Stages of relaxation of an isolated Bose gas

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Attractors and thermalization in nuclear collisions
and cold quantum gases
ECT* – 26 Sept 2025

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Zoran Hadzibabic group

Current members

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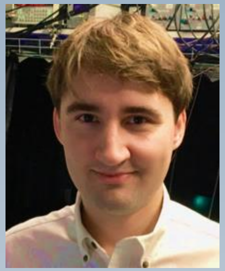

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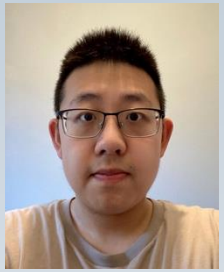

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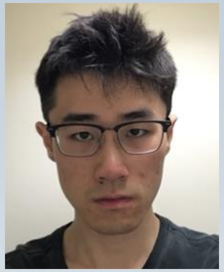
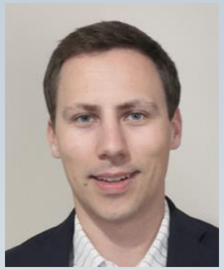


Paul Wong

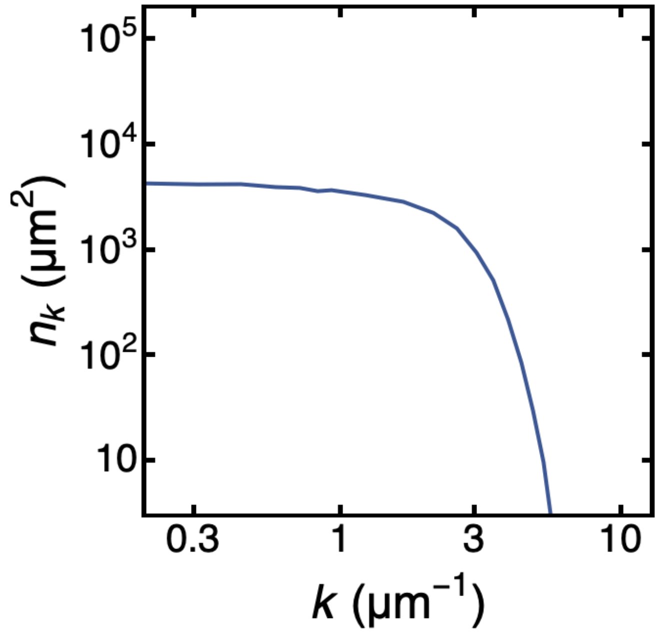


Jiří Etrych

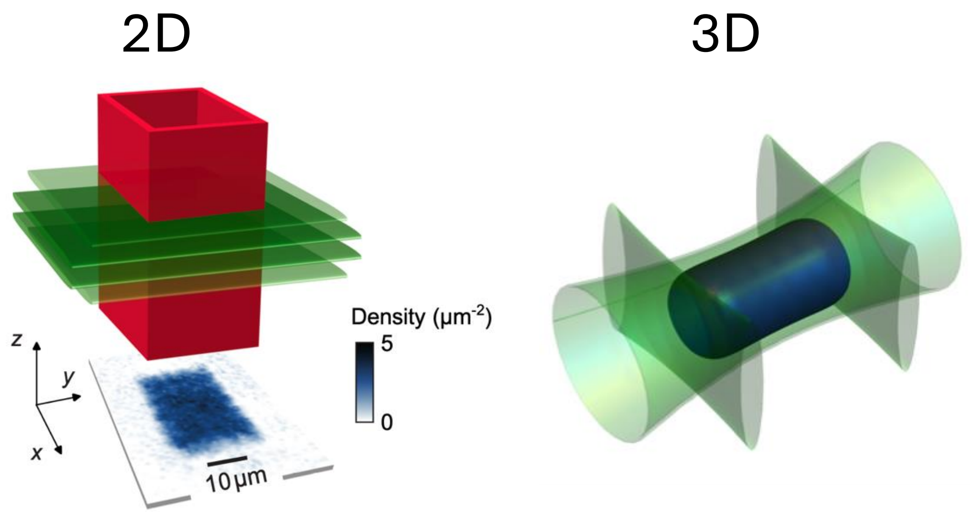
Andrey Karailiev

Our experiments: a Box in a Box

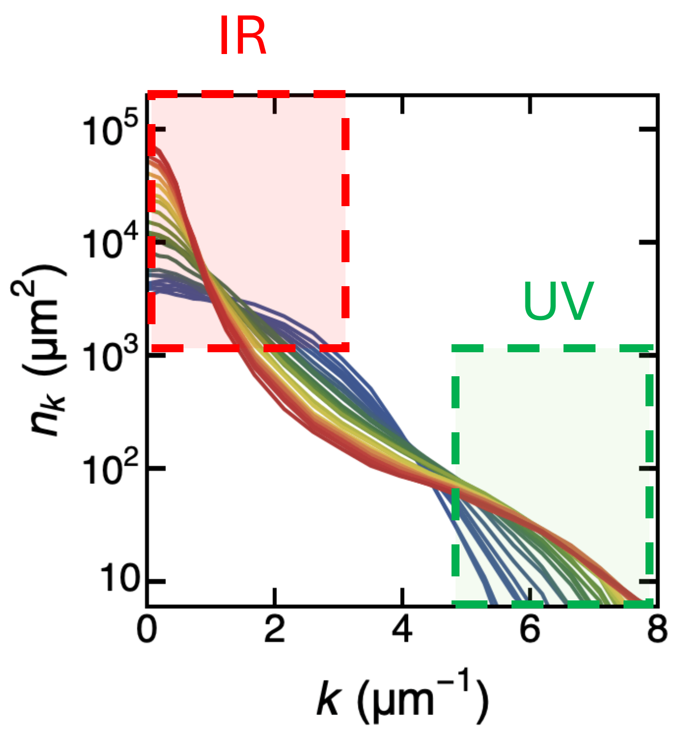
Box initial condition:



Box trap:

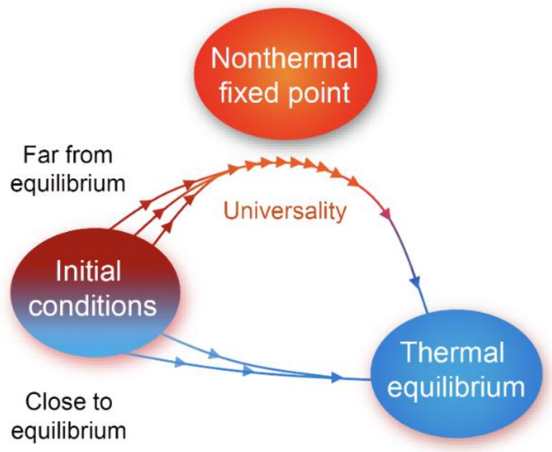


Formation of BEC: Universal dynamics near NTFP



Dynamic self-similar scaling (IR and UV separately):

$$n_k(k,t) = \left(\frac{t}{t_0}\right)^{\alpha} n_k\left[\left(\frac{t}{t_0}\right)^{\beta} k, t_0\right]$$



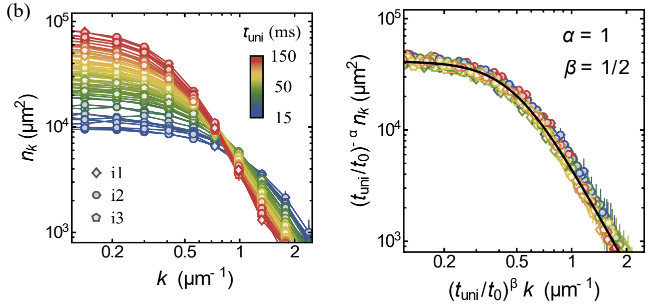
J. Berges *et al.*, **PRL. 101**, 041603 (2008).

Overview of the talk

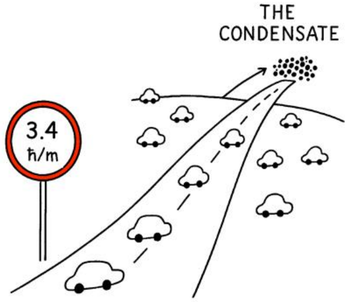
Previously: (Zoran's talk)

Dynamic scaling

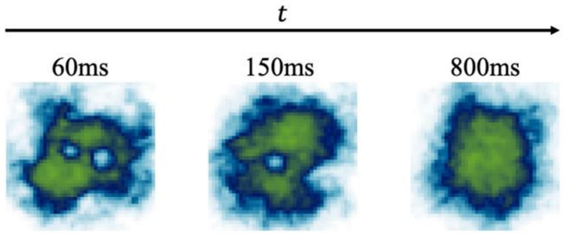
$\beta = 1/2$ in 2D and 3D)



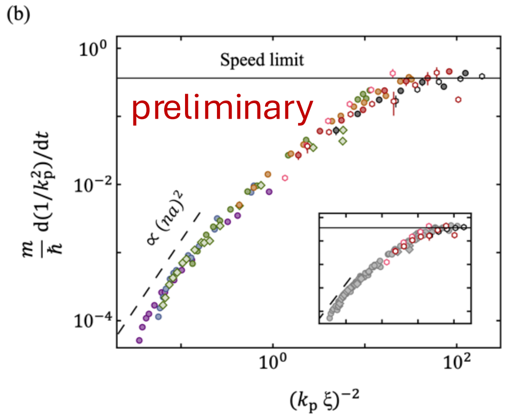
Speed limit



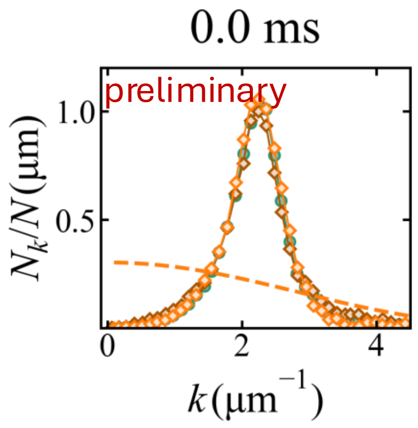
Part 1: Topological defects



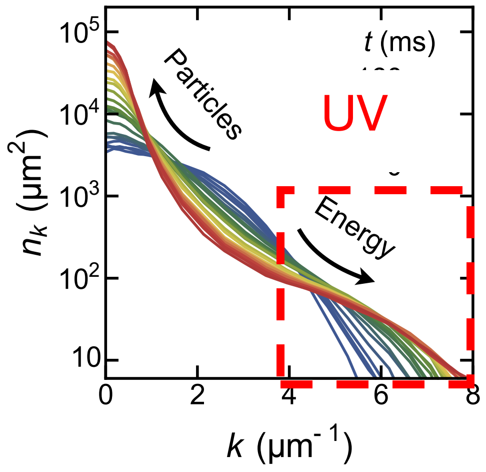
Part 2: Inverse WWT



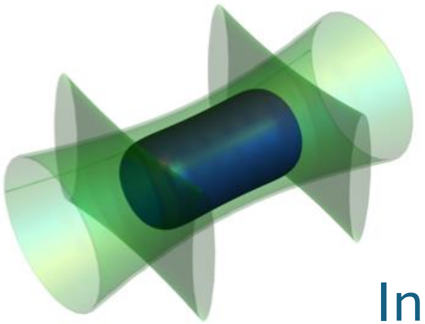
Part 3: Breakdown of classical-field theory



Part 4: UV transport

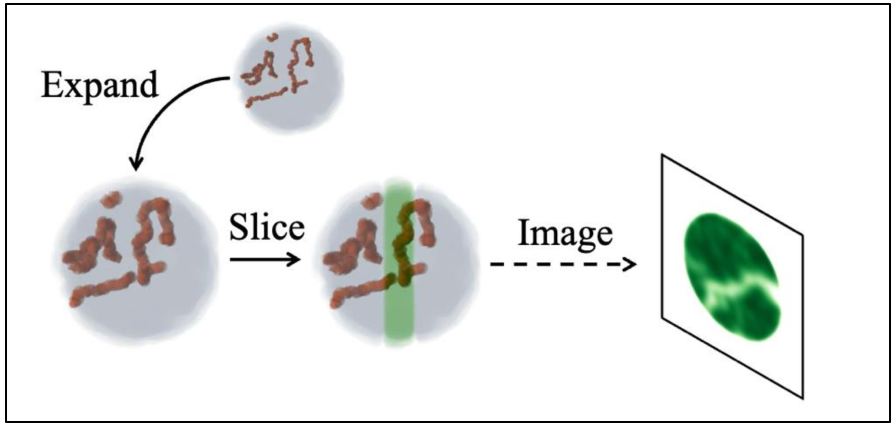


Part 1: Two probes



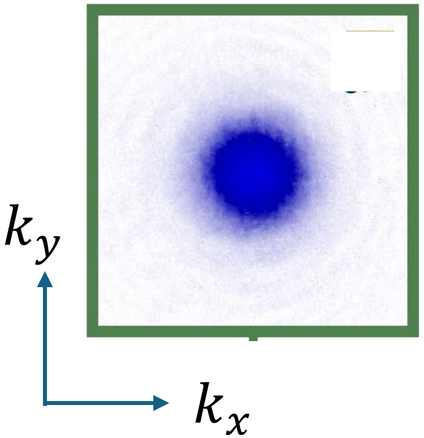
Noninteracting expansion
(Time-of-flight)

Interacting expansion

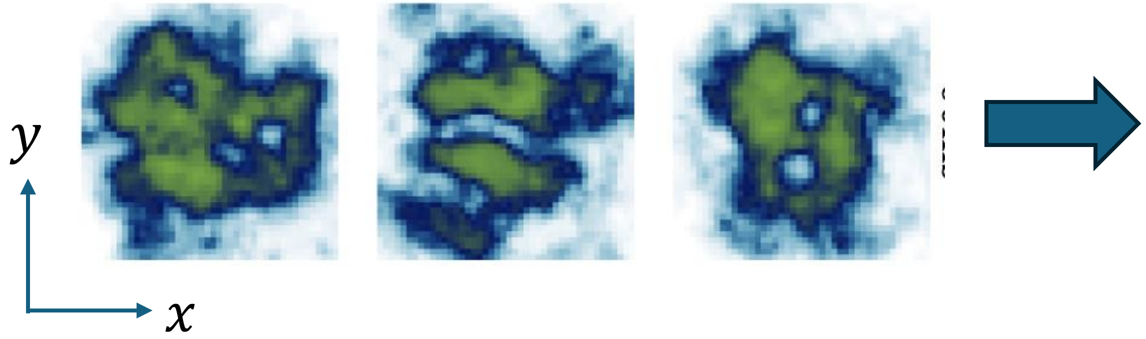


Momentum distribution,

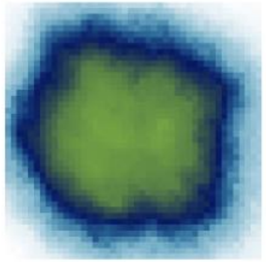
$$n_k = \langle \psi_k^\dagger \psi_k \rangle$$



Repeats (@ same time):

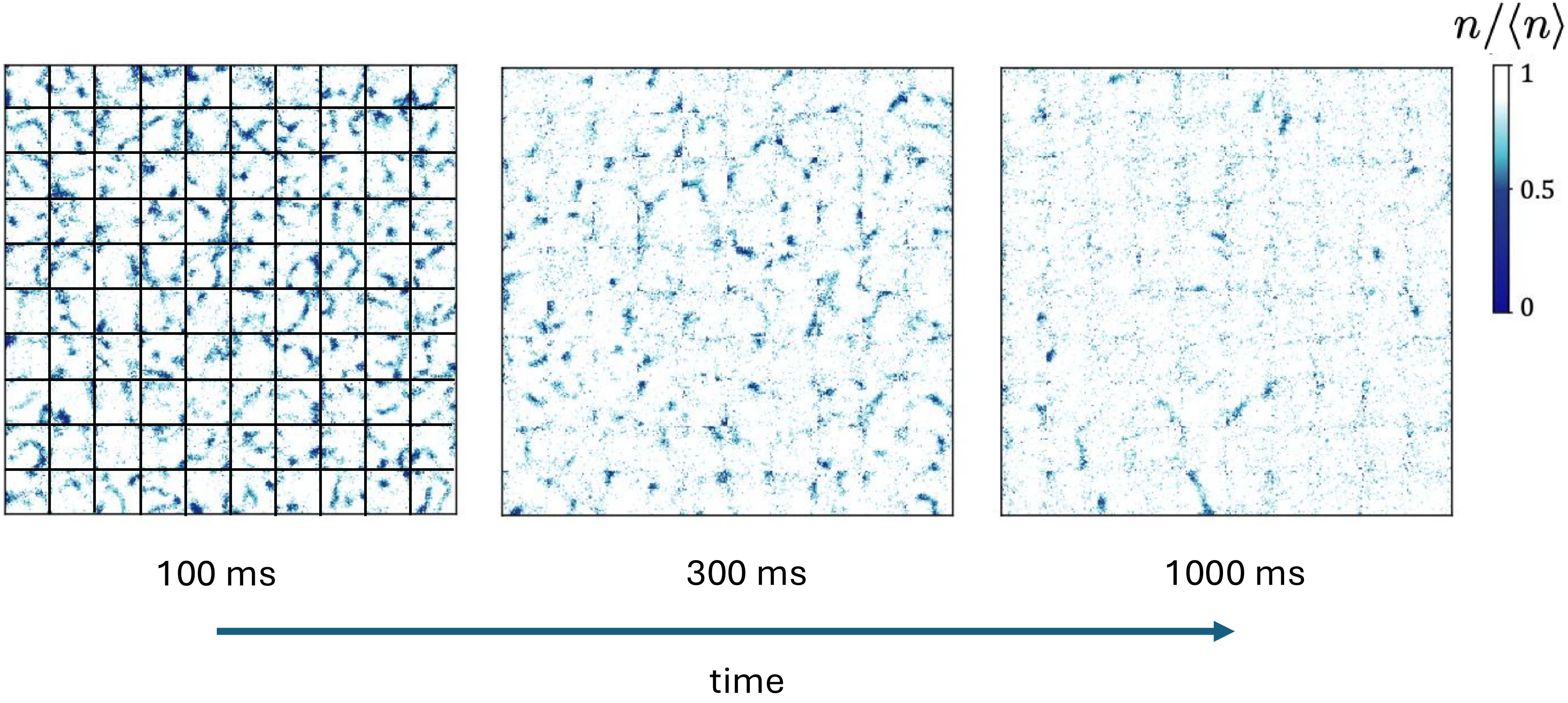


Average:



→ Vortex-line length density, $\mathcal{L}$

How does it look in practice?



Vortex density: $\mathcal{L} \sim t^{-1}$

t^* also enters: $t \to t - t^*$

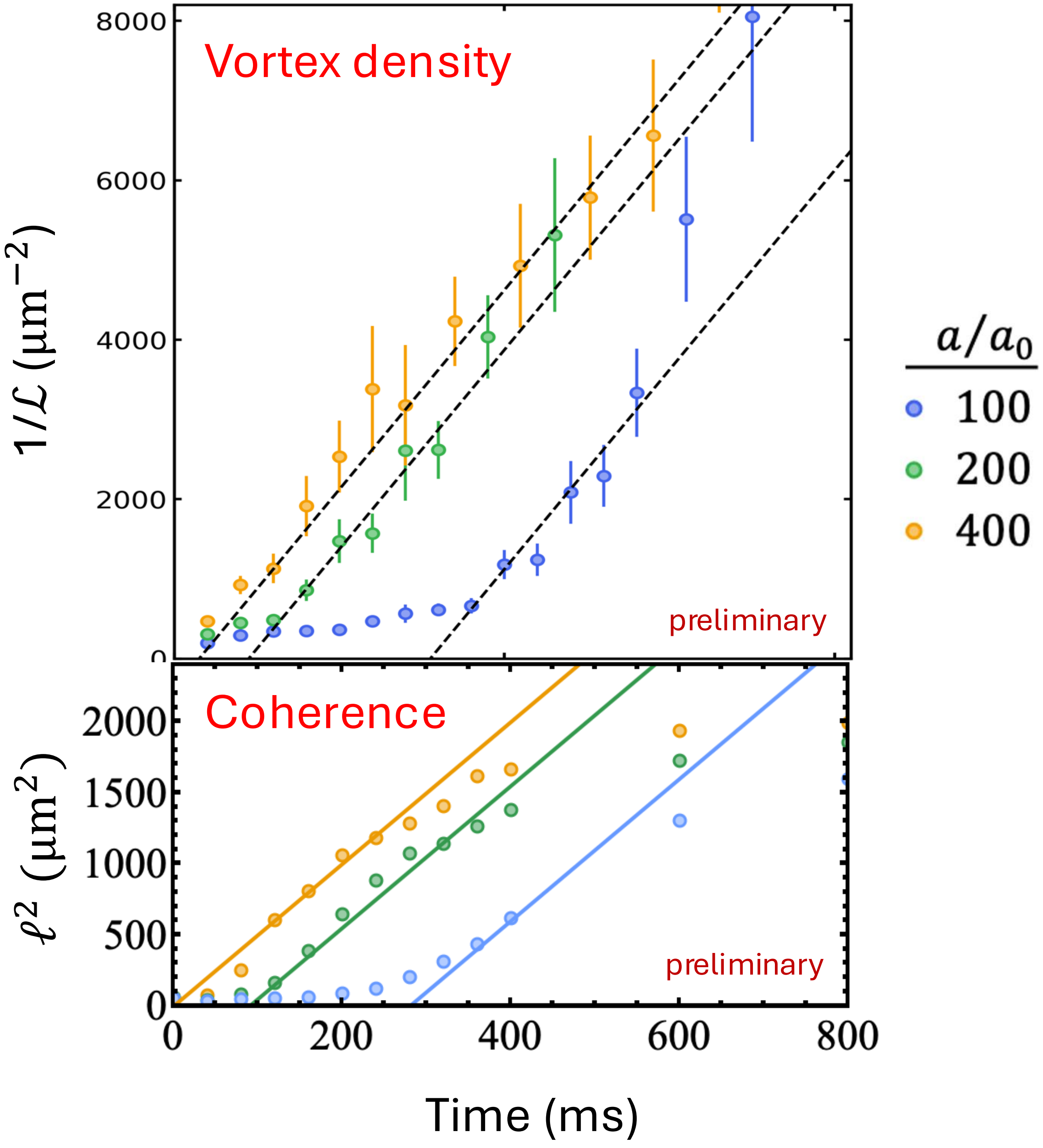
Plot: $1/\mathcal{L} \sim (t - t^*)$

(In Equilibrium: $\mathcal{L} \to 0$)

ℓ^2: $1/\mathcal{L}$:

	t^*(ms)	t^*_v(ms)
● (blue)	290	310
● (green)	90	90
● (orange)	30	30

Same offsets t* → Vortex coarsening?

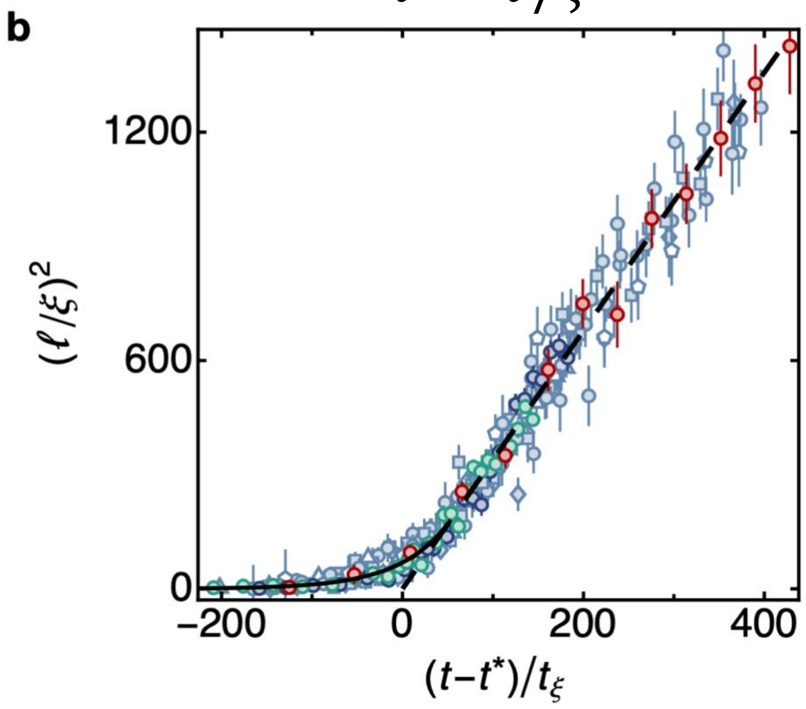


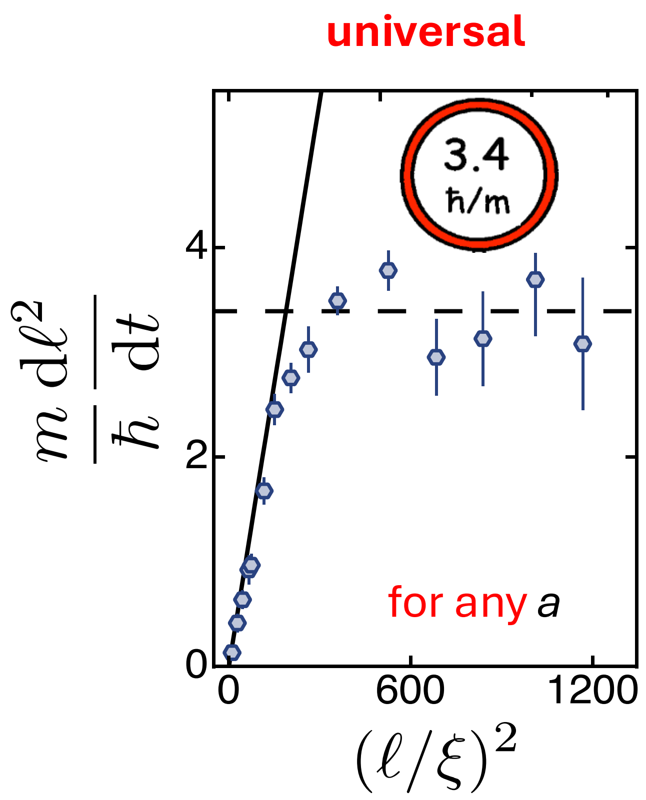
Part 2: Inverse weak-wave turbulence

all data, classical-field units:

$$t \to t/t_\xi$$

$$\ell \to \ell/\xi$$





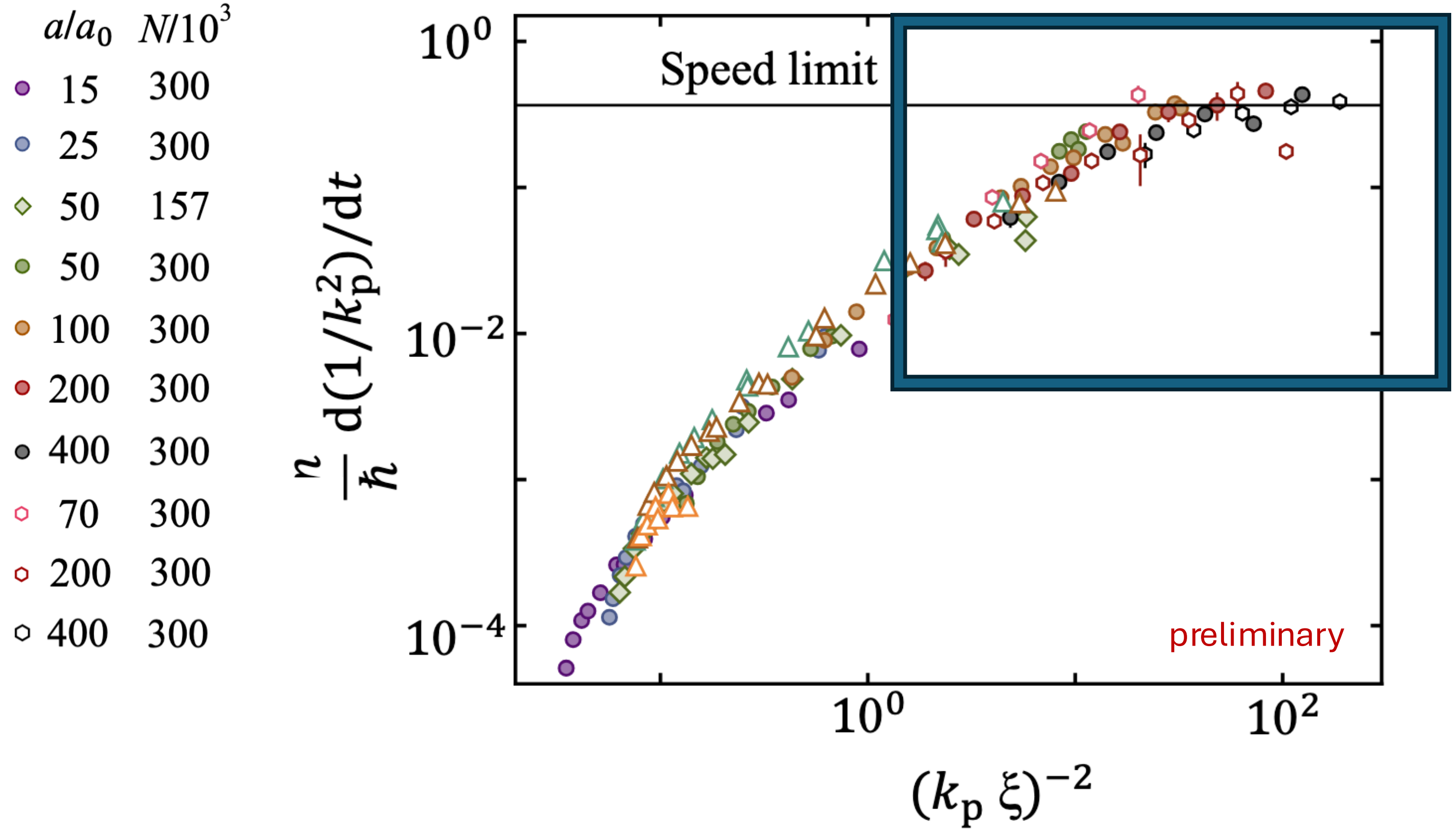
To reach the speed limit:

$\ell \gtrsim 15\,\xi \sim 1/\sqrt{na}$

What happens if $\ell < \xi$?

Towards the perturbative limit

Previously...

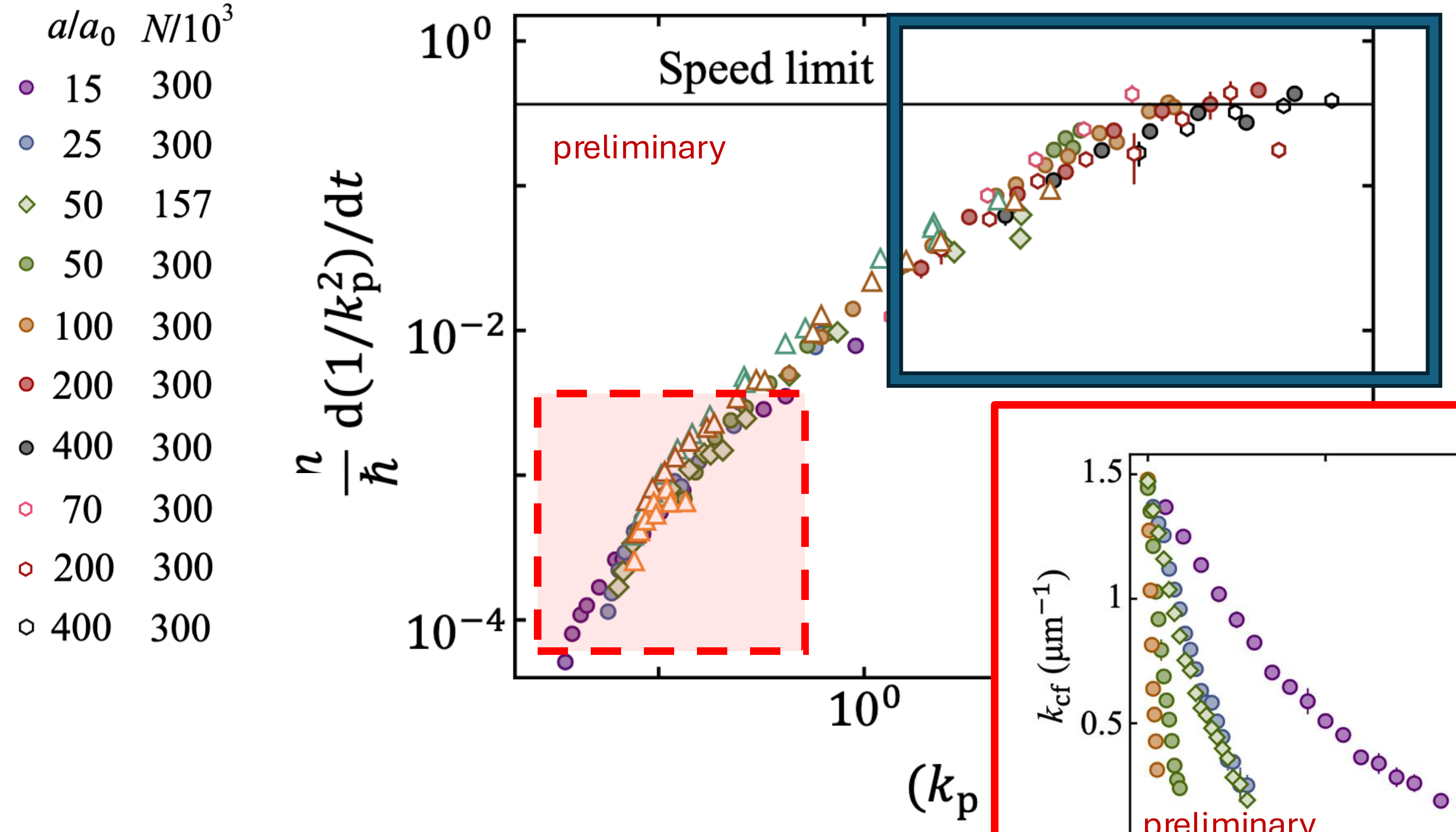


Speed limit 3.4 ħ/m

$$\frac{1}{k_p^2} \propto \ell^2 \propto D(t - t^*)$$

Towards the perturbative limit

Previously...



Speed limit 3.4 ħ/m

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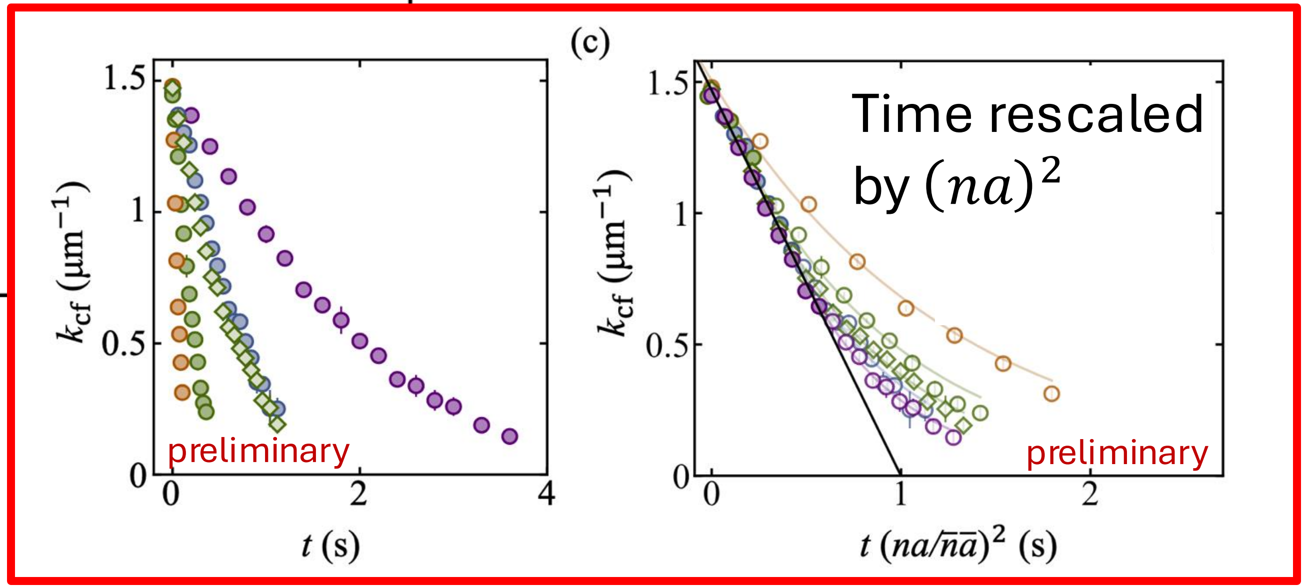
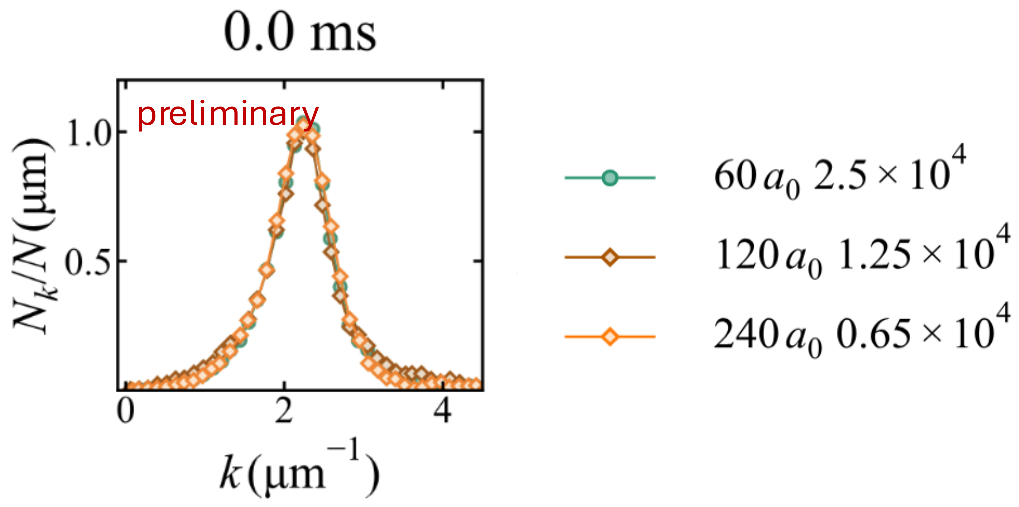


Part 3: Breakdown of classical field theory

Low energy Bose gas: 2 lengthscales $n^{-1/3}, a$

Classical field theory: na enters only together → single lengthscale ξ

Prepare states with the same na, but vary relative ratio between n, a



($N_k/N = 4\pi k^2\, n_k/\mathrm{N}$)

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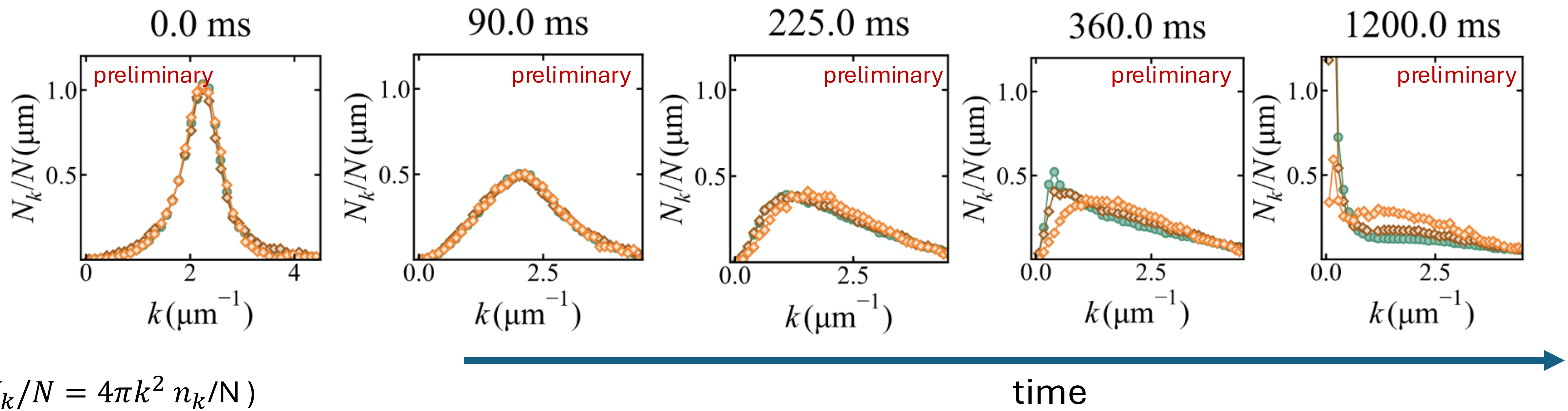

$(N_k/N = 4\pi k^2 n_k/\mathrm{N})$

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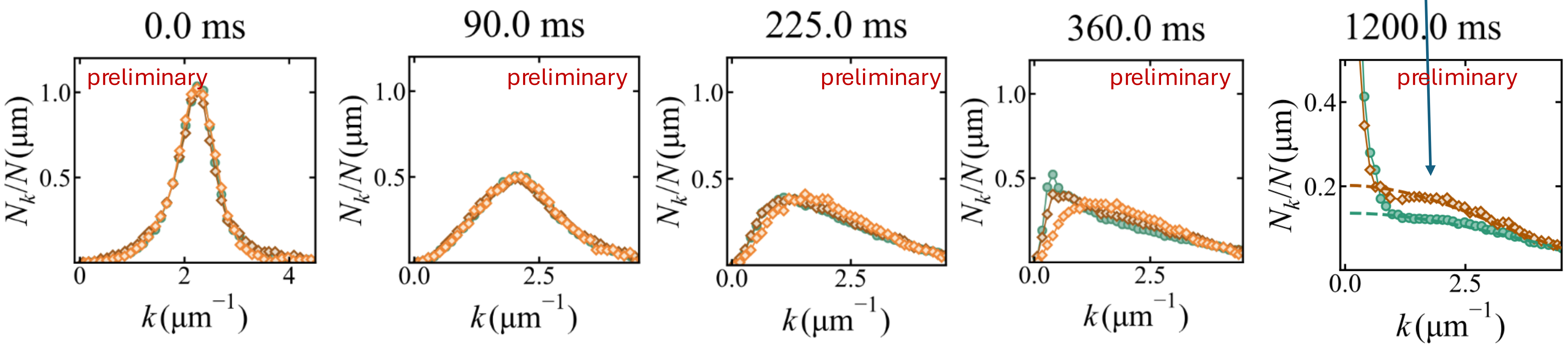
Dashed lines: thermodynamics



time

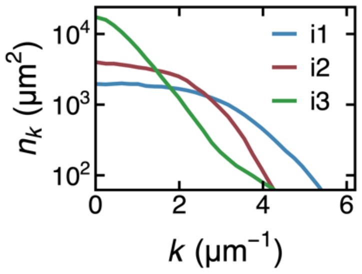
($N_k/N = 4\pi k^2 \, n_k/\mathrm{N}$)

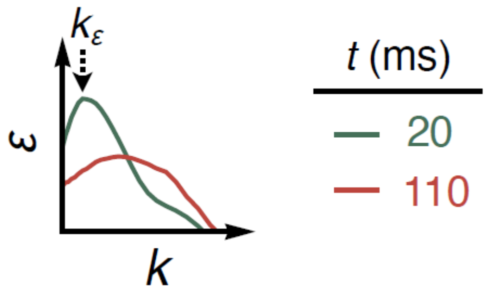
Part 4: UV dynamics (in 2D)

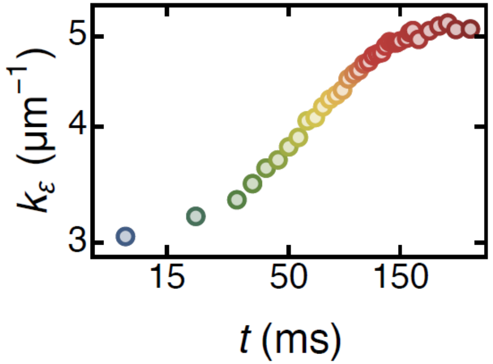
t^* different from IR

No k at which $n_k \propto (t - t^*)^\alpha$

Energy spectrum $\varepsilon_k \propto k^3 n_k$:



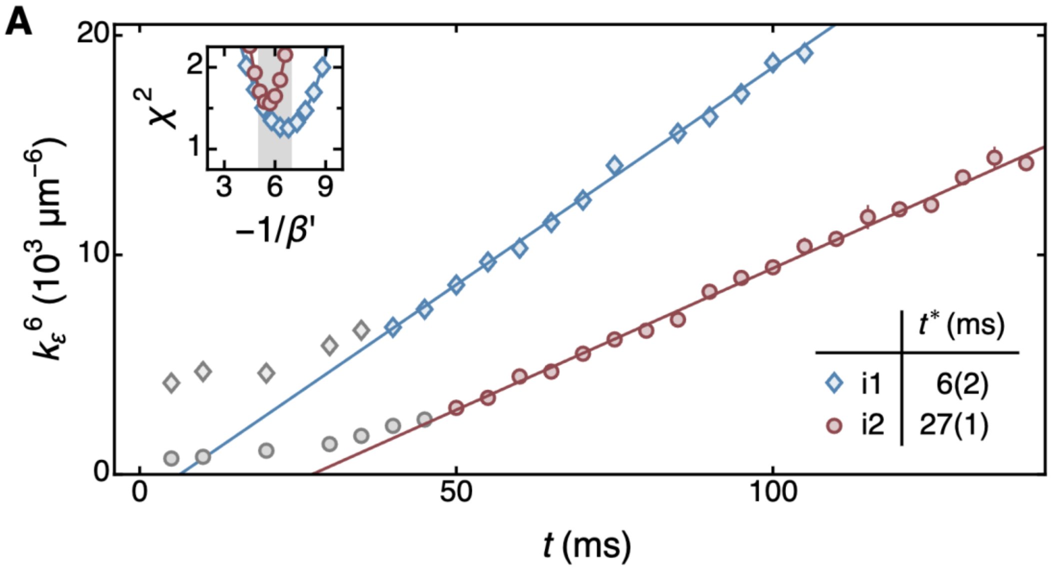




$\beta = -1/6$
$\alpha = -2/3$

Agrees with WWT:

$k_\varepsilon \sim (t - t^*)^{1/6}$

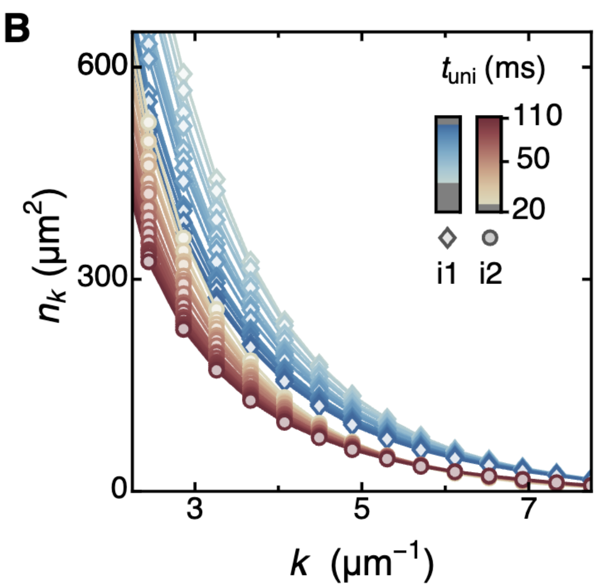


Different energies
➔ different slopes (clock speeds)

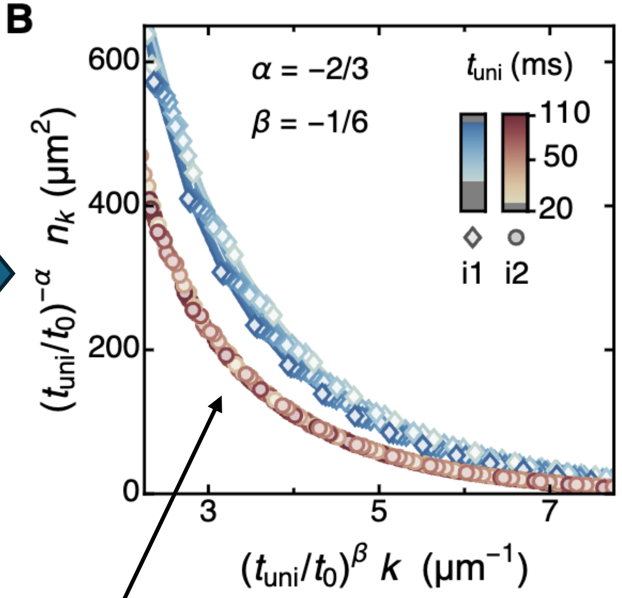
Gazo et. al. 2025

UV Dynamic Scaling

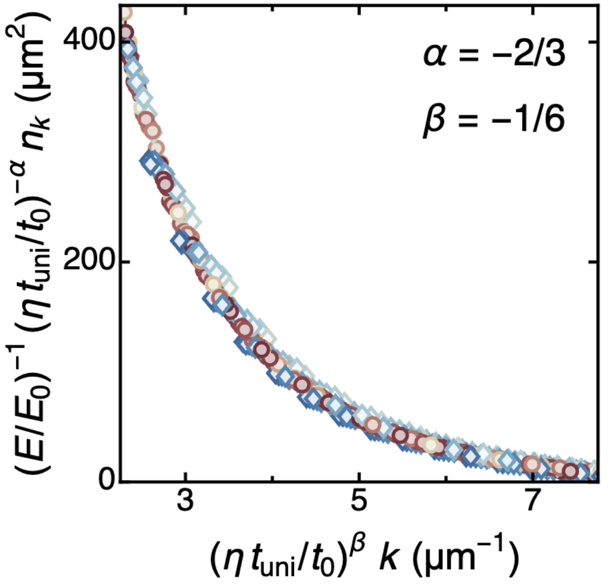
Unscaled

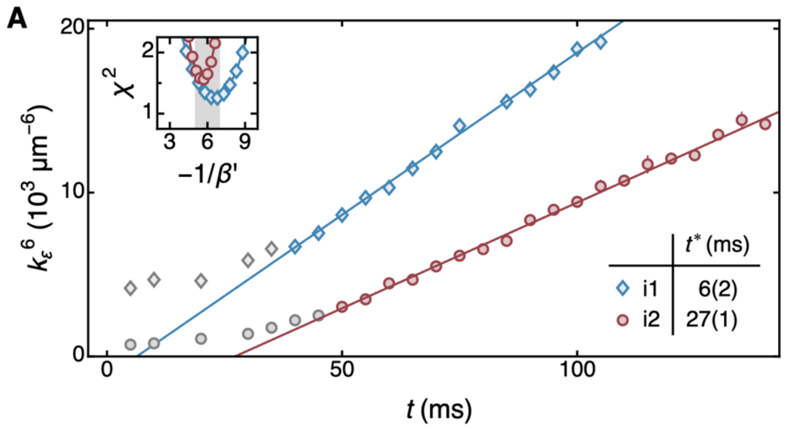


Dynamic scaling



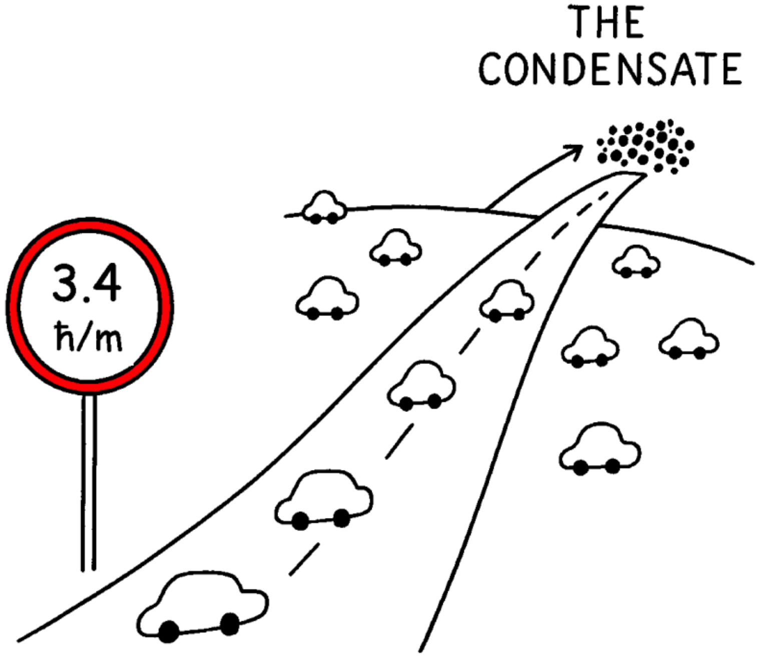
Normalise by E and scale time using slopes





Different energies do not collapse!

Thank you for you attention!



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