Improved modeling of superconducting circuits

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and

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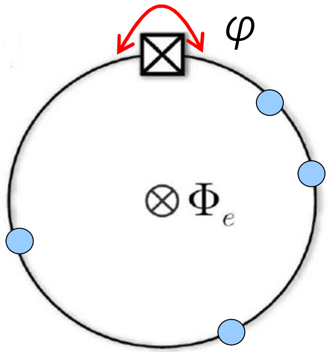
Federal Ministry
of Research, Technology
and Space

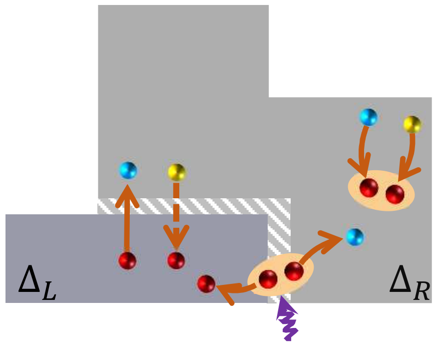
The Laboratory for Physical Sciences

Outline

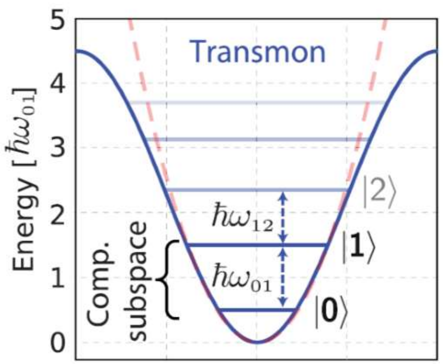
1. Review of quasiparticles in qubits



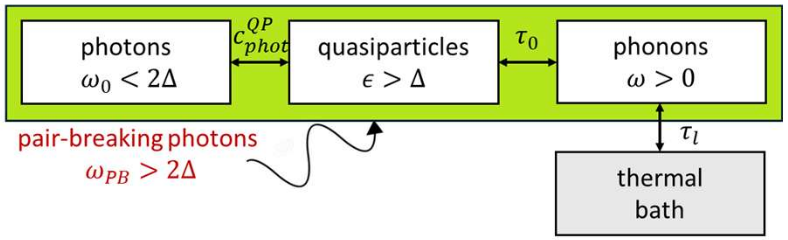
2. Modeling gap asymmetry



3. Modeling higher transmon levels



4. Quasiparticles in resonators



Superconductivity & Bogoliubov quasiparticles

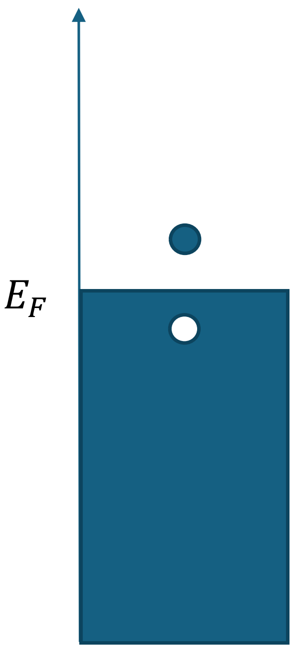
$$\mathscr{H}_M = \sum_{\mathbf{k}} E_{\mathbf{k}} (\gamma^*_{\mathbf{k}0}\gamma_{\mathbf{k}0} + \gamma^*_{\mathbf{k}1}\gamma_{\mathbf{k}1})$$

$$E_{\mathbf{k}} = (\xi_{\mathbf{k}}^2 + |\Delta_{\mathbf{k}}|^2)^{1/2}$$

Bogoliubov transformation

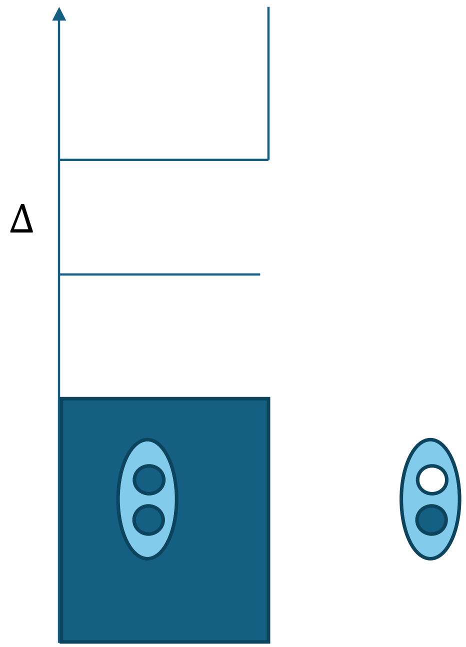
$$\gamma^*_{\mathbf{k}0} = u^*_{\mathbf{k}} c^*_{\mathbf{k}\uparrow} - v^*_{\mathbf{k}} c_{-\mathbf{k}\downarrow}$$
$$\gamma^*_{\mathbf{k}1} = u^*_{\mathbf{k}} c^*_{-\mathbf{k}\downarrow} + v^*_{\mathbf{k}} c_{\mathbf{k}\uparrow}$$

Normal metal

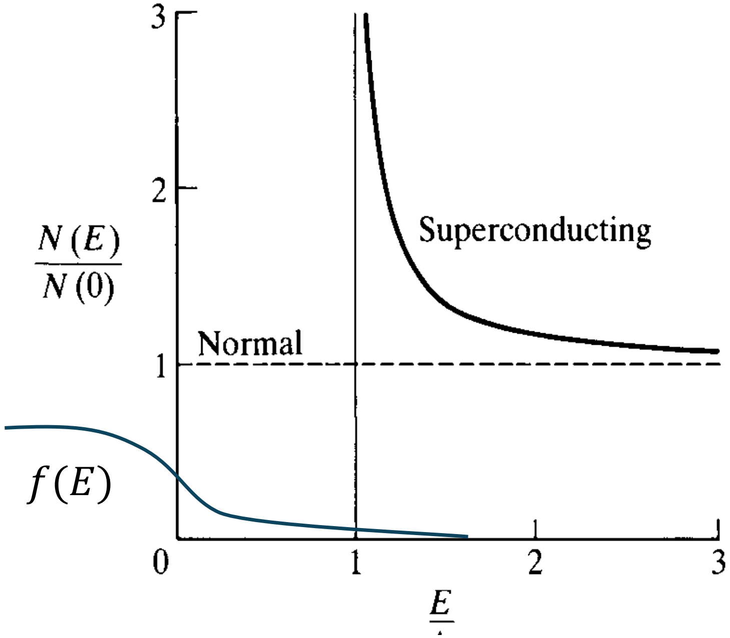


excitations:
electrons and holes

Superconductor



excitations:
quasiparticles

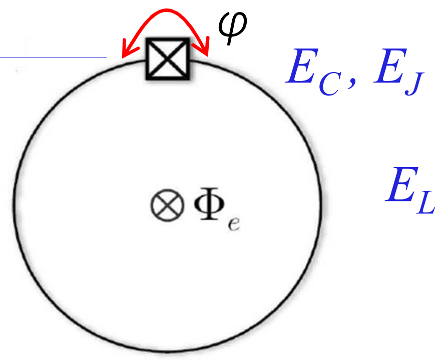


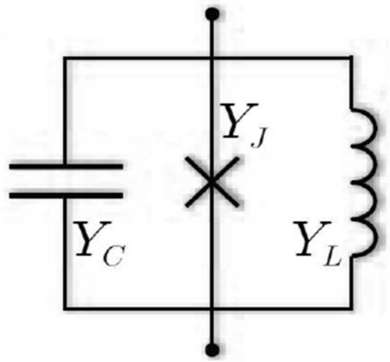
$$\frac{N_s(E)}{N(0)} = \frac{d\xi}{dE} = \begin{cases} \dfrac{E}{(E^2 - \Delta^2)^{1/2}} & (E > \Delta) \\ 0 & (E < \Delta) \end{cases}$$

Single-junction qubit (no QPs)

Josephson junction

superconducting loop + flux





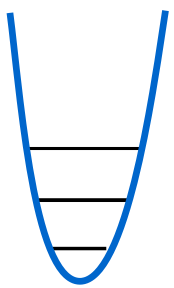
ideal circuit elements

qubit Hamiltonian (quantum dynamics of ***phase difference***)

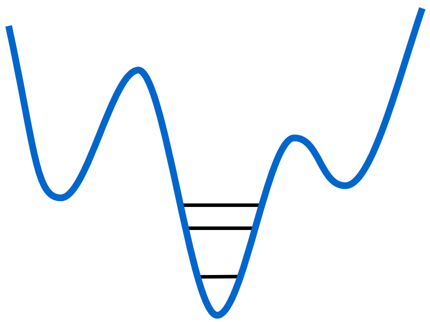
$$\hat{H}_\varphi = 4E_C\left(\hat{N} - \underline{n_g}\right)^2 - E_J\cos\hat{\varphi} + \frac{1}{2}E_L(\hat{\varphi} - \underline{2\pi\Phi_e/\Phi_0})^2$$

charging energy $e^2/2C$ | Josephson energy | inductive energy $(\Phi_0/2\pi)^2/L$

E_J=0



$E_J\neq0$

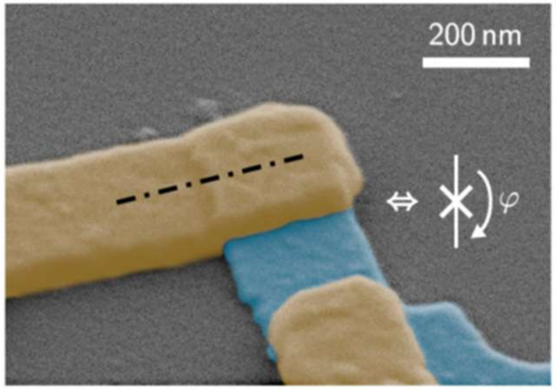


number operator: $\hat{N} = \frac{\hat{Q}}{2e} = -i\frac{d}{d\varphi}$

external flux: Φ_e

gate voltage: n_g

Tunnel junction



tunneling Hamiltonian $H_T \sim \tilde{t}\left(c_L^\dagger c_R + c_R^\dagger c_L\right)$

N: $I = GV \qquad G = 2\frac{e^2}{h}\sum_p T_p \qquad T_p \sim \tilde{t}^2 \ll 1$

S: Bogoliubov transformation $H_T = H_T^{\mathrm{p}} + H_T^{\mathrm{qp}}$

- pair tunneling $H_T^{\mathrm{p}} \sim \tilde{t}\left(e^{i\varphi/2}u_L v_R + e^{-i\varphi/2}v_L u_R\right)\gamma_L^\dagger\gamma_R^\dagger + \mathrm{H.c.}$
- qp tunneling $H_T^{\mathrm{qp}} \sim \tilde{t}\left(e^{i\varphi/2}u_L u_R - e^{-i\varphi/2}v_L v_R\right)\gamma_L^\dagger\gamma_R + \mathrm{H.c.}$ dissipation $\propto x_{qp}$

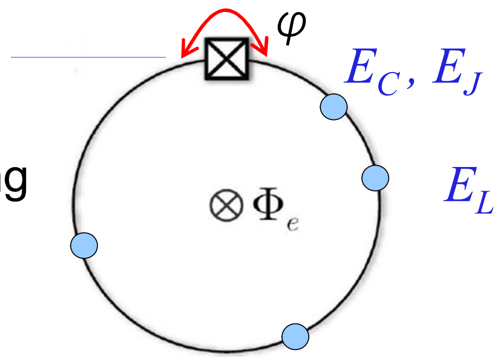
1st Josephson relation: $I = I_c \sin\varphi$ supercurrent (no dissipation)

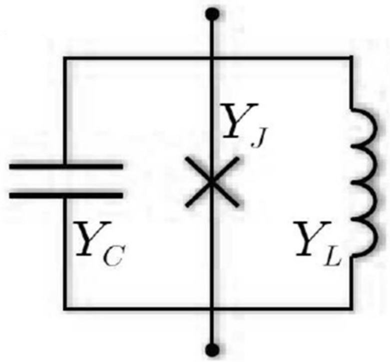
$$I = \dot{Q} = -\frac{\partial\mathcal{H}}{\partial\Phi} \qquad \varphi = 2\pi\frac{\Phi}{\Phi_0} \qquad \mathcal{H} = -E_J\cos\varphi \qquad I_c = \frac{2e}{\hbar}E_J \qquad E_J = \frac{\Delta}{8}\frac{G}{e^2/h}$$

Single-junction qubit (with QPs)

Josephson junction

superconducting loop + flux





lossy junction

$$\boxed{\hat{H} = \hat{H}_\varphi + \hat{H}_{\rm qp} + \hat{H}_T}$$

qubit Hamiltonian $$\hat{H}_\varphi = 4E_C\left(\hat{N} - n_g\right)^2 - E_J \cos\hat{\varphi} + \frac{1}{2}E_L\left(\hat{\varphi} - 2\pi\Phi_e/\Phi_0\right)^2$$

qp Hamiltonian $$\hat{H}_{\rm qp} = \sum_k E_k \hat{\gamma}_k^\dagger \hat{\gamma}_k \qquad E_k = \sqrt{\xi_k^2 + \Delta^2}$$

non-degenerate gas of excitations above gap

qp tunneling $$\hat{H}_T \sim \tilde{t}\sum\left(i\,\sin\frac{\hat{\varphi}}{2}\right)\hat{\gamma}_L^\dagger\hat{\gamma}_R + \text{H.c.}$$

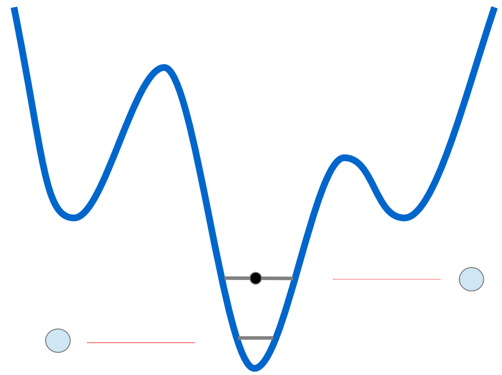
low-energy quasiparticles $T \ll \omega$

Transition rates and relaxation

$$\hat{H} = \hat{H}_\varphi + \hat{H}_{\rm qp} + \underline{\hat{H}_T}$$

qp tunneling

$$\hat{H}_T \sim \tilde{t} \sum \left(i \, \sin \frac{\hat{\varphi}}{2} \right) \hat{\gamma}_L^\dagger \hat{\gamma}_R + \mathrm{H.c.}$$



Fermi golden rule: $$\Gamma_{i\to f} = 2\pi \sum_{\{\lambda\}_{\rm qp}} \langle\langle |\langle f, \{\lambda\}_{\rm qp}| H_T |i, \{\eta\}_{\rm qp}\rangle|^2 \delta\left(E_{\lambda,\rm qp} - E_{\eta,\rm qp} - \omega_{if}\right)\rangle\rangle_{\rm qp}$$

$$\omega_{if} = E_i - E_f$$

qubit (phase) states

qp states

quantum statistical averaging

qp and phase dynamics factorize:

$$\Gamma_{i\to f} = \left|\langle f| \sin\frac{\hat{\varphi}}{2} |i\rangle\right|^2 S_{\rm qp}\left(\omega_{if}\right)$$

non-linear qubit-qp interaction

qp current spectral density

cold qp $T_{\rm eff} \ll \omega_{if}$

$$S_{\rm qp}(\omega) \propto \frac{1}{\sqrt{\omega}} x_{\rm qp} \propto \mathrm{Re}\, Y_{\rm qp}^{hf}$$

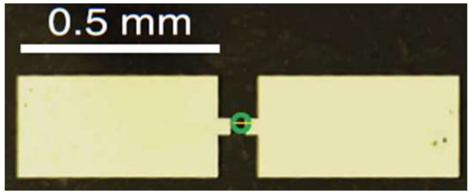
normalized qp density

admittance (magnitude)

PRL **106**, 077002 (2011), *PRB* **84**, 064517 (2011)

dephasing: master equation, *PRB* **86**, 184514 (2012)

QPs in transmon



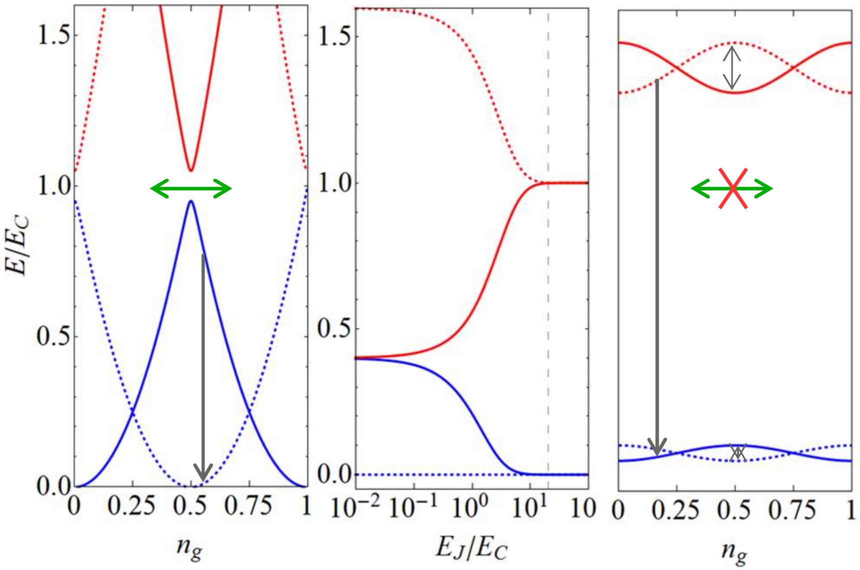
$$\hat{H}_\varphi = 4E_C\left(\hat{N} - \cancel{n_g}\right)^2 - E_J\cos\hat{\varphi} + \cancel{\frac{1}{2}E_L(\hat{\varphi} - 2\pi\Phi_e/\Phi_0)^2}$$

$$E_C \ll E_J$$

$$x_{eq} = \sqrt{\frac{2\pi T}{\Delta}}\, e^{-\Delta/T}$$

CPB $(E_C \gg E_J)$

transmon



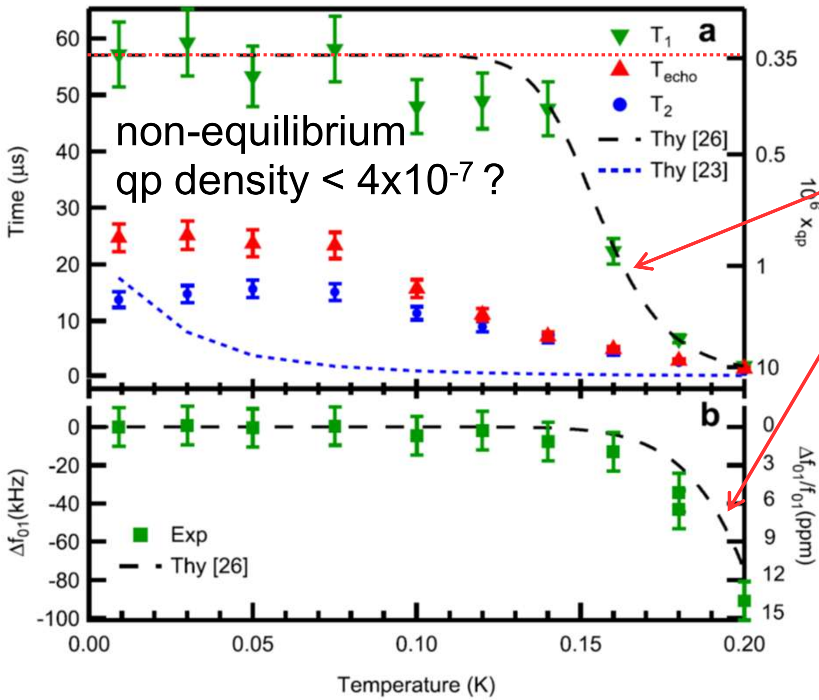
- charge noise
- quasiparticle poisoning

Lutchyn, Glazman, Larkin, *PRB* **72**, 014517 (2005)

Koch et al., *PRA* **76**, 042319 (2007)

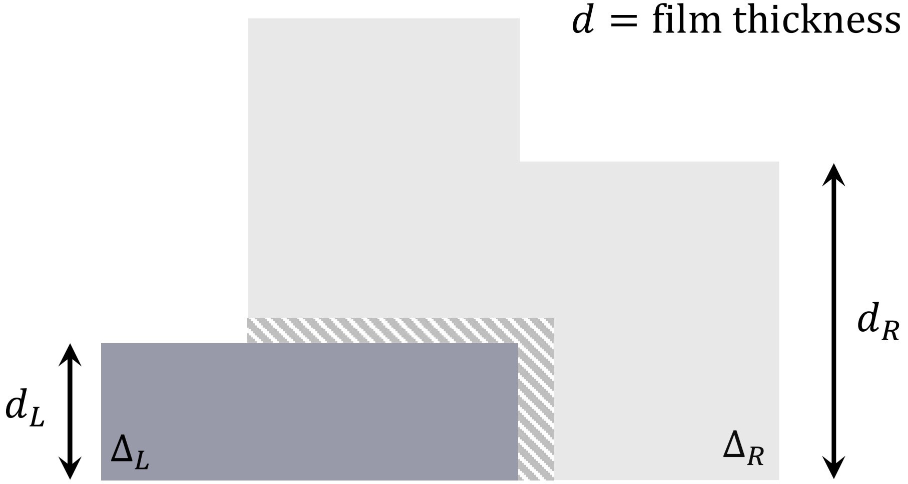
non-equilibrium qp density < 4×10^{-7} ?

thermal qp density

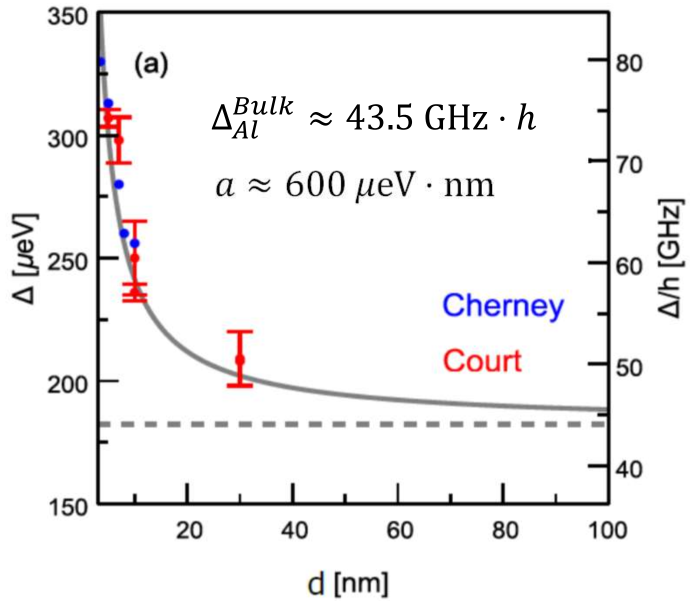


H. Paik et al., *PRL* **107**, 240501 (2011)

Al film thickness and superconducting gap



$$\Delta_{Al}(d) \approx \Delta_{Al}^{Bulk} + ad^{-1}$$



$$E_J \approx \frac{\Delta_L \Delta_R}{4(\Delta_L + \Delta_R)} \frac{G}{e^2/h}$$

New energy scale
$\hbar\omega_{LR} = \Delta_L - \Delta_R$

Cherney and J. Shewchun, *Can. J. Phys*. **47**, 1101 (1969)

Court et al., *Supercond. Sci. and Technol*. **21**, 015013 (2007)

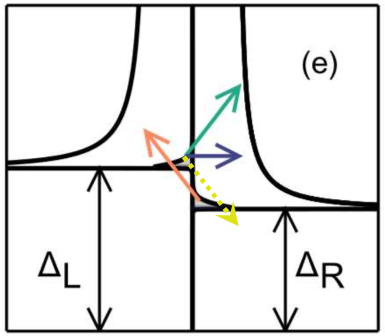
G. Marchegiani, L. Amico, G. Catelani, *PRX Quantum* **3**, 040338 (2022)

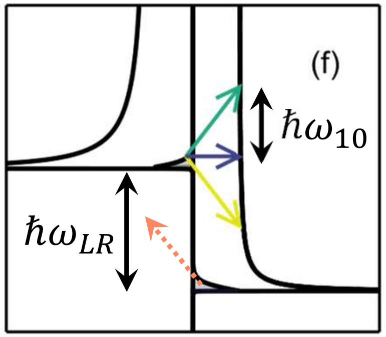
Quasiparticle transitions with different gaps

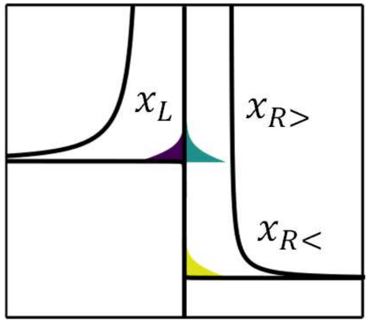
Two cases:

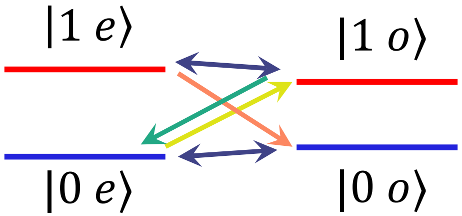
I: $\omega_{10} > \omega_{LR}$

II: $\omega_{LR} > \omega_{10}$





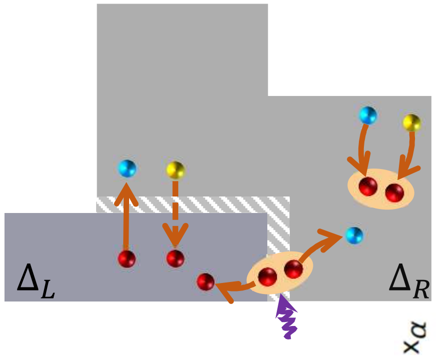




New energy scale
$\hbar\omega_{LR} = \Delta_L - \Delta_R$

$$\left.\begin{array}{l} x_L \\ x_{R>} \\ x_{R<} \end{array}\right\}\ \text{Nonequilibrium qp densities}$$

Quasiparticle-qubit nonequilibrium steady state

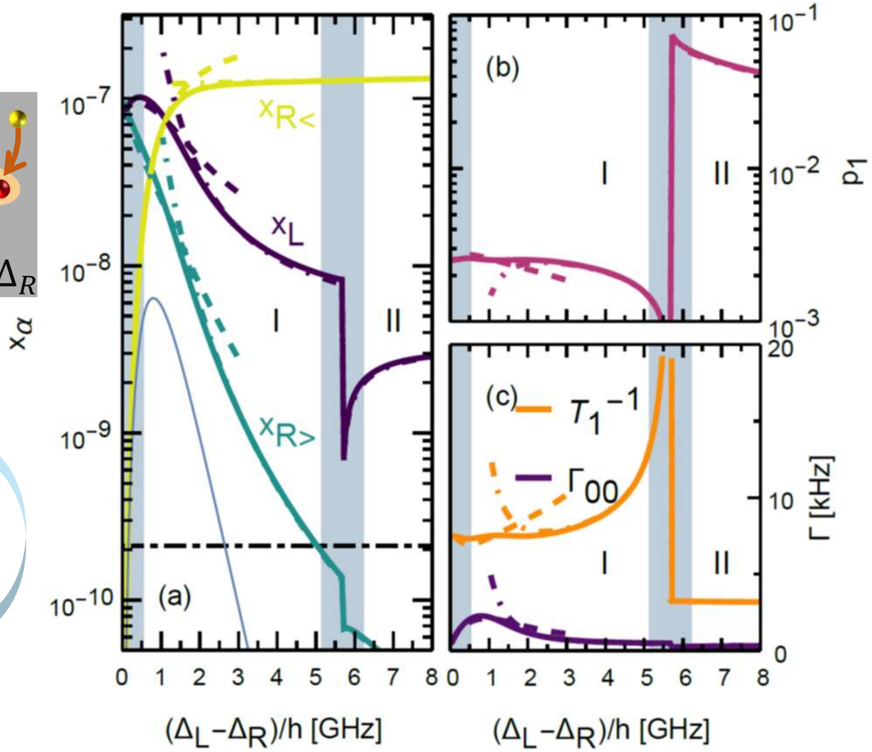


$$\dot{x}_\alpha = g_\alpha(p_i, x_\alpha)$$

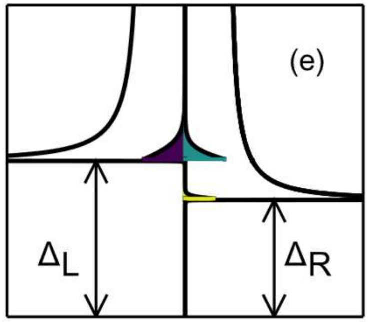
$$\dot{p}_i = f_i(p_i, x_\alpha)$$

coupled rate eqns for qp density and qubit population

$$\dot{p}_i = \dot{x}_L = \dot{x}_{R>} = \dot{x}_{R<} = 0$$



I : $\omega_{LR} \ll \omega_{10}$

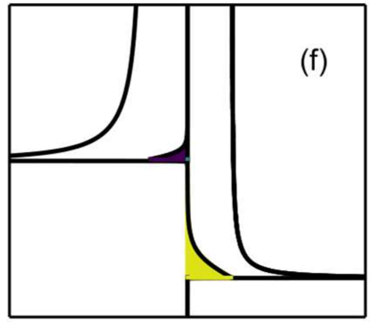


Quasiparticles located at energies $\sim \Delta_L$

✓ low values of p_1

✗ lower values of T_1

II : $\omega_{LR} > \omega_{10}$



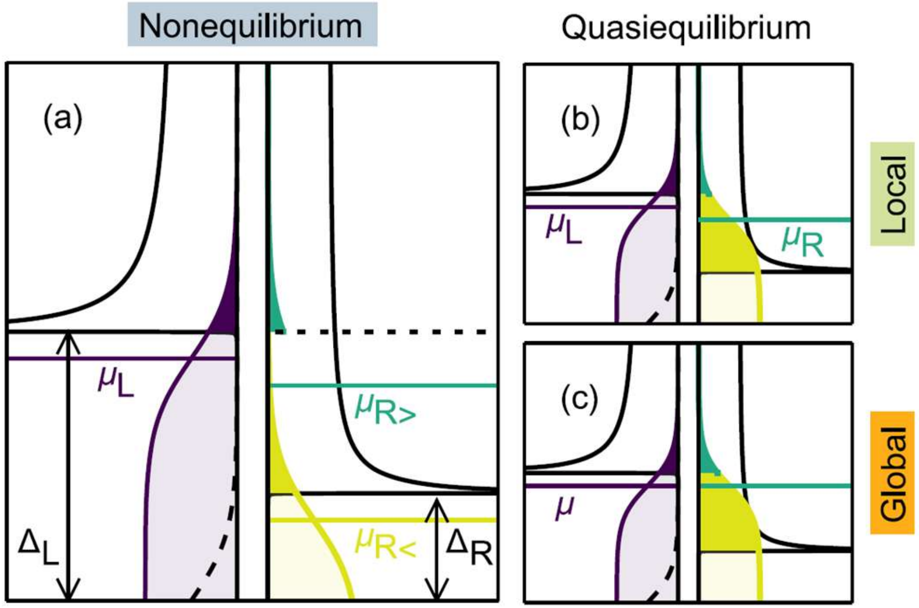
Quasiparticles trapped at energies $\sim \Delta_R$

✗ higher values of p_1

✓ improved T_1

Google: *PRL* **133**, 240601(2024)
& *Nature* **638**, 920 (2025)

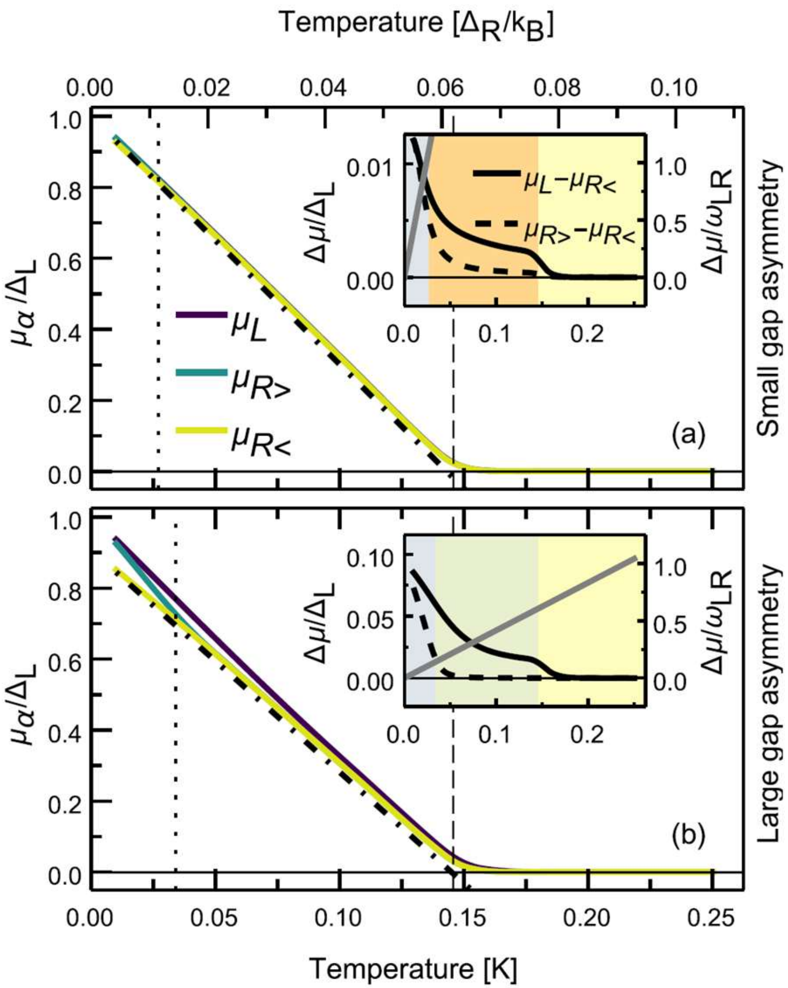
Nonequilibrium regimes vs T



Ansatz for distribution functions:

$$f_{\mathrm{L}}(\epsilon) = f_0(\epsilon - \mu_{\mathrm{L}})$$

$$f_{\mathrm{R}}(\epsilon) = f_0(\epsilon - \mu_{\mathrm{R}<})\theta(\Delta_{\mathrm{L}} - \epsilon) + f_0(\epsilon - \mu_{\mathrm{R}>})\theta(\epsilon - \Delta_{\mathrm{L}})$$



0.5 GHz

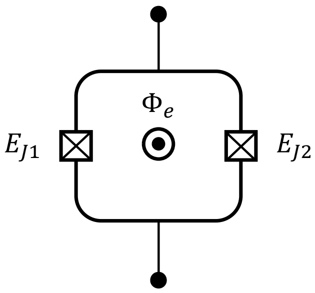
5 GHz

tunneling vs e-ph scattering

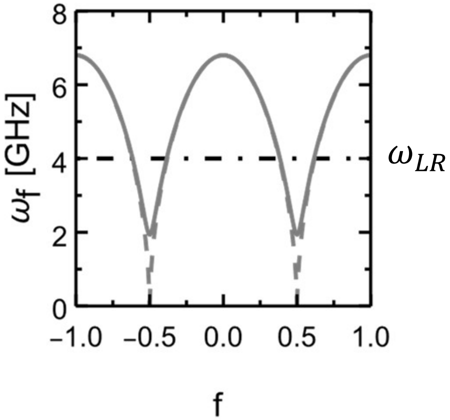
generation by thermal ph

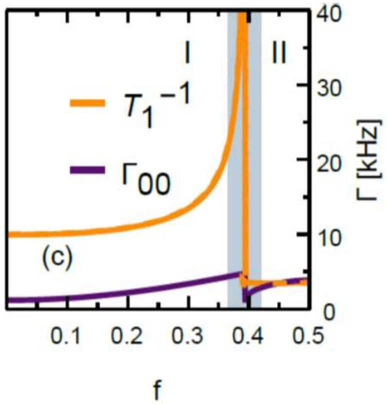
Marchegiani and GC, *Commun Phys* **8**, 120 (2025)

Probing gap-asymmetry: split-transmon

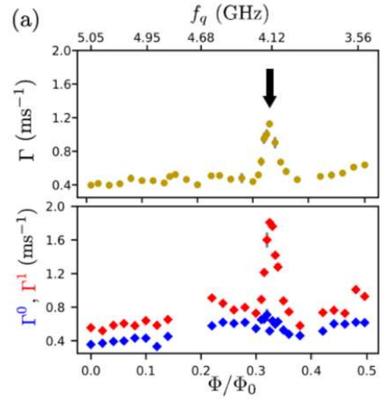


$$\omega_f \approx \omega_{f=0}\sqrt{G(f,d_0)}$$





$$\Gamma \approx 0.5(\Gamma^0 + \Gamma^1)$$
$$\Gamma^0 = \Gamma^{0\to0} + \Gamma^{0\to1}$$
$$\Gamma^1 = \Gamma^{1\to0} + \Gamma^{1\to1}$$



S. Diamond et al., *PRX Quantum* **3**, 040304 (2022)

Φ_e : external flux

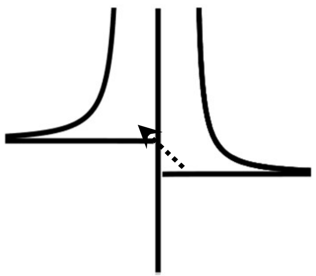
$$\Phi_0 = \frac{h}{2e}\text{: flux quantum}$$

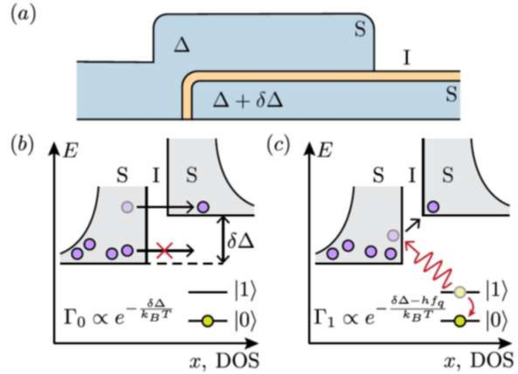
$$G(f,d_0) = \frac{E_J(f,d_0)}{E_{J\Sigma}} = \sqrt{\cos^2(\pi f) + d_0^2\sin^2(\pi f)}$$

$$E_{J\Sigma} = E_{J1} + E_{J2} \qquad d_0 = \frac{E_{J1} - E_{J2}}{E_{J1} + E_{J2}}$$

Diverging rate $\Gamma_{10}^{eo} \to +\infty$

$$\omega_{LR} = \omega_{10}(f)$$

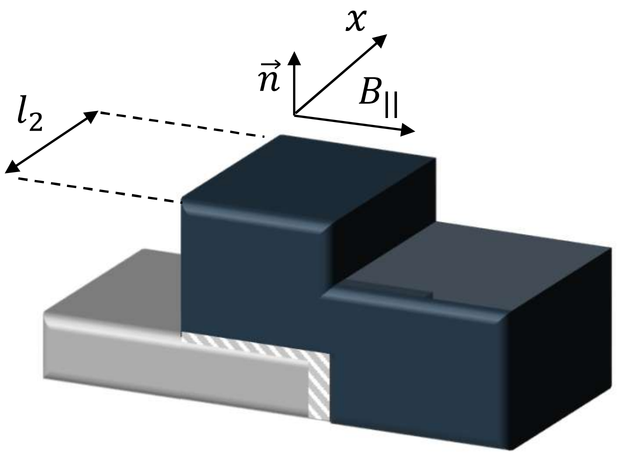


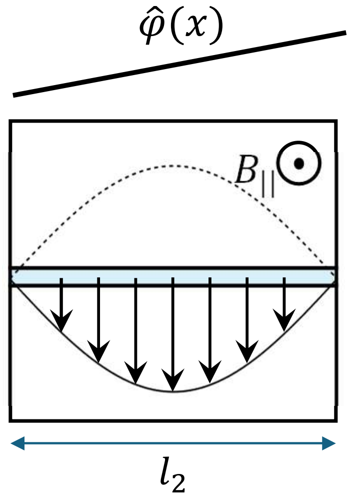


G. Marchegiani, L. Amico, G. Catelani, *PRX Quantum* **3**, 040338 (2022)

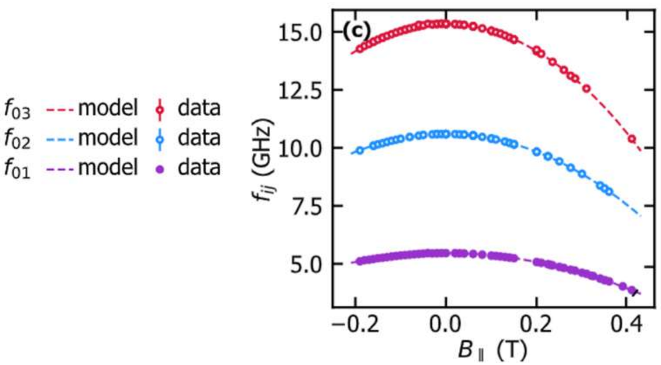
T. Connolly et al., *PRL* **132**, 217001 (2024)
[see also Marchegiani and GC,
Commun Phys **8**, 120 (2025)]

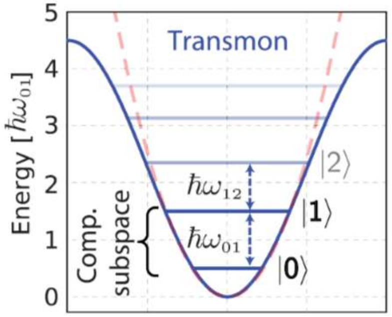
Probing gap-asymmetry: parallel magnetic field





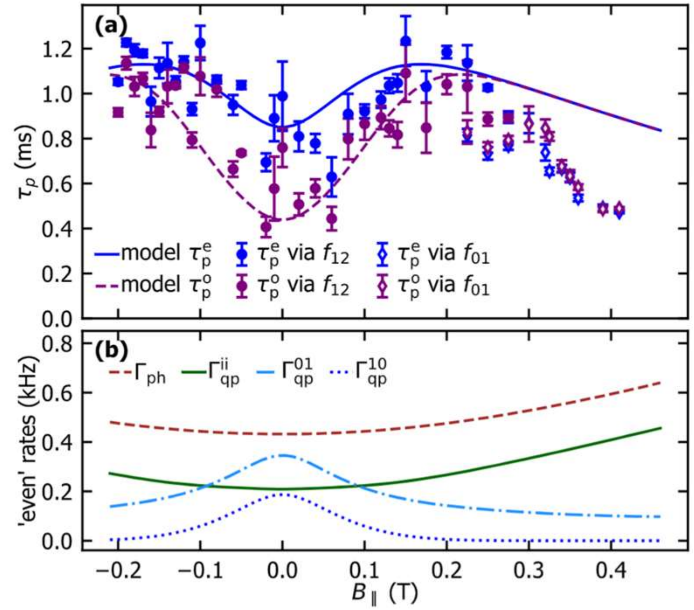
$$E_J \propto I_J(B_{||}) = I_J(0)\left|\mathrm{Sinc}\left(\pi\frac{B_{||}}{B_{\Phi 0}}\right)\right| \rightarrow \text{ reduces } f_{01}$$

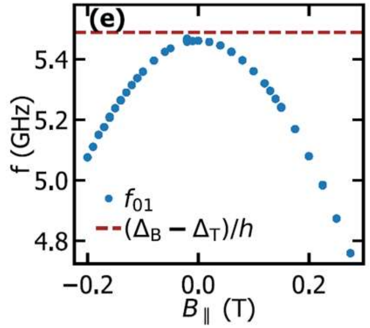




parity switching time

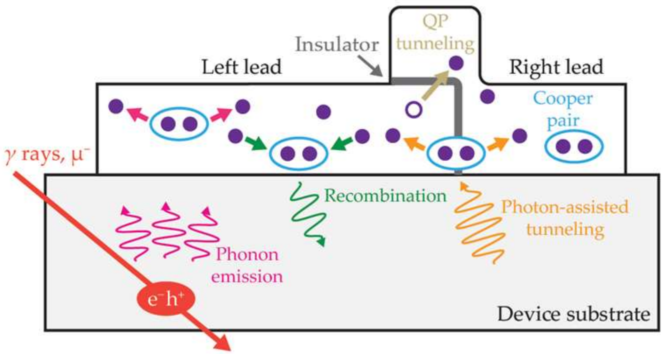
rates



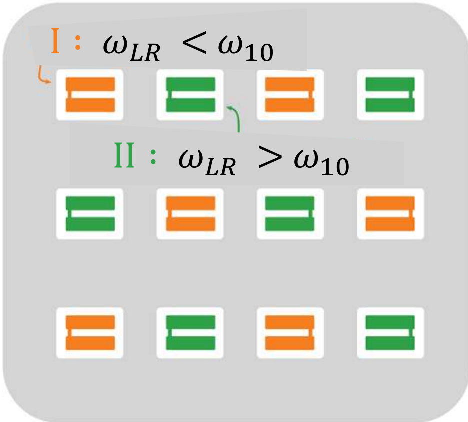


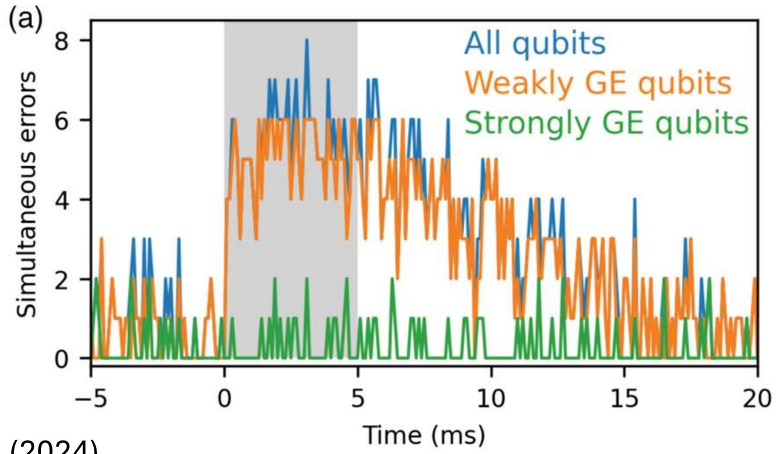
J. Krause, G. Marchegiani, L. M. Janssen, G. Catelani, Y. Ando, C. Dickel, *PRApp* **22**, 044063 (2024)

QP bursts and gap-asymmetry



J. Aumentado, GC, K. Serniak
Physics today **76**, 34 (2023)





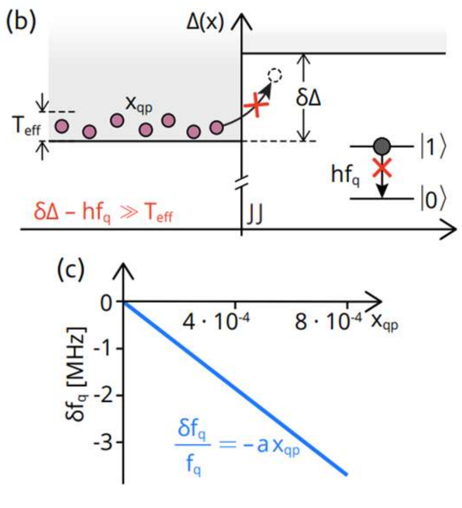
M. McEwen, et al., *PRL* **133**, 240601 (2024)

Quantum error correction below the surface code threshold

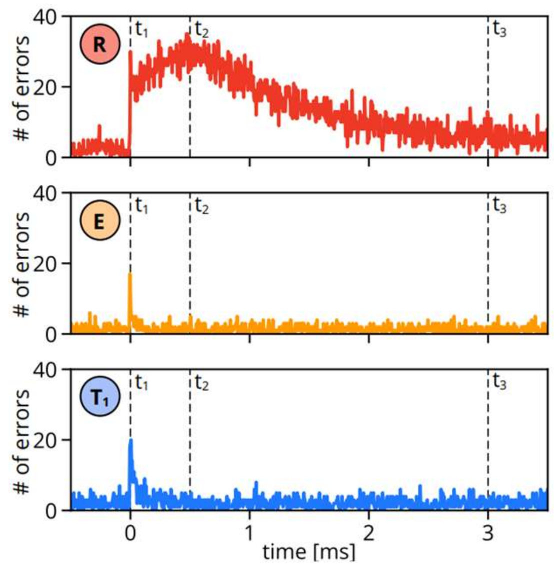
Google Quantum AI and Collaborators*

Notably, the error per cycle on the 72-qubit processor is suppressed far below 10^{-6}, breaking past the error floor observed previously. We attribute the mitigation of high-energy impact failures to gap-engineered Josephson junctions[39]. However, at code distances of $d \geq 15$, we observe a deviation from exponential error suppression at high distances culminating in an apparent logical error floor of 10^{-10}.

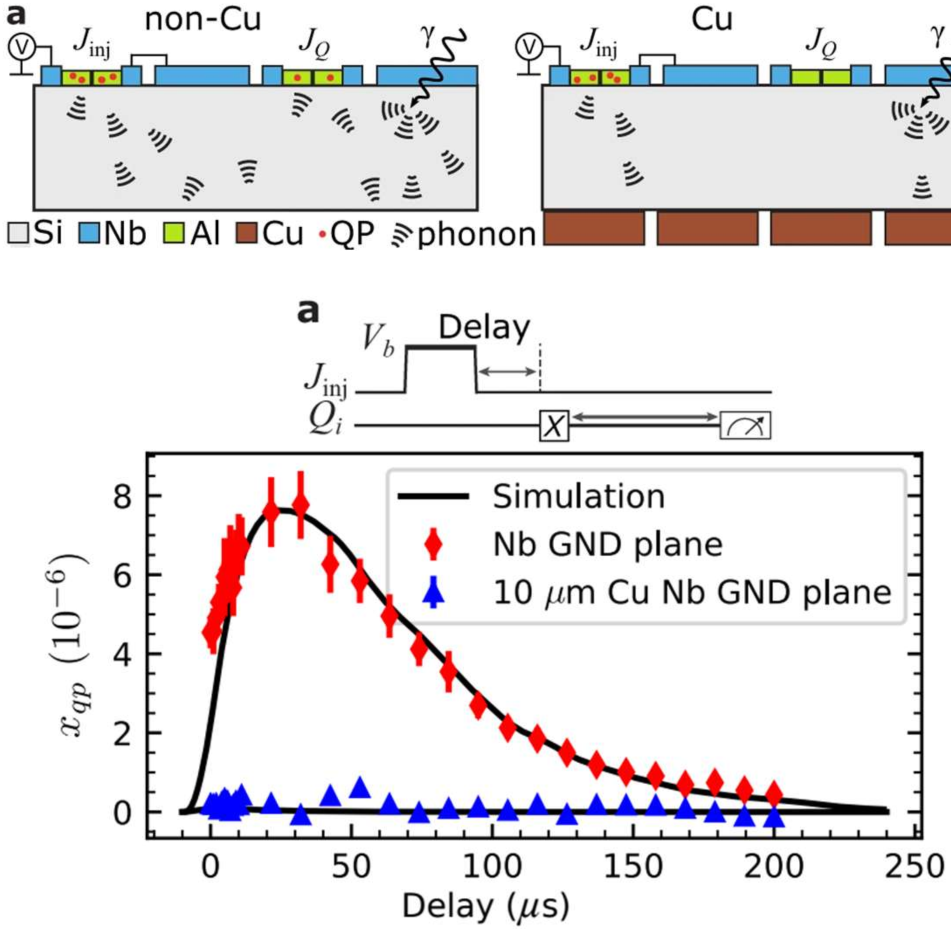
Nature **638**, 920 (2025)

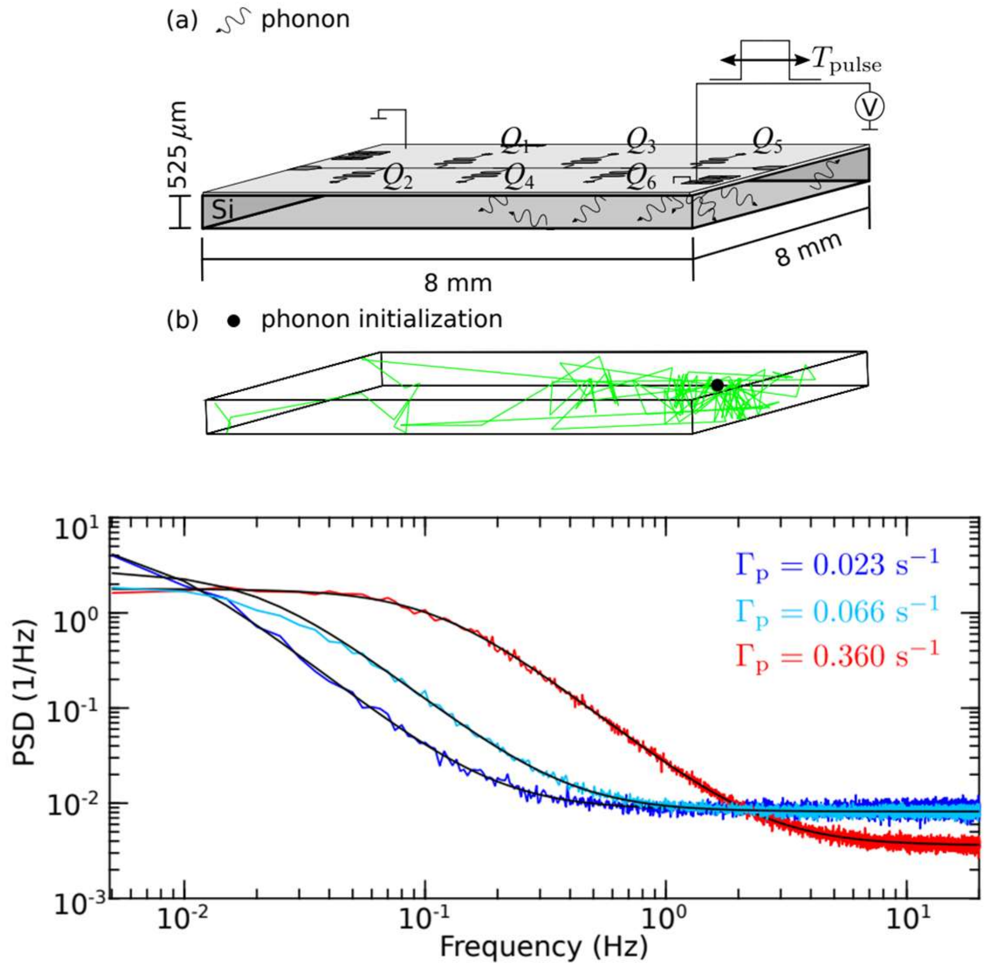


arXiv:2506.18228



Mitigating quasiparticle decoherence

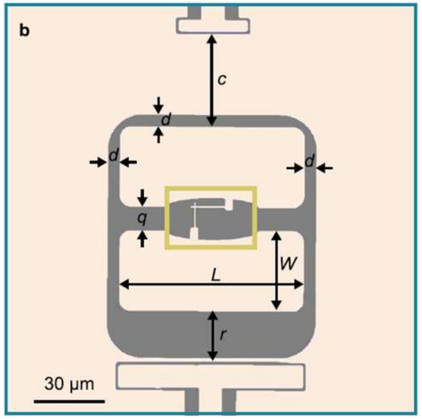


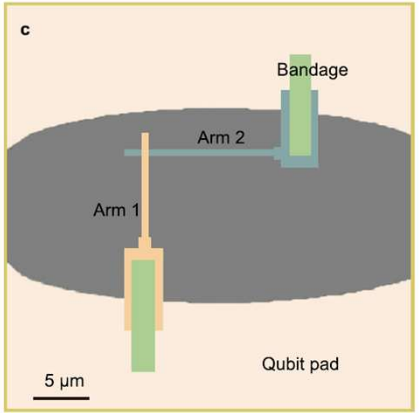


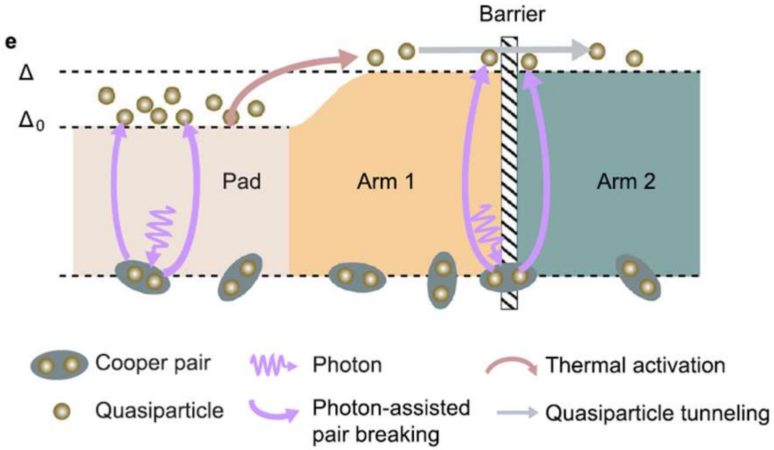
V. Iaia et al., *Nat Commun* **13**, 6425 (2022)

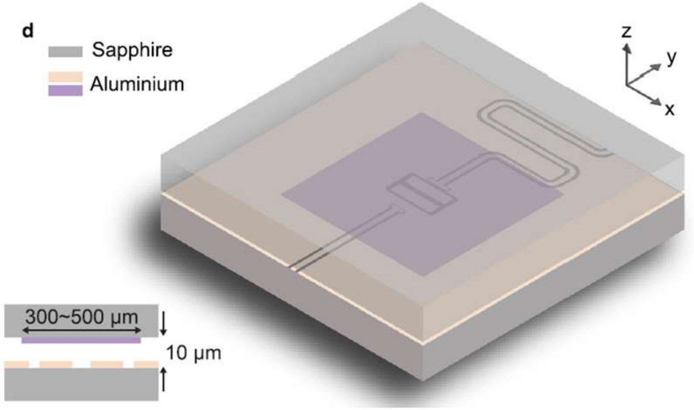
E. Yelton et al., *PRB* **110**, 024519 (2024)

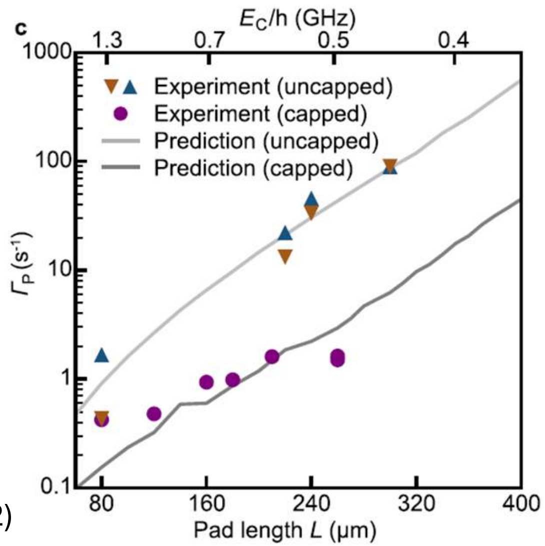
Mitigating quasiparticle decoherence

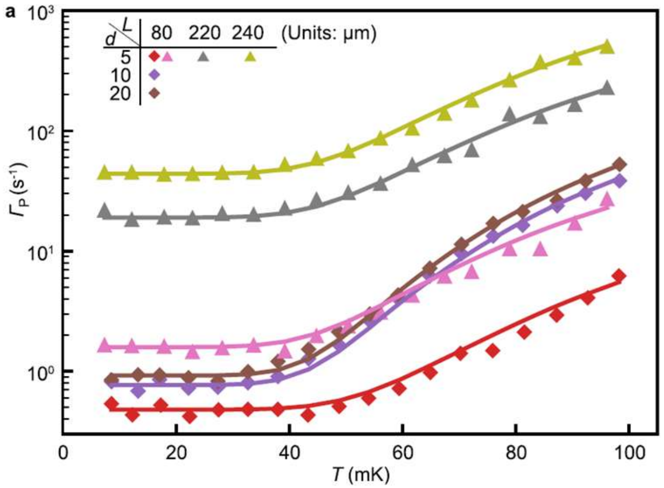






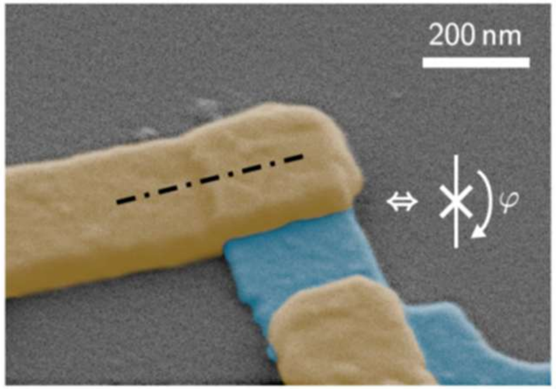






X. Pan et al., *Nat Commun* **13**, 7196 (2022)

Tunnel junction



tunneling Hamiltonian $H_T \sim \tilde{t}\left(c_L^\dagger c_R + c_R^\dagger c_L\right)$

N: $I = GV$ $\quad G = 2\frac{e^2}{h}\sum_p T_p$ $\quad T_p \sim \tilde{t}^2 \ll 1$

S: Bogoliubov transformation $H_T = H_T^{\mathrm{p}} + H_T^{\mathrm{qp}}$

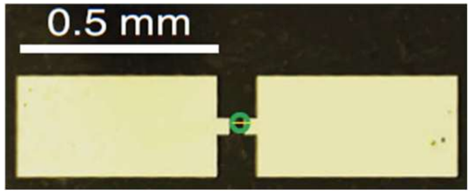
- pair tunneling $\quad H_T^{\mathrm{p}} \sim \tilde{t}\left(e^{i\varphi/2}u_L v_R + e^{-i\varphi/2}v_L u_R\right)\gamma_L^\dagger\gamma_R^\dagger + \mathrm{H.c.}$
- qp tunneling $\quad H_T^{\mathrm{qp}} \sim \tilde{t}\left(e^{i\varphi/2}u_L u_R - e^{-i\varphi/2}v_L v_R\right)\gamma_L^\dagger\gamma_R + \mathrm{H.c.}$ $\quad$ dissipation $\propto x_{qp}$

1st Josephson relation: $\quad I = I_c \sin\varphi$ $\quad$ supercurrent (no dissipation)

$$I = \dot{Q} = -\frac{\partial\mathcal{H}}{\partial\Phi} \qquad \varphi = 2\pi\frac{\Phi}{\Phi_0} \qquad \mathcal{H} = -E_J\cos\varphi \qquad I_c = \frac{2e}{\hbar}E_J \qquad E_J = \frac{\Delta}{8}\frac{G}{e^2/h}$$

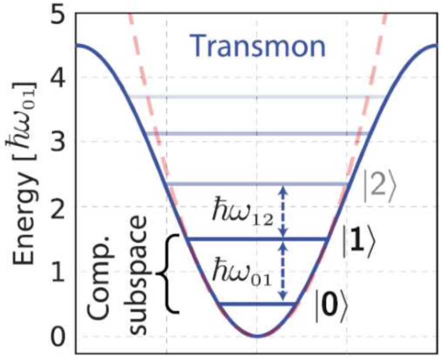
exact in the tunnel limit: $T_p \to 0$ and number of channel $\to \infty$ at constant G

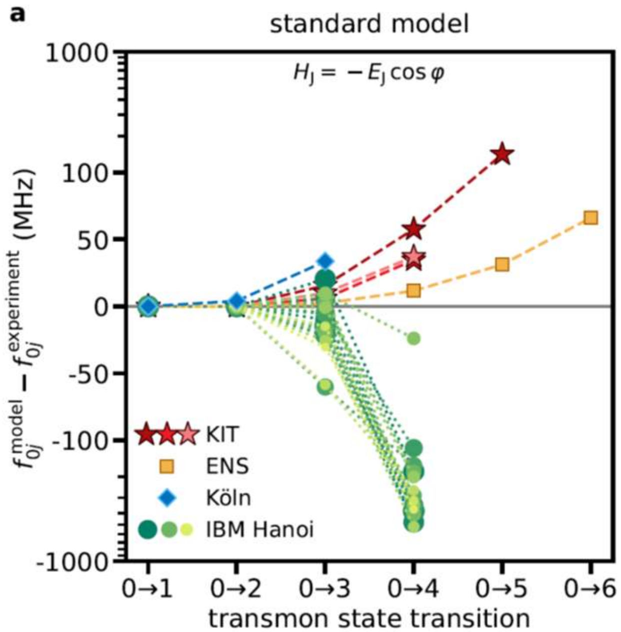
Transmon spectroscopy

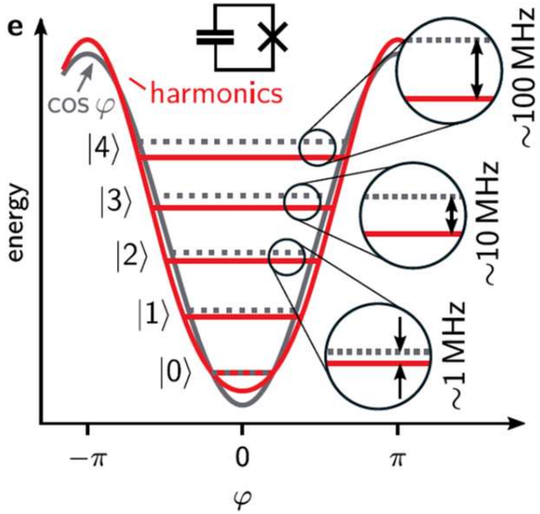


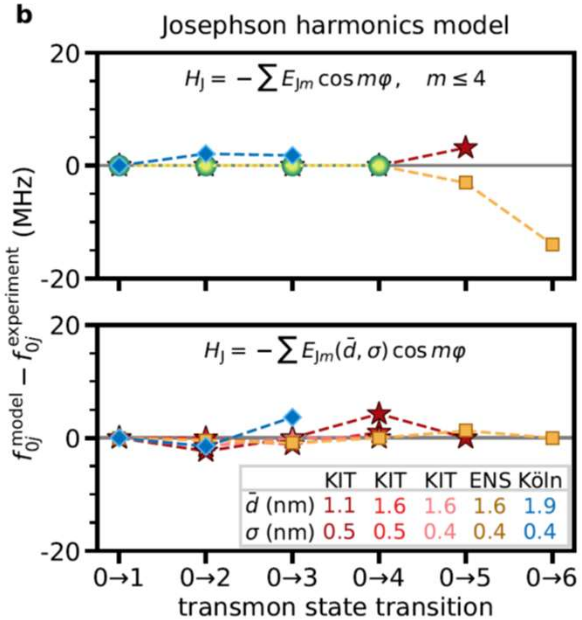
$$\hat{H}_\varphi = 4E_C\left(\hat{N} - \cancel{n_g}\right)^2 - E_J\cos\hat{\varphi} + \cancel{\frac{1}{2}E_L(\hat{\varphi} - 2\pi\Phi_e/\Phi_0)^2}$$

$$E_C \ll E_J$$

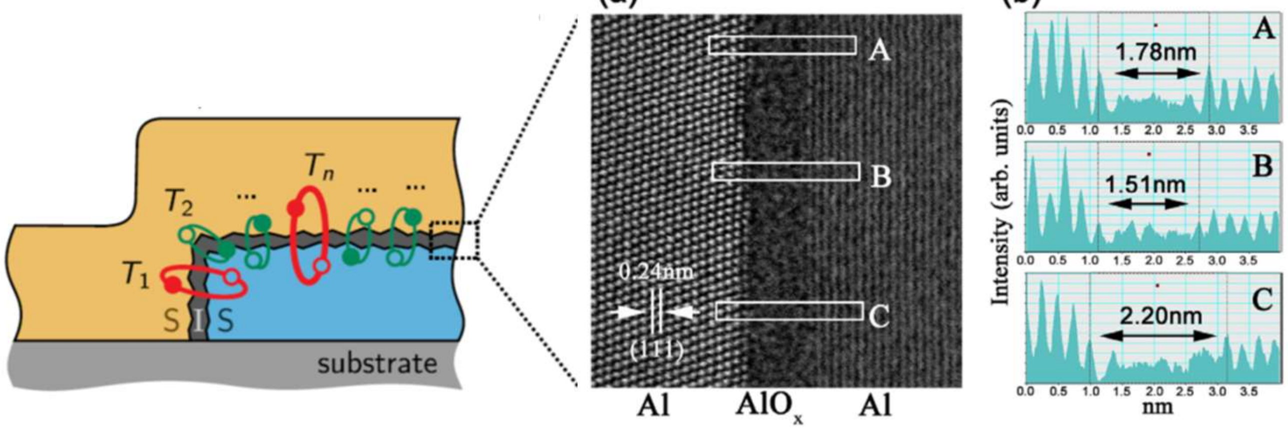


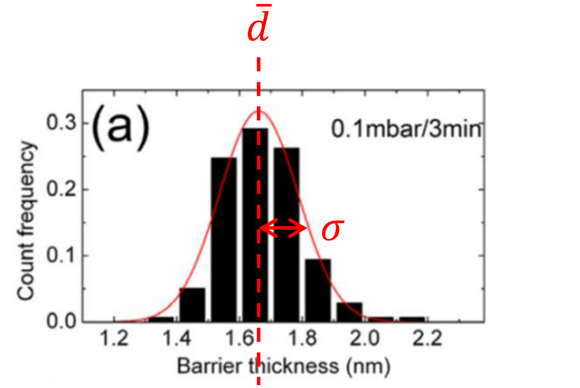




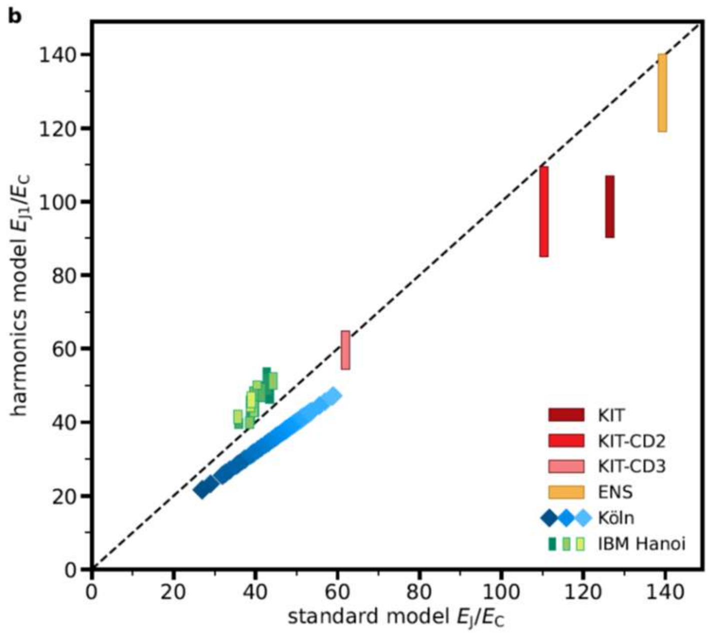


The “*d*-*σ*” model





Zeng *et al.*, *J. Phys. D* **48,** 395308 (2015)



Nat Phys **20**, 815 (2024)

current-phase relation

$$I_S(\varphi) = \frac{e\Delta}{2\hbar}\sum_{n=1}^{N}\frac{T_n \sin\varphi}{\sqrt{1 - T_n \sin^2(\varphi/2)}} = \frac{e\Delta}{2\hbar}\sum_{m=1}^{\infty}\sum_{n=1}^{N} c_m(T_n)\sin(m\varphi)$$

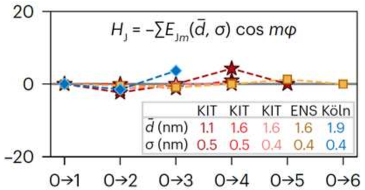
Josephson Hamiltonian

$$H_J = -\Delta\sum_{n=1}^{N}\sqrt{1 - T_n \sin^2\frac{\varphi}{2}} = -\sum_{m=1}^{\infty} E_{Jm}\cos(m\varphi)$$

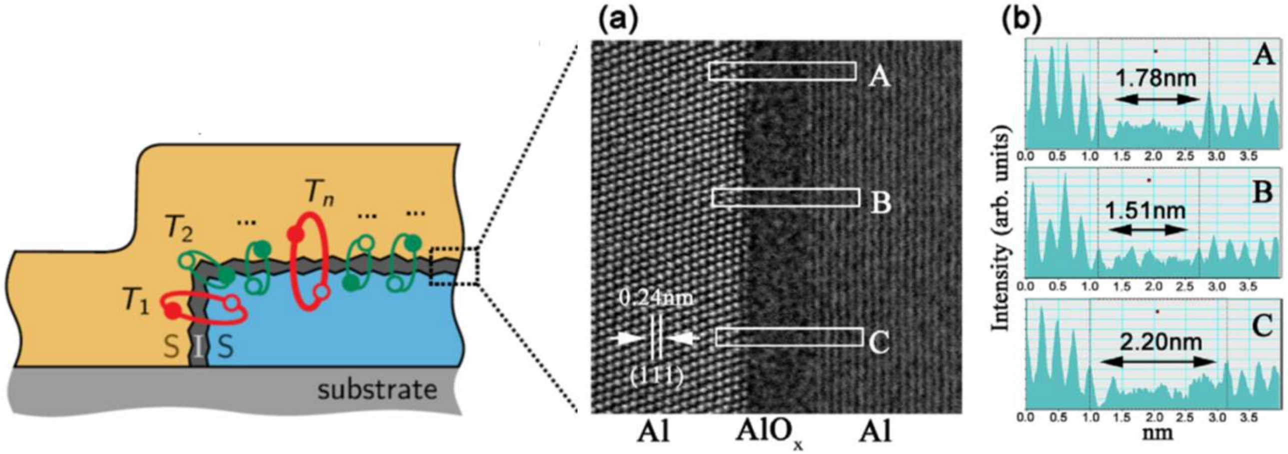
$$E_{Jm} = \frac{\Delta}{4}\frac{1}{m}\sum_{n=1}^{N} c_m(T_n) = \frac{\Delta}{4}\frac{N}{m}\int_0^1 dT\,\rho(T) c_m(T)$$

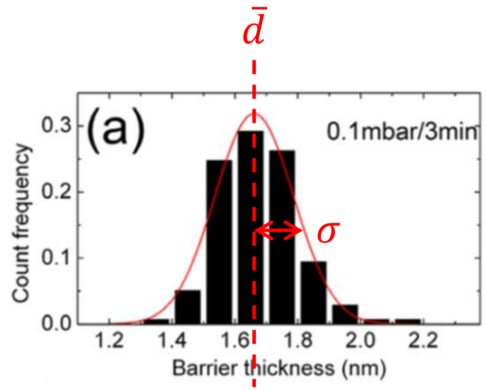
get $\rho(T)$ from

$$\rho(d) = \frac{2\Theta(d)}{1 + \mathrm{Erf}(\bar{d}/\sqrt{2}\sigma)}\frac{1}{\sqrt{2\pi}\sigma}e^{-(d-\bar{d})^2/(2\sigma^2)}$$



The "*d*-*σ*" model





Zeng *et al.*, *J. Phys. D* **48,** 395308 (2015)

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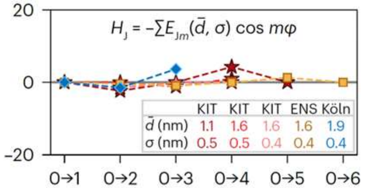
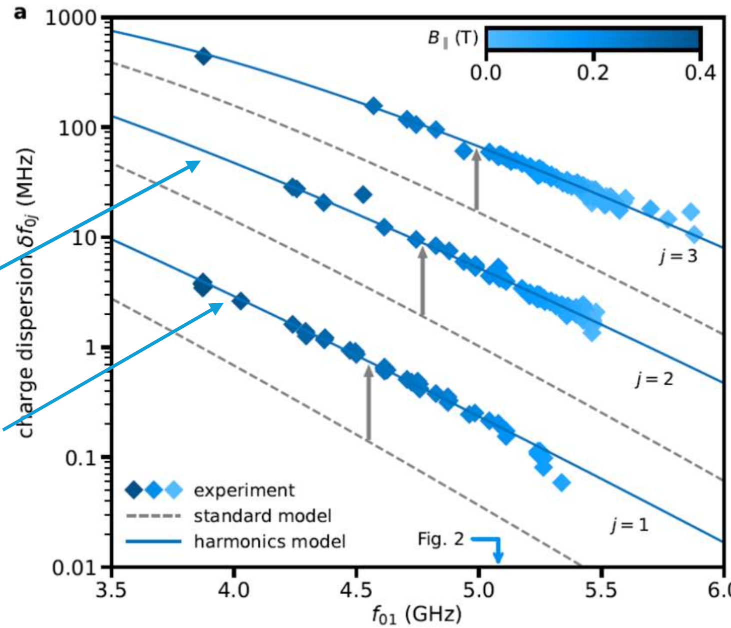


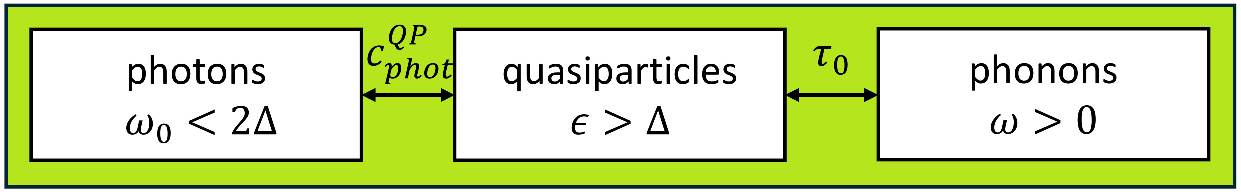
CZ gate fidelity

charge noise dephasing



Nat Phys **20**, 815 (2024)

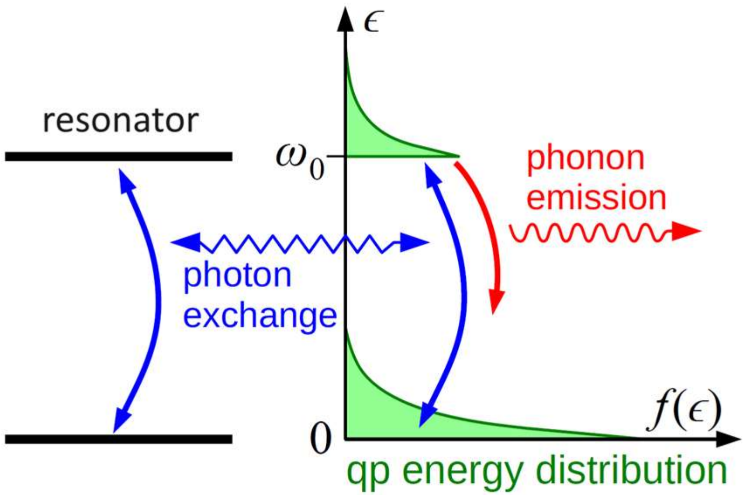
Quasiparticles in resonators



GC and D. Basko, *SciPost Phys* **6**, 013 (2019)

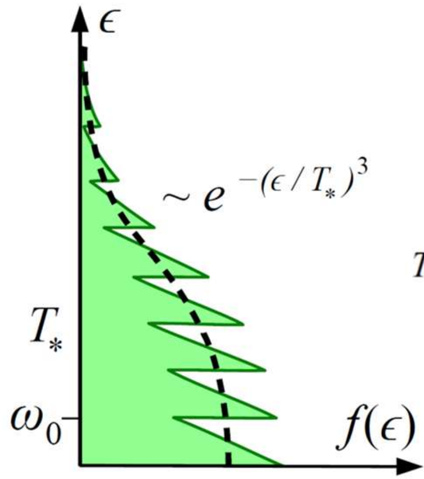
$$\frac{d}{dt} f(\epsilon) = St^{pho}\ \{f, n\} + St^{phot}\{f, \bar{n}\}$$

- emission/absorption



T=0

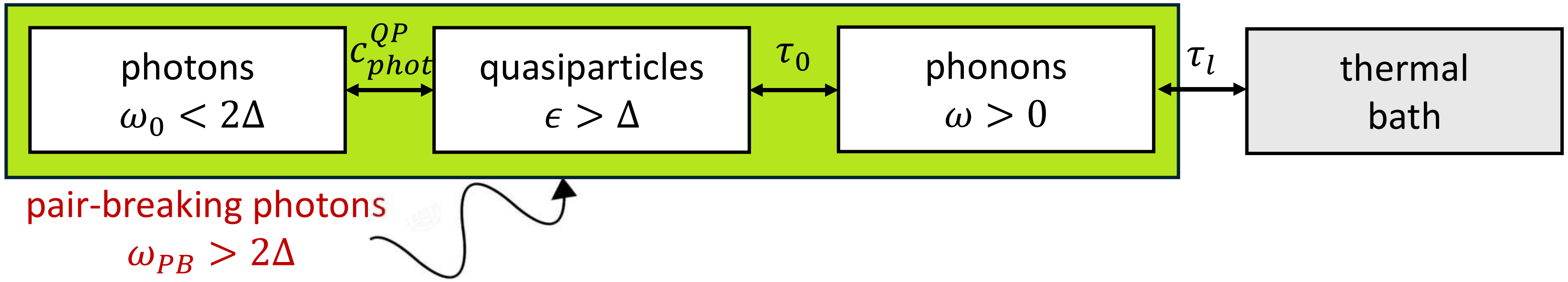
“cold” qp



$$T_* \equiv \left(\frac{105}{64} T_c^3 c_{\mathrm{phot}}^{\mathrm{QP}} \bar{n} \tau_0 \omega_0^2 \Delta\right)^{1/6}$$

“hot” qp

Quasiparticles in resonators



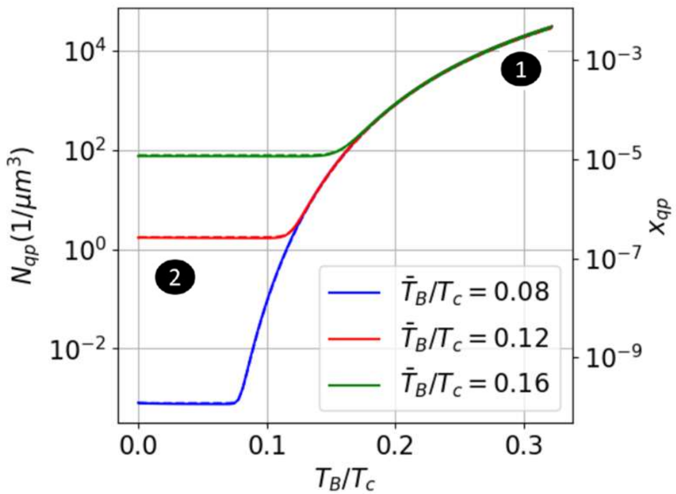
P. Fischer and GC, *Phys Rev Applied* **19**, 054087 (2023)

P. Fischer and GC, *SciPost Phys* **17**, 070 (2024)

$$\frac{d}{dt}f(\epsilon) = St^{phon}\{f,n\} + St^{phot}\{f,\bar{n}\} + St_{PB}^{phot}\{f,\bar{n}_{PB}\}$$

$$\frac{d}{dt}n(\omega) = St^{QP}\{f,n\} - \frac{1}{\tau_l}(n - n_T)$$

- emission/absorption
- generation/recombination
- thermalization



Generalized RT model:

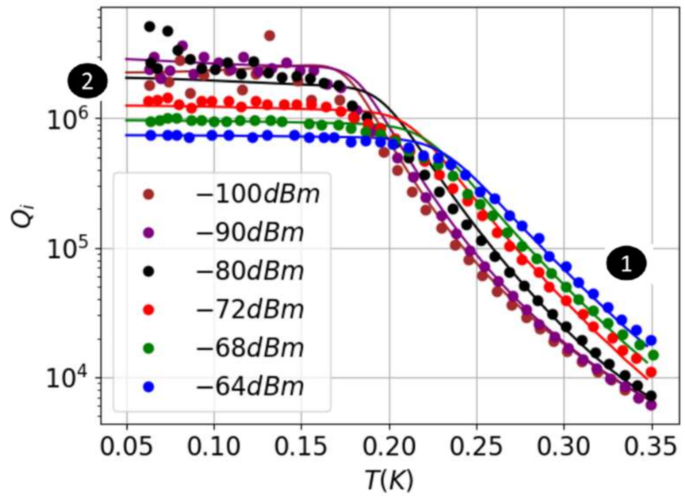
$$\frac{dN_{QP}}{dt} = G_T + \cancel{G(T_*/\Delta)N_{QP}} - RN_{QP}^2$$

hot qp

$$G_T = G_{T_B} + G_{\bar{T}_B}$$

pair-breaking photons 2

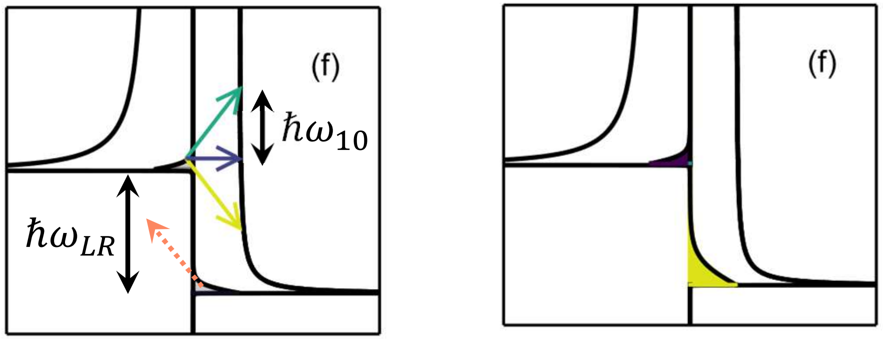
thermal phonons 1



exp: de Visser *et al.*, *PRL* **112**, 047004 (2014)

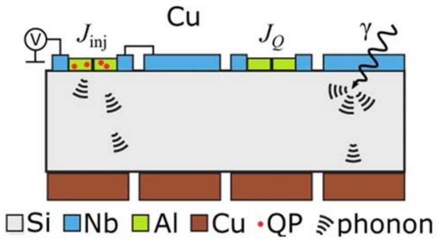
Summary

- standard fabrication leads to gap difference
- gap difference larger than qubit frequency decreases transition rates and protects against bursts

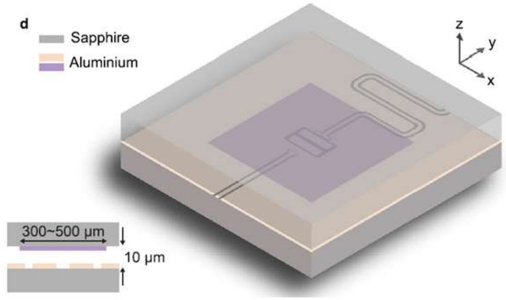


PRX Quantum **3**, 040338 (2022)

- generation mechanisms & mitigation

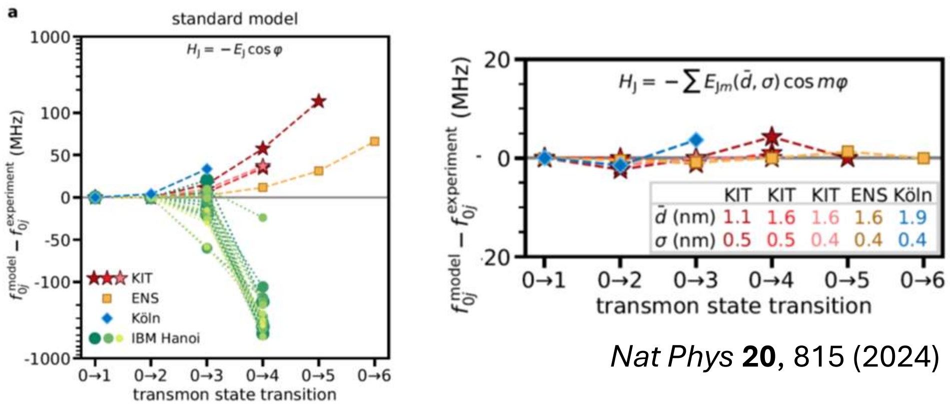


PRB **110**, 024519 (2024)



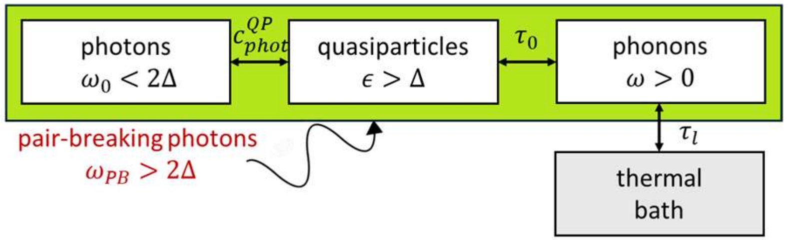
Nat Commun **13**, 7196 (2022)

- spectroscopic measurements in transmons are incompatible with standard tunnel model
- necessary to account for barrier inhomogeneity



Nat Phys **20**, 815 (2024)

- QP in resonators



PRApp **19**, 054087 (2023), *SciPost Phys* **17**, 070 (2024)