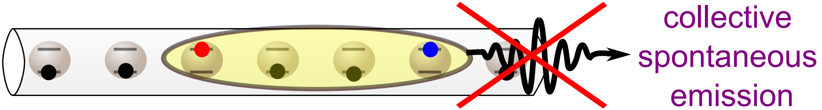
Subradiance in waveguide quantum electrodynamics

Jiaming Shi, Ran Tessler, **Alexander (Sasha) Poddubny**

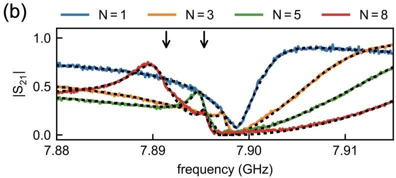
Weizmann Institute of Science



ECT Workshop on Superconducting Devices
for Quantum Optics and Quantum Simulations,
Trento, Oct 6-9 2025

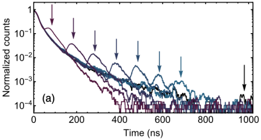
Subradiant states: experiments

Single-excited subradiant states



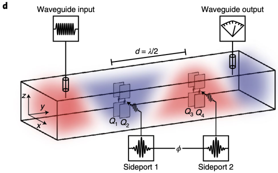
Brehm, ANP, Stehli, Wolz, Rotzinger, and Ustinov (KIT), npj Quantum Materials **6**, 10 (2021)

Storing and releasing light in subradiant states



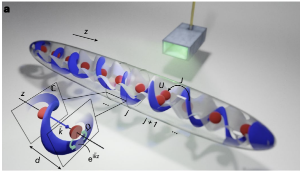
Ferioli, Glicenstein, Henriet, Ferrier-Barbut and Browaeys (CNRS), PRX **11**, 021031 (2021)

Double-excited subradiant states

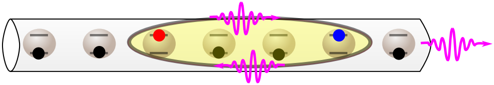


Zanner, Orell, Schneider, Albert, Oleschko, Juan, Silveri and Kirchmair (Innsbruck),Nat. Phys. **18**, 538 (2022)

Subradiance with matter waves in trapped atom arrays



Kim, Lanuza and Schneble (Stony Brook), Nat. Physics **21** 70 (2025)

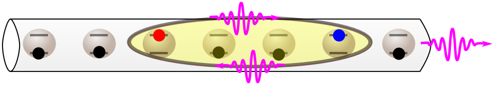


Generic Hamiltonian (1D, no polarization degrees of freedom)

$$H = \underbrace{\sum_{j=1}^{N} \hbar\omega_j \sigma_j^\dagger \sigma_j}_{\text{2-level atoms}} + \underbrace{\sum_k c|k| a_k^\dagger a_k}_{\text{photons}} + \underbrace{\sum_{k,j} (g_k e^{ikz_j} a_k^\dagger \sigma_j + g_k^* e^{-ikz_j} \sigma_j^\dagger a_k)}_{\text{coupling}}$$

lowering $\sigma_j |e_j\rangle = |g_j\rangle$ and raising $\sigma_j^\dagger |g_j\rangle = |e_j\rangle$ operators

Atom-Photon Interactions



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Eliminating photons ($c/|z_j - z_{j'}| \ll g^2/c$) we obtain an effective non-Hermitian Hamiltonian:

$$H_{\text{eff}} = \underbrace{\sum_{j=1}^{N} \hbar\omega_j \sigma_j^\dagger \sigma_j}_{\text{2-level atoms}} - \underbrace{\mathrm{i}\gamma_{1\text{D}} \sum_{j,j'=1}^{N} \sigma_j^\dagger \sigma_{j'} \mathrm{e}^{\mathrm{i}\omega_0|z_j - z_{j'}|/c}}_{\text{long-range coupling of atoms via photons}}$$

photon-mediated interactions are **long-ranged and dissipative**

Polariton-polariton interactions in double-excited sector

$$H_{\text{eff}} = -\mathrm{i}\gamma_{1\text{D}} \sum_{m,n=1}^{N} \sigma_m^\dagger \sigma_n \mathrm{e}^{\mathrm{i}\omega_0 |z_m - z_n|/c}, \quad H_{\text{eff}}\psi = \varepsilon\psi$$

$$\psi = \sum_{m,n=1}^{N} \psi_{nm} \sigma_n^\dagger \sigma_m^\dagger |0\rangle \ , \qquad \psi_{nm} \xrightarrow{\text{Fourier}} \psi(k_1, k_2)$$

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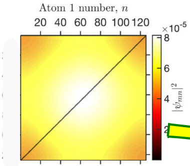
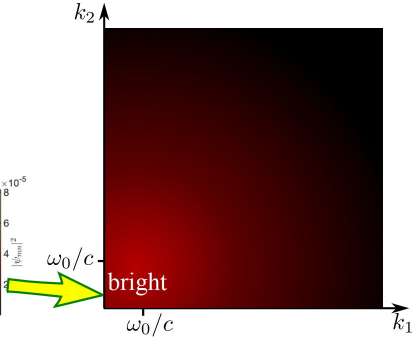


Superradiant state,
Dicke (1954)



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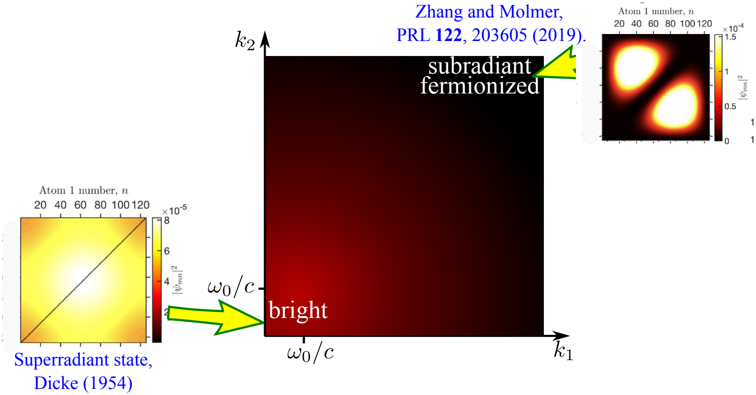

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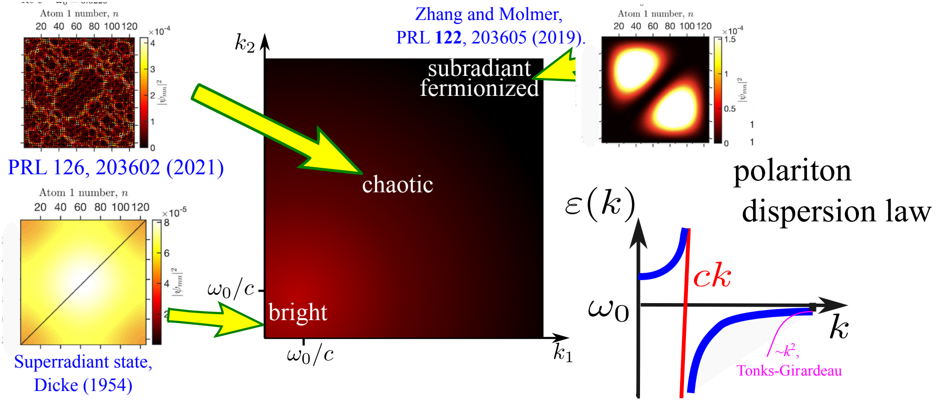

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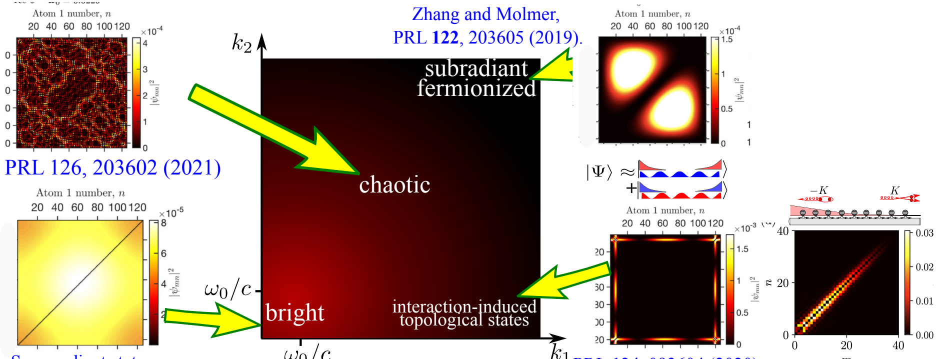

PRL 126, 203602 (2021)

Superradiant state, Dicke (1954)

Zhang and Molmer, PRL **122**, 203605 (2019).

PRL **124**, 093604 (2020)
Quant. Information **7**,34(2021)

PRL **134**, 233602(2025)

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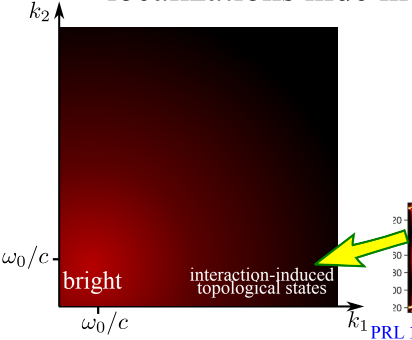
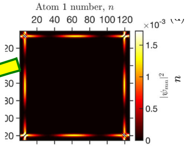
localizations hide in corners





PRL **124**, 093604 (2020)
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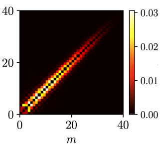

PRL **134**, 233602(2025)

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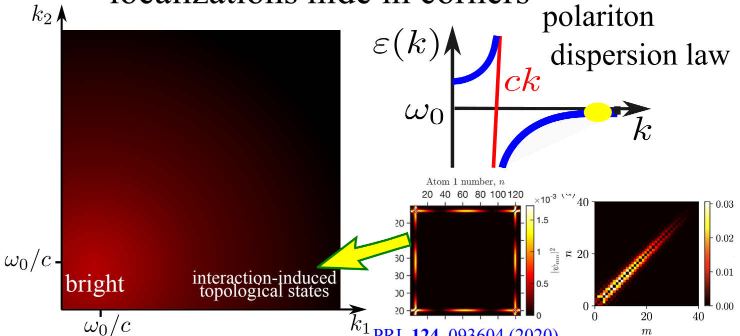

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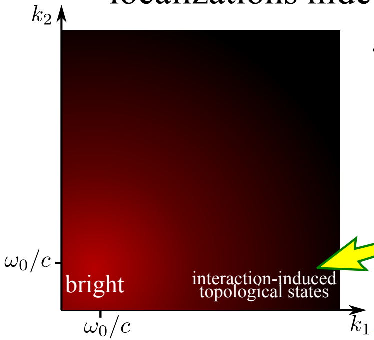

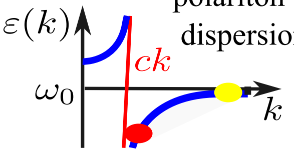

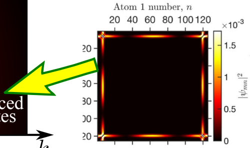

interaction-induced Aubry-Andre-Harper

no external magnetic fields or modulations!

PRL **124**, 093604 (2020)
Quant. Information **7**,34(2021)

What should we study?

Non-Hermitian H_{eff}:

$$H_{\text{eff}}\psi = \varepsilon\psi, \varepsilon = \omega - \underbrace{\mathrm{i}\Gamma}_{\text{decay rate}}$$

Collective dissipation:

$$H_{\text{eff}} - H_{\text{eff}}^{\dagger} = -\mathrm{i}\sum_{R} R^{\dagger}R$$

$$R_{\pm} = \sqrt{\gamma}\sum_{j}\sigma_j \mathrm{e}^{\mathrm{i}\omega_0 z_j/c}$$

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Where to look?

Lindblad Master equation:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \mathcal{L}\rho \equiv \mathrm{i}(\rho H_{\rm eff} - H_{\rm eff}^\dagger\rho) + \sum_R R\rho R^\dagger$$

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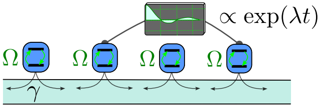

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- ~~ground state~~ (for 2-level atoms is trivial $|g_1\dots g_N\rangle$)
- brightest (shortest-living) state ? $\langle\psi|R^\dagger R|\psi\rangle\to\max$ (Dicke superradiance)
- darkest (longest-living) state ! $\langle\psi|R^\dagger R|\psi\rangle\to\min$ **(subradiance)**, e.g. $|\psi\rangle=(\sigma_1^\dagger-\sigma_2^\dagger)|0\rangle$

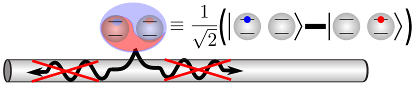
What is the most subradiant state under given conditions ?
including interactions (=potential anharmonicity), driving, disorder

Subradiant states and subradiant correlations

- Subradiant dimer excitation $\frac{1}{\sqrt{2}}(b_1^\dagger - b_2^\dagger)|0\rangle$

$$\psi = \psi_1\sigma_1^\dagger|0\rangle + \psi_2\sigma_2^\dagger|0\rangle, \quad -\mathrm{i}\gamma_{1\mathrm{D}}\begin{pmatrix} 1 & \mathrm{e}^{\mathrm{i}\omega_0 d/c} \\ \mathrm{e}^{\mathrm{i}\omega_0 d/c} & 1 \end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \omega\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

$$-\operatorname{Im}\omega_- = \gamma_{1\mathrm{D}}(1-\cos\frac{\omega_0 d}{c})$$



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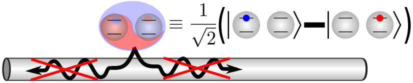
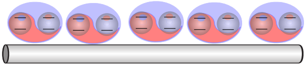


- Weakly-excited most-subradiant states are fermionized standing waves

Y.-X. Zhang and K. Mølmer, PRL **122**, 203605 (2019)

- Multiple-excited most subradiant states are dimer products with fill-factor$\leq 1/2$



PRL **123**, 253601 (2019); **127**, 173601 (2021); PRA **110**, 110, 053707 (2024)

- **Subradiant eigenstates of Liovillian superoperator $\mathcal{L}$: this talk**

Correlations in driven-dissipative dynamics

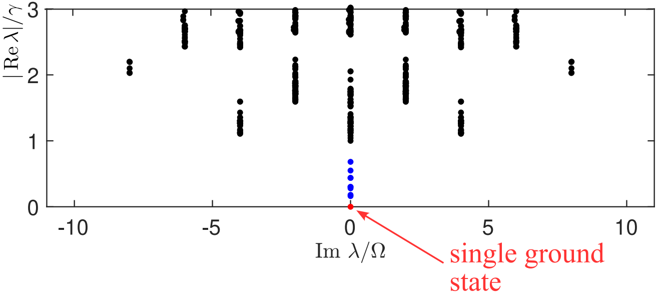
Lindblad master equation: $\partial_t \rho = \mathcal{L}\rho$

$$\mathcal{L}\rho = \mathcal{L}_0\rho + \underbrace{\mathrm{i}[\rho, V]}_{\text{driving}}, \quad V = \Omega \sum_{n=1}^{N} (\sigma_n^\dagger + \mathrm{H.c.})$$

Eigenstates of Lindbladian superoperator: $\mathcal{L}\rho = \lambda\rho$,
Stationary state: $\mathrm{Re}\,\lambda = 0$.

We look for long-living correlations, $|\,\mathrm{Re}\,\lambda| \ll \gamma_{\mathrm{1D}}$

Spectrum of Liovillian Superoperator at Strong Driving



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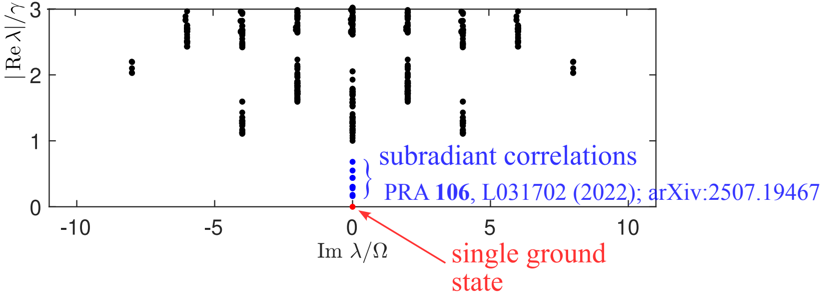

If $\mathrm{Im}\,\lambda/\Omega = 0$ one can have $\mathrm{Re}\,\lambda \ll \gamma$,
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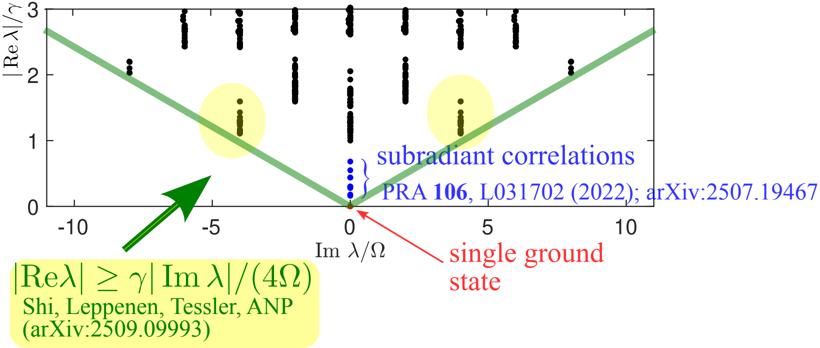

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Theorem: $|\,\mathrm{Re}\,\lambda| \geq \gamma|\,\mathrm{Im}\,\lambda|/4\Omega$,

Diagonal+Laplacian representation: $-\mathcal{L} = \underbrace{D}_{\text{diagonal, pos. def.}} + B^T B$

Similar-looking bounds in stat mech: Ohga et al., PRL **131**, 077101 (2023)

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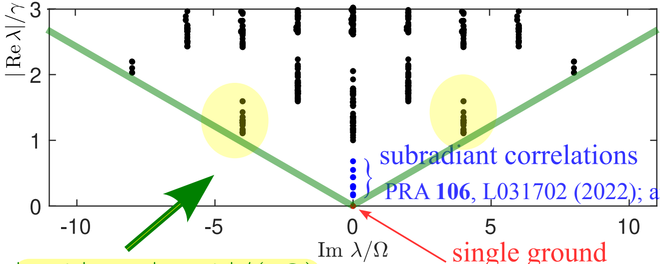

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Shi, Leppenen, Tessler, ANP (arXiv:2509.09993)

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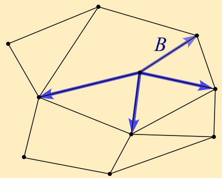
Laplacian of a graph, $L = B^T B$



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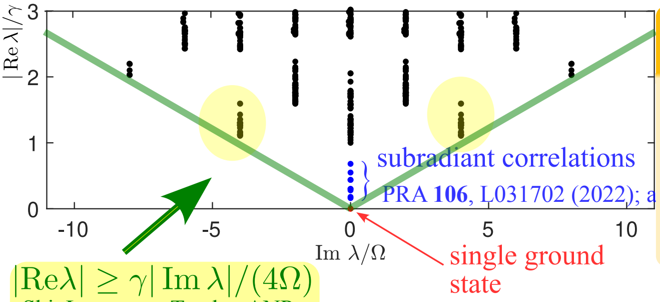

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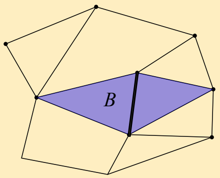
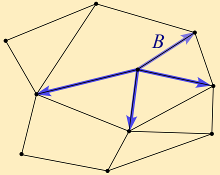
Laplacian of a simplicial complex



Laplacian of a graph, $L = B^T B$



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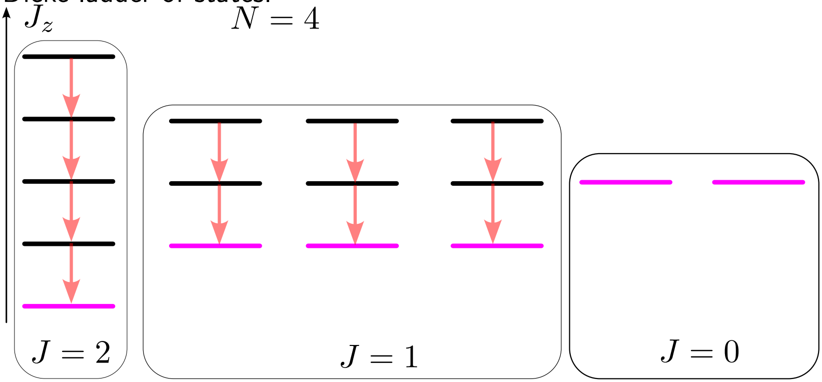
Subradiance is incompable with Rabi flips

$$J_z = \sum_m (\sigma_m^\dagger \sigma_m - \frac{1}{2}), \quad J_x = \frac{1}{2} \sum_m (\sigma_m^\dagger + \sigma_m)$$

Dicke ladder of states.



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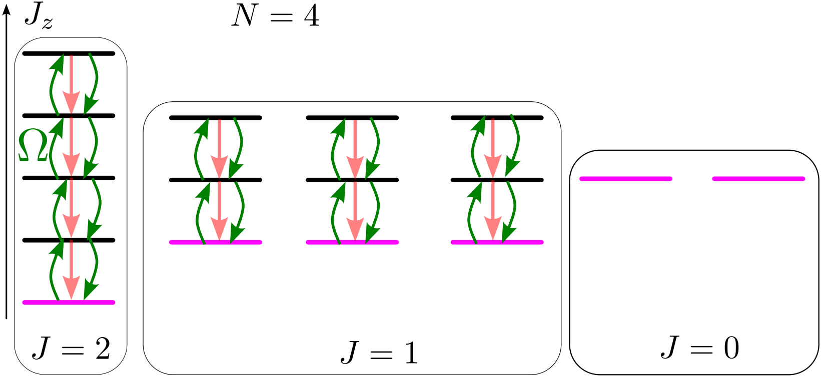
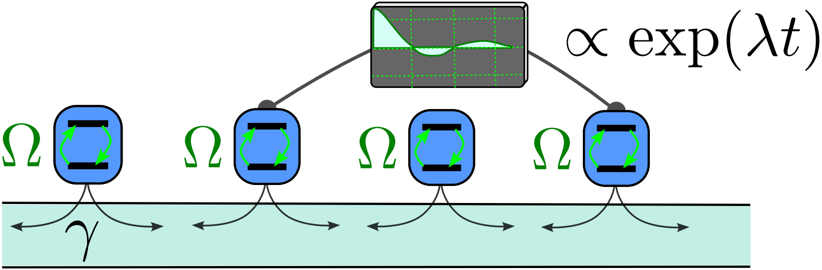
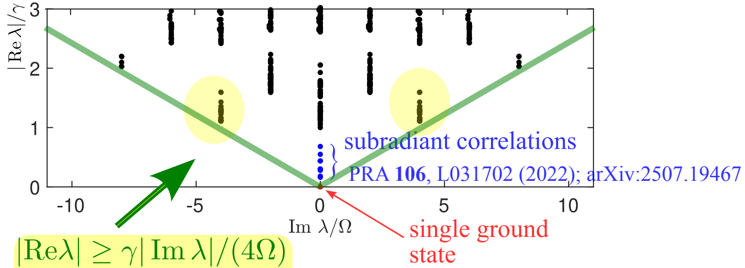


Driving mixes collective states with different number of excitations, which is incompatible with destructive interference





$|\mathrm{Re}\lambda| \geq \gamma |\,\mathrm{Im}\,\lambda|/(4\Omega)$

Shi, Leppenen, Tessler, ANP (arXiv:2509.09993)

Review on waveguide QED: Rev. Mod. Phys. **95**, 0105002 (2023)