Hierarchical Bayesian Unfolding in the Oslo Method

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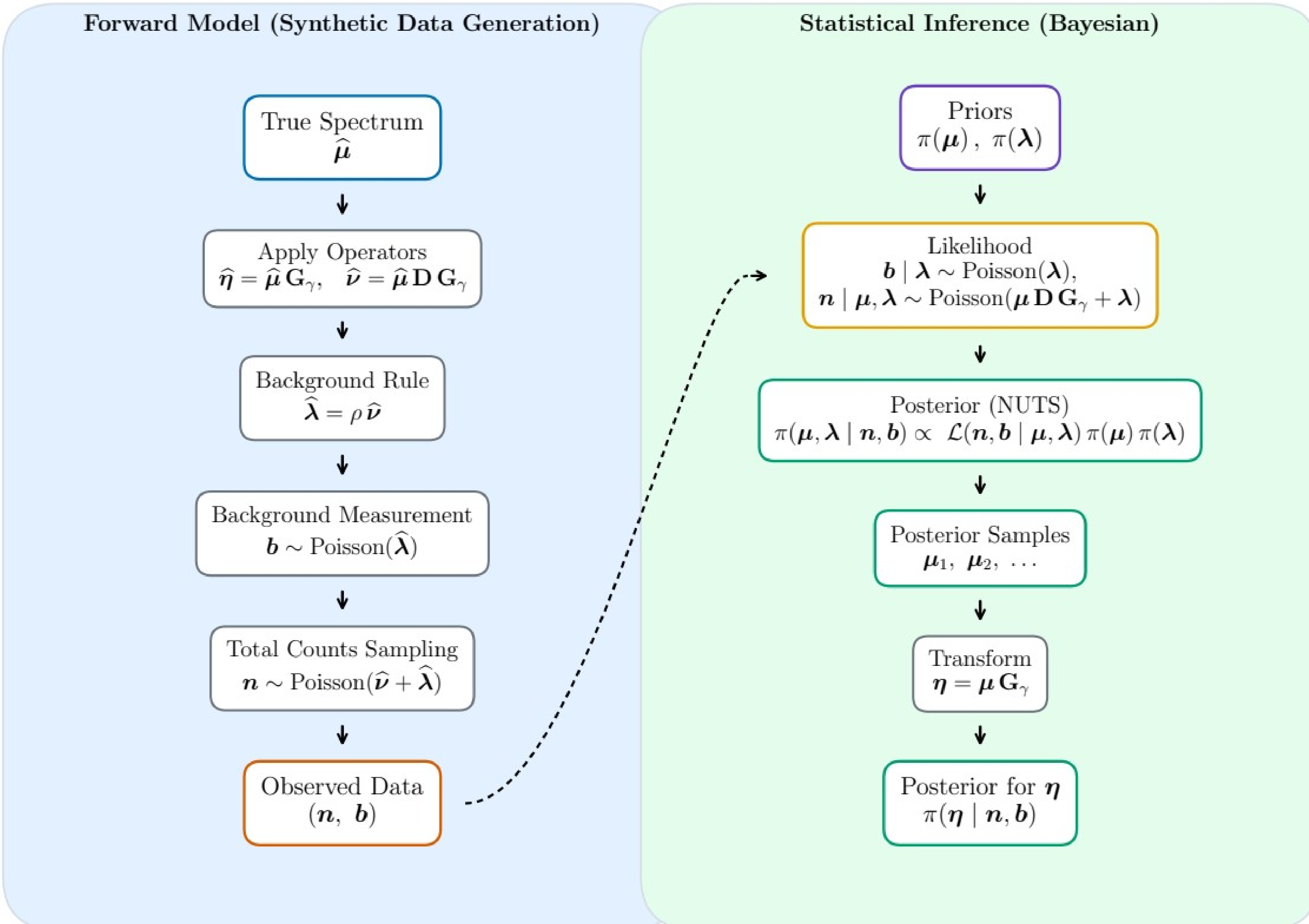
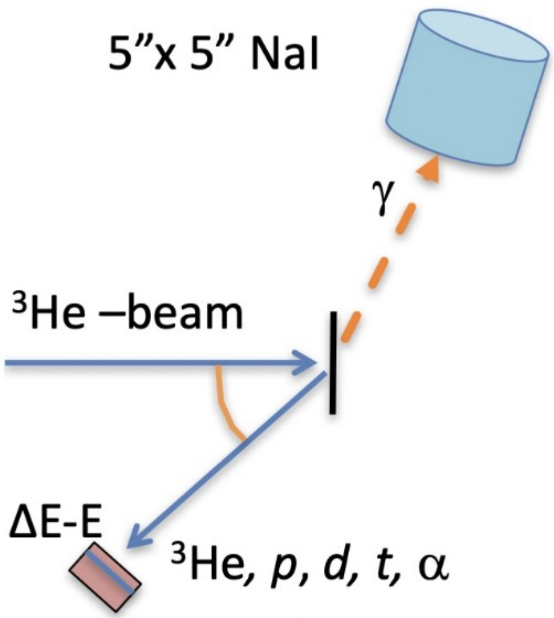
Overview

- The unfolding problem
- Hierarchical Bayesian Model
- Unfolding results and validity

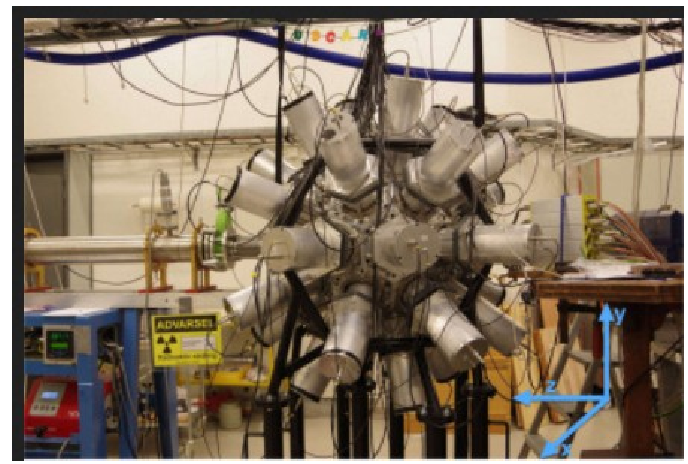
Bayesian Unfolding Framework (Signal + Background)



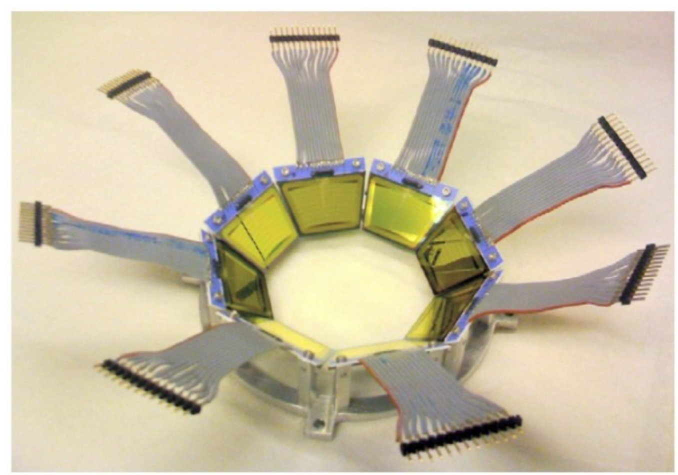
Experimental setup



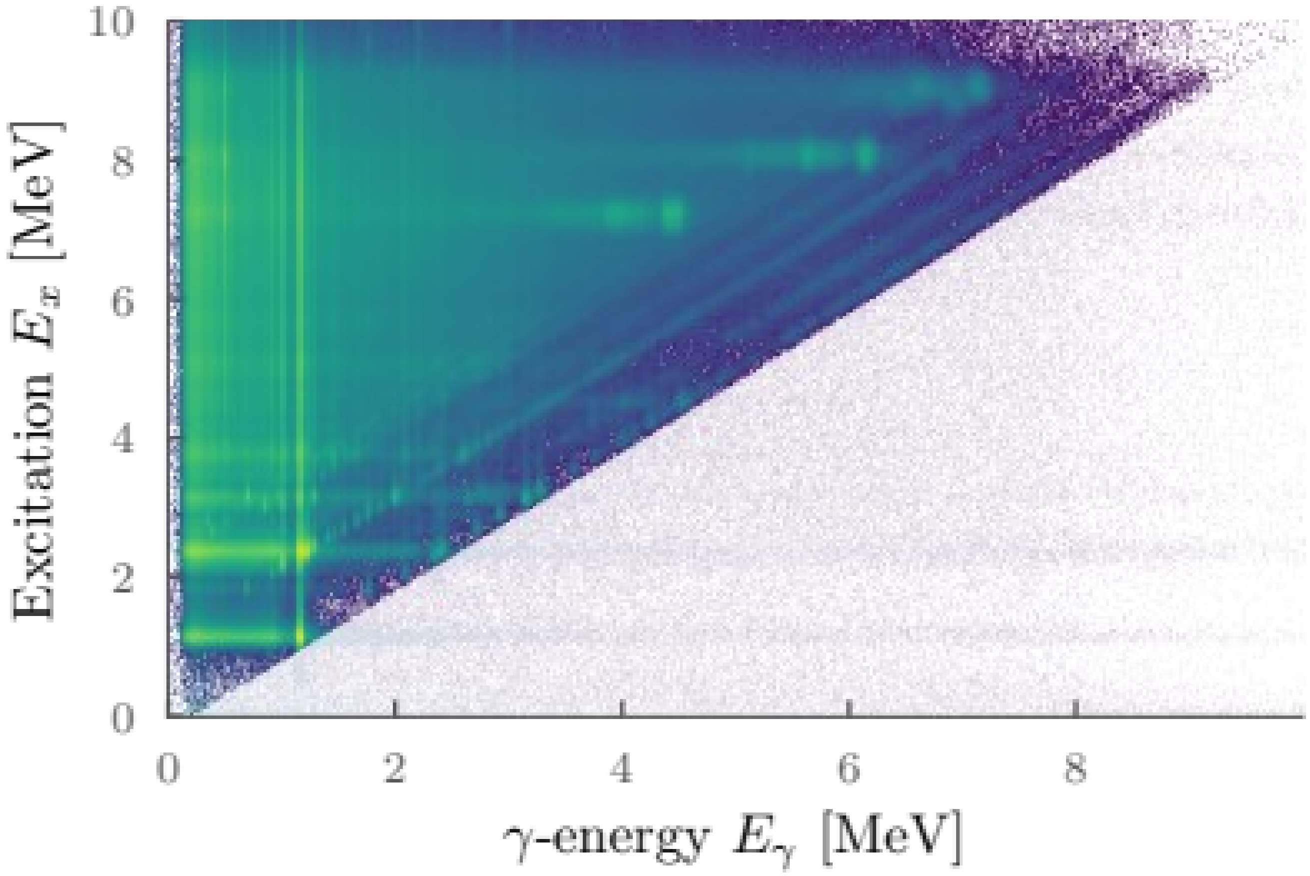
OSCAR



SiRi



Observed data



The Oslo Method

- Objective: *Simultaneous extraction of the nuclear level density (NLD) and the γ-ray strength function (γSF) of a nucleus*.
- The main steps:

1) **Unfolding of γ-ray spectra for each excitation energy to compensate for the detector response.**
2) Determination of first-generation γ-ray spectra.
3) Extraction of the NLD and the γSF.
4) Normalizing the NLD and the γSF to eliminate degrees of freedom.

- **A rigorous uncertainty propagation is needed for each step.**

The old Unfolding Method

1) Start with a trial function $\mathbf{u} = \mathbf{n}$ at iteration $i = 0$.
2) Calculate the folded spectrum $\mathbf{f}_i = \mathbf{R}\mathbf{u}_i$.
3) $\mathbf{u}_{i+1} = \mathbf{u}_i + (\mathbf{n} - \mathbf{f}_i)$.
4) Iterate until $\mathbf{f}_i \approx \mathbf{u}_i$

- *New frequentist method of unfolding by Erlend Lima, University of Oslo*

Why is unfolding hard: ill-conditioning, non-uniqueness

- $\mathbf{R} := \mathbf{D}\,\mathbf{G}_\gamma$
- The order matters: $\mathbf{D}\,\mathbf{G}_\gamma \neq \mathbf{G}_\gamma\,\mathbf{D}$
- The naive approach: $\boldsymbol{\mu} = \mathbf{R}^{-1}\,\mathbf{n}$ → very large fluctuations
- $\mathbf{R}$ is ill-conditioned → regularization is necessary
- The null space of $\mathbf{R}$ is non-empty in general:

$$\boldsymbol{\nu} = \mathbf{R}(\boldsymbol{\mu} + \mathbf{x}) \quad \forall \mathbf{x} : \mathbf{R}\mathbf{x} = 0$$

- The prior is a powerful tool to avoid unphysical solutions

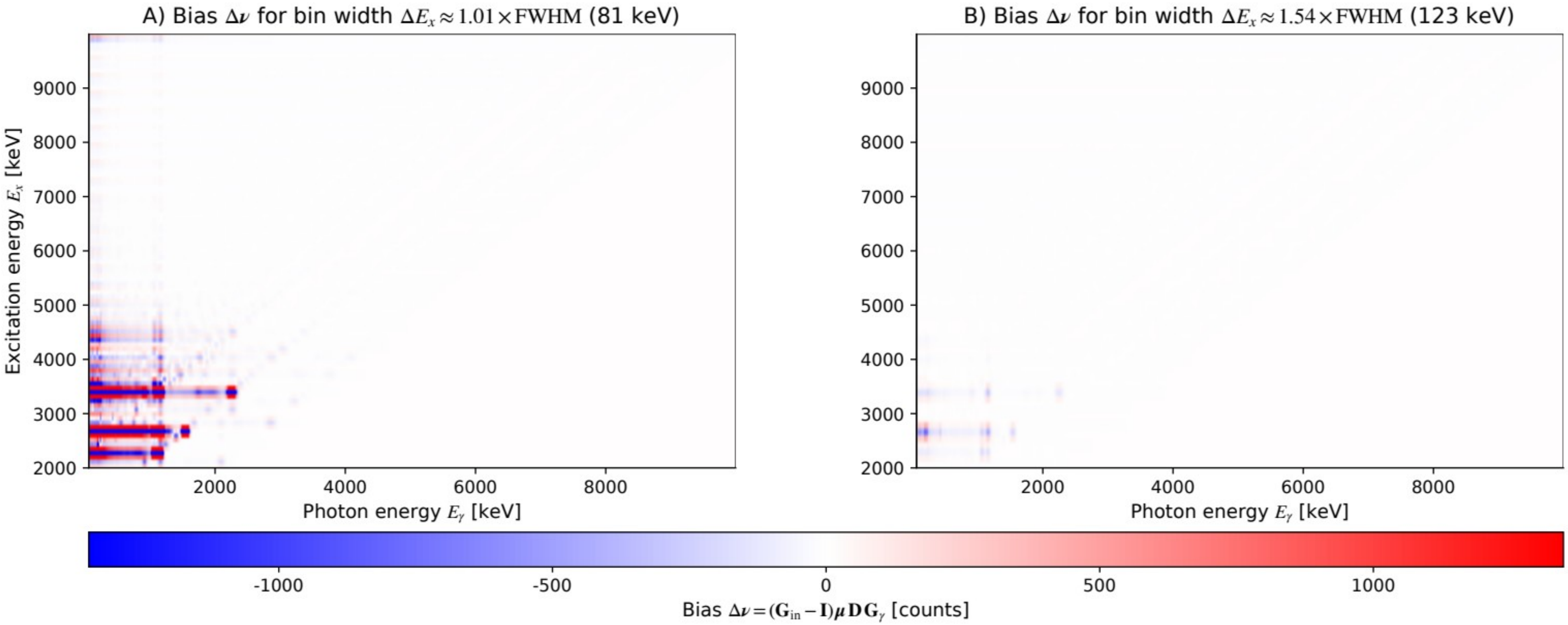
Assumptions

- Negligible “leakage” of counts between E_x bins.
- Perfect knowledge about the response .
- No systematic errors in the background between OFF and ON runs.

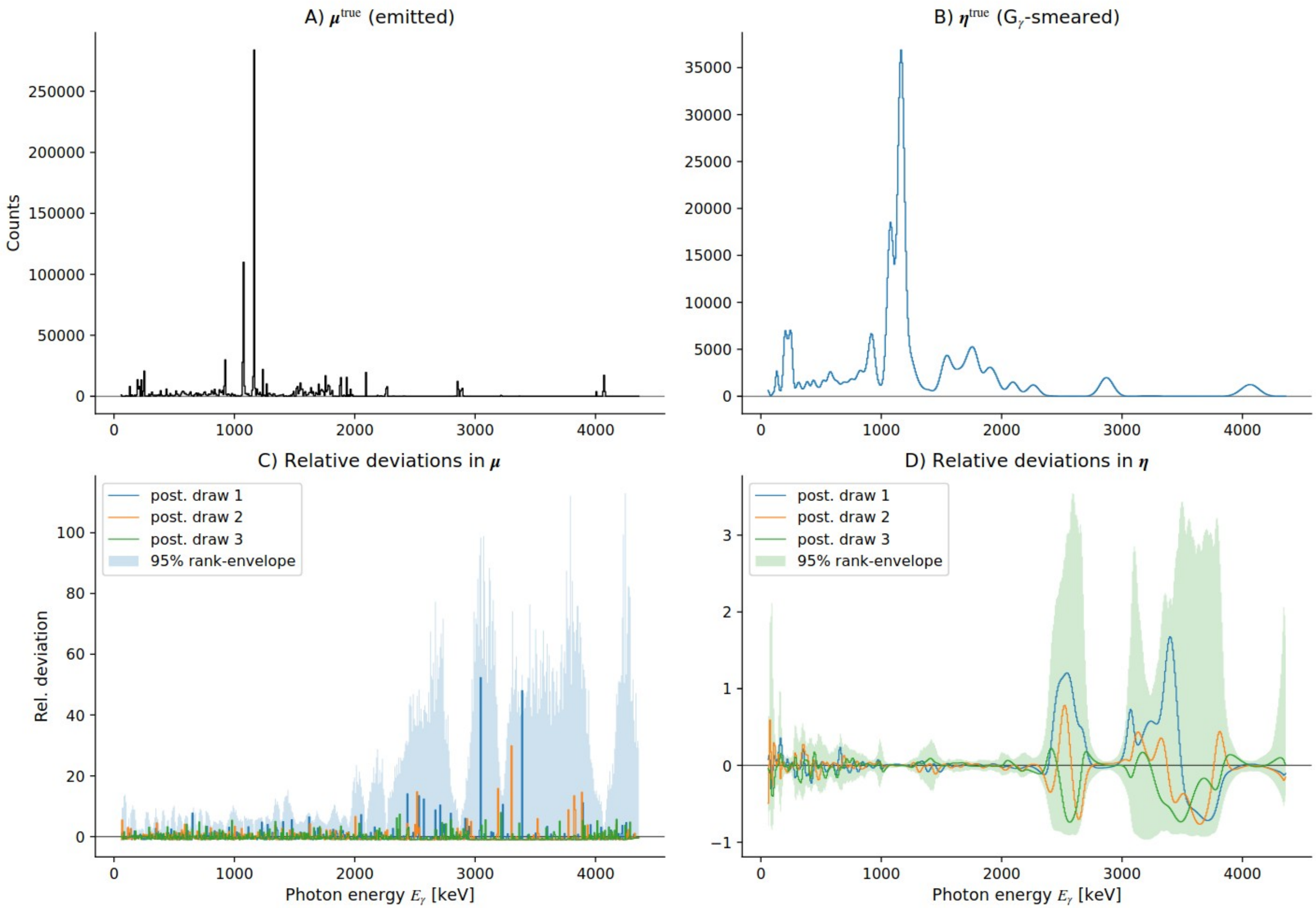
Beyond the scope of this work:

- Uncertainty in the response itself

The effect of the E_x-smearing



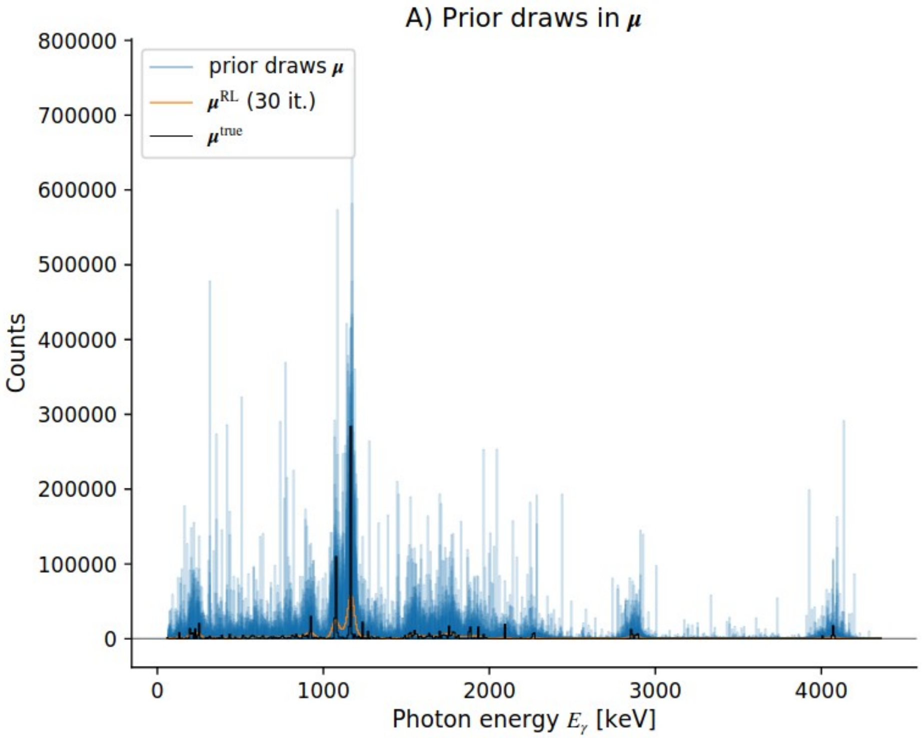
Identifiability **μ** vs **η**

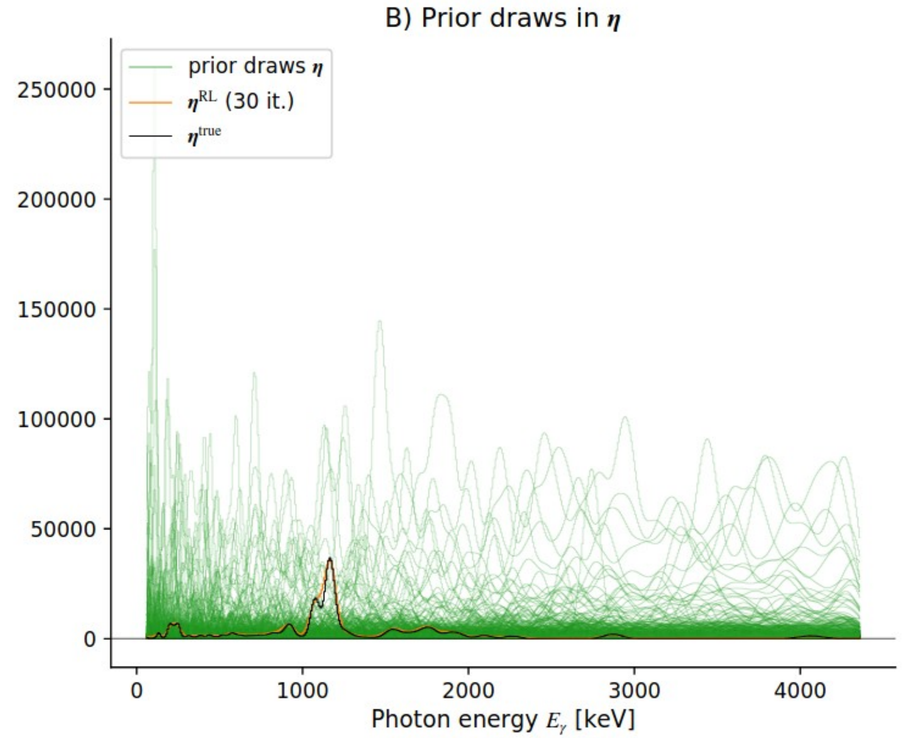


Richardson-Lucy (RL) estimate

$$\boldsymbol{\mu}^{(t+1)} = \boldsymbol{\mu}^{(t)} \circ \left[\left(\frac{\boldsymbol{n}-\boldsymbol{b}}{\boldsymbol{\mu}^{(t)}\mathbf{R}} \right) \mathbf{R}_{\gamma}^{\top} \right], \qquad t = 0, 1, \ldots$$

Citation: W. H. Richardson, Journal of the optical society of America, 1972 and L. B. Lucy - Astronomical Journal, 1974





Priors

$$\mu_j \mid M_j \sim \mathrm{Gamma}\Big(\alpha,\ \beta_j = \frac{\alpha}{M_j}\Big),$$

$$\log M_j \sim \mathcal{N}\big(\log m_j - \tfrac{1}{2}\sigma_j^2,\ \sigma_j^2\big),$$

where m_j is set to the RL-estimate μ_j^{RL}

$$\sigma_j = \sigma_{\min} + \frac{\sigma_{\max} - \sigma_{\min}}{1 + \frac{\mu_j^{\mathrm{RL}}}{\mathbb{E}[\boldsymbol{\mu}^{\mathrm{RL}}]}}.$$

$$\lambda_j \sim \mathrm{Gamma}(a_0, b_0)$$

typically with $a_0 = 1.0$

and $b_0 = a_0 / \mathrm{avg}\{\mathbf{b}^{\mathrm{obs}}\}$.

Likelihood

$$\mathcal{L}(\boldsymbol{n}, \boldsymbol{b} \mid \boldsymbol{\mu}, \boldsymbol{\lambda}) \;=\; \mathrm{Poisson}\big(\boldsymbol{b} \mid \boldsymbol{\lambda}\big)\,\mathrm{Poisson}\big(\boldsymbol{n} \mid \boldsymbol{\mu}\mathbf{DG}_\gamma + \boldsymbol{\lambda}\big).$$

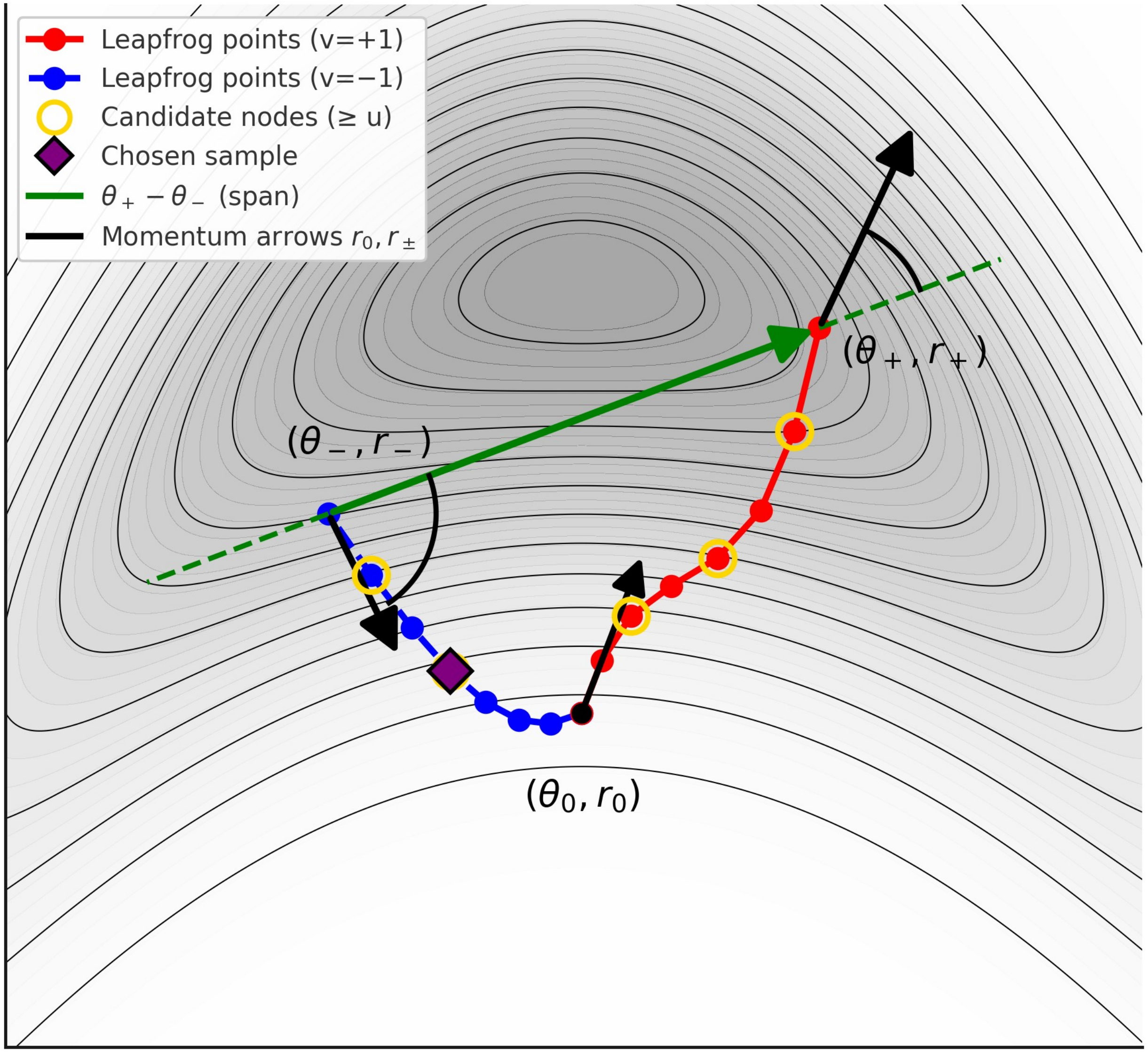
Hessian:

$$\mathbf{H}_{\log \boldsymbol{\mu}}^{(-\ell)} = -\,\mathrm{diag}(\boldsymbol{\mu})\,\mathbf{H}_{\boldsymbol{\mu}}\,\mathrm{diag}(\boldsymbol{\mu}) \;-\; \mathrm{diag}\big(\boldsymbol{\mu} \odot \mathbf{g}_{\boldsymbol{\mu}}\big),$$

$$\mathbf{g}_{\boldsymbol{\mu}} = (\mathbf{DG}_\gamma)^{\mathsf{T}} \left(\frac{\mathbf{n}}{\boldsymbol{\nu}} - \mathbf{1}\right), \qquad \boldsymbol{\nu} = \boldsymbol{\mu}\mathbf{DG}_\gamma + \boldsymbol{\lambda},$$

$$\mathbf{H}_{\boldsymbol{\mu}} = -(\mathbf{DG}_\gamma)^{\mathsf{T}}\,\mathrm{diag}\left(\frac{n}{\boldsymbol{\nu}^{\odot 2}}\right)(\mathbf{DG}_\gamma).$$

No-U-Turn Sampler (NUTS)

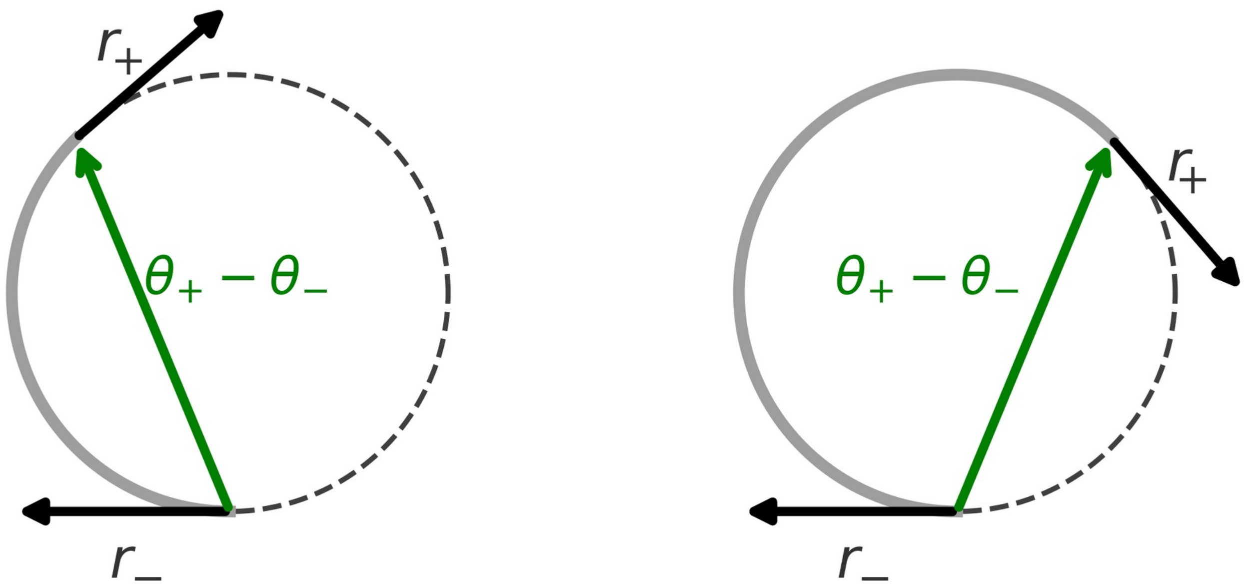


- v ∈ {-1, 1}
- 2^j leapfrog steps, j={0,1,..N}

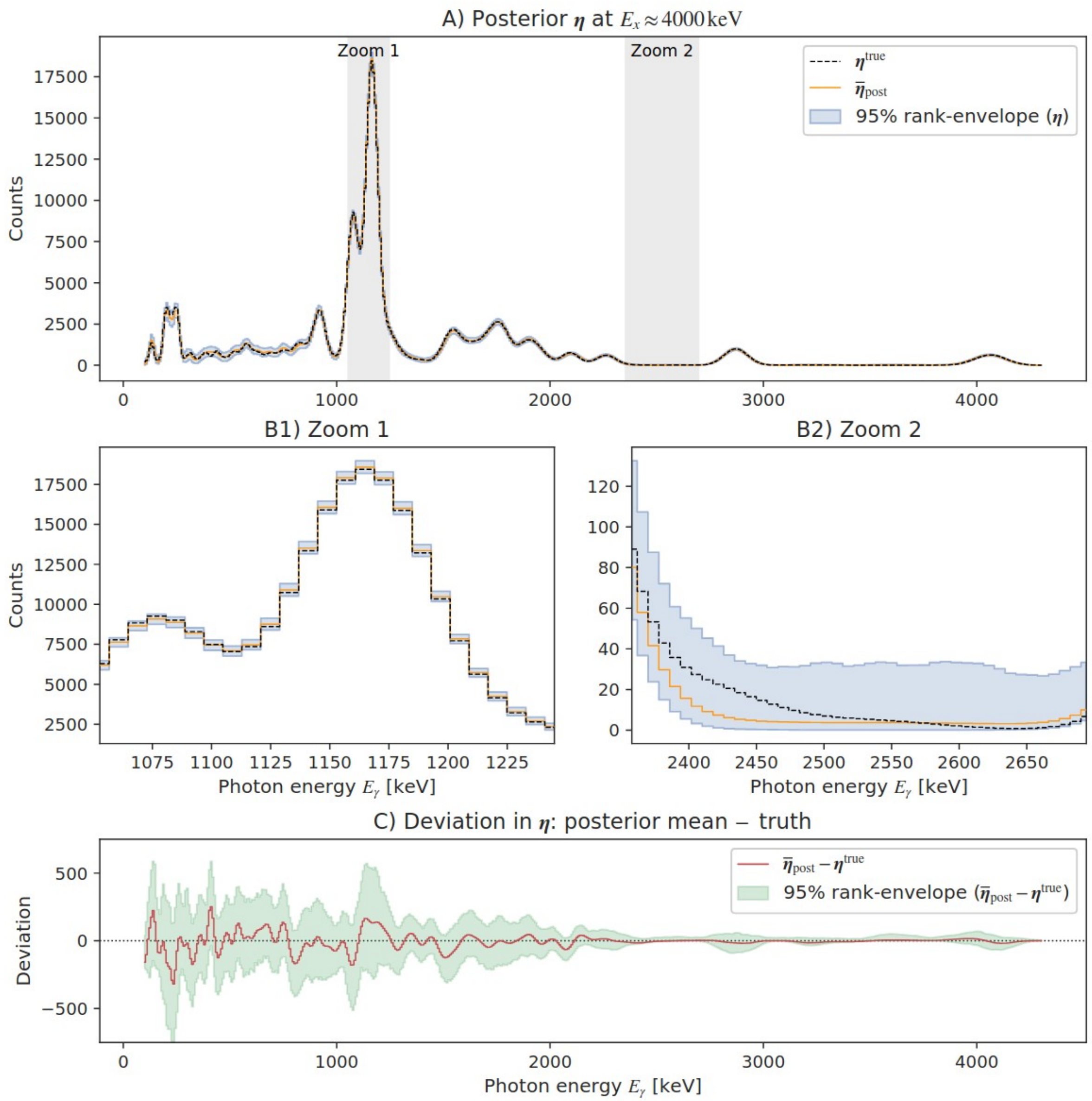
$$\mathrm{Uniform}(u; [0, \exp\{\mathcal{L}(\theta) - \tfrac{1}{2} r \cdot r\}])$$

No-U-Turn condition:

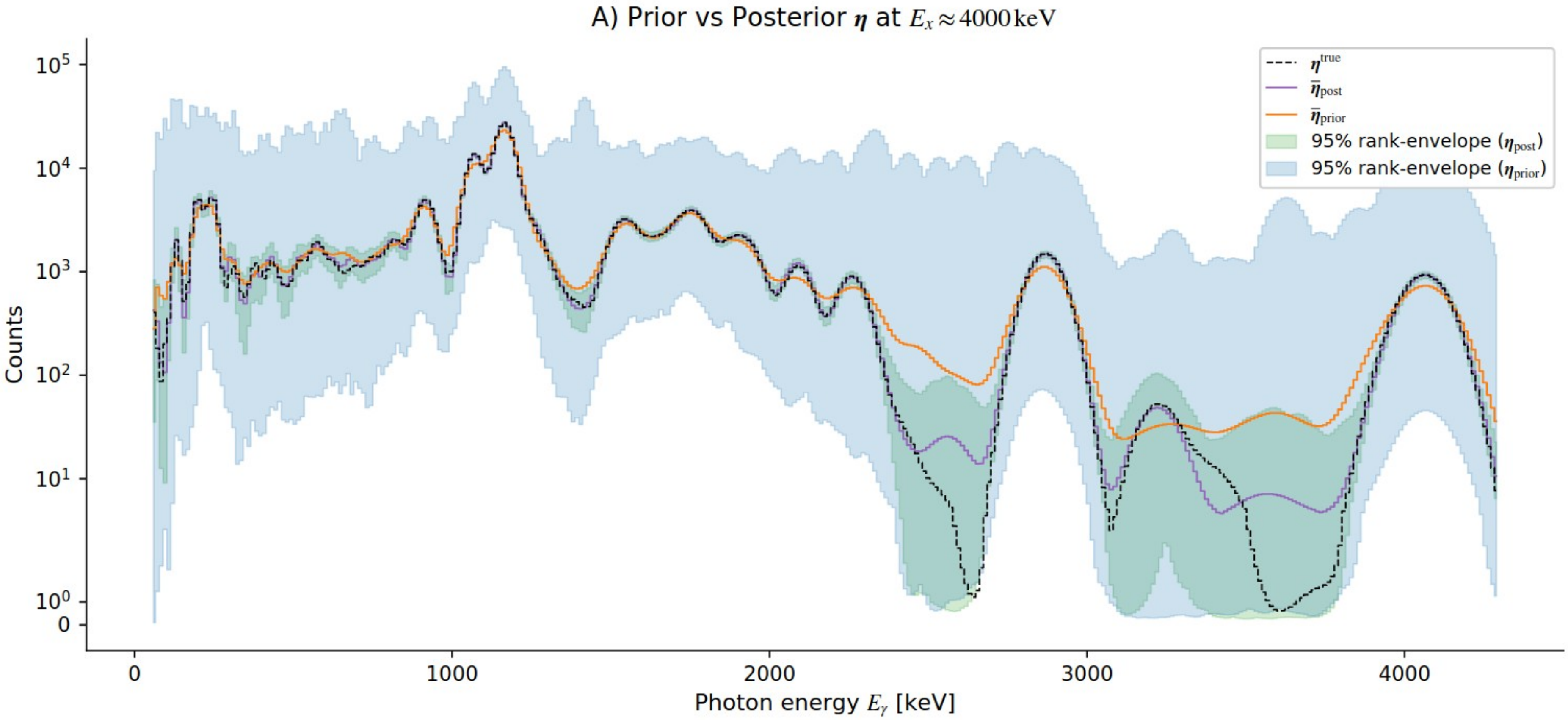
$$(\theta^{+} - \theta^{-}) \cdot r^{-} < 0 \quad \text{or} \quad (\theta^{+} - \theta^{-}) \cdot r^{+} < 0.$$



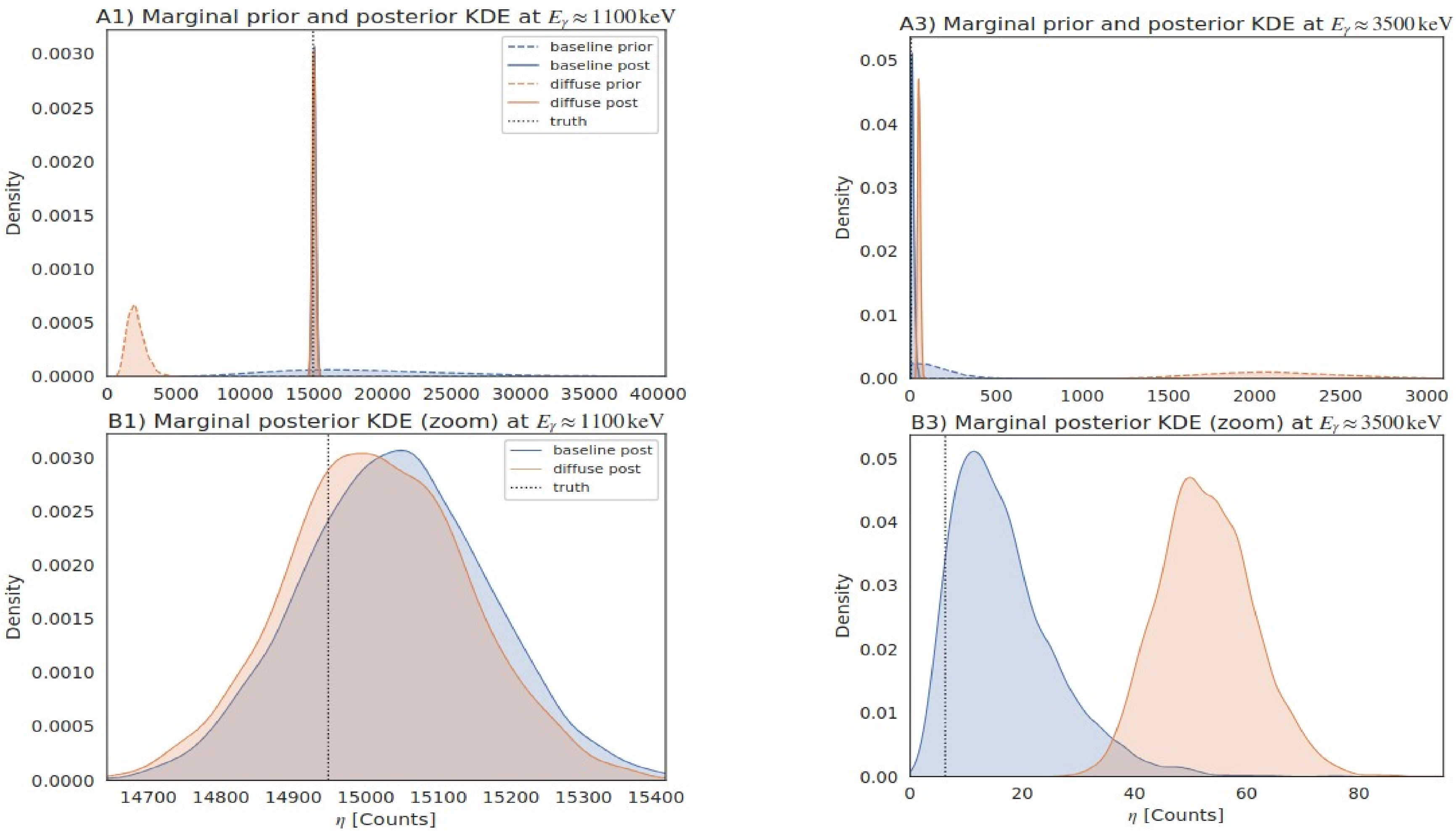
Posterior on unfolded solution



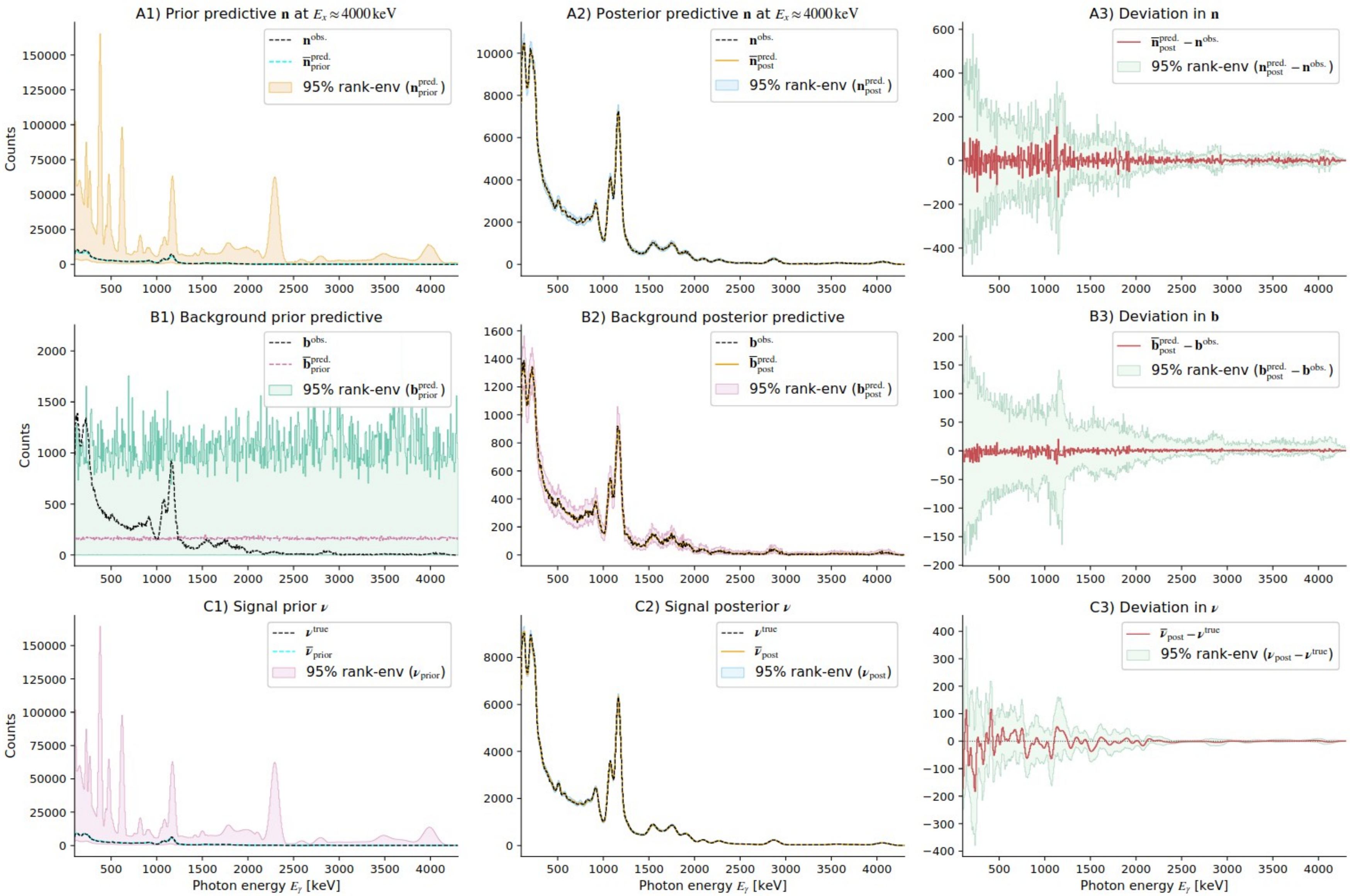
Prior vs posterior



Marginal prior vs posterior



Prior and posterior predictive checks



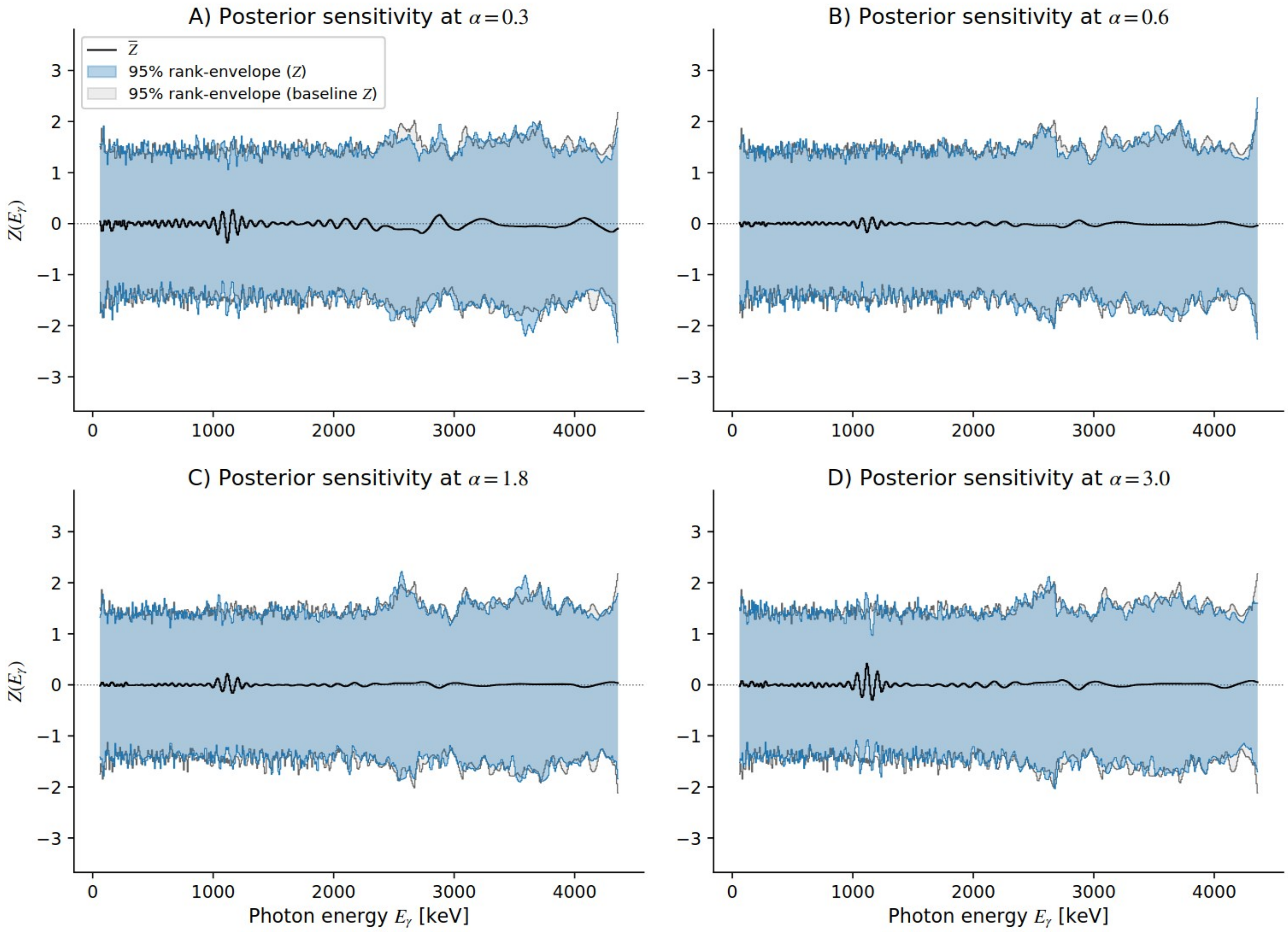
Sensitivity and robustness

- Sensitivity: how much does the posterior change if we *change a parameter of the prior/hyperprior*?
- Robustness: how much does the posterior change if we *change more fundamental properties of the prior*?
- Z-statistic (baseline prior vs changed prior):

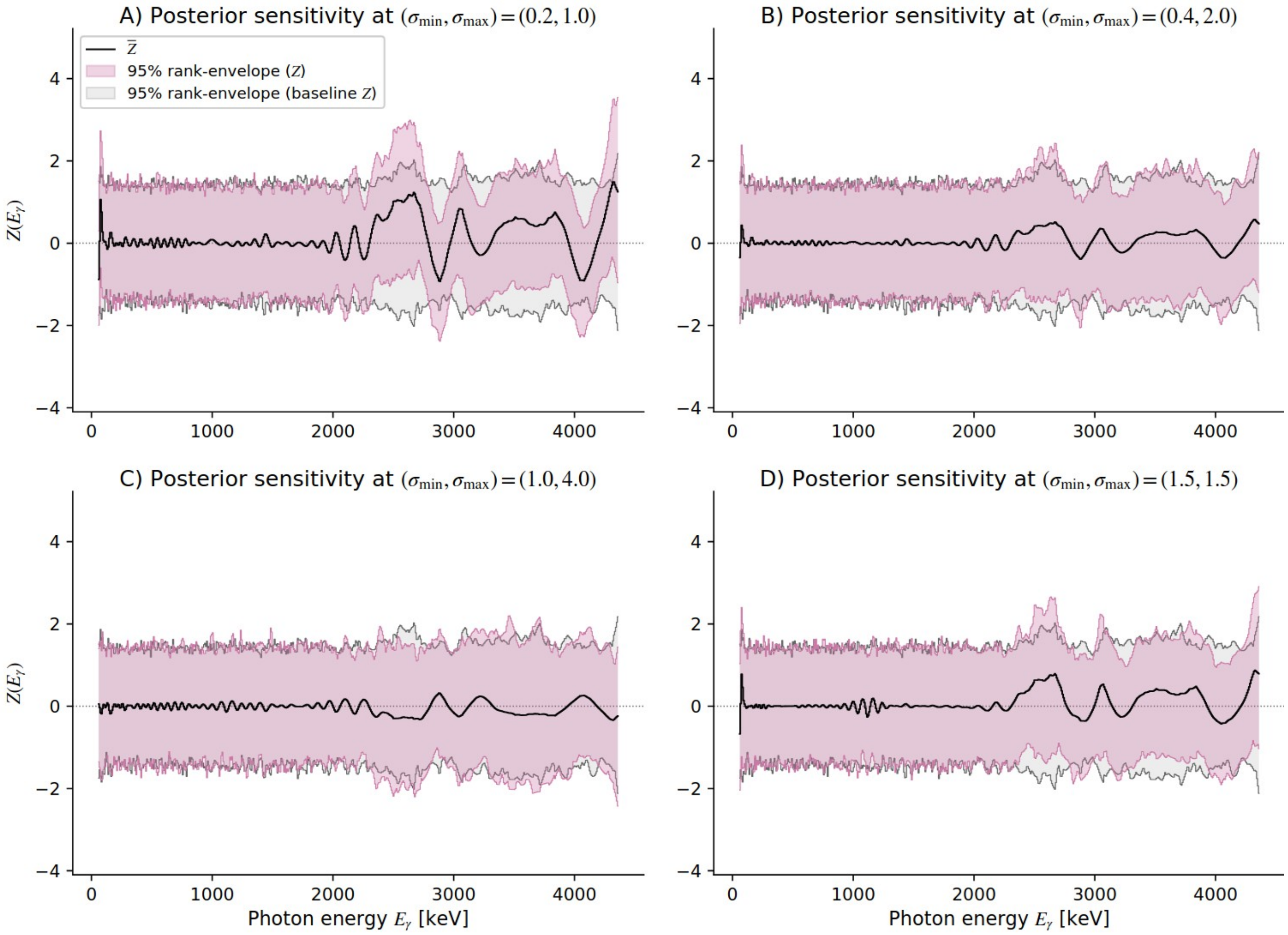
$$\boxed{Z_s(j) = \frac{C_s(j) - B_s(j)}{w(j)}}$$

$$Z_s^0(j) = \frac{B_s^{(2)}(j) - B_s^{(1)}(j)}{w(j)}$$

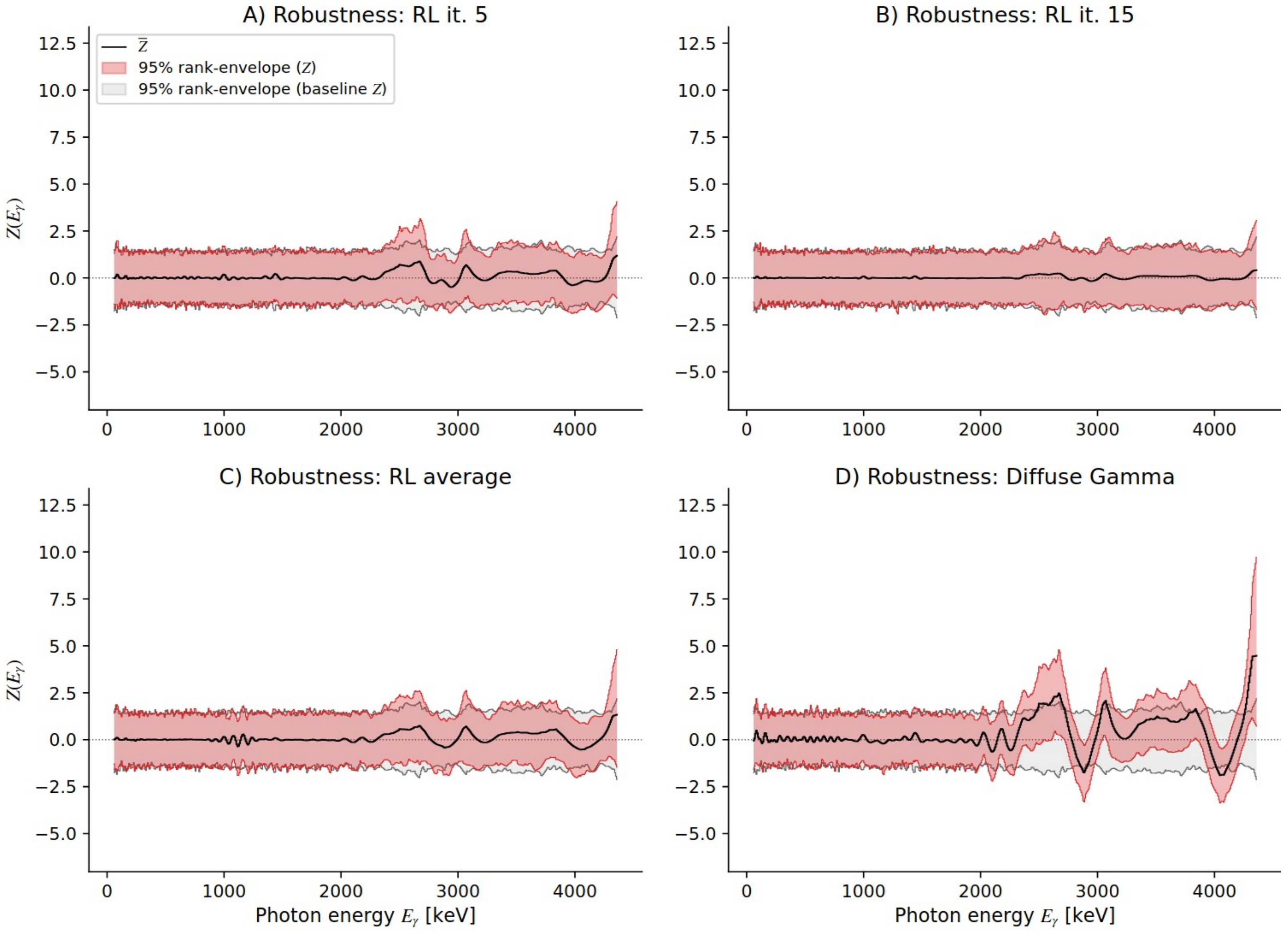
Sensitivity to prior α



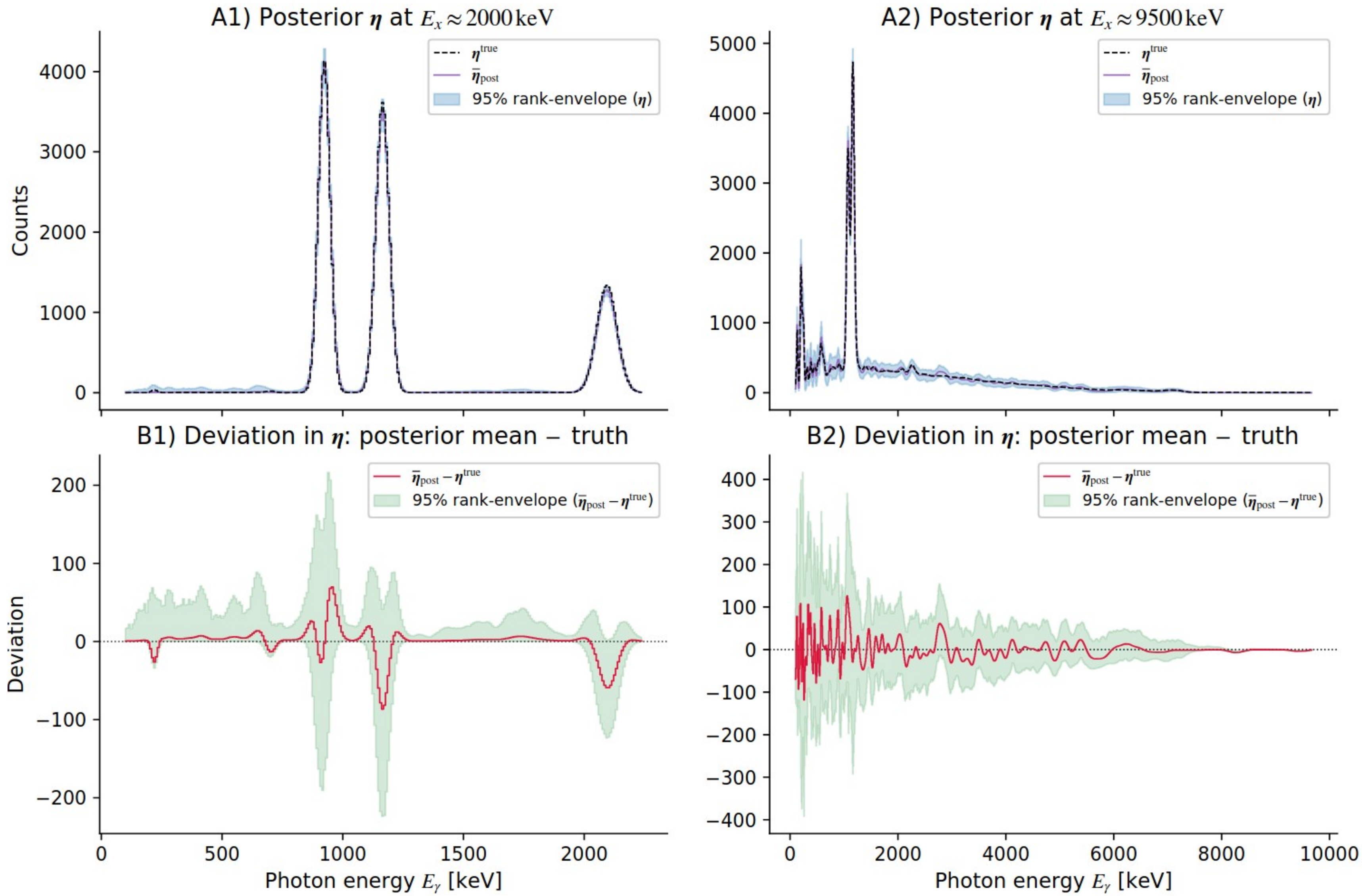
Sensitivity to hyperprior σ



Robustness to different priors



Unfolding results for different E_x



Summary and outlook

- The choice of prior is most important for low count energy bins, where the data is less informative.
- The prior needs to be sufficiently wide to avoid posterior bias.
- The RL estimate helps us determining the critical regions where the prior variance should be relatively larger.
- Outlook: Uncertainty propagation of posterior draws through the next steps of the Oslo Method