Advancing measurements in bosonic circuit quantum electrodynamics (cQED)

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6 October 2025


NUS
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Entrance


Cheers to 15 years!
CQT
Established 2007

Qcrew (our group)

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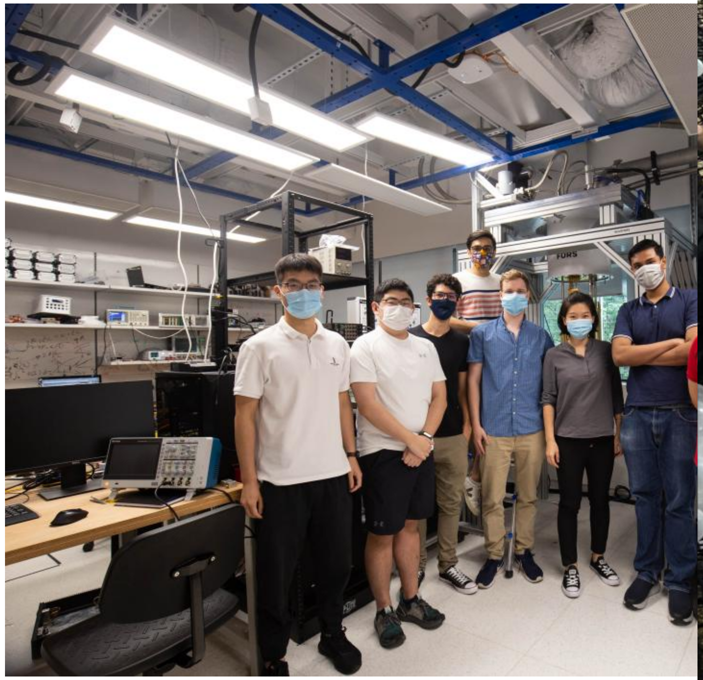
Dec 2020



May 2025

2 Research fellows
2 Research assistants
1 Research associate
9 PhD students
2 Master students
2 Interns

Bosonic systems

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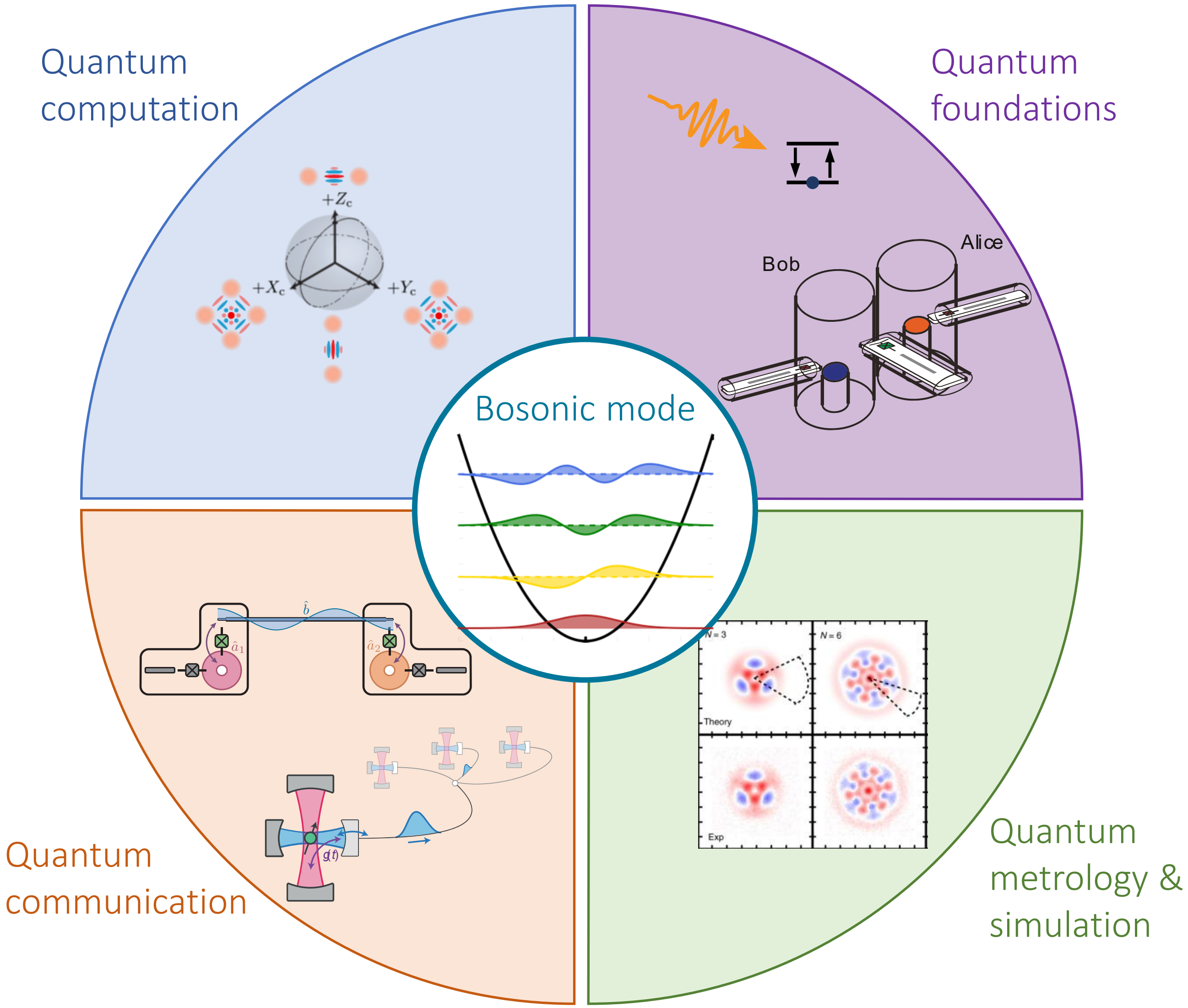

Kockum, *Nat. Rev. Phys.* (2019)
Gao, *Nature* (2019)

Ofek, *Nature* (2016)
Sivak, *Nature* (2023)
Ni, *Nature* (2023)

Pfaff, *Nat. Phys.* (2017)
Chou, *Nature* (2018)
Burkhart, *PRXQ* (2021)

McCormick, *Nature* (2019)
Wang, *Nat. Commun.* (2019)
Etc.

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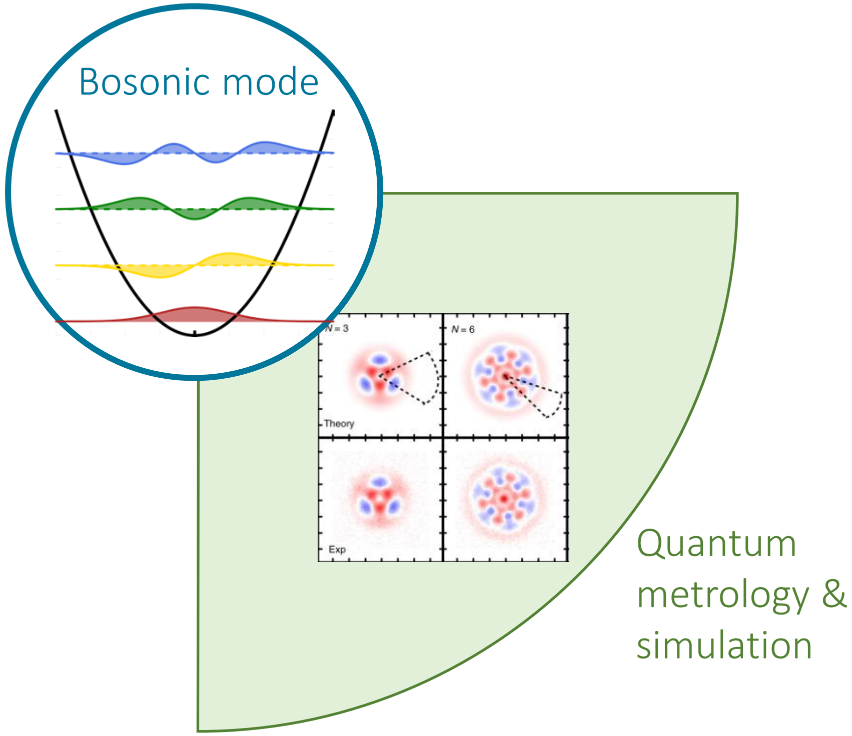
Outline



I. Standard bosonic cQED hardware

II. System characterization

- Efficient and robust tomography [1]
- Tomography with learning [2]
- Measuring any observable [3]

III. Quantum metrology

- Metrology with cat states [4]

IV. Summary

[1] TK, CYF, AC, PS et al., *PRX Quantum* 6, 010303 (2025)
[2] TK et al., *QST* 10, 035041 (2025)
[3] TK, FV, et al., *arXiv:2503.10436*
[4] XP, TK et al., *PRX Quantum* 6, 010304 (2025)

Standard bosonic cQED hardware

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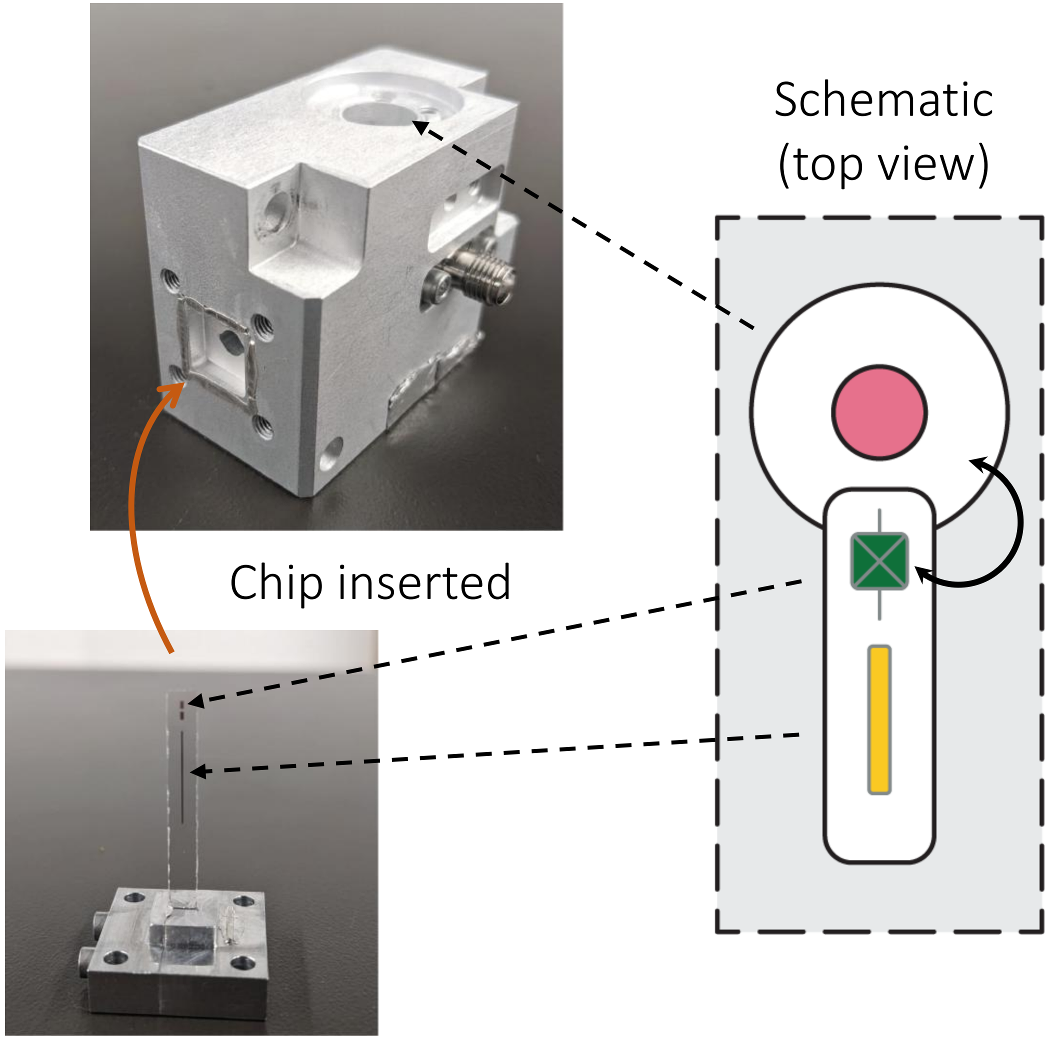

Rev. Mod. Phys. 93, 025005 (2021)

Basic components:

1. Cavity, long-lived bosonic mode (~1ms)
2. Transmon ancilla (nonlinear) (20-100μs)
3. Readout resonator (0.1-1μs)

Dispersive qubit-cavity coupling

$$\hat{H}_0/\hbar = (\chi/2)\hat{\sigma}_z\, \hat{c}^\dagger \hat{c}$$

Qubit and cavity drives

$$\hat{H}_d/\hbar = \epsilon_c(t)\hat{c} + \epsilon_c^*(t)\hat{c}^\dagger \\ +\epsilon_q(t)\hat{\sigma}_- + \epsilon_q^*(t)\hat{\sigma}_+$$

...and higher order terms (relatively weak)

Standard bosonic cQED hardware

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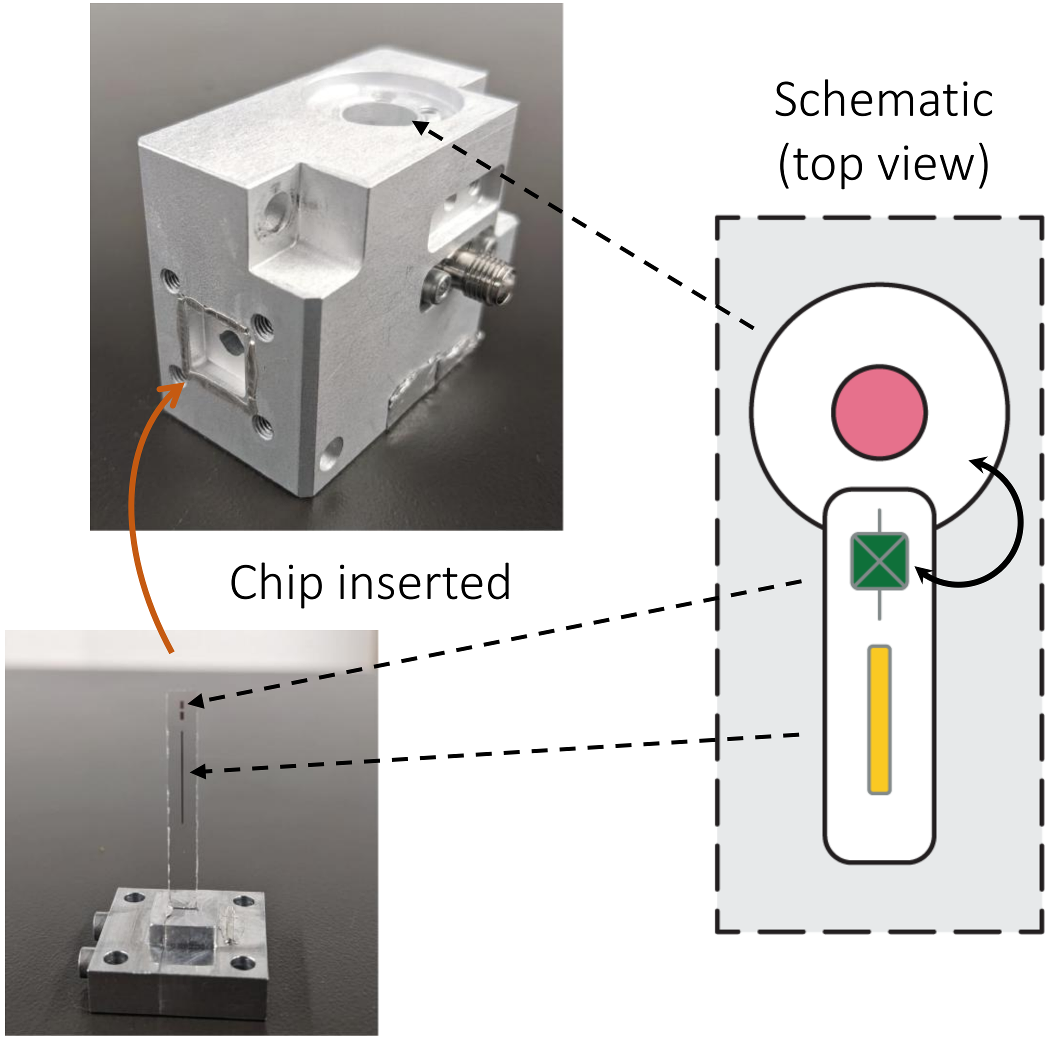

Universal control

State preparation

Initial state $|g\rangle|0\rangle$

Target state $|g\rangle|\psi\rangle$

Optimize the drives $\epsilon_c(t)$, $\epsilon_q(t)$ $\hat{U}|g\rangle|0\rangle = |g\rangle|\psi\rangle$

Nat. Commun. 8, 94 (2017)

Measurements

Qubit: single shot readout

Cavity: indirectly
(via a controlled unitary + qubit readout)

Photon number, parity, etc.

$\langle|n\rangle\langle n|\rangle$ $\langle\exp(i\hat{c}^\dagger\hat{c}\pi)\rangle$

Nature 445, 515 (2007) *Nature* 511, 444 (2014)

State characterization

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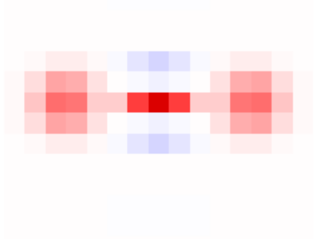
Quantum harmonic oscillators are attractive

Cat codes



PRA 59, 2631 (1999)

Binomial codes

PRX 6, 031006 (2016)

GKP codes

PRA 64, 012310 (2001)

Difficult to characterize

- As the system's dimension is high, it requires **a lot of different observables**

  Even more for multimode systems!

- Techniques: Wigner, Husimi-Q, etc. They have their limitations

Wish list for a bosonic reconstruction technique

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- **Fewest** measurements!

  $D^2 - 1$ observables, D is dimension of ρ

- Observables that are **robust against errors**!
- **High fidelities**, of course!
- **Versatility** across experimental platforms (bonus!)

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Optimized Reconstruction with Excitation Number Sampling (ORENS)

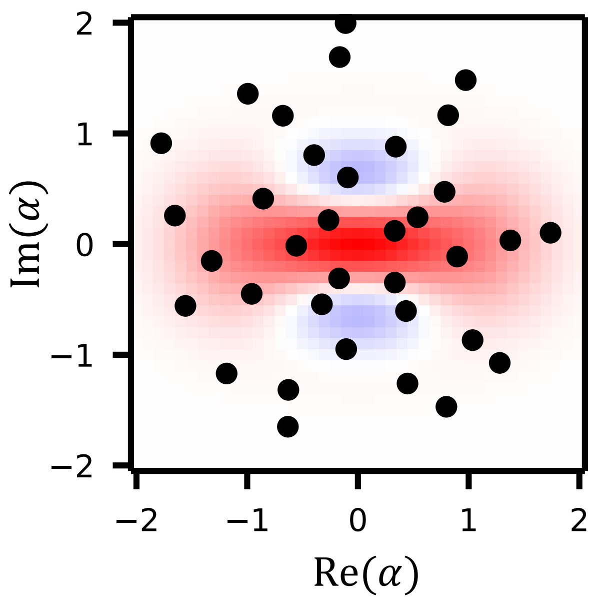
$Q_n(\alpha) = tr(|n\rangle\langle n|\, \hat{D}(-\alpha)\rho\hat{D}(\alpha))$

Choose a truncation dimension D

Optimize → $D^2 - 1$ different $\{(n, \alpha)\}$ to measure

Bayesian inference → ρ_{est}

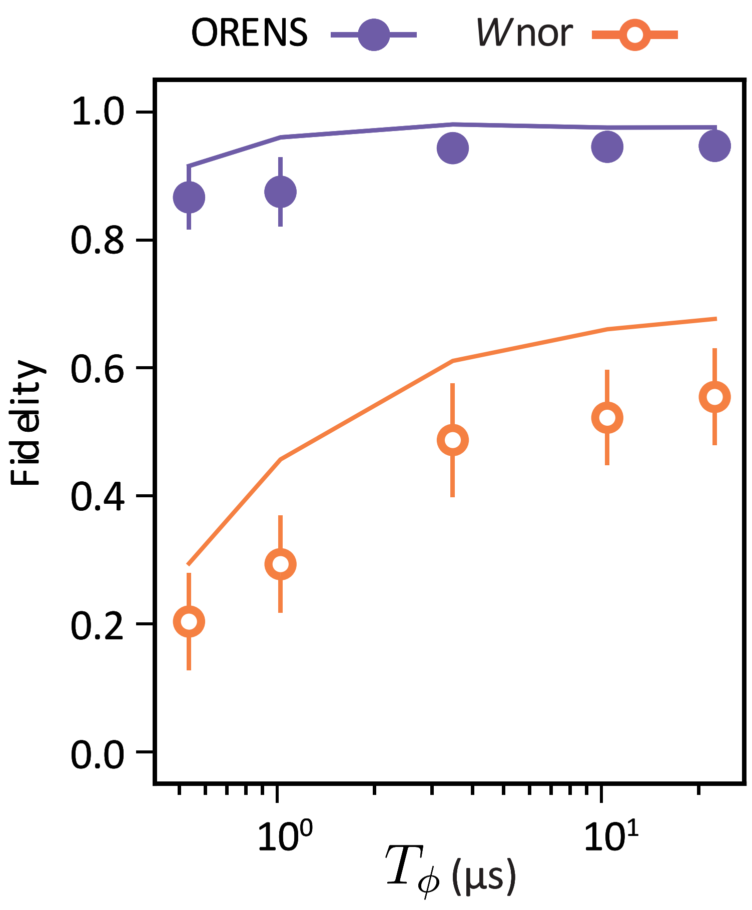
For example:
$D = 6$



Conventional:
Grid-based tomography
(measure parity)
$41 \times 41 = 1681$ points!

ORENS:
(Measure $\mathrm{p}(n = 5)$)
35 points

Tomography on cat states



- Test with 4 cat states: $\frac{|\alpha\rangle \pm |-\alpha\rangle}{N}, \frac{|\alpha\rangle \pm i|-\alpha\rangle}{N}$ with $\alpha = 1$
- ORENS protocol with $D = 6$
- Density matrix → fidelity with target state
- Artificially lower T_ϕ from 22us to 0.5us
- Compare to Wigner with $D^2 - 1$ sampling points
- ORENS is robust!

Why is normal Wigner tomography bad?

Bosonic tomography with learning

TK et al., *QST* 10, 035041 (2025)

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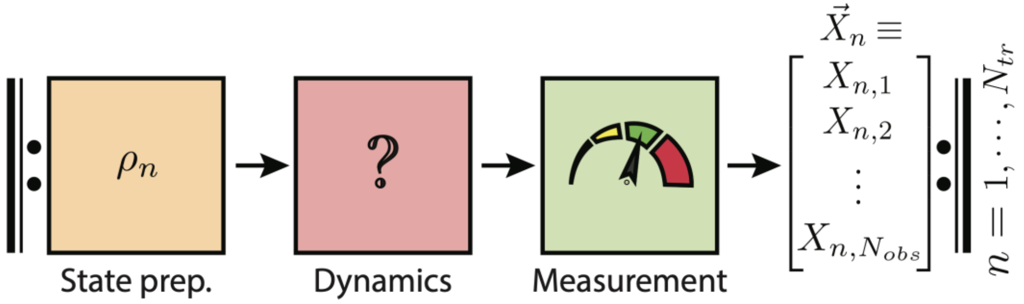
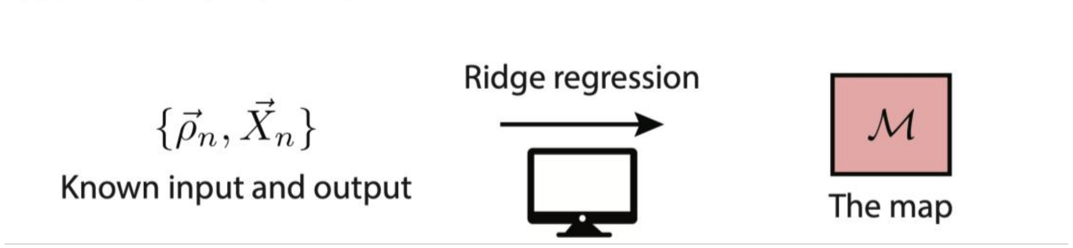
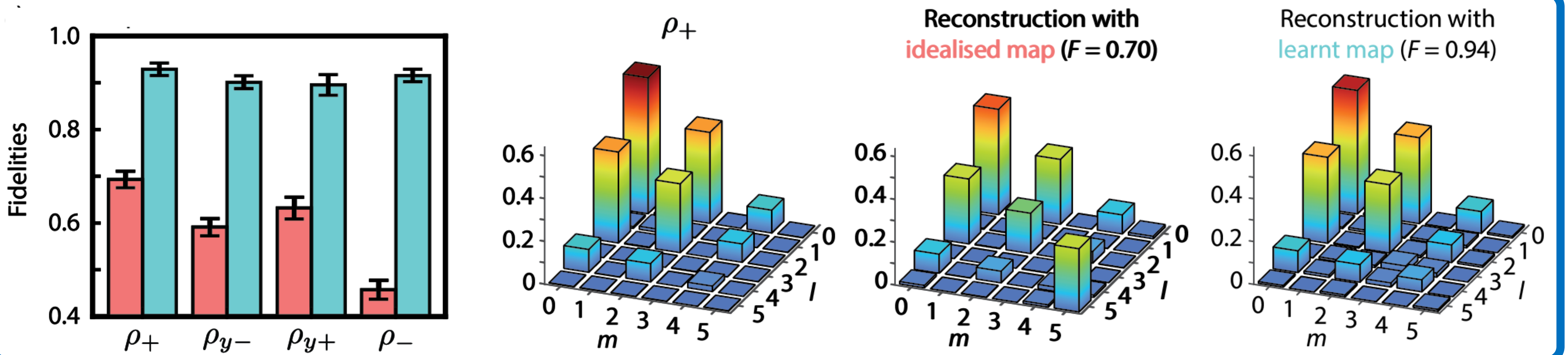
Perform a set of training experiments



Learn the dynamics via ridge regression





Measurement of any observable

TK, FV, et al., *arXiv:2503.10436*

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Tomography is great. However…

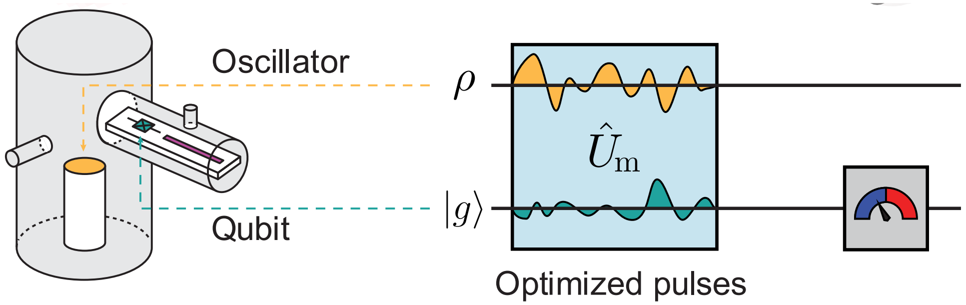
Quite often we only want $\langle \hat{O} \rangle$

$\langle \hat{x} \rangle, \langle \hat{p} \rangle, \langle |\psi\rangle\langle\psi| \rangle$, etc.

Reduce number of measurements??

Optimized Routine for Estimation of any Observable (OREO)

Given an arbitrary observable $\hat{O}$

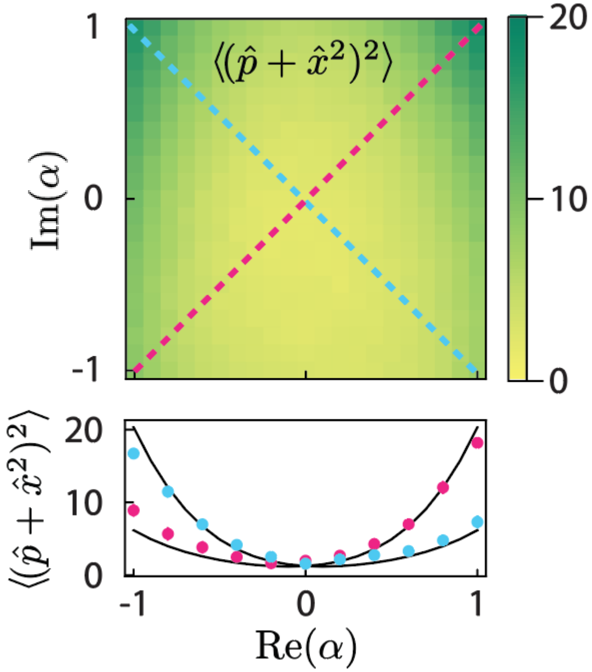


Optimize $\hat{U}_m$ such that $\langle \hat{\sigma}_z \rangle = \langle \hat{O} \rangle_\rho$

Works on any oscillator state ρ

Implementations

Quadratures of variable coherent states $|\alpha\rangle$



Quantum metrology

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Goal: Enhance precision when measuring $\boldsymbol{\beta}$

Classically:

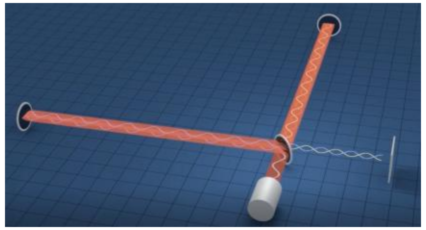
$\Delta\beta \propto 1/\sqrt{\bar{n}}$ The standard quantum limit (SQL)

Using quantum states:

$\Delta\beta \propto 1/\bar{n}$ The Heisenberg limit (HL)

Appealing

LIGO



Phys. Rev. X 13, 041021 (2023)

Magnetic sensor

Nat. Rev. Phys. 5, 157 (2023)

Typical challenges:

Requires multi-particle entangled quantum states

Non-deterministic state preparation

Non-versatile, focus on specific states tailored to hardware

Wish list for metrology in a bosonic cQED

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- A scheme! Using a single bosonic mode
- Versatility. Different classes of states
- Efficient. Deterministic state preparation
- High metrological gain, compared to classically-behaving coherent states

Experimental protocol

XP, TK et al., *PRX Quantum* 6, 010304 (2025)

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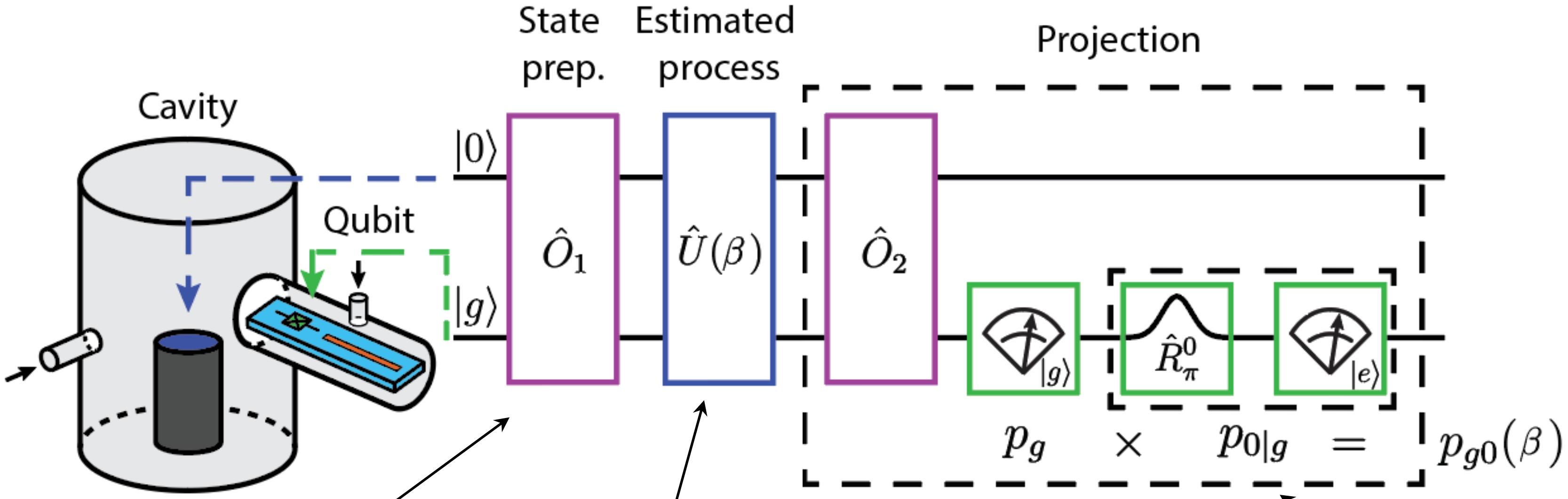


State preparation

$\hat{O}_1|g\rangle|0\rangle = |g\rangle|\psi\rangle$

Flexible & deterministic ✓

Estimated process

Phase estimation:

$\hat{U}(\theta) = e^{i\hat{a}^\dagger\hat{a}\theta}, \theta = \chi t/2$

Amplitude estimation:

$\hat{U}(\gamma) = e^{i(\hat{a}+\hat{a}^\dagger)\gamma}$

Etc.

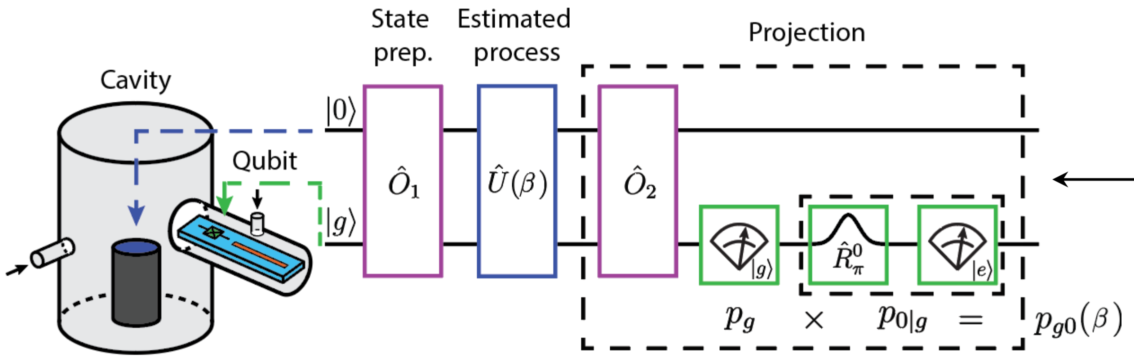
Measurement

$\hat{O}_2|g\rangle|\psi\rangle = |g\rangle|0\rangle$

$p_{g0}(\beta) = \langle\psi(\beta)|\psi\rangle\langle\psi|\psi(\beta)\rangle$

$\boldsymbol{\Delta\beta}$??

Experimental protocol



Measurement

Fisher information [1]:

$$\mathrm{FI}(\beta) = \frac{1}{p_{g0}(1-p_{g0})}\left(\frac{\partial p_{g0}}{\partial \beta}\right)^2$$

Cramér-Rao bound [2]: $\Delta\beta \geq \frac{1}{\sqrt{\mathrm{FI}(\beta)}}$

[1] *Rev. Mod. Phys.* 89, 035002 (2017)

[2] *Phys. Rev. Lett.* 72, 3439 (1994)

Experimental protocol

XP, TK et al., *PRX Quantum* 6, 010304 (2025)

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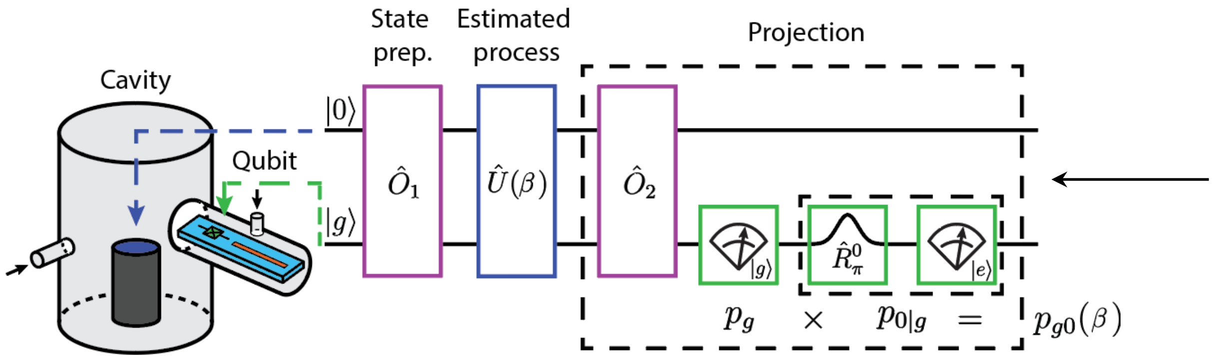

Measurement

Fisher information [1]:

$$\mathrm{FI}(\beta) = \frac{1}{p_{g0}(1 - p_{g0})} \left(\frac{\partial p_{g0}}{\partial \beta} \right)^2$$

Cramér-Rao bound [2]: $\Delta\beta \geq \frac{1}{\sqrt{\mathrm{FI}(\beta)}}$

Error propagation:

$$\Delta\beta = \frac{\Delta p_{g0}}{\left| \frac{\partial p_{g0}}{\partial \beta} \right|}$$

[1] *Rev. Mod. Phys.* 89, 035002 (2017)
[2] *Phys. Rev. Lett.* 72, 3439 (1994)

Quantum resource states?

XP, TK et al., *PRX Quantum* 6, 010304 (2025)

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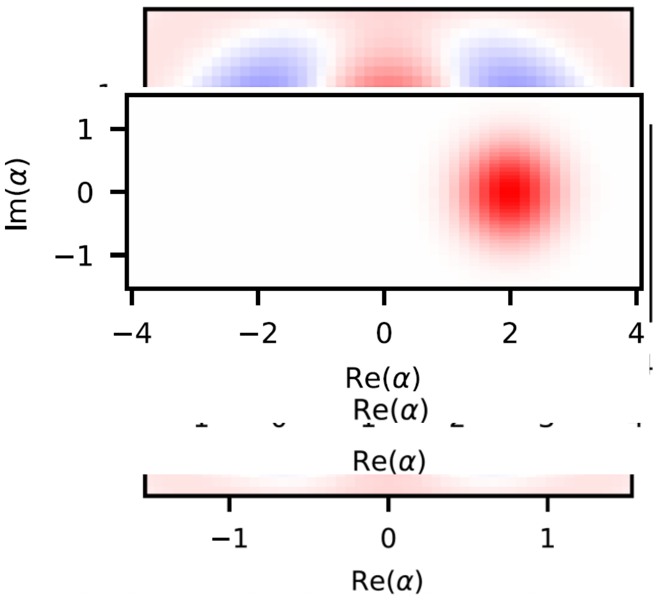
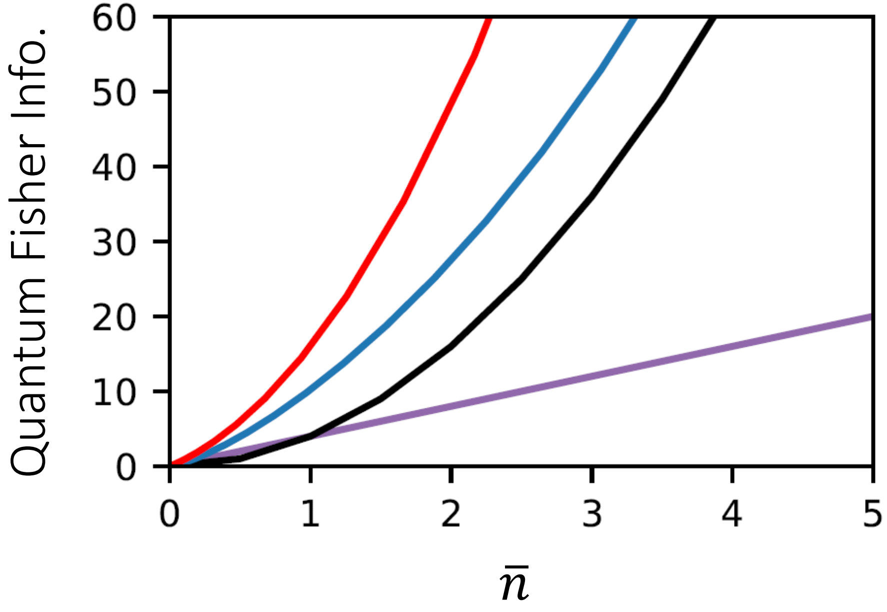
Unitary operator $\hat{U} = \mathrm{e}^{i\hat{a}^\dagger\hat{a}\,\theta}$ Process generator $\hat{a}^\dagger\hat{a} = \hat{n}$

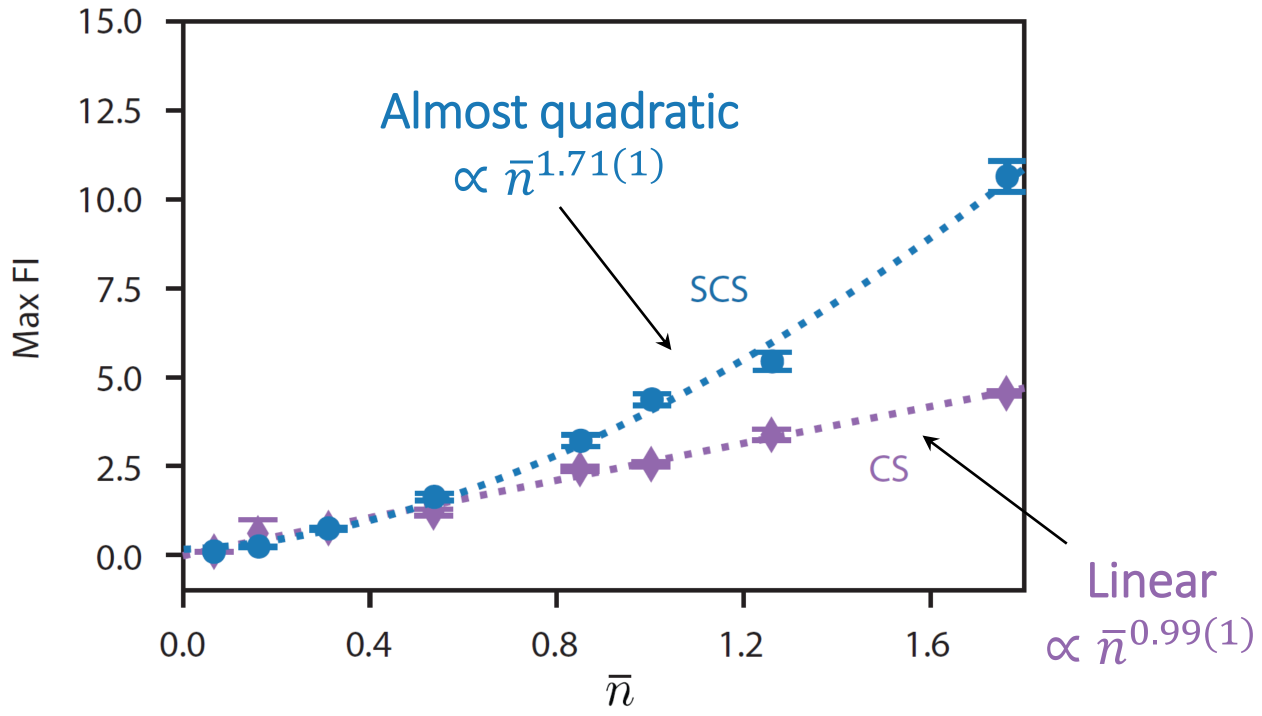
States with high variance $(\Delta\hat{n})^2$ $\longrightarrow$ High sensitivity



- Superposition of Fock states $(|0\rangle + |N\rangle)/\sqrt{2}$
- Superposition of coherent states $\mathcal{N}_\alpha(|0\rangle + |\alpha\rangle)$
- Squeezed vacuum $\hat{S}(r)|0\rangle$
- Coherent states (CS) $|\alpha\rangle$



Max FI vs states with different $\bar{n}$



Directly estimate $\Delta\theta$

XP, TK et al., *PRX Quantum* 6, 010304 (2025)

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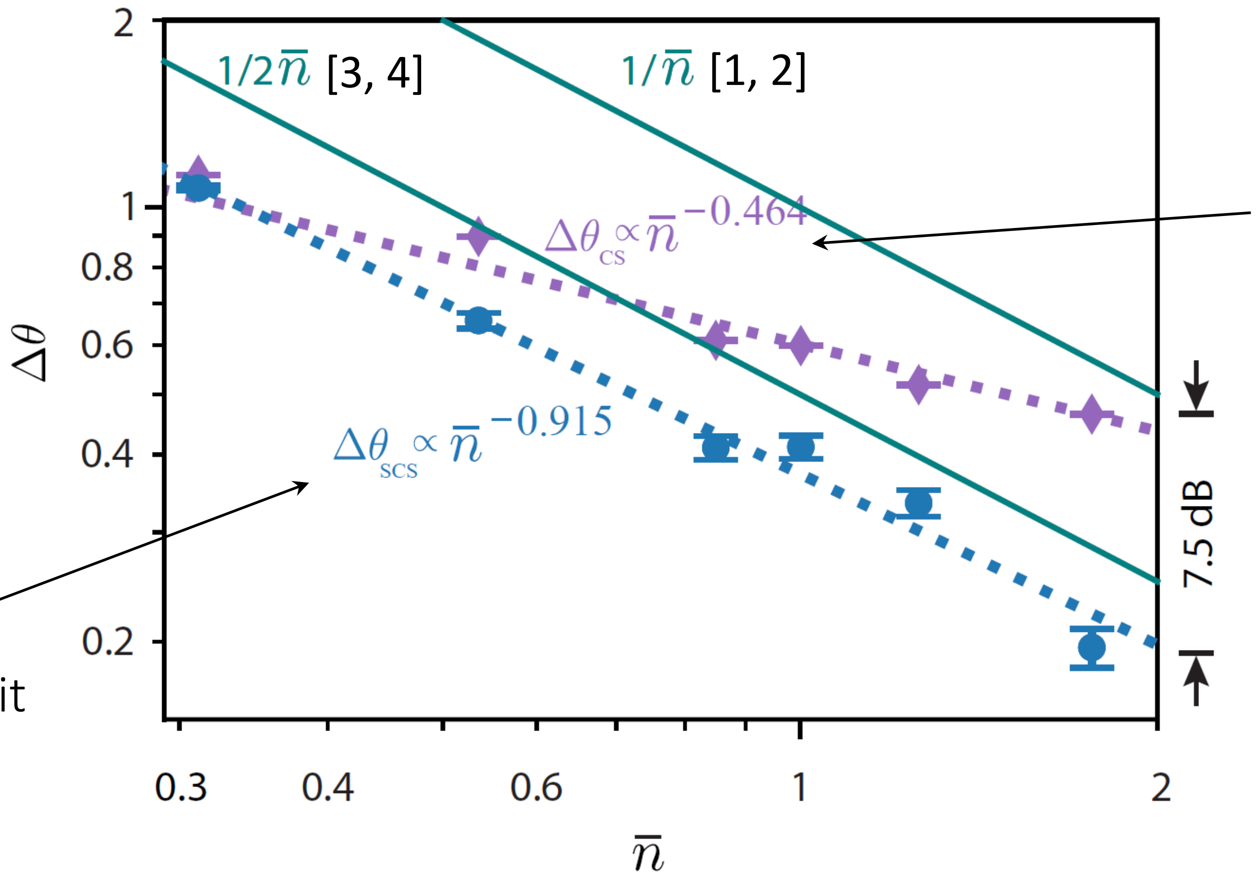
Error propagation

$$\Delta\theta = \frac{\Delta p_{g0}}{\left|\frac{\partial p_{g0}}{\partial \theta}\right|}$$



The standard quantum limit

Approaching the Heisenberg limit

$$\text{Gain} = 20\log_{10}\frac{\Delta\theta_{cs}}{\Delta\theta_{scs}} \approx 7.5 \text{ dB}$$

High metrological gain ✓

[1] *Science* 316, 726 (2007)
[2] *Nat. Phys.* 18, 925 (2022)
[3] *Nature* 572, 86(2019)
[4] *Nat. Commun.* 10, 4382 (2019)

Summary

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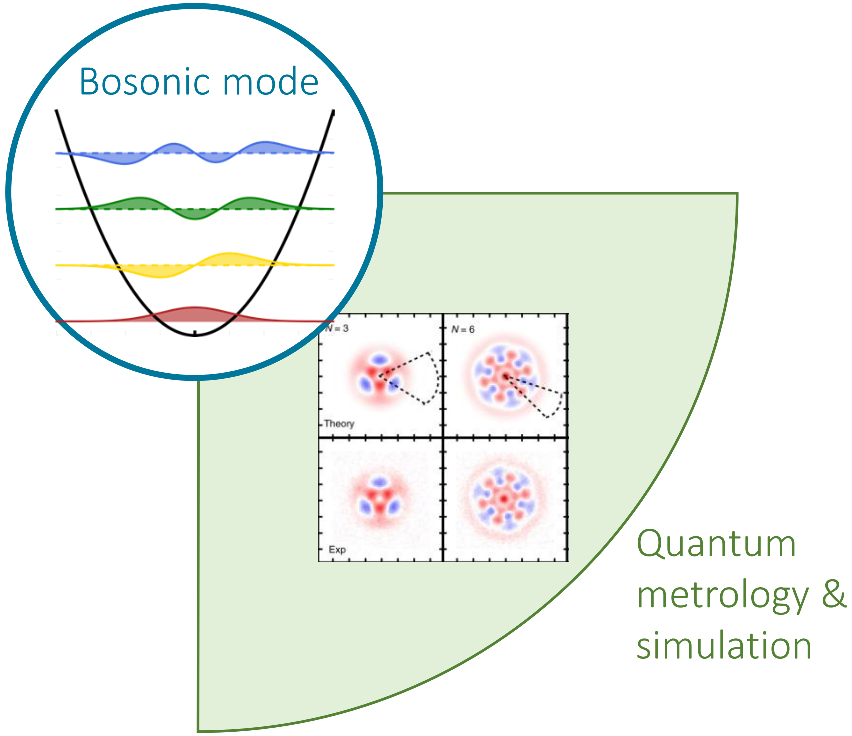

System characterization

- Robust tomography with fewest measured observables [1]
- Improved tomography with learning [2]
- Measurement of any observable [3]

Quantum metrology

- State-of-the-art metrology with cat states [4]

[1] TK, CYF, AC, PS et al., *PRX Quantum* 6, 010303 (2025)
[2] TK et al., *QST* 10, 035041 (2025)
[3] TK, FV, et al., *arXiv:2503.10436*
[4] XP, TK et al., *PRX Quantum* 6, 010304 (2025)

Ongoing explorations

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- Two-cavity device
    - Two-mode squeezed vacuum for metrology
    - C-phase gate
    - Qudit-qudit entanglement
- Three-cavity device
    - GHZ-cat and W-cat states

www.quantumcrew.org