Two-photon LZSM effect

Superconducting Devices
for Quantum Optics and Quantum Simulations
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Observation of the Two-Photon Landau-Zener-Stückelberg-Majorana Effect
Phys. Rev. Lett. **134**, 060602 (2024)

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MiSS Microwave Squeezing with Superconducting (meta) materials
RADES
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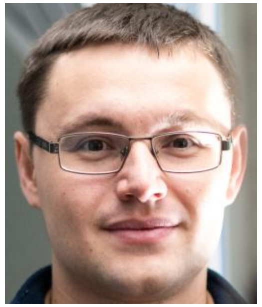

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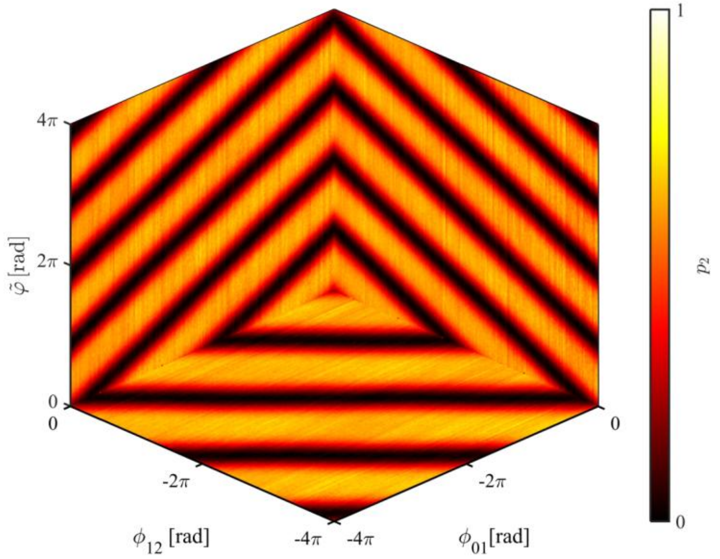
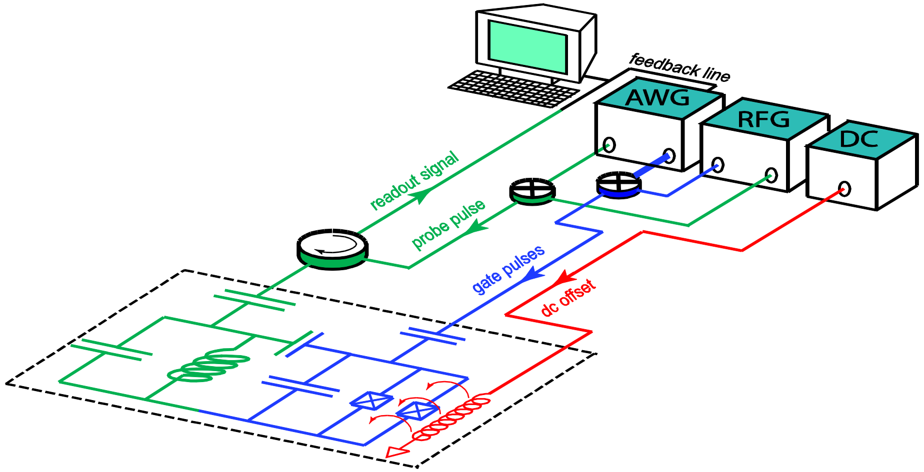
RESEARCH DIRECTIONS

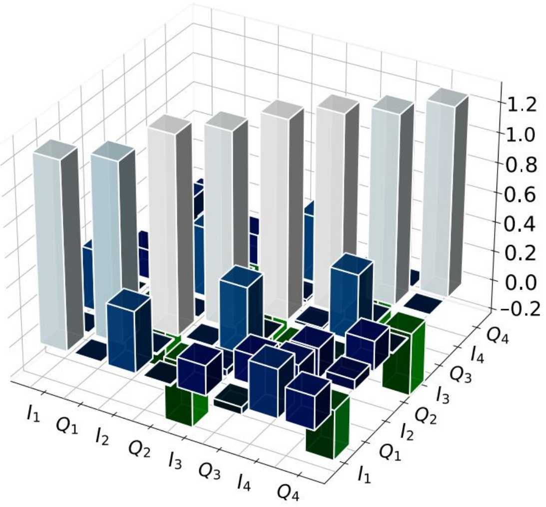
Analog simulations



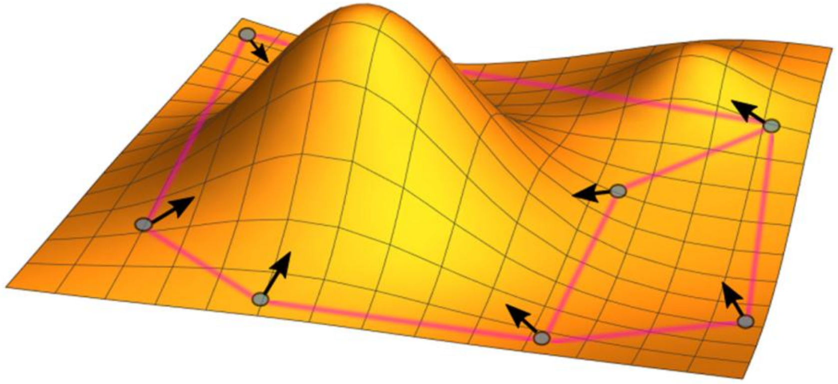
Quantum-enhanced magnetometry



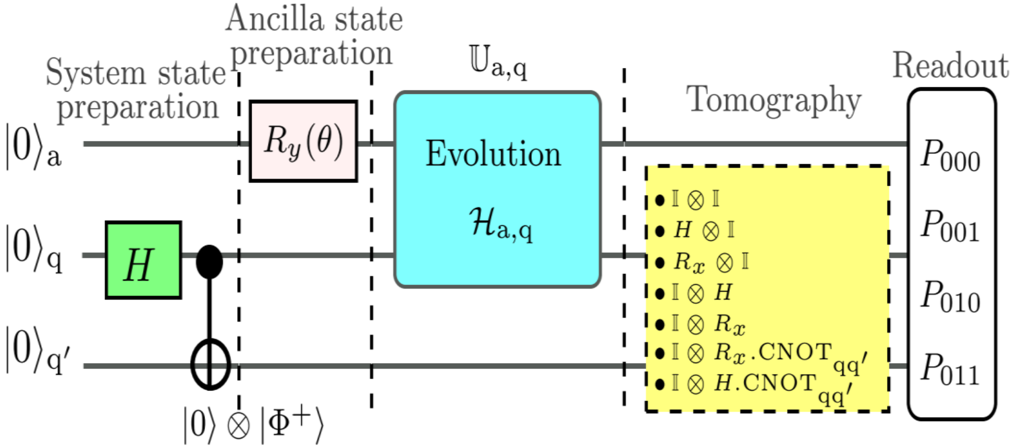
Entanglement in parametric devices



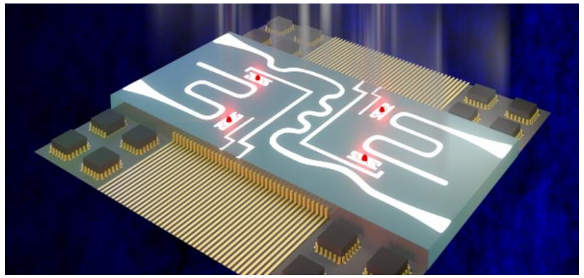
Machine learning and optimal control



Quantum algorithms



Quantum information



Single-photon Landau-Zener-Stückelberg-Majorana

M. Kuzmanovic , I. Björkman , J.J. McCord, S. Dogra, and G. S. Paraoanu,
Phys Rev Research **6**, 013188 (2024)

Landau-Zener-Stückelberg-Majorana effect

- Introduce finite coupling Ω between levels
- System Hamiltonian reads

$$\hat{H} = \frac{1}{2}\begin{pmatrix} vt & \Omega \\ \Omega & -vt \end{pmatrix}$$

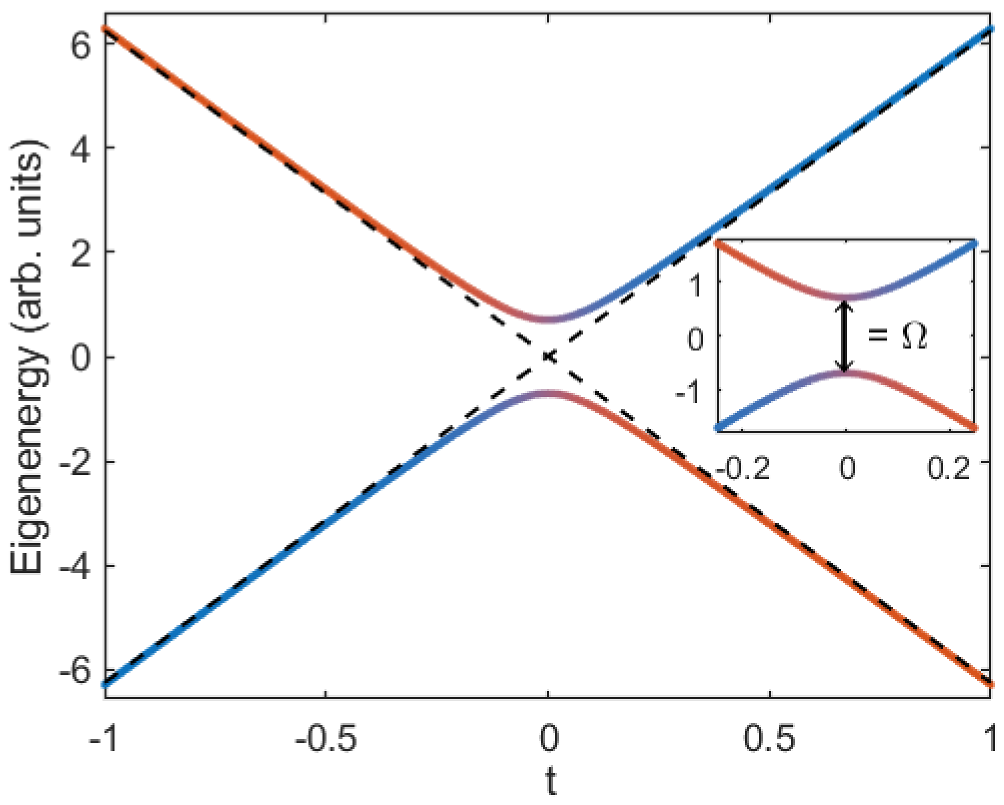
- Instantaneous eigenenergies:

$$\epsilon_{1/2} = \mp\frac{1}{2}\sqrt{(vt)^2 + \Omega^2}$$

- Dynamics changes
- Avoided crossing at $t = 0$

$$\epsilon_1 = -\epsilon_2 = -\frac{\Omega}{2}$$

- Eigenstates gradually exchange character
  - Equal superposition at avoided crossing
  - Completely interchanged eigenstates after process



Non-adiabatic transition probability

- Adiabatic transfer if infinitely slow changes
- In practice, gates faster than qubit decoherence, yet slow enough to have approximately adiabatic transfer
  - Finite probability that the transfer is non-adiabatic
- Analogous to driving with chirped drive $\Omega\cos(\omega_d t)\sigma_x$ sweeping across ω_{ge} with drive frequency $\omega_d(t) = \omega_{\mathrm{ge}} - vt/2 + \delta$
  - System Hamiltonian $$H = -\frac{\hbar}{2}(vt - \delta)\sigma^z + \frac{\hbar\Omega}{2}\sigma^x$$
- If $t \in [-T/2, T/2]$ and $|\delta/v| \ll T$ the non-adiabatic transfer probability is

$$P_{\mathrm{LZSM}} = \exp\left[-\pi\frac{\Omega^2}{2|v|}\right]$$

Beyond Rabi pulses: phase modulation for qubit control -

M. Kuzmanovic , I. Björkman , J.J. McCord,
S. Dogra, and G. S. Paraoanu,
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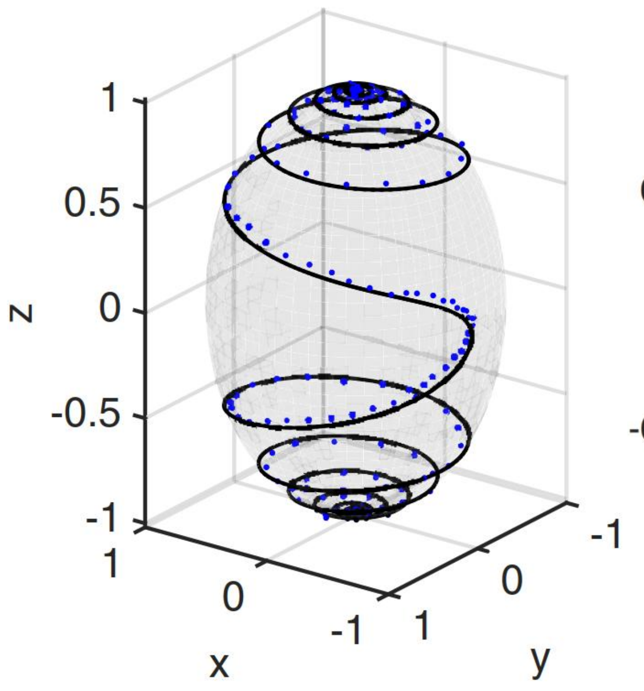
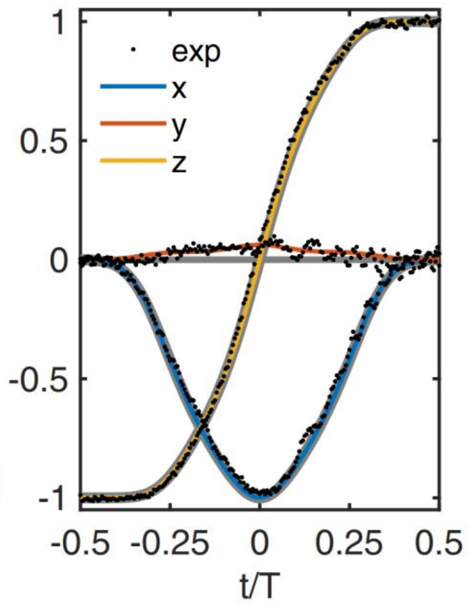
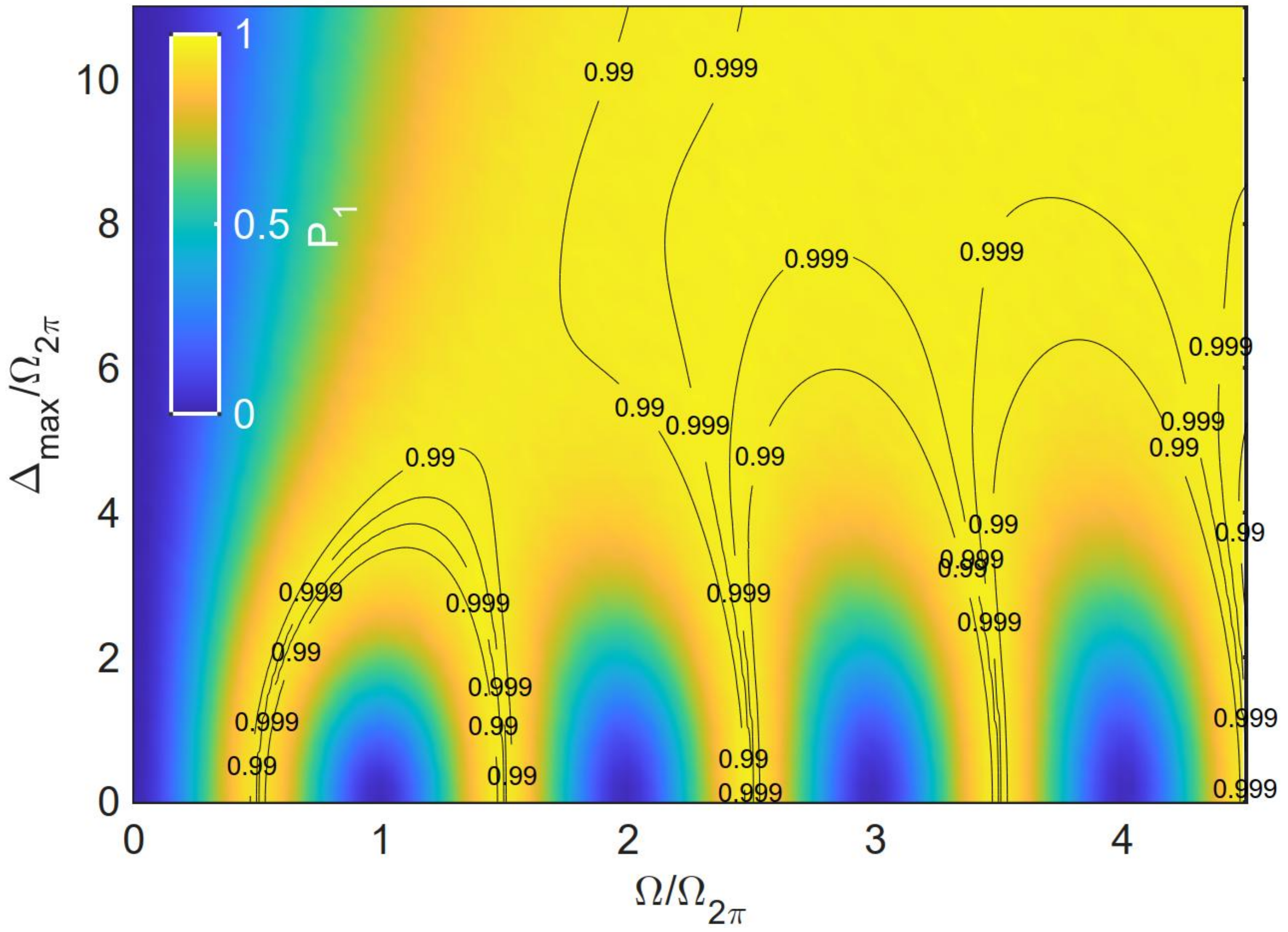
$$H = \frac{\hbar}{2}\begin{pmatrix} -\Delta(t) & \Omega(t) \\ \Omega(t) & \Delta(t) \end{pmatrix}$$

- **transfer of population** $\Delta(t) = \Delta_{\max}\frac{2t}{T}$



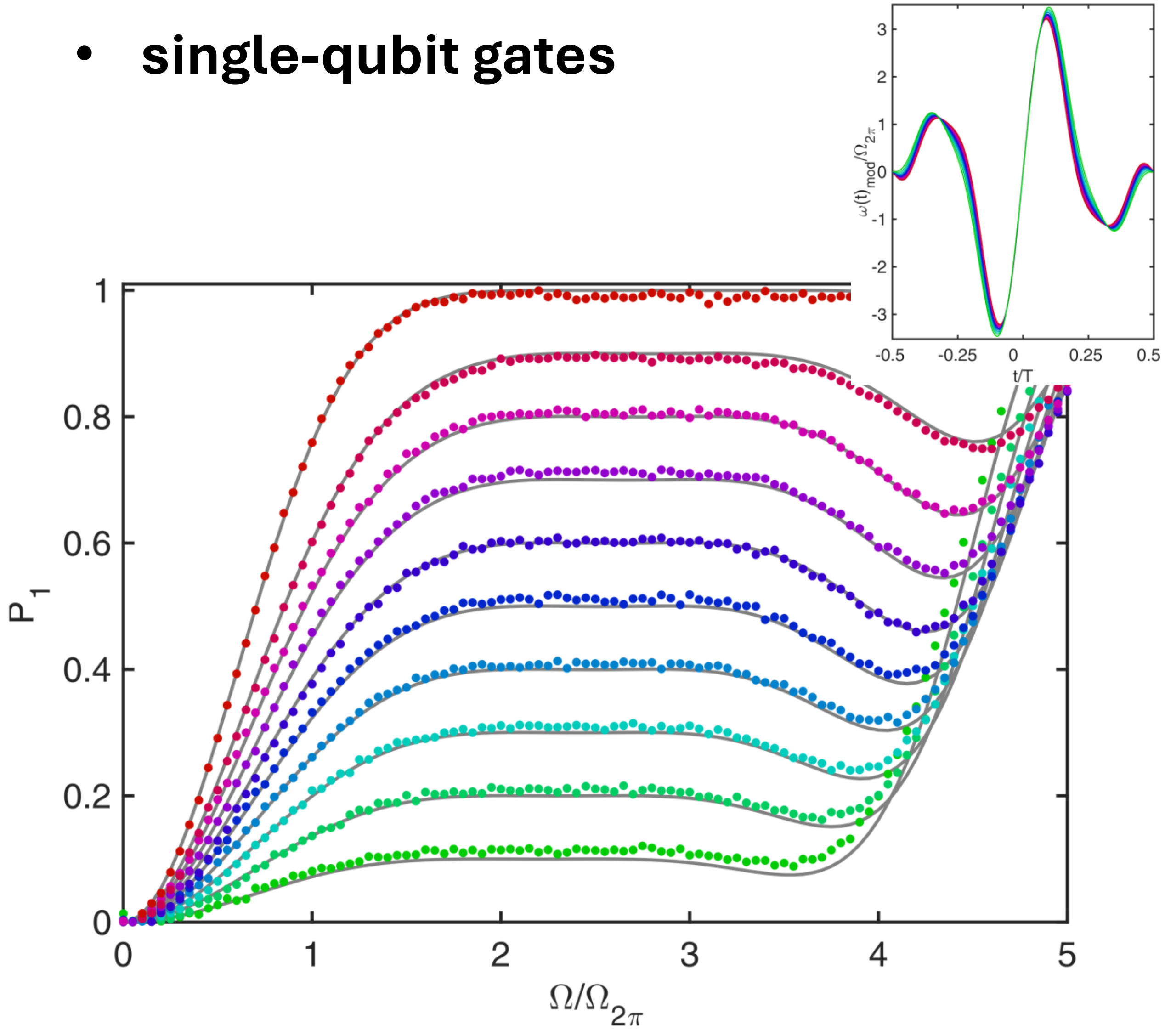


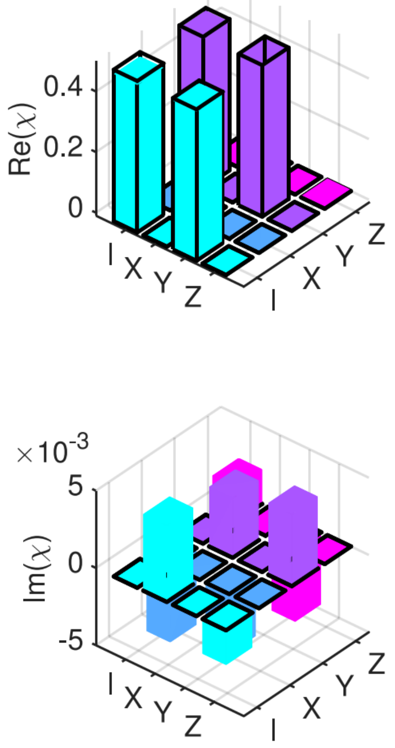


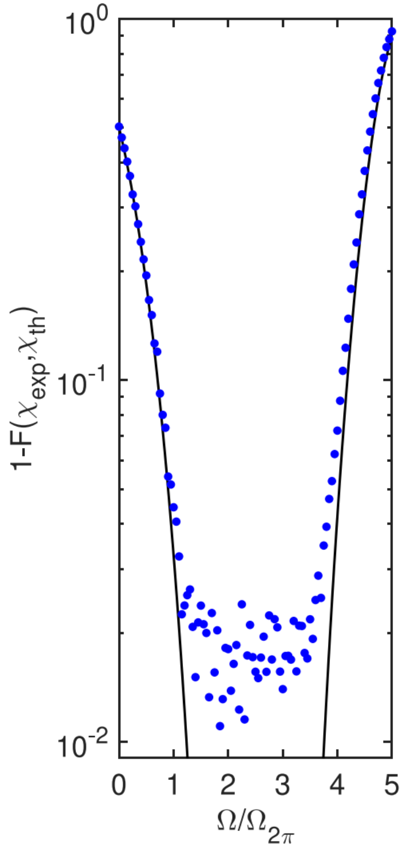
- **single-qubit gates**

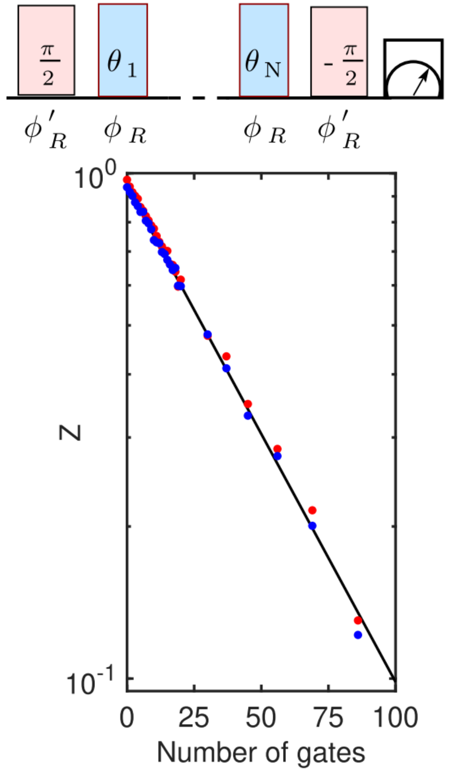
$$\|U_0 - U(\Omega, \Delta(t))\|_{\mathrm{F}} \quad \text{minimized}$$

$$\Delta(t) = \sum_{k\in\mathbb{N}} a_k \left[\sin(kt) + \frac{2\pi kt}{T}\cos(kt) \right]$$









randomized benchmarking

Marko Kuzmanović, Isak Björkman , John J. McCord , Shruti Dogra, and Gheorghe Sorin Paraoanu,
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Two-photon Landau-Zener-Stückelberg-Majorana

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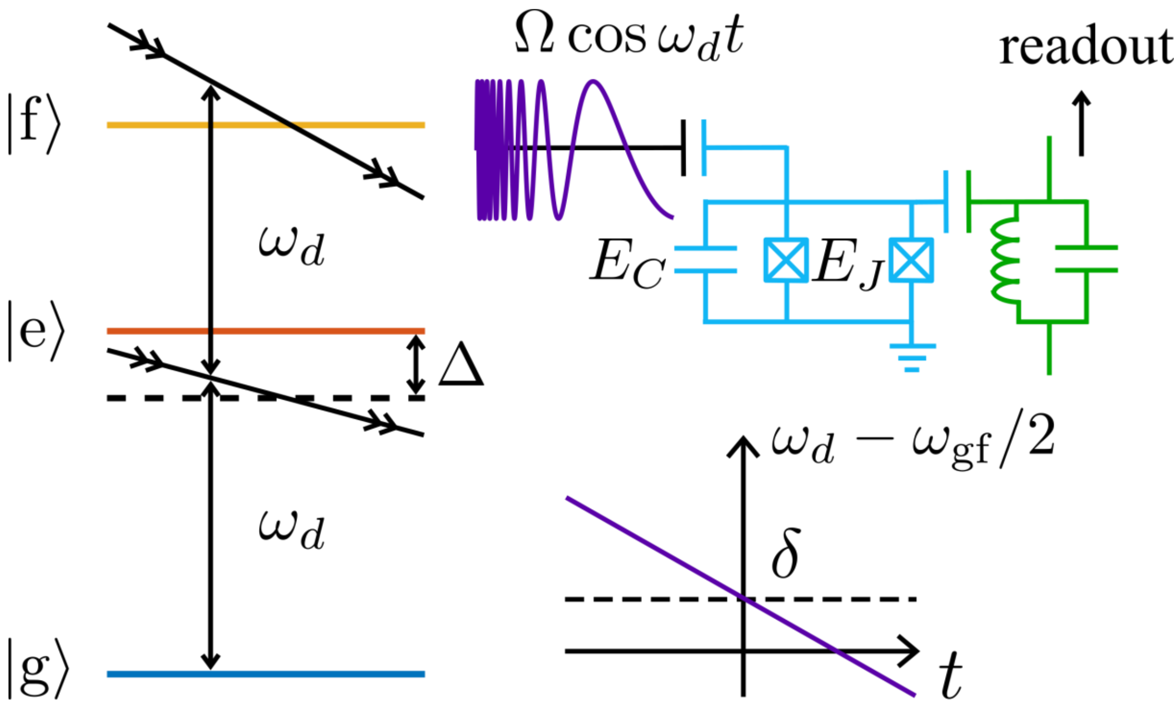
- Three level system (transmon) driven by

$$\omega_d(t) = \omega_{\mathrm{gf}}/2 - vt/2 + \delta$$
$$t \in [-T/2, T/2]$$

- In a frame co-rotating with the drive and after RWA, the system Hamiltonian is

$$H = -\hbar\Delta_{\mathrm{ge}}|\mathrm{g}\rangle\langle\mathrm{g}| + \frac{\hbar\Omega_{\mathrm{ge}}}{2}(|\mathrm{g}\rangle\langle\mathrm{e}| + |\mathrm{e}\rangle\langle\mathrm{g}|) + \frac{\hbar\Omega_{\mathrm{ef}}}{2}(|\mathrm{e}\rangle\langle\mathrm{f}| + |\mathrm{f}\rangle\langle\mathrm{e}|) + \hbar\Delta_{\mathrm{ef}}|\mathrm{f}\rangle\langle\mathrm{f}|.$$



- Detunings: $\Delta_{\mathrm{ge}} = \omega_{\mathrm{ge}} - \omega_d(t) - (d\omega_d(t)/dt)t \quad \Delta_{\mathrm{ef}} = \omega_{\mathrm{ef}} - \omega_d(t) - (d\omega_d(t)/dt)t$

- Resulting in $\Delta_{\mathrm{ge}}(t) = -\Delta - vt + \delta \qquad \Delta_{\mathrm{ef}}(t) = -\Delta + vt - \delta$

  where $\Delta = \omega_{\mathrm{ge}} - \omega_{\mathrm{gf}}/2 = \omega_{\mathrm{gf}}/2 - \omega_{\mathrm{ef}} \approx E_{\mathrm{C}}/2\hbar$ set by transmon anharmonicity

Ideal two-Photon LZSM Hamiltonian

- System Hamiltonian

$$\Omega \equiv \Omega_{\mathrm{ge}} = \Omega_{\mathrm{ef}}/\sqrt{2}$$

$$H = \hbar \begin{pmatrix} -\Delta - vt + \delta & \frac{\Omega_{\mathrm{ge}}}{2} & 0 \\ \frac{\Omega_{\mathrm{ge}}}{2} & 0 & \frac{\Omega_{\mathrm{ef}}}{2} \\ 0 & \frac{\Omega_{\mathrm{ef}}}{2} & -\Delta + vt - \delta \end{pmatrix}$$

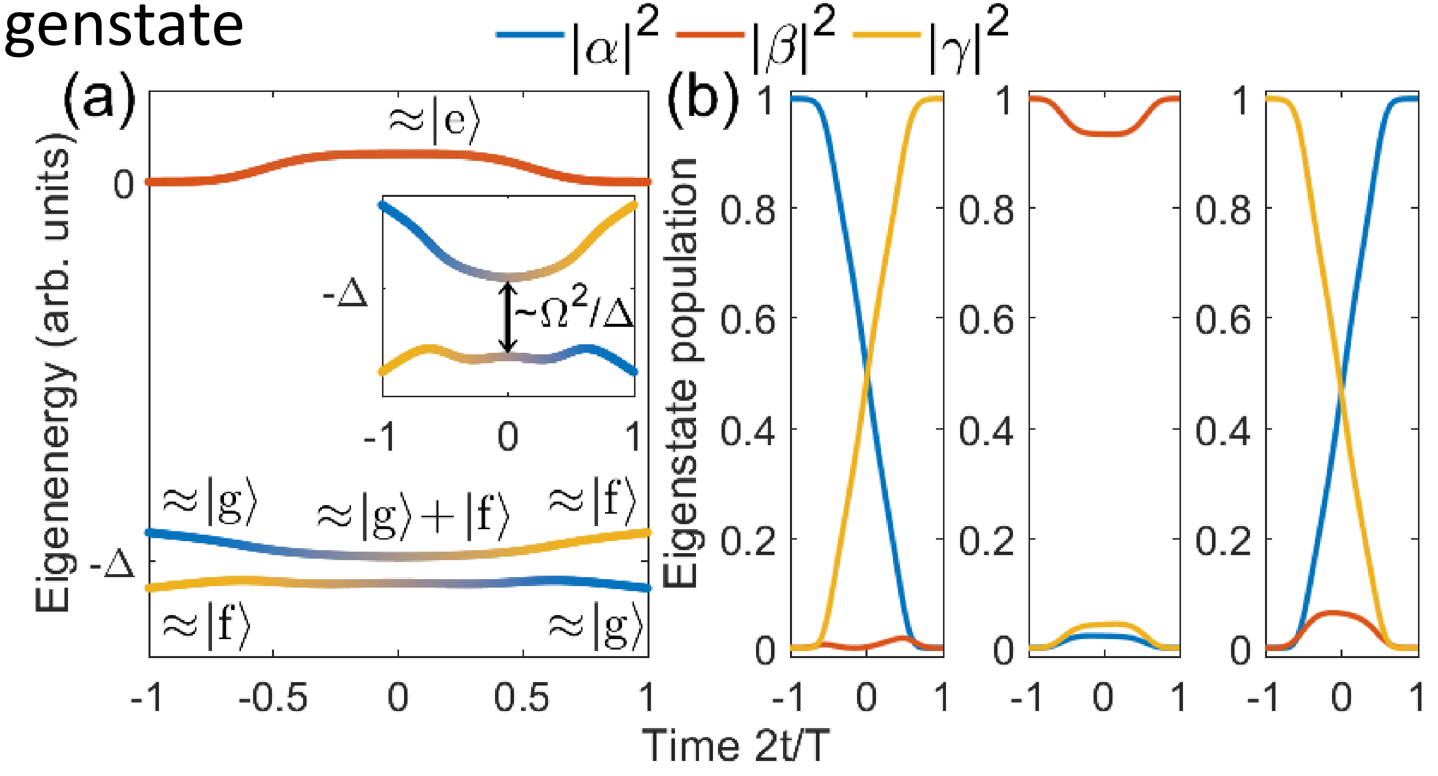
- Technical issue: a transfer from $|\mathrm{g}\rangle$ to $|\mathrm{f}\rangle$ cannot be realized with constant Ω as none of the instantaneous eigenstates start in $|\mathrm{g}\rangle$ nor end in $|\mathrm{f}\rangle$ exactly

- Solution: $\Omega(t = \pm T/2) \approx 0$ $\qquad$ $\Omega(t) = \Omega \exp\left[K_c (2t/T)^n\right]$

- We choose $K_c = \ln(0.01)$ $\qquad$ $n = 4$

Eigensystem

- Each instantaneous eigenstates are expanded in the generic form
$$\alpha|\mathrm{g}\rangle + \beta|\mathrm{e}\rangle + \gamma|\mathrm{f}\rangle$$
- Instantaneous eigenenergy colored based on the weight of each component in corresponding eigenstate
- Two eigenenergies at $\approx -\Delta$ form an avoided crossing
- Absorption of two photons using $|\mathrm{e}\rangle$ as intermediary state



Transmon under drive

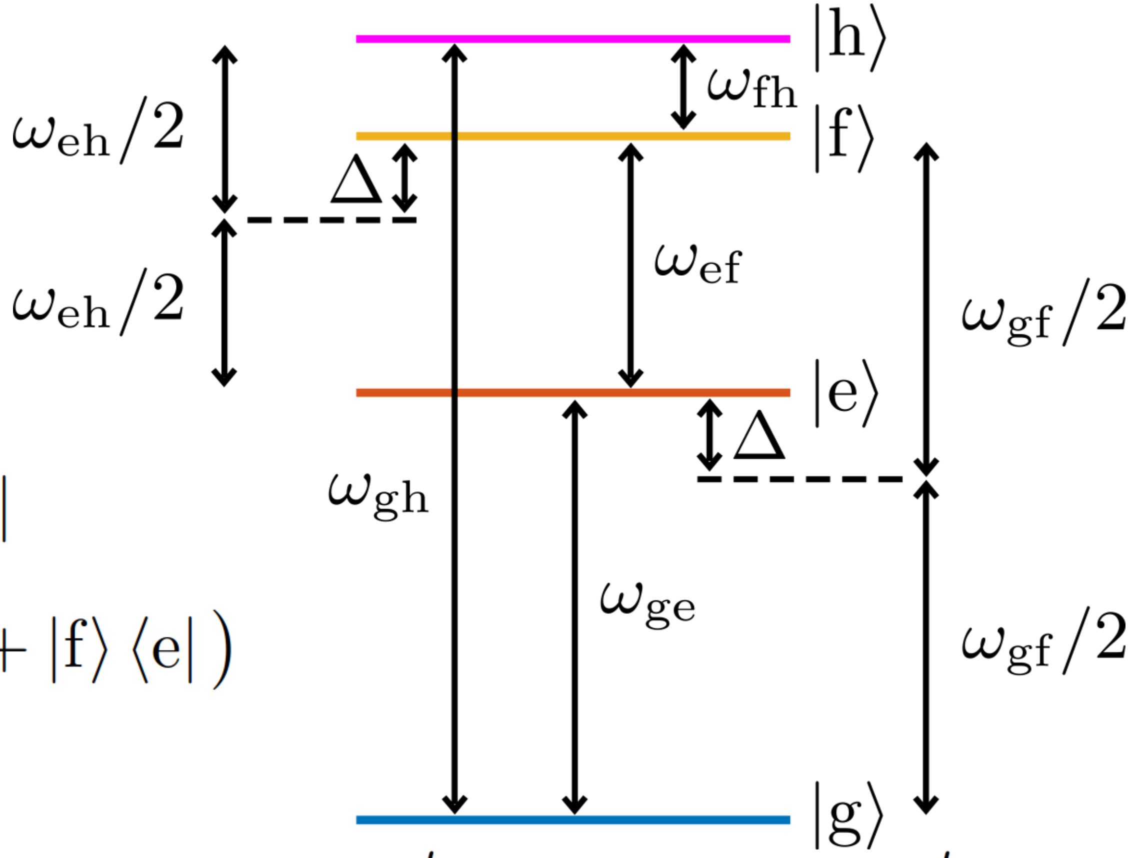
- External microwave $E = E_0 \cos(\omega_d t + \phi)$
- Interaction Hamiltonian $\hat{H}_{int} = \hbar \sum_{\mathrm{kl}} \Omega_{\mathrm{kl}} \cos(\omega_d t + \phi)$
- Rabi frequency between states k and l

$$\Omega_{\mathrm{kl}} = -d_{\mathrm{kl}} E_0/\hbar \qquad d_{\mathrm{kl}} \sim \langle \mathrm{k}|\hat{n}|\mathrm{l}\rangle$$

$$\hat{n} = -i(E_\mathrm{J}/8E_\mathrm{C})^{1/4}(\hat{b} - \hat{b}^\dagger)/\sqrt{2}$$

- First four levels RWA Hamiltonian:

$$\begin{aligned} H = &- \hbar\Delta_{\mathrm{ge}} \left|\mathrm{g}\right\rangle \left\langle\mathrm{g}\right| + \hbar\Delta_{\mathrm{ef}} \left|\mathrm{f}\right\rangle \left\langle\mathrm{f}\right| + \hbar\Delta_{\mathrm{eh}} \left|\mathrm{h}\right\rangle \left\langle\mathrm{h}\right| \\ &+ \frac{\hbar\Omega_{\mathrm{ge}}}{2}\left(\left|\mathrm{g}\right\rangle \left\langle\mathrm{e}\right| + \left|\mathrm{e}\right\rangle \left\langle\mathrm{g}\right| \right) + \frac{\hbar\Omega_{\mathrm{ef}}}{2}\left(\left|\mathrm{e}\right\rangle \left\langle\mathrm{f}\right| + \left|\mathrm{f}\right\rangle \left\langle\mathrm{e}\right| \right) \\ &+ \frac{\hbar\Omega_{\mathrm{fh}}}{2}\left(\left|\mathrm{f}\right\rangle \left\langle\mathrm{h}\right| + \left|\mathrm{h}\right\rangle \left\langle\mathrm{f}\right| \right) \end{aligned}$$



$$\Delta_{\mathrm{eh}}(t) = \omega_{\mathrm{fh}} - \omega_{\mathrm{ge}} - 4D\frac{t}{T} - 2\delta = -4\Delta - 4D\frac{t}{T} - 2\delta$$

Effective Hamiltonian

James, D. F., and Jonathan Jerke. "Effective Hamiltonian theory and its applications in quantum information." *Canadian Journal of Physics* 85.6 (2007): 625-632.

- Time-averaged effective Hamiltonian
  - (i) $\omega_{\mathrm{ge}} - \omega_d, \omega_d - \omega_{\mathrm{ef}} > 0 \ \forall \ t$
  - (ii) change in drive frequency over time is slow in comparison to everything else
  - (iii) weak interactions s.t. 4^{th} and higher order effects negligible
- The ac Stark shift in frequency between $|\mathrm{g}\rangle$ and $|\mathrm{f}\rangle$ depends on presence of fourth level $|\mathrm{h}\rangle$

$$H_{\mathrm{eff}} \approx -\frac{\hbar}{2}\left(-2\epsilon - 2t\frac{d\epsilon}{dt} + \frac{3\Omega_{\mathrm{ge}}^2 - 3\Omega_{\mathrm{ef}}^2 + \Omega_{\mathrm{fh}}^2}{12\Delta}\right)\sigma_{\mathrm{gf}}^z - \frac{\hbar}{2}\frac{\Omega_{\mathrm{ge}}\Omega_{\mathrm{ef}}}{2\Delta}\sigma_{\mathrm{gf}}^x$$

$$\epsilon(t) \ll \Delta$$

$$\epsilon(t) = \omega_d(t) - \omega_{\mathrm{gf}}/2 = -vt/2 + \delta$$

  - But $\Omega_{\mathrm{ge}} = \Omega_{\mathrm{ef}}/\sqrt{2} = \Omega_{\mathrm{fh}}/\sqrt{3}$, leading to the cancellation of the ac Stark shift
- Final effective Hamiltonian

$$\boxed{H_{\mathrm{LZSM}}^{(2\mathrm{ph})} \approx -\frac{\hbar}{2}\left(2vt - 2\delta\right)\sigma_{\mathrm{gf}}^z + \frac{\hbar\Omega_{\mathrm{gf}}}{2}\sigma_{\mathrm{gf}}^x}$$

$$\sigma_{\mathrm{gf}}^z = |\mathrm{g}\rangle\langle\mathrm{g}| - |\mathrm{f}\rangle\langle\mathrm{f}|$$

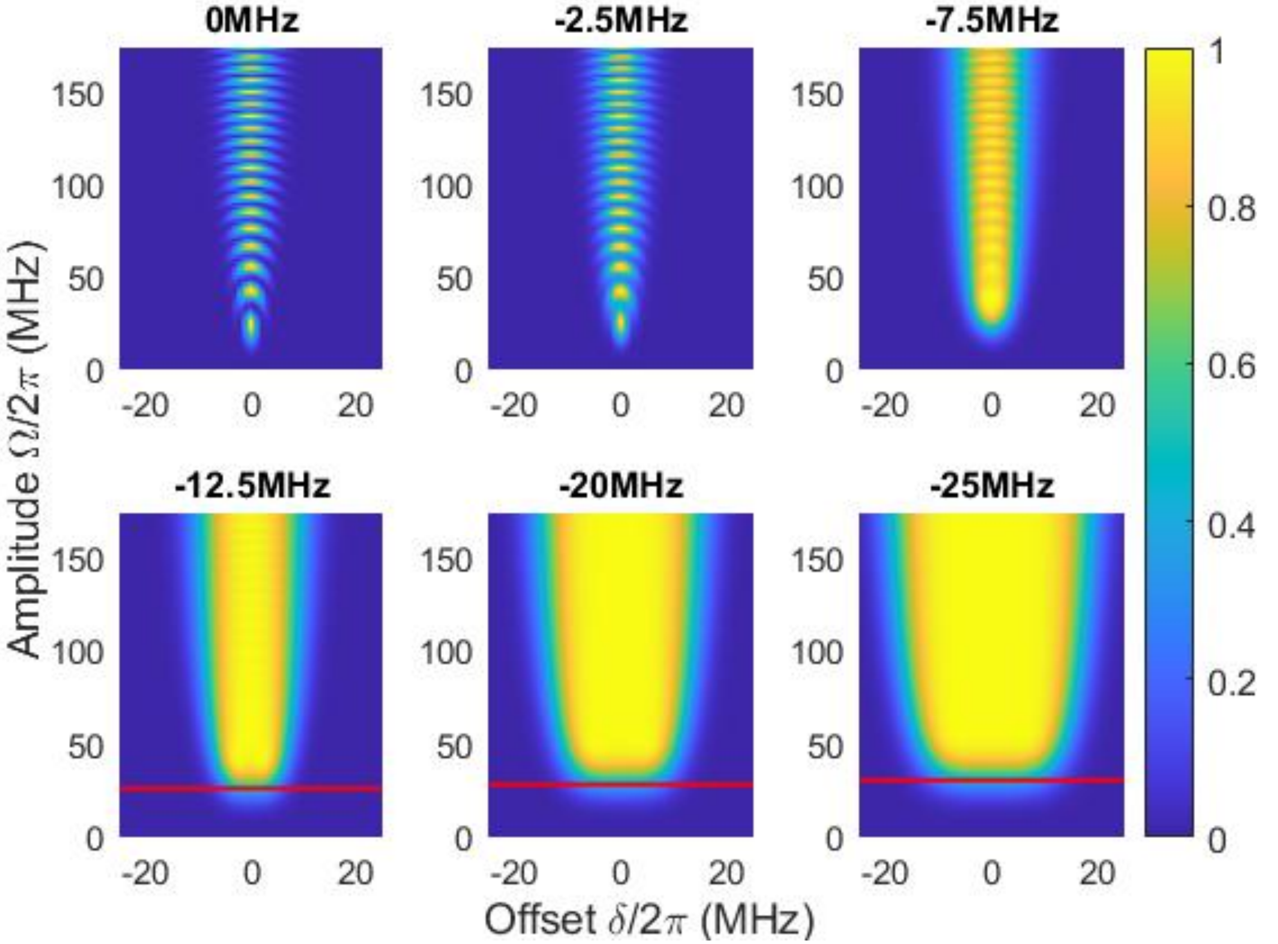
$$\sigma_{\mathrm{gf}}^x = |\mathrm{g}\rangle\langle\mathrm{f}| + |\mathrm{f}\rangle\langle\mathrm{g}|$$

  - effective coupling $\Omega_{\mathrm{gf}} = -\Omega_{\mathrm{ge}}\Omega_{\mathrm{ef}}/2\Delta$
- Has form of LZSM, yielding for the nonadiabatic probability

$$\boxed{P_{\mathrm{LZSM}}^{(2\mathrm{ph})} = \exp\left[-\pi\frac{\Omega_{\mathrm{gf}}^2}{4|v|}\right]}$$

Robustness

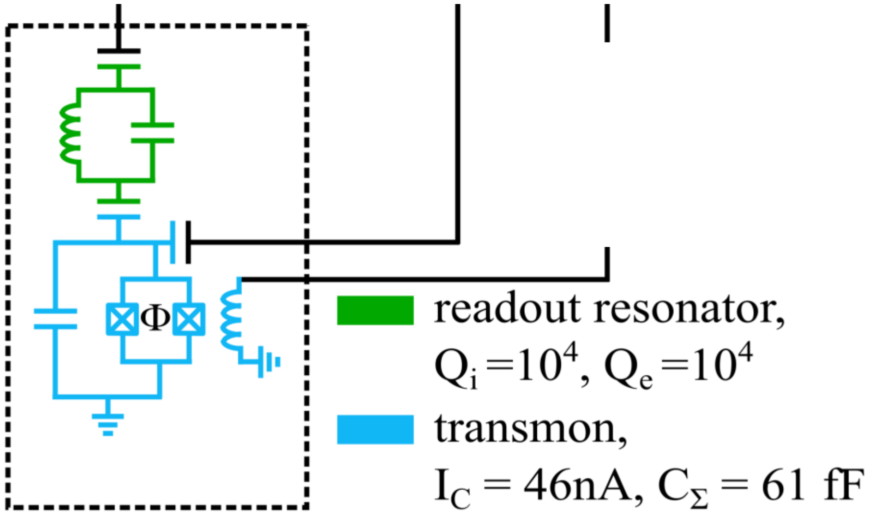
- $|\mathrm{f}\rangle$ population after drive for different $D = \omega_d(T/2) - \omega_d(-T/2) = -vT/2$
- Increased $|D|$ means greater frequency robustness but also an increase in the threshold of amplitude robustness
- An offset δ only shifts the time when resonant with $\omega_{\mathrm{gf}}/2$ (avoided crossing)
  - Greater $|D|$ covers a broader frequency range
  - Better satisfies $-vt + \delta \approx 0$ for some t



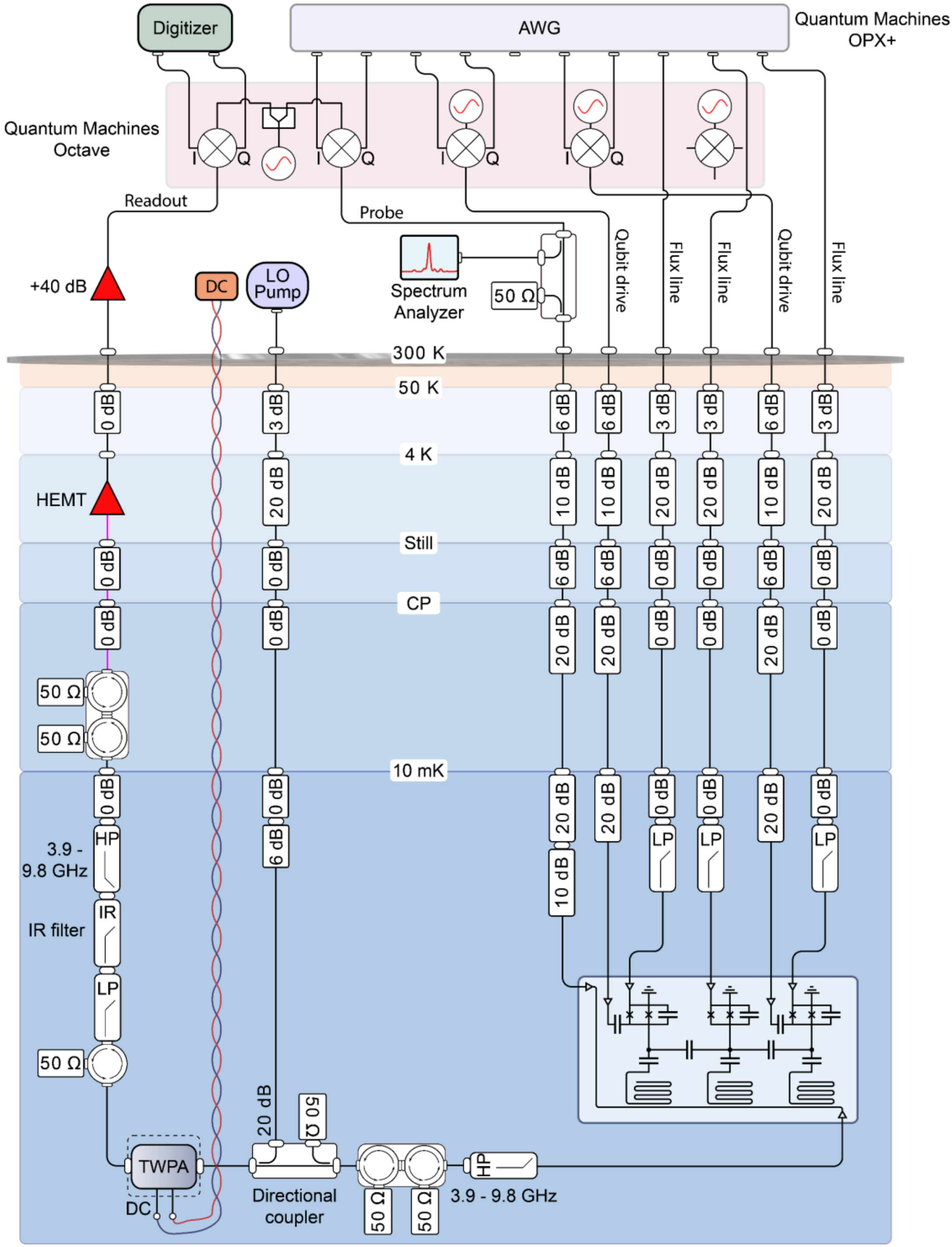
- From effective Hamiltonian: $P_{\mathrm{LZSM}}^{(2\mathrm{ph})} = \exp\left[-\pi \frac{\Omega_{\mathrm{gf}}^2}{4|v|}\right]$ $\Omega_{\mathrm{gf}} = -\Omega^2/\sqrt{2}\Delta$
  - Robustness to amplitude; impact of amplitude offset exponentially reduced
  - If $|v|$ increases, equal increase in Ω^4 needed for same probability

Experiment

- Transmon
  - $\omega_{\rm ge}/2\pi = 7.24$ GHz $\quad \omega_{\rm ef}/2\pi = 6.90$ GHz
  - $E_{\rm J} = h \times 23.0$ GHz $\quad E_{\rm C} = h \times 318$ MHz

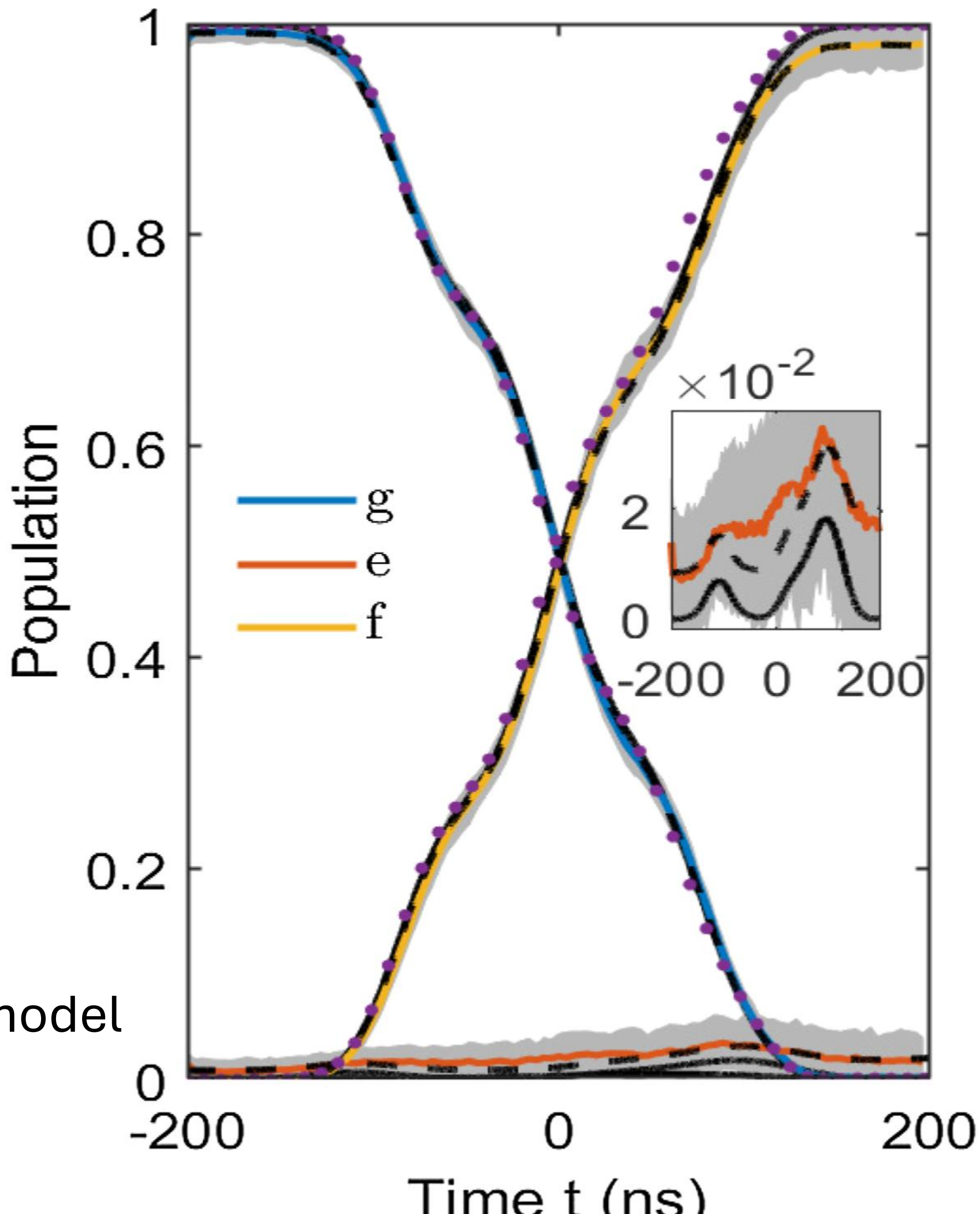


- Duration $\quad T = 400$ ns $\quad \omega_d(t) = \omega_{\rm gf}/2 + Dt/T + \delta$
- We choose $\quad D/2\pi = -12.5$ MHz
  - Larger $|D|$ gets us closer to $\omega_{\rm ge}$ and $\omega_{\rm ef}$
  - Smaller $|D|$ and we recover Rabi oscillations

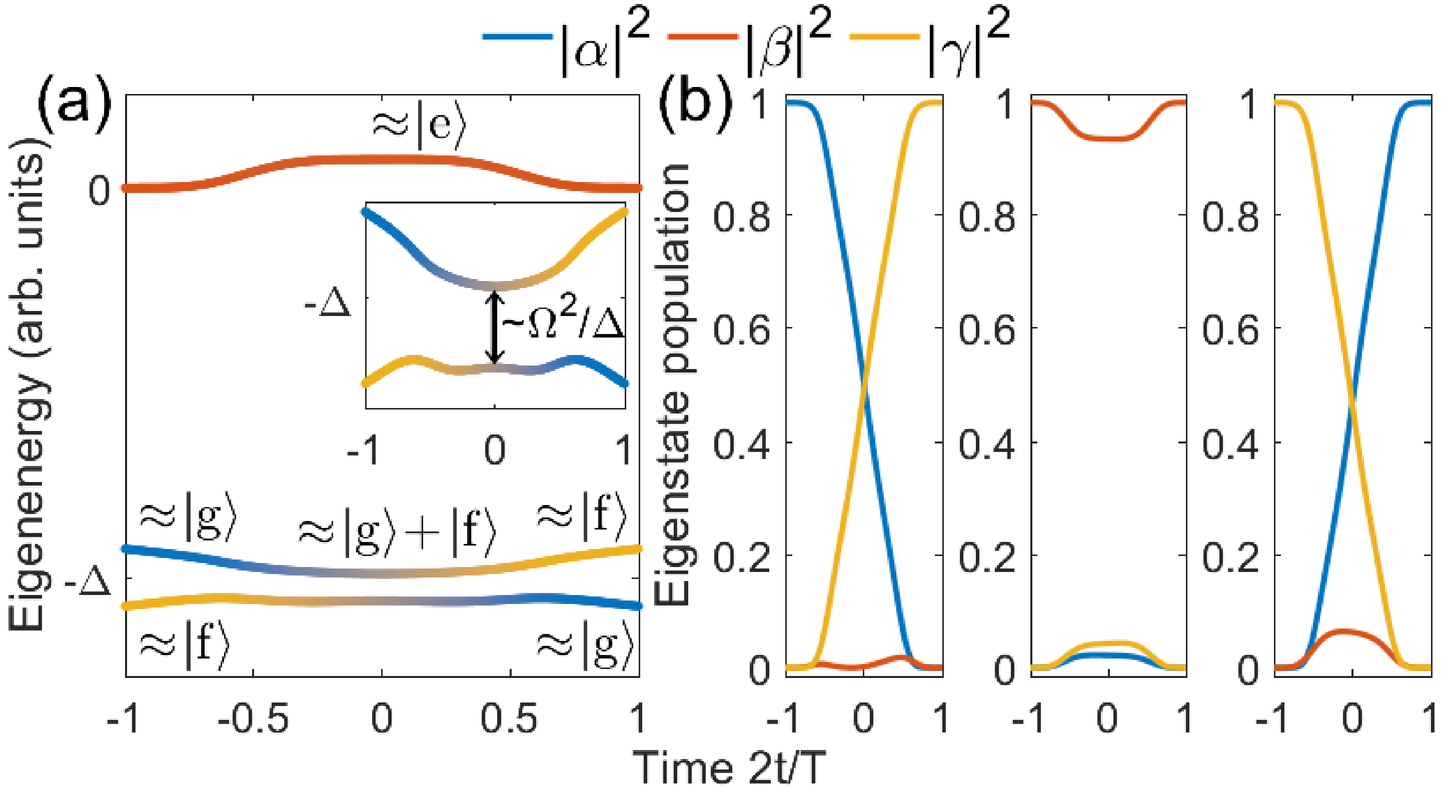


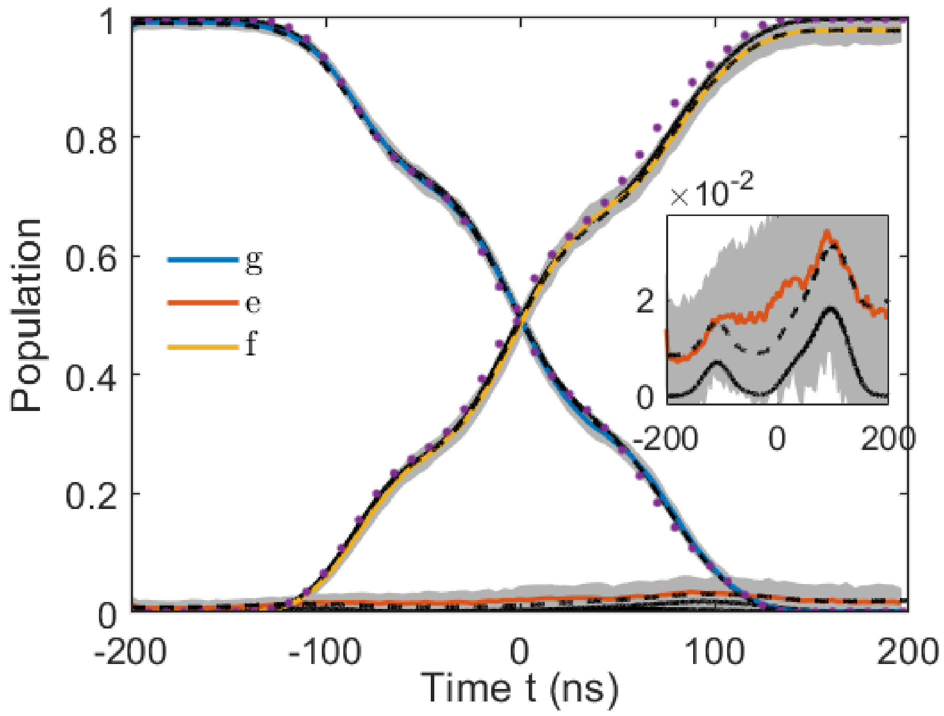
Result

- Adiabatic transfer
  from $|\mathrm{g}\rangle$ to $|\mathrm{f}\rangle$
- Parameters
  $\Omega/2\pi = 55.6\ \mathrm{MHz}$ and $\delta/2\pi = 0\ \mathrm{MHz}$
- Experimentally
  $p_{\mathrm{f}} \approx 98.0\%$ and $p_{\mathrm{e}} < 3.48\%$
- Ideally
  $p_{\mathrm{f}} > 99.9\%$ and $p_{\mathrm{e}} < 1.87\%$
- Effective model (dotted lines) and Lindblad model (dashed lines) in agreement



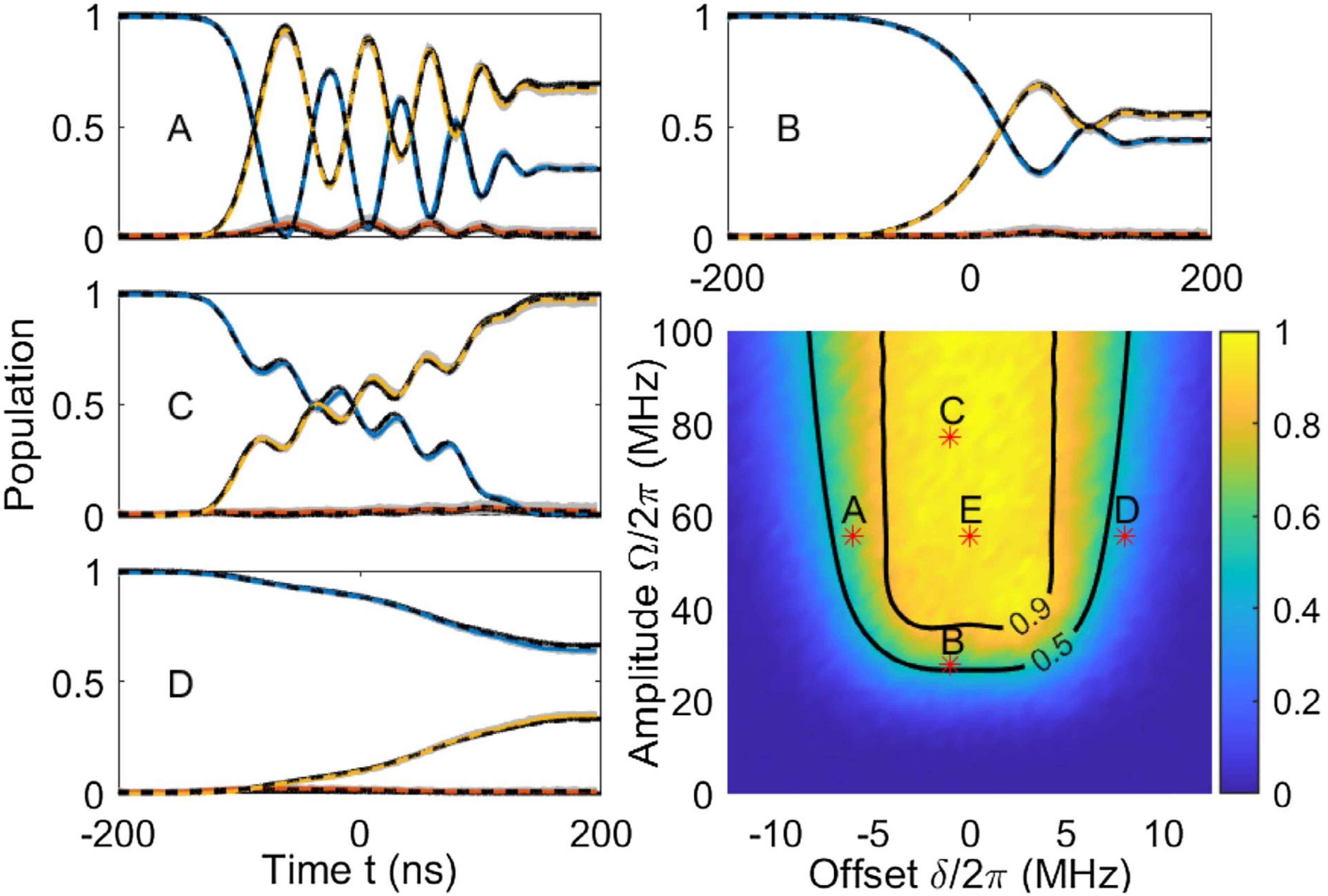
Comparison with ideal three-level model





Robustness

- Probability to measure $|f\rangle$ with different Ω and δ



Two-photon LZSM process

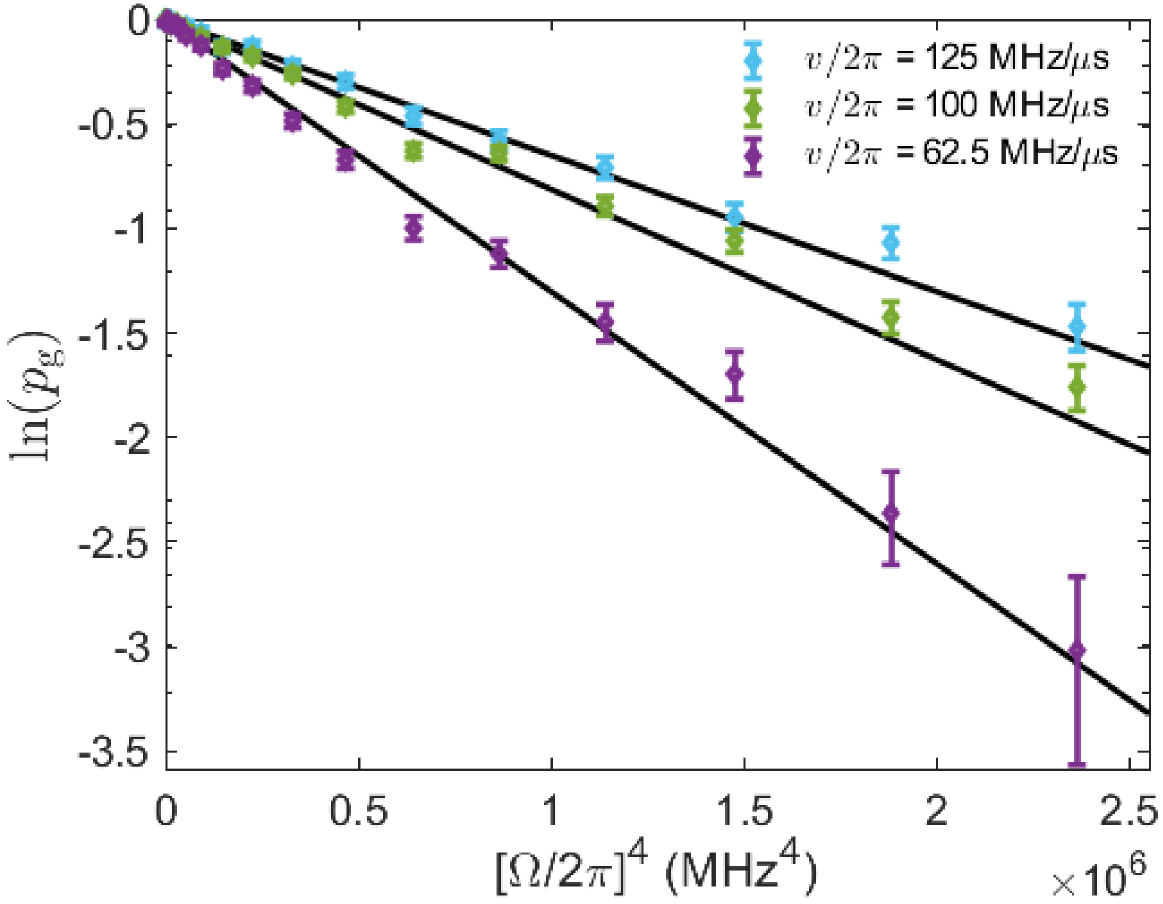
- A two-photon LZSM process with double LZSM velocity
  - Compare non-adiabatic transfer probability to experimental data

$$p_g = P_{\mathrm{LZSM}}^{(2\mathrm{ph})} = \exp\left[-\pi\frac{\Omega_{\mathrm{gf}}^2}{4|v|}\right]$$

$$\Omega_{\mathrm{gf}} = -\Omega^2/\sqrt{2}\Delta$$

- Should scale like $\ln p_g \propto \Omega^4$

- The agreement is a clear signature of the two-photon LZSM process



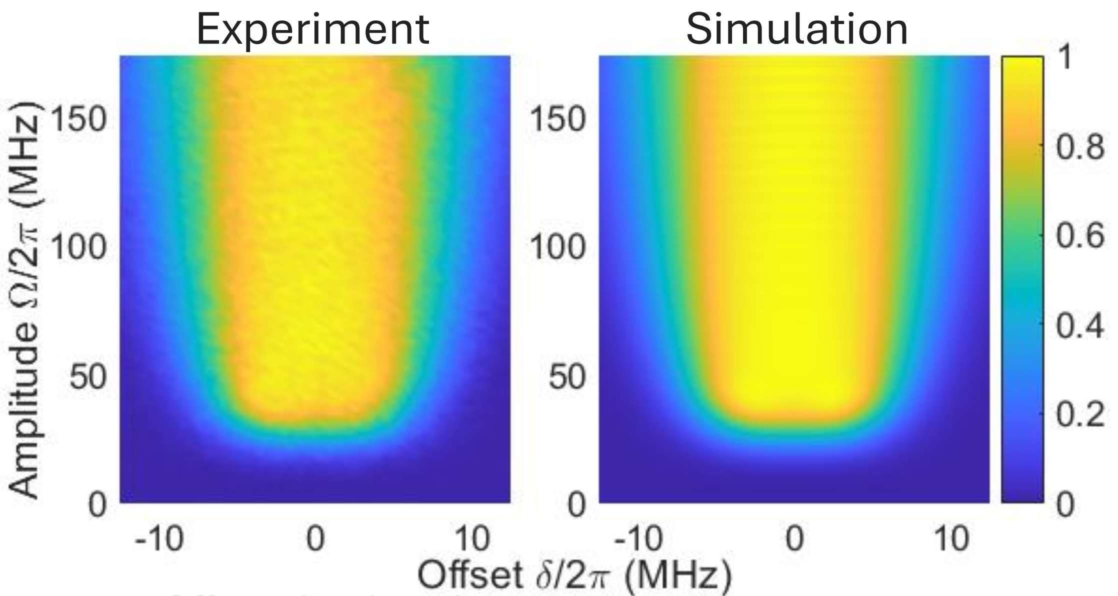
Negative LZSM velocity

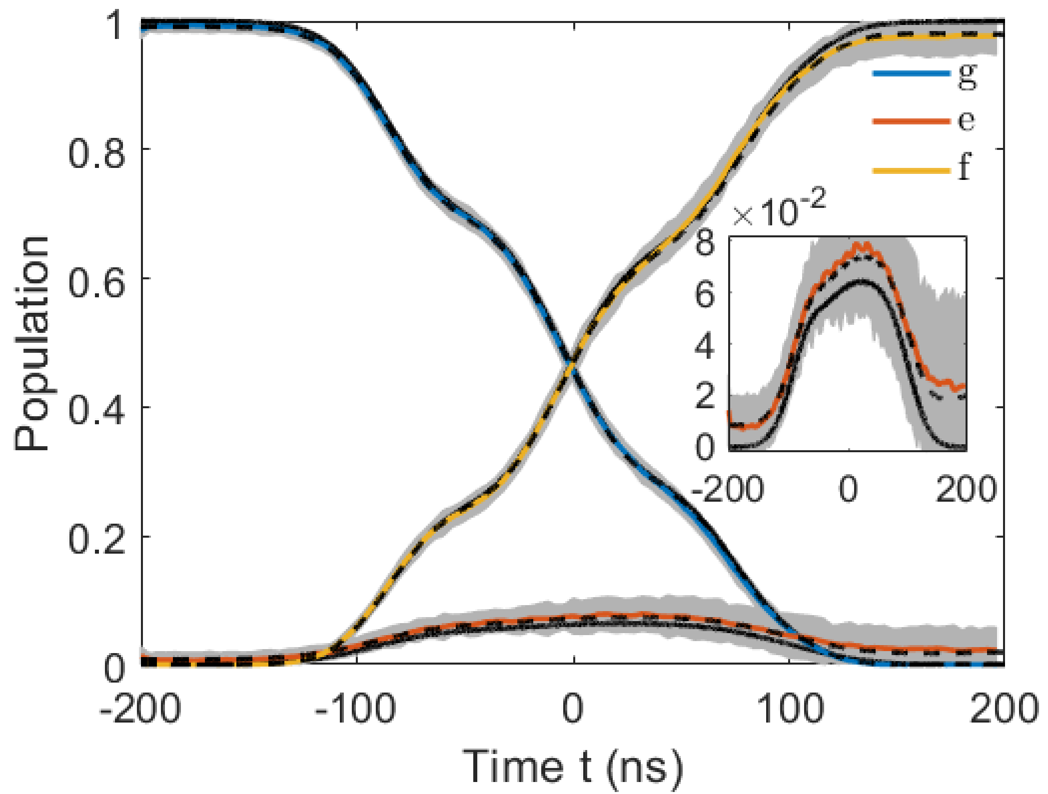
- Hamiltonian invariant under simultaneous exchange of $v \leftrightarrow -v$ and $\mathrm{g} \leftrightarrow \mathrm{f}$

$$H = -\hbar\Delta_{\mathrm{ge}}|\mathrm{g}\rangle\langle\mathrm{g}| + \frac{\hbar\Omega_{\mathrm{ge}}}{2}(|\mathrm{g}\rangle\langle\mathrm{e}| + |\mathrm{e}\rangle\langle\mathrm{g}|) + \frac{\hbar\Omega_{\mathrm{ef}}}{2}(|\mathrm{e}\rangle\langle\mathrm{f}| + |\mathrm{f}\rangle\langle\mathrm{e}|) + \hbar\Delta_{\mathrm{ef}}|\mathrm{f}\rangle\langle\mathrm{f}|.$$

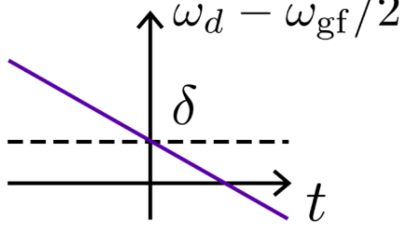
$$\Delta_{\mathrm{ef}}(t) = -\Delta + vt - \delta$$
$$\Delta_{\mathrm{ge}}(t) = -\Delta - vt + \delta$$

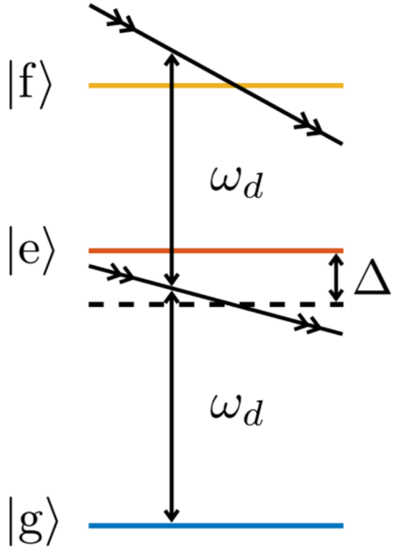
- Experimentally $p_{\mathrm{f}} \approx 97.7\%$ and to $|\mathrm{e}\rangle$ is $p_{\mathrm{e}} < 7.92\%$
- Ideally $p_{\mathrm{f}} > 99.9\%$ and $p_{\mathrm{e}} < 6.48\%$.
- Similar robustness

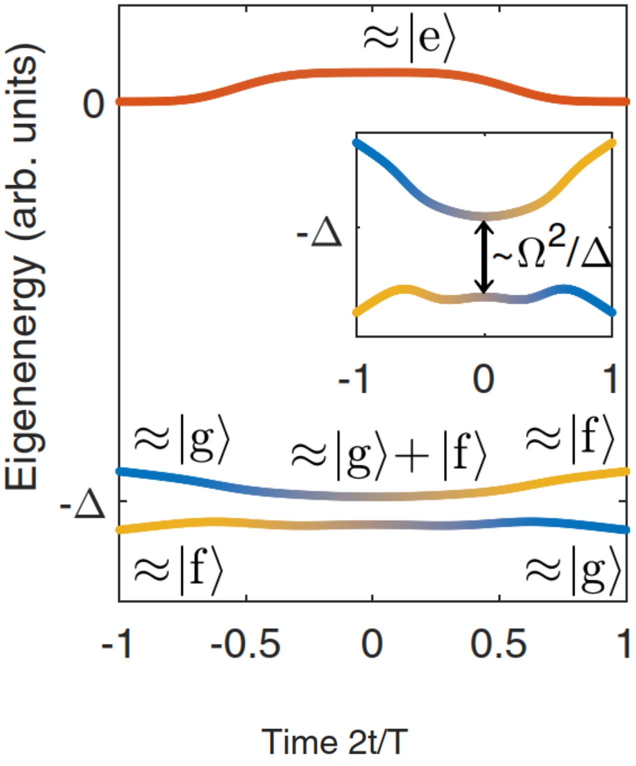




Positive and negative velocities

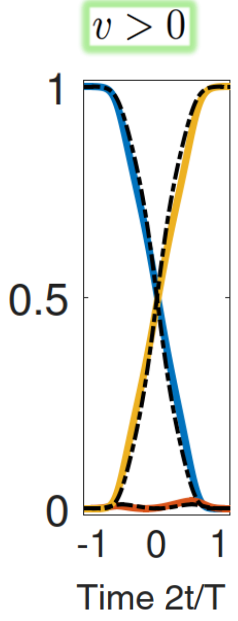


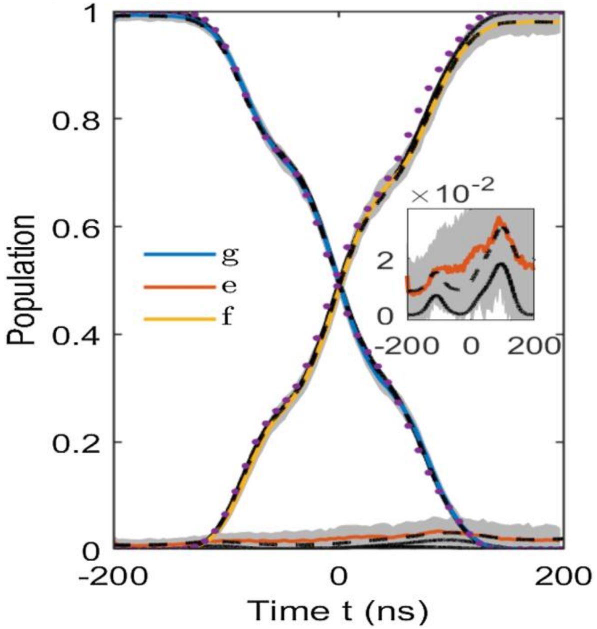


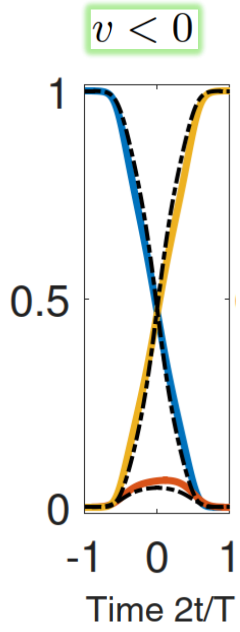


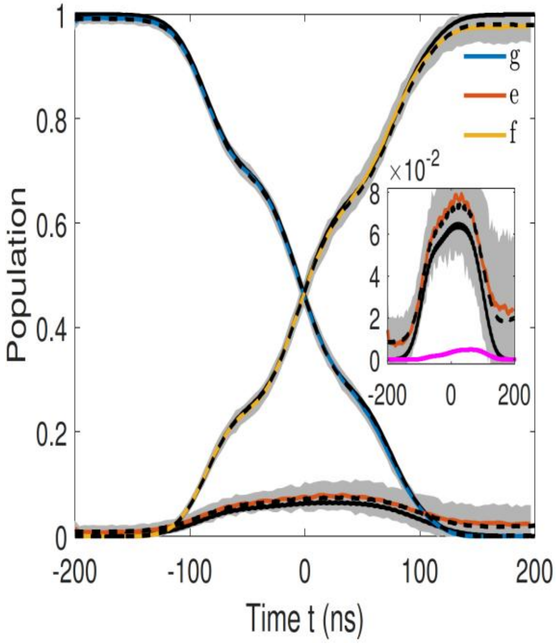
$\alpha|g\rangle + \beta|e\rangle + \gamma|f\rangle$

$|\alpha|^2$
$|\beta|^2$
$|\gamma|^2$









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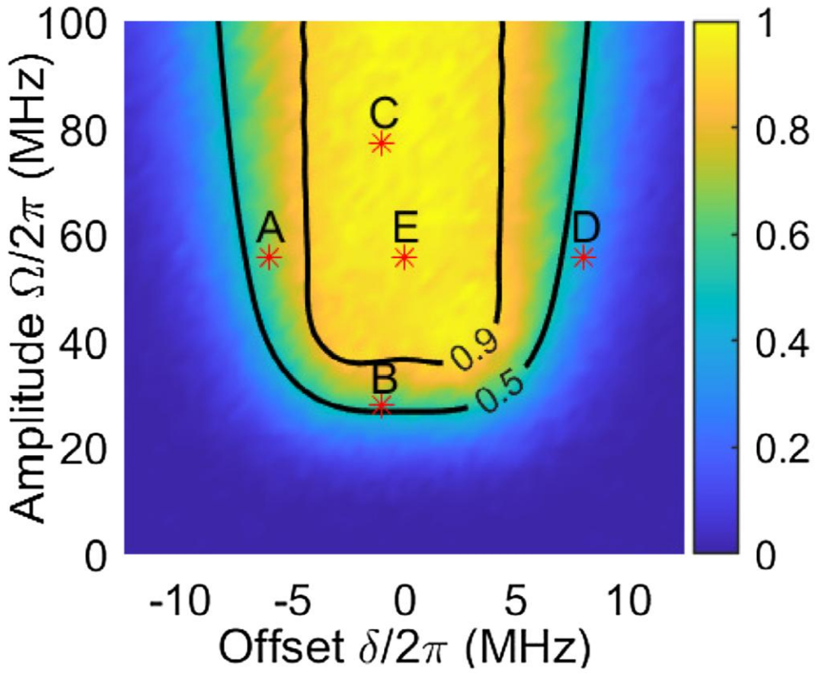
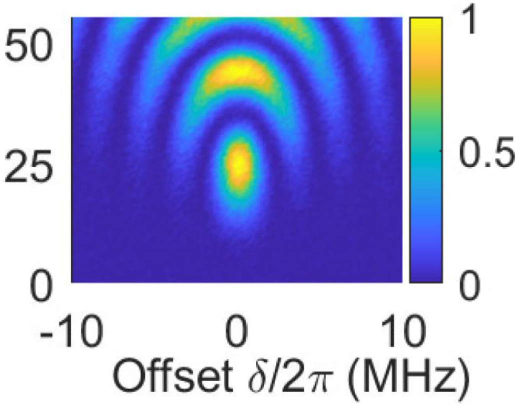
Conclusions

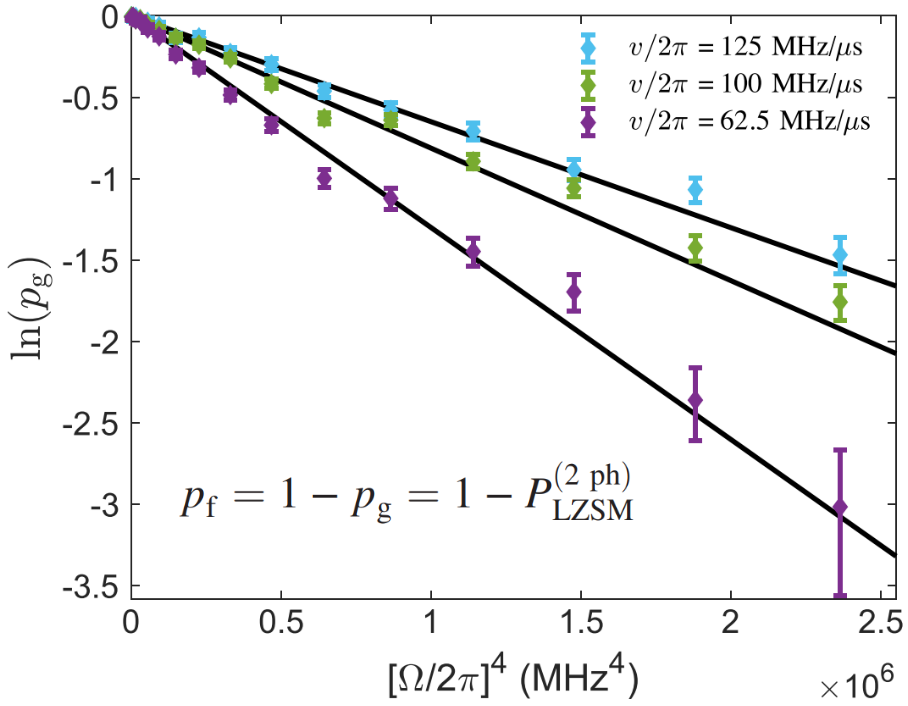
Robustness:



Compare with two-photon Rabi



Doubling of LZSM speed:



Comparison with single-photon LZSM:
(qubit q with chirped X-drive)

$$\Omega_q \cos(\omega_d t)\sigma_x \qquad \omega_d(t) = \omega - vt/2 + \delta$$

$$H^{(1\text{ ph})}_{\text{LZSM}} = -\frac{\hbar}{2}(vt - \delta)\sigma^z + \frac{\hbar\Omega_q}{2}\sigma^x$$

$$P^{(1\text{ ph})}_{\text{LZSM}} = \exp\left[-\pi\frac{\Omega_q^2}{2|v|}\right]$$

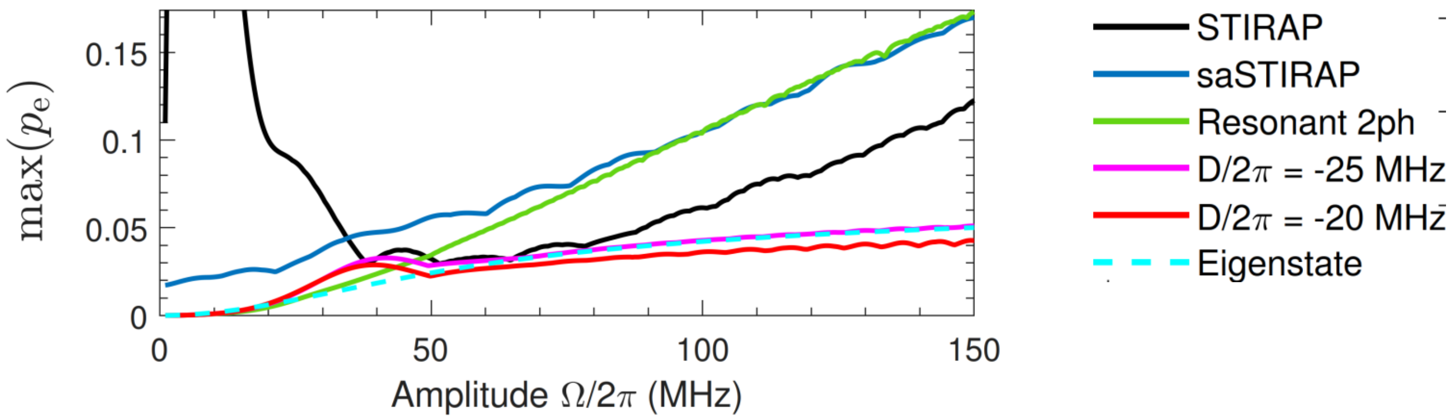
$$\omega_d(t) = \omega_{gf}/2 - vt/2 + \delta$$

$$H^{(2\text{ ph})}_{\text{LZSM}} \approx -\frac{\hbar}{2}(2vt - 2\delta)\sigma^z_{gf} + \frac{\hbar\Omega_{gf}}{2}\sigma^x_{gf}$$

$$P^{(2\text{ ph})}_{\text{LZSM}} = \exp\left[-\pi\frac{\Omega_{gf}^2}{4|v|}\right]$$

Two-Photon Landau-Zener-Stückelberg-Majorana

- Reduce number of qubits in large-scale quantum computer by increasing information per qubit
  - include third level of the transmon
- Transfer from $|\mathrm{g}\rangle$ to $|\mathrm{f}\rangle$ is robust
- Utilizing all levels require ability to transfer between each state
  - direct transfer prohibited
  - existing protocols (Raman process, (sa)STIRAP) sensitive to offsets or increasingly complex
  - Lowest $|e\rangle$ population



Generalizations

- Swap gate utilizing LZSM techniques
  - three transmons, one is a flux-tunable coupler
- Generalizations to multilevel systems