Soft pions and condensate growth in the chiral phase transition

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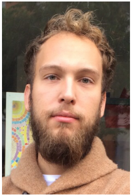
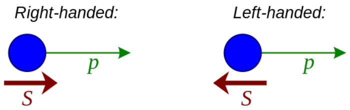
- E. Grossi, A. Soloviev, DT, F. Yan: PRD, arXiv:2005.02885, arXiv:2101.10847
- A. Florio, E. Grossi, DT, PRD, arXiv:2111.03640, arXiv:2306.06887
- A. Florio, E. Grossi, A. Mazeliauskas, A. Soloviev, DT, 2504.03514, 2504.03516






QCD and Chiral Symmetry:

$SU_L(2) \times SU_R(2) \simeq O(4)$



u_R, d_R $\qquad$ u_L, d_L

$$\begin{pmatrix} u'_L \\ d'_L \end{pmatrix} = U_L \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

and ditto for right

QCD is (almost) symmetric between, left and right, and up and down:

$$\mathcal{L}_{QCD} = \sum_{q=u,d} \bar{q}_L(i\not{D})q_L + \bar{q}_R(i\not{D})q_R \; - \underbrace{m_q\left(\bar{q}_L q_R + \bar{q}_R q_L\right)}_{\text{small}}$$

Then one would expect four approx. conservation laws, u_L, d_L, u_R, d_R:

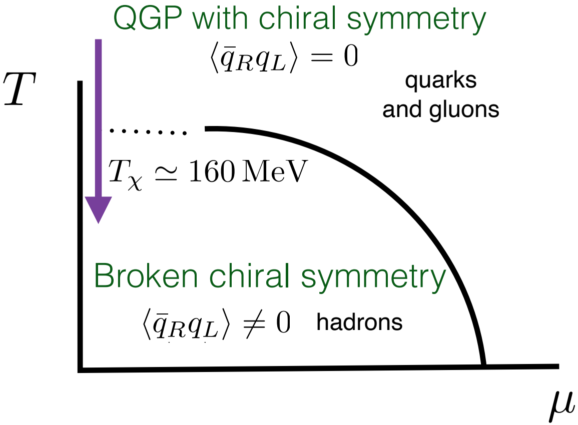
$$\vec{n}_V \sim (u_L + u_R) - (d_L + d_R) \qquad \text{Isovector charge}$$

$$\vec{n}_A \sim (u_L - u_R) - (d_L - d_R) \qquad \text{Isoaxial vect. charge}$$

Chiral symmetry breaking and heavy ion collisions

Pisarski, Wilczek

The system has thermalized and QGP expands *slowly* through the chiral transition ...



The quark mass smooths the 2nd order transition to a crossover, like a magnetic field in the Ising model

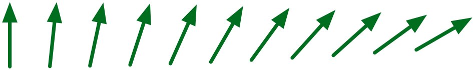
Chiral symmetry breaking plays no role in current hydro model ...

What is a pion?

Our cold world: T< Tcritical

$$\langle \bar{q}_R q_L \rangle = \bar{\sigma}\, \mathbb{I}_{2\times 2}$$

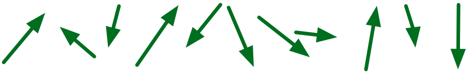
Order parameter $\langle \bar{q}_R q_L \rangle$ is like the magnetization. $q = u, d$



$$\bar{q}_R q_L = \bar{\sigma}\, e^{i\vec{\tau}\cdot\vec{\varphi}(x)}$$

The slow modulation of the $SU_A(2)$ phase of $\bar{q}_R q_L$ is a pion, $\vec{\pi} = \bar{\sigma}\vec{\varphi}$

The hot world: T> Tcritical



State is disordered: pion propagation is frustrated

Want to predict pions during a quench, *close to the phase transition.*

Thermodynamics and the Chiral $O(4)$ Transition

- The $O(4)$ order parameter fluctuates in amplitude and phase:

$$\phi_a = (\phi_0, \phi_1, \phi_2, \phi_3) = (\sigma, \vec{\pi})$$

The quark condensate scales as

$$\bar{q}_R q_L \sim \sigma e^{i\vec{\tau}\cdot\vec{\varphi}} \simeq \sigma + i\vec{\tau}\cdot\vec{\pi}$$

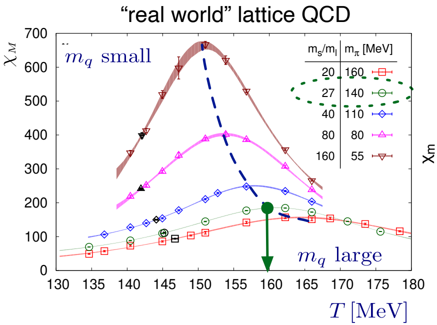
- The Landau Ginzburg function for the $O(4)$ order parameter is: $\phi^2 \equiv \phi_a \phi_a$

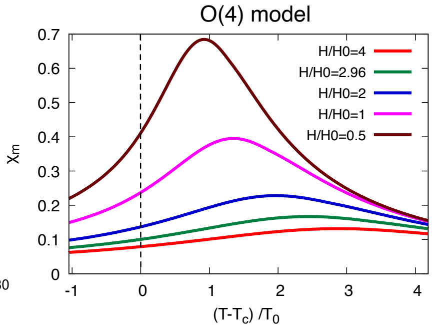
$$\mathcal{H} = \int d^3x \, \frac{1}{2}\nabla\phi_a \cdot \nabla\phi_a + \frac{1}{2} m_0^2(T)\,\phi^2 + \frac{\lambda}{4}\,\phi^4 - \underbrace{H}_{\propto\, m_q}\,\sigma$$

- The model has a critical mass $m_c^2(T) < 0$, $m_0 - m_c \equiv c(T - T_c)$

Tune the bare mass $m_0(T)$, so the renormalized mass $m_\sigma \equiv \xi^{-1}$ is zero.

magnetic susceptibility $\chi_M \equiv \langle \sigma^2 \rangle - \langle \sigma \rangle^2$





Hydrodynamics of the $O(4)$ transition:

Rajagopal and Wilczek '92, Son '99, Son and Stephanov '01, and finally us, arxiv:2101.10847.

1. The order parameter

$$\phi_a = (\sigma, \vec{\pi})$$

2. The approximately conserved charges quantities:

$$\vec{n}_V = \underbrace{\bar{\psi}\gamma^0\vec{\tau}\psi}_{\text{isovect chrg}} \qquad \textit{and} \quad \vec{n}_A = \underbrace{\bar{\psi}\gamma^0\gamma^5\vec{\tau}\psi}_{\text{isoaxial-vect chrg}}$$

which are combined into an anti-symmetric $O(4)$ tensor n_{ab}

$$n_{ab} = (\vec{n}_A, \vec{n}_V)$$

The charge n_{ab} generates $O(4)$ rotations, $\phi \to \phi_c + \frac{i}{\hbar}\theta_{ab}[n_{ab}, \phi_c]$, implying a Poisson bracket between the hydrodynamic fields:

$$\{n_{ab}(\boldsymbol{x}), \phi_c(\boldsymbol{y})\} = \epsilon_{abcd}\,\phi_d(\boldsymbol{x})\,\delta(\boldsymbol{x}-\boldsymbol{y})$$

The Landau-Ginzburg Hamiltonian for the $O(4)$ transition:

The Hamiltonian is tuned to the crit. point with $m_0^2(T) < 0$ and $H \propto m_q$:

$$\mathcal{H} = \int d^3x \, \frac{1}{2}\nabla\phi_a \cdot \nabla\phi_a + \frac{1}{2}m_0^2(T)\phi^2 + \frac{\lambda}{4}\phi^4 - H\sigma + \frac{n_{ab}^2}{4\chi_I}$$

and gives the equilibrium distribution with the correct critical EOS:

$$Z = \int D\phi \, Dn \, e^{-\mathcal{H}[\phi,n]/T_c}$$

The hydro equations (Model G) take the form

$$\frac{\partial \phi}{\partial t} + \{\phi, \mathcal{H}\} = 0 + \text{visc. corrections} + \text{noise}$$

$$\frac{\partial n_{ab}}{\partial t} + \{n_{ab}, \mathcal{H}\} = 0 + \text{visc. corrections} + \text{noise}$$

The equations and the simulations:

see also Schlichting, Smekal

We have a charge diffusion equation coupled to order parameter:

$$\partial_t n_{ab} + \underbrace{\nabla \cdot (\nabla \phi_{[a}\phi_{b]})}_{\text{poisson bracket}} = \underbrace{D_0 \nabla^2 n_{ab}}_{\text{diffusion}} + \underbrace{\nabla \cdot \xi_{ab}}_{\text{noise}}$$

and a rotation of the order parameter induced by the charge:

$$\partial_t \phi_a + \underbrace{\frac{n_{ab}}{\chi_I}\phi_b}_{\text{poisson bracket}} = \underbrace{\Gamma_0 \frac{\delta \mathcal{H}}{\delta \phi_a}}_{\text{dissipation}} + \underbrace{\xi_a}_{\text{noise}}$$

Captures two hydrodynamic limits:

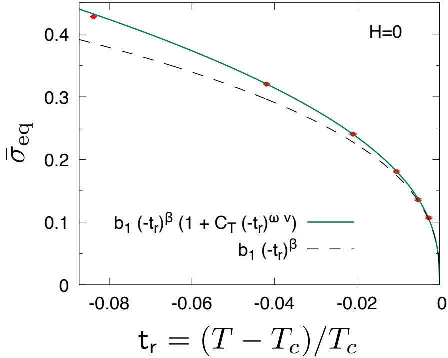
1. Simple diffusion and relaxation above T_c
2. Non-abelian (spin on three sphere) superfluid dynamics below T_c

Equilibrium: statics for $T < T_c$:

$$M_a(t) \equiv \frac{1}{V} \sum_{\boldsymbol{x}} \phi_a(t, \boldsymbol{x}) \equiv \text{order parameter}$$

The condensate:

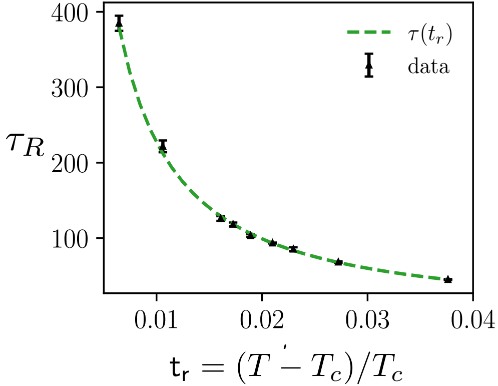
$$\bar{\sigma}_{\rm eq} \equiv \lim_{H \to 0} \langle M_0 \rangle \propto (-\mathsf{t_r})^\beta$$



Equilibrium: Dynamics for $T > T_c$ and critical slowing down

- The order parameter relaxes increasingly slowly near T_c
  - Analyzing $\langle M_a(t) M_a(0) \rangle \sim e^{-t/\tau_R}$ we define and then determine:

$$\tau_R \sim \xi^z$$



Relaxation Time of Order Parameter

$$\tau_R \propto \xi^{d/2}$$

Equilibrium: Pions and non-abelian superfluid dynamics $T < T_c$

Focus on the ideal eom coming from the Poisson Brackets

- Charge conservation:

$$\partial_t n_{ab} + \nabla \cdot (\nabla \phi_{[a} \phi_{b]}) = 0$$

- The Josephson constraint and precession

$$\partial_t \phi_a + \frac{n_{ab}}{\chi_I} \phi_b = 0$$

The magnitude of ϕ_a is fixed and the direction (or "spin") fluctuates

$$\phi_a \sim f s_a \qquad s_a s_a = 1$$

The linearized equations predicts spin waves, aka pions, with velocity $v^2 = f^2/\chi_I$

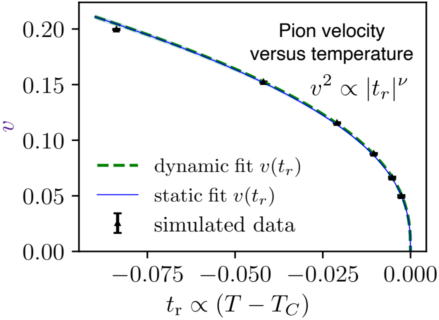
Equilibrium: analysis of pion dispersion curve

- Scaling dictates how the dispersion curve scales with ξ and $\tau_R \propto \xi^{d/2}$

$$\omega(k) = vk - iD_A k^2$$

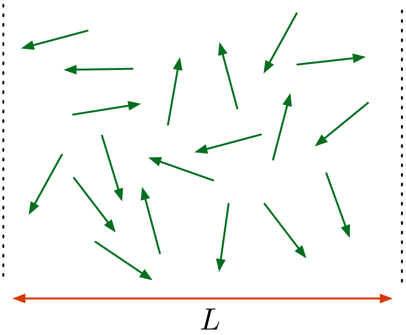
with pions propagating at speed v:

$$v \sim \frac{\xi}{\tau_R}$$

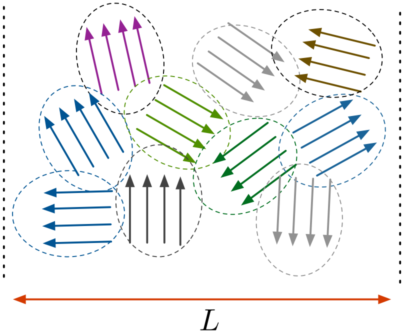


Condensate Growth Following a Quench: $t_r \to -t_r$

$T > T_c$ at time $t = 0$



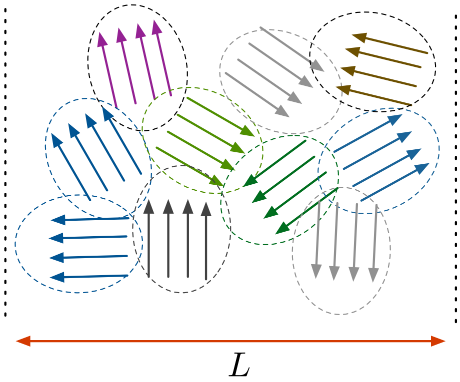
Nonequilibrium state at time t



We will monitor

$$G_0(t, \xi, L) = \frac{V}{4} \langle M_a(t) M_a(t) \rangle = \begin{cases} \sim 1 & \text{for } M_a \text{ random} \\ \sim V & \text{for } M_a \text{ constant} \end{cases}$$

Condensate Growth – Limits



$$G_0(t,\xi,L) = \frac{V}{4}\left\langle M_a(t) M_a(t) \right\rangle$$

- Initial conditions

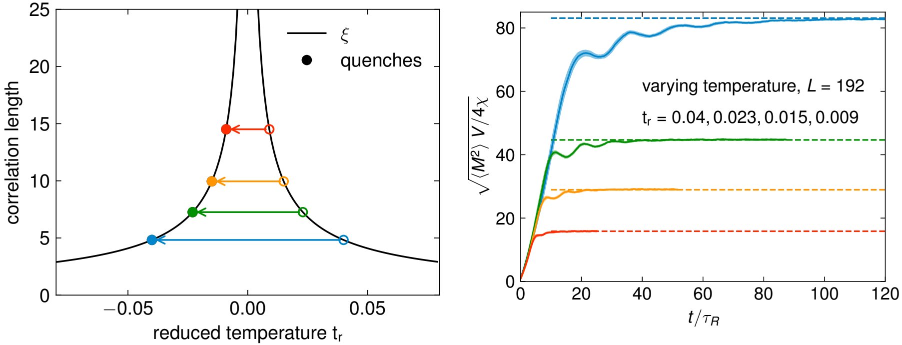
$$G_0(t)\big|_{t=0} = \chi \propto \xi^{2-\eta}$$

- Final conditions

$$\lim_{t\to\infty} G_0(t) = \frac{V\bar{\sigma}_{\rm eq}^2}{4} \propto \xi^{2-\eta}\left(\frac{L}{\xi}\right)^d$$

Naive dynamical critical scaling in the large volume limit $L/\xi \gg 1$

$$
\begin{aligned}
G_0(t,\xi,L) =& \xi^{2-\eta}\,\mathcal{F}(t/\tau_R,\xi/L) \\
\approx& \,\xi^{2-\eta}\,\mathcal{F}(t/\tau_R)
\end{aligned}
$$

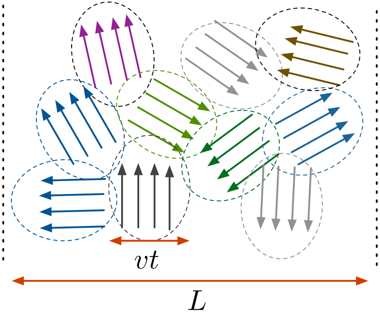


Good data collapse at short times, but neglects secularity for $t \sim \tau_R(L/\xi)$

Condensate Growth with large volume $L/\xi \gg 1$ and scaling $v \sim \xi/\tau_R$

Interested in late times $t \gg \tau_R$ with $t \sim L/v \sim \tau_R(L/\xi) \sim 1$:

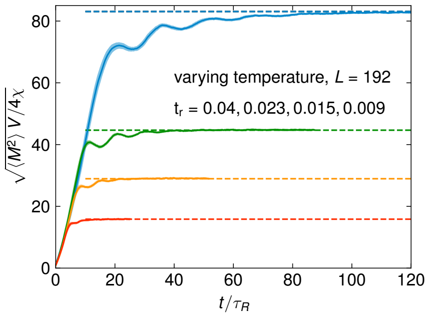
$$
\begin{aligned}
G_0(t,\xi,L) =& \xi^{2-\eta}\,\mathcal{F}(t/\tau_R, \xi/L) \\
=& \xi^{2-\eta}\left(\frac{L}{\xi}\right)^d \mathcal{F}(vt/L, \xi/L) \\
\simeq& \xi^{2-\eta}\left(\frac{L}{\xi}\right)^d \mathcal{F}(vt/L)
\end{aligned}
$$



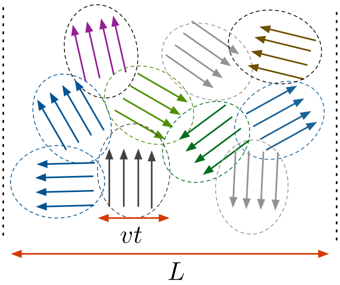
At early times, an L independent result:

$$
G_0(t,\xi,L) \Rightarrow \quad \underbrace{\xi^{2-\eta}\left(\frac{t}{\tau_R}\right)^d}_{\text{early time growth}}
$$

Condensate Growth



Condensate Growth and the Ideal Superfluid Regime



This is in the (ideal) superfluid regime

The scaling form is valid in a hydro regime

$$t \gg \tau_R \qquad \text{and} \qquad vt \gg \xi$$

For intermediate times dissipation is unimportant . . .

(but a bit needed to remove pions in the UV)

Growth of Pion Correlators

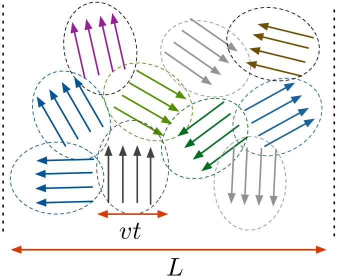
Note:

- Wavelengths $kL \sim 1$ are not parametrically different from $k = 0$
- The decomposition of $\phi_a(\mathbf{k}) = (\sigma(\mathbf{k}), \vec{\pi}(\mathbf{k}))$ is arbitrary.

Soft pions scale like the condensate:

$$G_{\pi\pi} \equiv \frac{V}{3} \langle \vec{\pi}(t, \mathbf{k})\, \vec{\pi}(t, -\mathbf{k}) \rangle$$

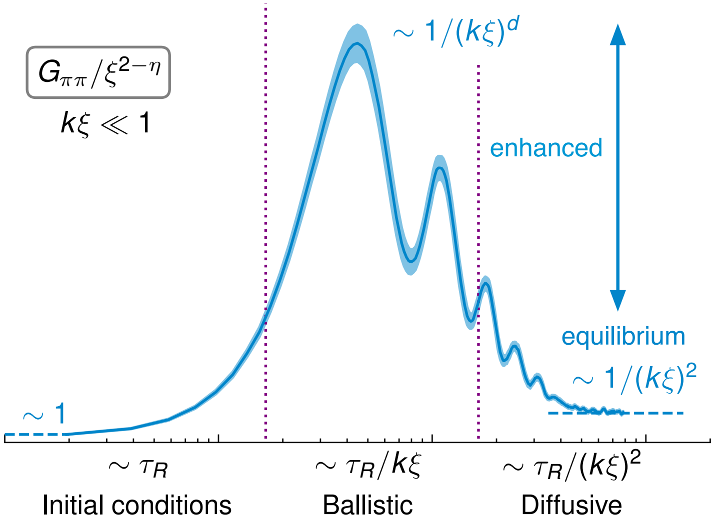
$$= \xi^{2-\eta} \left(\frac{L}{\xi}\right)^d F(vtk, kL)$$



For $kL \gg 1$, the correlation $G_{\pi\pi}$ is independent of L , and we find

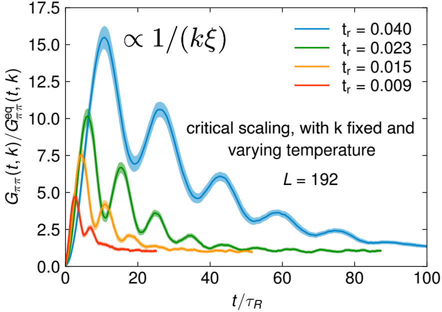
$$G_{\pi\pi}(t, \mathbf{k}) = \frac{\xi^{2-\eta}}{(k\xi)^d} \mathcal{F}(vtk)$$

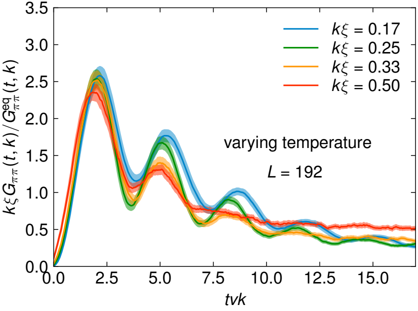
Pion evolution schematic at *fixed* k:



Pion Evolution:

$G^{\rm equil}_{\pi\pi}(k) \propto 1/(k\xi)^2$





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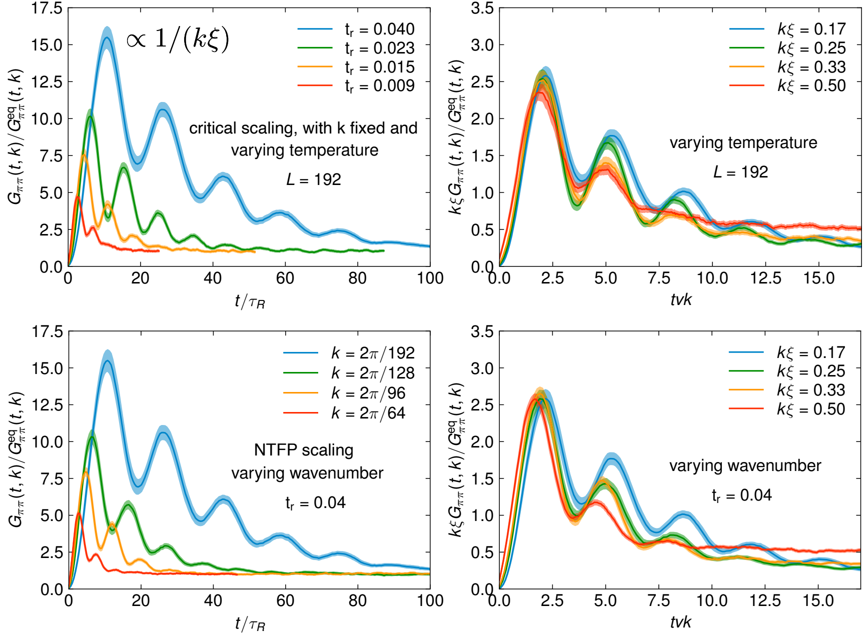
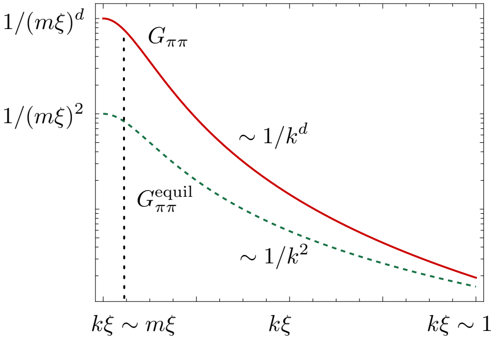


Summary:

Qualitative expectation for soft pion yields, $k\xi \ll 1$ and $d = 3$

$$
\begin{aligned}
n_\pi(t,k) &\propto vk\, G_{\pi\pi}(t,k) \\
&= \frac{T}{\tau_R}\left[\frac{1}{(vk)^2}\mathcal{F}(vkt)\right] \longrightarrow \left(\frac{\text{const}}{k\xi}\right) \underbrace{\left(\frac{T}{vk}\right)}_{\text{equilib.}}
\end{aligned}
$$

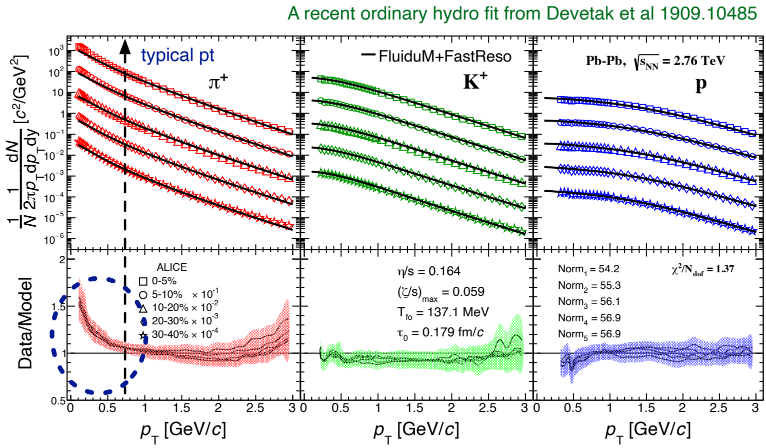


Non-equilibrium enhancement $\sim 1/(m\xi)$ which lives a long time $\sim \tau_R/(m\xi)^2$

This is a generalized NTFP with

$\alpha = 2 \qquad \beta = 1$

Quench Dynamics in the Heavy Ion Data?



See also, Guillen&Ollitrault arXiv:2012.07898; Schee, Gürsoy, Snellings: arXiv:2010.15134

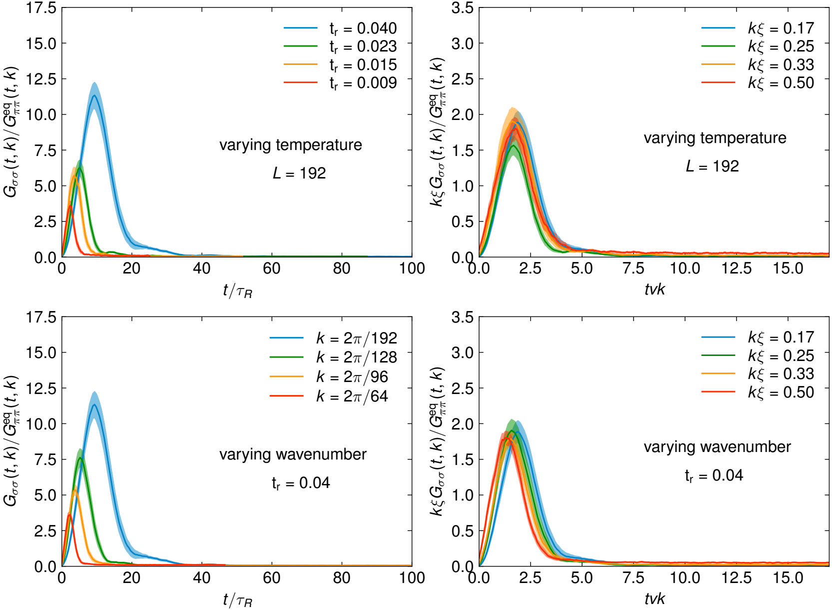
Summary

- Wrote down a critical model of the chiral transition:
  - Plain diffusion of left and right handed quarks for $T \gg T_c$
  - Pion superfluid and propagation for $T \ll T_c$:
- During a quench condensate growth exhibits dynamical-critical scaling
  - The non-linear (non-abelian) superfluid theory describes this growth.
- Pions which are tied to the condensate must track this growth
  - Pions are enhanced ($\propto 1/m$), before relaxing (slowly) to equilibrium.

I hope this is seen in experiment!

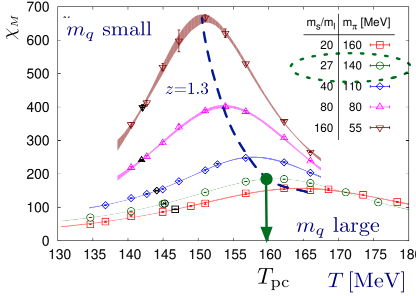
Backup

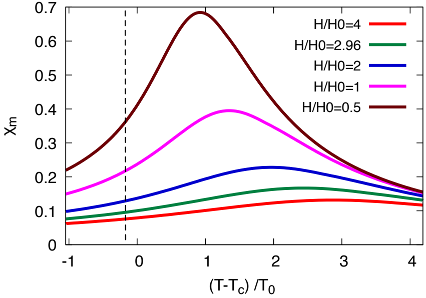
Sigma Evolution:



Scaling predictions and Real QCD

magnetic susceptibility $\chi_M \equiv \langle \sigma^2 \rangle - \langle \sigma \rangle^2$





Scaling predictions reasonably describe how the peak rises and shifts.

$$\chi_M \propto m_q^{1/\delta - 1} f_\chi(u) \qquad u \propto \left(\frac{T - T_C}{T_C}\right) m_q^{-1/\beta\delta}$$