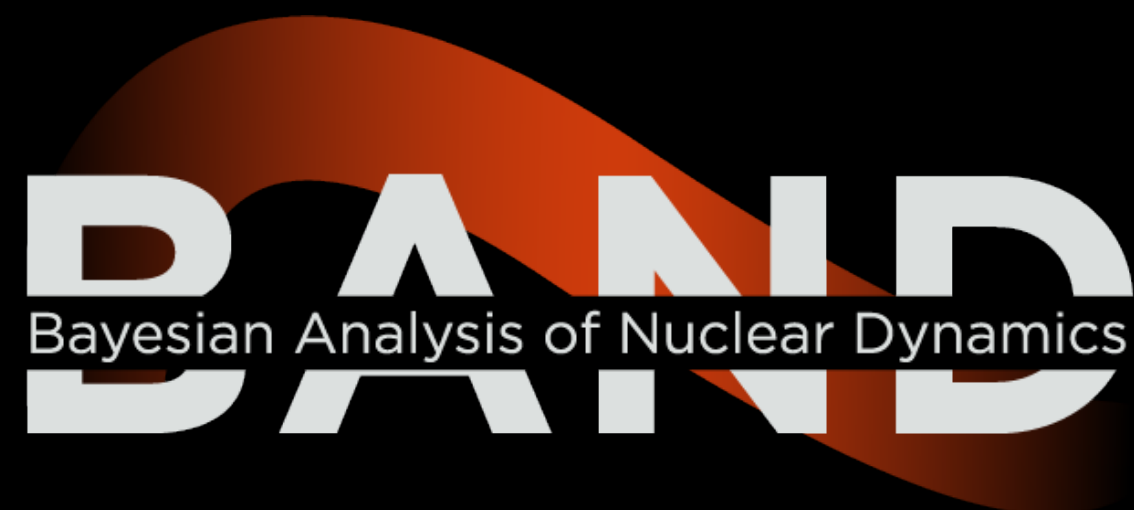
H. Roch, S. A. Jahan and C. Shen, Phys. Rev. C 110, 044904 (2024)

$$\mathcal{H} = \ln \left(\sqrt{\left\langle \left(\frac{\text{prediction} - \text{truth}}{\text{pred. uncertainty}} \right)^2 \right\rangle} \right)$$

$\mathcal{H} = 0$: Best

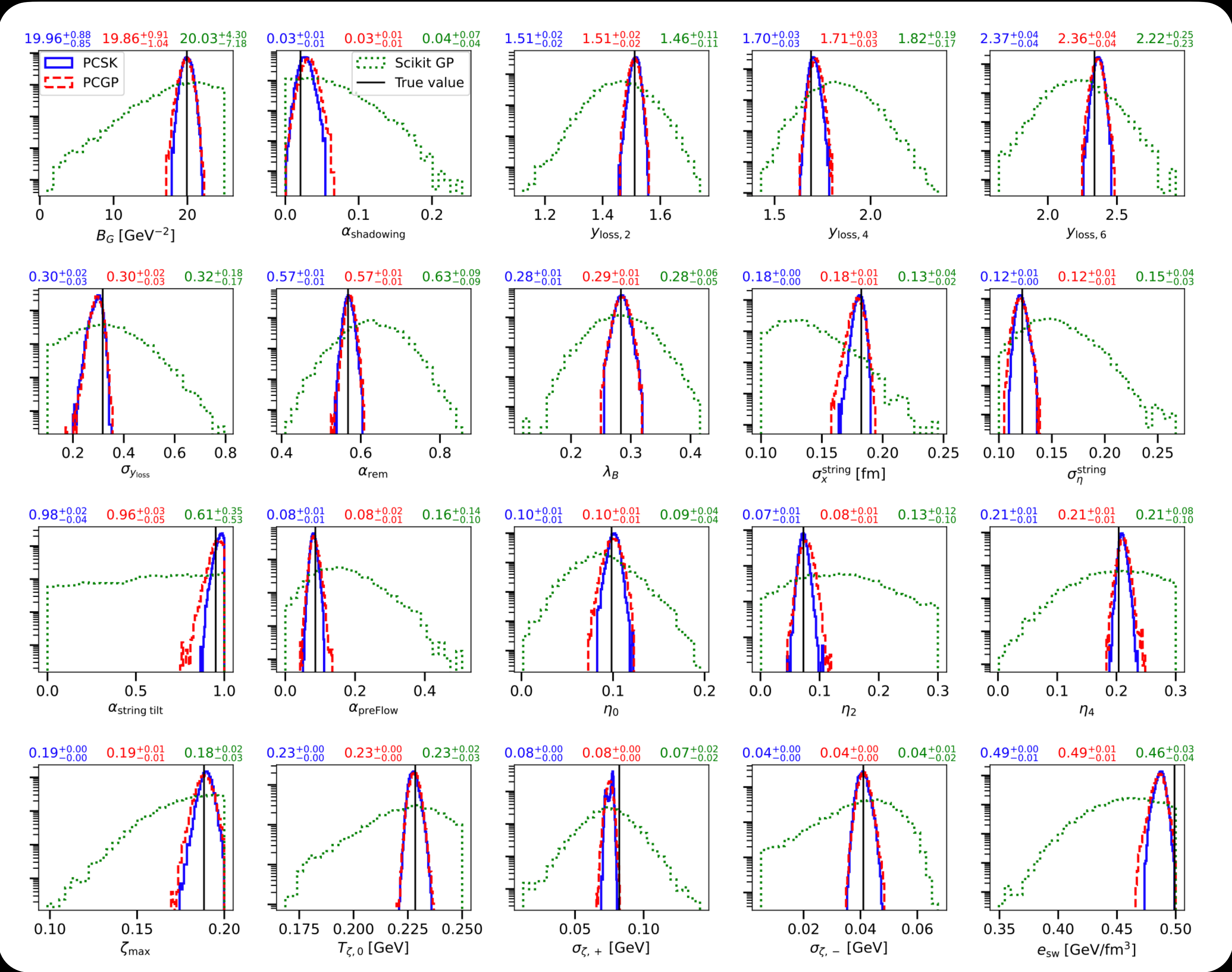
$\mathcal{H} > 0$: Predicted uncertainty is too small

$\mathcal{H} < 0$: Predicted uncertainty is too large



- The PCGP and PCSK emulators also give more reliable uncertainty estimation than that from the Scikit GP

IMPACTS OF EMULATOR PRECISION ON CLOSURE TESTS



- The more accurate PCGP and PCSK emulators give tighter posterior on model parameters than that from the Scikit GP

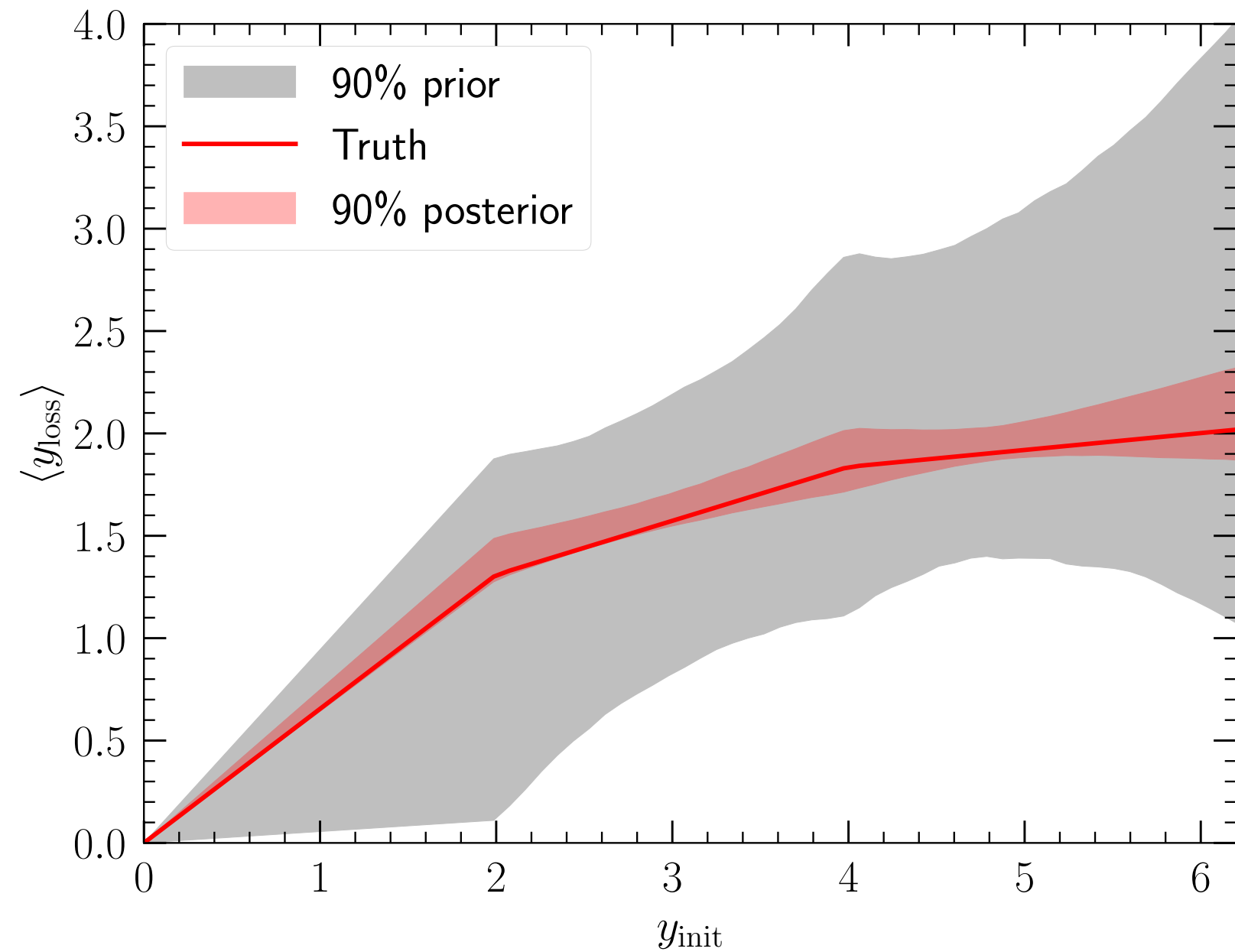
$$\Delta \equiv \frac{1}{N_{\text{param.}}} \int \left| \frac{\theta - \theta_{\text{truth}}}{\theta_{\text{max}} - \theta_{\text{min}}} \right|^2 p(\theta) d\theta,$$

Emulator	Δ
PCGP	4.5×10^{-4}
PCSK	5.7×10^{-4}
Scikit GP	2.5×10^{-2}

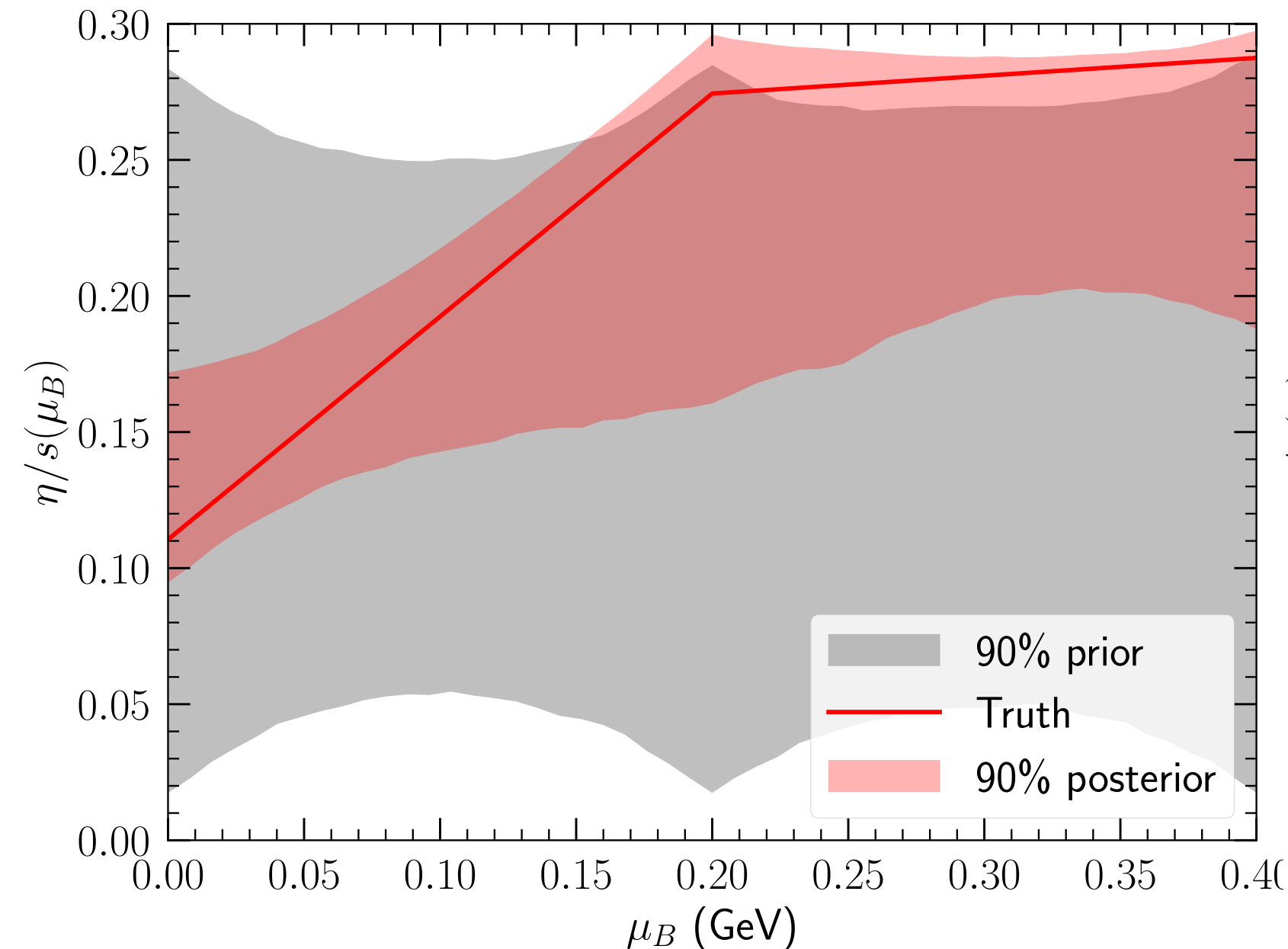
The smaller the better

BAYESIAN VALIDATION: CLOSURE TEST

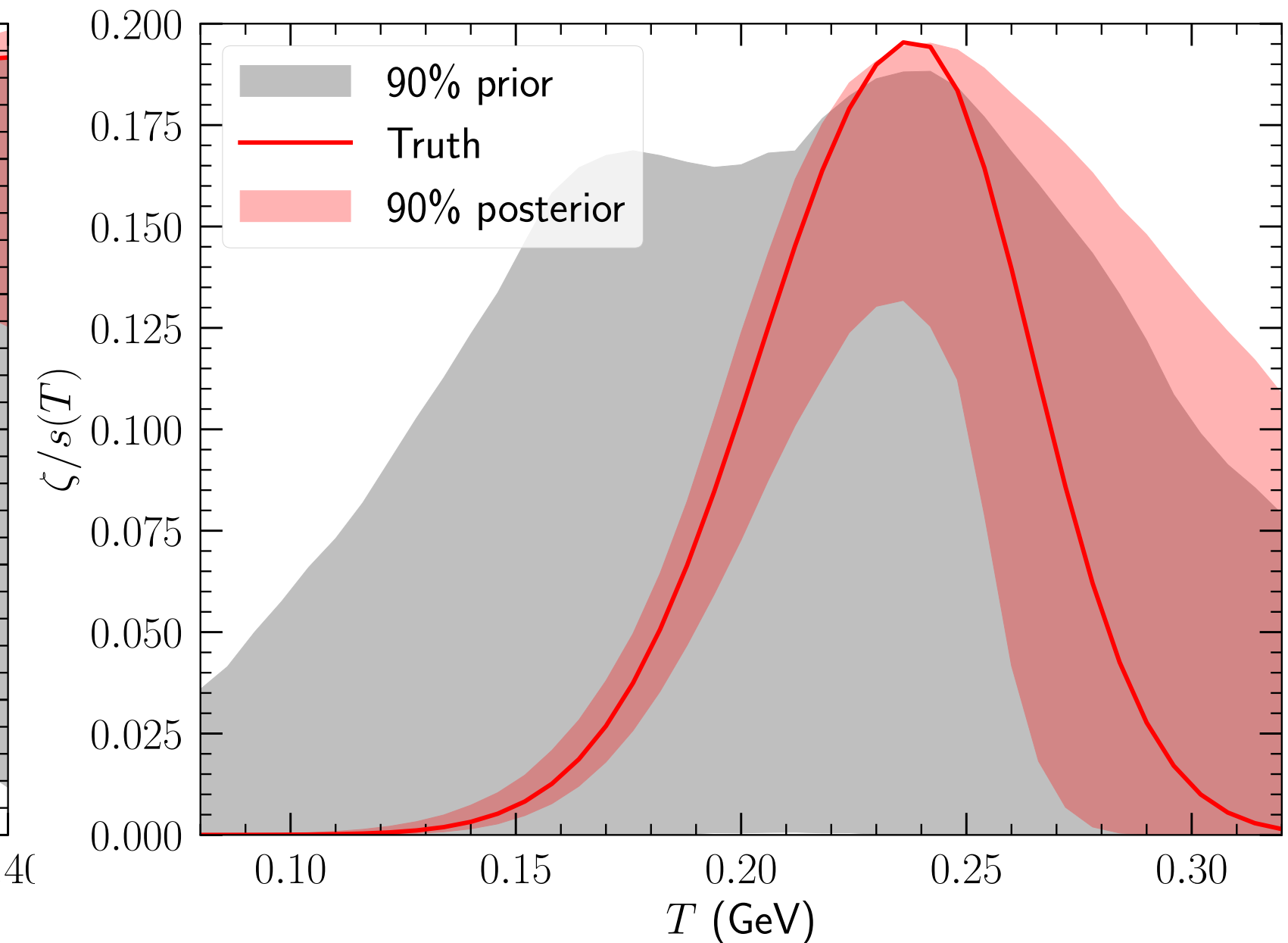
Initial-state stopping



$\eta/s(\mu_B)$



$\zeta/s(T)$

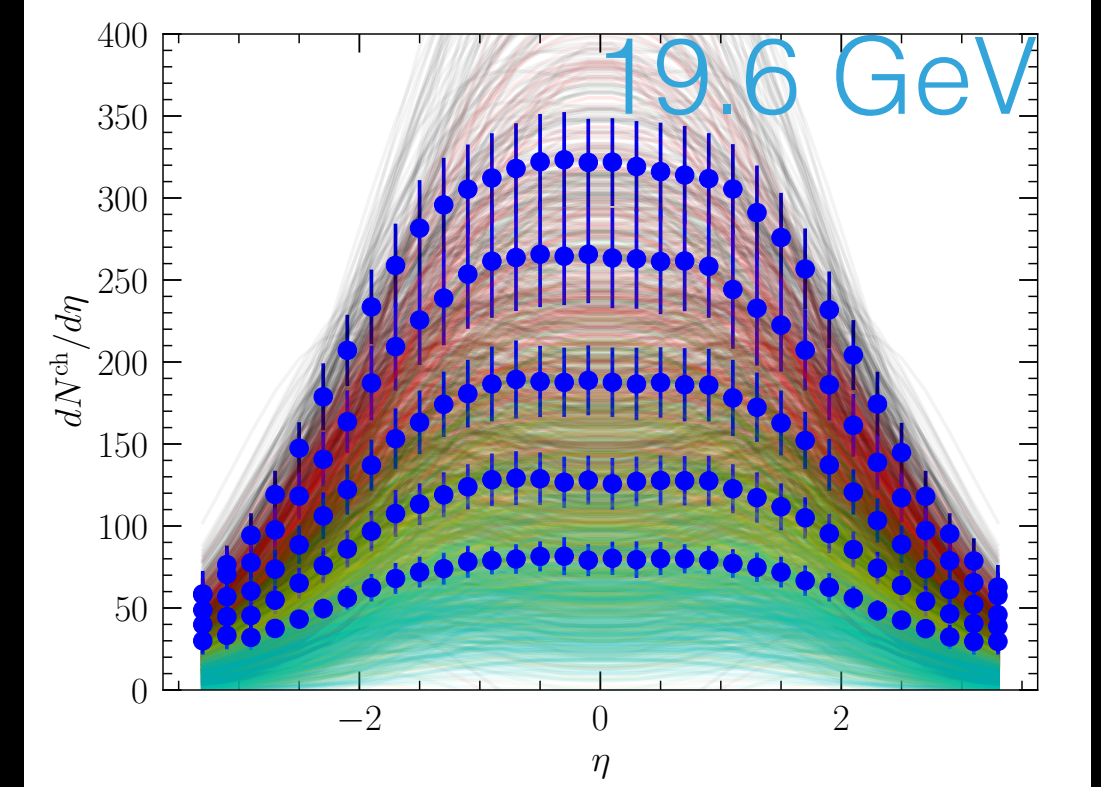
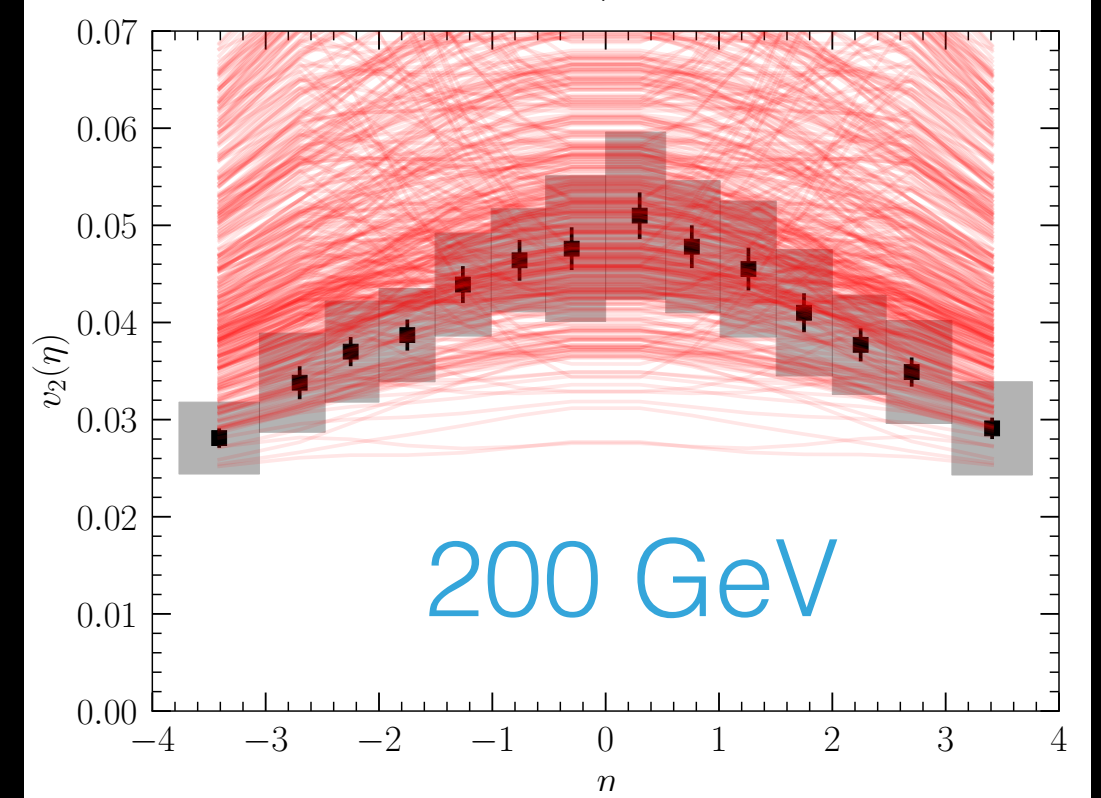
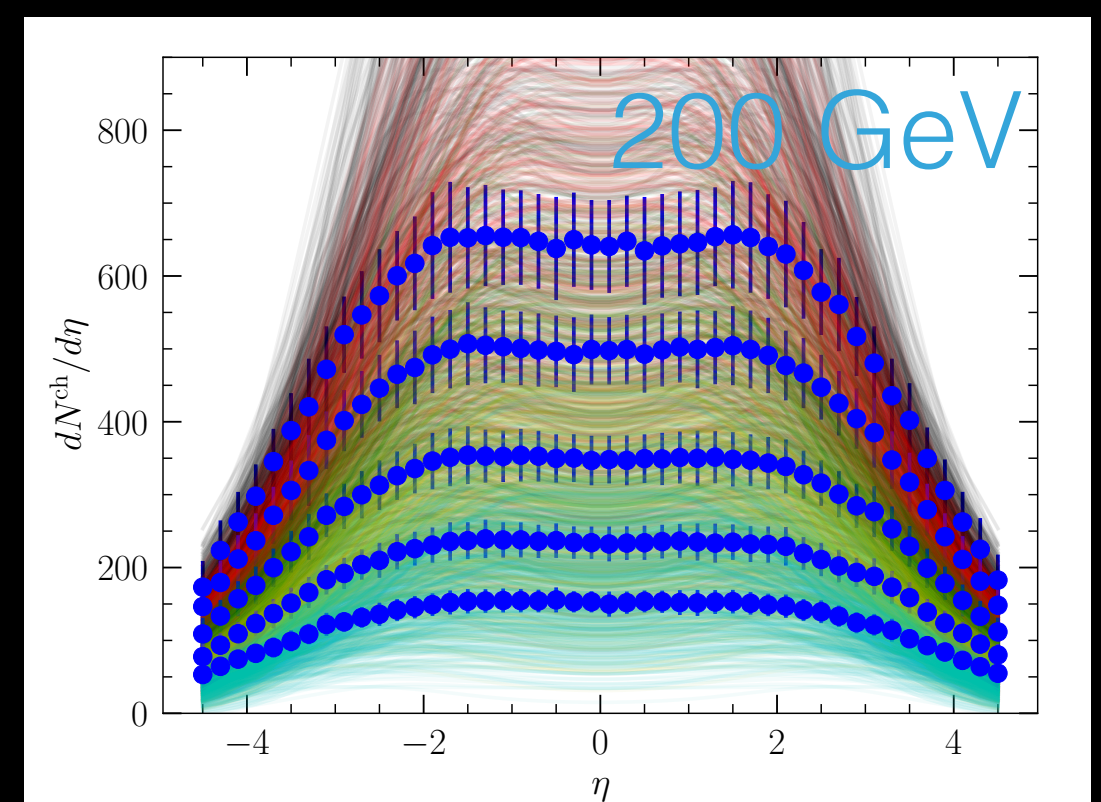
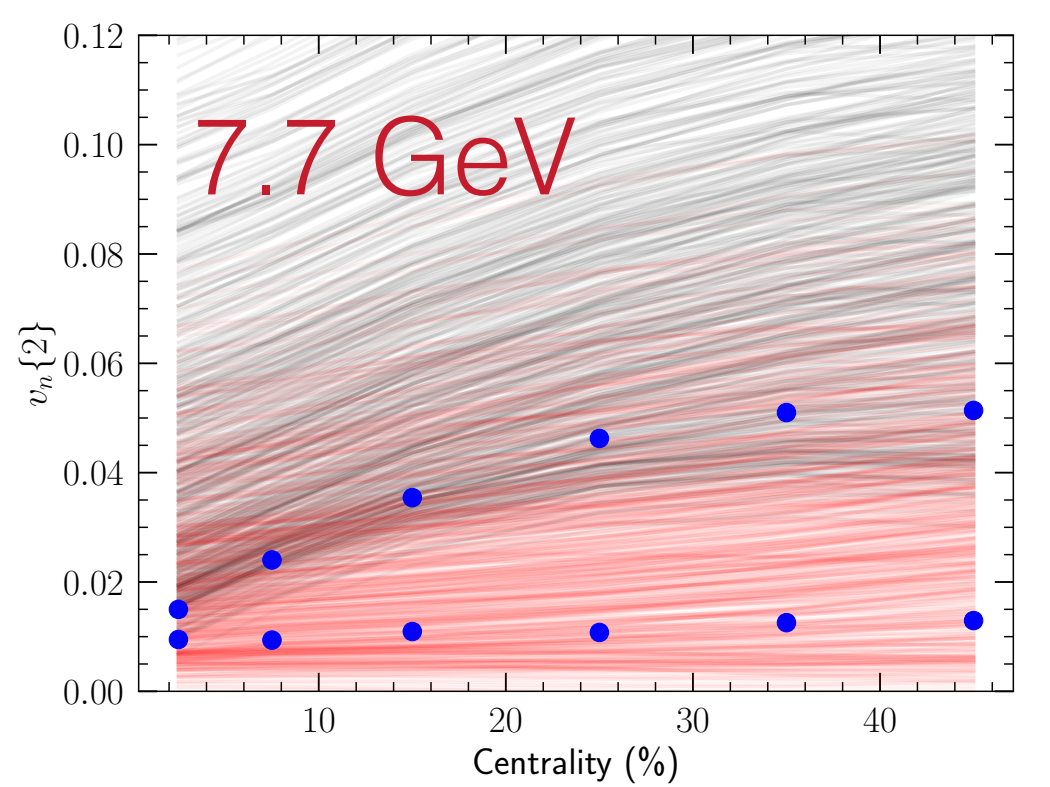
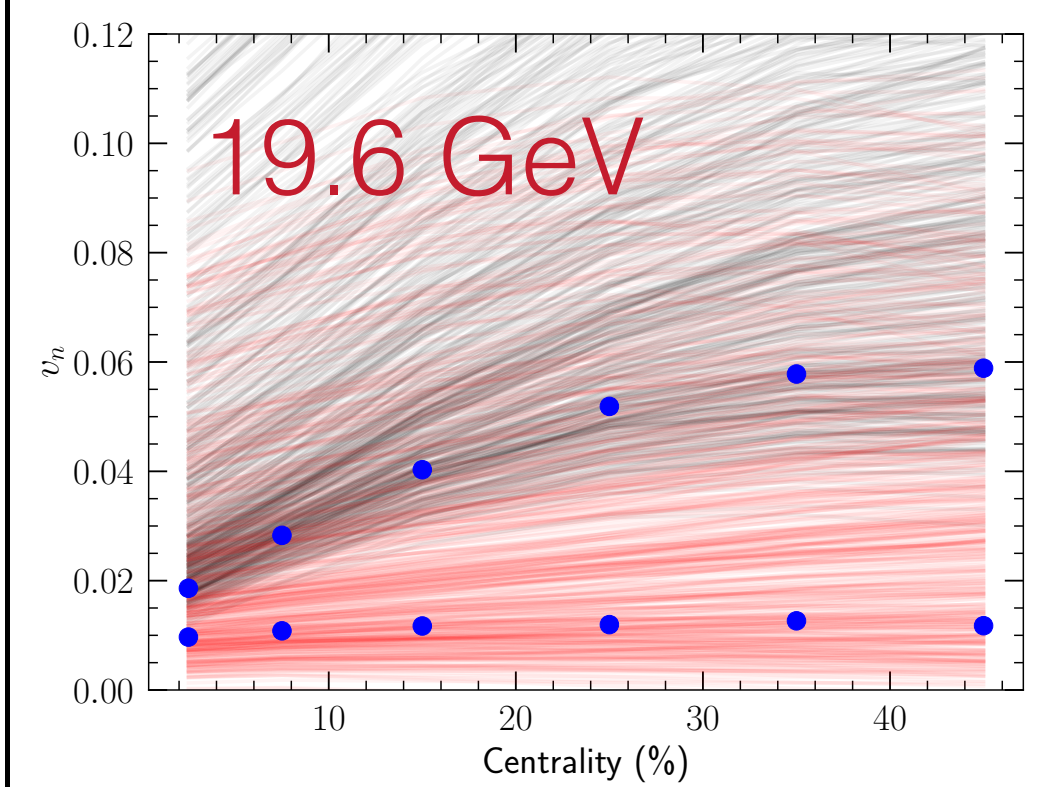
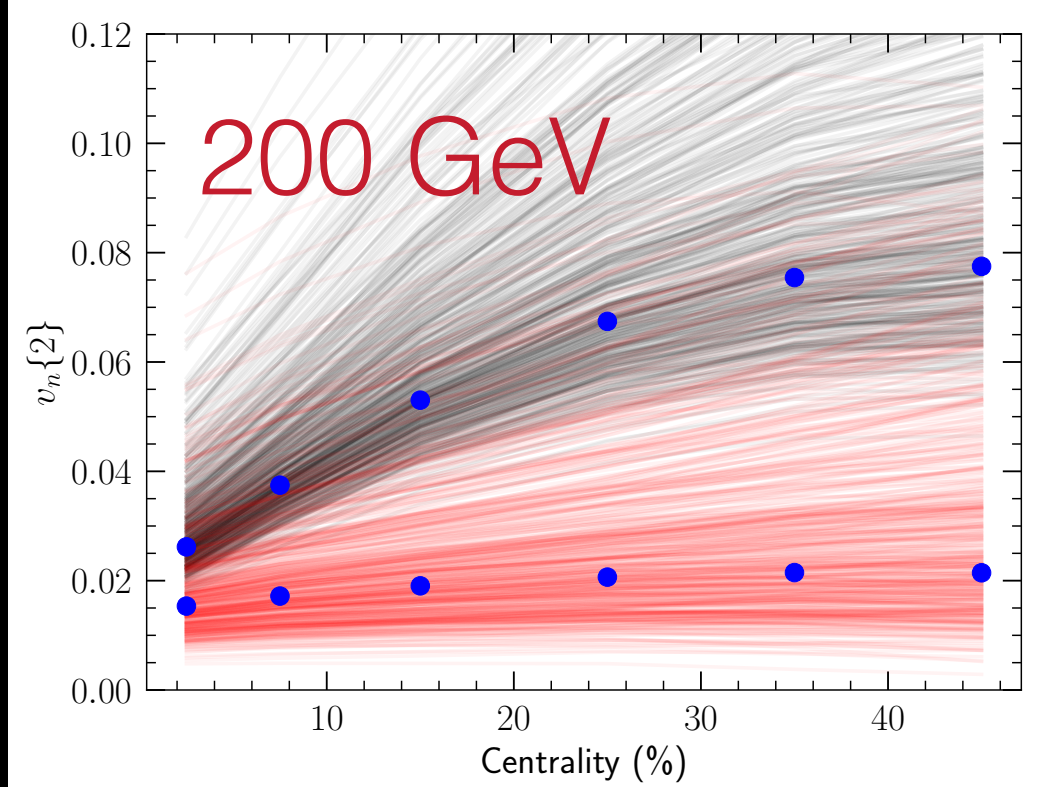
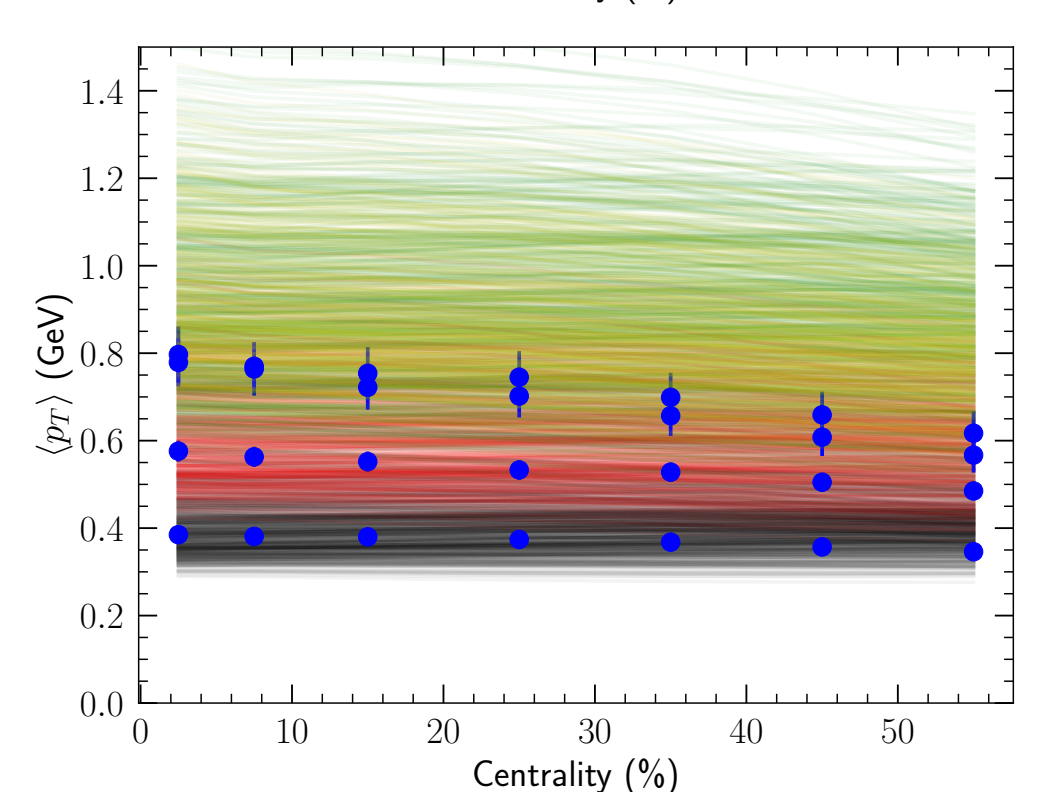
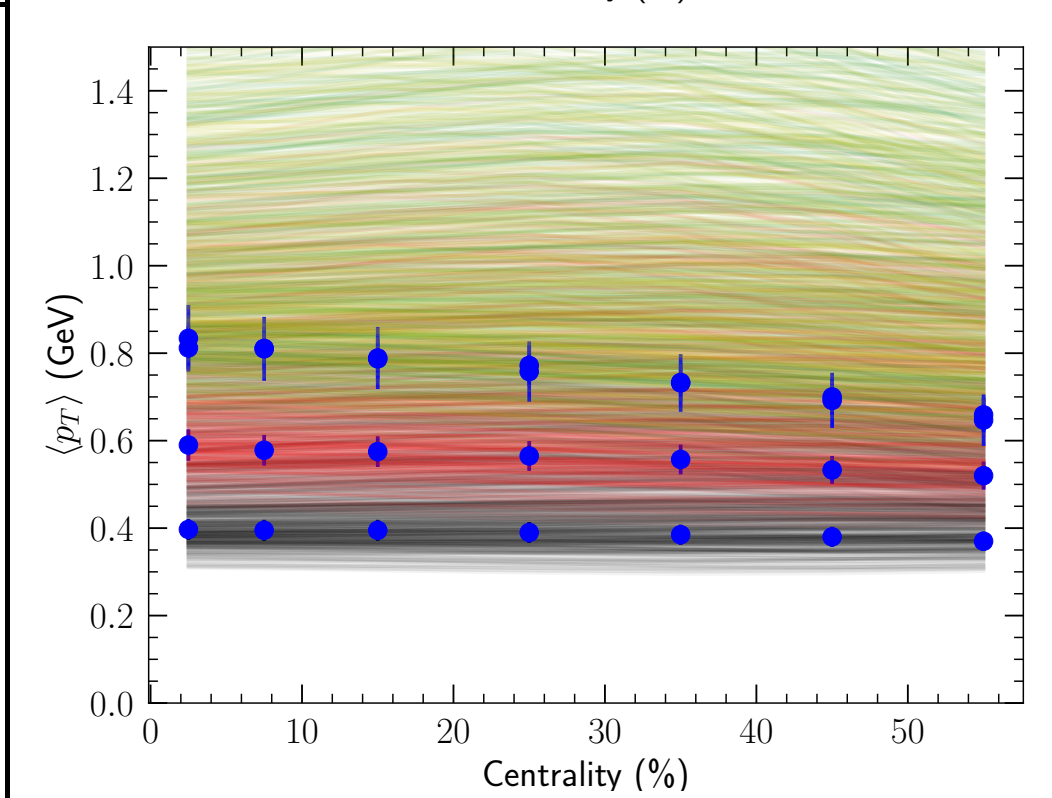
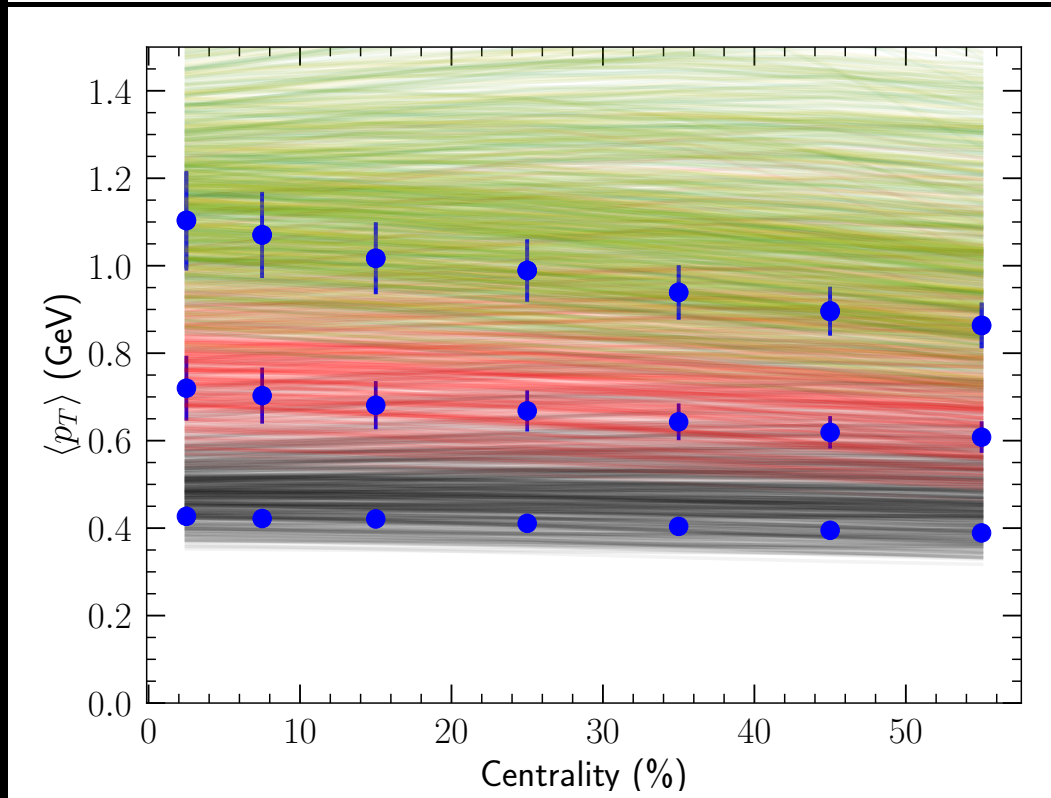
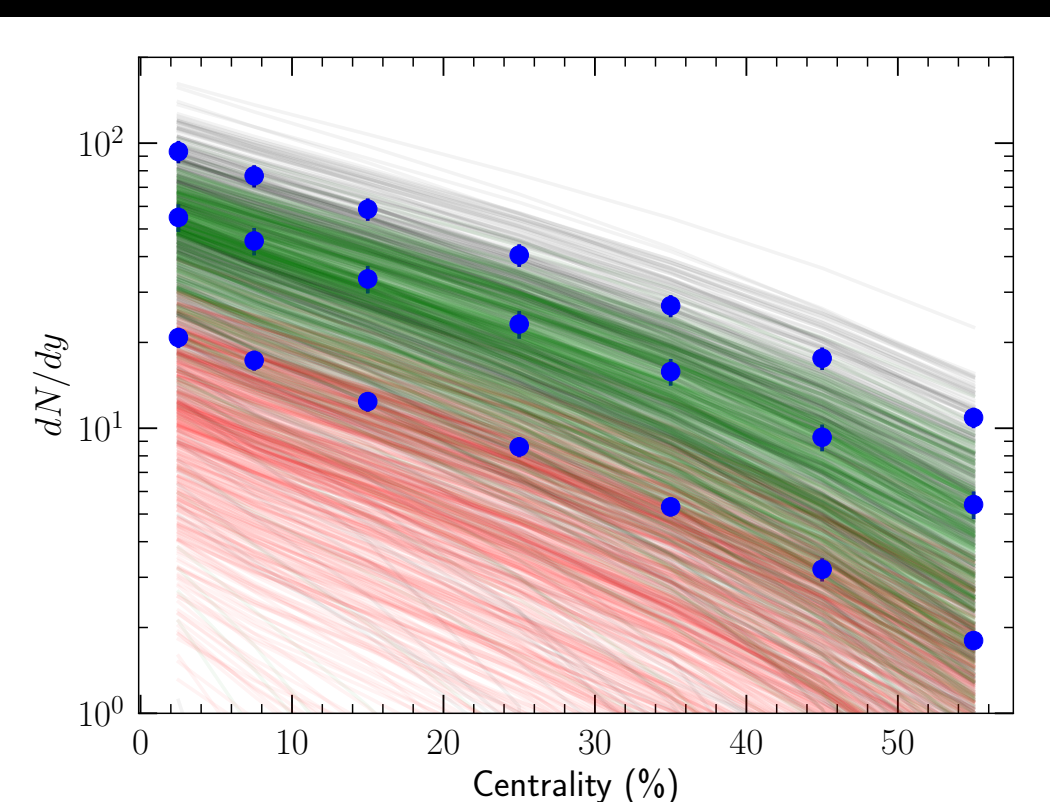
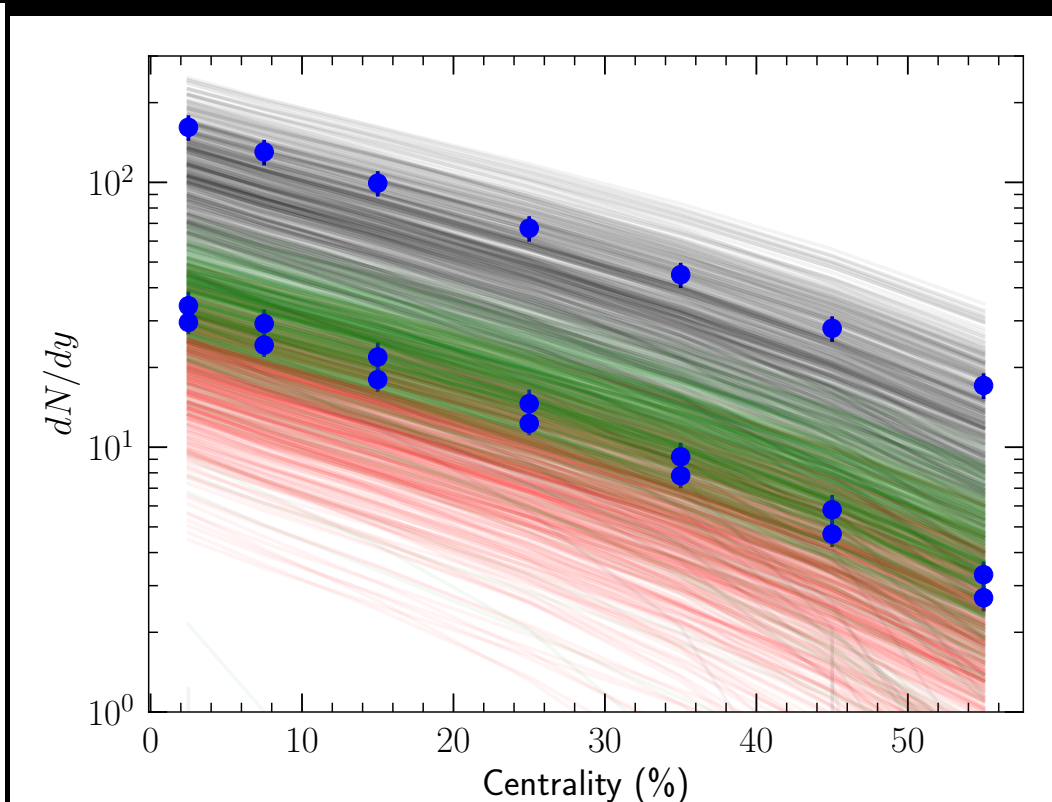
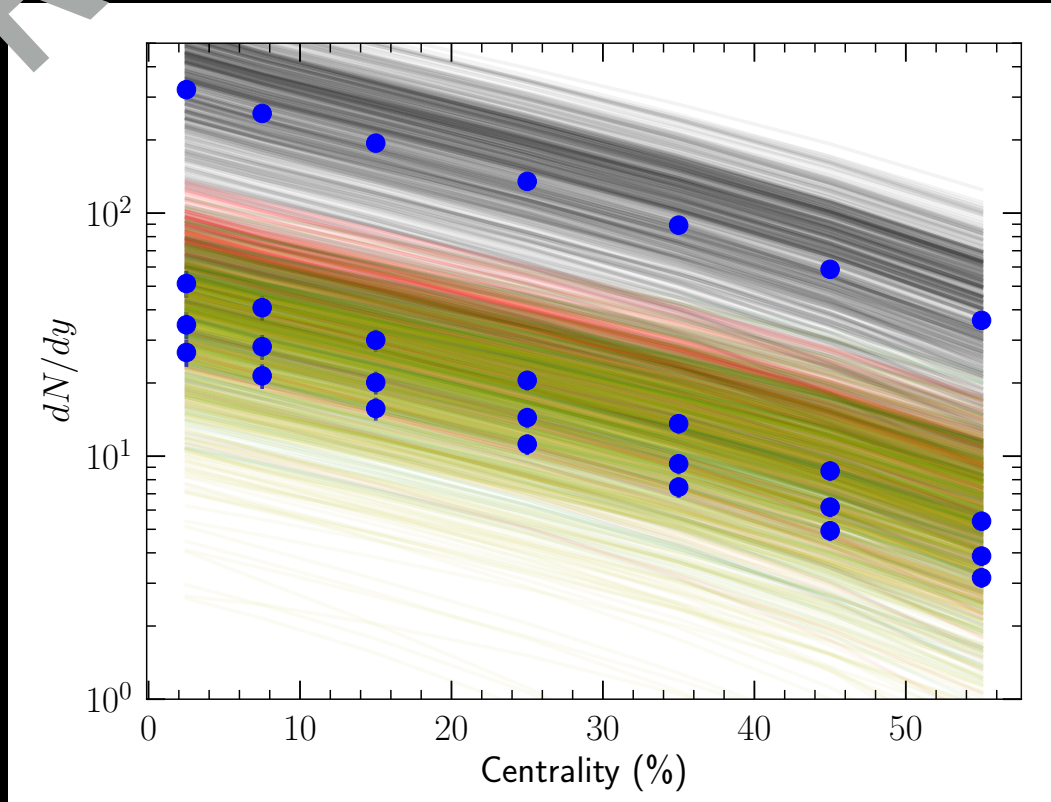


- Model emulation with Markov Chain Monte Carlo (MCMC) is verified with a closure test for initial-state stopping $y_{\text{loss}}(y_{\text{init}})$, $\eta/s(\mu_B)$, and $\zeta/s(T)$
- The selected observables can give strong constraints on the QGP properties at RHIC BES energies

BAYESIAN INFERENCE AT RHIC BES ENERGIES

PRIOR

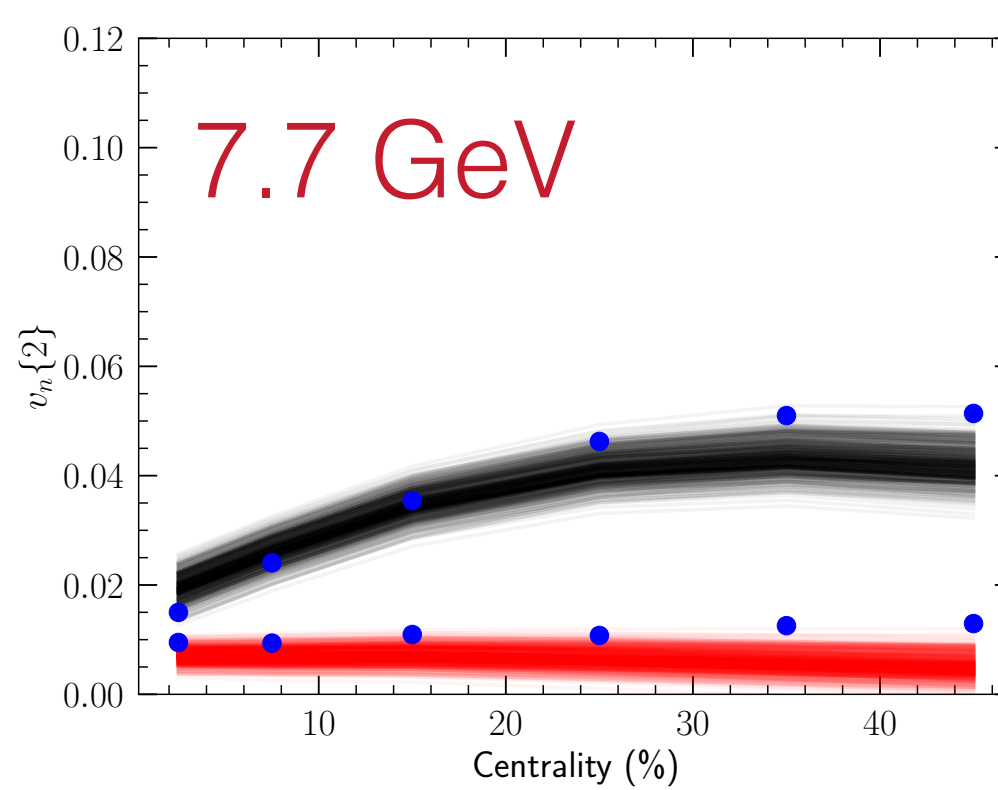
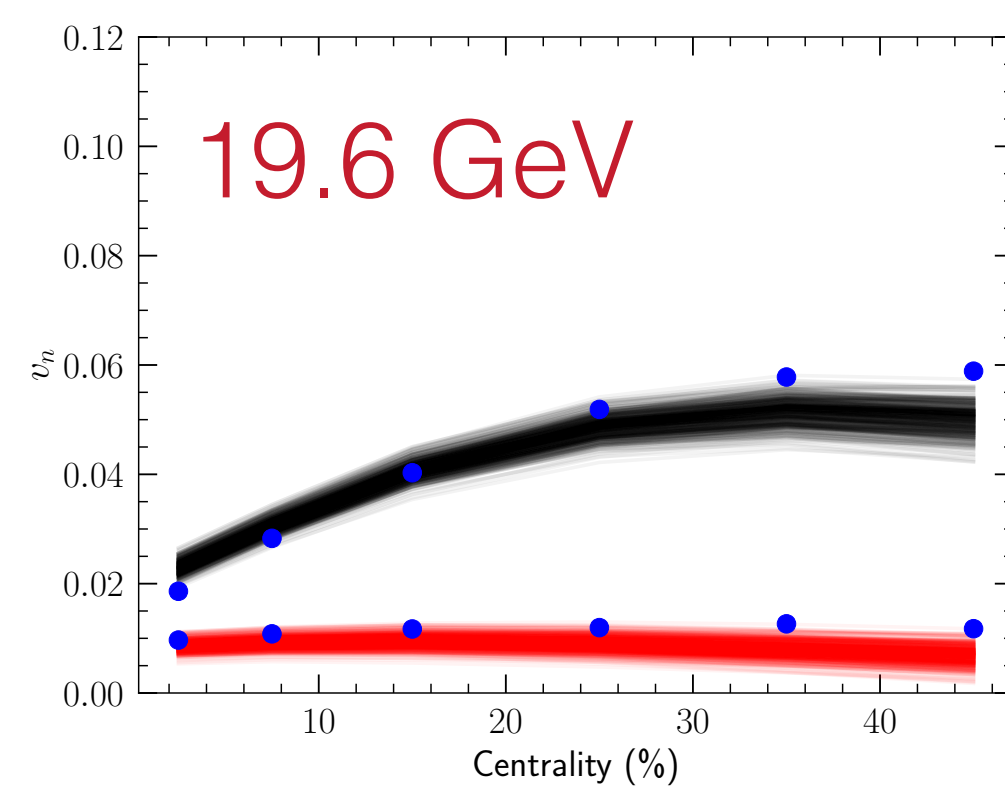
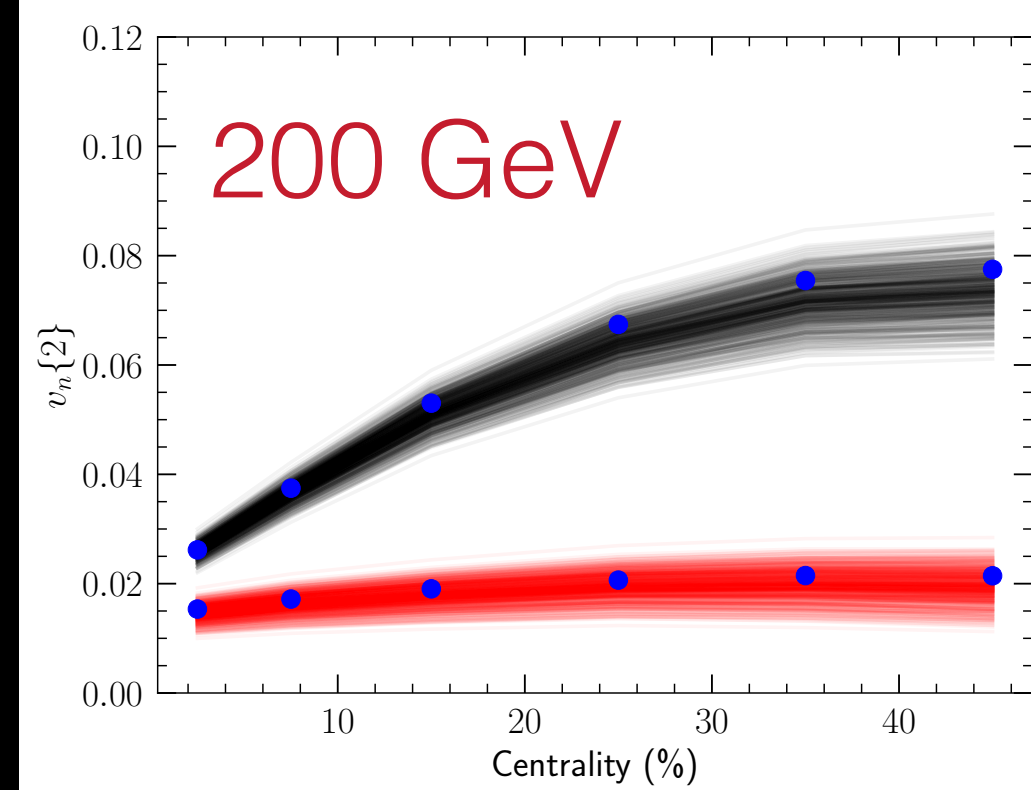
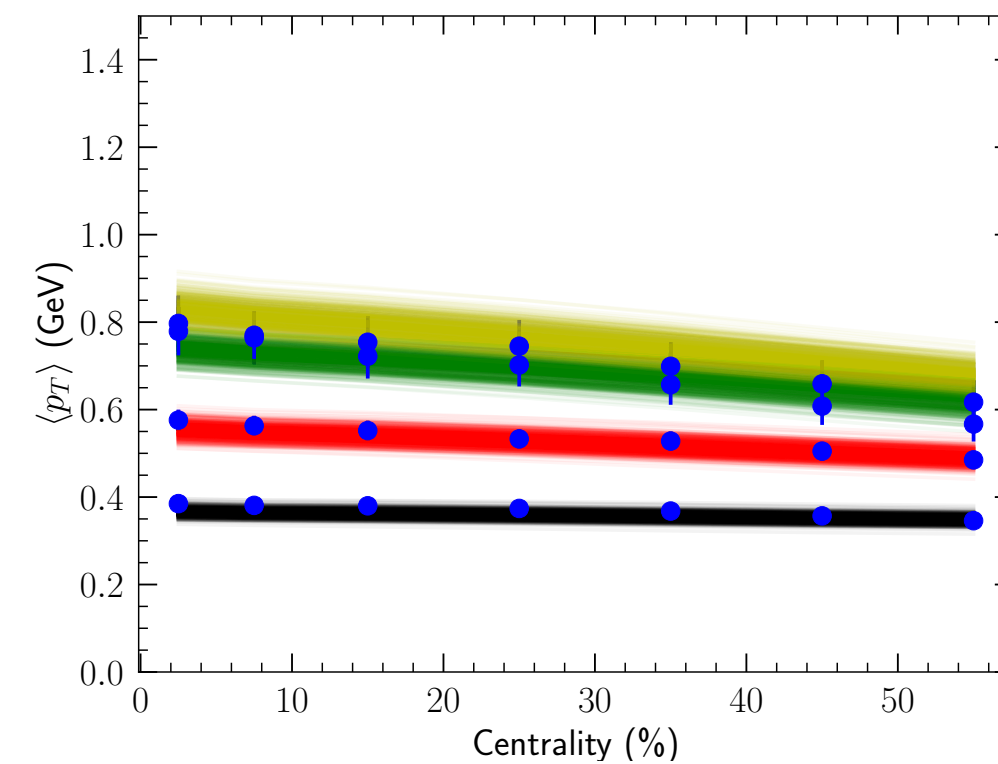
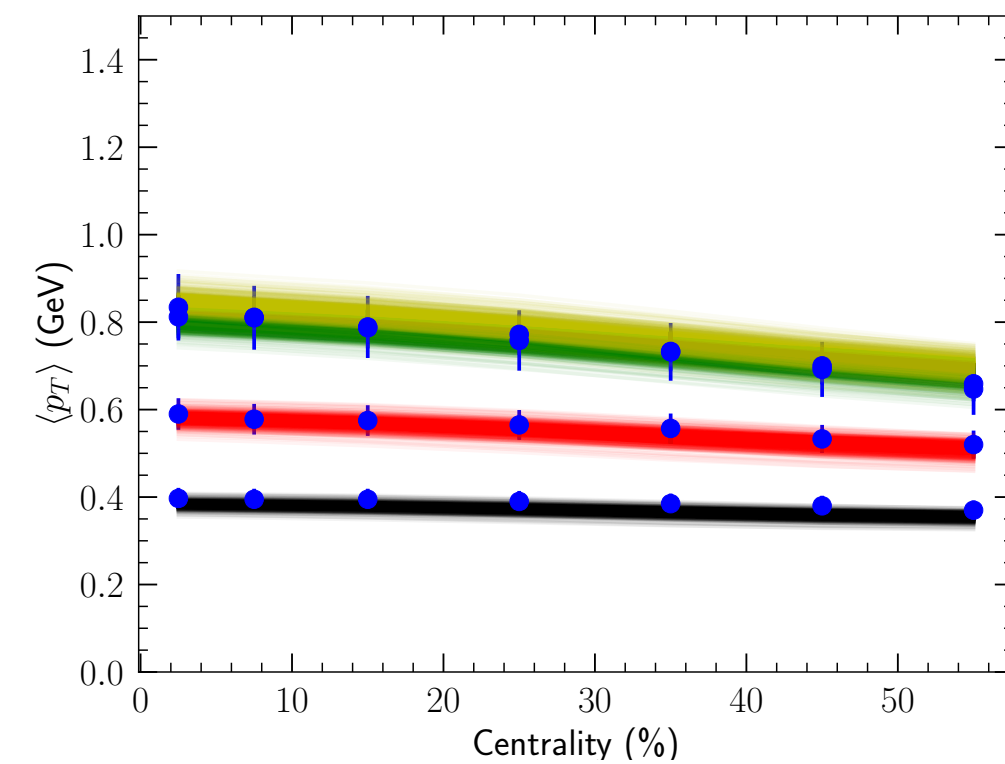
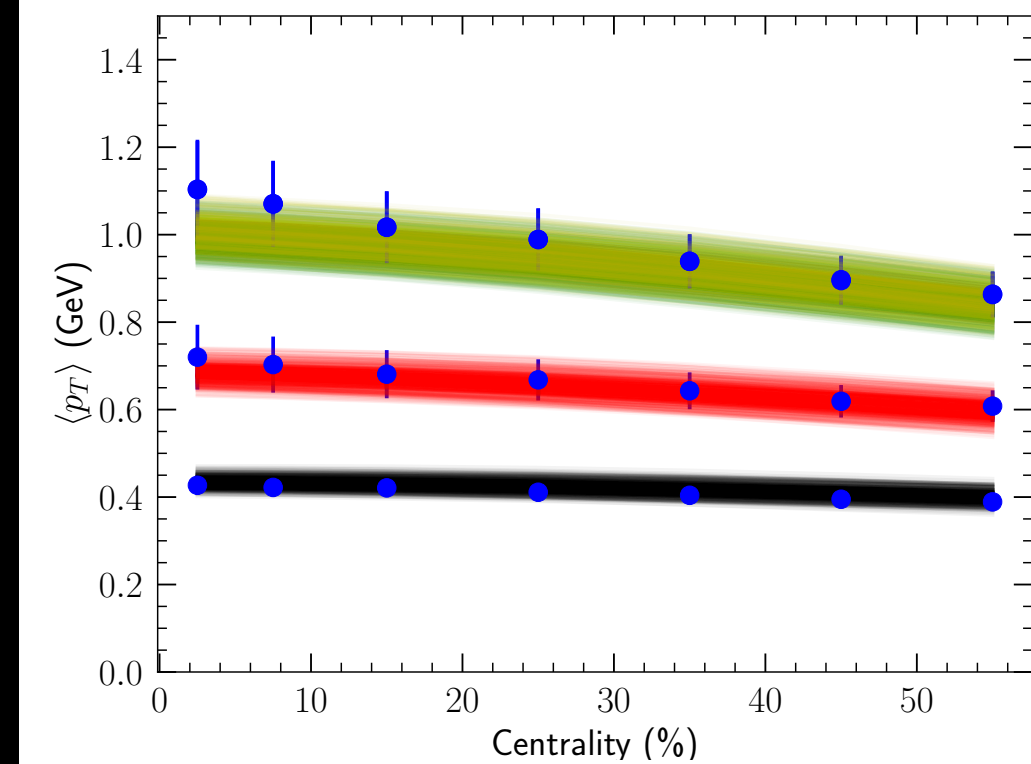
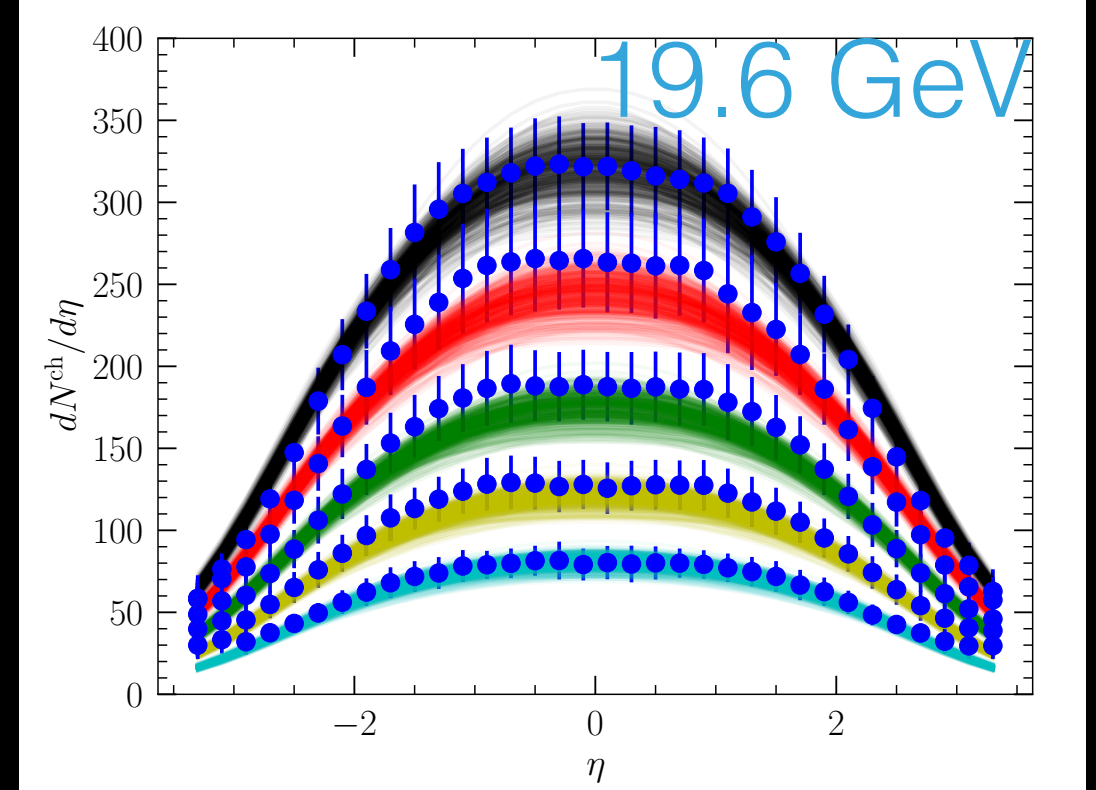
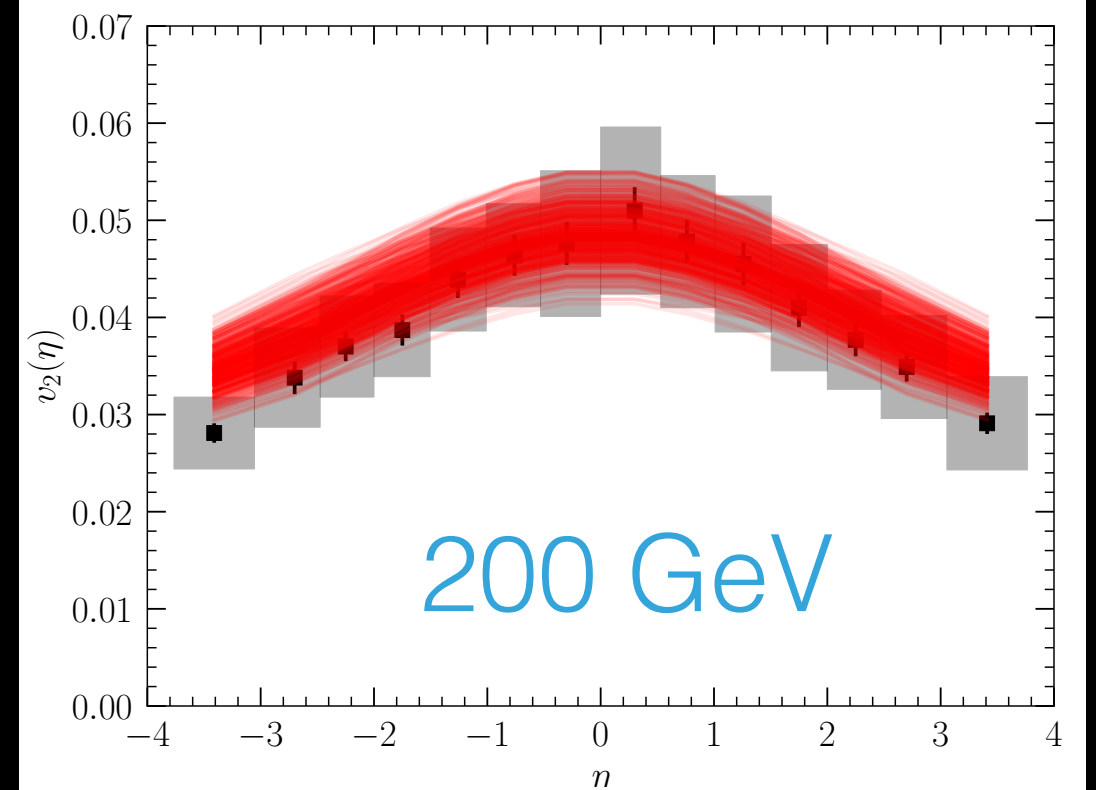
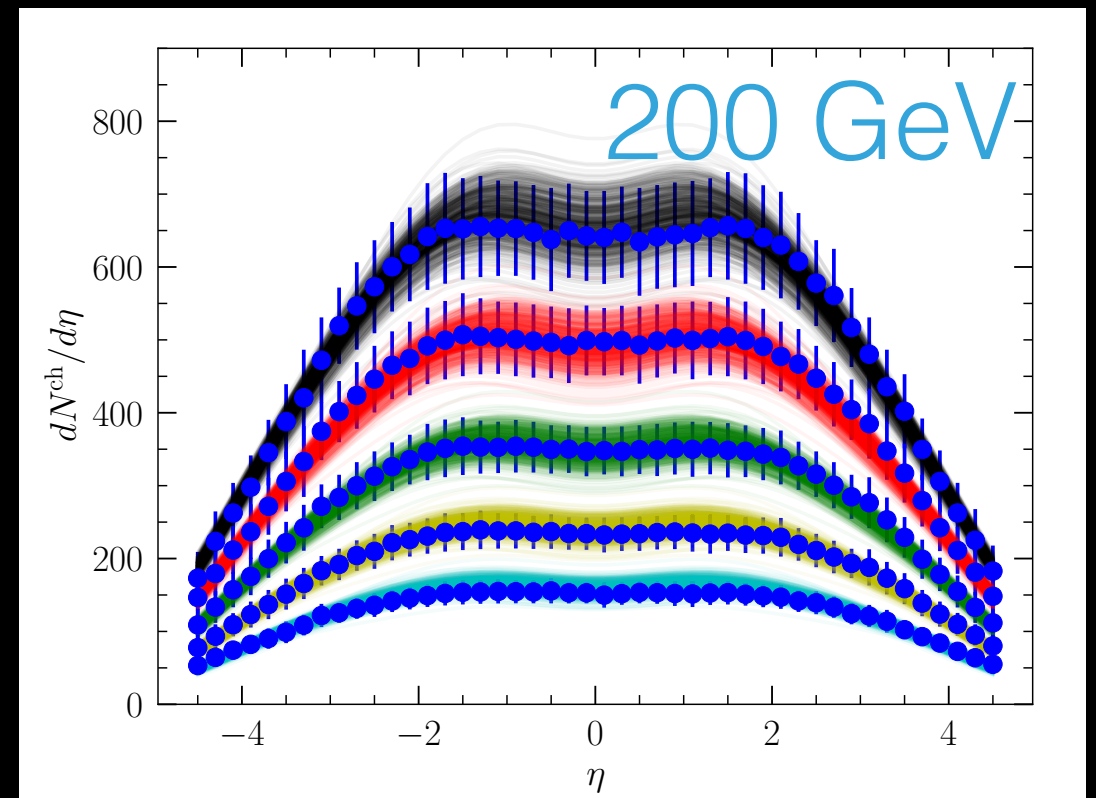
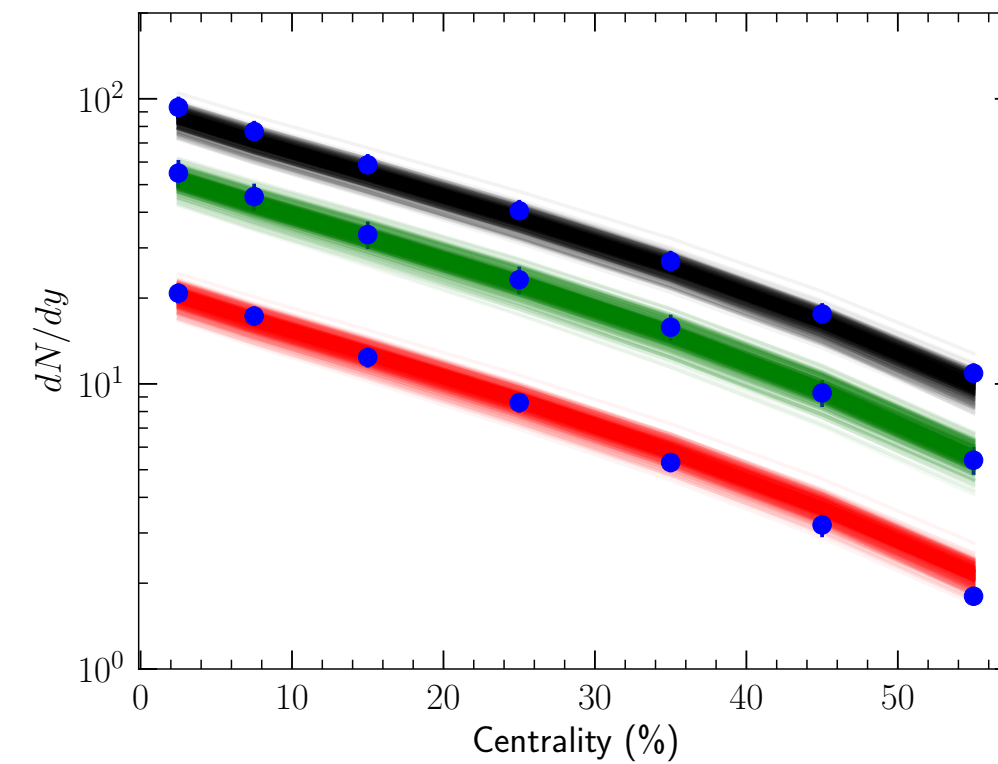
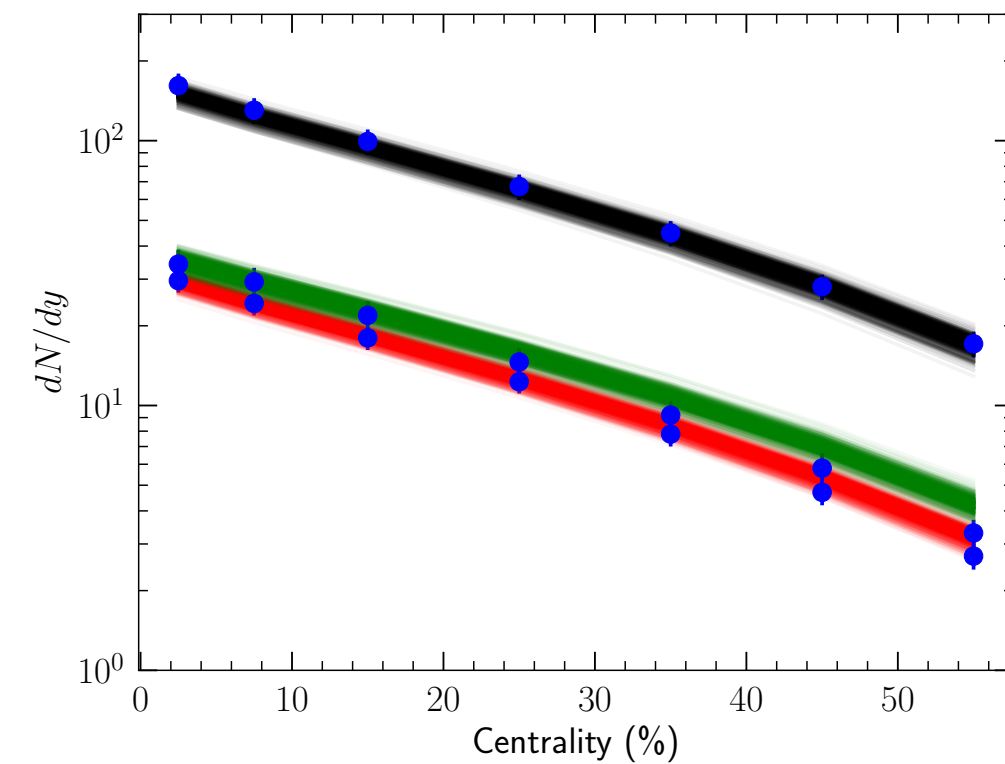
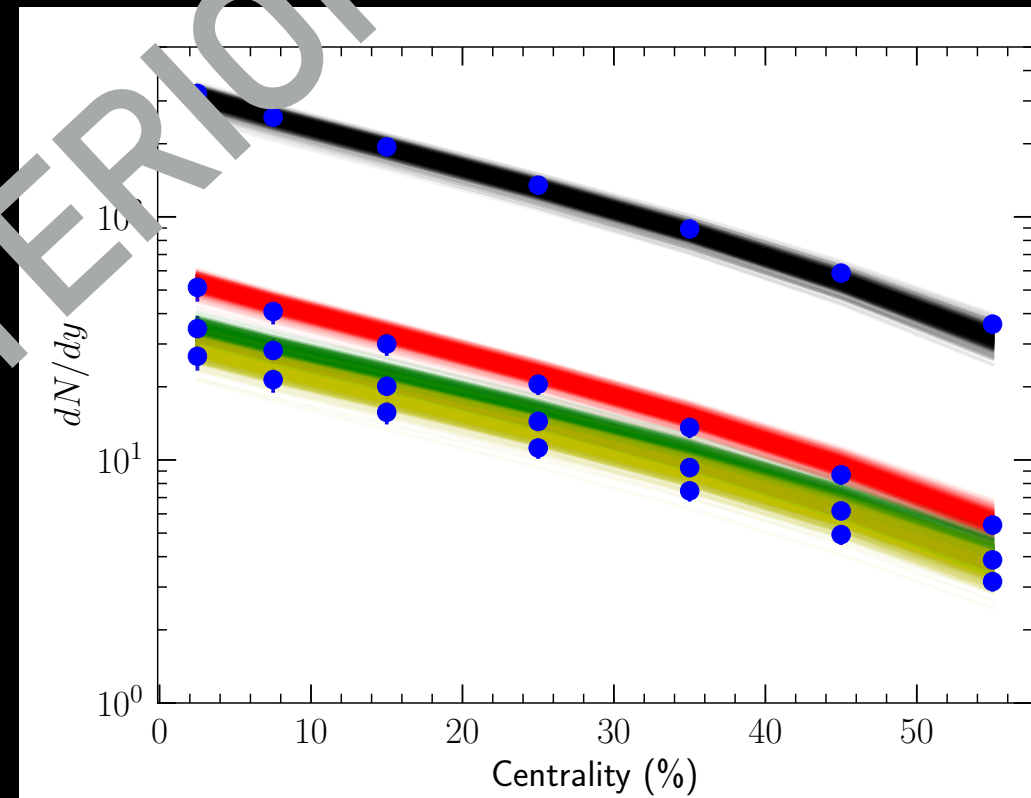
STAR



PHOBOS

BAYESIAN INFERENCE AT RHIC BES ENERGIES

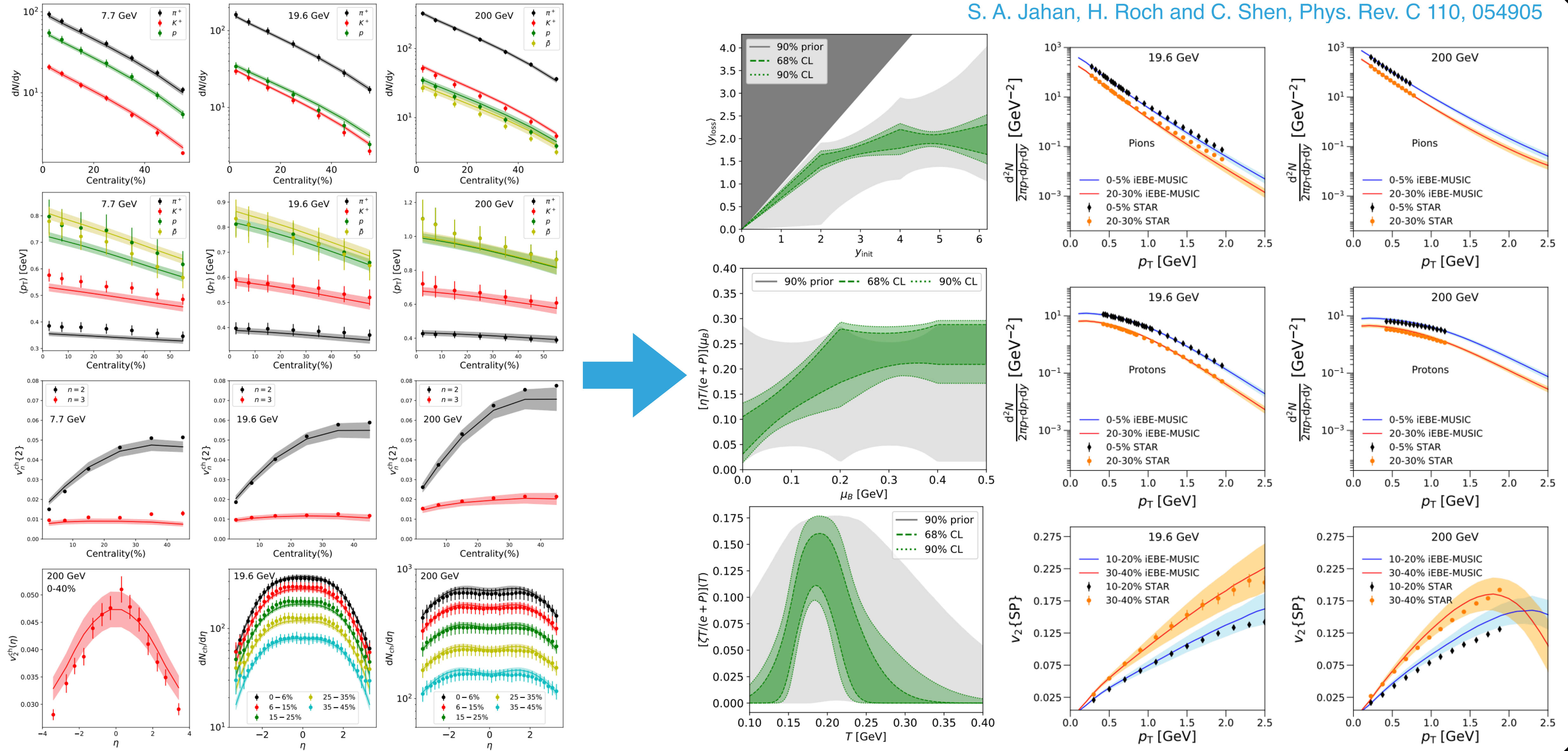
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PHOBOS

GLOBAL BAYESIAN CONSTRAINTS ON QGP PROPERTIES

S. A. Jahan, H. Roch and C. Shen, Phys. Rev. C 110, 054905



BAYESIAN MODEL SELECTION

Can we do better by introducing energy-dependence on model parameters?

$$\theta \rightarrow \theta(\sqrt{s})$$

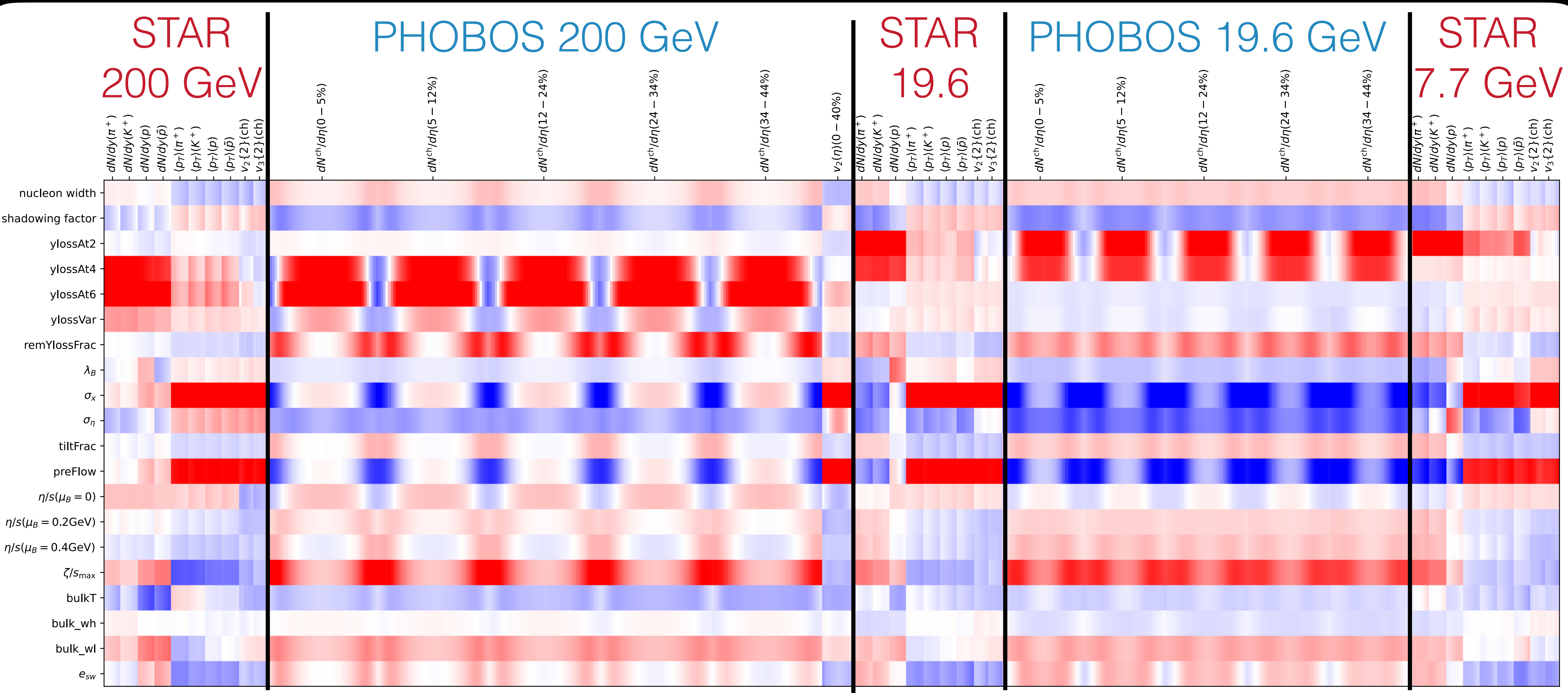
Select the optimal model using the Bayes factor $\mathcal{B}_{A/B} = \frac{P(y_{\text{exp}} | A)}{P(y_{\text{exp}} | B)}$

Model A	Model B	$\ln(\mathcal{B}_{A/B})$
no $\sqrt{s_{\text{NN}}}$	$\alpha_{\text{shadowing}}(\sqrt{s_{\text{NN}}})$	0.88 ± 0.09
no $\sqrt{s_{\text{NN}}}$	$\alpha_{\text{rem}}(\sqrt{s_{\text{NN}}})$	0.2 ± 0.3
no $\sqrt{s_{\text{NN}}}$	$\lambda_B(\sqrt{s_{\text{NN}}})$	0.2 ± 0.08
no $\sqrt{s_{\text{NN}}}$	$\sigma_x(\sqrt{s_{\text{NN}}})$	-2.00 ± 0.10
no $\sqrt{s_{\text{NN}}}$	$\sigma_\eta(\sqrt{s_{\text{NN}}})$	-1.6 ± 0.2
no $\sqrt{s_{\text{NN}}}$	$\alpha_{\text{preFlow}}(\sqrt{s_{\text{NN}}})$	-0.2 ± 0.2
no $\sqrt{s_{\text{NN}}}$	$e_{\text{sw}}(\sqrt{s_{\text{NN}}})$	0.57 ± 0.08
no $\sqrt{s_{\text{NN}}}$	$\sigma_x(\sqrt{s_{\text{NN}}}), \sigma_\eta(\sqrt{s_{\text{NN}}})$	-3.01 ± 0.08

Introducing $\sigma_x(\sqrt{s})$ and $\sigma_\eta(\sqrt{s})$ are favored, while the Bayes factor penalizes other irrelevant parameters

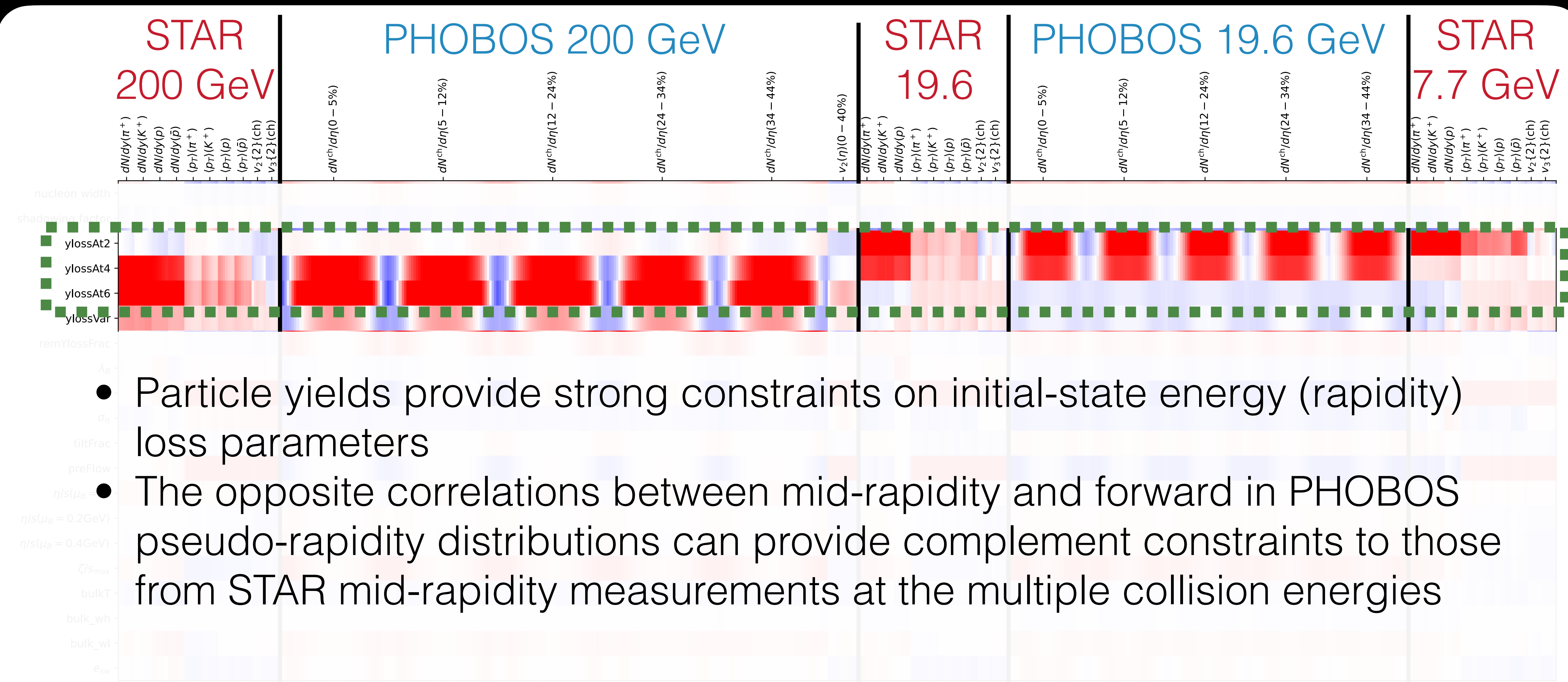
- $\ln(\mathcal{B}) > 0$: Model B disfavored
- $-5 < \ln(\mathcal{B}) < 0$: Model B moderately favored
- $\ln(\mathcal{B}) < -5$: Model B strongly favored

OBSERVABLE RESPONSES TO MODEL PARAMETERS



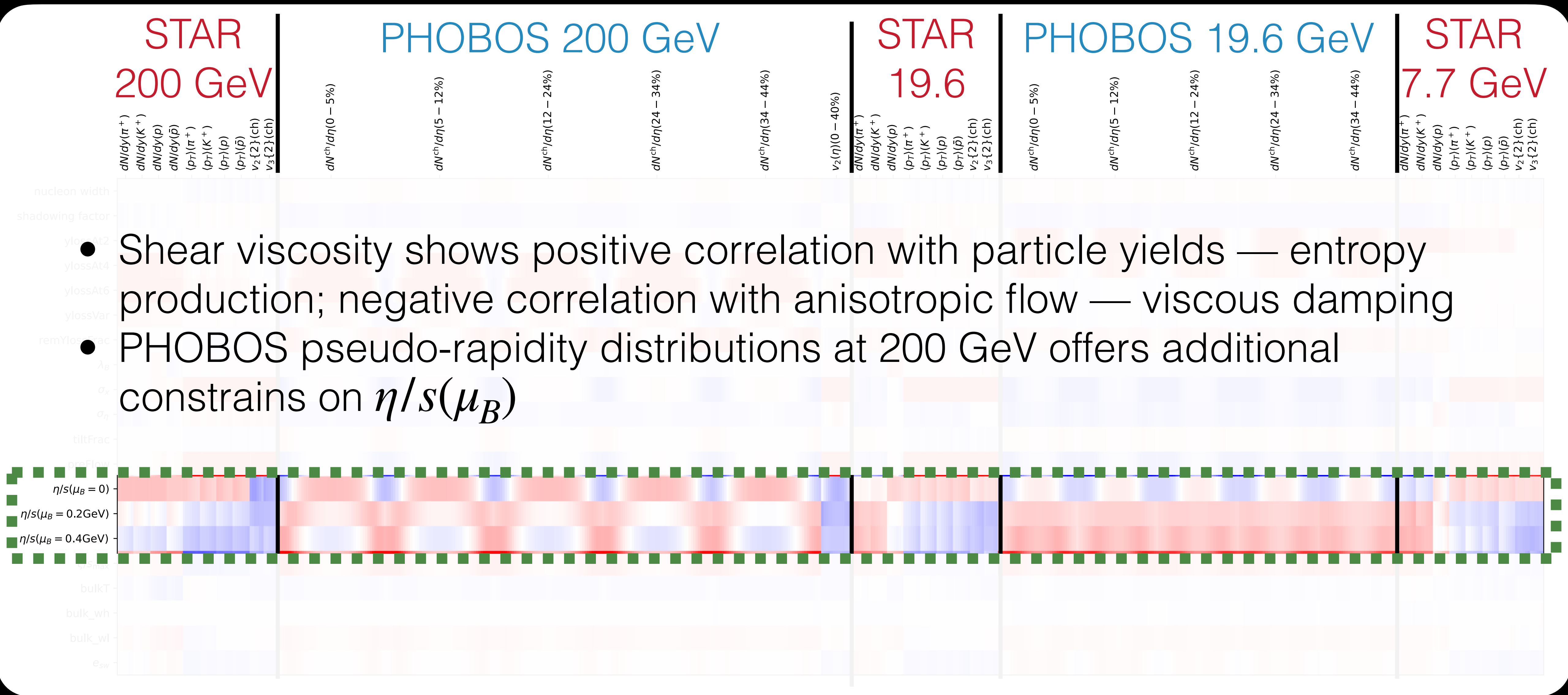
Red: Positive correlation; Blue: Negative correlation

OBSERVABLE RESPONSES TO MODEL PARAMETERS



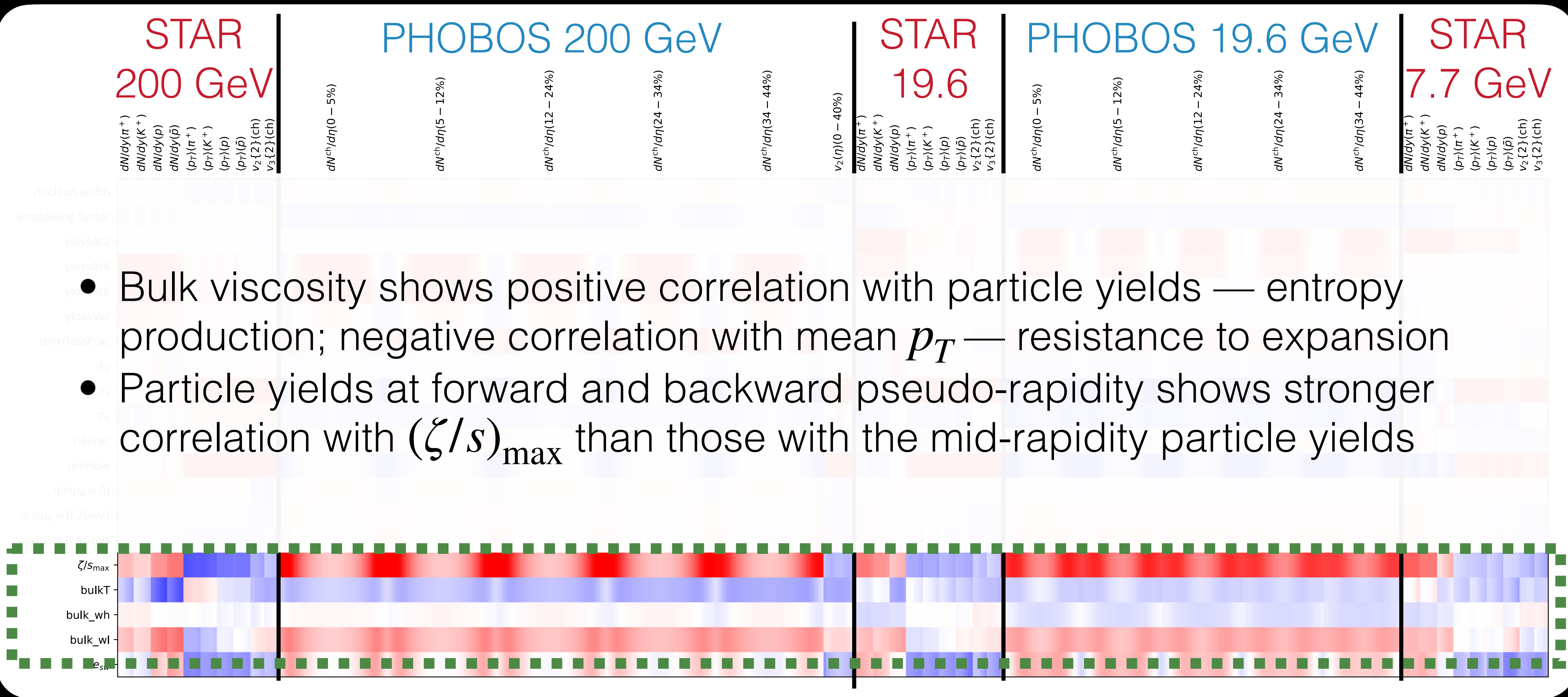
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OBSERVABLE RESPONSES TO MODEL PARAMETERS



Red: Positive correlation; Blue: Negative correlation

OBSERVABLE RESPONSES TO MODEL PARAMETERS



Red: Positive correlation; Blue: Negative correlation

MORE ON SENSITIVITY ANALYSES

E. Sangaline and S. Pratt, Phys. Rev. C 93, 024908 (2016)

$$\begin{aligned}\frac{\partial}{\partial y_a^{(\text{exp})}} \langle\langle f(\vec{x}) \rangle\rangle &= \frac{\partial}{\partial y_a^{(\text{exp})}} \frac{\int d\vec{x} f(\vec{x}) \mathcal{L}(\vec{x})}{\int d\vec{x} \mathcal{L}(\vec{x})} \\ &= \langle\langle f(\vec{x}) \frac{1}{\mathcal{L}} \frac{\partial}{\partial y_a^{(\text{exp})}} \mathcal{L} \rangle\rangle - \langle\langle f(\vec{x}) \rangle\rangle \langle\langle \frac{1}{\mathcal{L}} \frac{\partial}{\partial y_a^{(\text{exp})}} \mathcal{L} \rangle\rangle.\end{aligned}$$

$$\mathcal{L} \propto \exp[-\delta y_a \Sigma_{ab}^{-1} \delta y_b] \quad \delta y_a = y_a(x_i) - y_{\text{exp}}$$

$$\frac{\partial x_i}{\partial y_a} = \langle \delta x_i \delta y_b \rangle \langle \delta y_b \delta y_a \rangle^{-1}$$

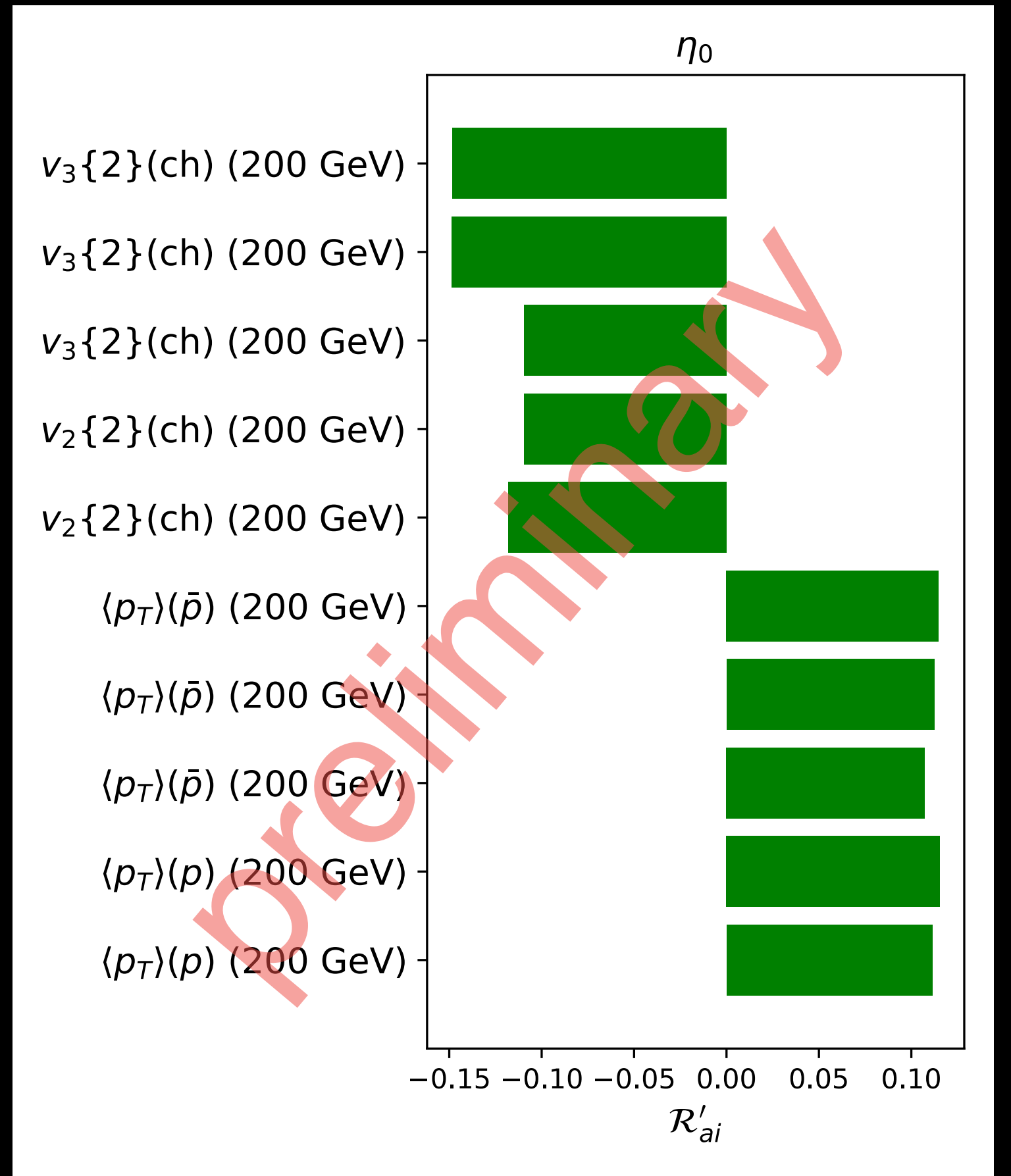
MORE ON SENSITIVITY ANALYSES

E. Sangaline and S. Pratt, Phys. Rev. C 93, 024908 (2016)

$$\begin{aligned} \frac{\partial}{\partial y_a^{(\text{exp})}} \langle \langle f(\vec{x}) \rangle \rangle &= \frac{\partial}{\partial y_a^{(\text{exp})}} \frac{\int d\vec{x} f(\vec{x}) \mathcal{L}(\vec{x})}{\int d\vec{x} \mathcal{L}(\vec{x})} \\ &= \langle \langle f(\vec{x}) \frac{1}{\mathcal{L}} \frac{\partial}{\partial y_a^{(\text{exp})}} \mathcal{L} \rangle \rangle - \langle \langle f(\vec{x}) \rangle \rangle \langle \langle \frac{1}{\mathcal{L}} \frac{\partial}{\partial y_a^{(\text{exp})}} \mathcal{L} \rangle \rangle. \end{aligned}$$

$$\mathcal{L} \propto \exp[-\delta y_a \Sigma_{ab}^{-1} \delta y_b] \quad \delta y_a = y_a(x_i) - y_{\text{exp}}$$

$$\frac{\partial x_i}{\partial y_a} = \langle \delta x_i \delta y_b \rangle \langle \delta y_b \delta y_a \rangle^{-1}$$



MOVING TOWARDS NEXT-GENERATION BAYESIAN INFERENCE

TRANSFER LEARNING

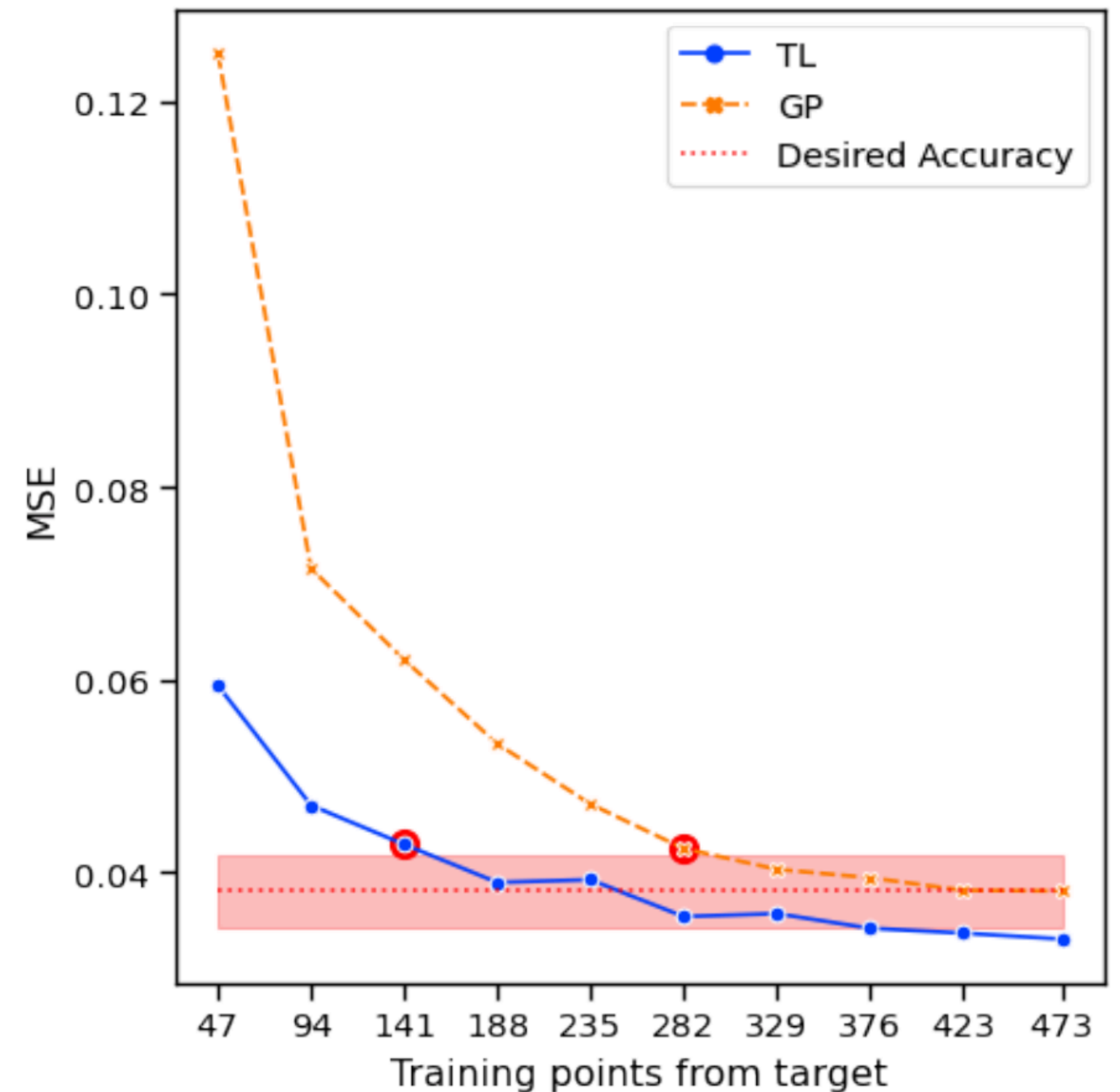
D. Liyanage, Y. Ji, D. Everett, M. Heffernan, U. Heinz, S. Mak and J. F. Paquet,
Phys. Rev. C105, 034910 (2022)

For comparable model variations,
transfer learning offers a way to save
the training cost for the target model,

$$f_T(\theta) = \rho f_S(\theta) + \delta(\theta)$$

Learning only the difference from a
source model can reduce the training
cost significantly

The current framework is limited to the
same model parameter space between
the target and source models



THEORETICAL UNCERTAINTY

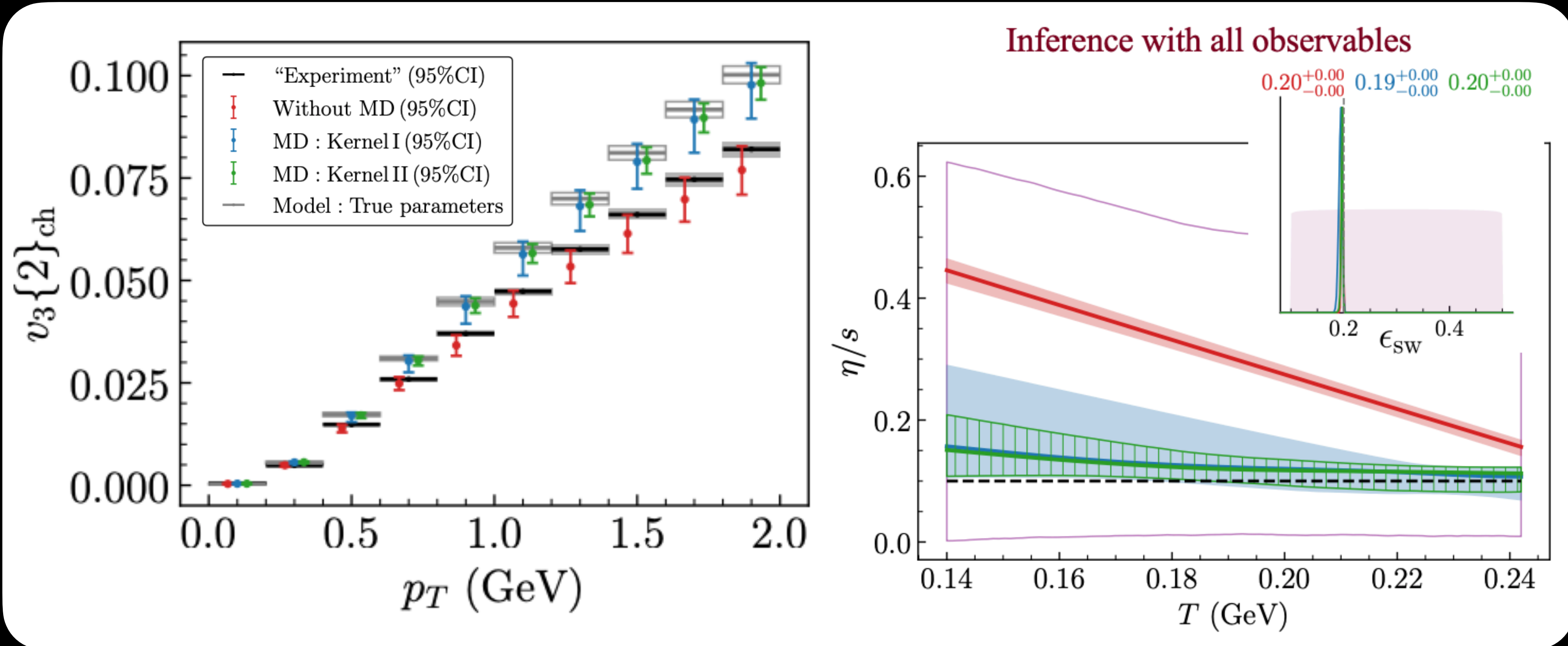
S. Jaiswal, C. Shen, R. J. Furnstahl, U. Heinz and M. T. Pratola, arXiv:2504.13144 [hep-ph]

Theoretical models are not perfect. Modeling its error $\delta(p_T)$ with Gaussian Process in a data-driven approach,

$$y(p_T) = \eta(p_T, \theta) + \delta(p_T) + \epsilon$$

Validate this framework with mock data

Enabled robust inference with observables with different theoretical fidelities



ITERATIVE BAYESIAN INFERENCE

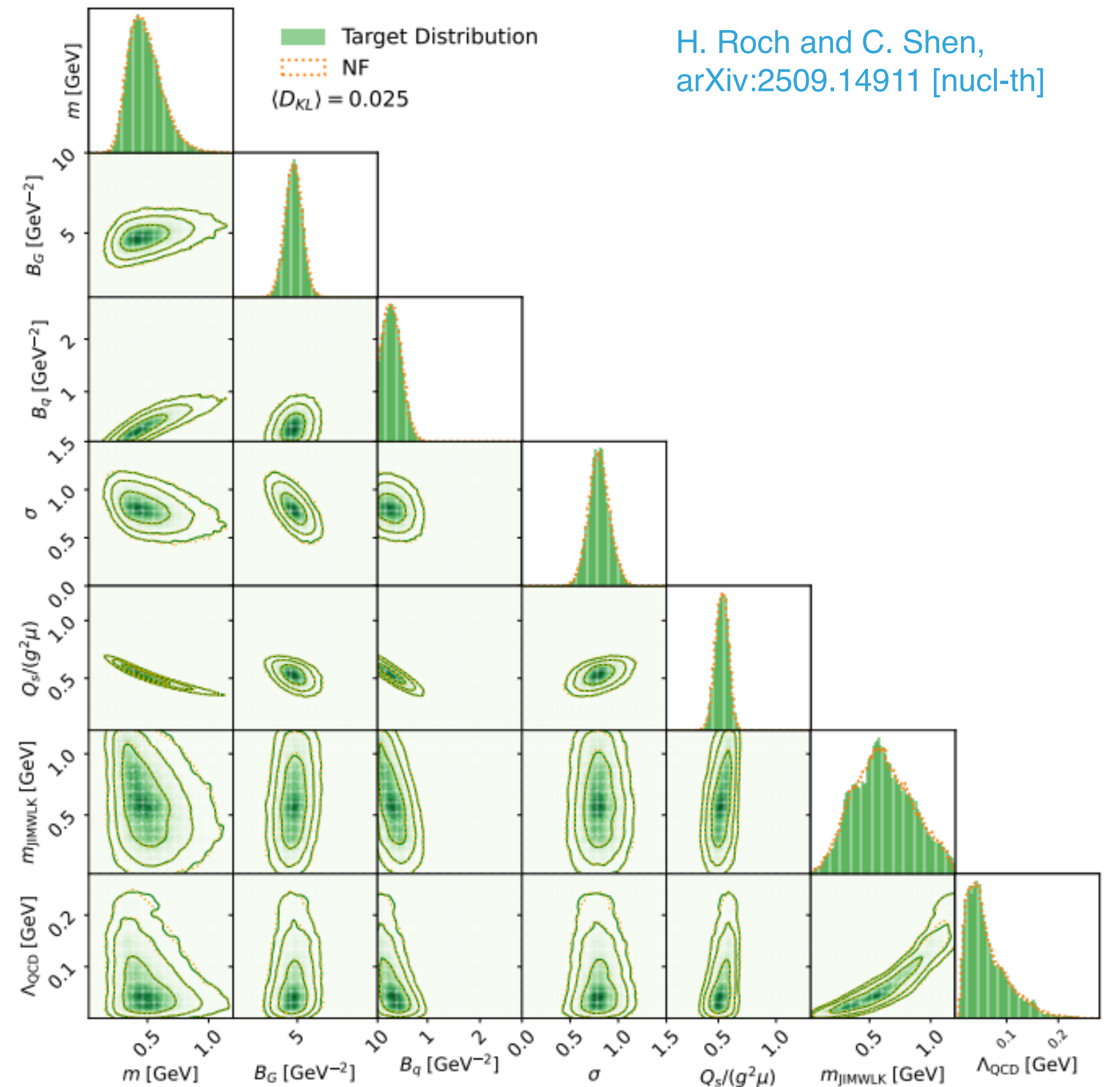
Today's posterior is tomorrow's prior

$$\begin{aligned}
 P(\theta | D_1, D_2) &= \frac{P(D_1, D_2 | \theta) P(\theta)}{P(D_1, D_2)} \\
 &= \frac{P(D_2 | \theta) P(D_1 | \theta) P(\theta)}{P(D_2) P(D_1)} \\
 &= \frac{P(D_2 | \theta) P(\theta | D_1)}{P(D_2)}
 \end{aligned}$$

The distribution $P(\theta | D_1)$ is highly correlated and non-trivial to sample

A normalizing-flow generative model learns $P(\theta | D_1)$

Y. Yamauchi, L. Buskirk, P. Giuliani and K. Godbey, arXiv:2310.04635 [nucl-th]

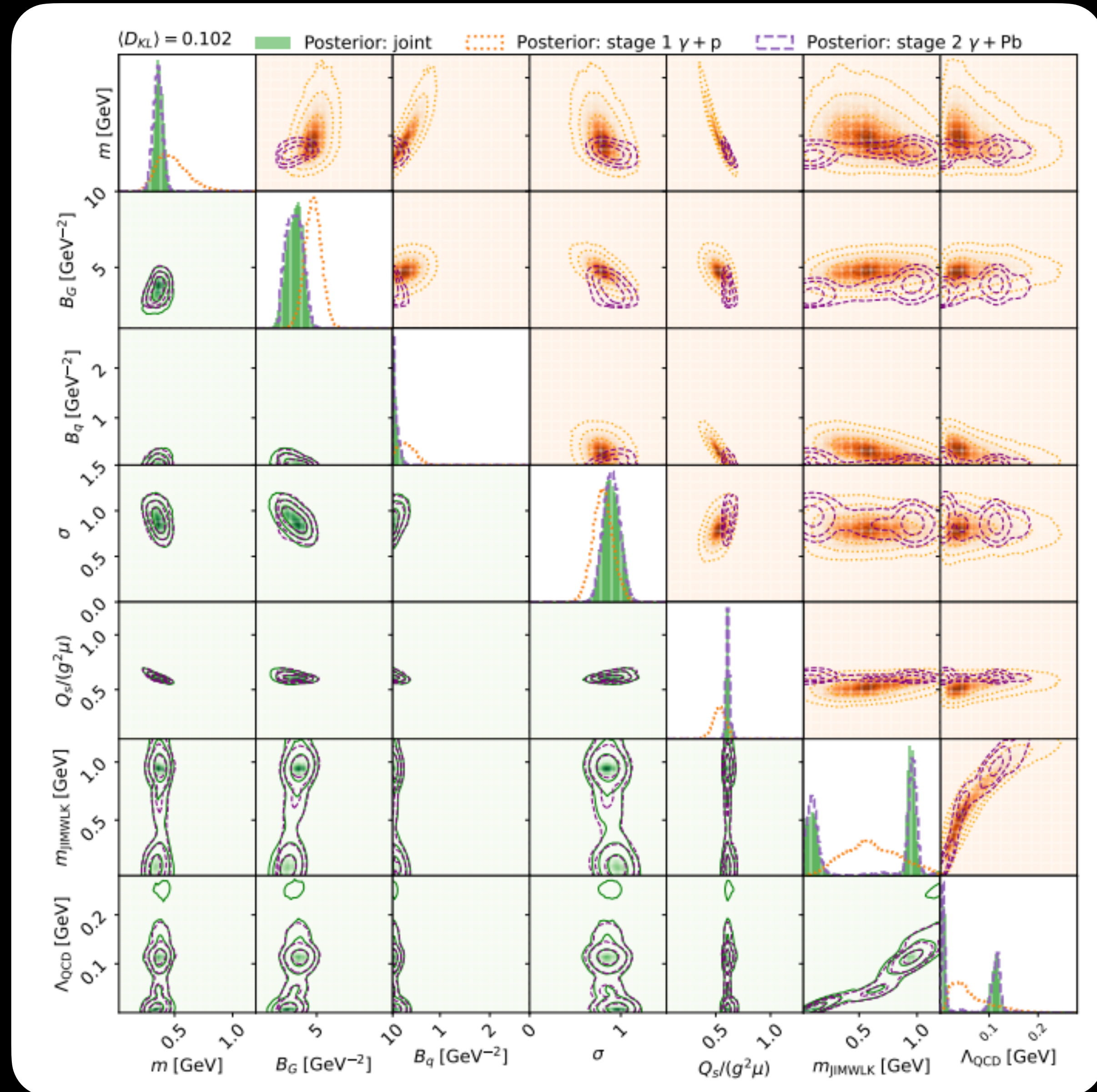


ITERATIVE BAYESIAN INFERENCE

Today's posterior is tomorrow's prior

$$\begin{aligned}
 P(\theta | D_1, D_2) &= \frac{P(D_1, D_2 | \theta) P(\theta)}{P(D_1, D_2)} \\
 &= \frac{P(D_2 | \theta) \color{red}{P(D_1 | \theta)} P(\theta)}{P(D_2) \color{red}{P(D_1)}} \\
 &= \frac{P(D_2 | \theta) \color{red}{P(\theta | D_1)}}{P(D_2)}
 \end{aligned}$$

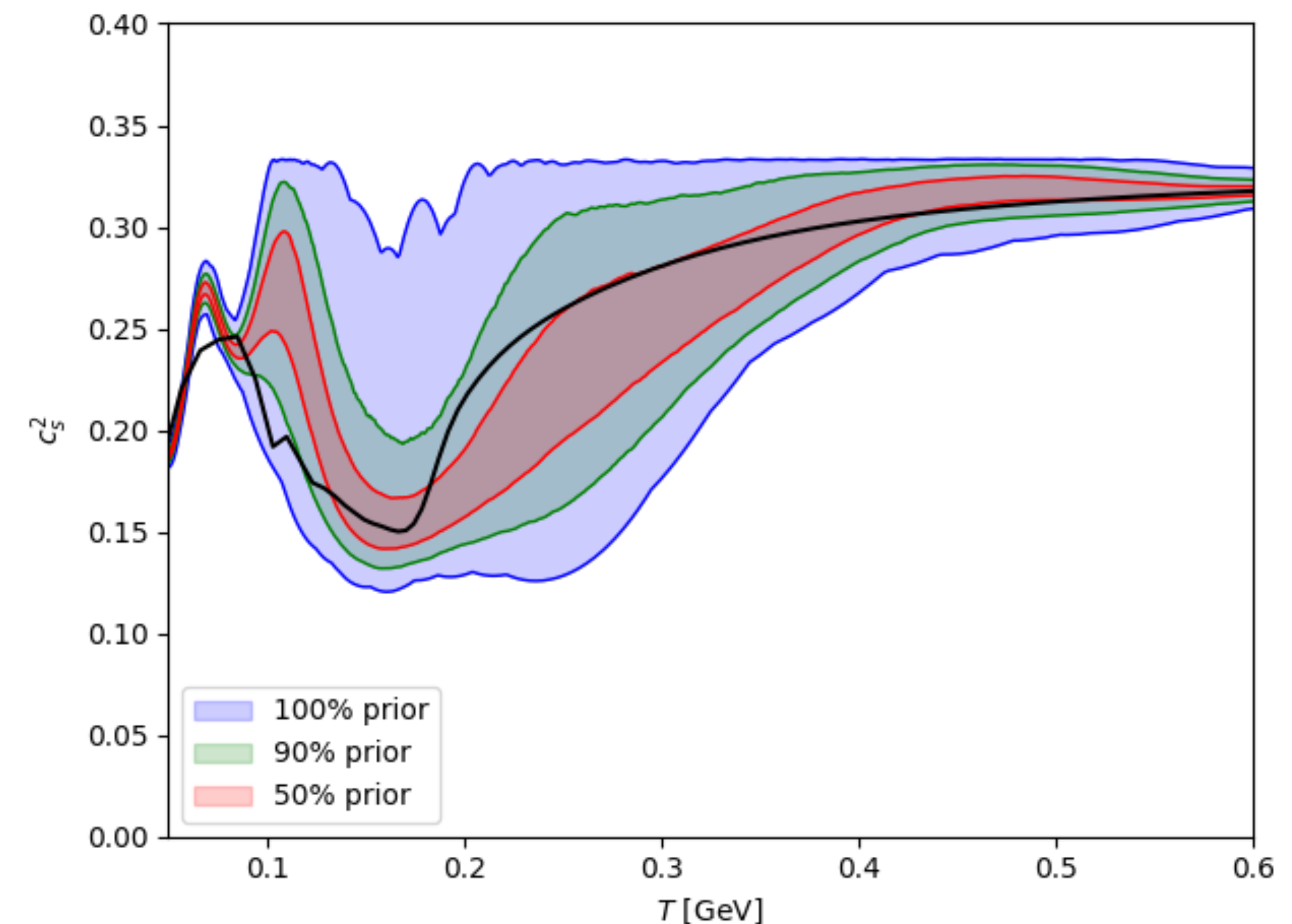
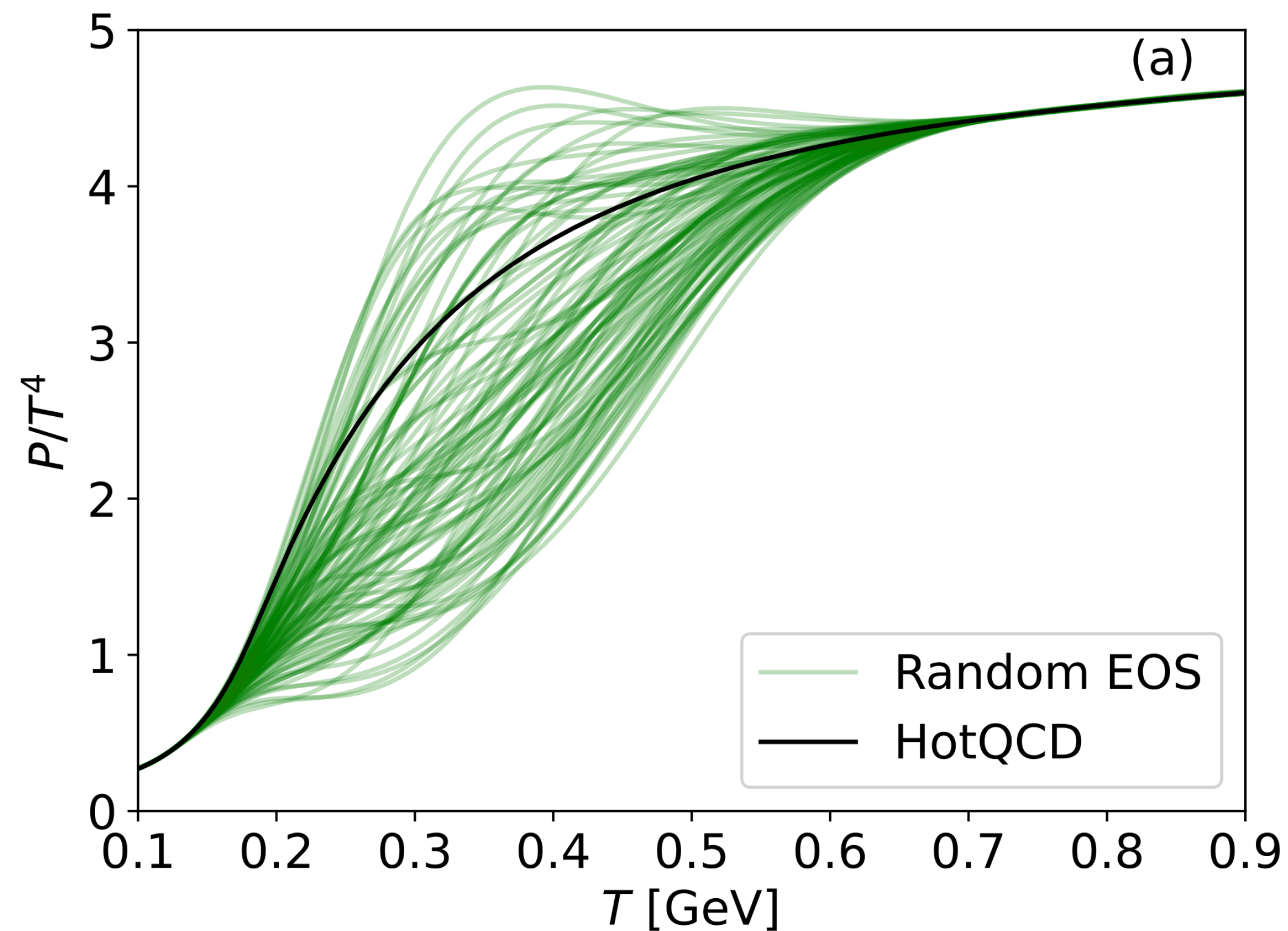
Robust Markov chain Monte Carlo (MCMC) sampler (pocoMC) is required for iterative Bayesian inference when the posterior is bimodal



CONSTRAIN QCD EQUATION OF STATE

- Confront with the RHIC BES phase II data
 - ➔ Constrain the speed of sound $c_s^2(T, \mu_B)$
- First step: introduce nonparametric GP prior to reproduce lattice QCD EoS at $\mu_B = 0$

J. Gong, H. Roch and C. Shen, arXiv:2410.22160 [nucl-th]



SUMMARY

- The vibrant experimental programs at the RHIC and LHC have been driving heavy-ion physics to a **precision** era

Unified theoretical + Bayesian statistical frameworks are *essential* to elucidate the many-body QCD physics



- We performed a comprehensive Bayesian Inference study at multiple RHIC BES energies with a state-of-the-art event-by-event (3+1)D hybrid framework

Initial stopping, $\eta/s(\mu_B)$, $\zeta/s(T, \mu_B)$

- Looking forward to the next-generation of Bayesian studies in heavy-ion physics

Quantify theoretical uncertainties
Transfer learning, Iterative Bayesian constraints
and more machine learning enhancements