Quantum Solitons in SNAIL TWPAs

Miles Blencowe

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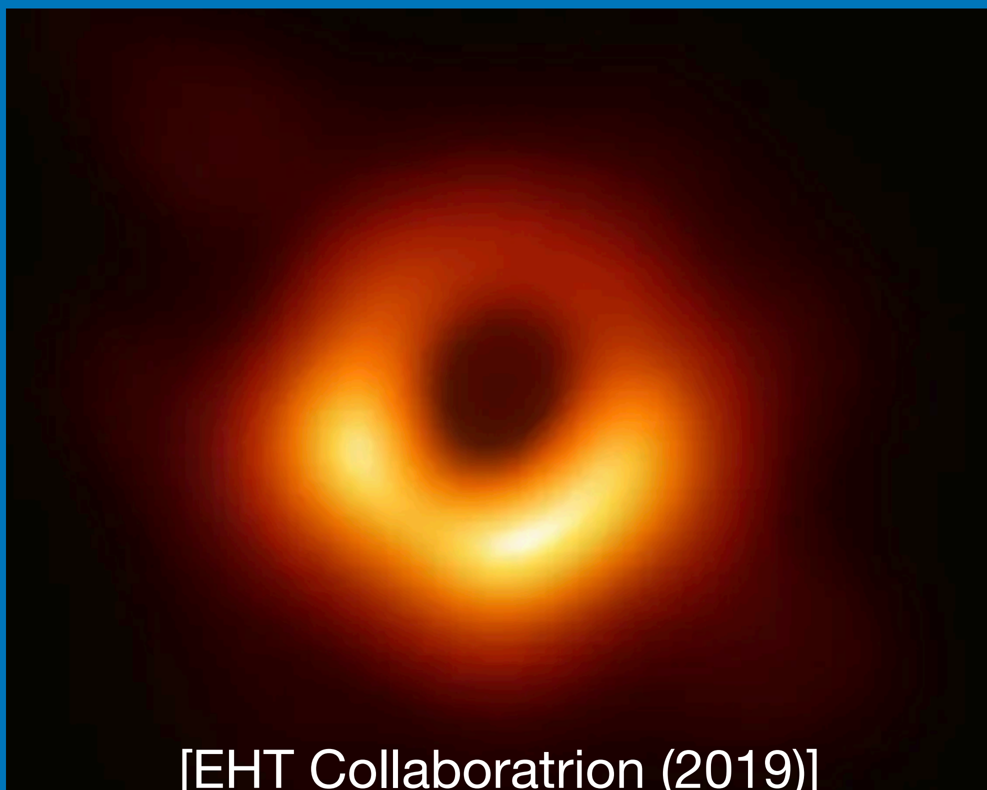
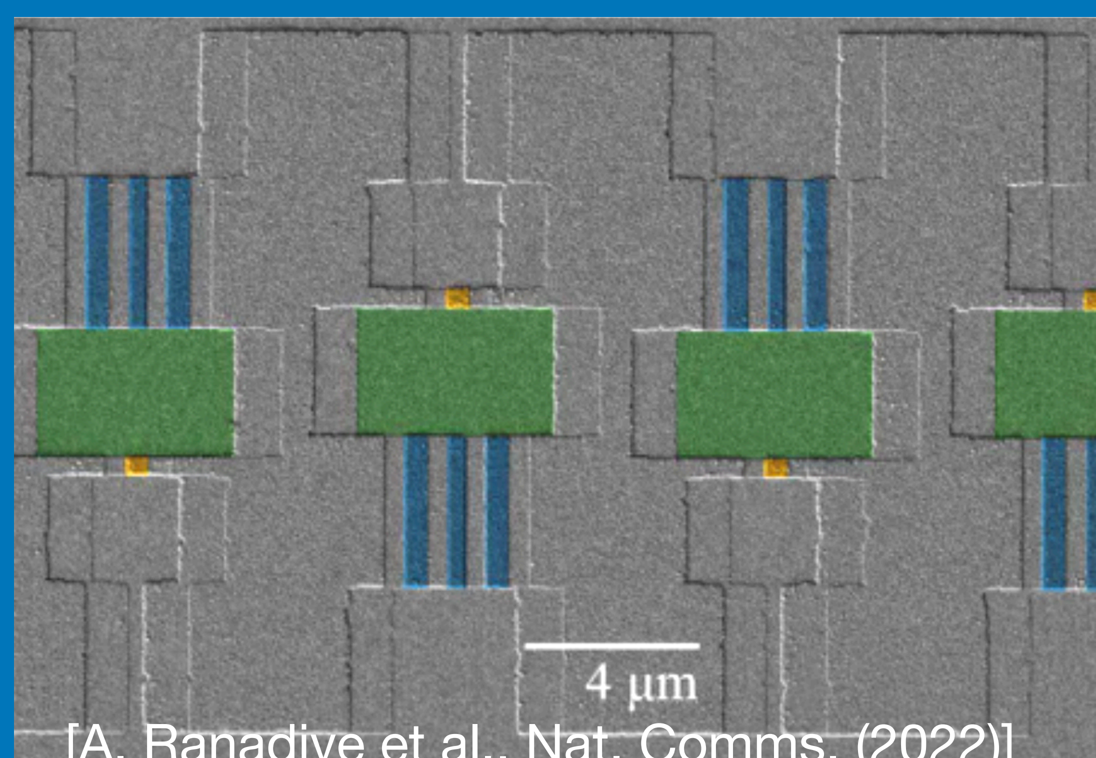


[EHT Collaboratrion (2019)]

[R. Griffith, YouTube (2024)]



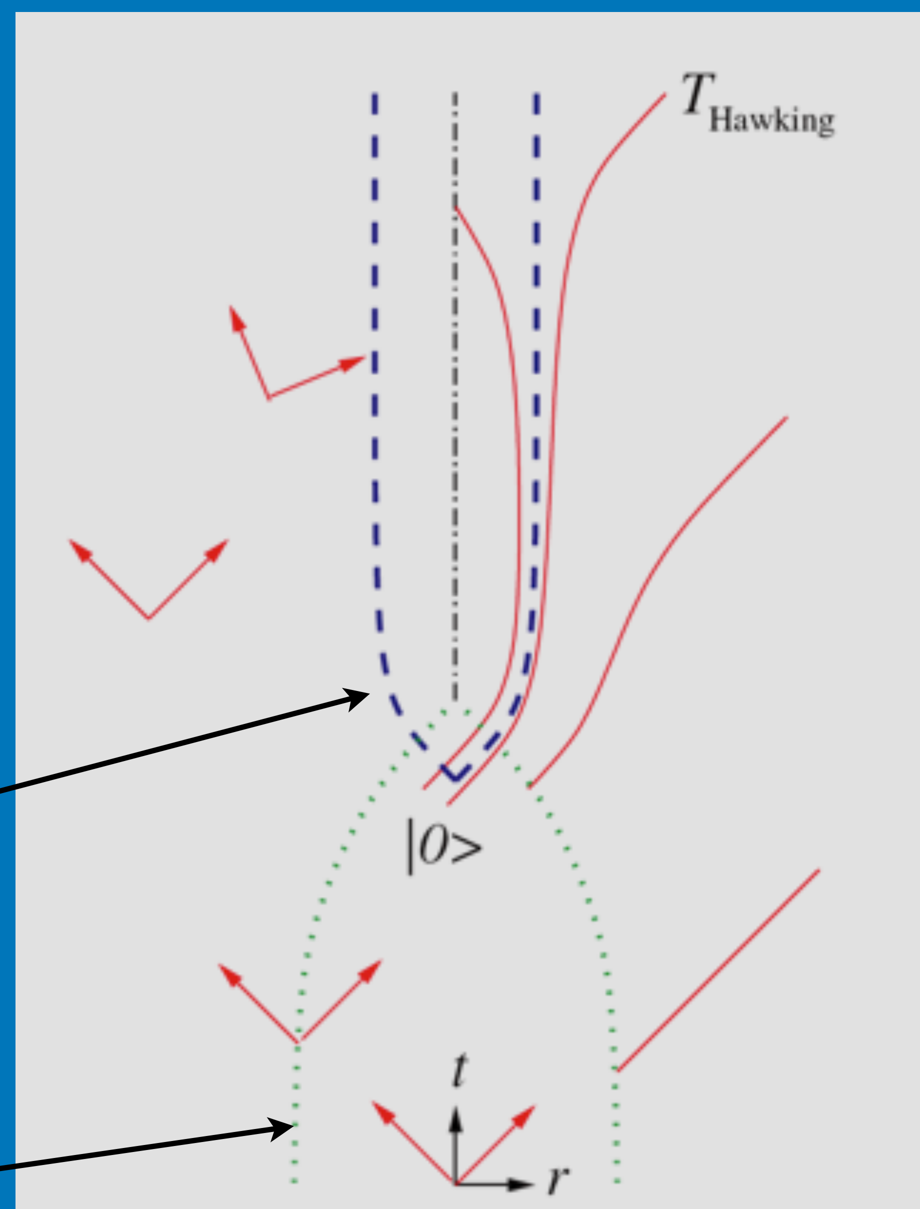
[A. Ranadive et al., Nat. Comms. (2022)]

Hawking (1974): Black holes radiate through photon pair creation in the vicinity of the event horizon.

To a distant observer, appears as a black body with temperature:

$$T_{\mathrm{Hawking}} = \frac{\hbar c^3}{8\pi k_B G M}$$

Derivation: quantize electromagnetic field in the space-time background of a collapsing spherical star. Assume that the electromagnetic field is initially in a vacuum state $|0\rangle$ (i.e., no photons present).
Find that photon pairs are *spontaneously produced from the vacuum* in the vicinity of the event horizon.



Event horizon

Surface of collapsing star

[R. Schützhold, Lect. Notes Phys. (2007])

But original derivation has problems: tracking the emitted Hawking radiation back in time, we find that the radiation wavelength gets infinitely blue-shifted as one approaches the horizon.

In particular, the radiation wavelength gets smaller than the Planck length, where our classical notion of space-time as a continuum is expected to break down: the “trans-Planckian problem”.

$$L_{\mathrm{Planck}} = \sqrt{\frac{\hbar G}{c^3}} \approx 10^{-35}\,\mathrm{meters}$$

Thus, either

- The Hawking thermal radiation prediction is invalid and we need a full quantum gravity description to figure out what happens.

Or

- The Hawking thermal radiation prediction is valid: the radiation is insensitive to the short distance physics.

Can we test Hawking's prediction for real black holes?

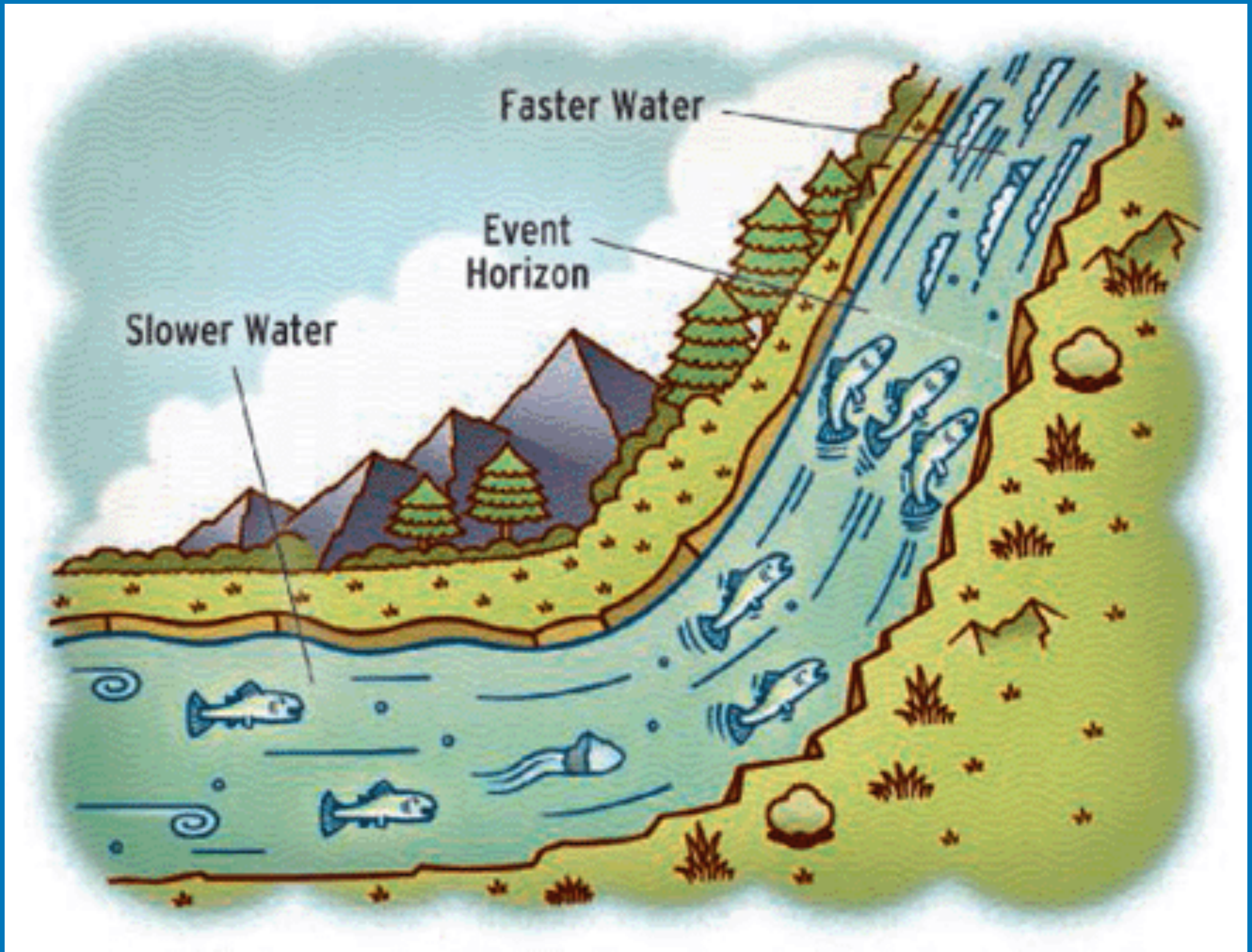
For, e.g., a solar mass:

$$T_{\rm Hawking} = \frac{\hbar c^3}{8\pi k_B G M_\odot} \sim 10^{-7}\,{\rm K}$$

Negligible compared to the cosmic microwave background.

Black Hole Analogues

Address validity of Hawking's derivation using low energy, laboratory analogues of a black hole event horizon where the short distance physics is known [Unruh, PRL (1981)].



[P. Hoey, Science (2008)]

An *acoustic (or sonic) horizon* forms at the boundary where the steady state fluid velocity exceeds the sound velocity.

•“Hawking” approach: quantize the sound wave fluctuations (phonons) about the background classical fluid flow in the vicinity of the sonic horizon: get *Hawking radiation*!

Fluid flow velocity

$$T_{\mathrm{Hawking}} = \frac{\hbar}{2\pi k_B} \left. \frac{dv}{dx} \right|_{\mathrm{horizon}}$$

“Trans-Planckian” problem: breakdown of continuum fluid approximation at intermolecular distance scales.

•"Quantum gravity" approach: solve (in principle) Schrödinger's equation for the known microscopic, molecular Hamiltonian of the fluid system. Does one obtain thermal Hawking radiation?

But, difficult to perform such a calculation. Alternatively, perform experiment...

Sound speed

Horizon size

Assume $dv/dx|_{\text{horizon}} \sim c/L_{\text{horizon}}$

Then $T_{\text{Hawking}} \sim 10^{-7}\,\mathrm{K} \left(\dfrac{c}{300\,\mathrm{m}\cdot\mathrm{s}^{-1}}\right)\left(\dfrac{1\,\mathrm{mm}}{L_{\text{horizon}}}\right)$

Nontrivial to maintain transonic flows at such small *L*'s without turbulence developing.

Alternative approach to a moving medium:

What matters are the effective properties of the medium; not necessary to physically move a medium to form an event horizon.

E.g., use *nonlinear optical fibre* where effective index of refraction depends on light intensity [Philbin et al., Science (2008)]:

$$n = n_0 + \delta n,\ \delta n \propto I(x, t)$$

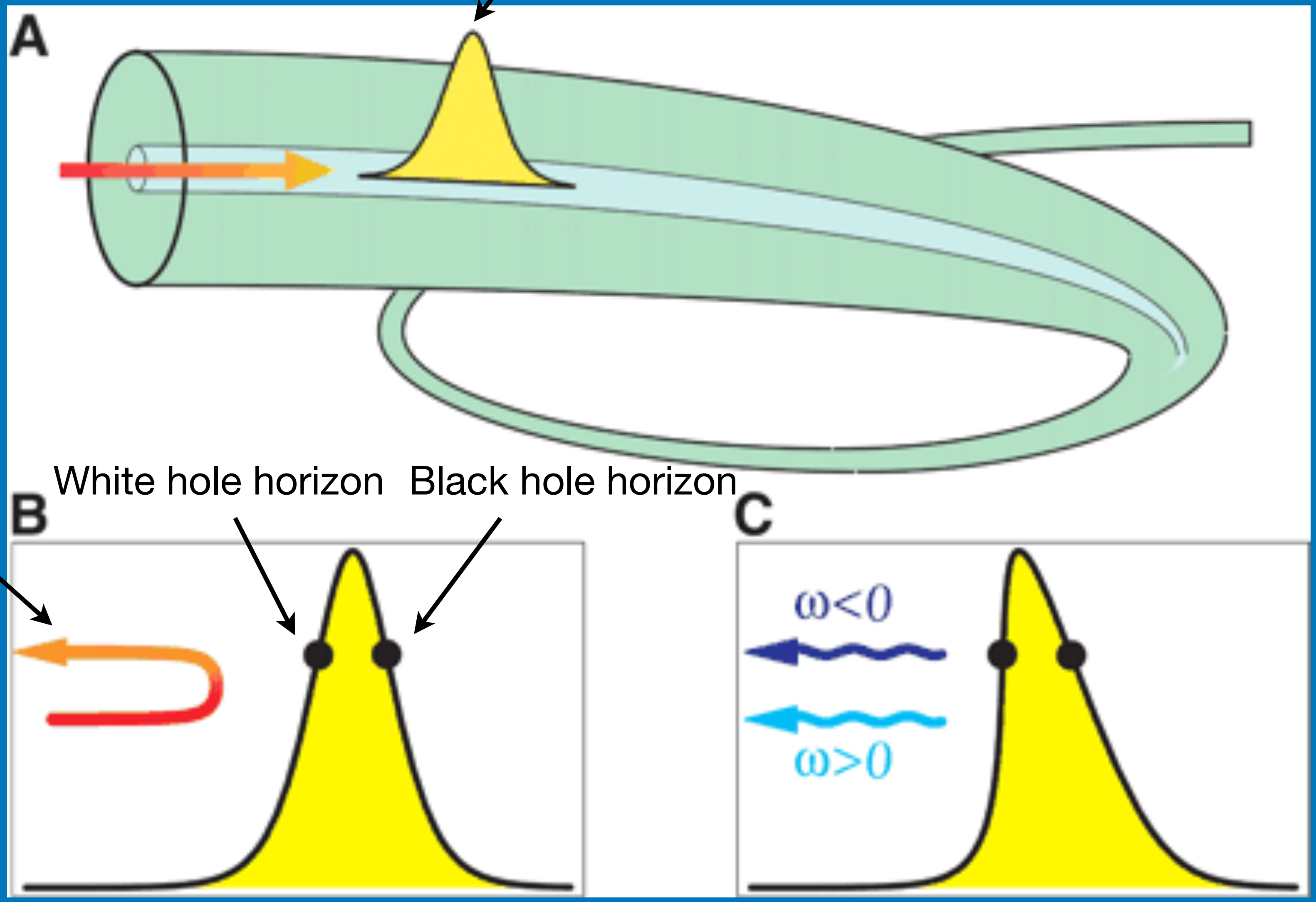
[Recall, $n = c/v_p$, *c* is the velocity of light in vacuum, v_p is the phase velocity of light in the medium]

“Fiber-Optical Analogue of the Event Horizon”

[Philbin et al., Science (2008)]:

Effective medium altering pump pulse (soliton)

“Trans-Planckian” physics: probe light gets blue shifted as approaches horizon. Leveling of dispersion curve results in probe light slowing down and falling behind the pulse



Rest frame of pulse

An alternative effective medium proposal using a *microwave* transmission line:

PRL **95,** 031301 (2005) PHYSICAL REVIEW LETTERS week ending 15 JULY 2005

Hawking Radiation in an Electromagnetic Waveguide?

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(Received 23 August 2004; published 15 July 2005)

It is demonstrated that the propagation of electromagnetic waves in an appropriately designed waveguide is (for large wavelengths) analogous to that within a curved space-time—such as around a black hole. As electromagnetic radiation (e.g., microwaves) can be controlled, amplified, and detected (with present-day technology) much easier than sound, for example, we propose a setup for the experimental verification of the Hawking effect. Apart from experimentally testing this striking prediction, this would facilitate the investigation of the trans-Planckian problem.

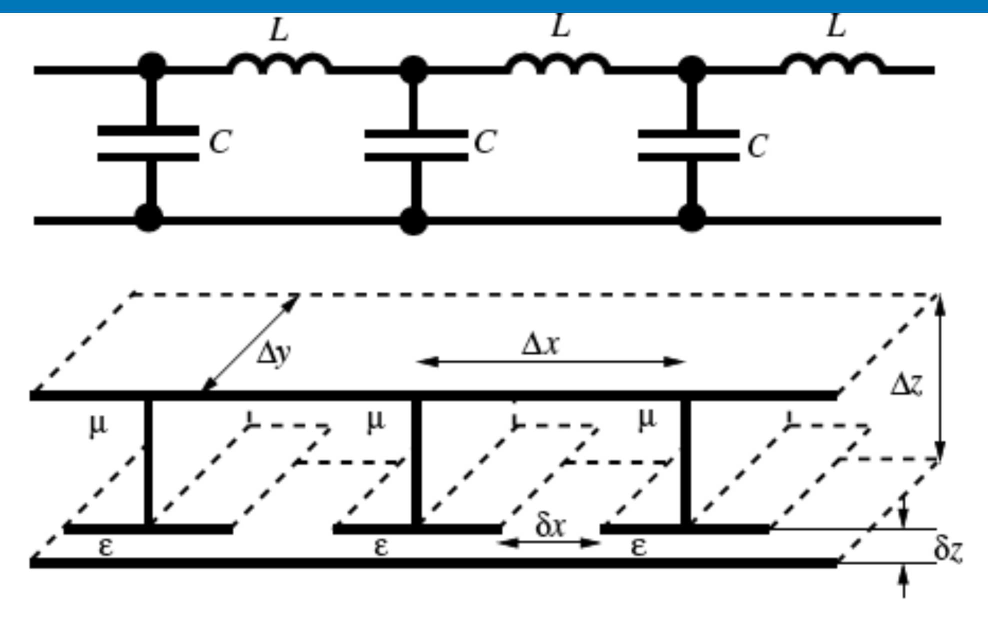

FIG. 1. Circuit diagram and sketch of the waveguide.

Dielectric between capacitor plates controlled by an external laser beam: sweep along the cavity length

$$T_{\mathrm{Hawking}} \sim 10 - 100\,\mathrm{mK}$$

But, have the problem of heating due to laser

An alternative effective medium proposal using a *superconducting*, microwave *nonlinear* transmission line:

PRL **103,** 087004 (2009) PHYSICAL REVIEW LETTERS week ending 21 AUGUST 2009

Analogue Hawking Radiation in a dc-SQUID Array Transmission Line

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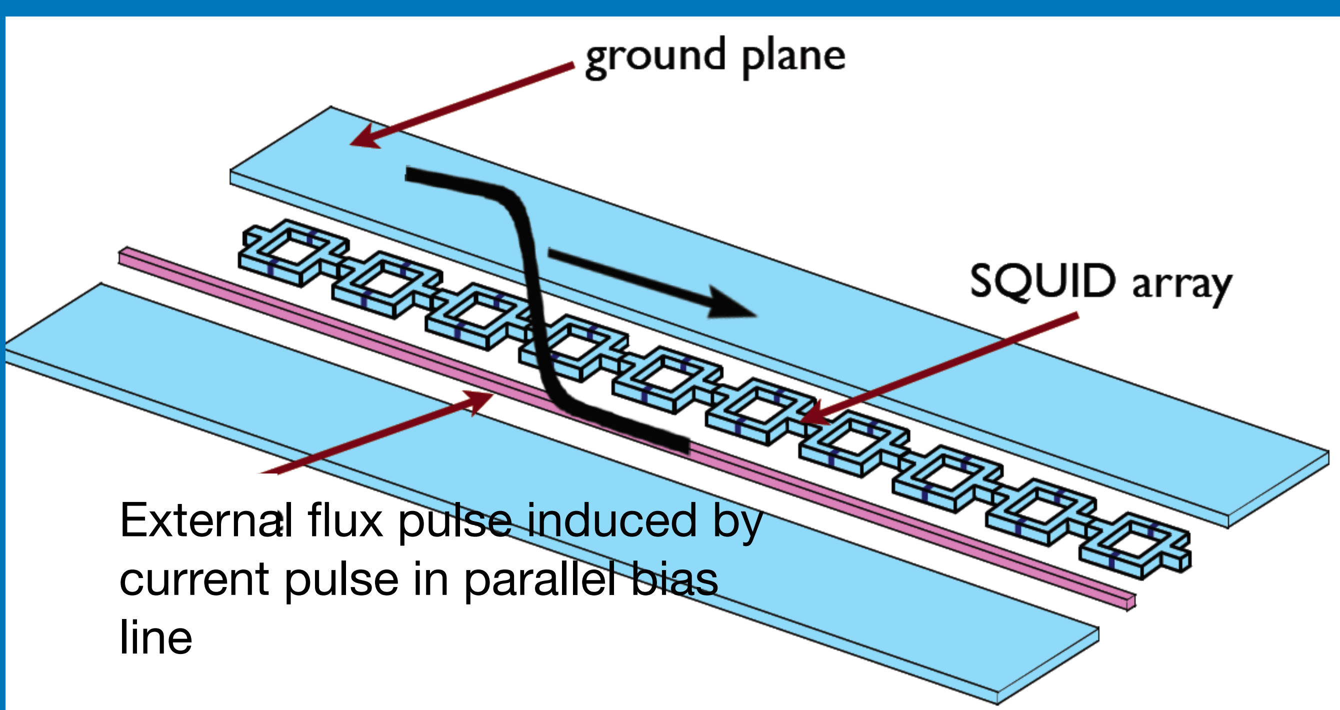
(Received 23 April 2009; published 20 August 2009)

We propose the use of a superconducting transmission line formed from an array of direct-current superconducting quantum interference devices for investigating analogue Hawking radiation. Biasing the array with a space-time varying flux modifies the propagation velocity of the transmission line, leading to an effective metric with a horizon. Being a fundamentally quantum mechanical device, this setup allows for investigations of quantum effects such as backreaction and analogue space-time fluctuations on the Hawking process.



$$T_{\mathrm{Hawking}} \sim 10 - 100\,\mathrm{mK}$$

Contents lists available at SciVerse ScienceDirect

Computer Physics Communications

www.elsevier.com/locate/cpc

QuTiP: An open-source Python framework for the dynamics of open quantum systems ☆

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Paul Nation (Principle Research Scientist, IBM Quantum)

On Mar 27, 2020, at 10:56 AM, William D Oliver <wi18222@mit.edu> wrote:

I would suggest increasing the fabrication cost. A standard run for us is 150K (loaded). Or, we should add a student at MIT.

Is all experimental measurement being done at Dartmouth?

Will

Instead, use an *existing* superconducting, microwave nonlinear transmission line device

PHYSICAL REVIEW RESEARCH **5**, L022055 (2023)

Letter

Analog black-white hole solitons in traveling wave parametric amplifiers with superconducting nonlinear asymmetric inductive elements

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Haruna Katayama (Assistant Professor, Hiroshima University)

A traveling wave parametric amplifier (TWPA) using superconducting nonlinear asymmetric elements (SNAILs):

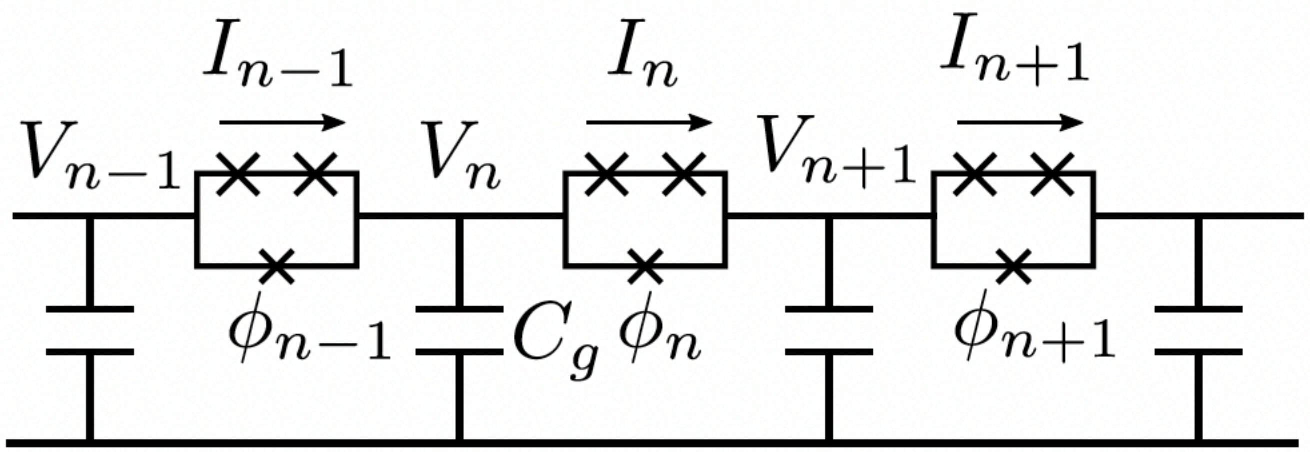
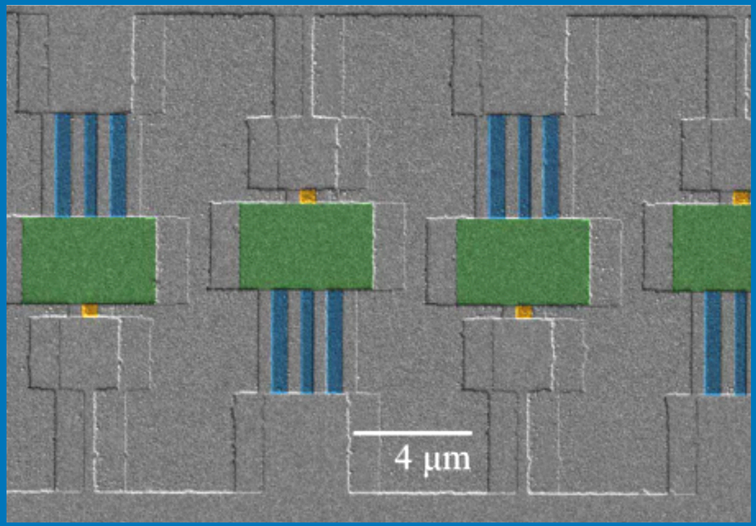
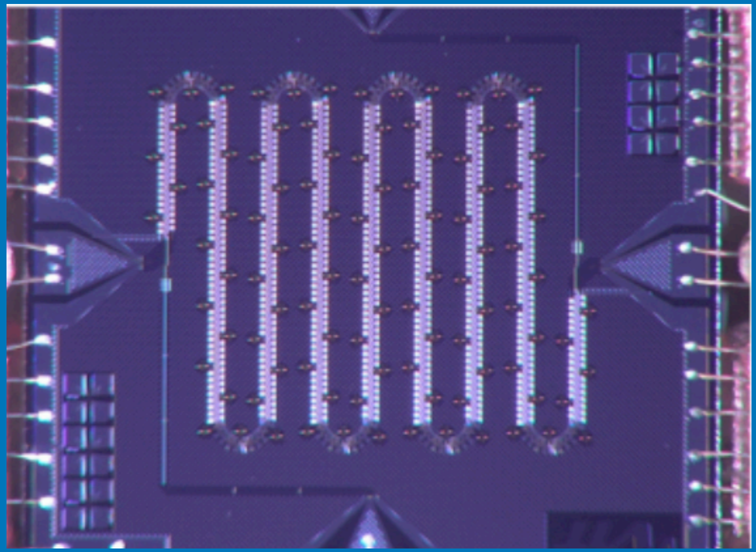


FIG. 1. Schematic diagram of the traveling wave parametric amplifier (TWPA) transmission line with superconducting nonlinear asymmetric inductive element (SNAIL) unit cells in series, alternating with shunt capacitors C_g in parallel (all assumed to be identical). The quantities I_n, V_n, and ϕ_n represent the current, voltage, and phase difference of the nth SNAIL, respectively. Each SNAIL has a small JJ with Josephson energy αE_J ($\alpha < 1$) in one branch and two larger JJs (with Josephson energy E_J) in the other, parallel branch, forming a loop that is threaded by an external applied magnetic flux $\Phi_{\rm ext}$.





Used, e.g., as:

- Broadband microwave amplifiers close to the quantum limit
- Generators of broadband squeezed and entangled microwave states

[M. Esposito et al., PRL (2022)]

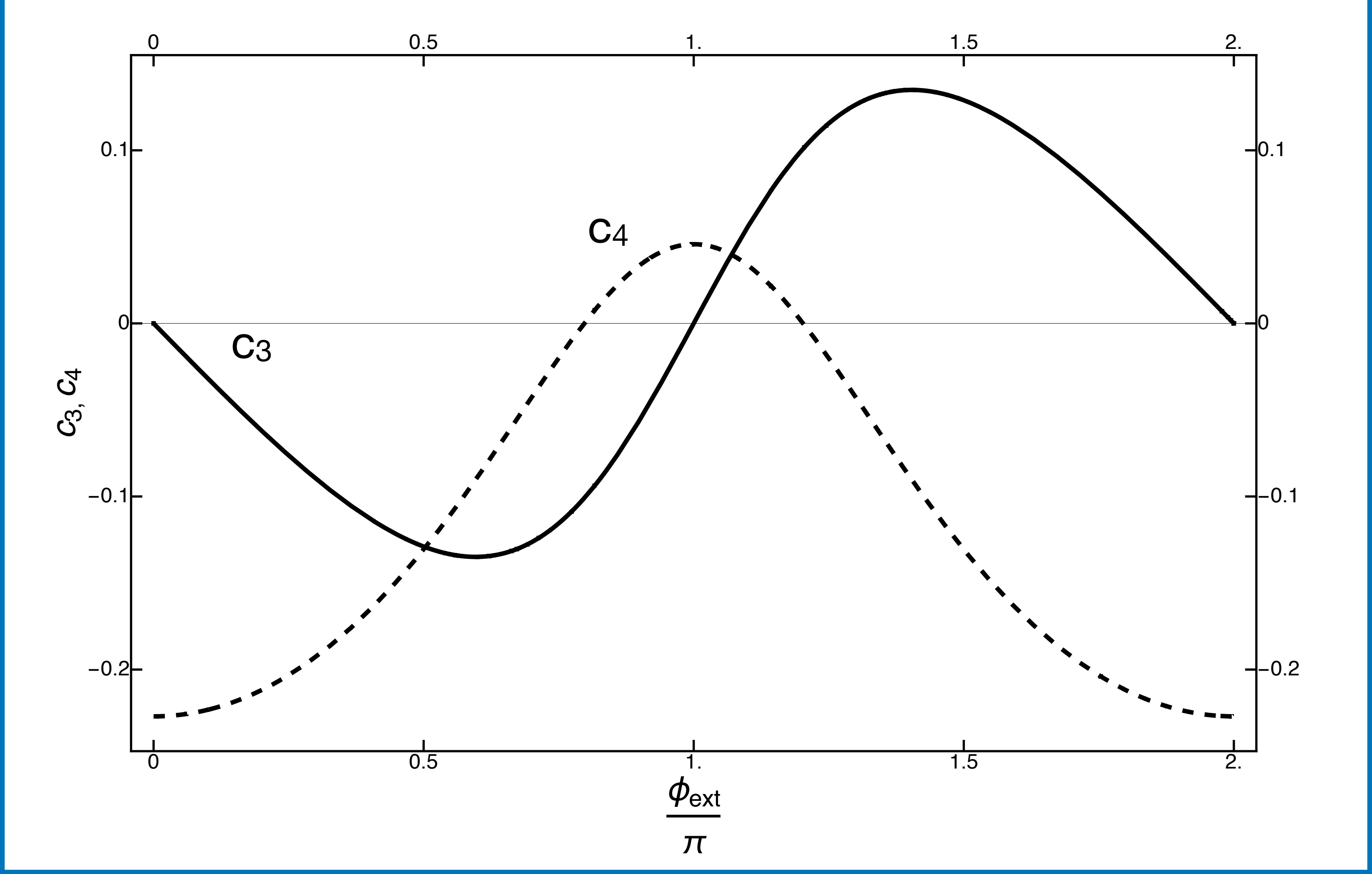
[M. Perelshtein et al., PRApplied (2022)]

SNAIL-TWPA circuit equations in the continuum approximation:

$$\frac{\partial^2 \phi}{\partial t^2} - \underbrace{r a^2 \frac{\partial^4 \phi}{\partial x^2 \partial t^2}}_{\text{Dispersion}} - v_0^2 \frac{\partial^2}{\partial x^2}\left(\phi + \underbrace{\frac{c_3}{2!}\phi^2 + \frac{c_4}{3!}\phi^3}_{\text{Nonlinearity}}\right) = 0$$

SNAIL unit cell size

$$r = C_J/C_g \qquad v_0 = a\omega_0 \qquad \omega_0 = 1/\sqrt{C_g L_0}$$



Can selectively tune between 3-wave and 4-wave mixing by varying the external magnetic flux bias

$$\phi_{\text{ext}} = 2\pi\Phi_{\text{ext}}/\Phi_0$$

$$\Phi_0 = h/(2e)$$

[Parameters from Esposito et al., PRL (2022)]

Presence of wave *dispersion* and *nonlinearity* can give rise to the existence of stable, localized propagating wave solutions: *solitons*. [John Scott Russell (1834)]

Idea: Consider the possible use of soliton solutions in SNAIL TWPAs for generating analogue event horizons (in the soliton's co-moving frame).

$$\frac{\partial^2 \phi}{\partial t^2} - \underbrace{r a^2 \frac{\partial^4 \phi}{\partial x^2 \partial t^2}}_{\text{Dispersion}} - v_0^2 \frac{\partial^2}{\partial x^2}\left(\phi + \underbrace{\frac{c_3}{2!}\phi^2 + \frac{c_4}{3!}\phi^3}_{\text{Nonlinearity}}\right) = 0$$

$$\phi = \bar{\phi} + \delta\phi$$

Strong classical background field pulse

Weak “probe” signal

Tune external magnetic flux such that $c_4 = 0 \;\; (c_3 \neq 0)$

Apply reductive perturbation method [H. Leblond, J.Phys.B (2018)]:

$$\bar{\phi}(\xi, \tau) = \sum_{n=1}^{\infty} \varepsilon^n \bar{\phi}^{(n)}(\xi, \tau) \qquad \varepsilon = (ka)^2$$

“Stretched” coordinates:

$$\xi = \varepsilon^{1/2}(x - v_0 t)$$

$$\tau = \varepsilon^{3/2} t$$

Obtain the Korteweg-de Vries (KdV) equation to lowest order ε^3:

$$\frac{\partial \bar{\phi}^{(1)}}{\partial \tau} + \frac{c_3 v_0}{2} \bar{\phi}^{(1)} \frac{\partial \bar{\phi}^{(1)}}{\partial \xi} + \frac{r}{2} a^2 v_0 \frac{\partial^3 \bar{\phi}^{(1)}}{\partial \xi^3} = 0$$

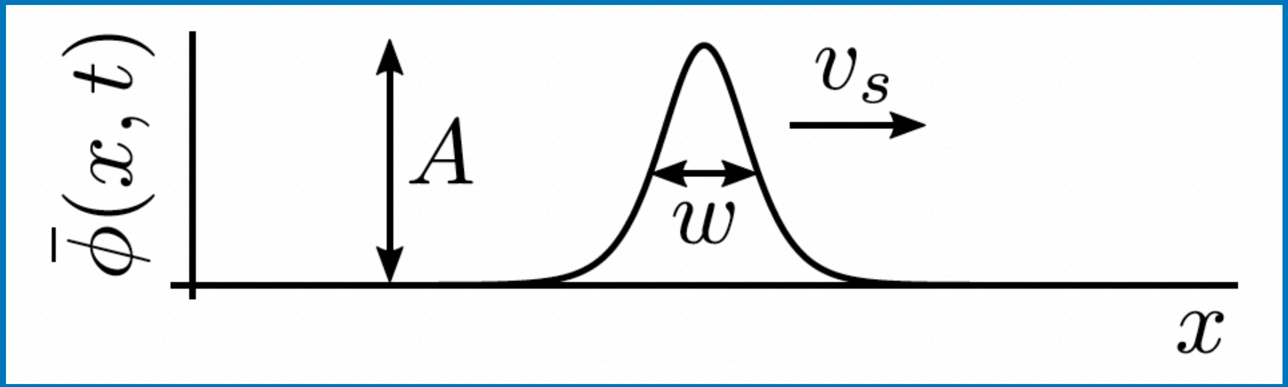
Known to have soliton solutions [Kivshar and Malomed, RMP (1989)]:

Single soliton solution:

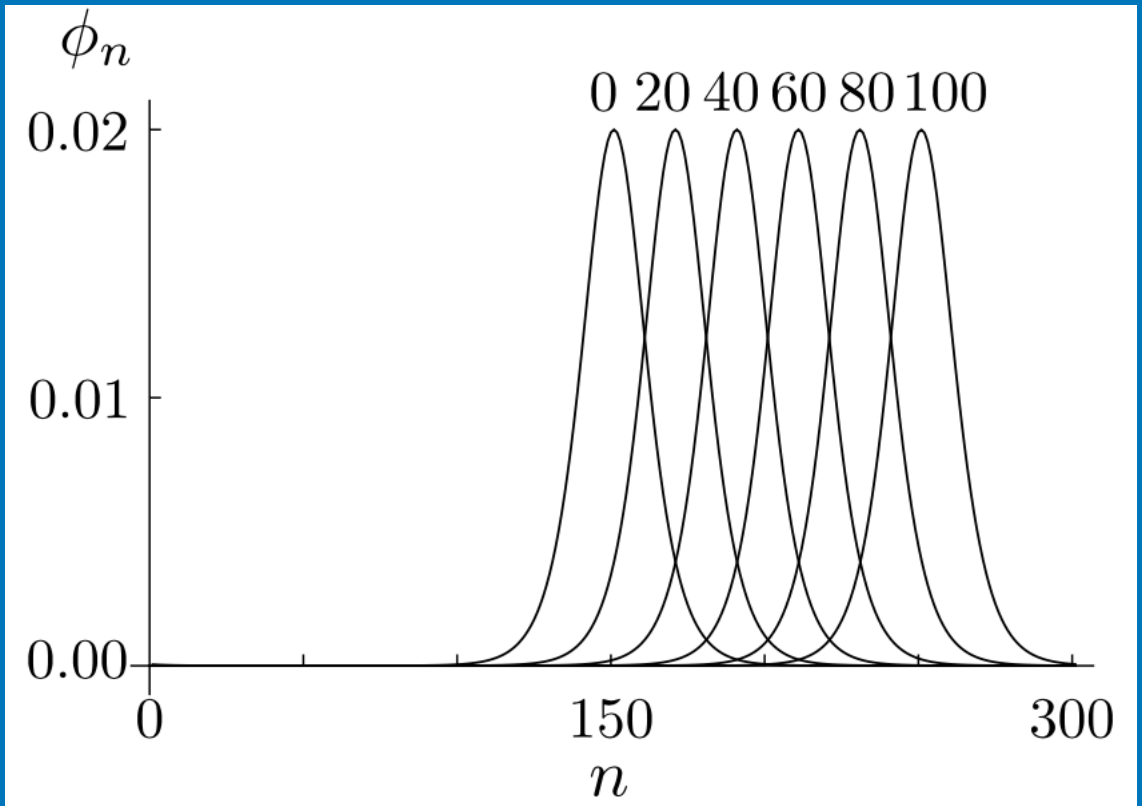
$$\bar{\phi}(x,t) = A \operatorname{sech}^2 \left[\frac{1}{a} \sqrt{\frac{c_3 A}{12 r}} \left(x - v_s t\right) \right] \qquad c_3 > 0 \, (A > 0)$$

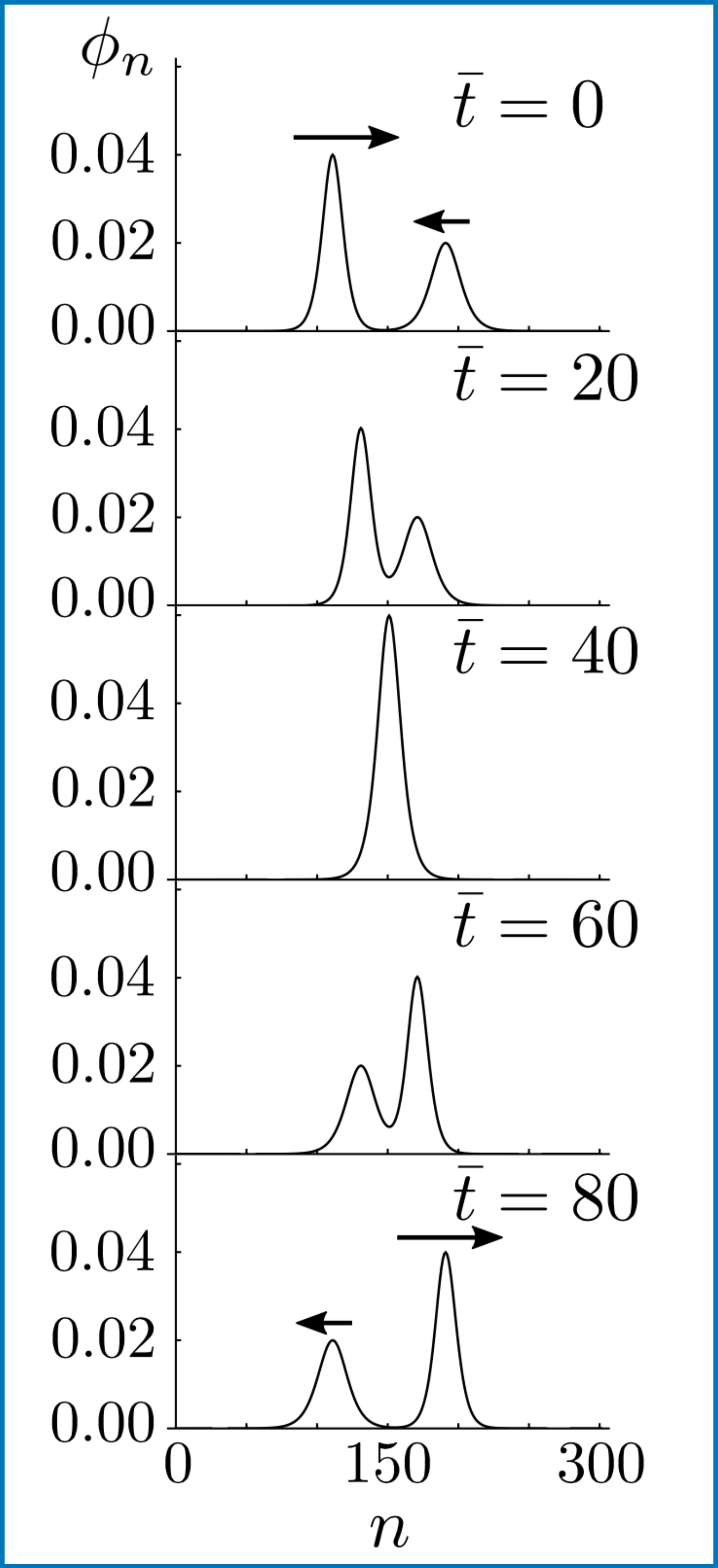
Velocity: $v_s = v_0 (1 + c_3 A/6)$

Half-width: $w \sim 2a \sqrt{12r / (c_3 A)}$



Verified by numerical solutions of original discrete space equations, where soliton width $w \gg a$





Wave equation of weak probe field $\delta\varphi$, expressed in comoving frame of background soliton pulse $\eta = x - v_s t$:

Phase coordinate across SNAIL unit cell

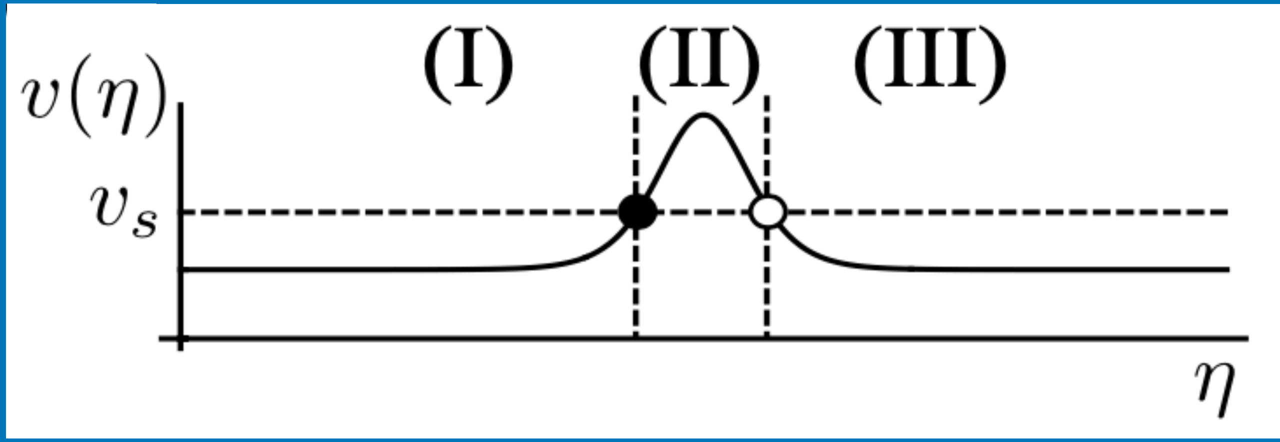
$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\delta\varphi\right) = 0 \qquad \delta\phi \approx -a\,\partial\delta\varphi/\partial x$$

Phase coordinate across C_J

$$g^{\mu\nu} = \frac{1}{v(\eta)}\begin{pmatrix} -1 & v_s & 0 & 0 \\ v_s & v^2(\eta) - v_s^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

event horizons where $v^2(\eta) = v_s^2$

Have analogue black-white hole pair



[H. Katayama et al., PRR (2023)]

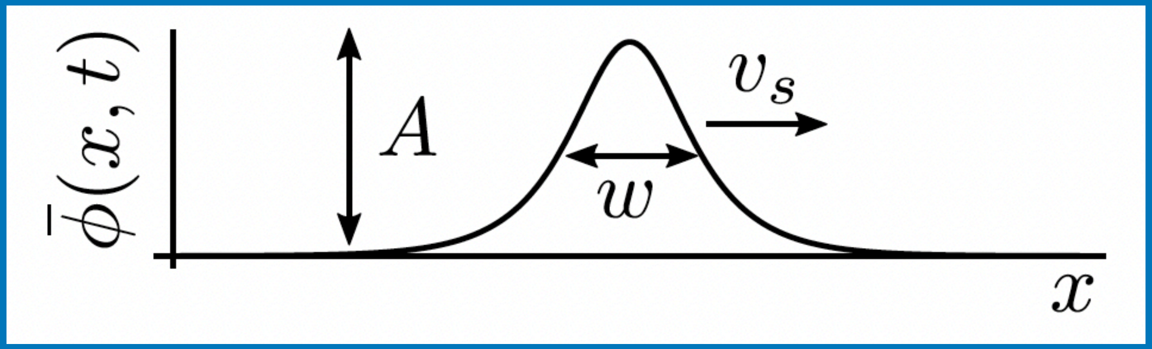
Other KdV-type soliton solutions for $c_4 \neq 0\,(c_3 = 0)$:

$$\frac{\partial \bar{\phi}^{(1)}}{\partial \tau} + \frac{c_4 v_0}{4}\left(\bar{\phi}^{(1)}\right)^2 \frac{\partial \bar{\phi}^{(1)}}{\partial \xi} + \frac{r}{2} a^2 v_0 \frac{\partial^3 \bar{\phi}^{(1)}}{\partial \xi^3} = 0$$

"Modified" KdV equation

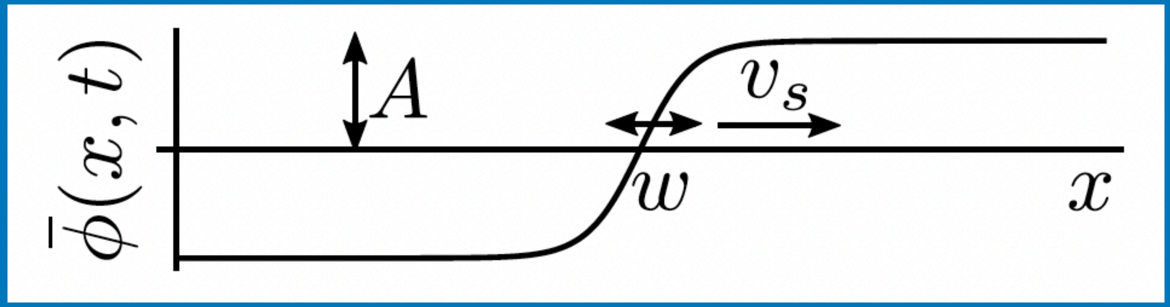
$c_4 > 0:$

$$\bar{\phi}(x,t) = A\,\mathrm{sech}\left[\frac{A}{a}\sqrt{\frac{c_4}{12r}}\,(x - v_s t)\right]$$



$c_4 < 0:$

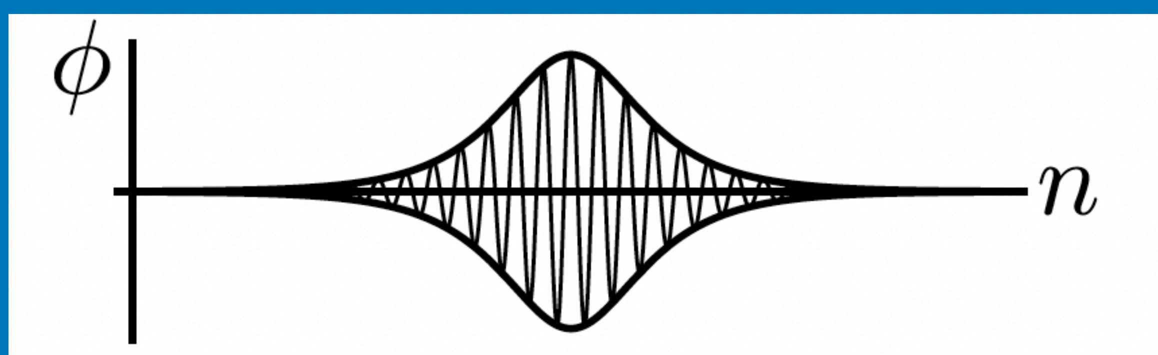
$$\bar{\phi}(x,t) = A\tanh\left[\frac{A}{a}\sqrt{\frac{|c_4|}{12r}}\,(x - v_s t)\right]$$



If instead, introduce pump *carrier* wave $\omega_p \sim$ few GHz with suitable *modulation* pulse shape envelope, can have yet another class of solitons ("Nonlinear Schrödinger Eq."):

$c_4 > 0\,(c_3 = 0):$ analogous to nonlinear optical fiber solitons

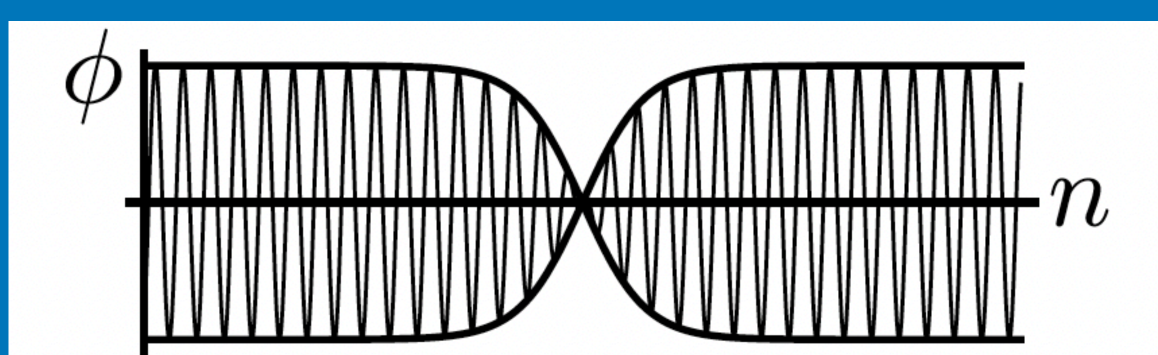
$$\phi_n(t) = A\,\mathrm{sech}\left[\frac{\sqrt{c_4}A}{4}\left(\frac{v_g}{\omega_p}\left|\frac{dv_g}{d\omega_p}\right|\right)^{-1/2}(na - v_g t)\right]\cos\left[\left(k_p + \frac{c_4 A^2 \omega_p}{32 v_g}\right)na - \omega_p t\right]$$



Fundamental "Bright" soliton

$c_4 < 0\,(c_3 = 0):$

$$\phi_n(t) = A\,\tanh\left[\frac{\sqrt{|c_4|}A}{4}\left(\frac{v_g}{\omega_p}\left|\frac{dv_g}{d\omega_p}\right|\right)^{-1/2}(na - v_g t)\right]\cos\left[\left(k_p + \frac{|c_4| A^2 \omega_p}{16 v_g}\right)na - \omega_p t\right]$$



"Black" soliton

Unique advantage of SNAIL TWPAs: can tune nonlinearity via magnetic flux) and select frequency range such as to generate different types of soliton using the same device [e.g., KdV, modified KdV$^{\pm}$, NLSE bright (dark)]

Which soliton type is most suitable for observing Hawking radiation?

Note: NLSE solitons expected to parametrically generate squeezed/entangled microwave photons

[M. Esposito et al., PRL (2022)]

[M. Perelshtein et al., PRApplied (2022)]

In progress…

Thank you!