



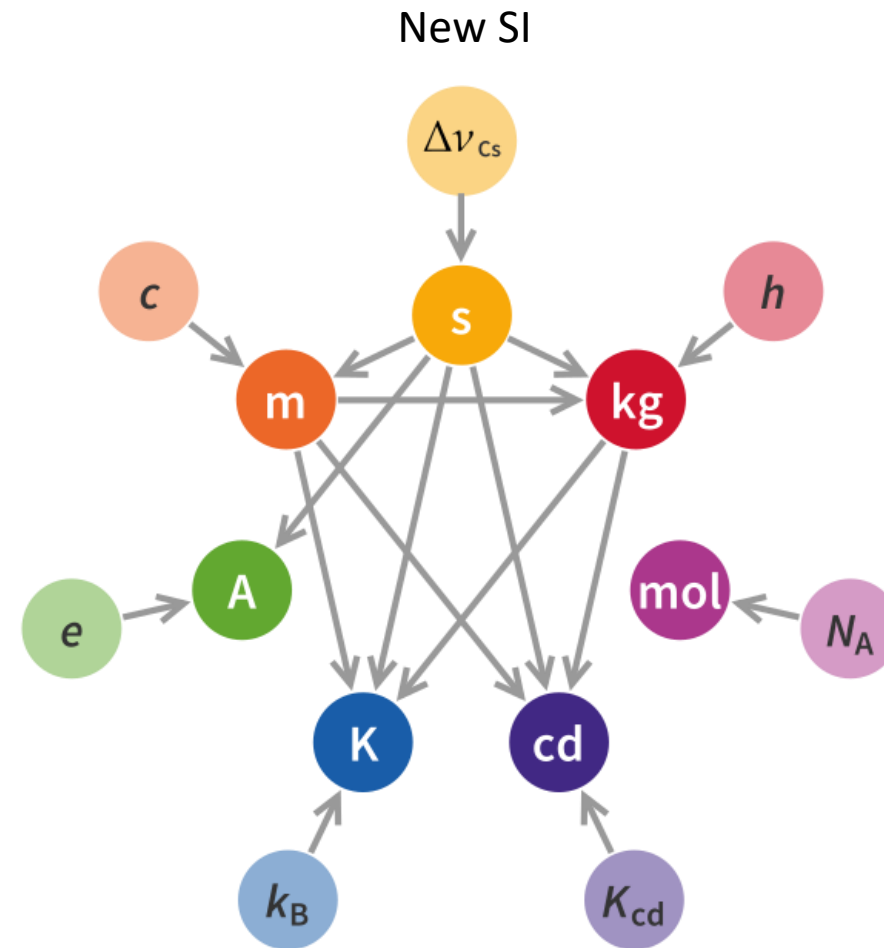
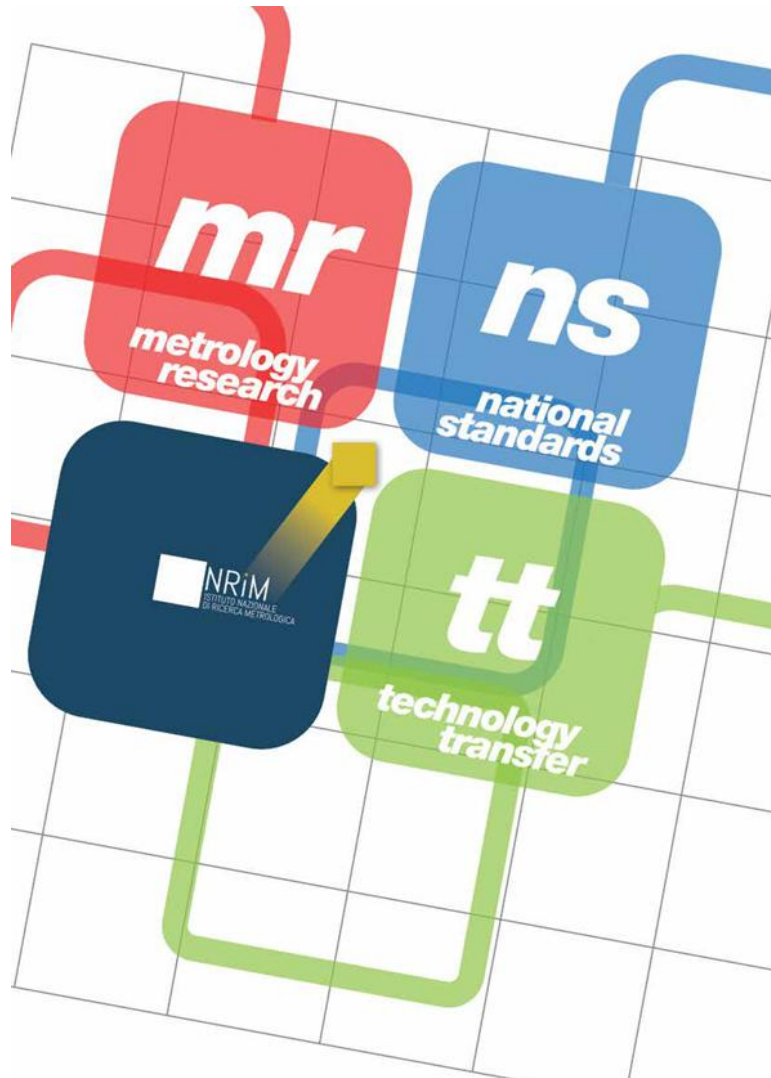
Programmable Cluster State with Josephson Metamaterials

Superconducting Devices for Quantum Optics
ECT* - Trento 8 October 2025

Emanuele ENRICO (e.enrico@inrim.it)

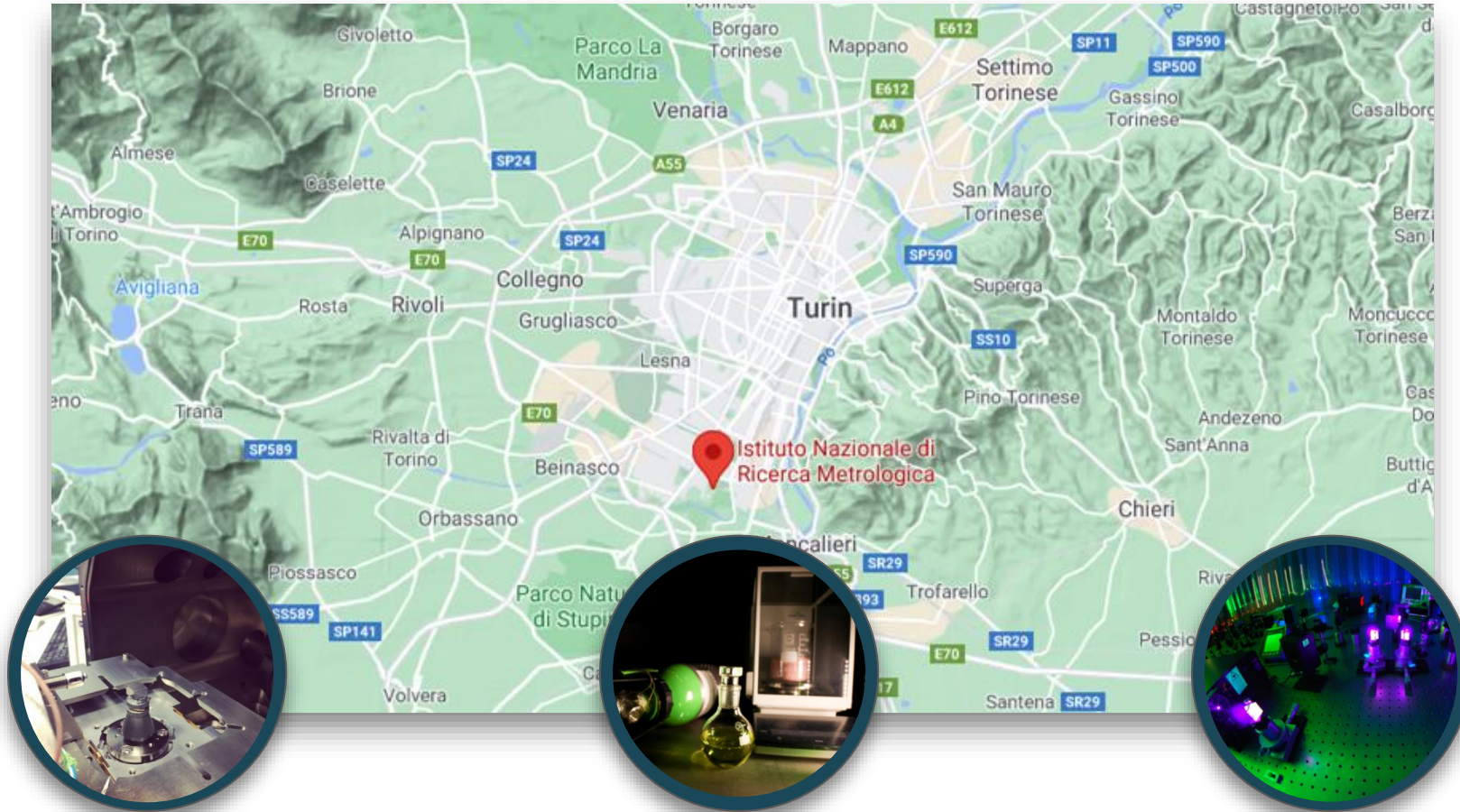
INRiM - Introduction

1



INRiM - Divisions

2

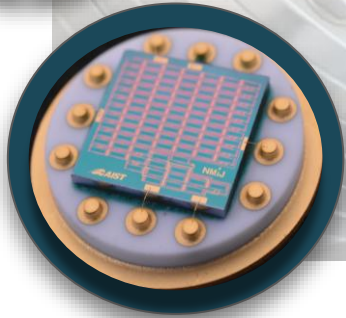


Advanced materials
metrology and life science

Applied metrology and
engineering

Quantum metrology and
nano technologies

Quantum Electronics - NanoTech

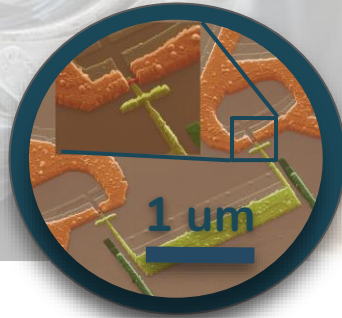


Quantum Hall Array
Resistance Standard

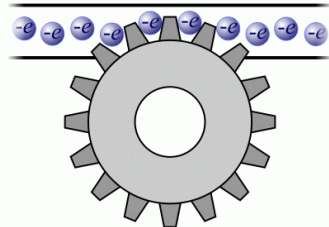
$$R_H = R_K / i$$

with $R_K = h/e^2$

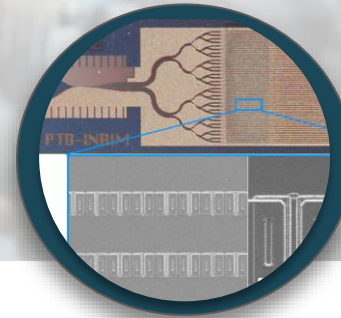
and $i = 1, 2, 3, 4, \dots$



Quantized-charge transport
DC current standard



$$I = ne f$$

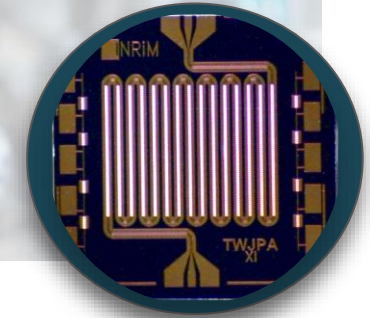


Josephson
Voltage Standard

$$V_J = n K_J f$$

with $K_J = h/2e$

and $k = 1, 2, 3, 4, \dots$



Microwave photonics

ν	1 GHz $\rightarrow$ 20 GHz
$\lambda = c/\nu$	300 cm $\rightarrow$ 15 cm
$E = h/\nu$	4 μ eV $\rightarrow$ 80 μ eV
$T = E/k_B$	50 mK $\rightarrow$ 1K

Superconducting Quantum Electronics

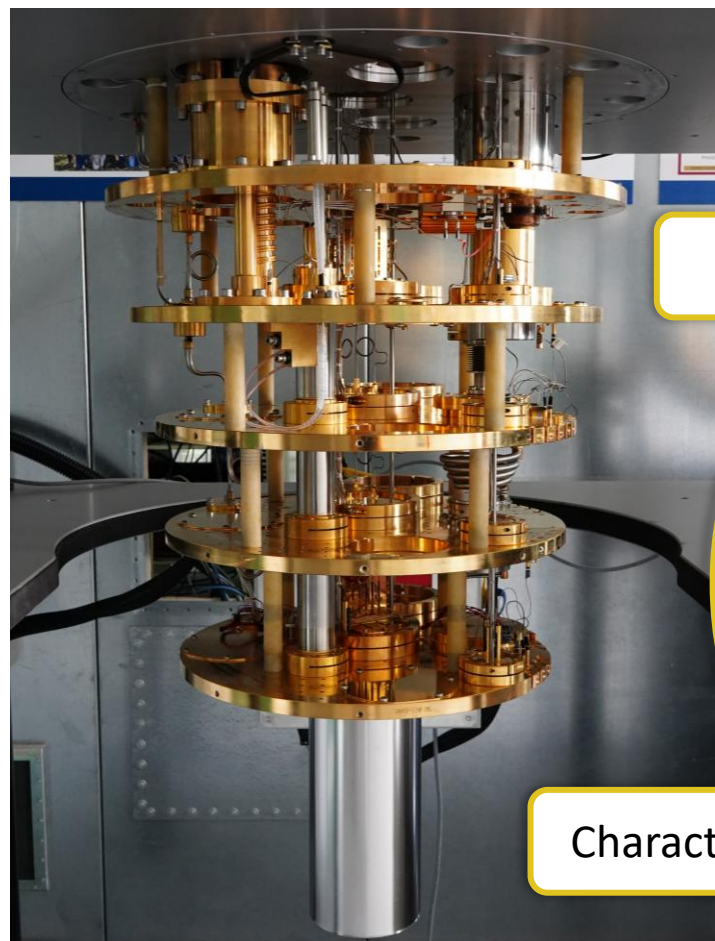
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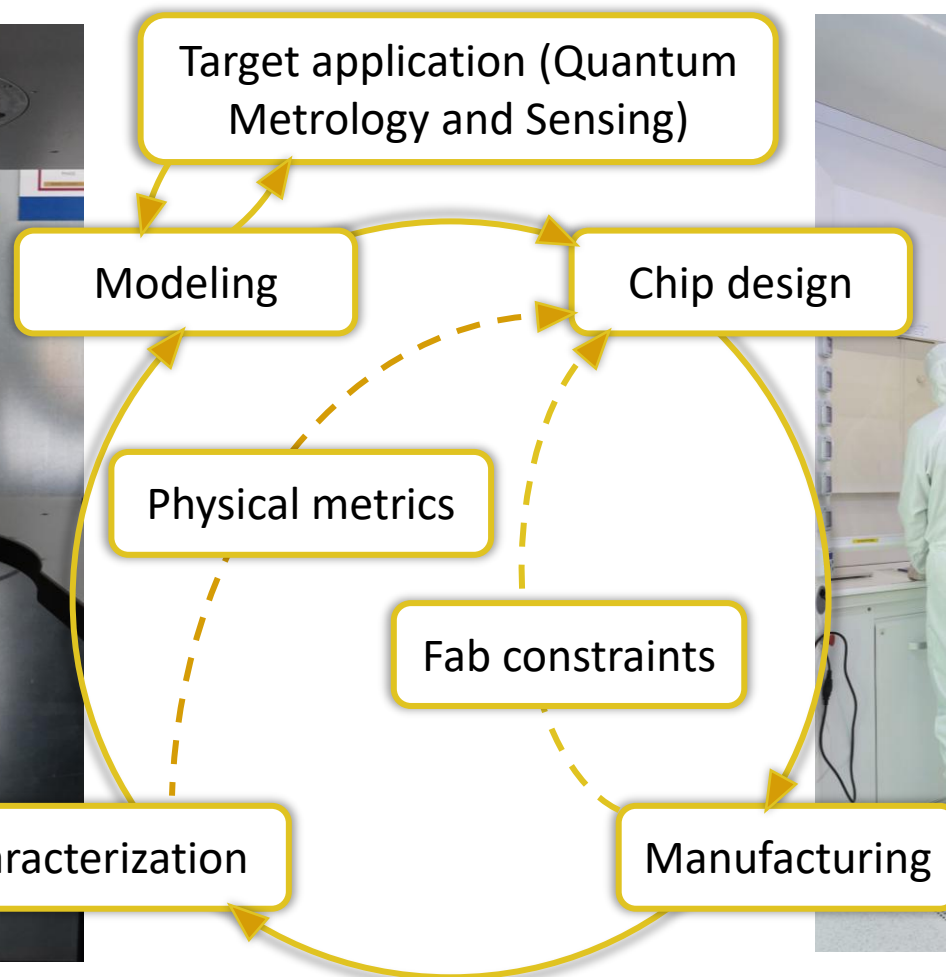
- Researchers
 - Emanuele ENRICO (PI)
 - Luca FASOLO
 - Luca OBERTO
 - Luca CALLEGARO
- PhD Students
 - **Alessandro ALOCCO (PoliTO)**
 - Andrea CELOTTO (PoliTO)
 - Emanuele PALUMBO (PoliTO)
 - Bernardo GALVANO (UniPA)
- Master/Bachelor students
 - Fabrizio BISOGNO (UniTO)
 - Tommaso SABBADINI (PoliTO)
- CleanRoom Technician
 - Start on July

Our workflow

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Quantum Circuit for Metrology laboratory
hosting a dilution refrigerator ($T < 20$ mK)



PiQuET Cleanroom facility

Quantum Circuits for Metrology Lab.



Dilution Refrigerator

- Leiden CF-CS110
- ($400 \mu\text{W}$ @ 100 mK, BT < 20 mK)
- Warm insertable probe for RF
- $\mu\text{Metal(RT)}$ and CryoPhy(BT) shields
- Warm insertable probe for QHE
- 9T magnet option

Shielded and thermally stable room

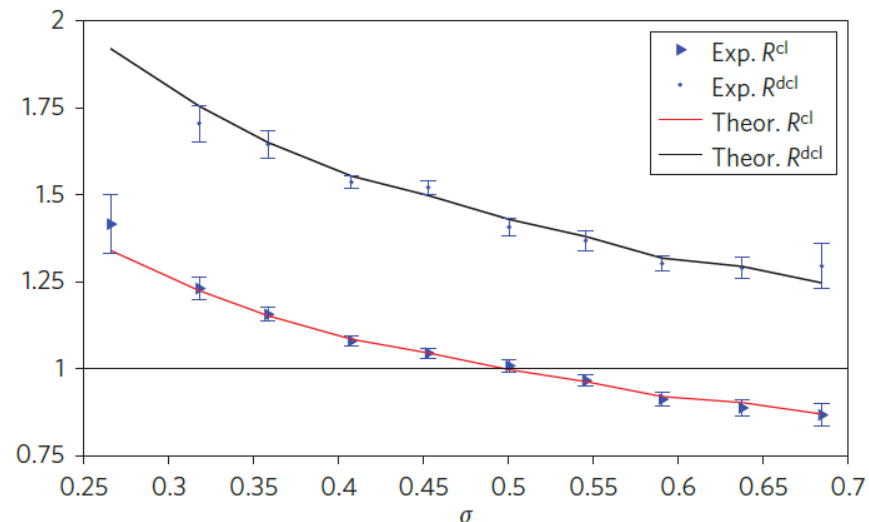
- Control electronics (DC+RF)
- CW (PNA-X) and pulsed protocols (QCS)

Superconducting Devices for **Quantum Optics**

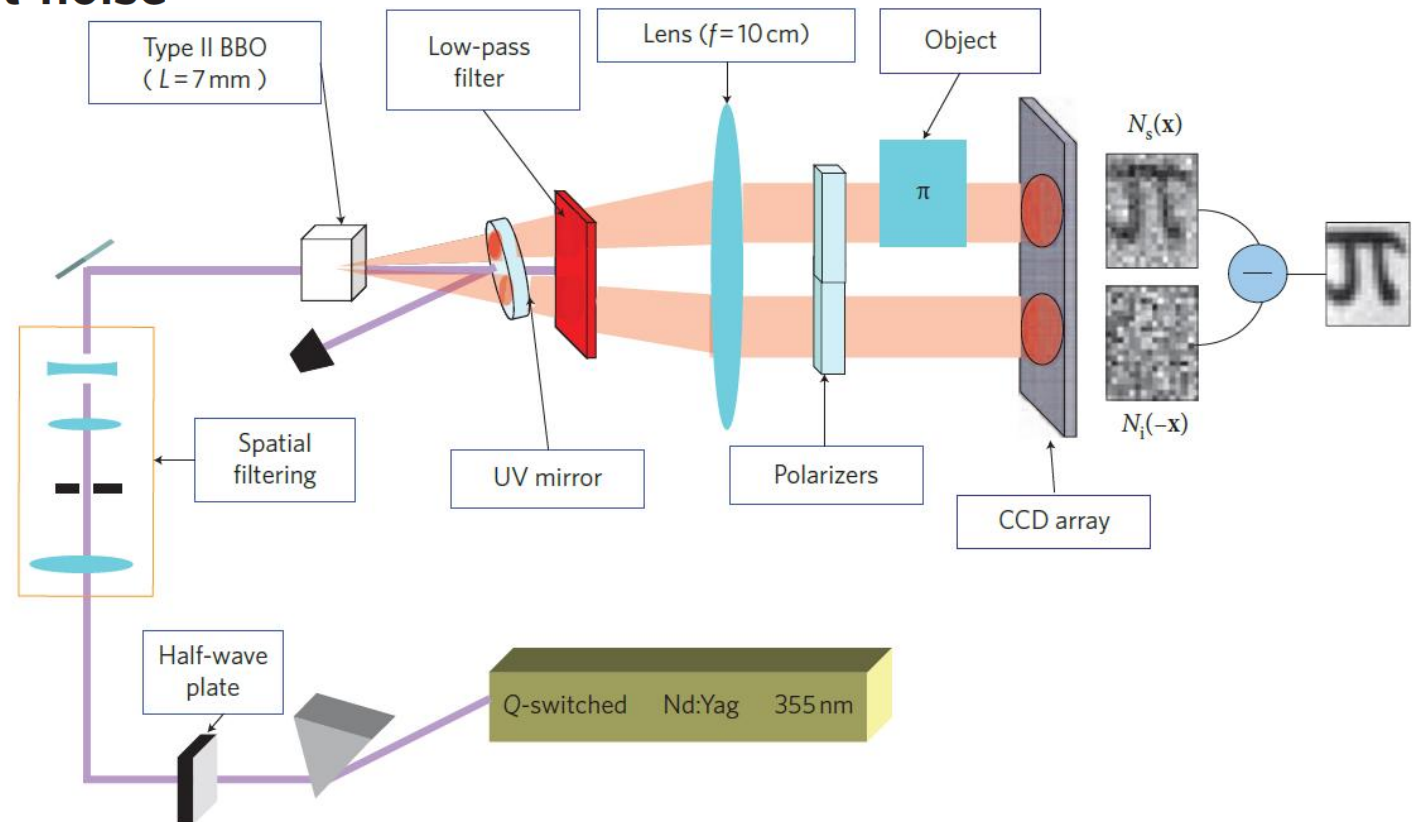
(with a **metrological** flavor)

Experimental realization of sub-shot-noise quantum imaging

G. Brida, M. Genovese and I. Ruo Berchera*



Ratio R between the **signal-to-noise ratio in quantum imaging**, and **differential (dcl) and direct (cl) classical imaging**. R is plotted as a function of the degree of correlation



SPDC and Metrology – Counting

8

APPLIED PHYSICS LETTERS **101**, 221112 (2012)

An extremely low-noise heralded single-photon source: A breakthrough for quantum technologies

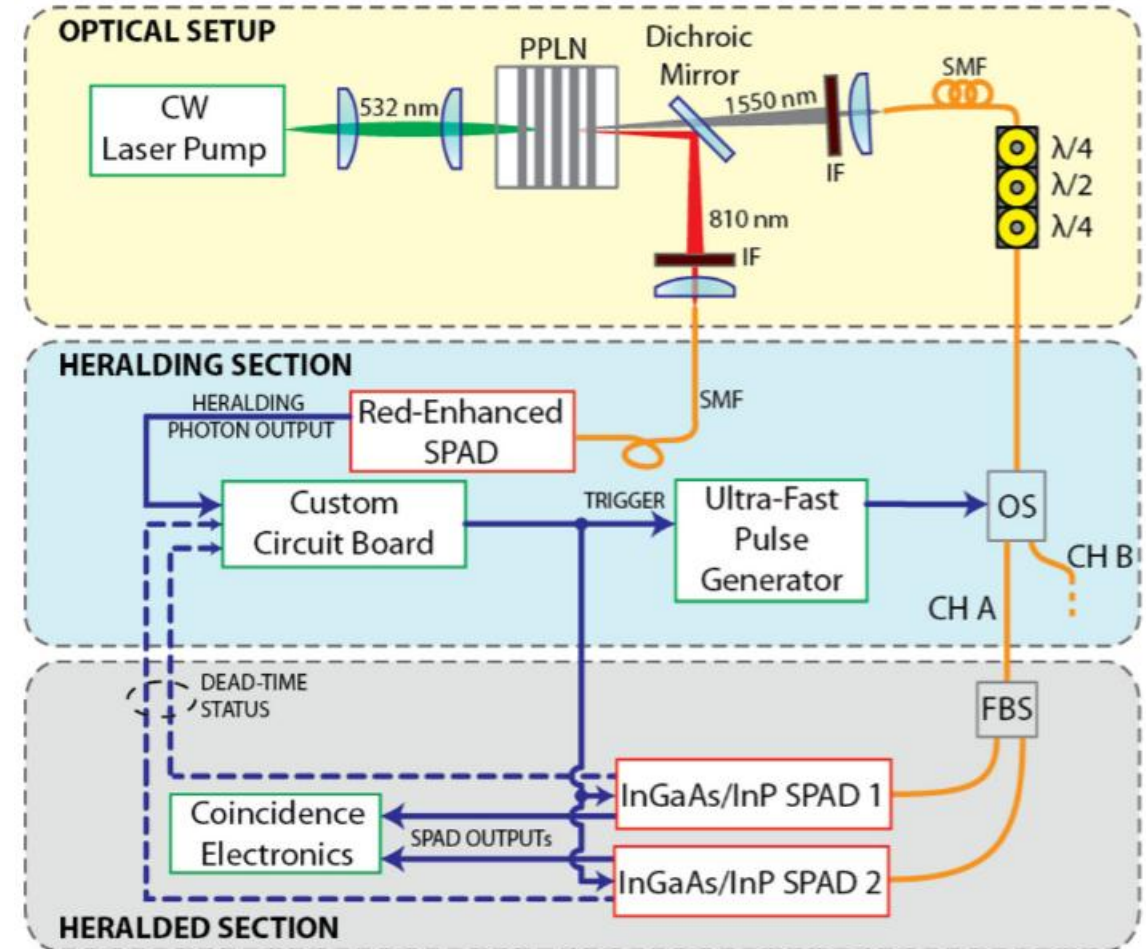
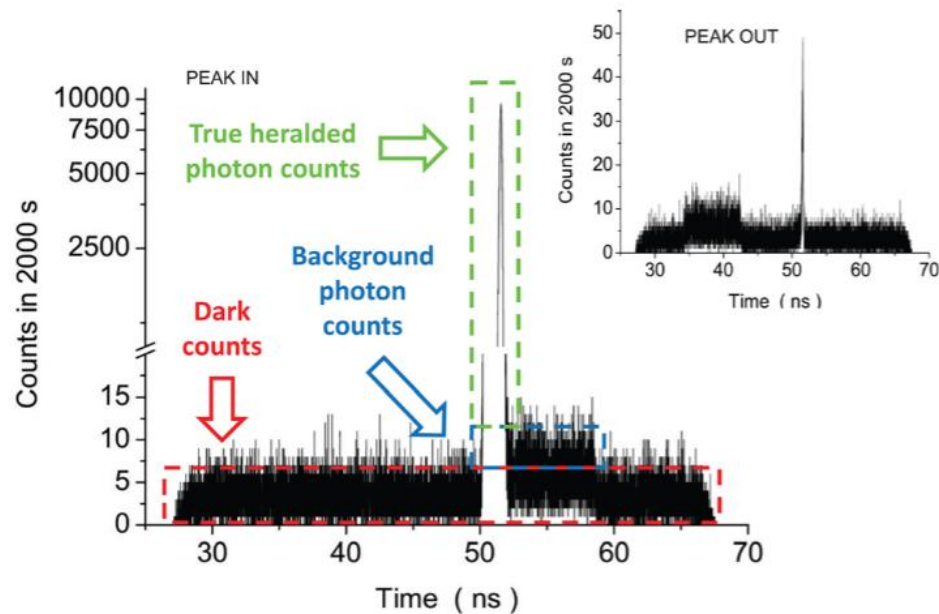
G. Brida,¹ I. P. Degiovanni,¹ M. Genovese,¹ F. Piacentini,¹ P. Traina,¹ A. Della Frera,² A. Tosi,² A. Bahgat Shehata,² C. Scarcella,² A. Gulinatti,² M. Ghioni,² S. V. Polyakov,³ A. Migdall,³ and A. Giudice⁴

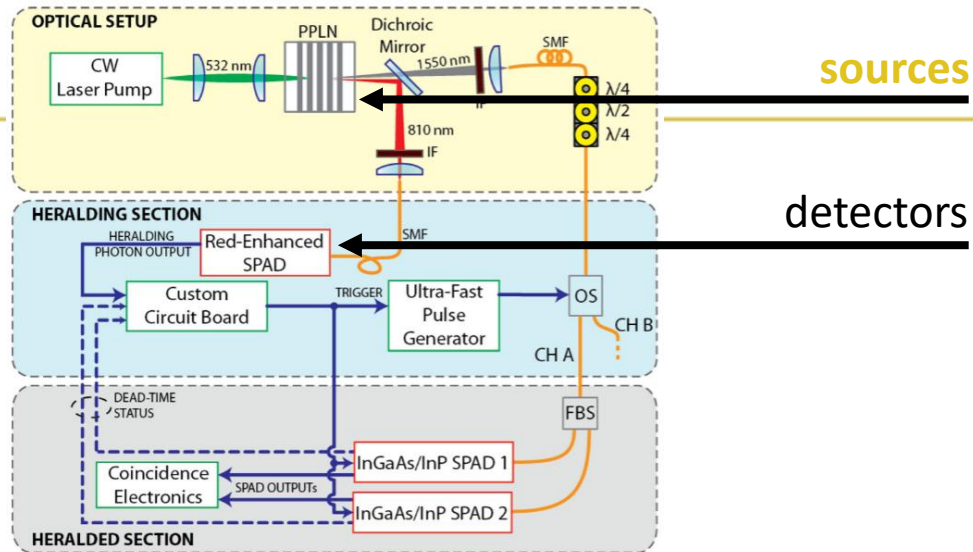
¹Istituto Nazionale di Ricerca Metrologica-I.N.R.I.M., Strada delle Cacce 91, 10135 Torino, Italy

²Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133 Milano, Italy

³Joint Quantum Institute, University of Maryland, and National Institute of Standards and Technology, 100 Bureau Dr. Stop 8410, Gaithersburg, Maryland 20899, USA

⁴Micro Photon Devices Srl, Via Stradivari 4, 39100 Bolzano, Italy



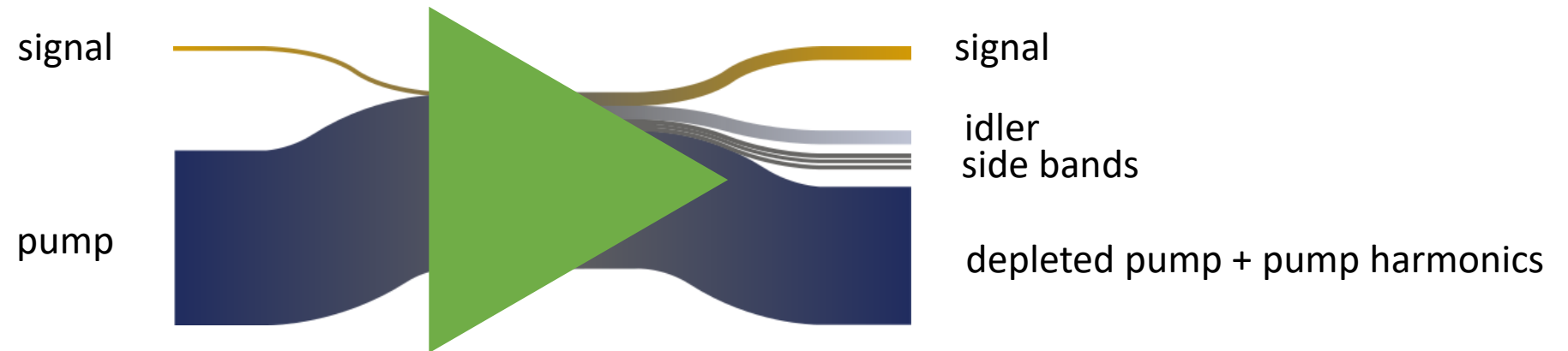


Superconducting Devices for Quantum Optics

(with a **metrological** flavor)

Josephson Nonlinear Metamaterials

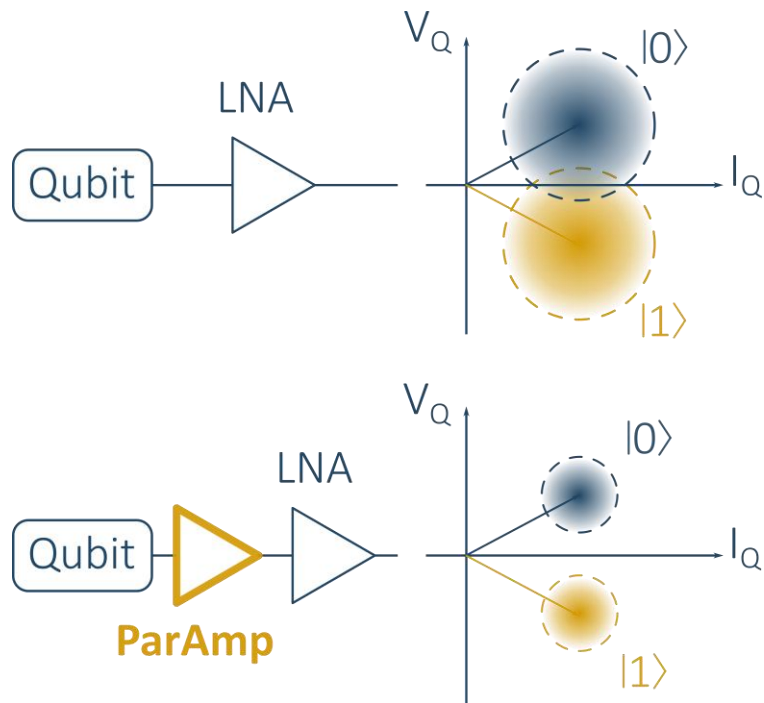
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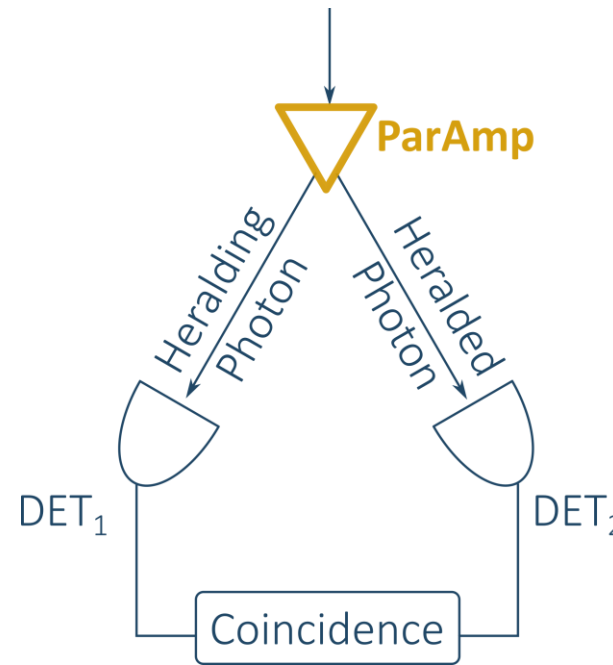
J-Metamaterials and Metrology

10

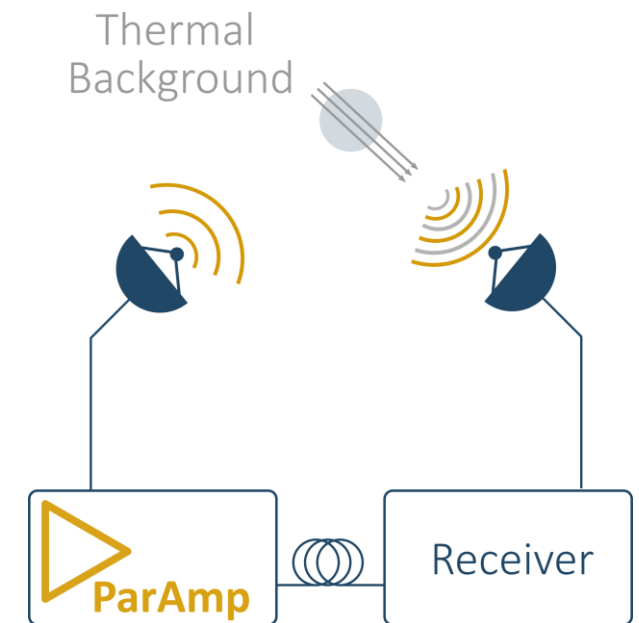
Improvement of readout of weak microwave signals with a **quantum limited amplifier**



Realization of **absolute calibration** technique of microwave **number resolved photon detectors**



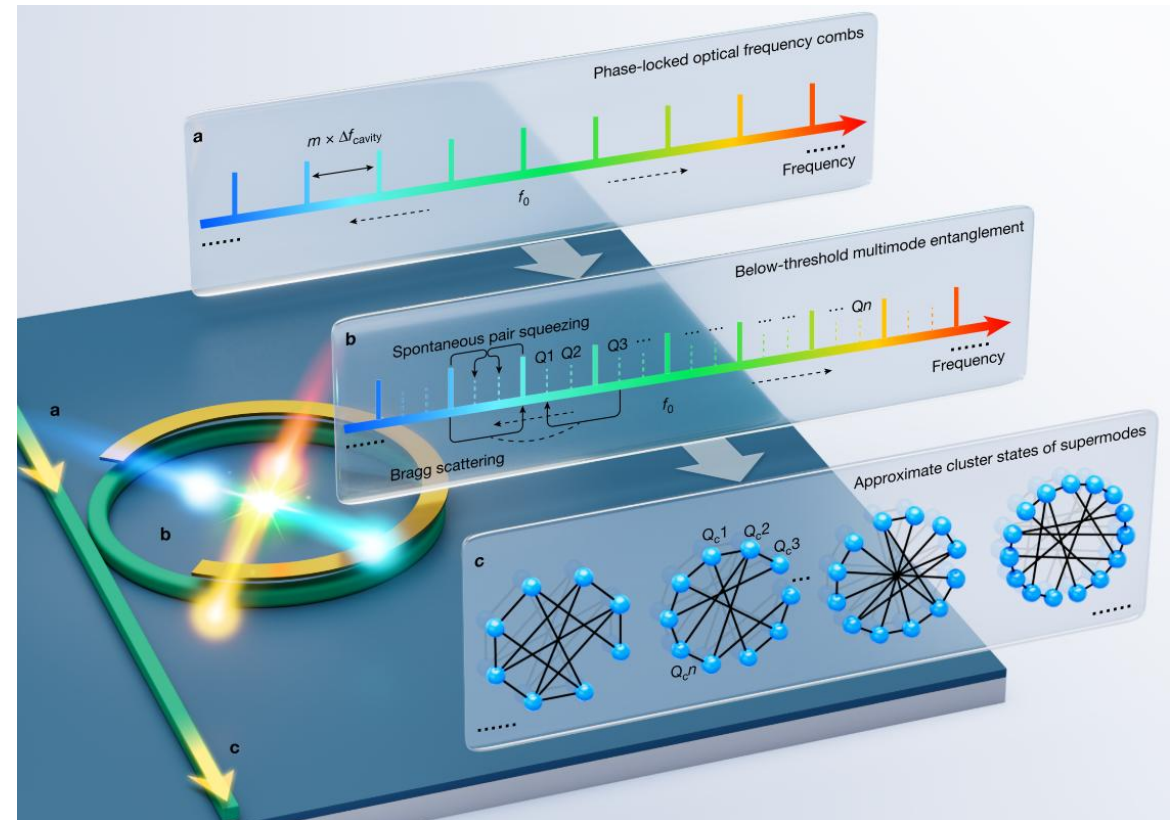
Microwave **Quantum Illumination** to improve detection or sensing in noisy environments



Cluster States - Overview

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- **Qubit** Cluster States: Highly entangled states on discrete-variable systems (two-level qubits)
- **CV** Cluster States: Analogous entangled states in continuous-variable systems (e.g., squeezed light modes)
- Both serve as **universal resources** for measurement-based quantum computation (**MBQC**)
- Built via entangling operations on graph-like structures (e.g., controlled-Z for qubits, beam splitters + squeezing for CV)

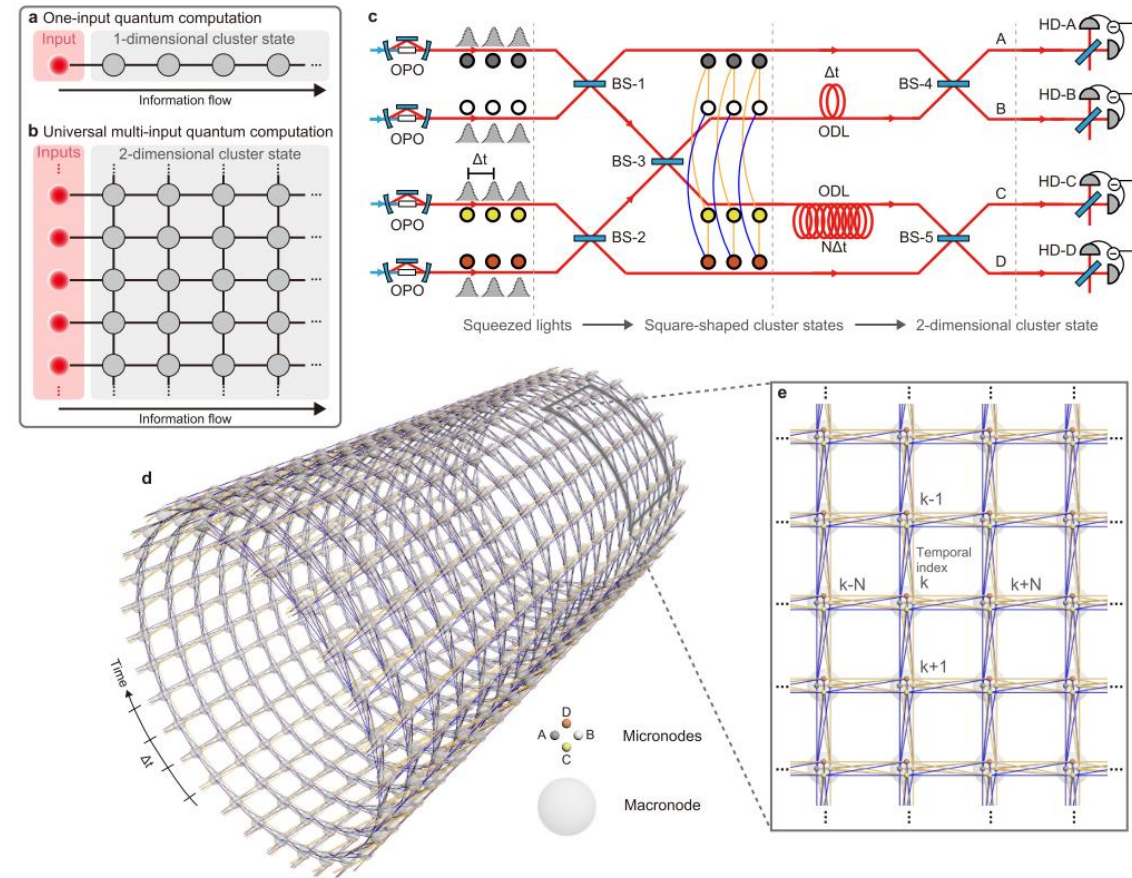


Jia, X., Zhai, C., Zhu, X. et al. Continuous-variable multipartite entanglement in an integrated microcomb. *Nature* 639, 329–336 (2025). <https://doi.org/10.1038/s41586-025-08602-1>

Cluster States - Applications

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- **MBQC**: Perform quantum algorithms via local measurements on a pre-prepared entangled state
- Quantum Communication: Enable one-way quantum repeaters and teleportation networks (qubit & CV)
- Quantum Simulation: Emulate complex many-body systems and quantum dynamics
- Quantum Error Correction: Foundation for topological codes (qubit) and bosonic codes (CV)
- **Quantum Sensing & Metrology**: Exploit multipartite entanglement for enhanced sensitivity (especially in CV systems)

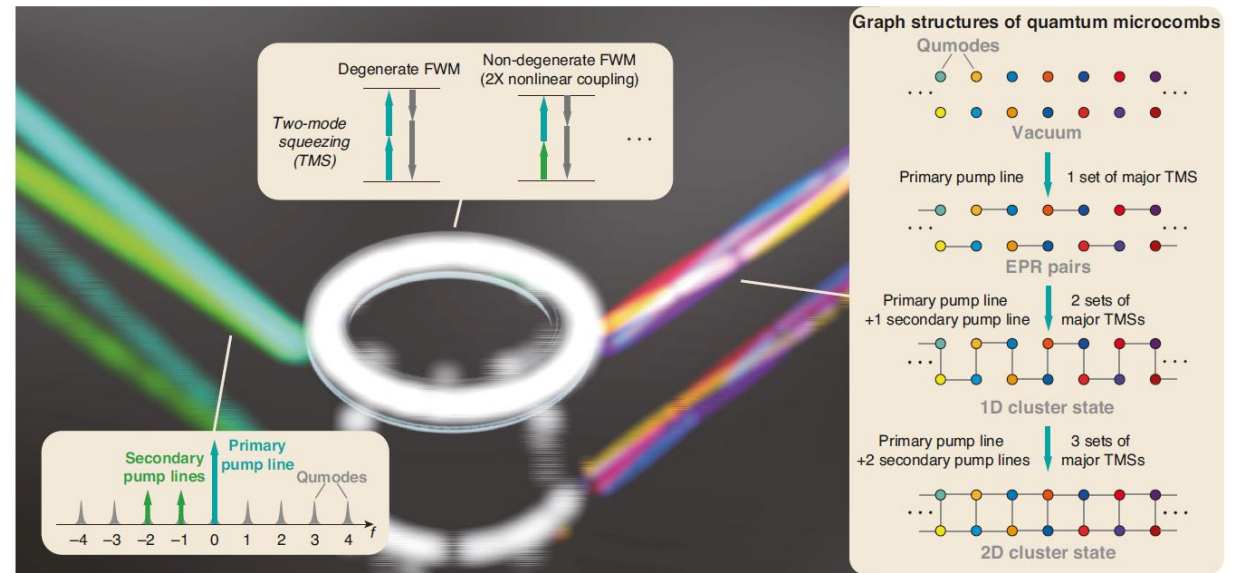


Warit Asavanant et al., Generation of time-domain-multiplexed two-dimensional cluster state. Science 366,373-376 (2019). DOI:10.1126/science.aay2645

Cluster States – Advantages

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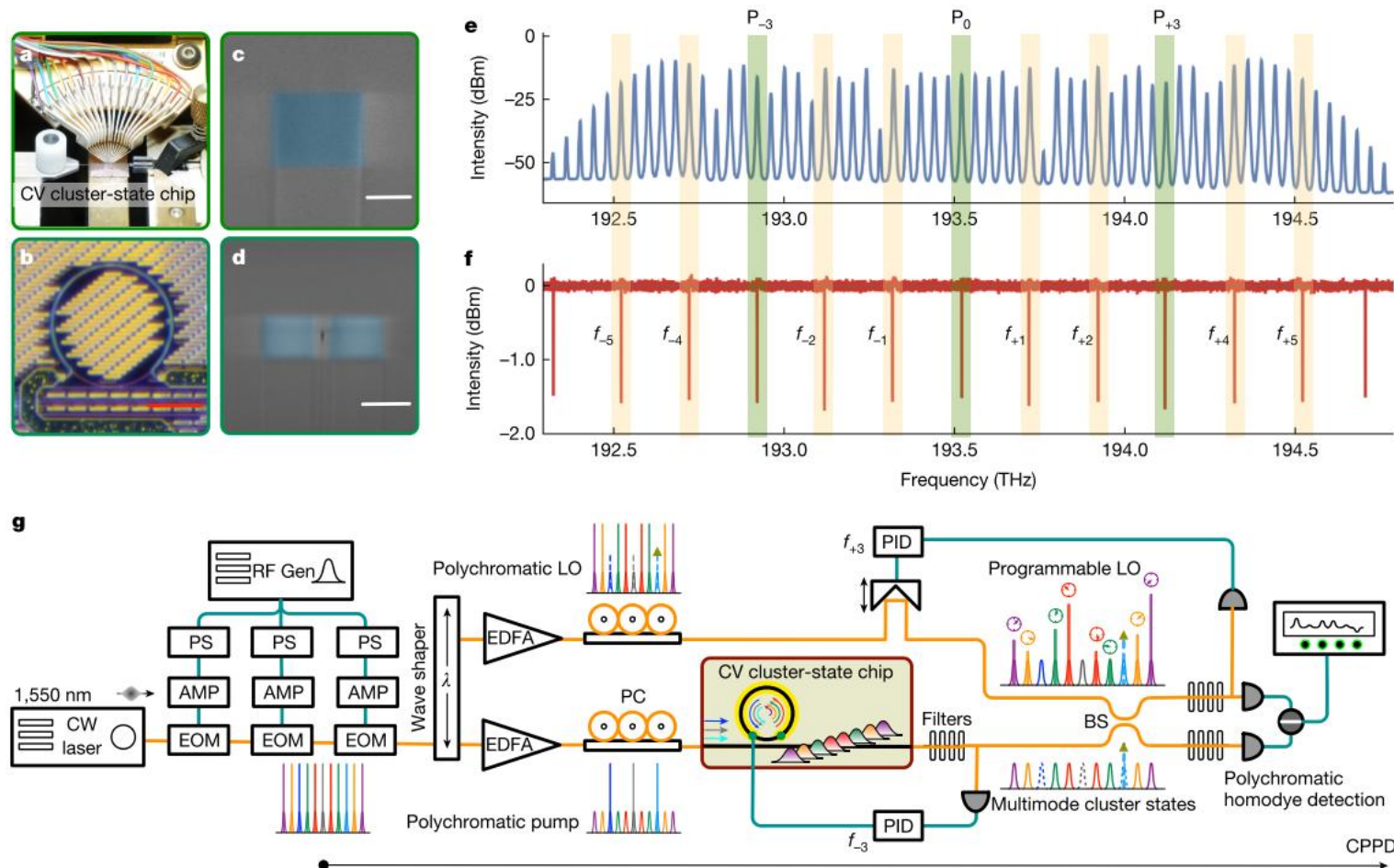
- Decoupled Architecture: Prepare entanglement once, **compute via measurements** → simplifies control
- Parallelism: **Many measurements** can be performed **simultaneously** in spatial or temporal structures
- Scalability: CV cluster states can be generated in **large scale** (e.g., using optical frequency combs or time-domain multiplexing)
- Hardware Efficiency: Measurement-based protocols **reduce gate complexity** and hardware overhead
- Noise Robustness: **Topological and fault-tolerant constructions** possible for both qubit and CV implementations



Wang, Z., Li, K., Wang, Y. et al. Large-scale cluster quantum microcombs. *Light Sci Appl* 14, 164 (2025).
<https://doi.org/10.1038/s41377-025-01812-2>

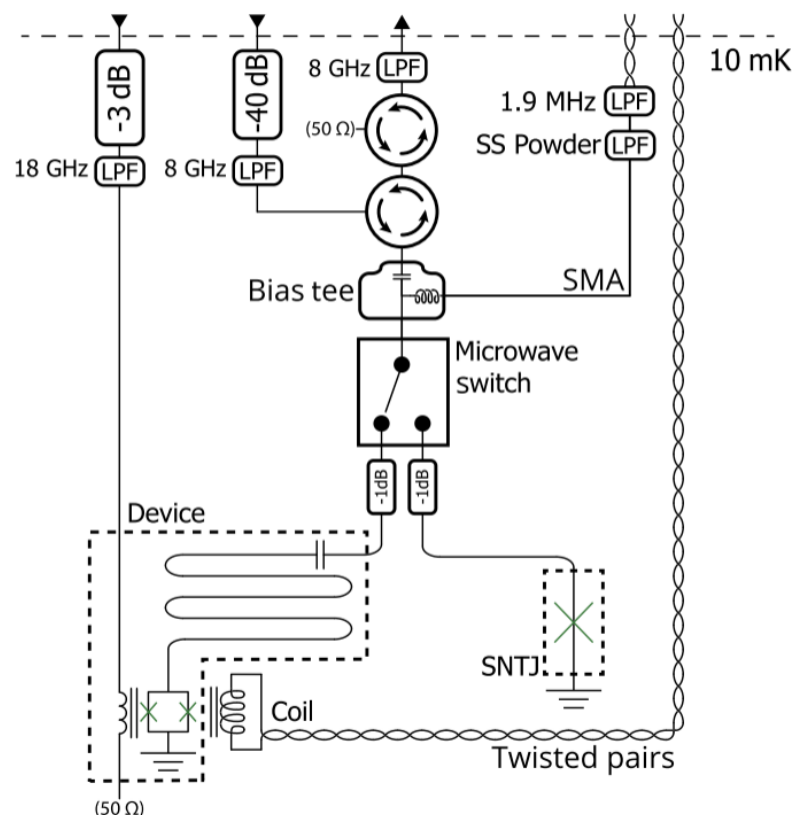
Cluster States – Optics Regime

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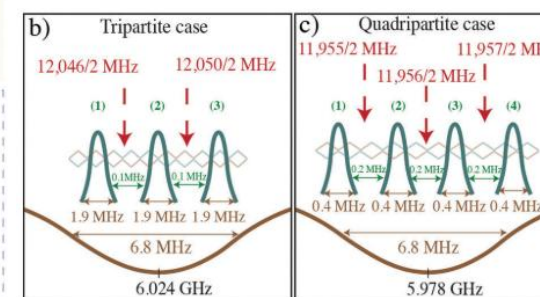
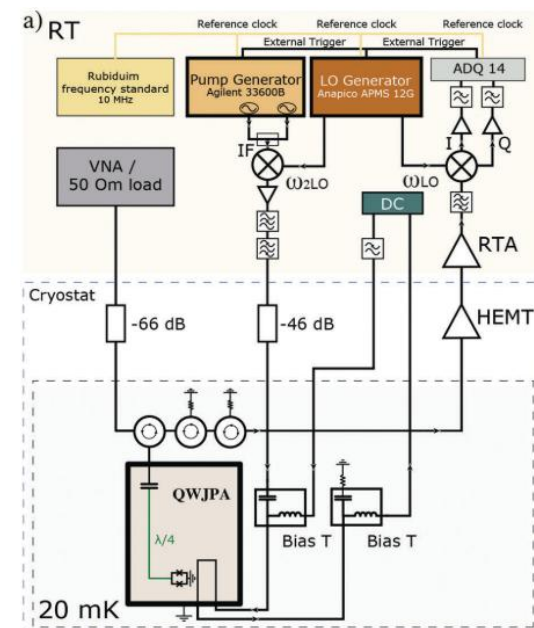


Jia, X., Zhai, C., Zhu, X. et al. Continuous-variable multipartite entanglement in an **integrated microcomb**. *Nature* 639, 329–336 (2025). <https://doi.org/10.1038/s41586-025-08602-1>

Cluster States – Microwave Regime



Sandbo Chang, C. W., Simoen, M., Aumentado, J., et.al. (2018).
 Generating Multimode Entangled Microwaves with a Superconducting
 Parametric Cavity. *Physical Review Applied*, 10(4).
<https://doi.org/10.1103/PhysRevApplied.10.044019>



Petrovnnin, K. V., Perelshtein, M. R., Korkalainen, T., Vesterinen, V., Lilja, I.,
 Paraoanu, G. S., & Hakonen, P. J. (2023). Generation and Structuring of
 Multipartite Entanglement in a Josephson Parametric System. *Advanced
 Quantum Technologies*, 6(1). <https://doi.org/10.1002/qute.202200031>

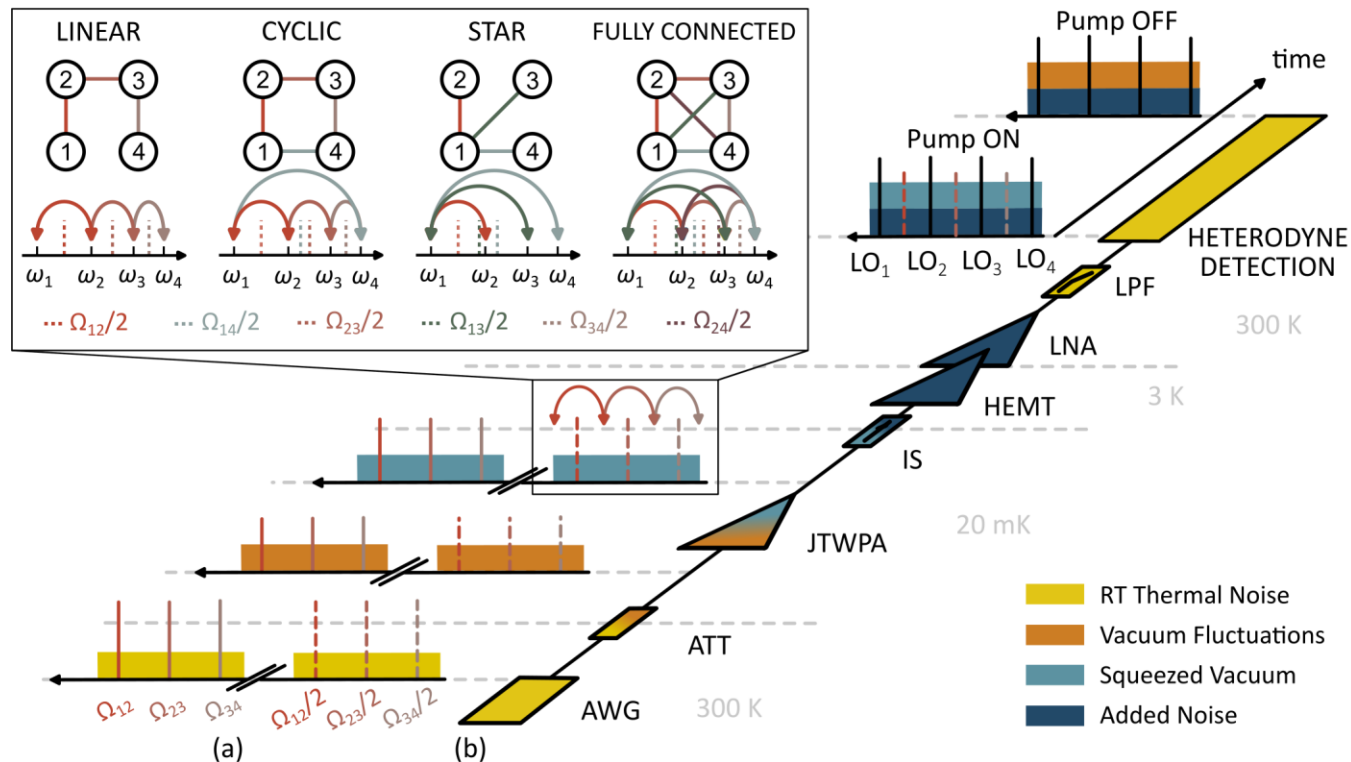
Programmable Microwave CS

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(a) Multiple pump tones from an AWG drive the JTWPA[1] into a **three-wave mixing regime**, generating CV cluster states in the frequency domain.

(b) Experimental setup: broadband mode synthesis, cryogenic routing, and **multiplexed heterodyne detection** with noise rejection via **reference state protocol**.

[1] VTT /Arctic TWPA from



EUROPEAN
PARTNERSHIP

Co-funded by
the European Union

METROLOGY
PARTNERSHIP

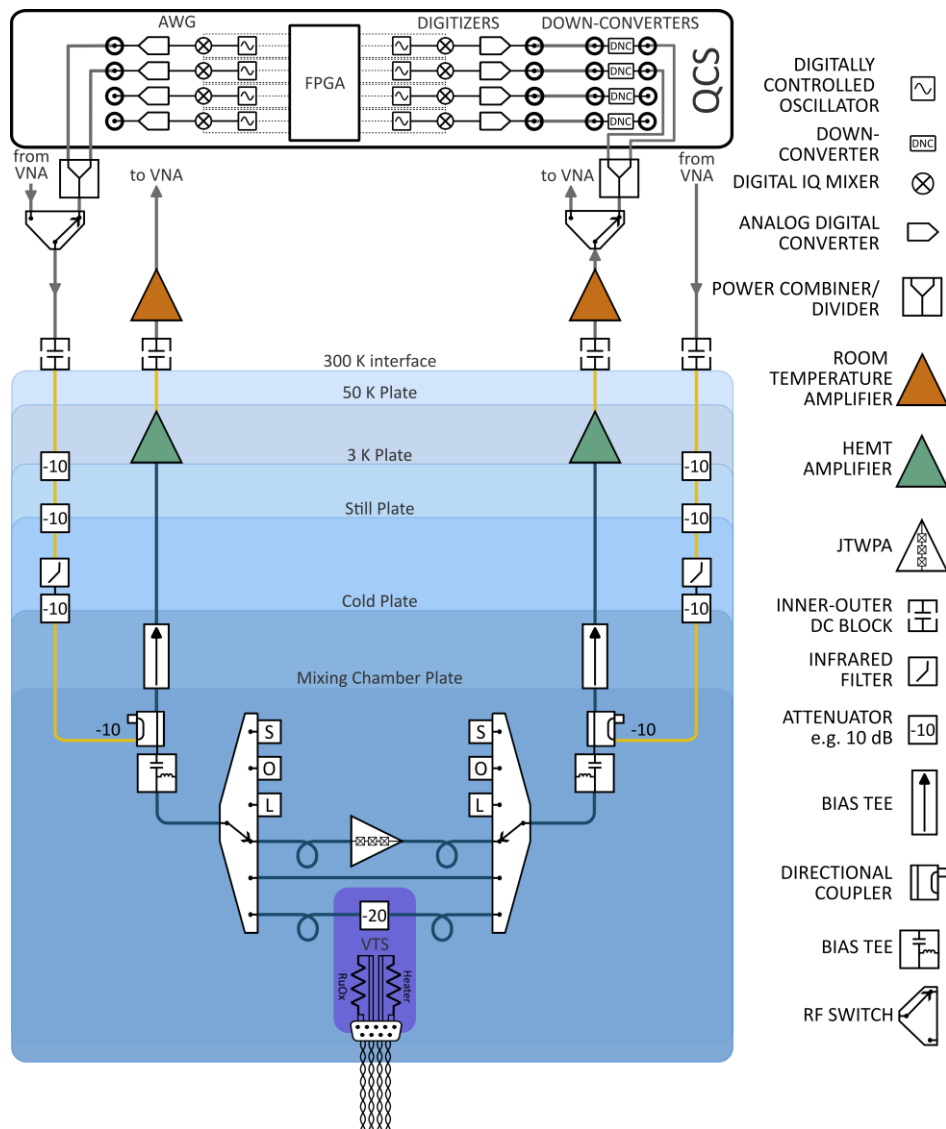


$$\hat{H}_{\text{eff}} = \sum_{k=1}^n \sum_{j \leq l} \left(r_{jl}^{(k)} \hat{a}_j^\dagger \hat{a}_l^\dagger + r_{jl}^{(k)*} \hat{a}_j \hat{a}_l \right)$$

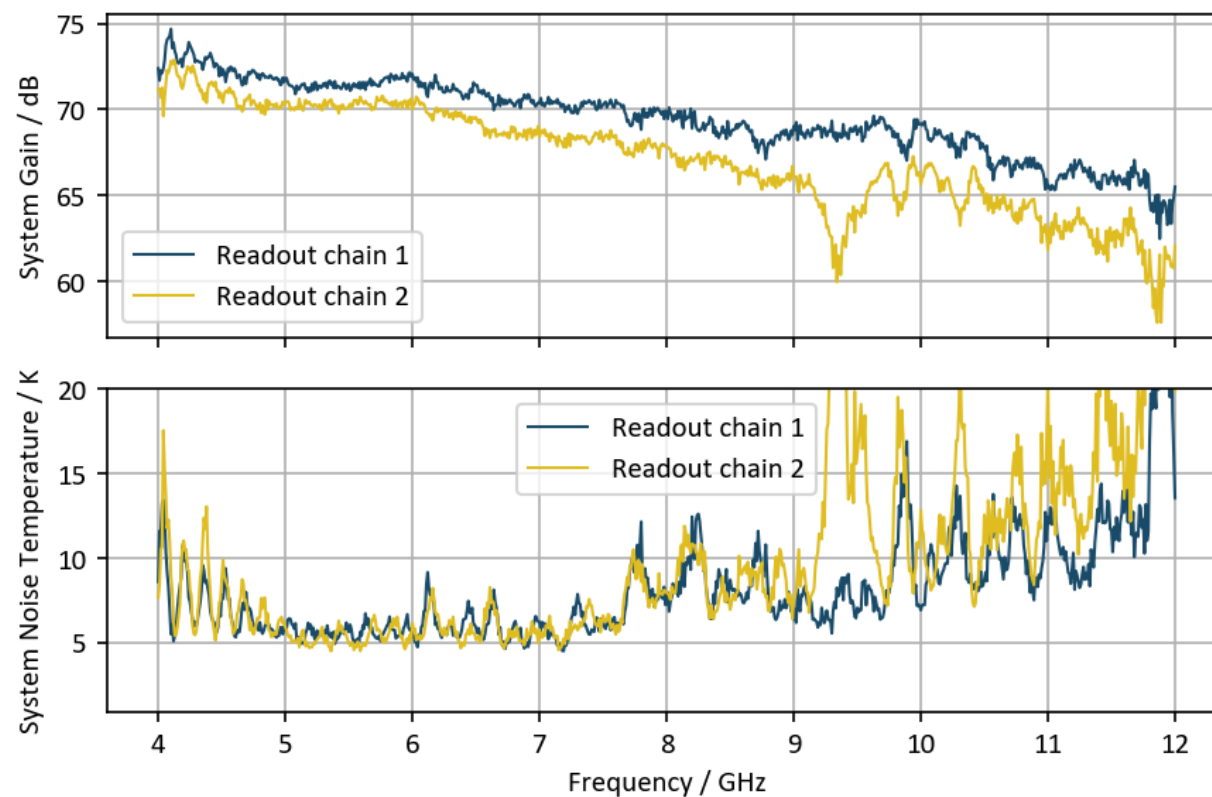
$$r_{jl}^{(k)} = g_{jl}^{(k)} \alpha_k e^{-i\phi_k}$$



Cryogenic setup and Plank spectroscopy

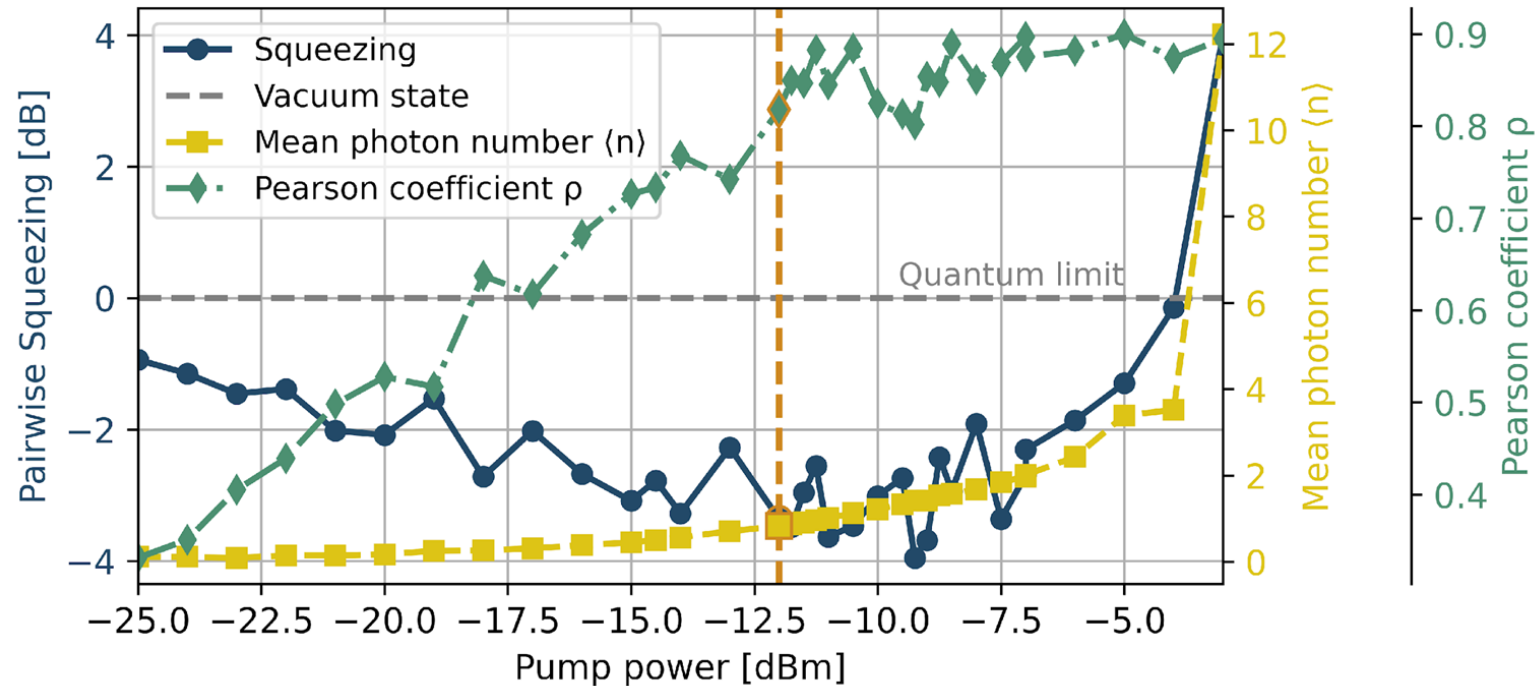


$$\frac{\langle I_j^2 + Q_j^2 \rangle}{Z_0 \tau_j^{-1}} = G_j \left[\frac{\hbar \omega_j}{2} \coth \left(\frac{\hbar \omega_j}{2 k_B T_{\text{SYS},j}} \right) + k_B T_{\text{SYS},j} \right]$$

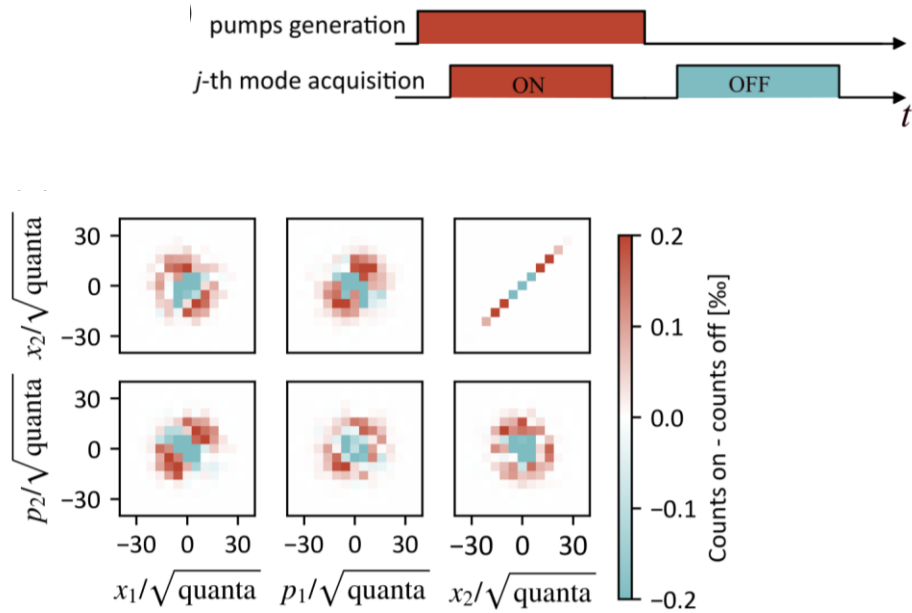


Single pump setup validation

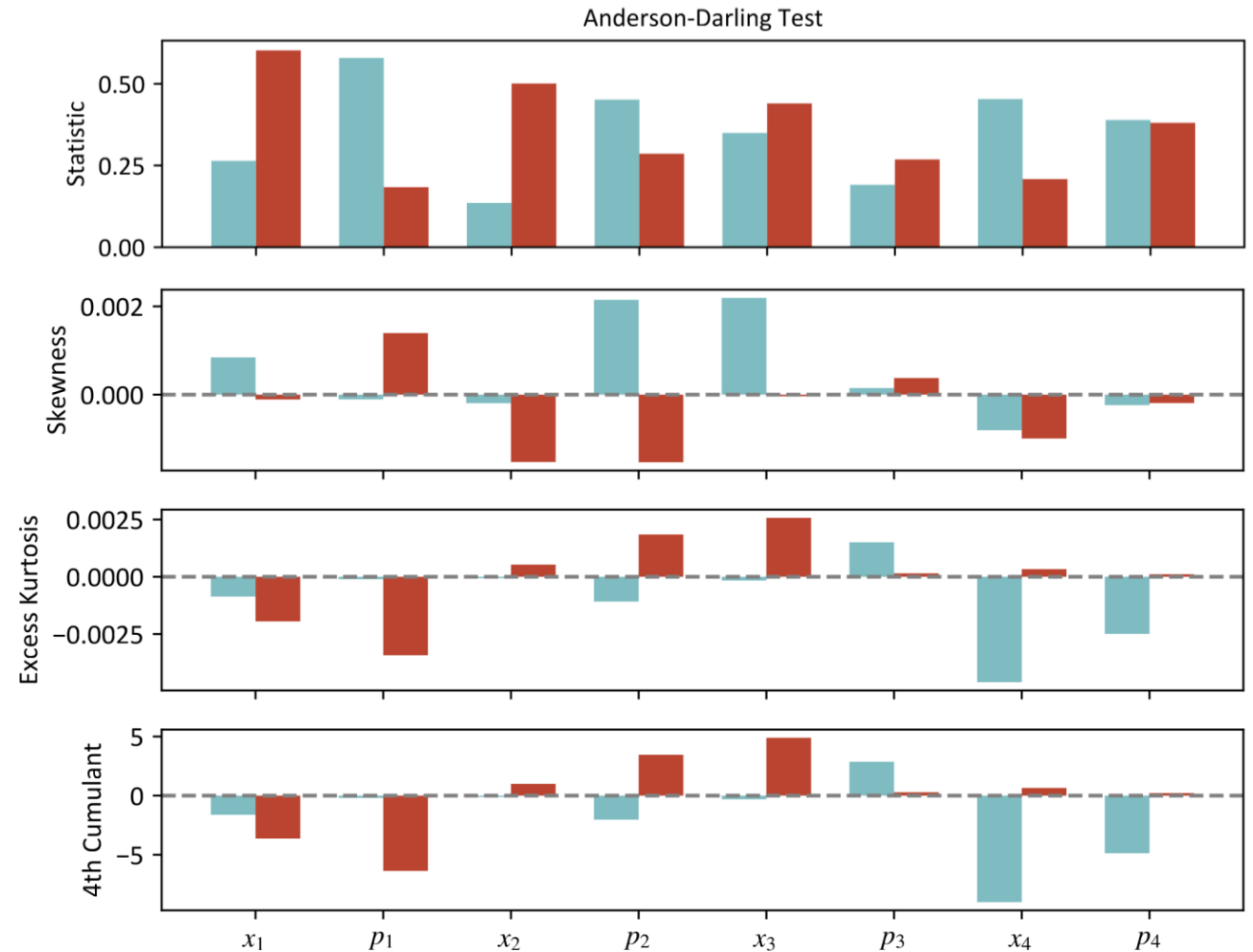
18



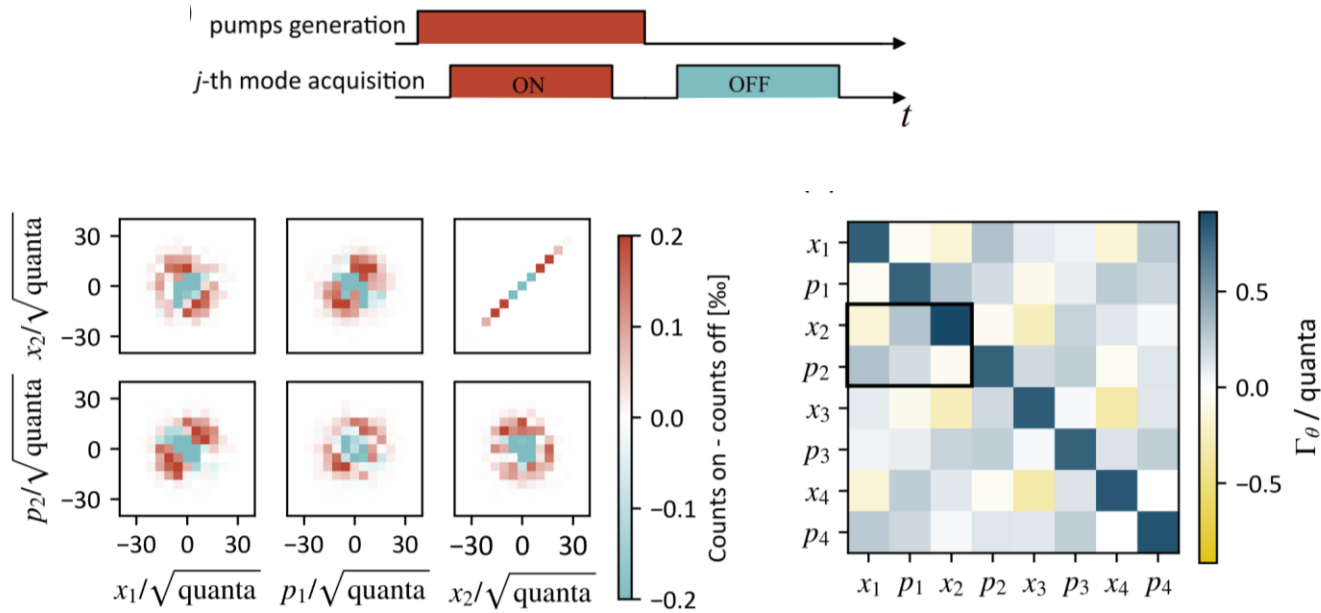
Gaussianity Validation



- **Standard normality tests**, namely the Shapiro–Wilk and Anderson–Darling tests.
- **Moment analysis up to fourth order**, including skewness, excess kurtosis, and fourth-order cumulants.



Covariance Matrix Reconstruction



Measure output quadratures with pump OFF
→ **reference vacuum state**.

Measure with pump ON
→ signal (**CS**) + TWPA-added noise.

Comparing the two
→ reconstruction of the output covariance matrix, isolating the TWPA's effect (**amplification, squeezing, added noise**).

$$i_j = \frac{I_j}{\sqrt{2G_j Z_0 \hbar \omega_j / 2\tau^{-1}}}, \quad q_j = \frac{Q_j}{\sqrt{2G_j Z_0 \hbar \omega_j \tau^{-1}}}$$

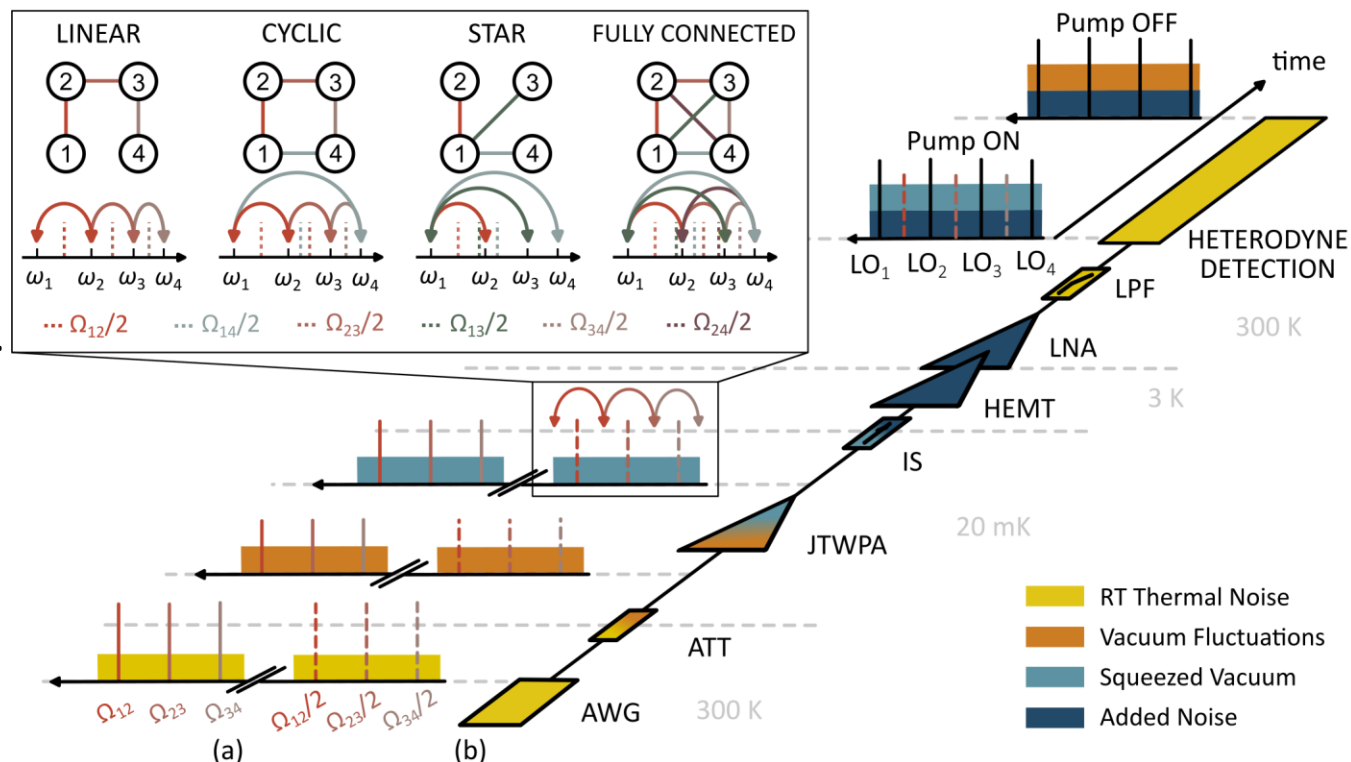
$$\Gamma_{j,l}^{\text{exp}} = \frac{1}{2} \langle M_j M_l + M_l M_j \rangle - \langle M_j \rangle \langle M_l \rangle$$

$$\mathbf{M} = (i_1, q_1, \dots, i_m, q_m)^T$$

$$\Gamma_{\theta} = (\Gamma_{\text{ON}}^{\text{exp}} - \Gamma_{\text{OFF}}^{\text{exp}}) + \sigma^{\text{in}}$$

Het. Det. with Unknown Phase Offsets

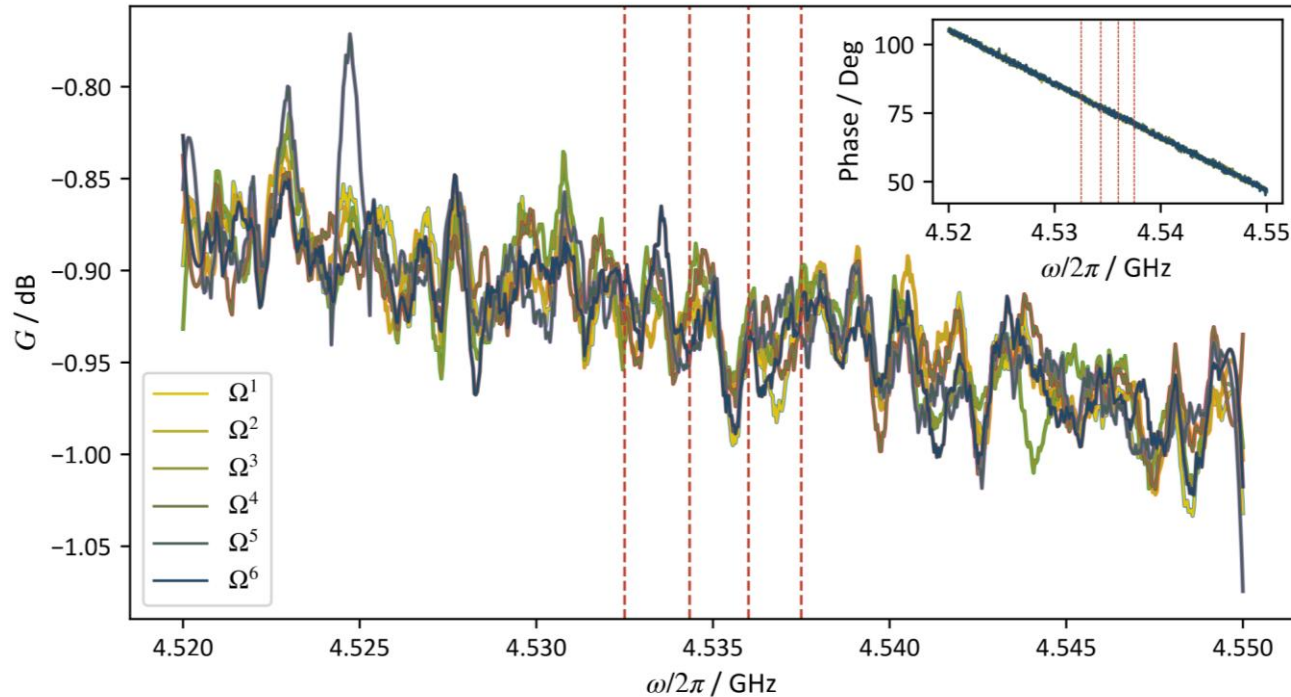
- LOs are **phase-locked** to the JTWPA pumps.
- Unknown **phase shifts arise** from setup components, cable dispersion, and group delays.
- These cause effective quadrature rotation across frequency.
- Covariance matrix affected.



$$\Gamma_{\theta} = \mathbf{R}_{\theta} \cdot \Gamma \cdot \mathbf{R}_{\theta}^T, \quad \mathbf{R}_{\theta} = \bigoplus_{j=1}^m \begin{pmatrix} \cos \theta_j & -\sin \theta_j \\ \sin \theta_j & \cos \theta_j \end{pmatrix}$$

Assumptions & Squeezing Optimization

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- Covariance matrix optimized **via local phase-space rotations** $R_j(\theta_j)$.
- Pumps phase-locked, frequencies close
 $\Rightarrow$ stable relative phase drift.
- Symmetric (almost flat) squeezing
 $\Rightarrow$ no need for local squeezing or shear.
- Restriction to orthogonal symplectic transforms **preserves entanglement structure**.

$\rightarrow$ Goal: reorient squeezing axes to maximize observed two-mode correlations.

Squeezing Optimization Procedure

- Optimization aims to maximize $p_j - x_l$ correlations over the **set of LO phases θ** .
- Define the **average covariance** function:

$$\mathcal{C}_{px}(\theta) = \frac{1}{m^2} \sum_{j=1}^m \sum_{l=1}^m \Gamma_{\theta, p_j, x_l}$$

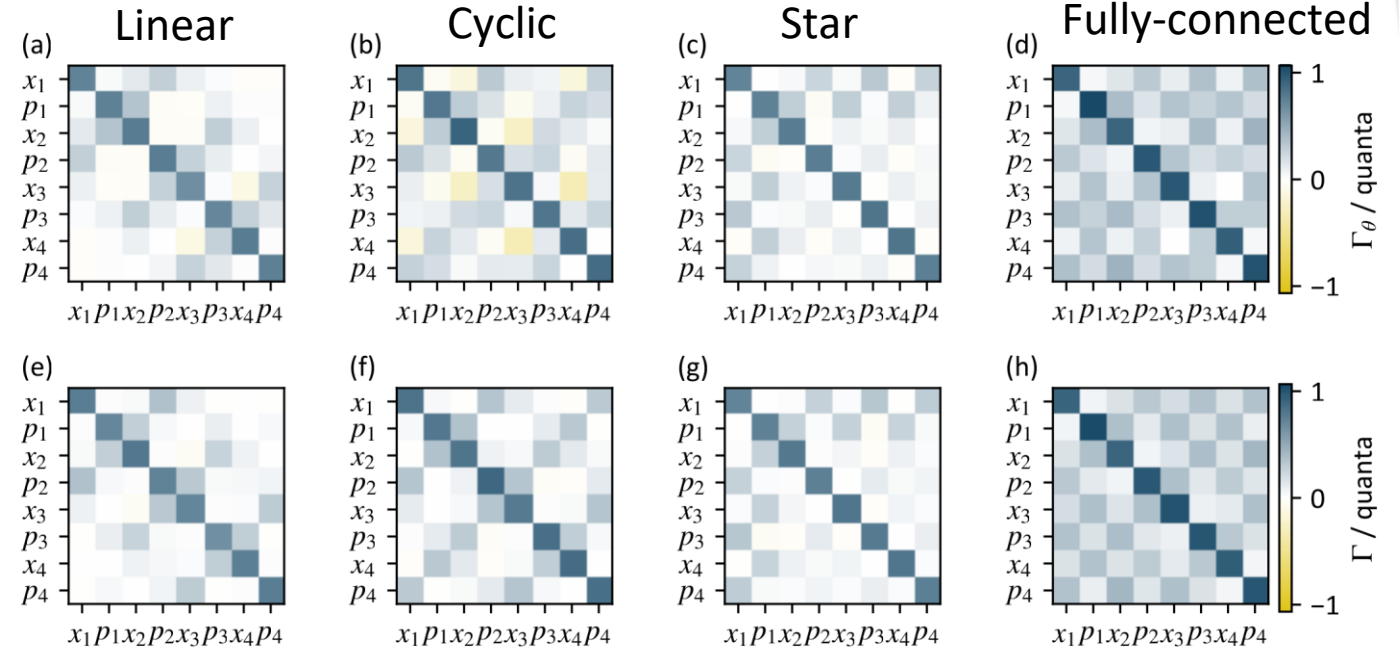
- The optimal phase set θ_{opt} **maximizes** this correlation measure.

$$\theta_{\text{opt}} = \arg \max_{\theta} \mathcal{C}_{px}(\theta)$$

- This identifies the **global quadrature basis** where the reconstructed covariance matrix Γ_{θ} best matches an ideal cluster state.

$$\Gamma = \mathbf{R}_{\theta_{\text{opt}}}^{-1} \Gamma_{\theta} \left(\mathbf{R}_{\theta_{\text{opt}}}^{-1} \right)^{\top}$$

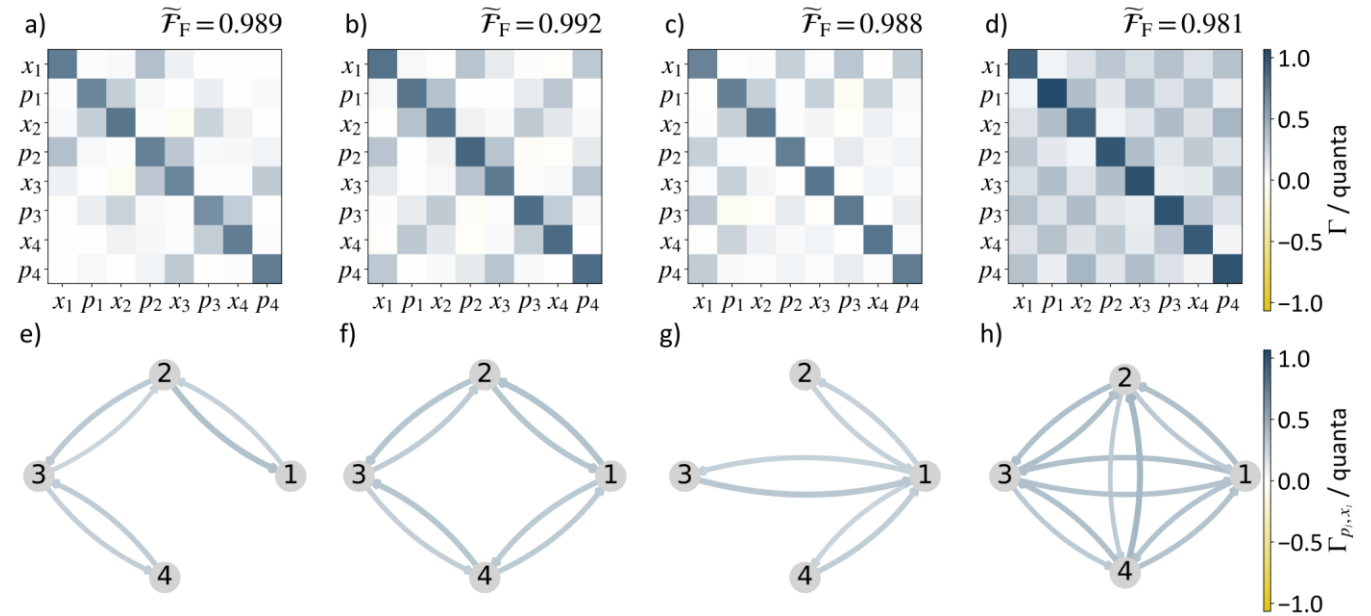
PRE



POST

Squeezing Optimization Procedure

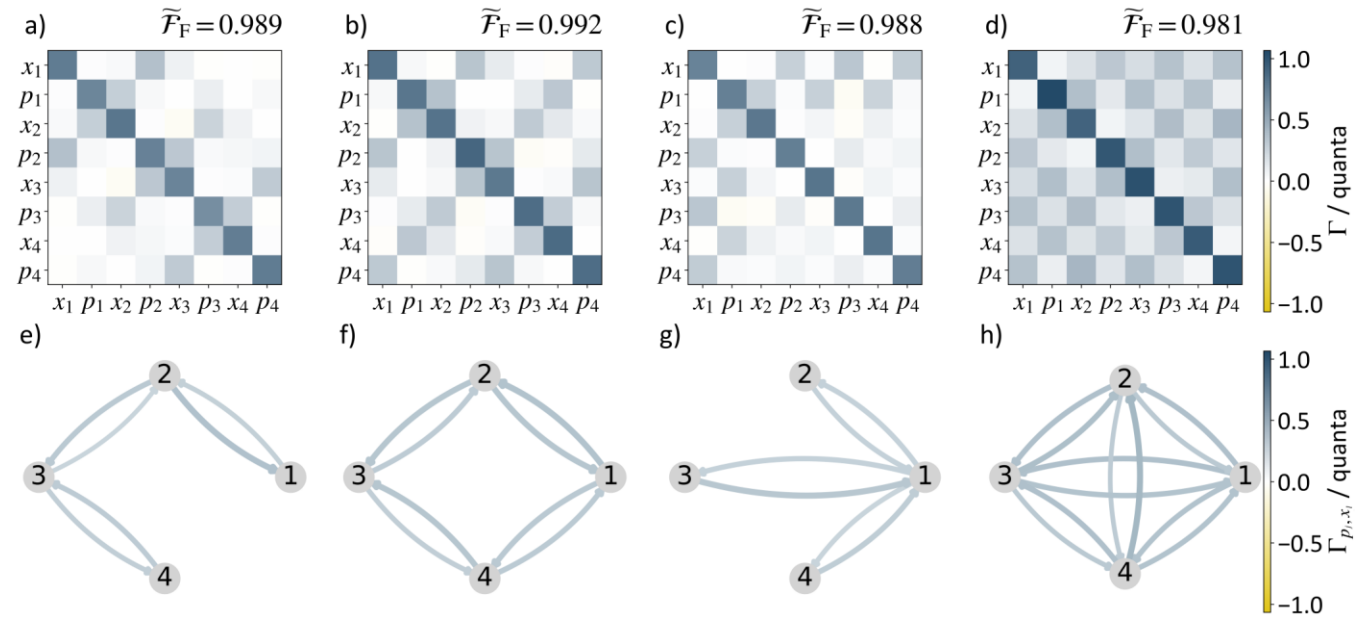
- Captures and **compensates unknown pump-induced phase shifts**
- Ensures **physically consistent phase alignment across modes**—key for coherent protocols (e.g. cluster states, error correction)
- The **optimal global quadrature basis** reveals maximal two-mode squeezing and **graph structure**



Squeezing Optimization Procedure

Is there a trivial way to compare with the theory?

- Captures and **compensates unknown pump-induced phase shifts**
- Ensures **physically consistent phase alignment across modes**—key for coherent protocols (e.g. cluster states, error correction)
- The **optimal global quadrature basis** reveals maximal two-mode squeezing and **graph structure**

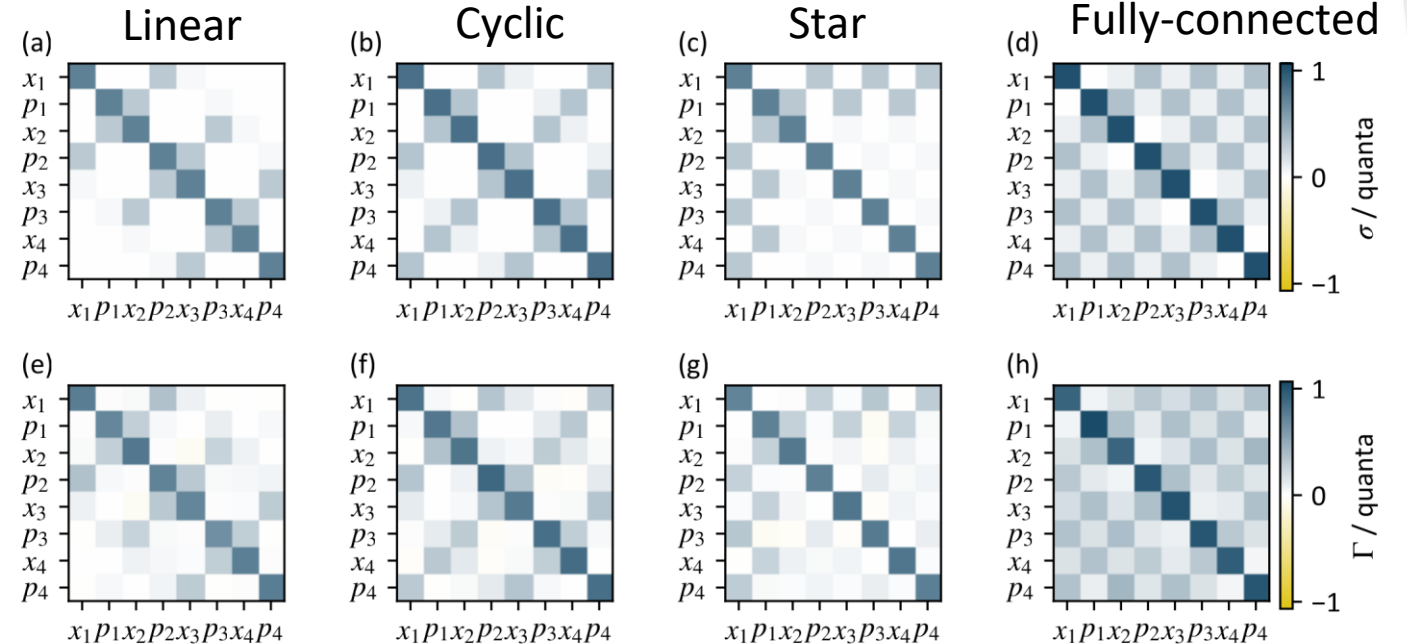


Comparison with Theory

- The reconstructed covariance matrix Γ is compared to its **theoretical model** σ
- The **Frobenius distance** quantifies the similarity between the two

$$\widetilde{\mathcal{F}}_F(\sigma, \Gamma) = \exp \left(- \frac{\|\sigma - \Gamma\|_F^2}{\|\sigma\|_F^2} \right)$$

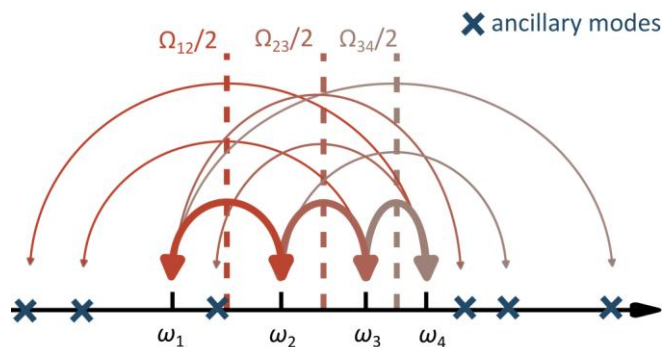
THEORY



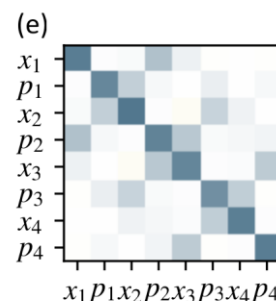
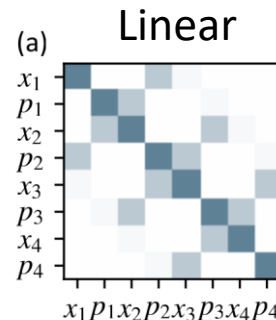
MEASUREMENT

Role of Ancillary Modes: e.g. Linear CS

- **Principal modes:** 1–4 connected linearly via TMS links
(1,2),(2,3),(3,4)
- **Ancillary modes:** 5–10, connected to the principal chain via TMS links:
(1,5), (1,6), (2,7), (3,8), (4,9), (4,10).



THEORY



MEASUREMENT

Generalized input-output relations

$$\hat{a}_{\text{out},i} = \sum_j \left(\mathbf{U}_{ij} a_i + \mathbf{V}_{ij} \hat{a}_j^\dagger \right)$$

Bogoliubov matrices

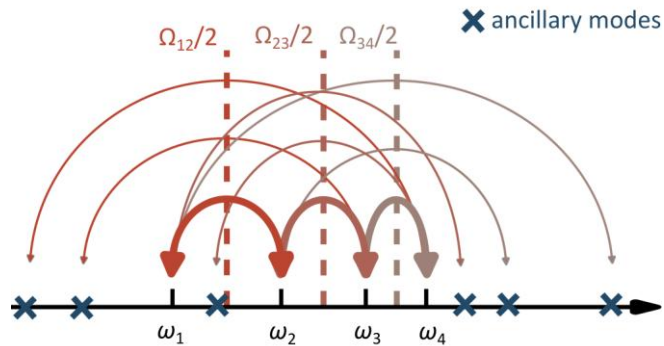
$$\mathbf{U} = \mathbb{I}_m + (\cosh |r| - 1) \cdot \text{diag}(\mathbf{D}), \quad \mathbf{V} = \sinh |r| \cdot \mathbf{A}$$

Covariance matrix within symplectic formalism

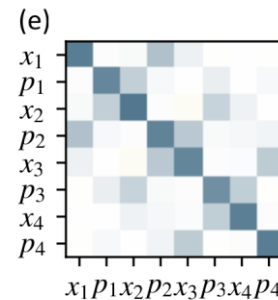
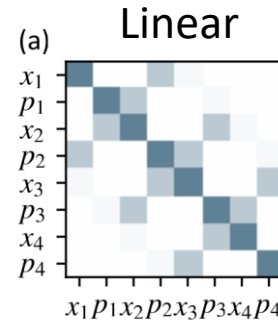
$$\sigma = \frac{1}{2} \mathbf{S}_X \mathbf{S}_X^\top = \frac{1}{2} \begin{pmatrix} (\mathbf{U} + \mathbf{V})^2 & 0 \\ 0 & (\mathbf{U} - \mathbf{V})^2 \end{pmatrix}$$

Role of Ancillary Modes: e.g. Linear CS

- Principal modes:** 1–4 connected linearly via TMS links
(1,2),(2,3),(3,4)
- Ancillary modes:** 5–10, connected to the principal chain via TMS links:
(1,5), (1,6), (2,7), (3,8), (4,9), (4,10).



THEORY



MEASUREMENT

Principal

Ancillary

$$\sigma_{\text{red}}^{(1:4)} = \begin{bmatrix} D & 0 & 0 & A & E & 0 & 0 & 0 \\ 0 & D & A & 0 & 0 & E & 0 & 0 \\ 0 & A & D & 0 & 0 & A & E & 0 \\ A & 0 & 0 & D & A & 0 & 0 & E \\ E & 0 & 0 & A & D & 0 & 0 & A \\ 0 & E & A & 0 & 0 & D & A & 0 \\ 0 & 0 & E & 0 & 0 & A & D & 0 \\ 0 & 0 & 0 & E & A & 0 & 0 & D \end{bmatrix}$$

...

...

$$D = -6 \cosh(|r|) + 3 \cosh(2|r|) + \frac{7}{2}$$

$$A = (3 \cosh(|r|) - 2) \sinh(|r|),$$

$$E = \frac{1}{2} \sinh^2(|r|).$$

Validation via Nullifier Measurements

- To verify multipartite entanglement and validate the cluster graph, we use **nullifier operators** based on the **adjacency matrix**.

$$\hat{\delta}_j = \hat{p}_j - \sum_{l=1}^m A_{jl} \hat{x}_l, \quad j = 1, \dots, m$$

$$\hat{\delta} = \mathbf{N} \cdot \hat{\mathbf{X}}, \quad \text{with } \mathbf{N} = (-\mathbf{A} \mid \mathbb{I}_m)$$

Nullifier covariance matrix

$$\Delta = \mathbf{N} \sigma \mathbf{N}^\top$$

Diagonal elements

$$\Delta_{jj} = \langle \hat{\delta}_j^2 \rangle - \langle \hat{\delta}_j \rangle^2 = \text{Var}(\hat{\delta}_j)$$

Validation via Nullifier Measurements

- To verify multipartite entanglement and validate the cluster graph, we use **nullifier operators** based on the **adjacency matrix**.

$$\hat{\delta}_j = \hat{p}_j - \sum_{l=1}^m A_{jl} \hat{x}_l, \quad j = 1, \dots, m$$

- In an **ideal** cluster state, **nullifiers vanish**; in practice, finite squeezing leads to small but nonzero variances.

$$\hat{\delta} = \mathbf{N} \cdot \hat{\mathbf{X}}, \quad \text{with } \mathbf{N} = (-\mathbf{A} \mid \mathbb{I}_m)$$

Nullifier covariance matrix

$$\Delta = \mathbf{N} \sigma \mathbf{N}^\top$$

Diagonal elements

$$\Delta_{jj} = \langle \hat{\delta}_j^2 \rangle - \langle \hat{\delta}_j \rangle^2 = \text{Var}(\hat{\delta}_j)$$

Eg. 3-modes Cyclic cluster state

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$\hat{\delta}_1 = \hat{p}_1 - \hat{x}_2 - \hat{x}_3$$

$$\hat{\delta}_2 = \hat{p}_2 - \hat{x}_1 - \hat{x}_3$$

$$\hat{\delta}_3 = \hat{p}_3 - \hat{x}_1 - \hat{x}_2$$

Validation via Nullifier Measurements

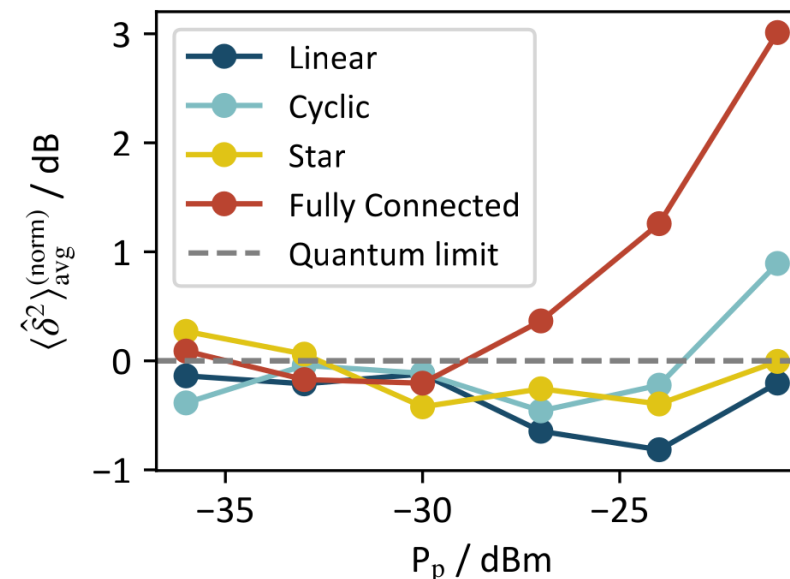
- To verify multipartite entanglement and validate the cluster graph, we use **nullifier operators** based on the **adjacency matrix**.

$$\hat{\delta}_j = \hat{p}_j - \sum_{l=1}^m A_{jl} \hat{x}_l, \quad j = 1, \dots, m$$

- In an **ideal** cluster state, **nullifiers vanish**; in practice, finite squeezing leads to small but nonzero variances.
- A state is identified as a cluster state if all **nullifier variances** are below the shot noise level (vacuum limit).

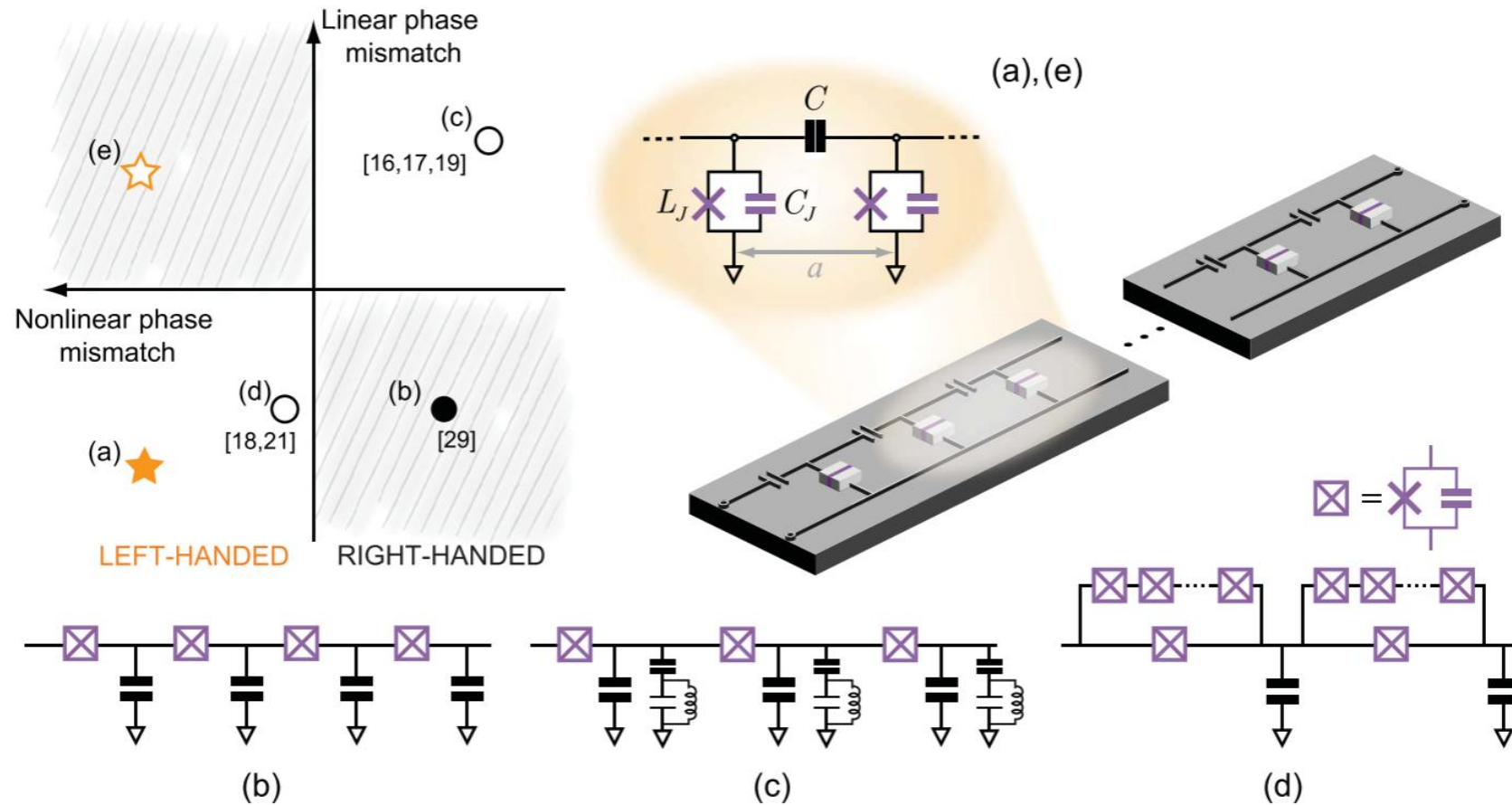
$$\langle \hat{\delta}_j^2 \rangle = \langle \hat{p}_j^2 \rangle + \sum_l A_{jl}^2 \langle \hat{x}_l^2 \rangle = \frac{1}{2} \left(1 + \sum_l A_{jl}^2 \right)$$

$$\langle \hat{\delta}^2 \rangle_{\text{avg}}^{(\text{norm})} = \frac{1}{m} \sum_{j=1}^m \frac{2\Delta_{jj}}{1 + \sum_l A_{jl}^2}$$



Squeezing-Gain trade-off

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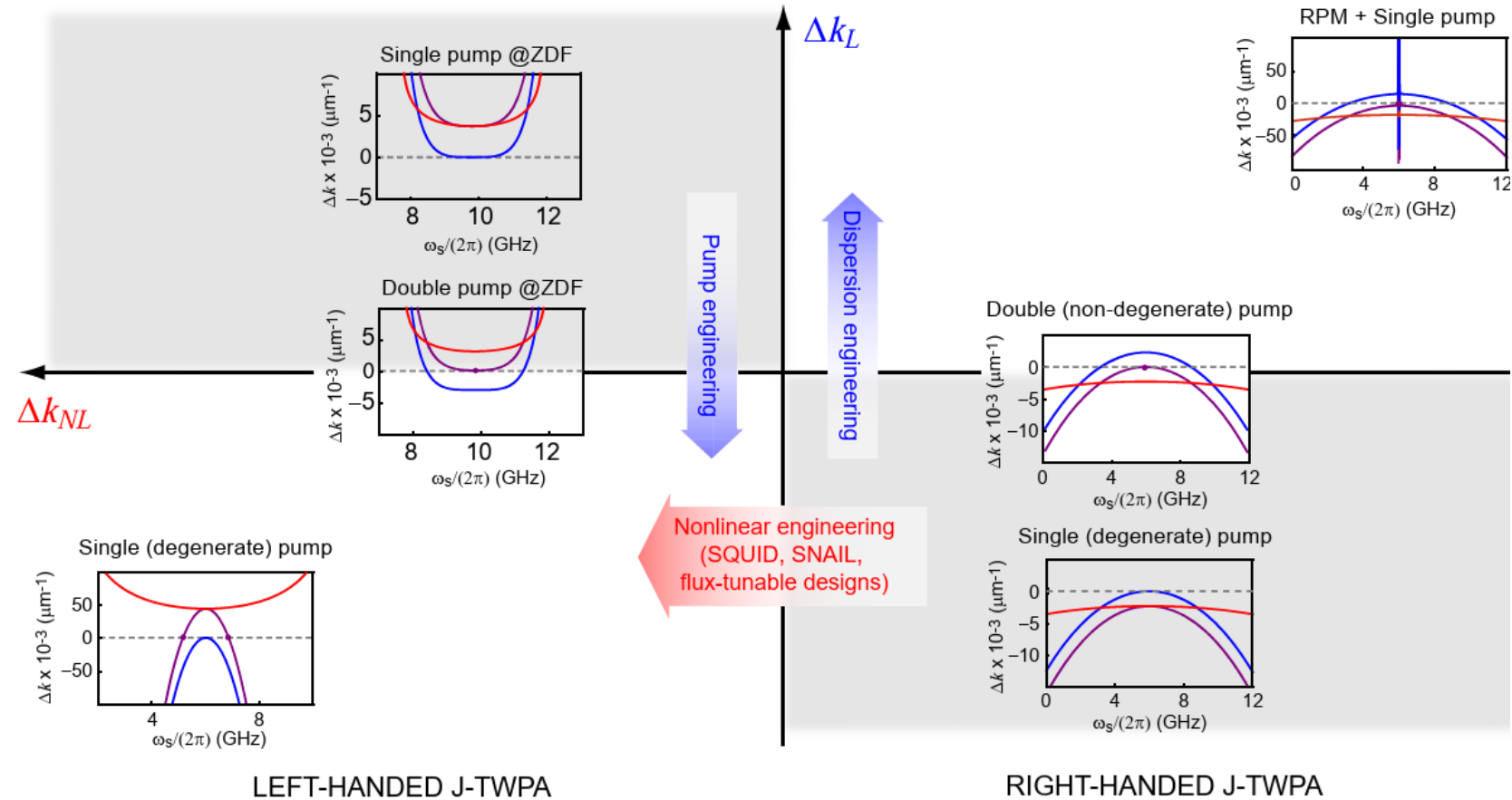


Kow, C., Podolskiy, V., & Kamal, A. (2024). Self phase-matched broadband amplification with a **left-handed Josephson transmission line**. <http://arxiv.org/abs/2201.04660>

Squeezing-Gain trade-off

Recall on the squeezing parameter

$$r_{jl}^{(k)} = g_{jl}^{(k)} \alpha_k e^{-i\phi_k}$$

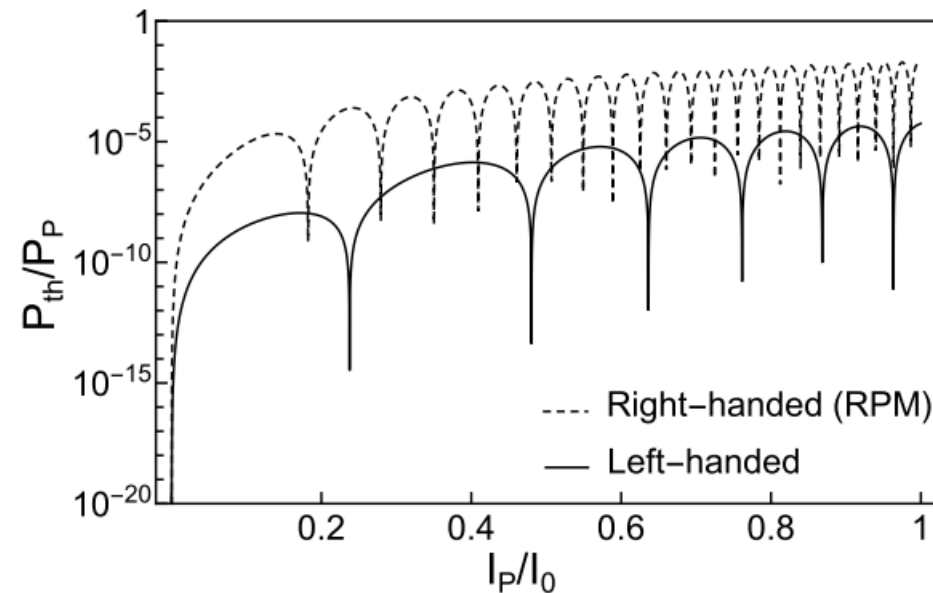


Kow, C., Podolskiy, V., & Kamal, A. (2024). Self phase-matched broadband amplification with a **left-handed Josephson transmission line**. <http://arxiv.org/abs/2201.04660>

Squeezing-Gain trade-off

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Pump harmonics (and relative sidebands) dilute entanglement over multiple modes and increase complexity

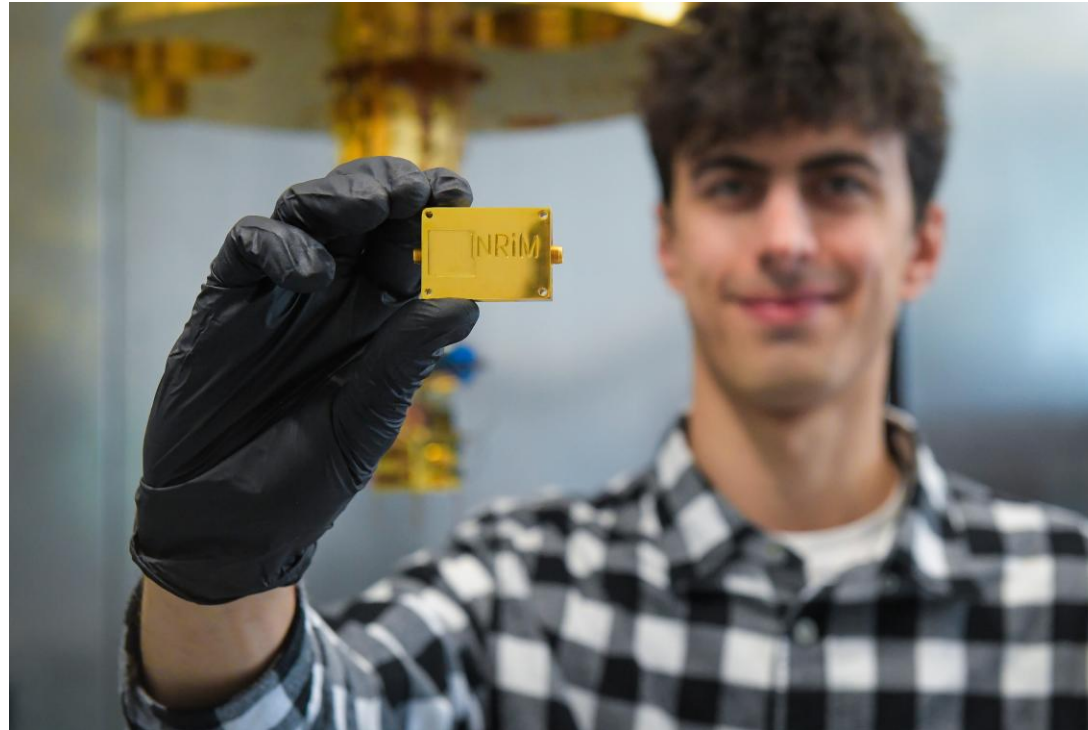


Kow, C., Podolskiy, V., & Kamal, A. (2024). Self phase-matched broadband amplification with a **left-handed Josephson transmission line**. <http://arxiv.org/abs/2201.04660>

Conclusions and Perspectives

- We demonstrate a reconfigurable, scalable platform for generating continuous-variable cluster states using programmable pump engineering in a superconducting **Josephson Travelling Wave Parametric Amplifier**.
- Our method **leverages the frequency domain** to create large entangled graphs with **minimal hardware overhead**.
- Graph structure and entanglement are **validated via nullifier** measurements and **covariance-based reconstruction**.
- This approach bridges optical CV techniques and microwave superconducting hardware, enabling hardware-efficient measurement-based quantum computation.
- Future integration with **LH-TWPAs and feedback** could enable quantum algorithm execution.





Thanks for your attention!

Acknowledgments



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