Simulating Quantum Impurity Models in High Impedance Circuits

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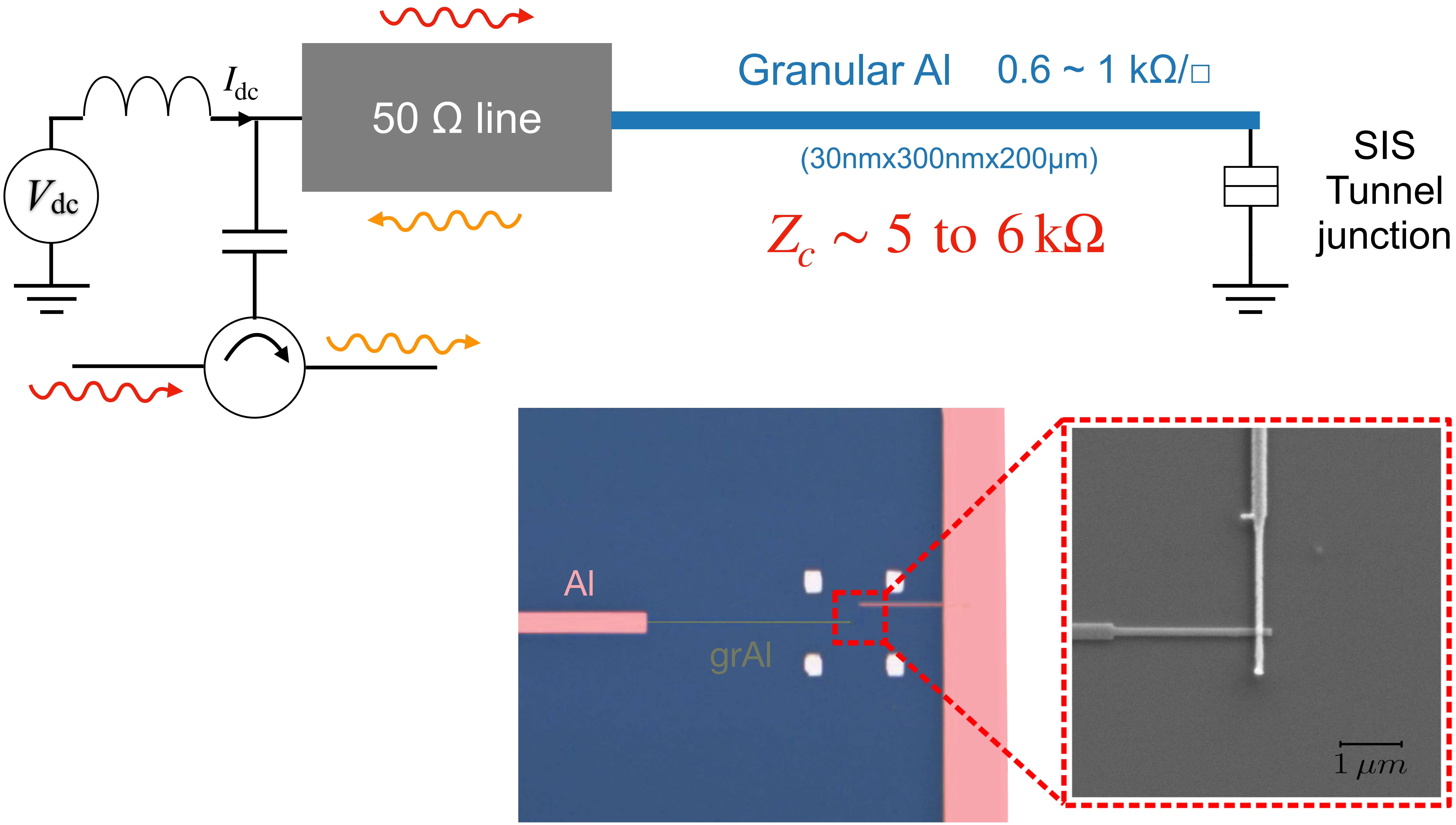
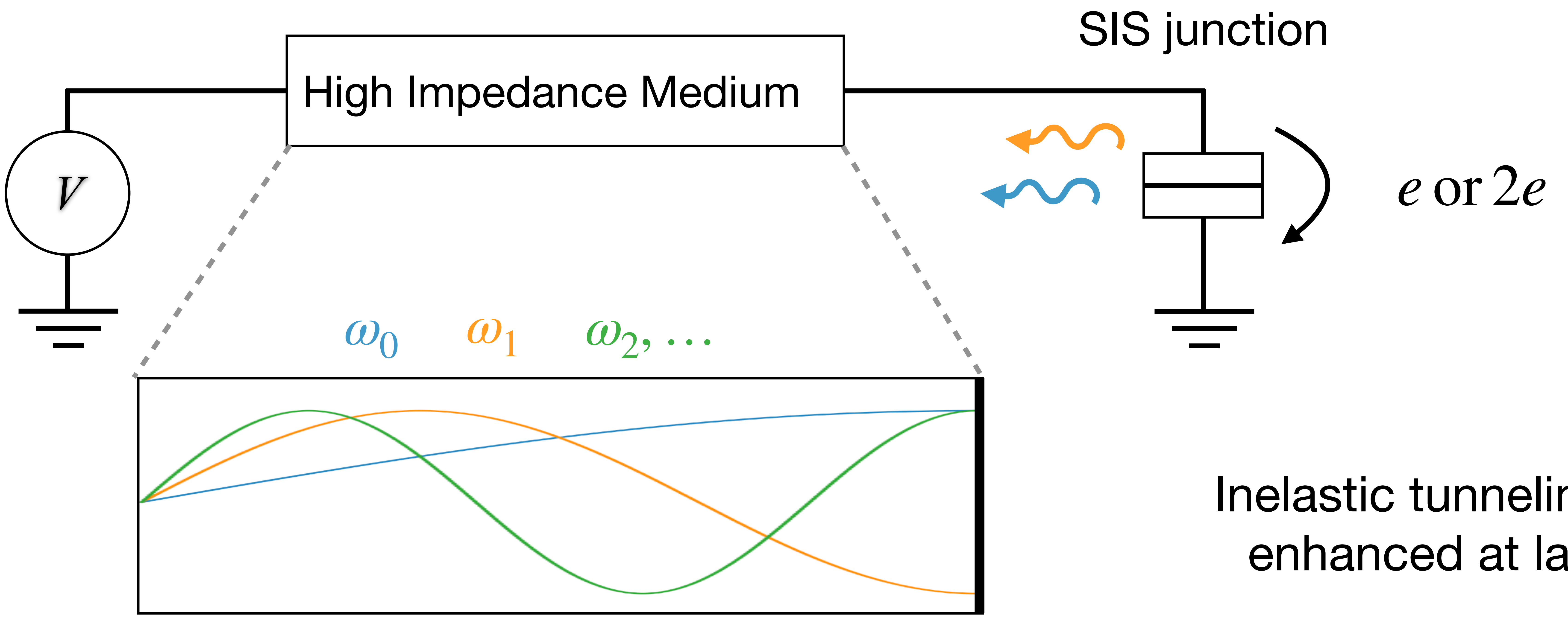
Sample & Setup

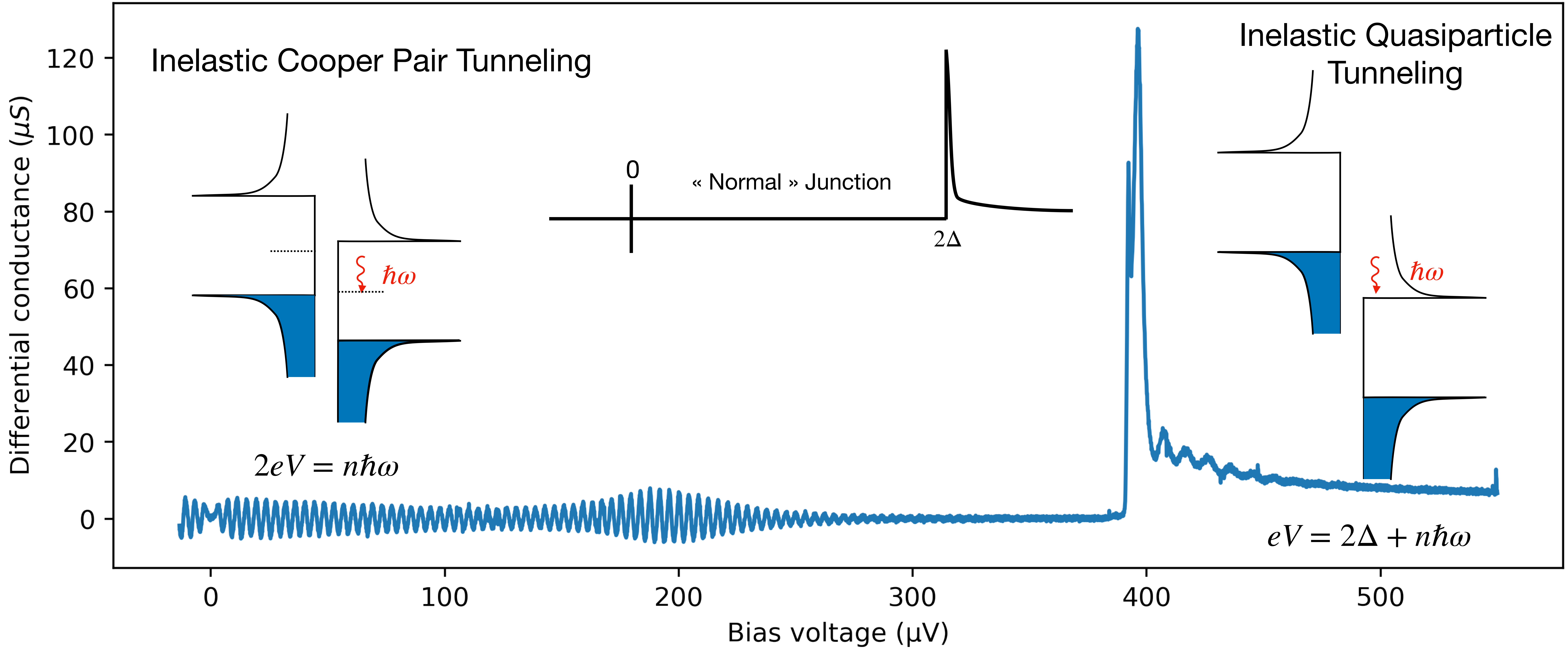


Electronic Transport in a High Impedance Environment

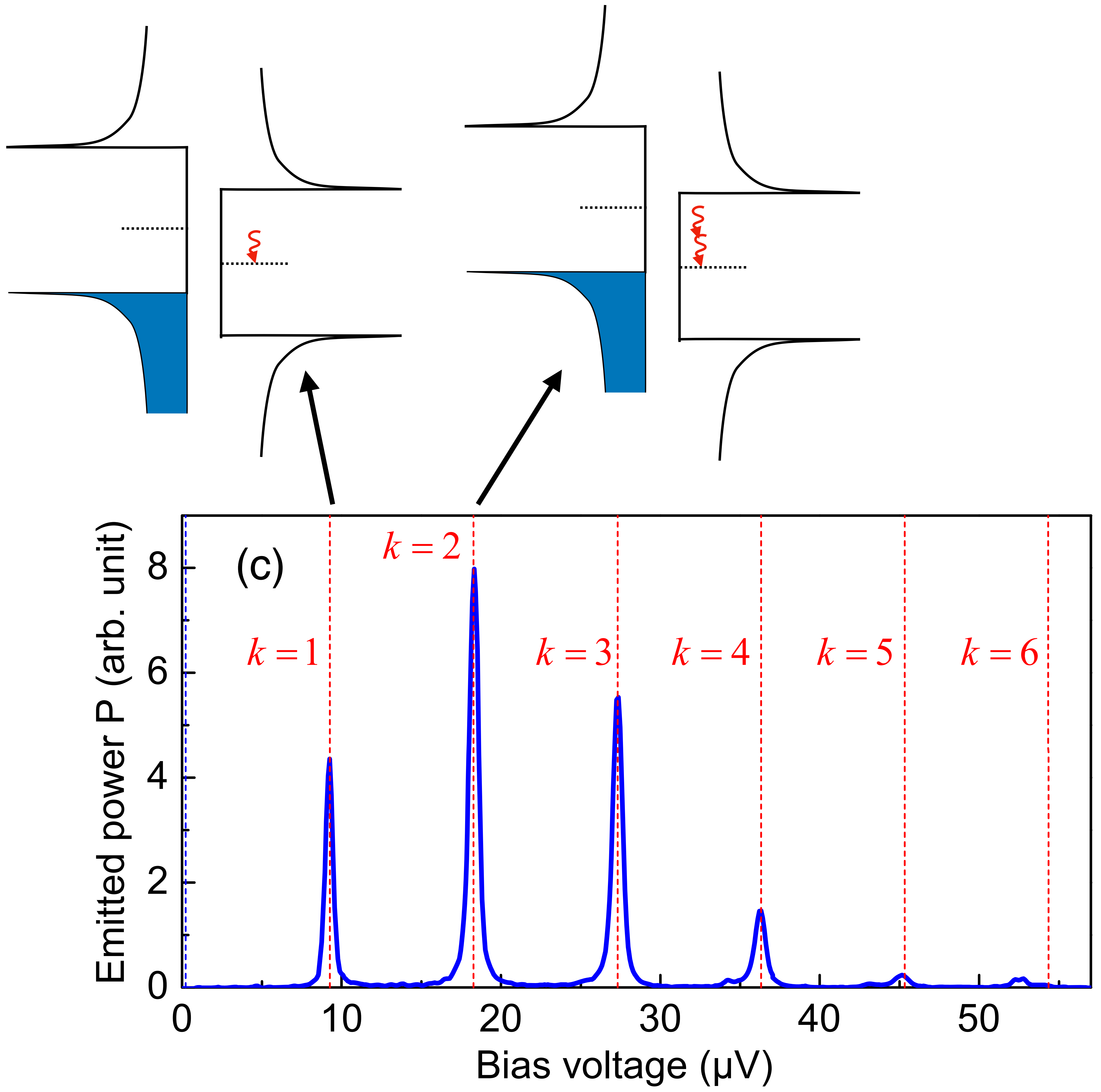


Inelastic tunneling processes are enhanced at large impedance

Junction Conductance



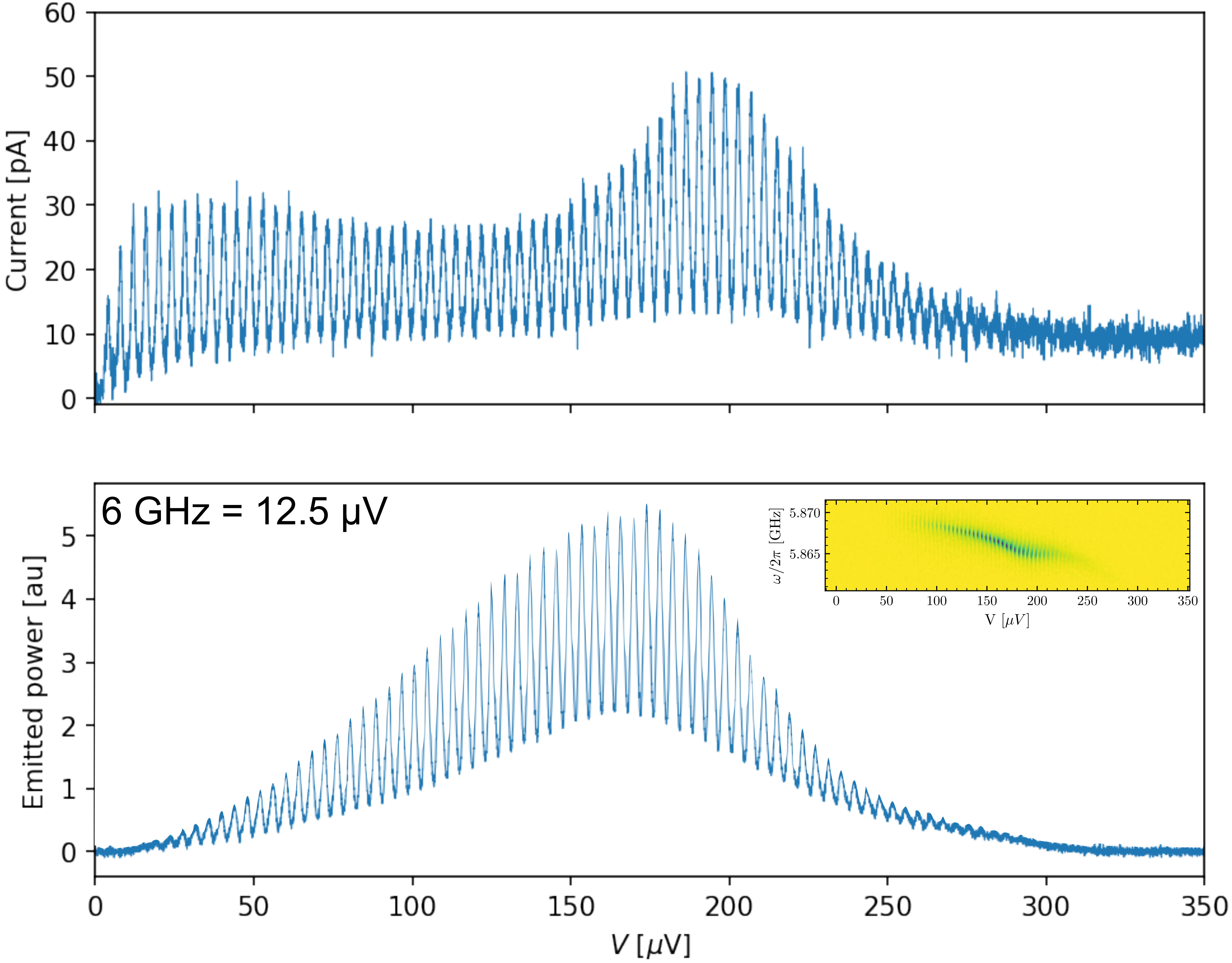
Inelastic Cooper Pair Tunneling in a Single Mode Environment

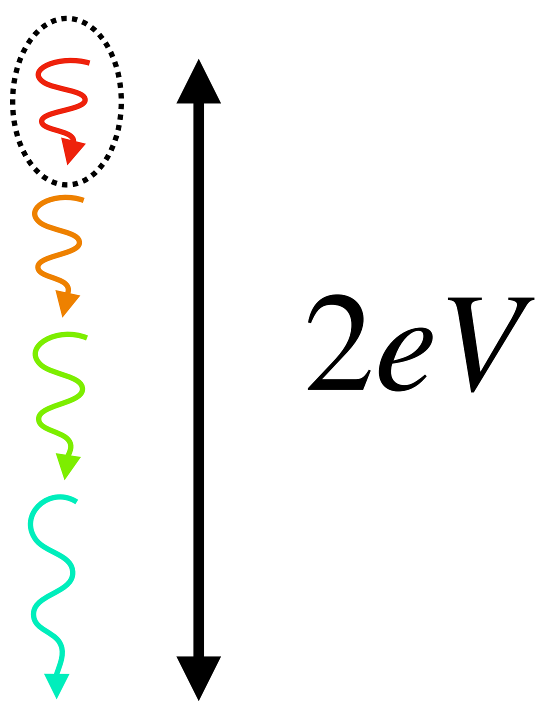


One Cooper pair gives k photons when $2eV = k\,\hbar\omega$

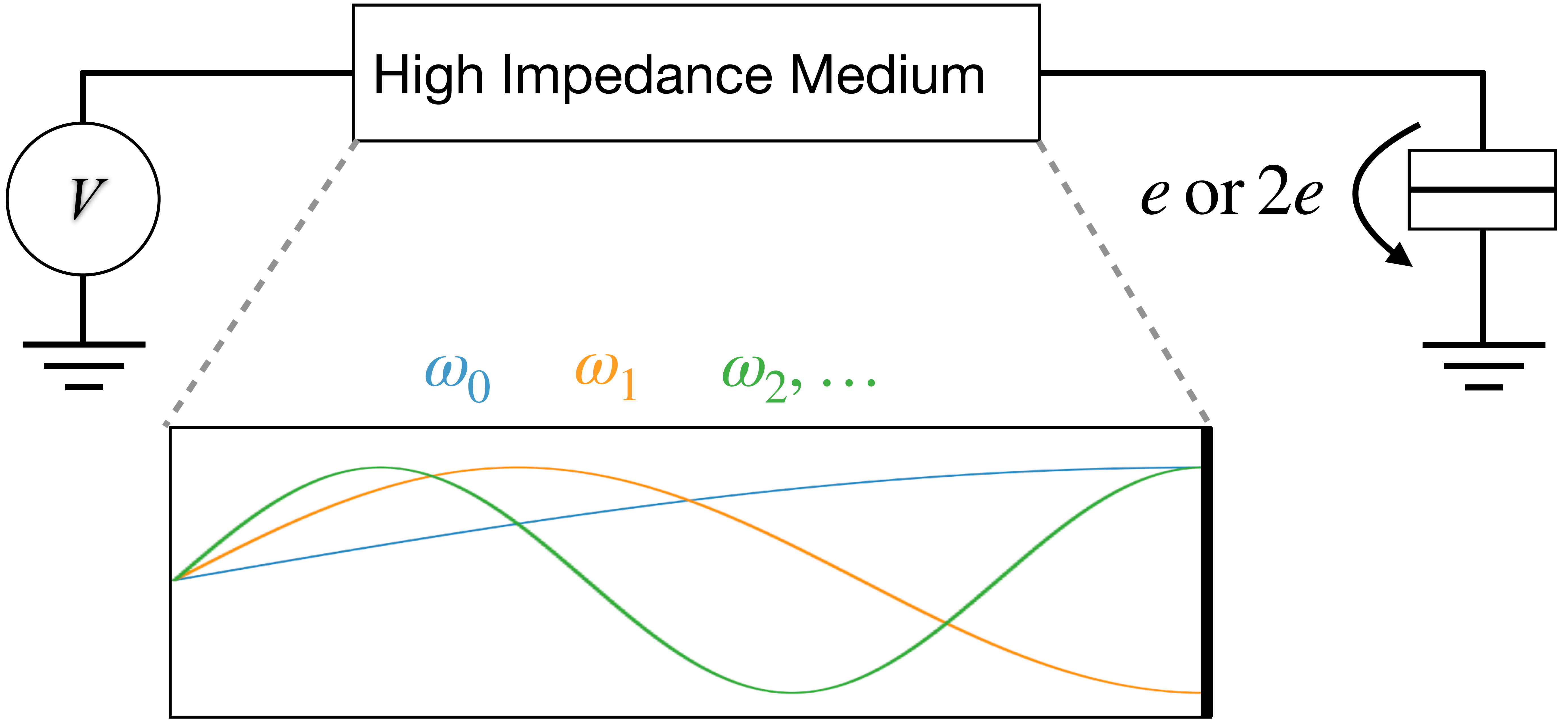
Hofheinz et al. PRL **106**, 217005 (2011)
Rolland et al. PRL **122**, 186804 (2019)
Ménard et al. PRX **12**, 021006 (2022)

Inelastic Cooper Pair Tunneling in a Multimode Environment





Electronic Transport in a High Impedance Environment



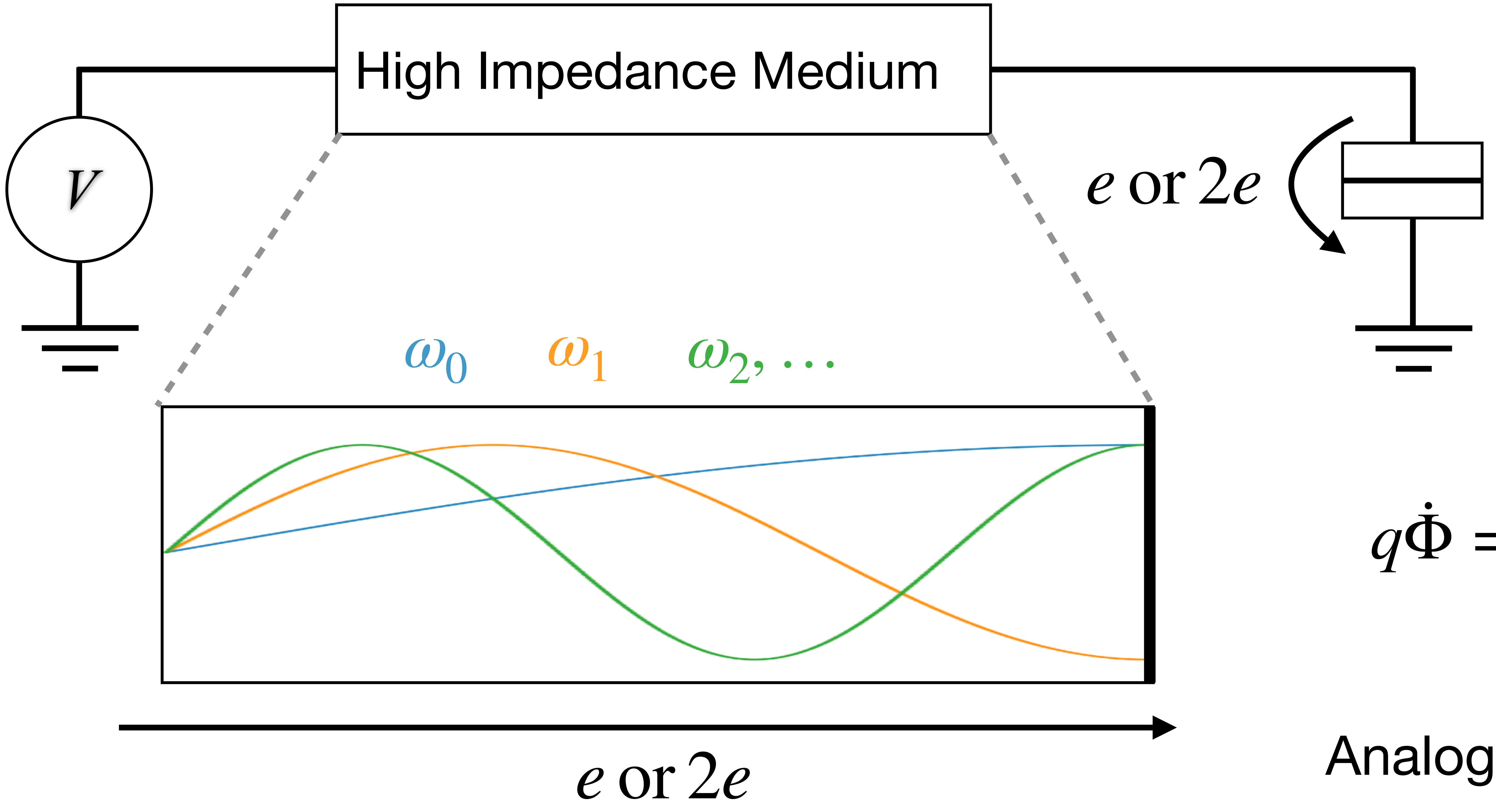
$$\Phi = Vt + \frac{\Phi_0}{2\pi} \sum_m \lambda_m (a_m + a_m^\dagger)$$

Light Matter coupling constant

$$\lambda_m^2 = \frac{4Z_c/R_K}{2m+1}$$

$Z_c = 5 \text{ to } 6\,\mathrm{k}\Omega \Rightarrow$ Strong coupling regime $\lambda_m \sim 1$

Electronic Transport in a High Impedance Environment



$$\Phi = Vt + \frac{\Phi_0}{2\pi} \sum_m \lambda_m (a_m + a_m^\dagger)$$

Light-Matter Coupling :

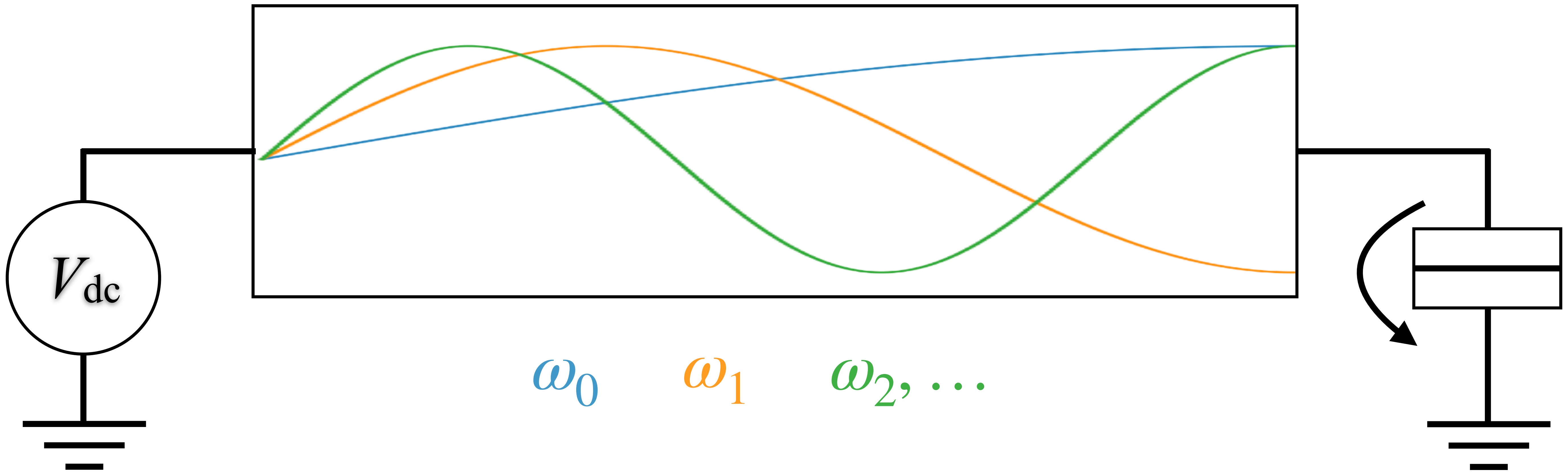
$$q\dot{\Phi} = -e \sum_{kq} (c_k^\dagger c_k - d_q^\dagger d_q) \sum_m i\omega_m \lambda_m (a_m - a_m^\dagger)$$

Analogous to electron-phonon coupling (Holstein)

Tunneling : $c_k^\dagger d_q + \mathrm{hc}$ $\xrightarrow[\text{transform}]{\text{Polaron}}$ $c_k^\dagger \hat{d}_q e^{ie\hat{\Phi}/\hbar} + \mathrm{hc} = \hat{c}_k^\dagger \hat{d}_q e^{\sum_m i\lambda_m(\hat{a}_m + \hat{a}_m^\dagger) + ieVt/\hbar} + \mathrm{hc}$

Charge translation of the environment modes

Electronic Transport in a High Impedance Environment



$$H = H^L_{BCS} + H^R_{BCS} + H_T + \sum_m \hbar\omega_m a^\dagger_m a_m$$

$$H_T = \sum_{kq} t_{kq} \hat{c}^\dagger_k \hat{d}_q e^{\sum_m i\lambda_m(\hat{a}_m + \hat{a}^\dagger_m) + ieVt/\hbar} + \mathrm{hc}$$

Inelastic Cooper pair tunneling at low energy :

$$H = \sum_m \hbar\omega_m a^\dagger_m a_m - E_J \cos\Big[\sum_m \lambda^{(\mathrm{CP})}_m (a^\dagger_m + a_m) + 2eVt/\hbar\Big]$$

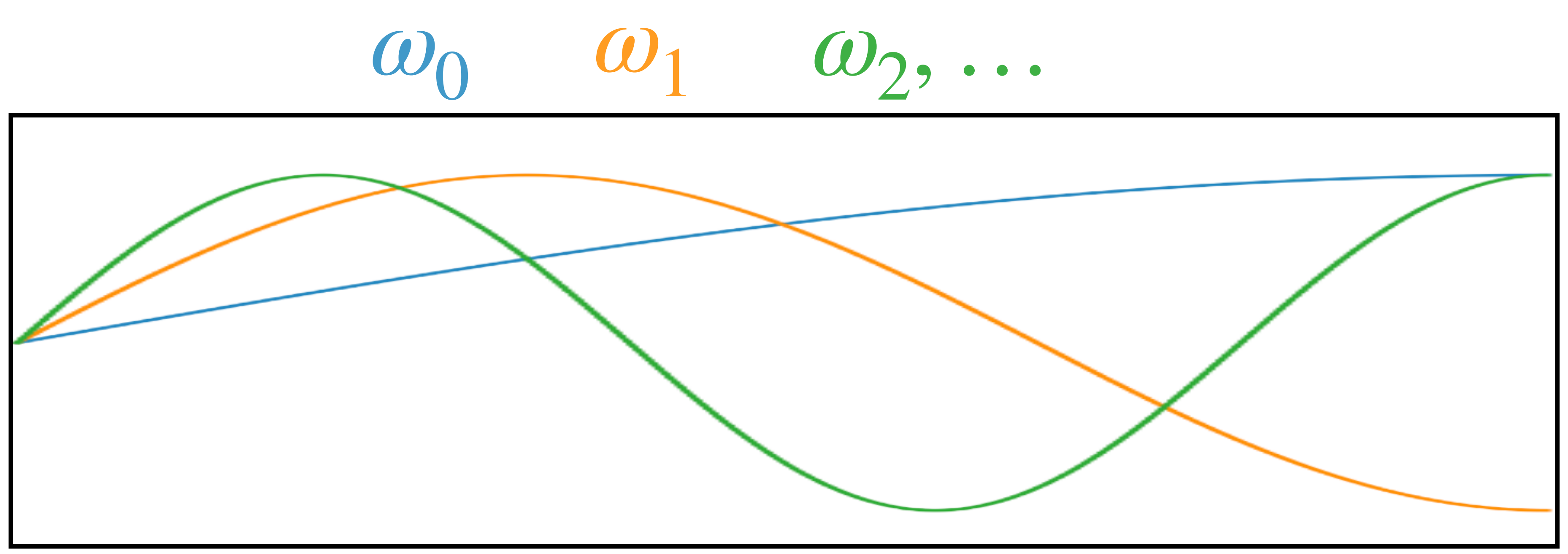
$$\lambda^{(\mathrm{CP})} = 2\lambda$$

Inelastic QP tunneling at high energy :

$$H = \sum_m \hbar\omega_m a^\dagger_m a_m + \sum_k \epsilon_k c^\dagger_k c_k + \sum_q \epsilon_q d^\dagger_q d_q$$

$$+ \sum_{kq} t_{kq} \hat{c}^\dagger_k \hat{d}_q e^{\sum_m i\lambda_m(\hat{a}_m + \hat{a}^\dagger_m) + ieVt/\hbar} + \mathrm{hc}$$

Perturbation Theory For Inelastic CP Tunneling



Ideal quarter wavelength configuration:

$$\omega_m = (2m+1)\omega_0 \qquad \lambda_m^2 = \frac{\lambda^2}{2m+1} \qquad \Lambda^2 = \sum_m \frac{\lambda^2}{2m+1}$$

Phase correlation function:

$$\langle e^{i\hat{\phi}(t_1)} e^{-i\hat{\phi}(t_2)} \rangle \approx e^{-\Lambda^2} \sum_n \frac{n^{\lambda^2/2-1}}{\Gamma(\lambda^2/2)} e^{in\omega_0(t_2-t_1)}$$

First order in E_J aka $P(E)$ theory

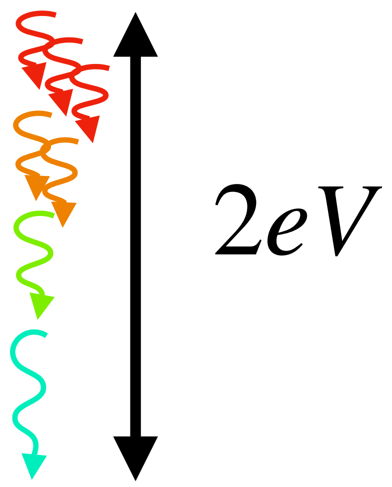
Inelastic current:

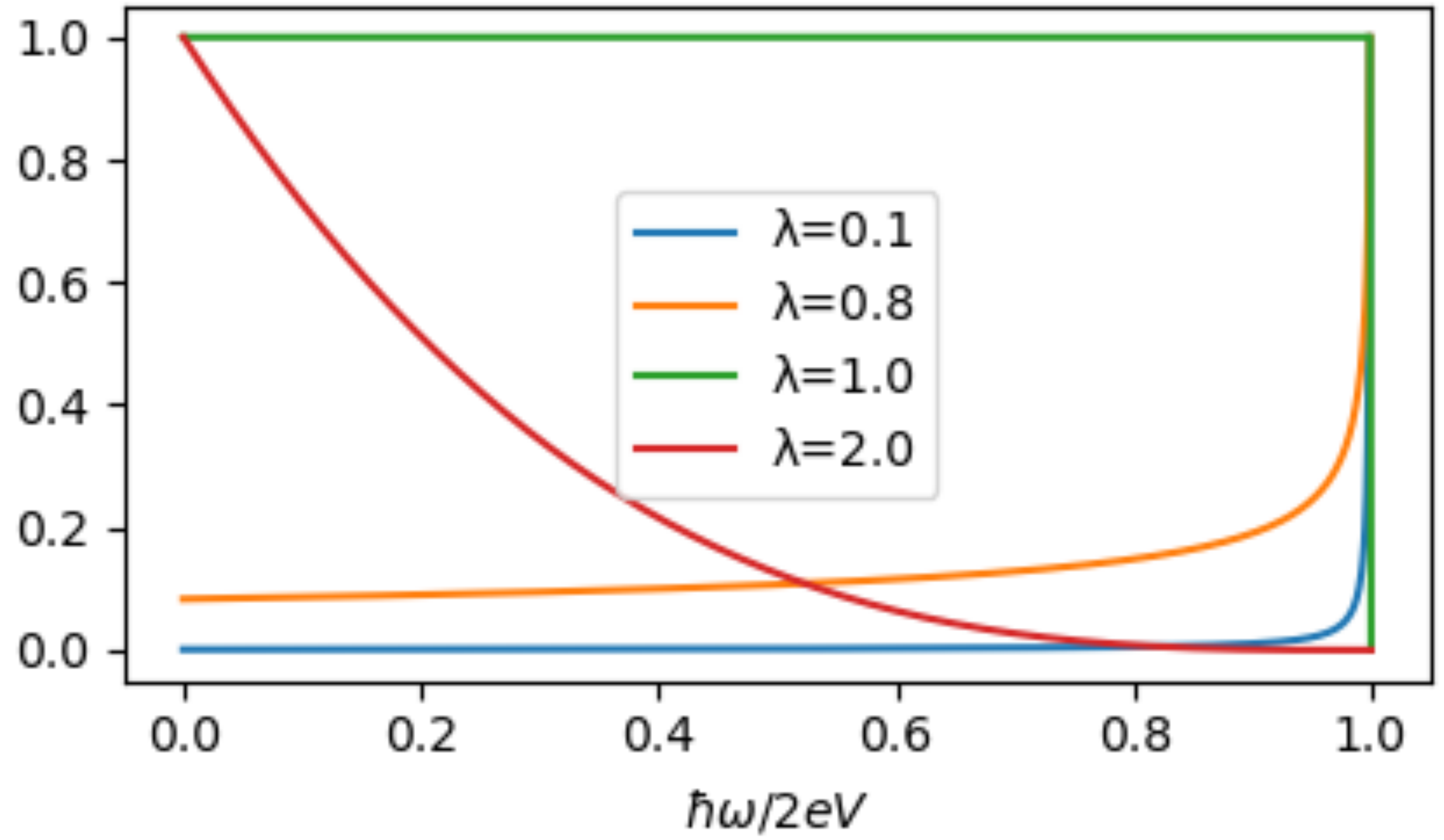
$$I \propto (2eV)^{2\lambda^2-1}$$

Spectral density of emitted photon:

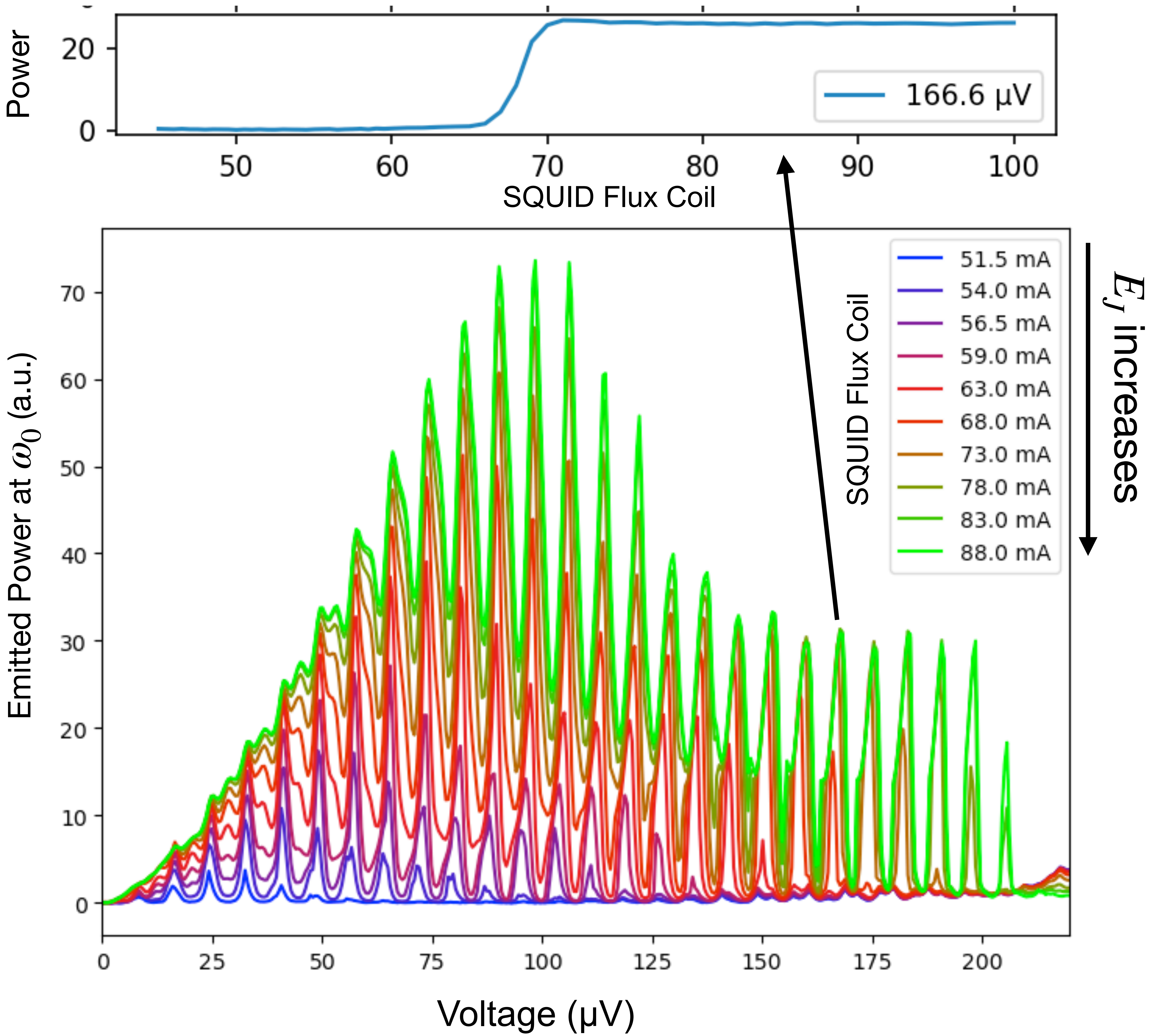
$$\rho(\omega) \propto (2eV - \hbar\omega)^{\lambda^2-1}\,\theta(2eV-\hbar\omega)$$

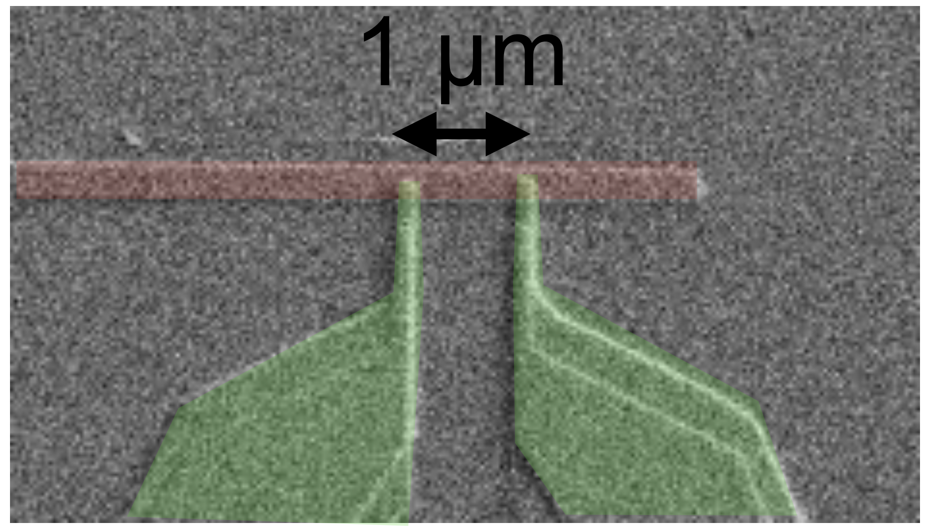
Large λ : emission everywhere !





Emission at low and high E_J





SQUID to vary E_J

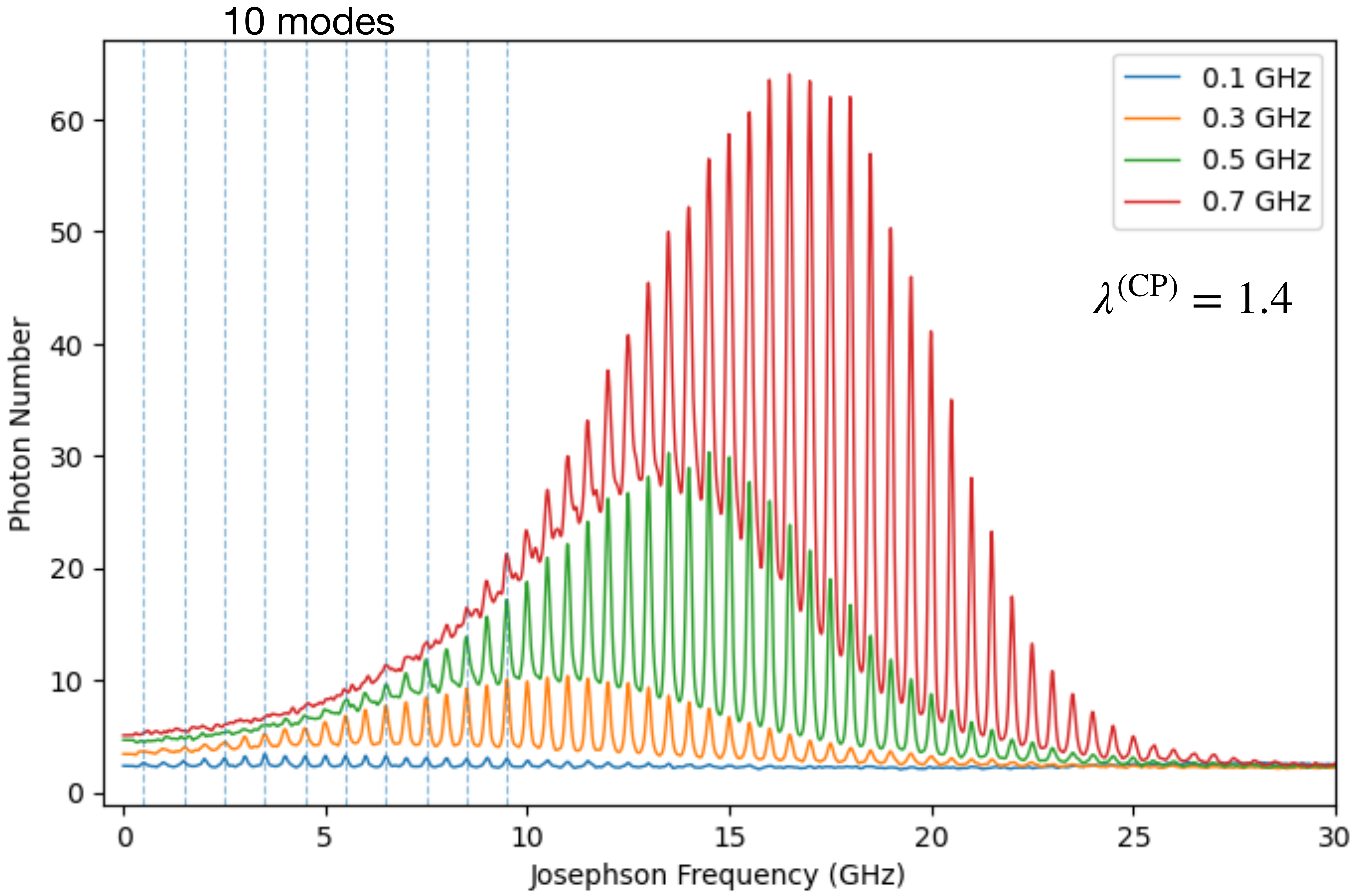
Broad emission is a non-linear effect at high E_J/κ

No broad emission in the perturbative limit

Finite Size Effects 😔

Truncated Wigner Approximation

$$W(x_1, \ldots, y_1 - \lambda_1, \ldots, y_M - \lambda_M) - W(x_1, \ldots, y_1 + \lambda_1, \ldots, y_M + \lambda_M) \approx - \sum_m 2\lambda_m \frac{\partial W}{\partial y_m}$$



Qualitative agreement:
Critical E_J/κ threshold

Truncated Wigner Approximation

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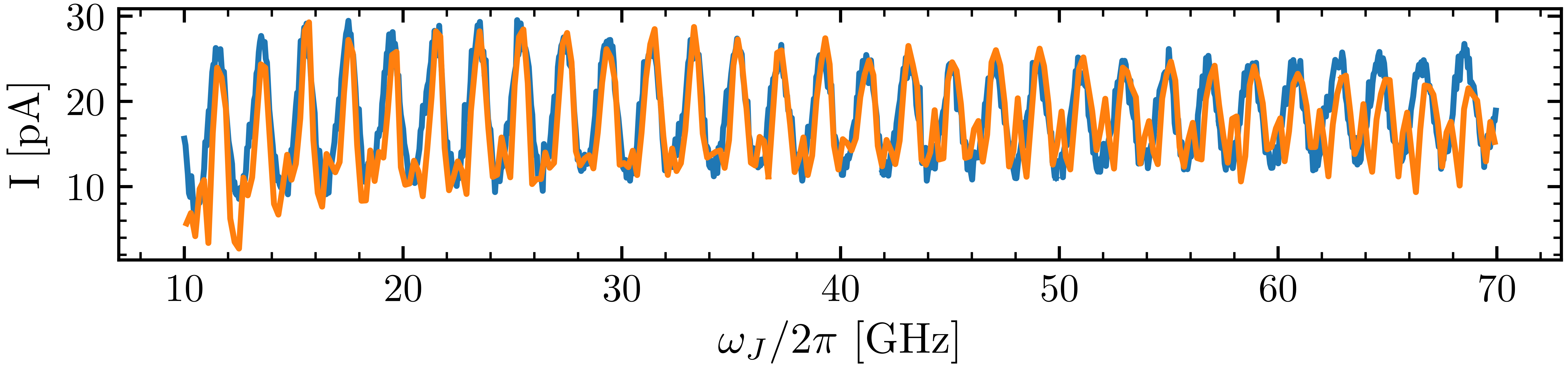
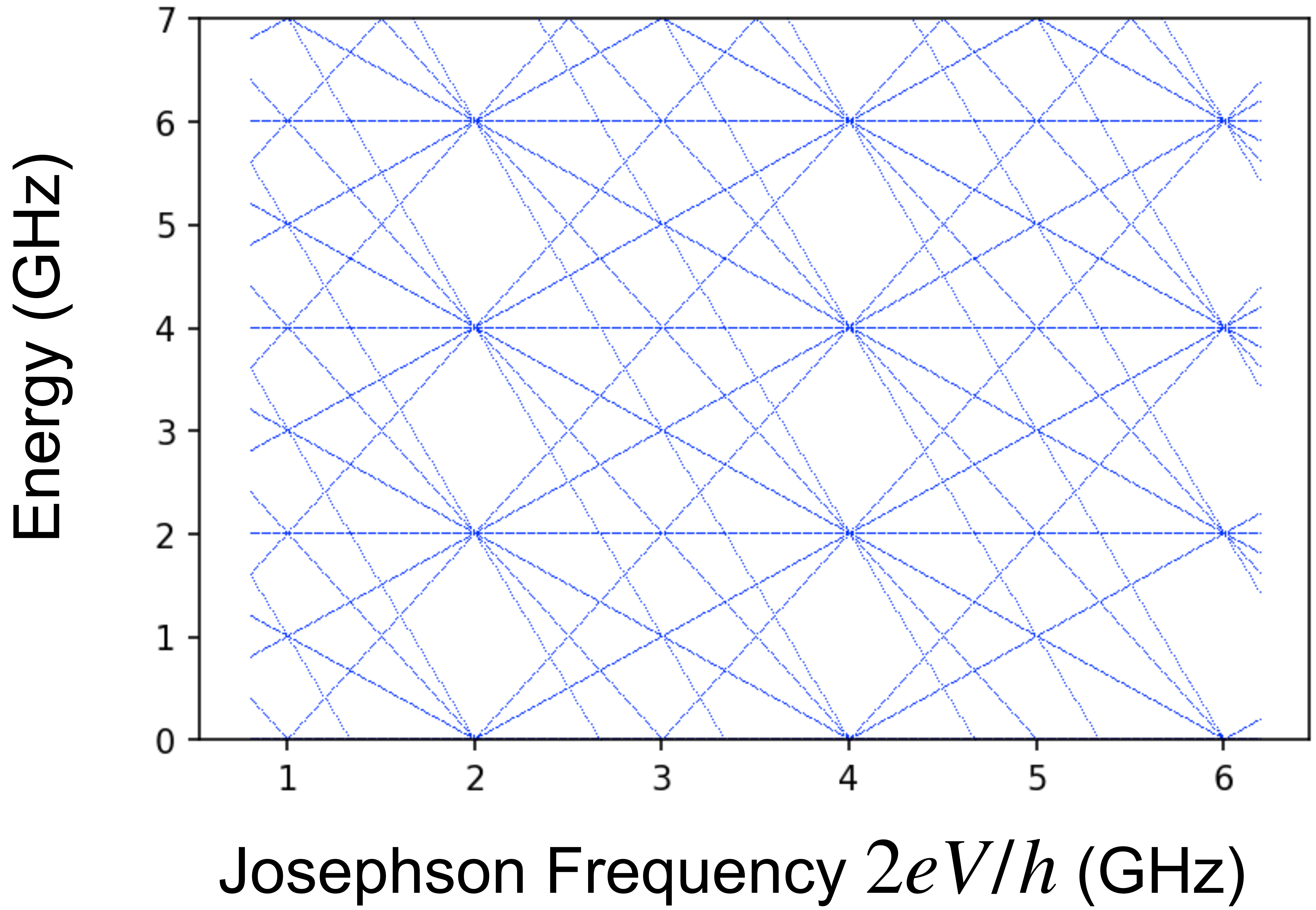


Quantitative agreement by tweaking mode parameters

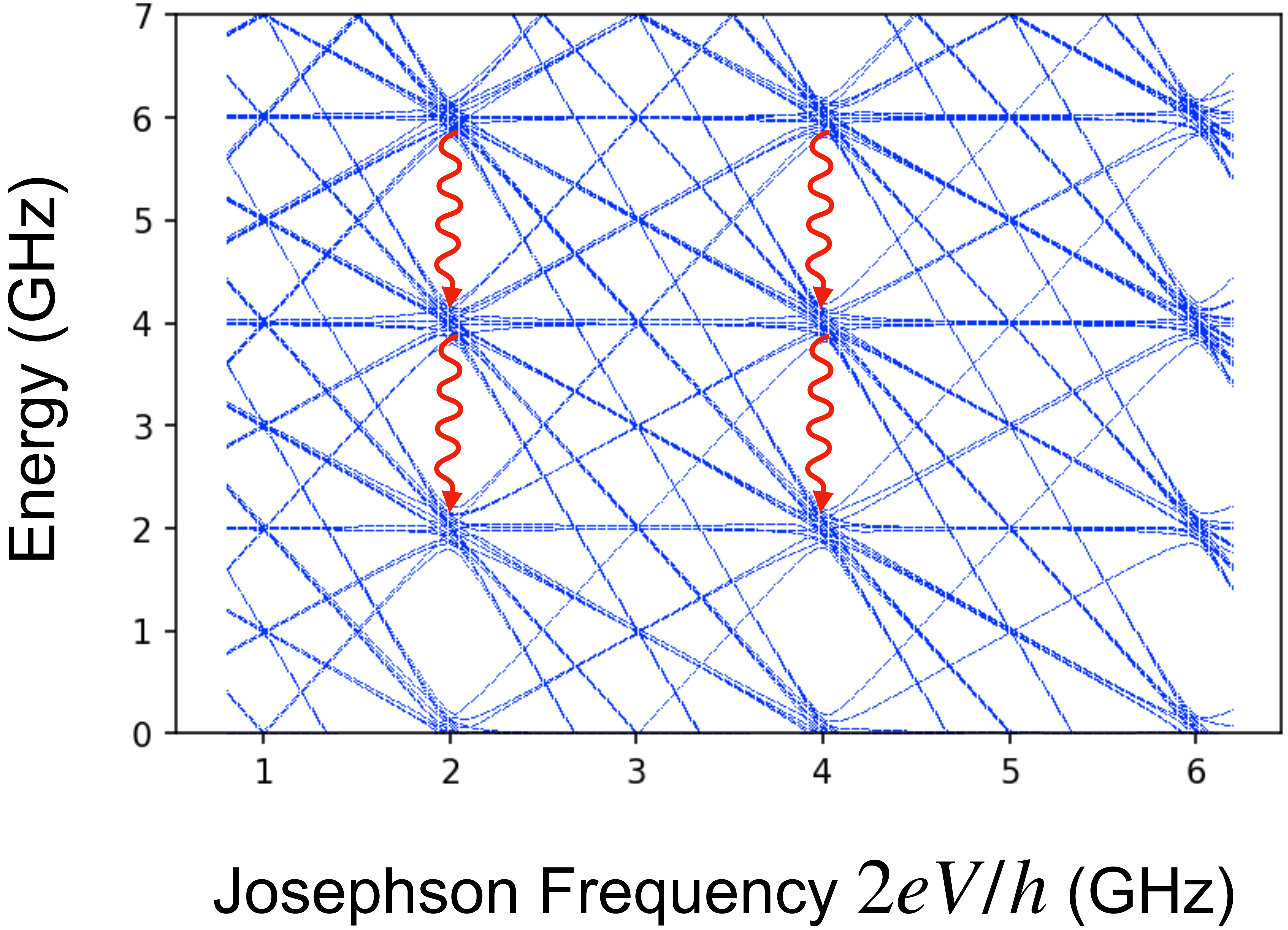
Dressed State Picture

$$|n_0, n_1, \ldots; N\rangle \quad \varepsilon = n_0\hbar\omega_0 + n_1\hbar\omega_1 + \ldots + N\hbar\omega_J$$

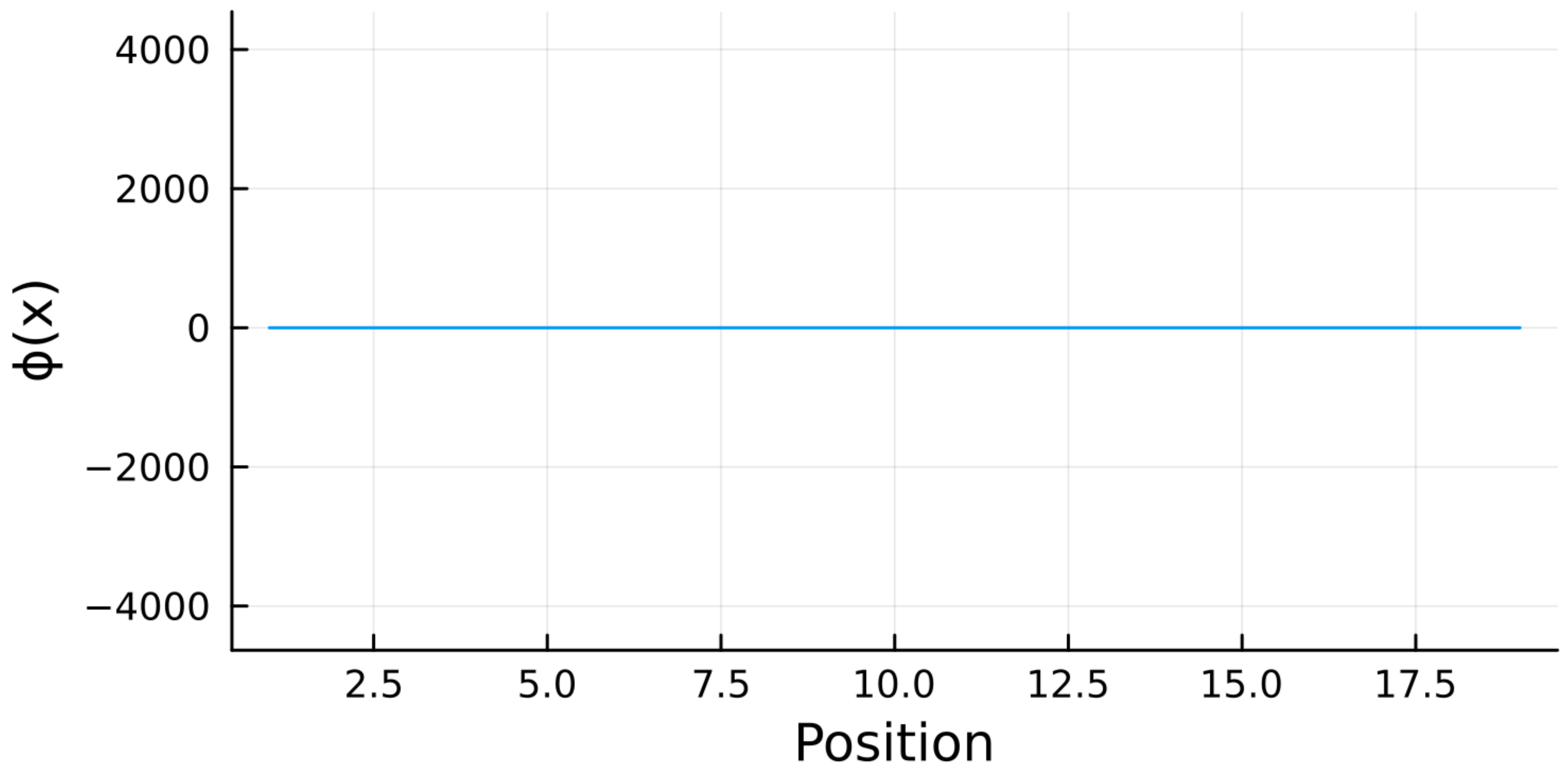


Dressed State Picture

Fluorescence at the fundamental frequency



Classical « Lasing » Trajectories at Large E_J



Free propagation + Interaction at the boundary :

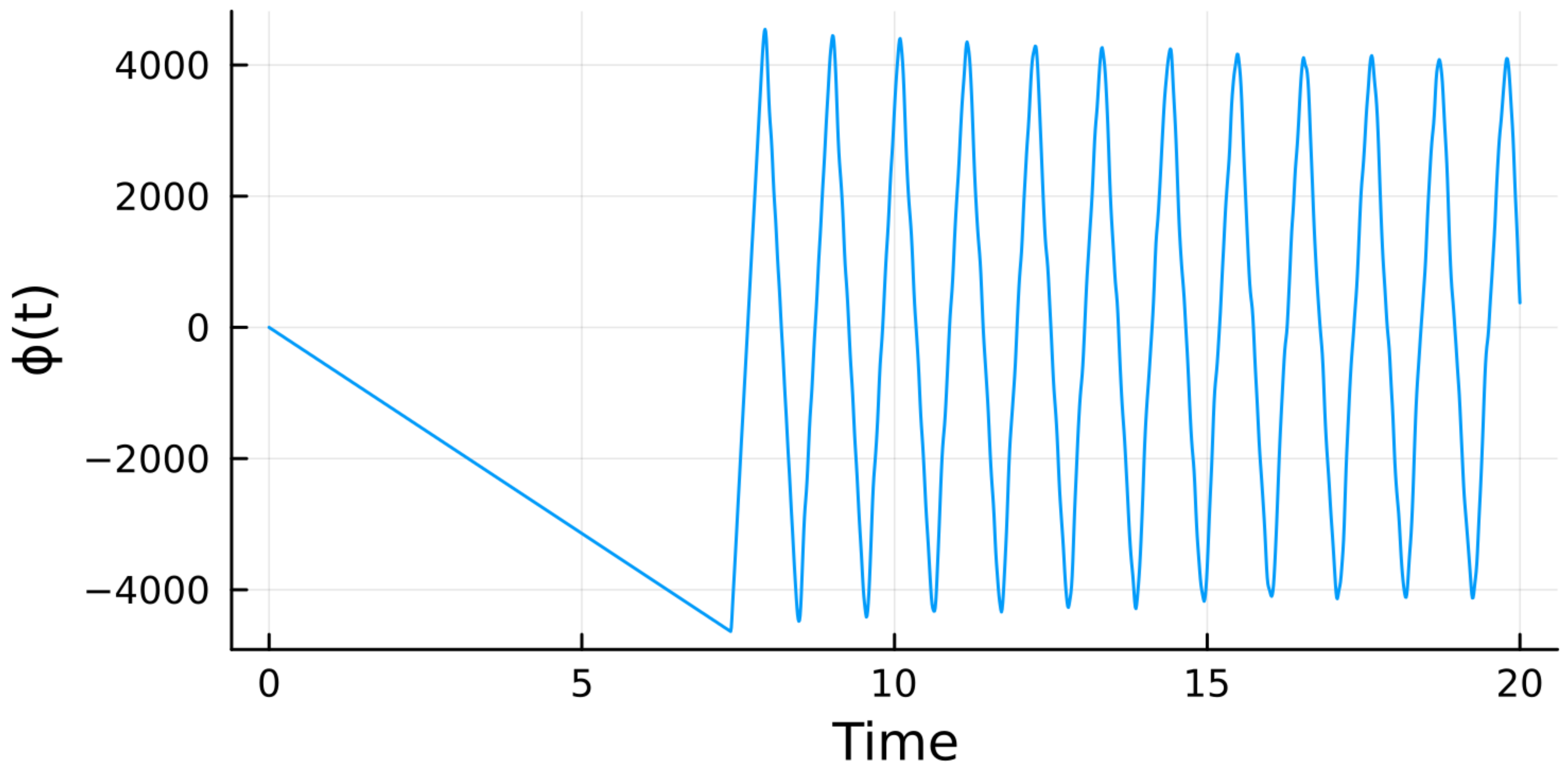
$$\Phi(t) = \Phi(t-T)e^{-\kappa T} + \chi \int_{t-T}^{t} K(t-t')\sin\left[2eVt'/\hbar + \Phi(t')\right] dt'$$

$$K(t) = \sum_m \lambda_m^2 \sin(\omega_m t)\ e^{-\kappa t}$$

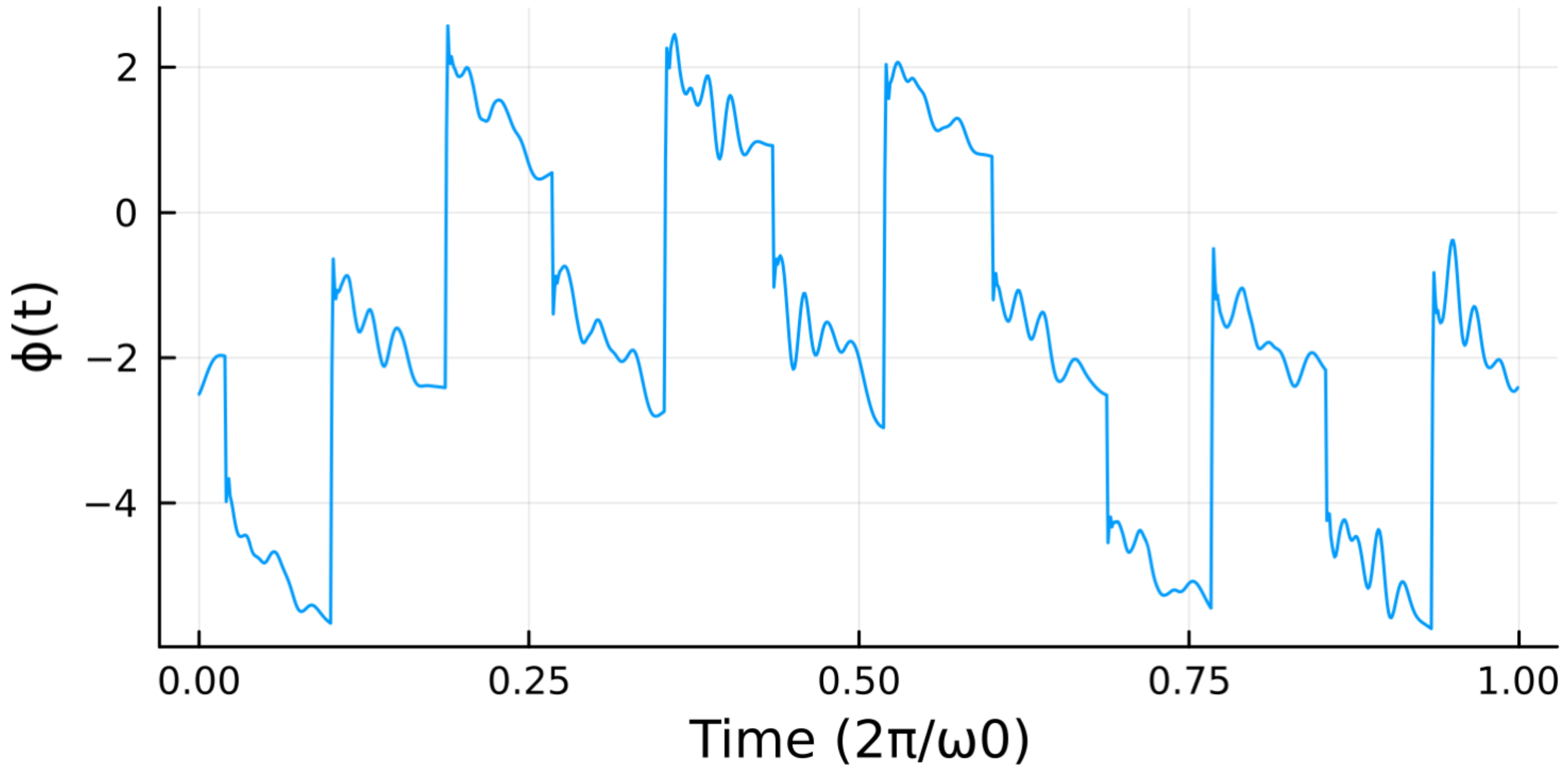
Simon et al. PRL **121** 027004 (2018)

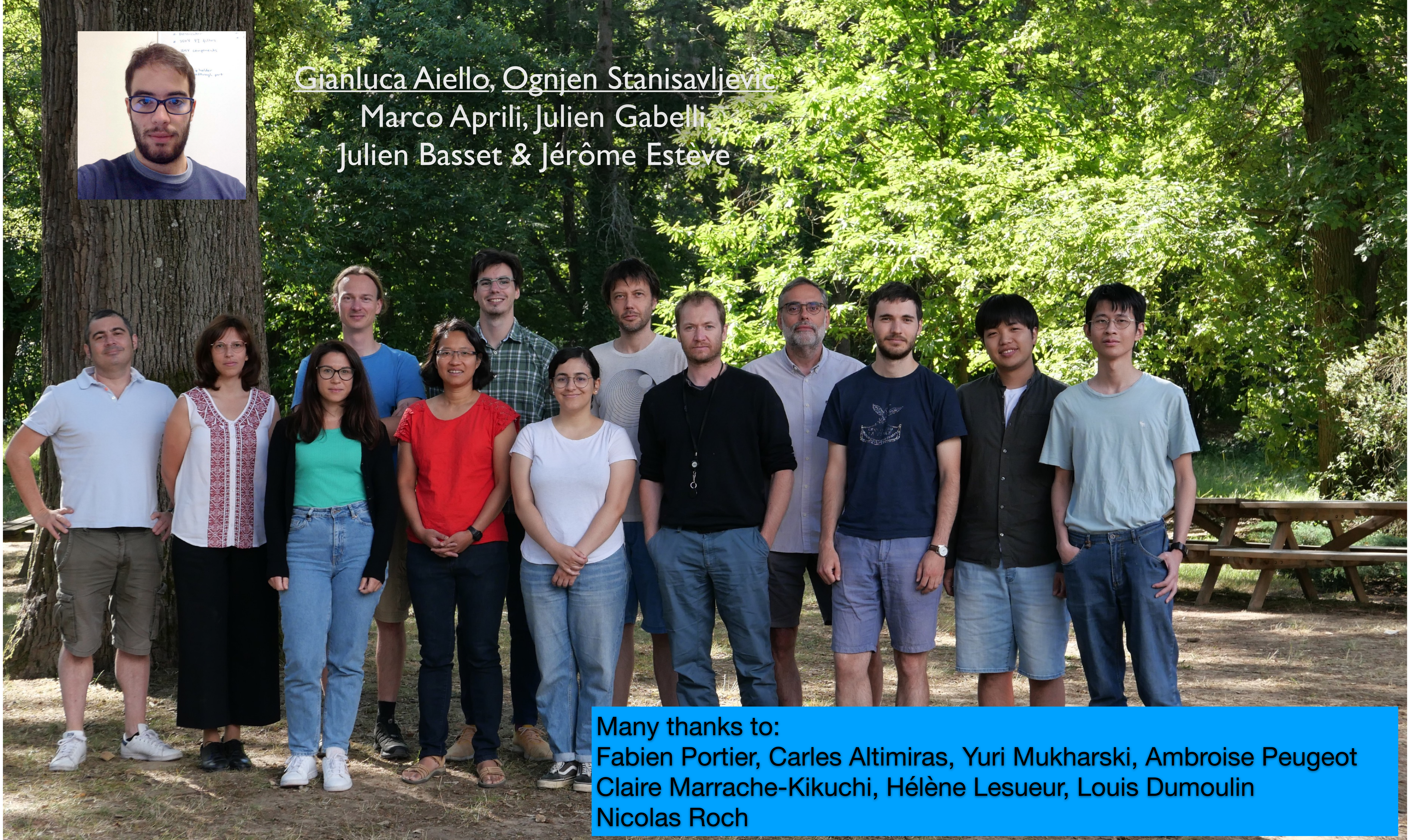
Cassidy et al. Science **355** 939 (2017)

Above threshold



Classical trajectory : π phase jumps followed by time linear/or constant evolution





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Nicolas Roch

Conclusion & Perspectives

Josephson side

$$\cancel{\mathscr{L} = \frac{\hbar R_Q}{4\pi Z}\int_0^{\ell} \frac{1}{v}(\partial_t\phi)^2 - v(\partial_x\Phi)^2 dx + E_J \cos\left[\phi(\ell) + \omega_J t\right]}$$

Multimode & multi photon open system

Hard to get an ohmic bath…

$$H = \sum_m \hbar\omega_m a_m^\dagger a_m - E_J \cos\Big[\sum_m \lambda_m^{(\mathrm{CP})}(a_m^\dagger + a_m) + 2eVt/\hbar\Big]$$

Nice concepts like refermionization are lost.

Energies are comparable to the gap ?

See also work by the Manucharyan group & Grenoble.
See Denis Basko later this week

QP side

Electronic degree of freedom is a « true » bath

$$H = \sum_m \hbar\omega_m a_m^\dagger a_m + \sum_k \epsilon_k c_k^\dagger c_k + \sum_q \epsilon_q d_q^\dagger d_q - g\sum_{kq}(c_k^\dagger c_k - d_q^\dagger d_q)\sum_m i\omega_m \lambda_m (a_m - a_m^\dagger)$$

« Sharp » DOS → Non Markovian effects ?

Test bench for HEOM, DNRG …

Collaboration with Vasilii Vadimov (Aalto)