

# Efficient Bayesian Model Combination With **pybmc**

Kyle Godbey

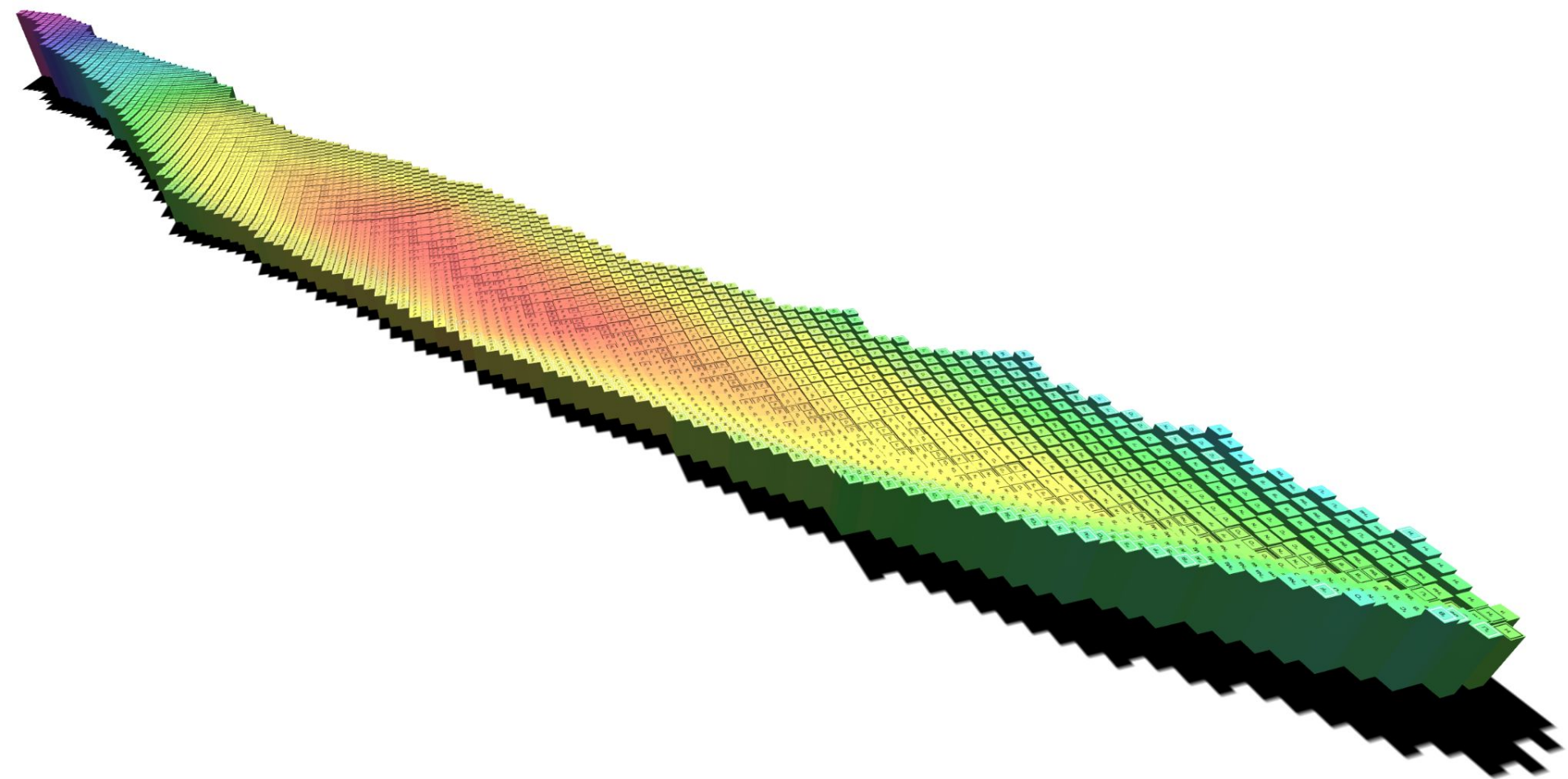
Slides with videos:

[https://docs.google.com/presentation/d/1xRZo72xdc\\_uwMq-Jl6Gk9myKCNaJ4l2mLoLP\\_HqvQcQo/edit?usp=sharing](https://docs.google.com/presentation/d/1xRZo72xdc_uwMq-Jl6Gk9myKCNaJ4l2mLoLP_HqvQcQo/edit?usp=sharing)

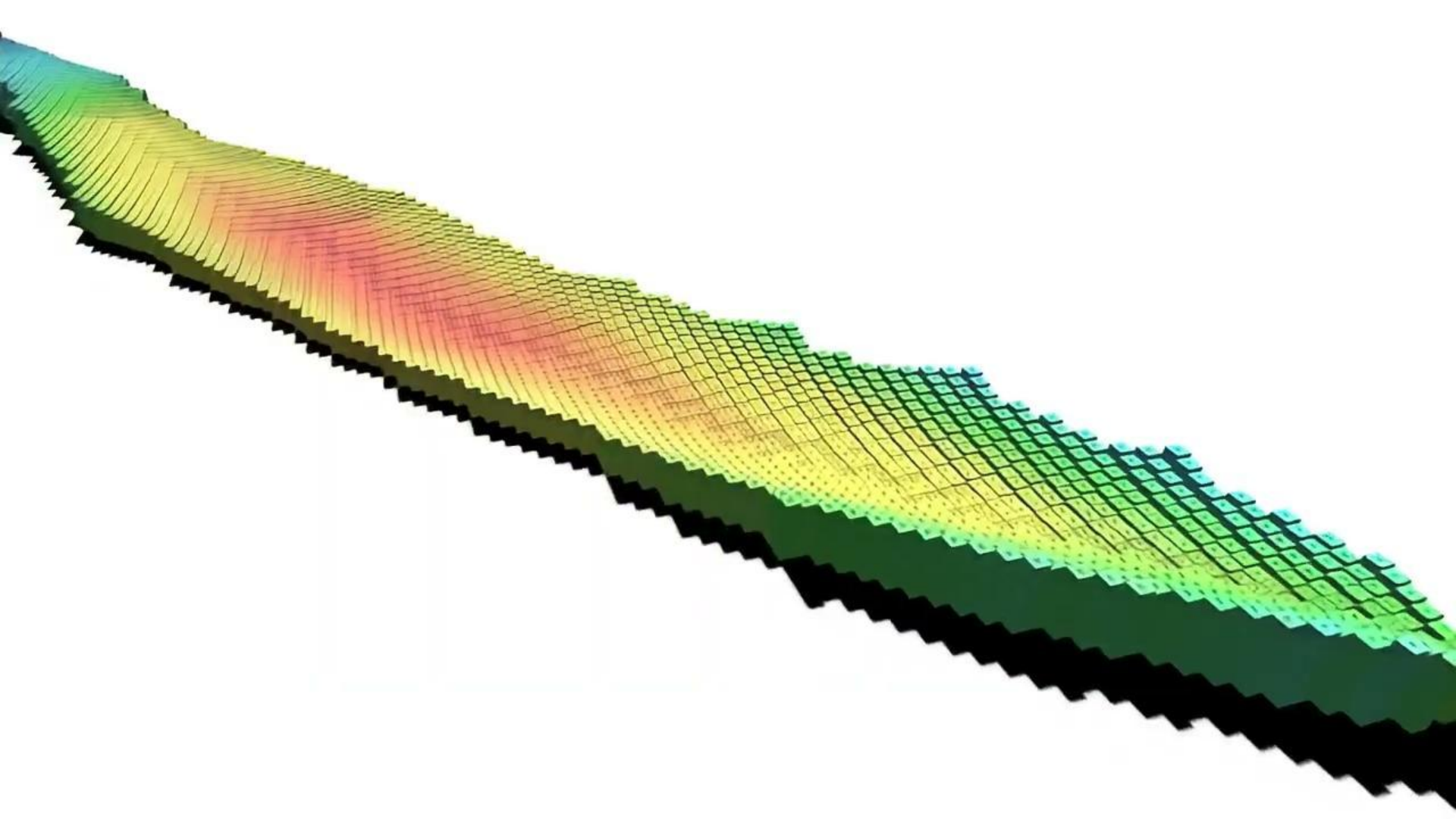


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# What's Model Combination?



# Model Combination

Imagine a scenario where you have many wrong (and potentially useful) models

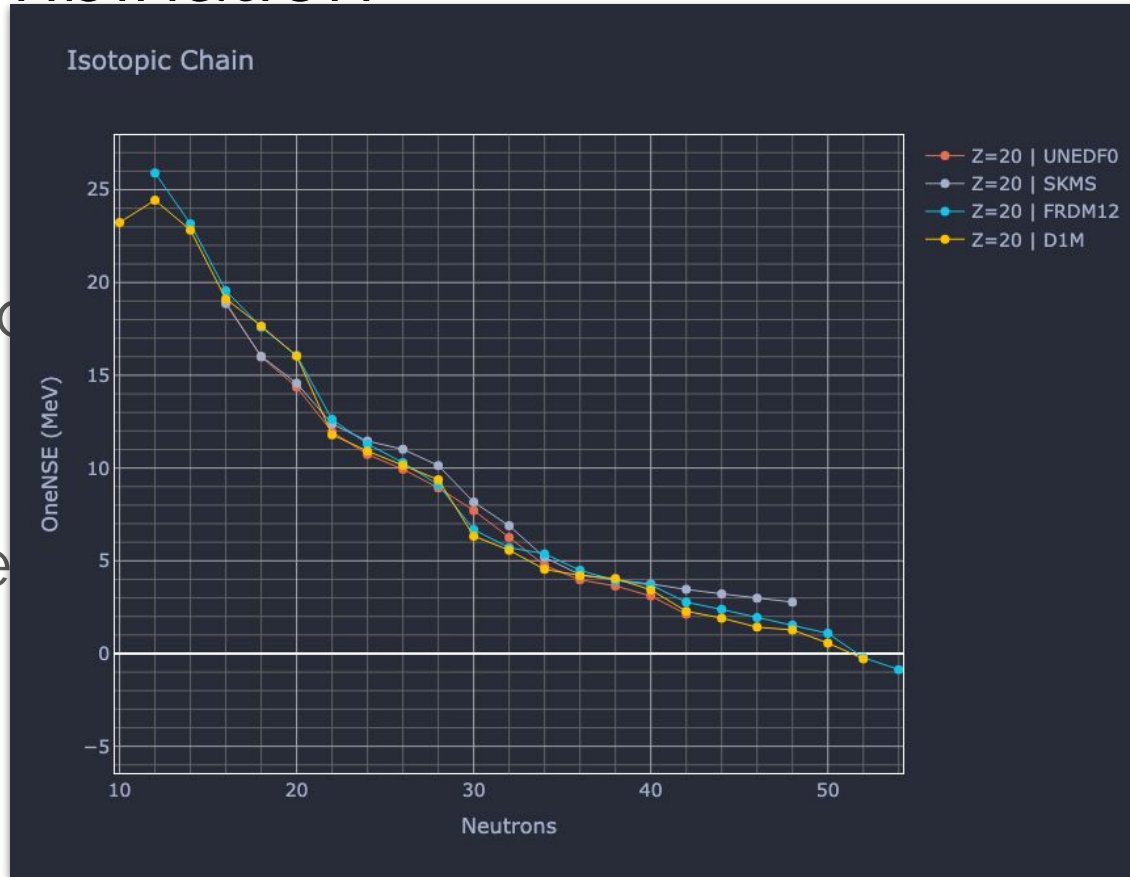
Will they be wrong in the same ways and places? Are they equally useful?



# Model Combination

Imagine a set of models that are all wrong (and

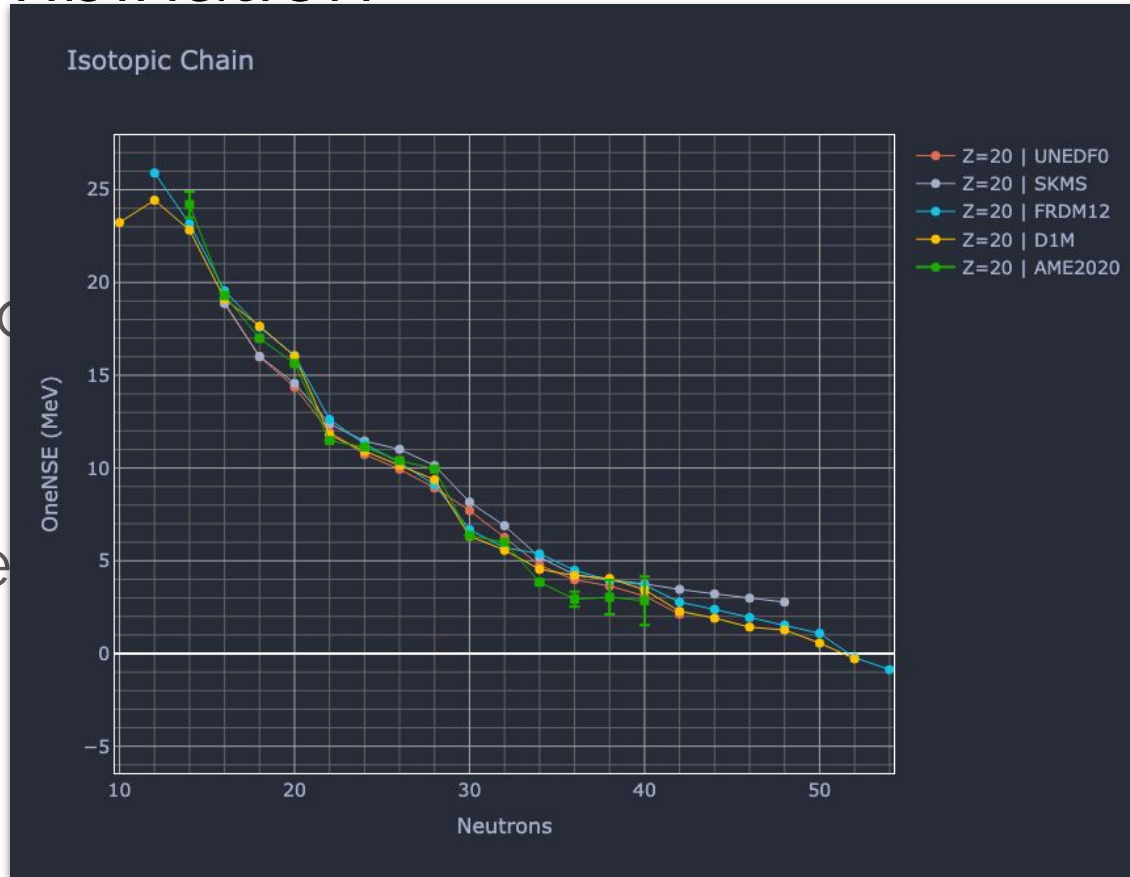
Will they be in the right places? Are



# Model Combination

Imagine a set of models that are all wrong (and

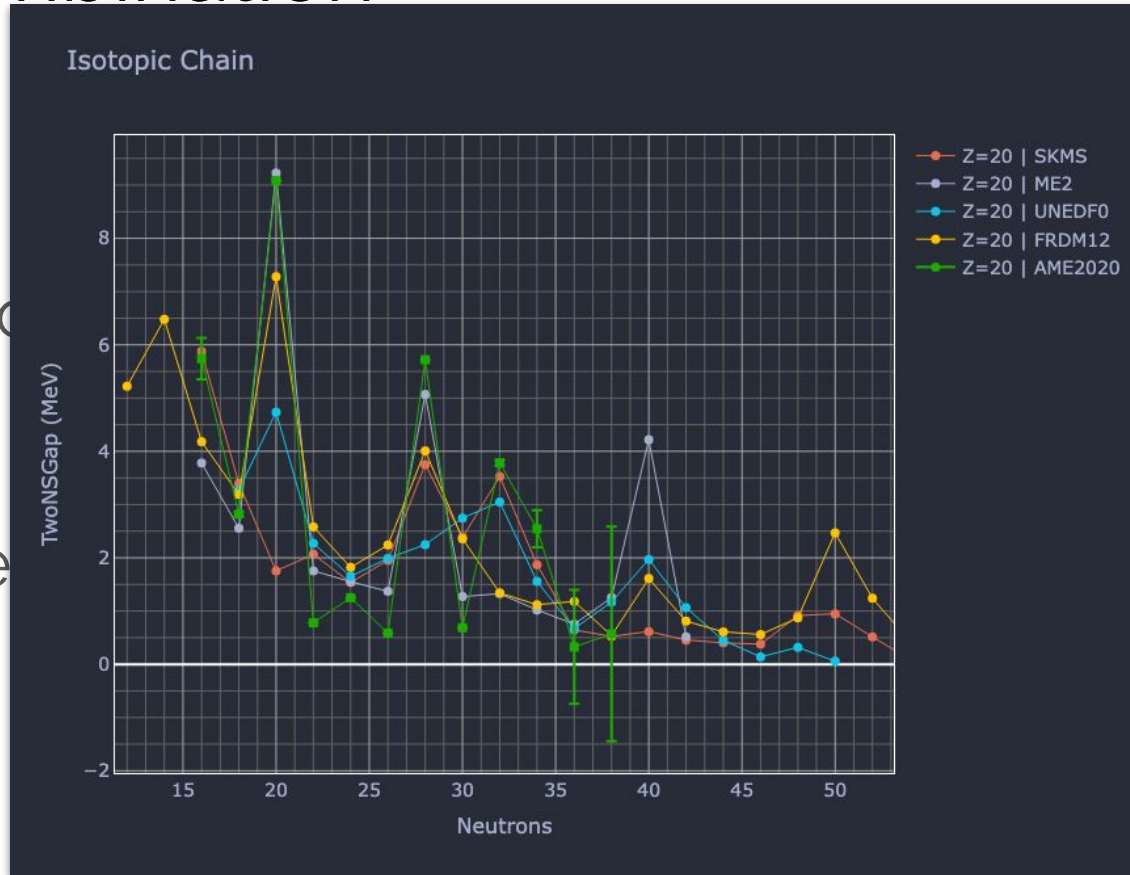
Will they be in the right places? Are



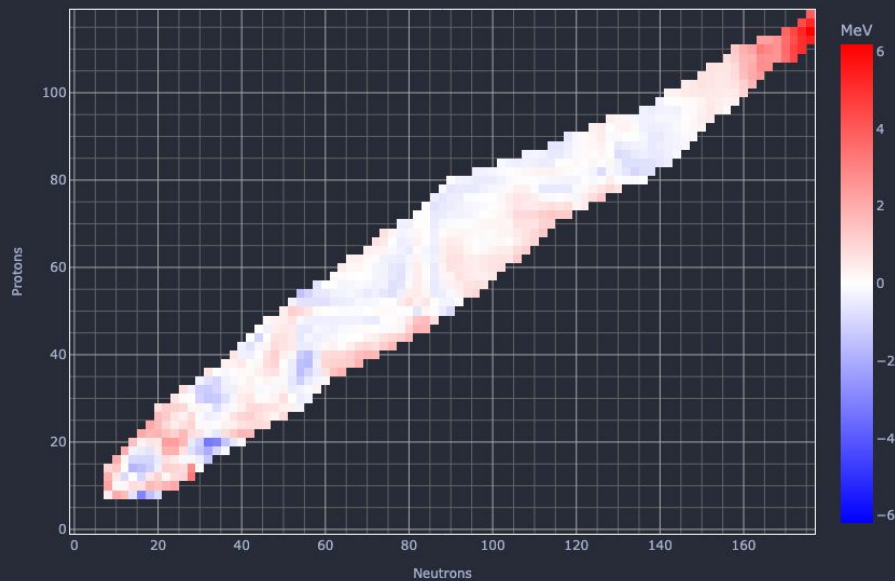
# Model Combination

Imagine a so  
wrong (and

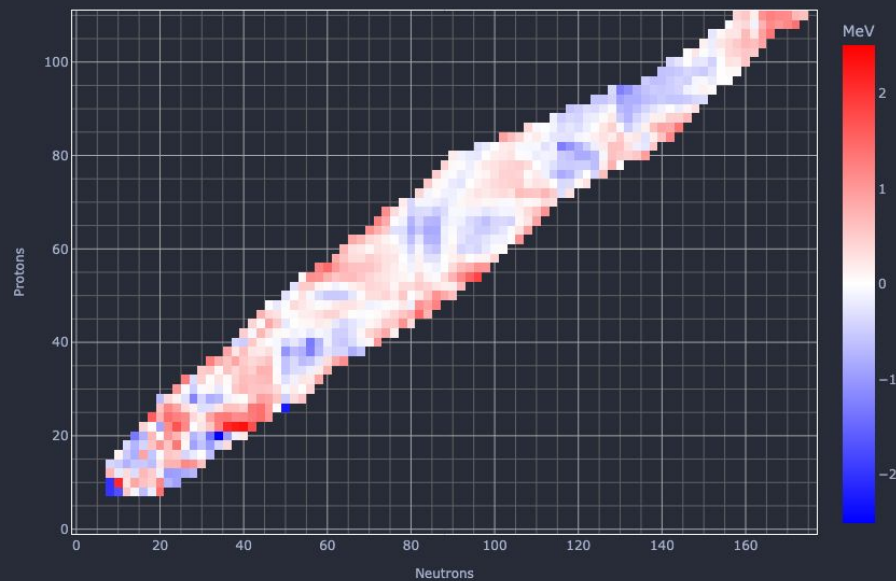
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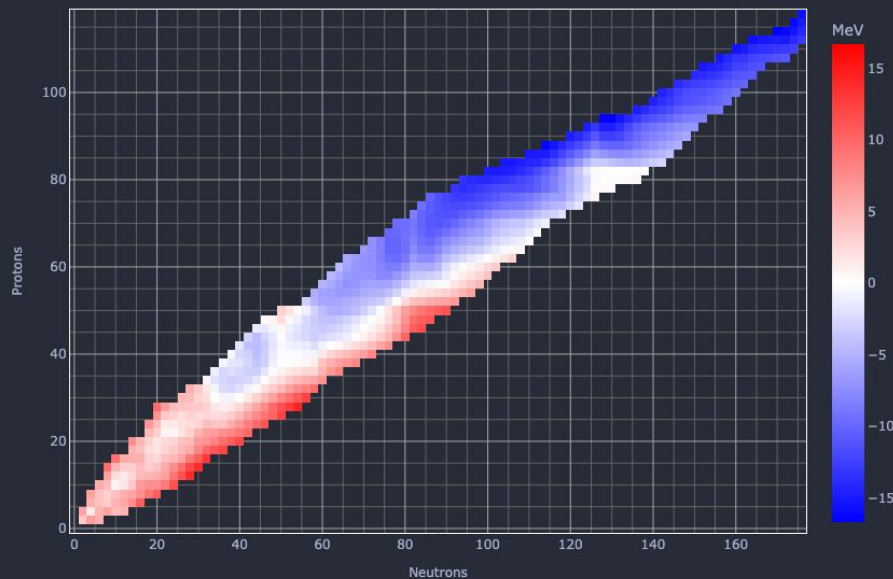
Binding Energy | FRDM12 - AME2020



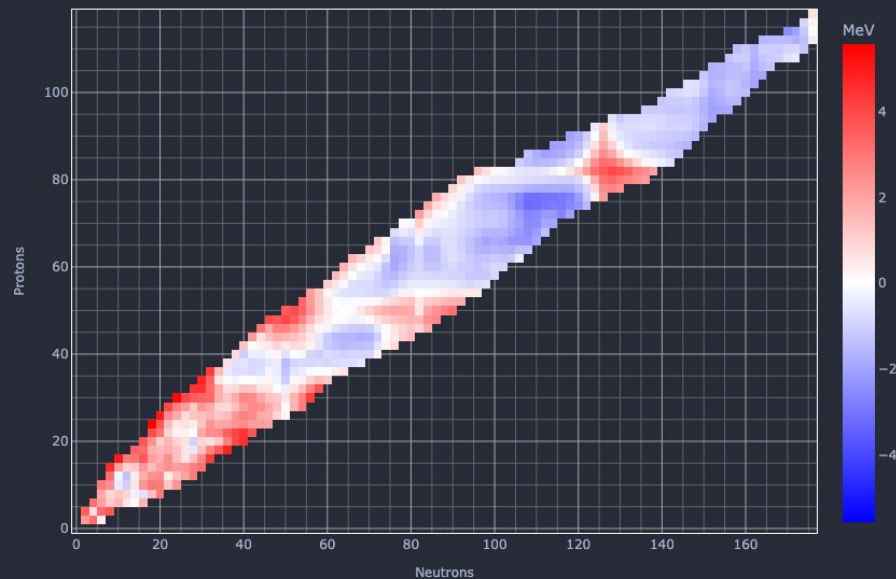
Binding Energy | HFB24 - AME2020



Binding Energy | SKMS - AME2020



Binding Energy | UNEDF0 - AME2020



# Model Combination

Model combination techniques provide a prescription on how to handle this!

This talk is going to focus on **forecast combination** specifically, i.e. where we already have independent model forecasts prepared



# Model Combination (+ orthogonalization)

Additionally, we'll orthogonalize our forecasts to remove model redundancies

Side benefit: we can truncate the principal components to get less overfitting



# Model Combination (+ orthogonalization)

First, center your data

$$X_{i,k}^c = X_{i,k}^0 - \phi_0(x_i)$$

where

$$\phi_0(x_i) = \frac{1}{m} \sum_{k=1}^m X_{i,k}^0 = \frac{1}{m} \sum_{k=1}^m f_k(x_i).$$



# Model Combination (+ orthogonalization)

Then, reduce the dimensionality:

$$\begin{aligned} \mathbf{X}^c_{n \times m} &= \mathbf{U}_{n \times n} \mathbf{S}_{n \times m} \mathbf{V}_{m \times m}^T \\ &\approx \hat{\mathbf{X}}^c_{n \times p} = \hat{\mathbf{U}}_{n \times p} \hat{\mathbf{S}}_{p \times p} \hat{\mathbf{V}}_{p \times m}^T. \end{aligned}$$

**n** = # of data points, **m** = # of models, **p** = # of kept components

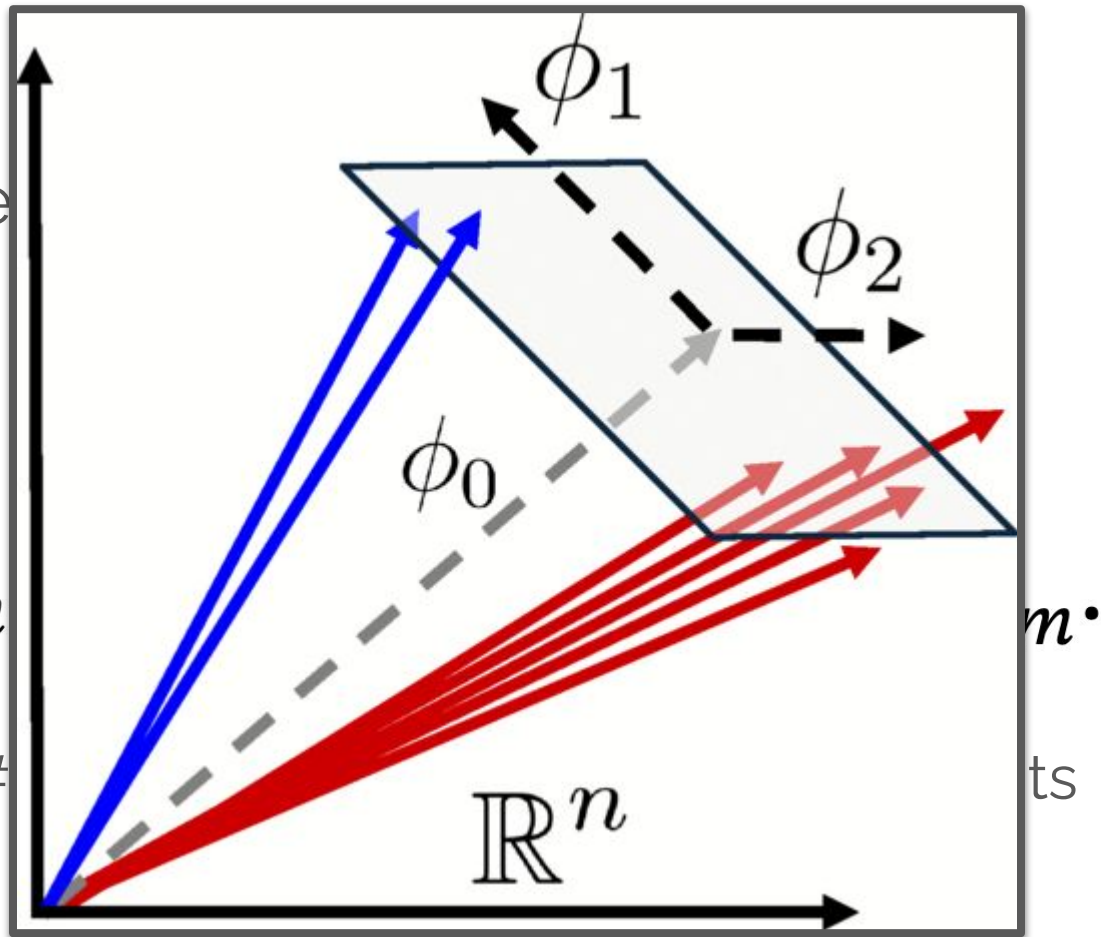


# Model Combination (+ orthogonalization)

Then, reduce the dimensionality

$$\mathbf{X}^c_{\textcircled{n} \times \textcircled{m}} = \mathbf{U}_{n \times n} \\ \approx \widehat{\mathbf{X}}^c_n$$

**n** = # of data points, **m** = # of features



# Model Combination (+ orthogonalization)

Now construct your combined forecast:

$$f^{\dagger}(\mathbf{x}; \mathbf{b}) = \phi_0(\mathbf{x}) + \sum_{j=1}^p b_j \phi_j(\mathbf{x})$$



# Bringing it Together



Dataset  
construction is  
important!

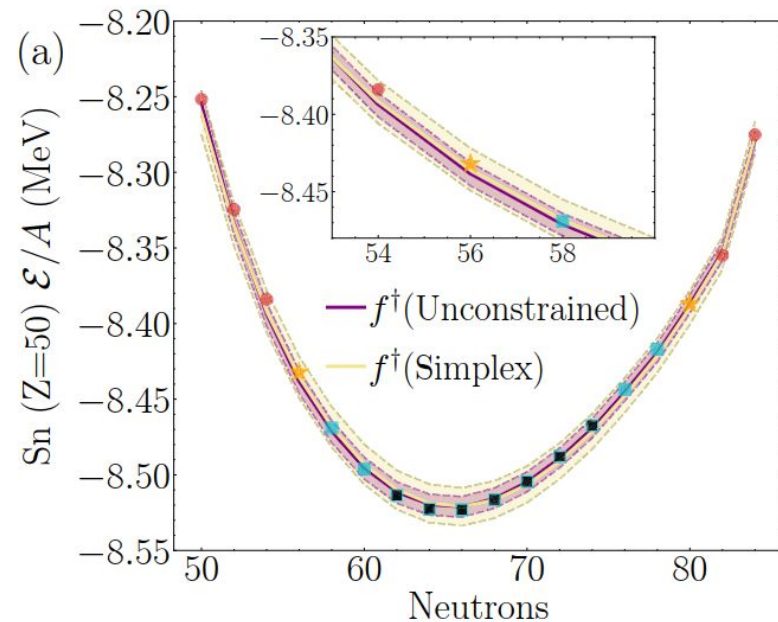
Once you determine  
the weights, you're  
ready to go!

P. Giuliani, K. Godbey, V. Kejzlar, and W. Nazarewicz  
Phys. Rev. Research 6, 033266

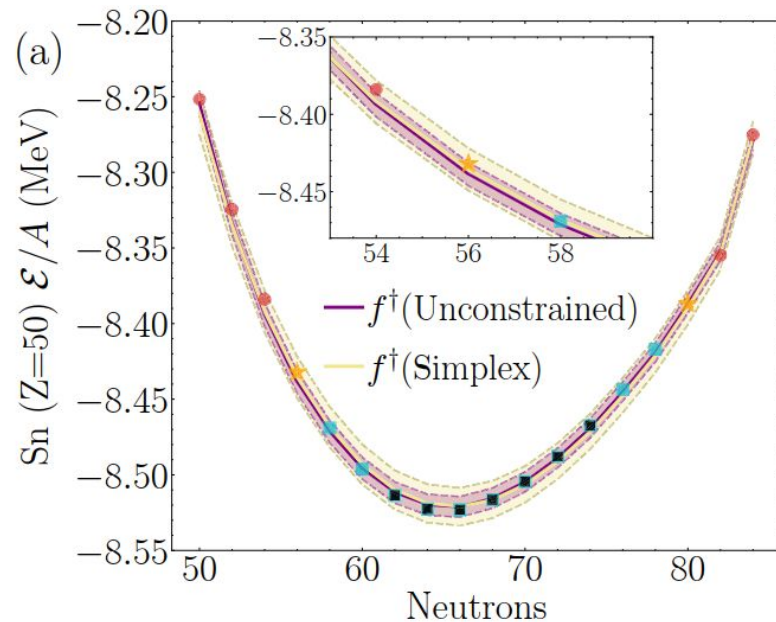
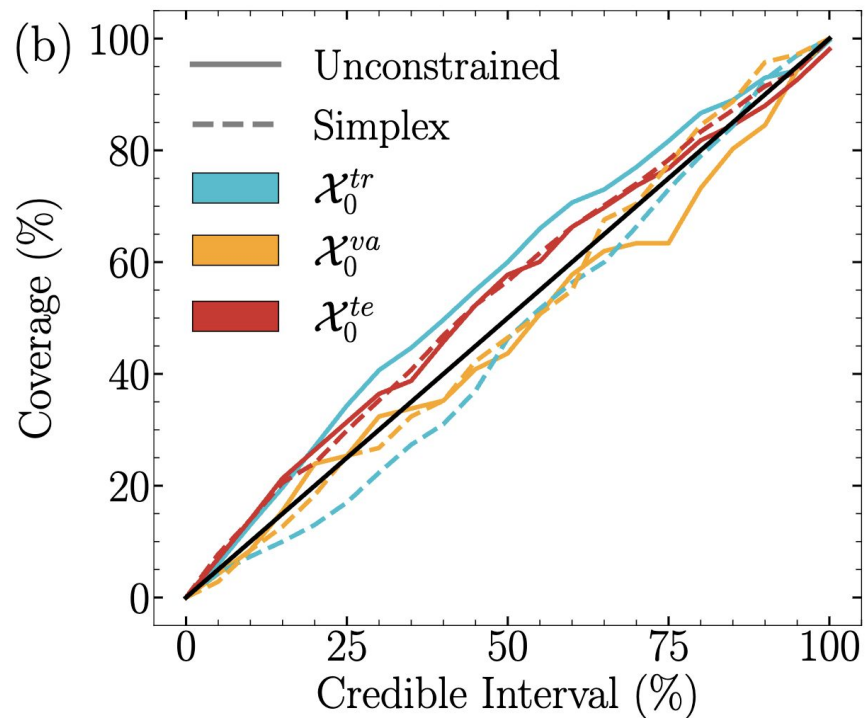


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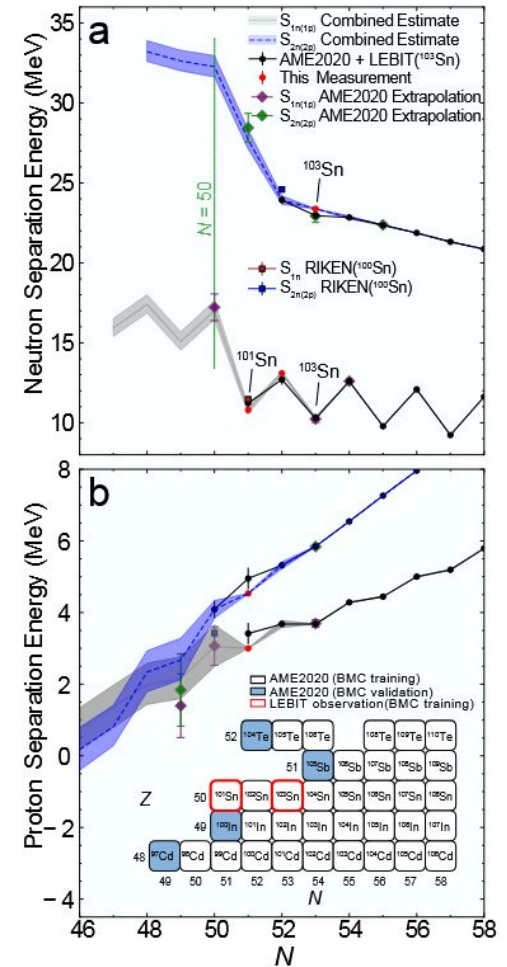
| Model                            | RMSE $\mathcal{X}_0^{\text{tr}}$ | RMSE $\mathcal{X}_0^{\text{va}}$ | RMSE $\mathcal{X}_0^{\text{te}}$ | RMSE $\mathcal{X}_0$ |
|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------|
| DD-ME2 [39]                      | 2.47                             | 2.48                             | 2.25                             | 2.38                 |
| DD-ME $\delta$ [40]              | 2.46                             | 2.19                             | 2.18                             | 2.32                 |
| DD-PC1 [41]                      | 1.94                             | 1.77                             | 2.19                             | 2.03                 |
| NL3* [42]                        | 2.18                             | 2.15                             | 3.59                             | 2.84                 |
| SkM* [43]                        | 4.91                             | 6.34                             | 9.75                             | 7.42                 |
| SkP [44]                         | 3.06                             | 3.50                             | 4.41                             | 3.72                 |
| SLy4 [45]                        | 4.53                             | 4.84                             | 6.27                             | 5.34                 |
| SV-min [46]                      | 2.97                             | 2.99                             | 3.98                             | 3.42                 |
| UNEDF0 [47]                      | 1.47                             | 2.02                             | 1.51                             | 1.56                 |
| UNEDF1 [48]                      | 1.82                             | 1.83                             | 2.06                             | 1.92                 |
| UNEDF2 [49]                      | 1.83                             | 1.66                             | 2.04                             | 1.90                 |
| FRDM-12 [38]                     | 0.62                             | 0.62                             | 0.65                             | 0.63                 |
| HFB-24 [37]                      | 0.52                             | 0.51                             | 0.52                             | 0.52                 |
| BCPM [50]                        | 2.57                             | 2.34                             | 2.44                             | 2.49                 |
| D1M [51]                         | 5.02                             | 4.91                             | 5.63                             | 5.27                 |
| $f^\dagger(p=7)$                 | 0.38                             | 0.46                             | 0.47                             | 0.43                 |
| $f^\dagger(\text{simplex}, p=7)$ | 0.74                             | 0.69                             | 0.71                             | 0.72                 |



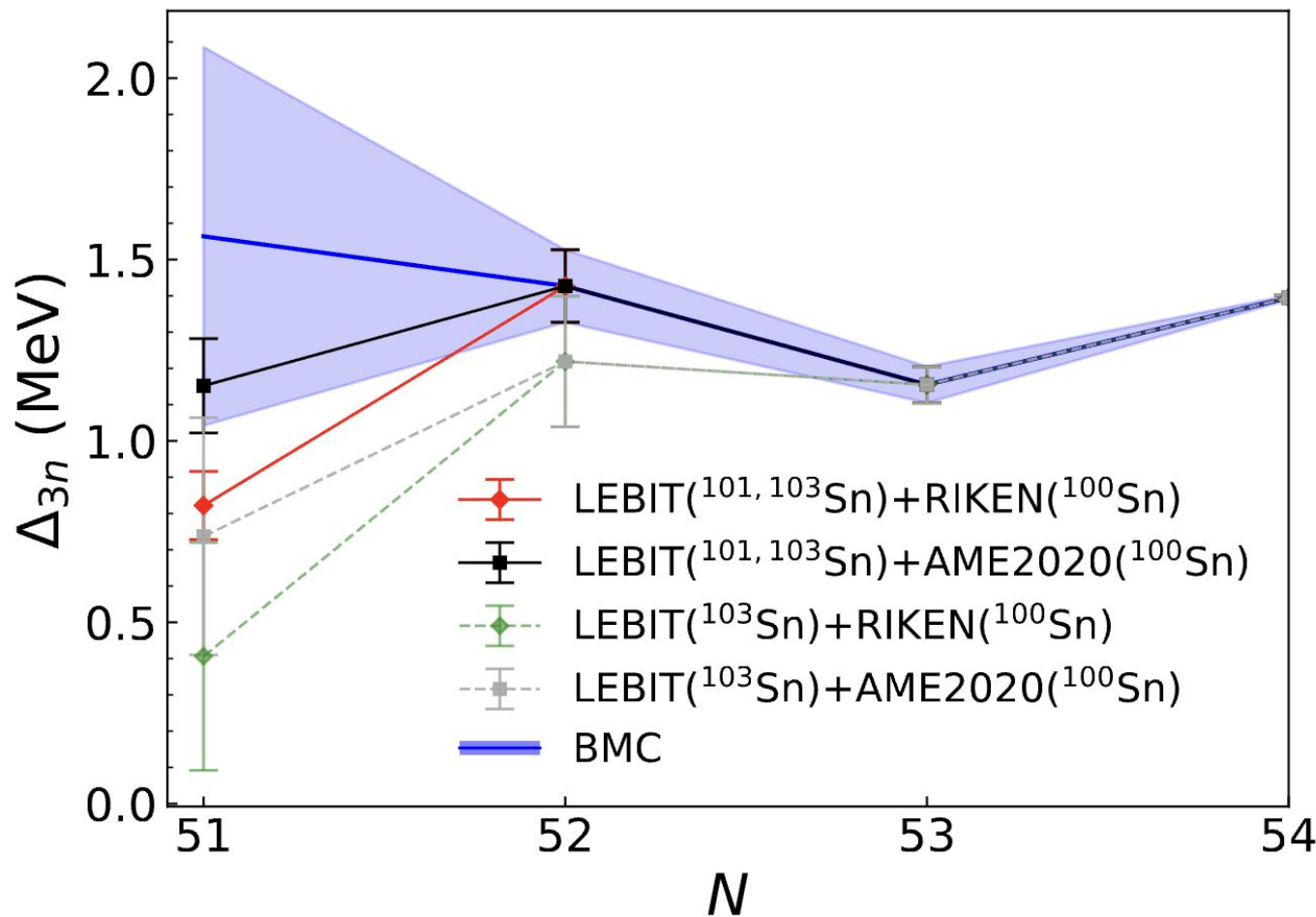
|                                  |      |      |      |      |
|----------------------------------|------|------|------|------|
| $f^\dagger(p=7)$                 | 0.38 | 0.46 | 0.47 | 0.43 |
| $f^\dagger(\text{simplex}, p=7)$ | 0.74 | 0.69 | 0.71 | 0.72 |



The combination scheme is \*very\* quick! New (or proposed) experimental data can be easily incorporated



The combined  
quick!  
data can



Combined Estimate  
Combined Estimate  
2020 + LEBIT( $^{103}\text{Sn}$ )  
Measurement  
AME2020 Extrapolation  
AME2020 Extrapolation

$^{100}\text{Sn}$   
RIKEN( $^{100}\text{Sn}$ )  
RIKEN( $^{100}\text{Sn}$ )

$^{103}\text{Sn}$

20 (BMC training)  
20 (BMC validation)  
observation (BMC training)

|    |                   |                   |                   |                   |
|----|-------------------|-------------------|-------------------|-------------------|
| Te | $^{128}\text{Te}$ | $^{129}\text{Te}$ | $^{130}\text{Te}$ | $^{131}\text{Te}$ |
| Sb | $^{125}\text{Sb}$ | $^{126}\text{Sb}$ | $^{127}\text{Sb}$ | $^{128}\text{Sb}$ |
| Sn | $^{122}\text{Sn}$ | $^{123}\text{Sn}$ | $^{124}\text{Sn}$ | $^{125}\text{Sn}$ |
| In | $^{119}\text{In}$ | $^{120}\text{In}$ | $^{121}\text{In}$ | $^{122}\text{In}$ |
| Cd | $^{116}\text{Cd}$ | $^{117}\text{Cd}$ | $^{118}\text{Cd}$ | $^{119}\text{Cd}$ |
| Zn | $^{114}\text{Zn}$ | $^{115}\text{Zn}$ | $^{116}\text{Zn}$ | $^{117}\text{Zn}$ |

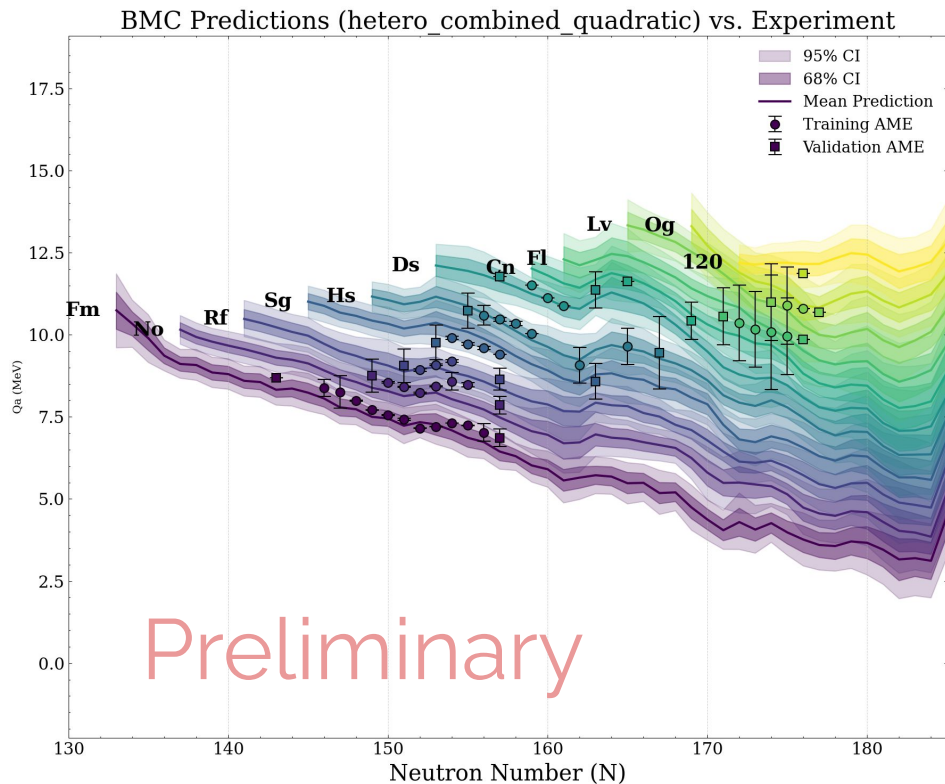
54 55 56 57 58

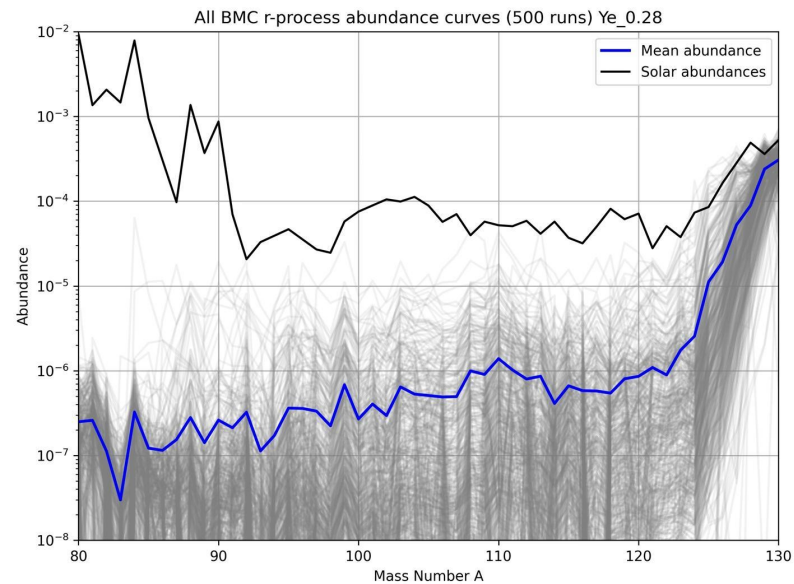
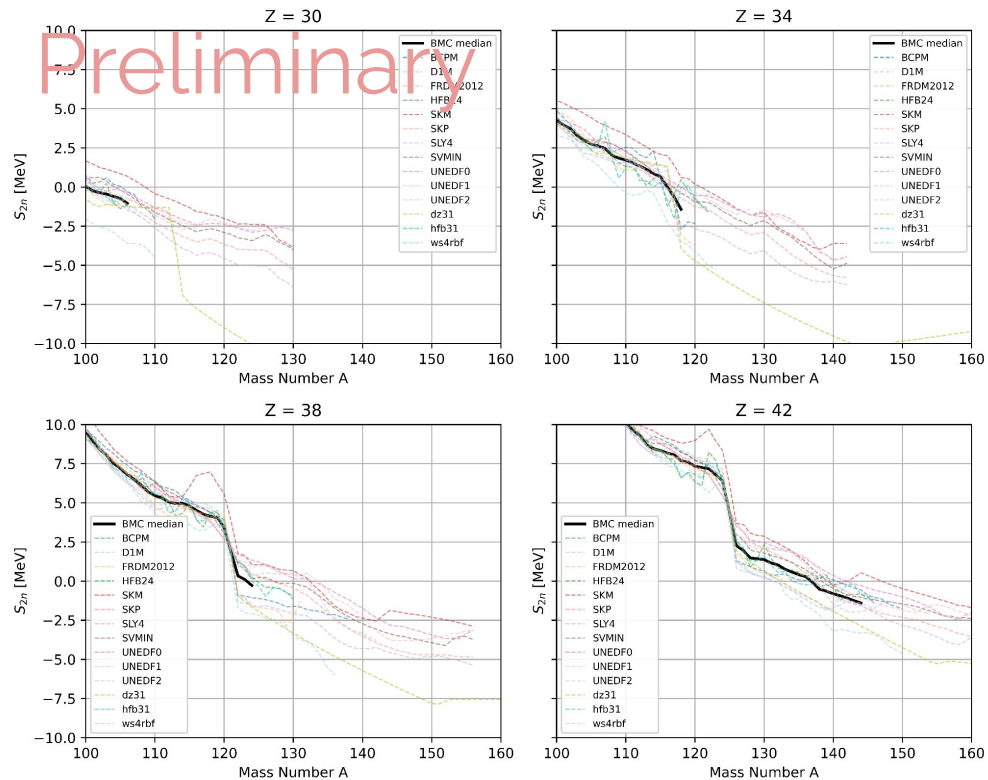


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<https://arxiv.org/pdf/2510.11815>





# Enter: pybmc

Lightweight Python package designed to streamline model orthogonalization and combination tasks

Currently, the core algorithm has been implemented by Undergraduate students Troy Dasher, An Le, and Pranav Agarwal



# Sidebar

Putting the effort into making (and releasing)  
software can pay off immensely!



# Wrapping Up: Known Issues

What if your original model space doesn't contain the right behavior (missing physics)?

Model contamination?

Should the error scale be global?

What about the use of global principal components?



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What if your original model space doesn't contain the right behavior (missing physics)?

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What about the use of global principal components?

Care to help?



# Wrapping Up: Known Issues

What if your original model space doesn't contain the right behavior (missing physics)?

Model contamination?

Should the error scale be global?

What about the use of global principal components?

Care to help?

New pull request





# ASCSN

Advanced Scientific Computing and Statistics Network



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# ASCSN

## Innovation Through Connection

A global community of researchers advancing the frontiers of computational science, statistics, and applied mathematics through collaboration.



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# GitHub

# YouTube



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Public



Repository for the 2023 FRIB-TA Summer School on practical uncertainty quantification and emulation!



Jupyter Notebook



6



56



**2024-FRIB-TA-Summer-School**

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Jupyter Notebook



7



21



**professionalwebsites**

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JavaScript



1



34



**bayesianprimer**

Public



Short ASCSN primer on Bayesian statistics



Jupyter Notebook



5



32



**theory-challenges**

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8



82



**nuclear-dimensionality-reduction-book**

Public



Jupyter Notebook



7



6



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# Dimensionality Reduction in Nuclear Physics

Presented by ASCSN



## Application 5: Black-Box Methods

Efficient Emulation of  
SECAR Beam

Non-linear and non-affine  
problem

<https://dr.ascsn.net>



# Dimensionality Reduction in Nuclear Physics

Presented by ASCSN

## Introduction to Dimensionality Reduction in Nuclear Physics

Introduction

Application 1: The Quantum Harmonic  
Oscillator

Application 2: Two body single channel  
nuclear scattering

Application 3: The Empirical  
Interpolation Method

Application 4: Time Dependent Systems  
(evolution in the reduced space)

Application 5: Black-Box Methods

Contributors



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## Dimensionality Reduction in Nuclear Physics

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### Application 5: Black-Box Methods

Efficient Emulation of  
SECAR Beam

Non-linear and non-affine  
problem

<https://dr.ascsn.net>

# Contributors #

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# Immense Gratitude to All Collaborators!

## Funding

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DOE Grant Nos. DE-SC0013365, DE-SC0023175

NSF CSSI Program No. 2004601

## Computing Resources

Australian National Computational Infrastructure Raijin and Gadi

Oak Ridge Leadership Computing Facility Summit and Frontier

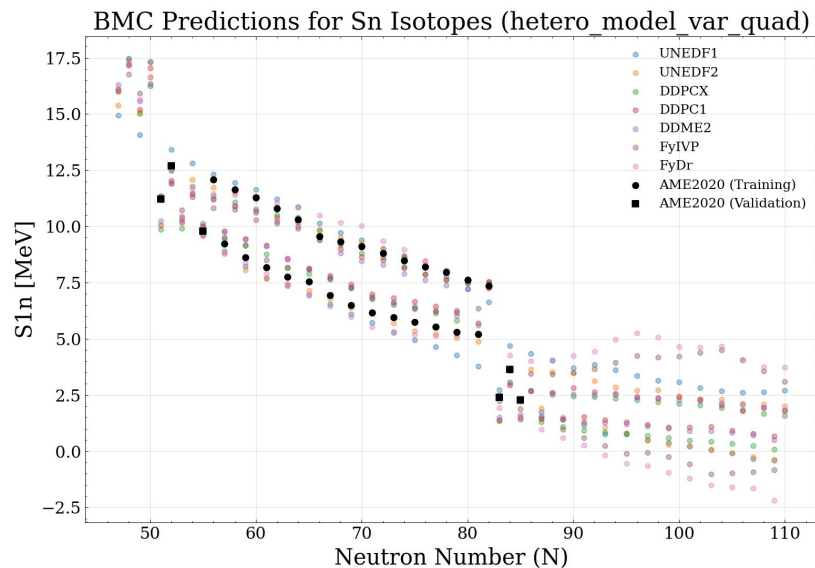
Argonne Leadership Computing Facility Polaris

Texas A&M High Performance Research Computing Terra and Ada

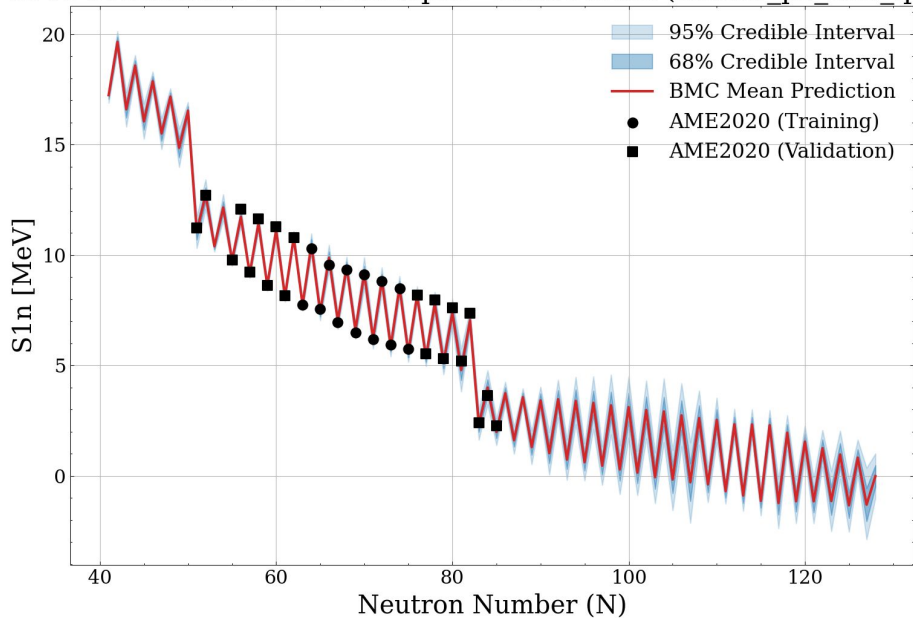
Michigan State University HPCC



# Heteroscedastic Uncertainty

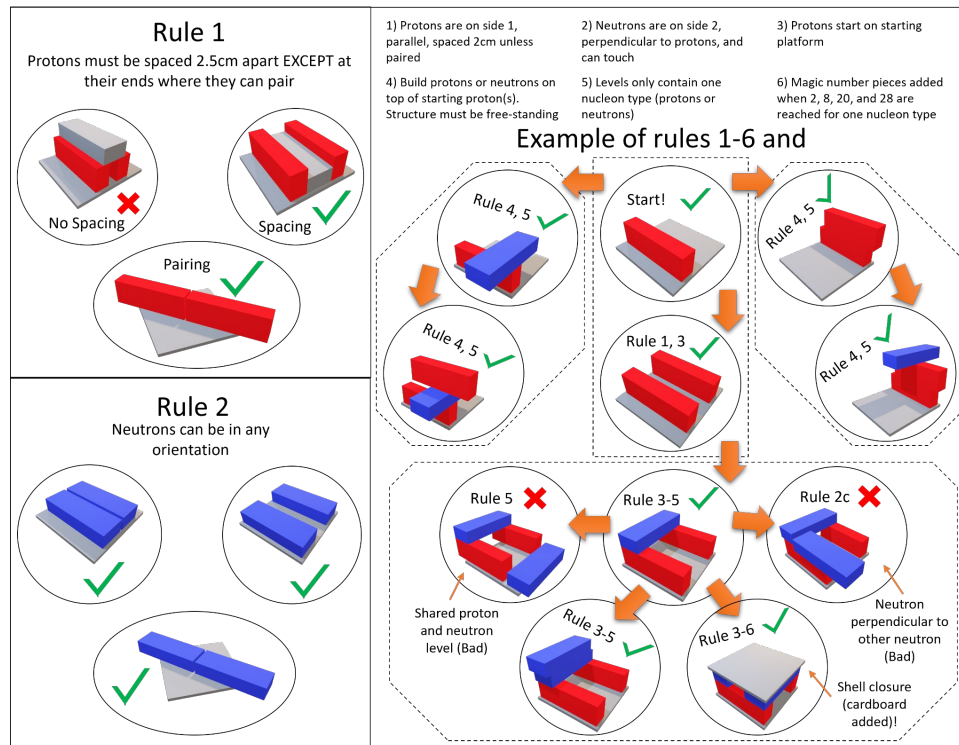


BMC Predictions for Sn Isotopes - Best Model (hetero\_pc\_dist\_quad)



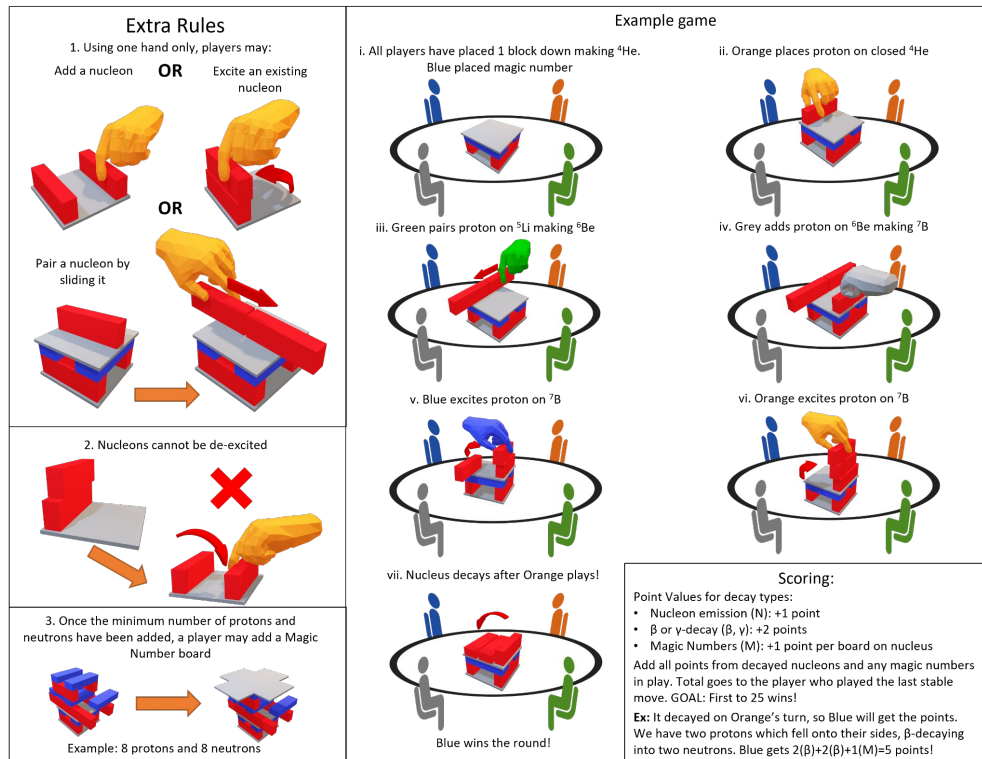
# Interface of Technology and Education

- We focus on highlighting structure with dynamic decays
- Rule sets for varying student level (younger students to adults)
  - Coulomb repulsion
  - Pairing
  - Nucleon Excitations
  - Magic numbers



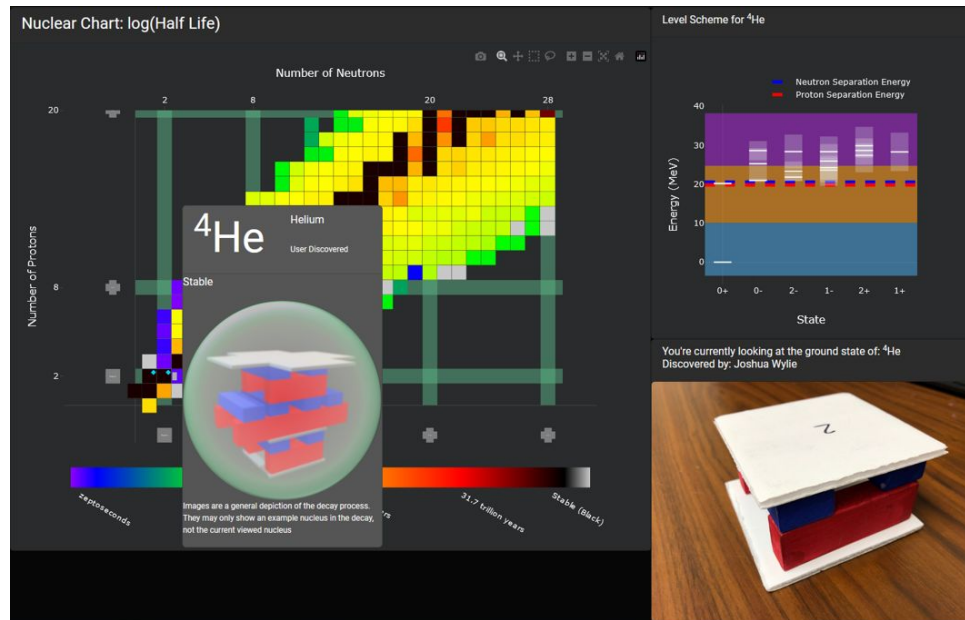
# Interface of Technology and Education

- We focus on highlighting structure with dynamic decays
- Alternate multiplayer rule set:
  - Highlights competing forces and effects that may make nuclei more unstable
  - Promotes focus on excitation or more proton or neutron-rich systems



# Interface of Technology and Education

- Website will be available:
  - User may click on a nucleus to see the relation between in-game view and real nuclear properties (a translation tool)
  - User-input for “discoveries” (blue diamonds) of ground state, excited state, and second excited state configurations
  - “Discovered” nuclear states are shown with user-submitted photos



# Interface of Technology and Education

- Website will be available:
  - User may click on a nucleus to see the relation between in-game view and real nuclear properties (a translation tool)
  - Supplement so users can see game representation of actual physical phenomena (decay modes)

