Hydrodynamic attractor near superfluid phase transitions

Based on *Phys.Rev.D* 103 (2021) 7, 076014 with Prof. A Mukhopadhyay, Dr. A Soloviev
arXiv:2410.01892 with Guri Buza and Dr. A Soloviev

ITP, University of Heidelberg

Toshali Mitra

ECT* workshop, Attractors and thermalisation in nuclear collisions and cold quantum gases, Trento, Italy

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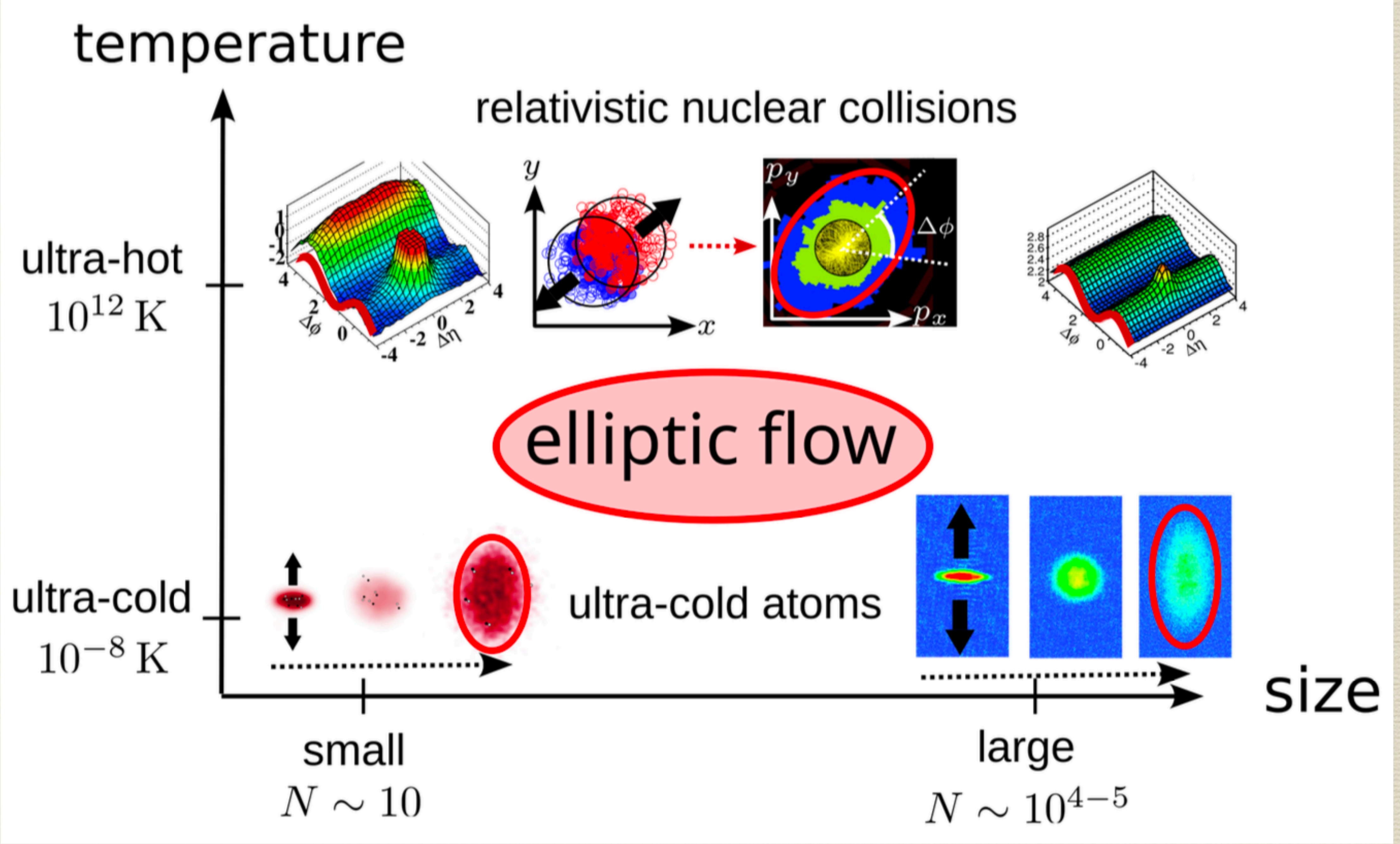
Motivation

- Hydrodynamics:
  - Emergent of collective dynamics in large and small systems.
  - Effective description of many-body physics in long wavelength limit.



EMMI Rapid Reaction Task Force 2025

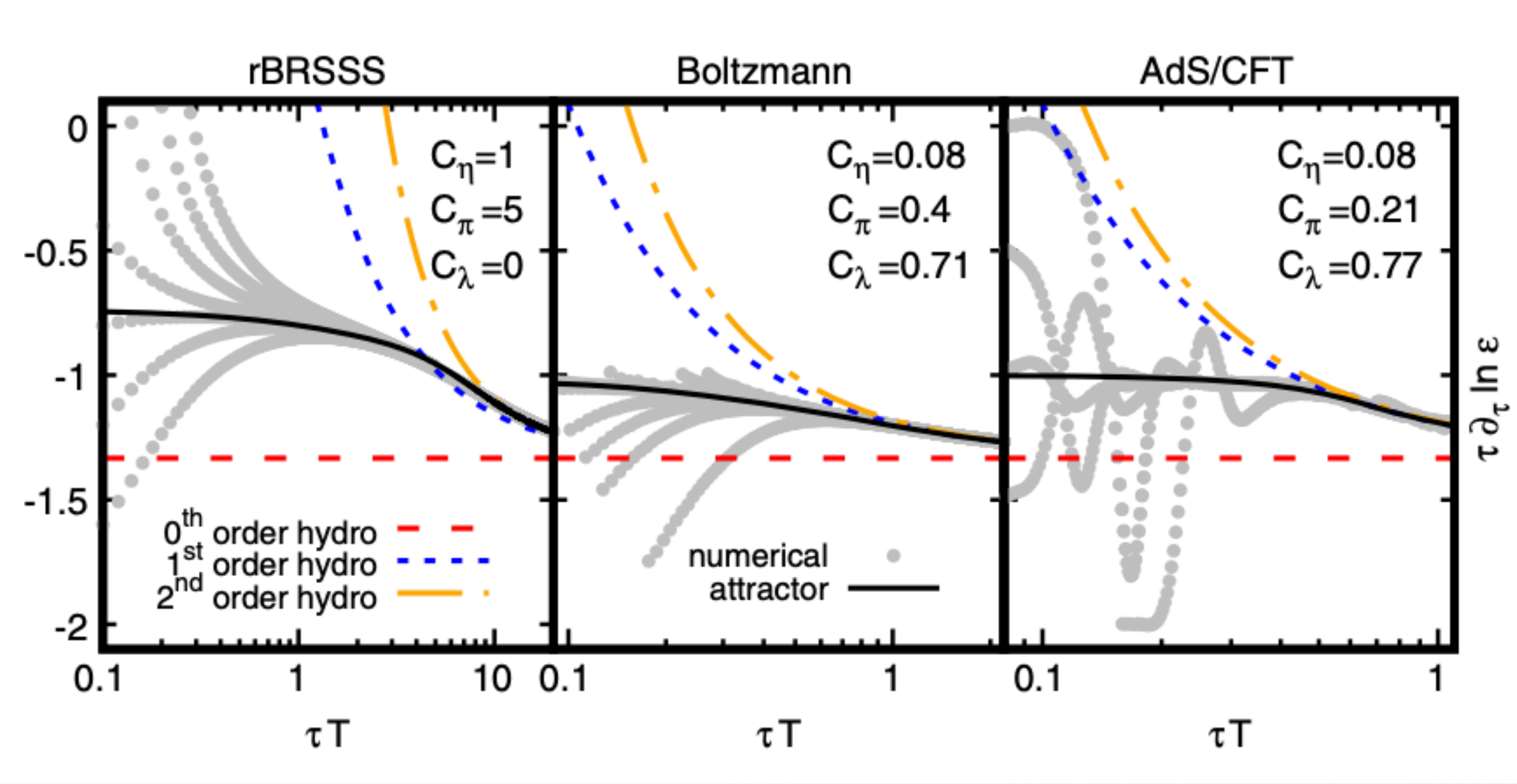
Talk by Soloviev, Enss, Mazeliauskas ,Fuji,
Inces

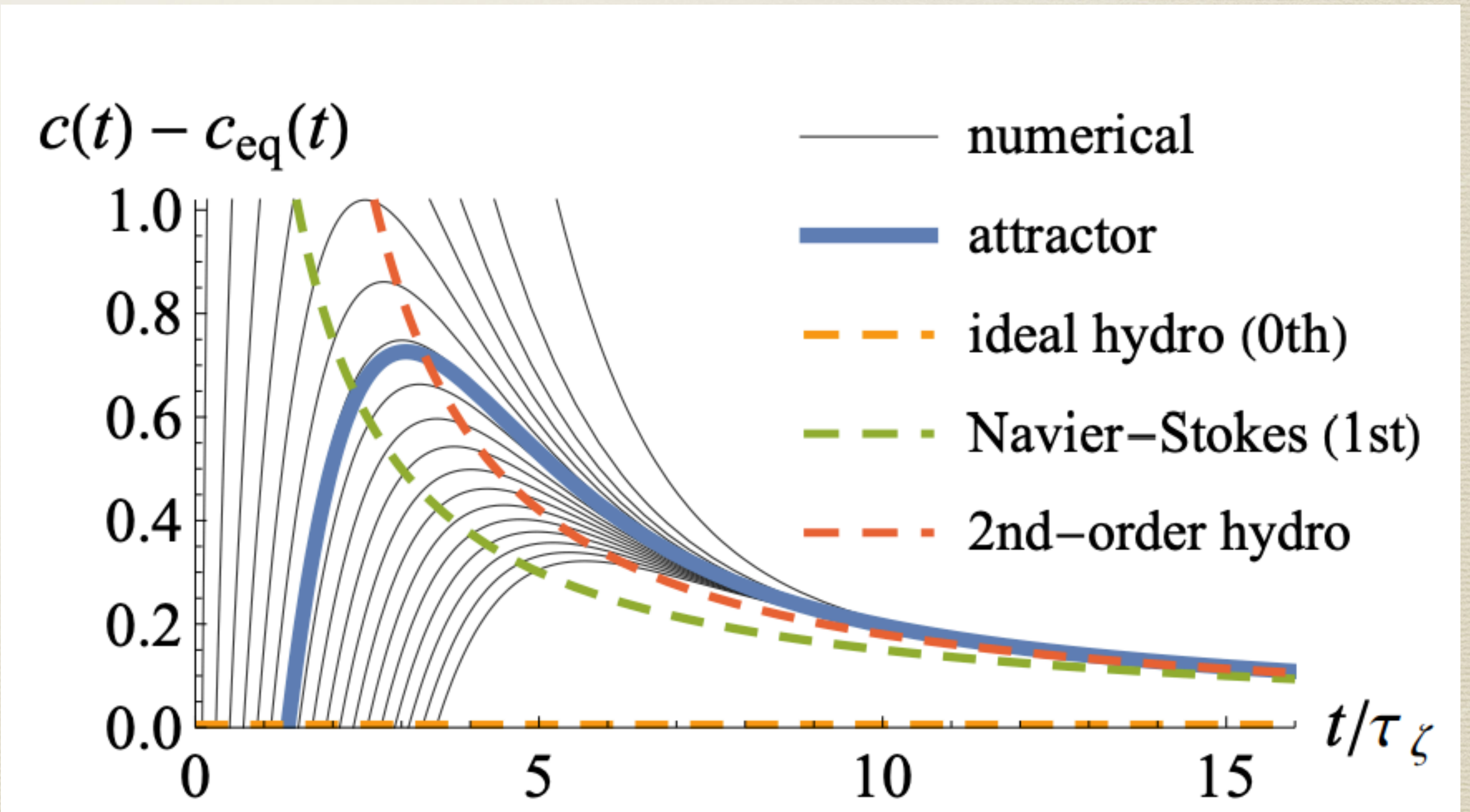
∗ Hydrodynamic attractor:

- Hydrodynamic in far from equilibrium- Hydrodynamic attractors

- Explored in symmetric theories.

Paul Romatschke (2018)

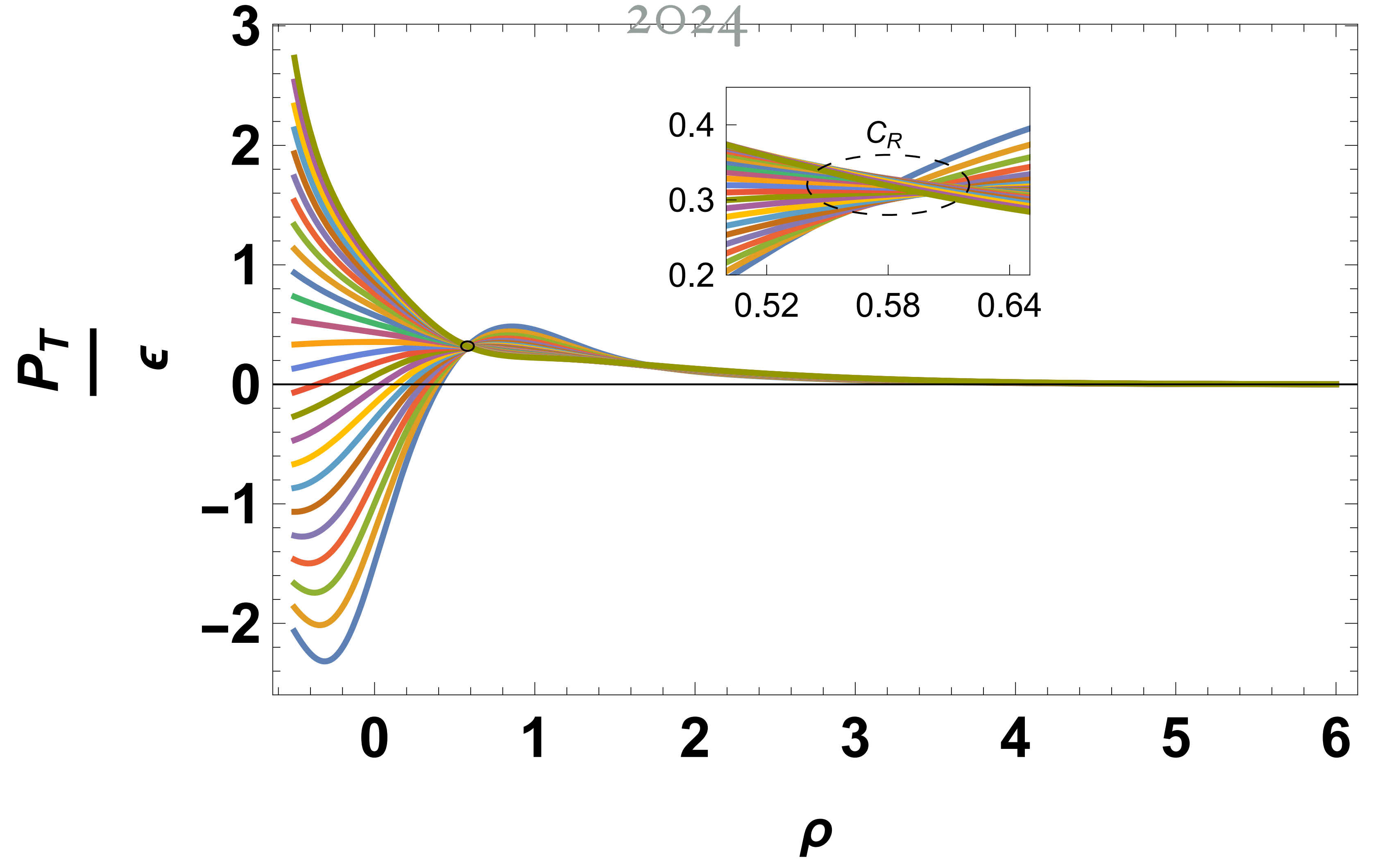


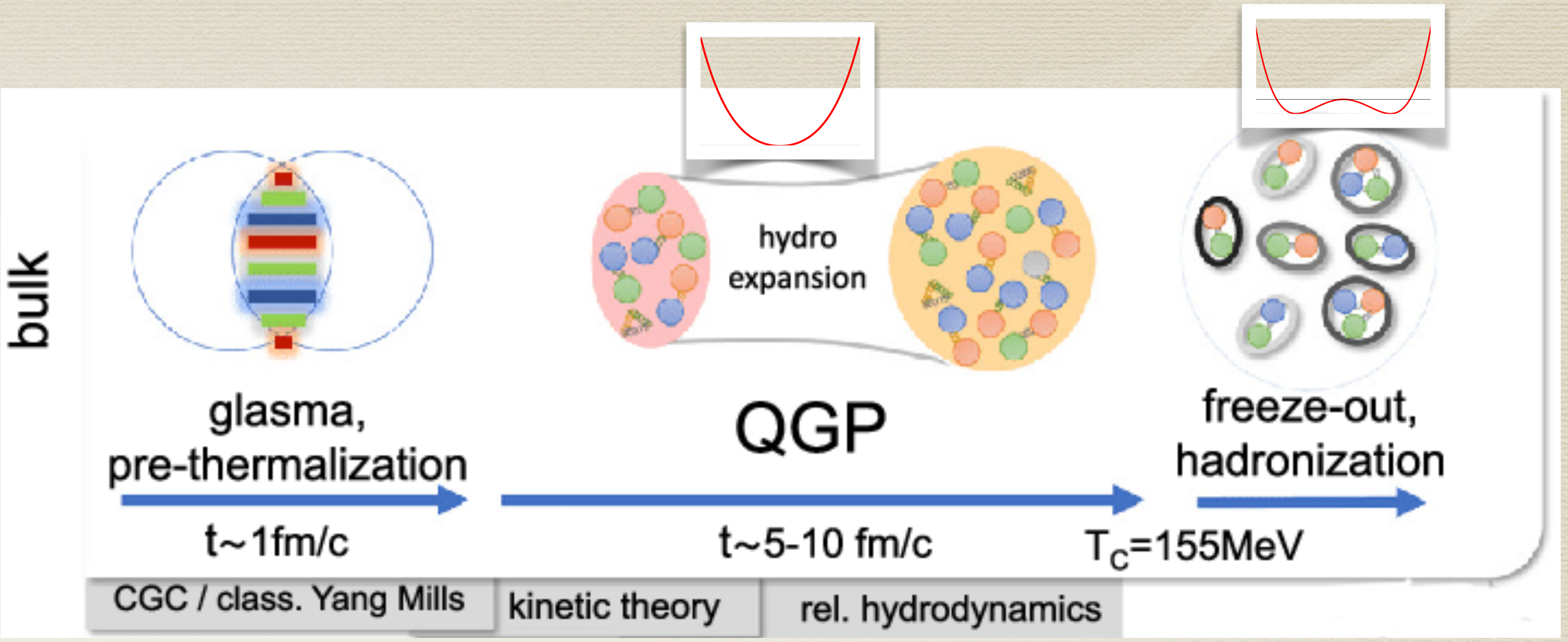


Enss, Fujii (2024)

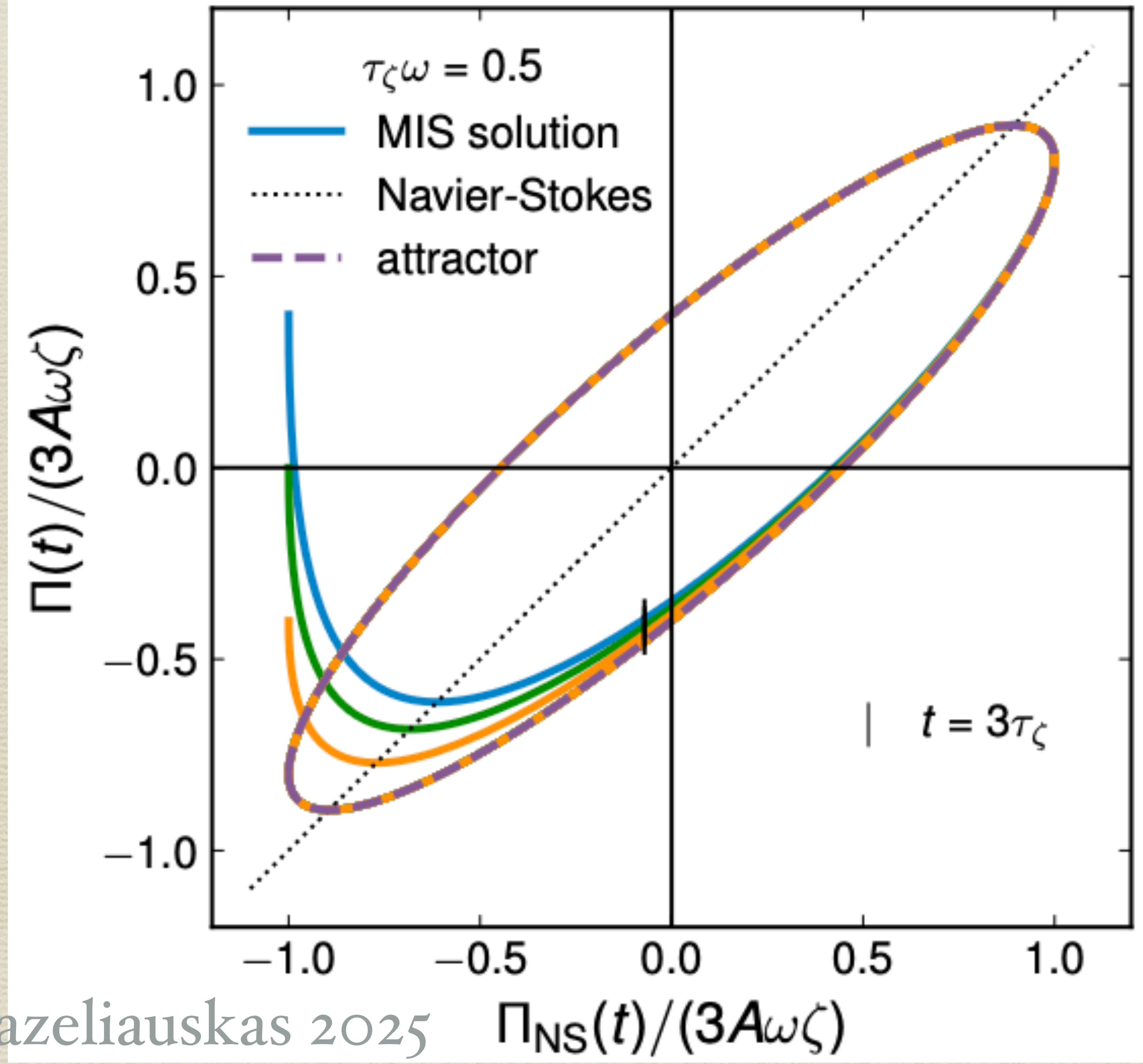
Phase transition and broken continuous symmetry

Mitra, Mukhopadhyay, Mondkar and Soloviev, 2024



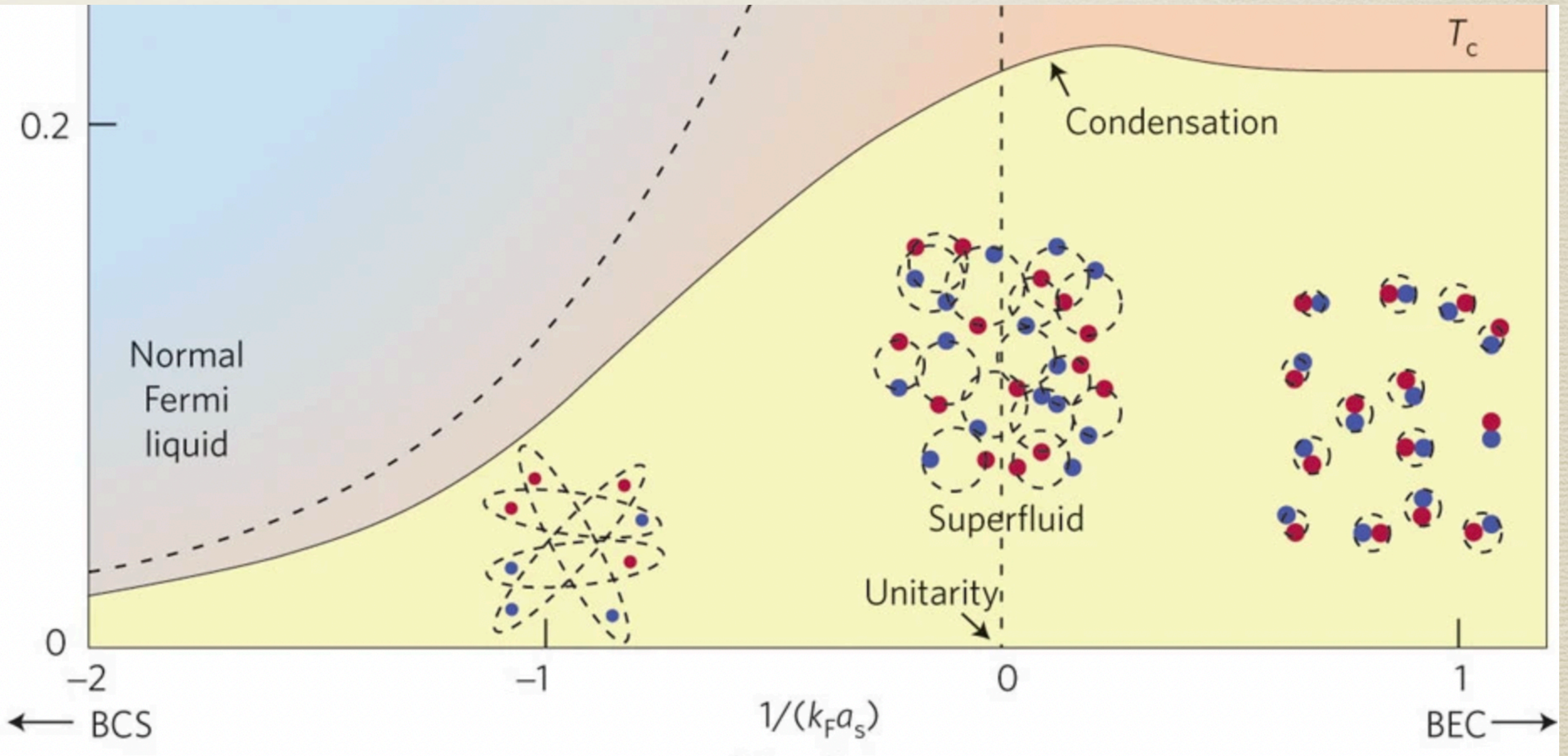


Heavy quarkonium in extreme conditions by Alexander Rothkopf



Enss, Mazeliauskas 2025

Randeria, Zwerger & Zwierlein (2012)



Superfluid phase

- Hydrodynamic theory with spontaneously broken symmetry - Superfluidity.
- Dissipative fluid with spontaneously broken $U(1)$ symmetry.
    - Merging effective field theory for relativistic superfluids with MIS formalism.
    - Expansion of the system in three different backgrounds.

Mitra, Mukhopadhyay, Soloviev

MIS-formulation of Superfluid

- Couple a dissipative fluid with a scalar field $\Sigma = \sigma e^{i\psi}$ of mass $m = m_0(T - T_c)$.

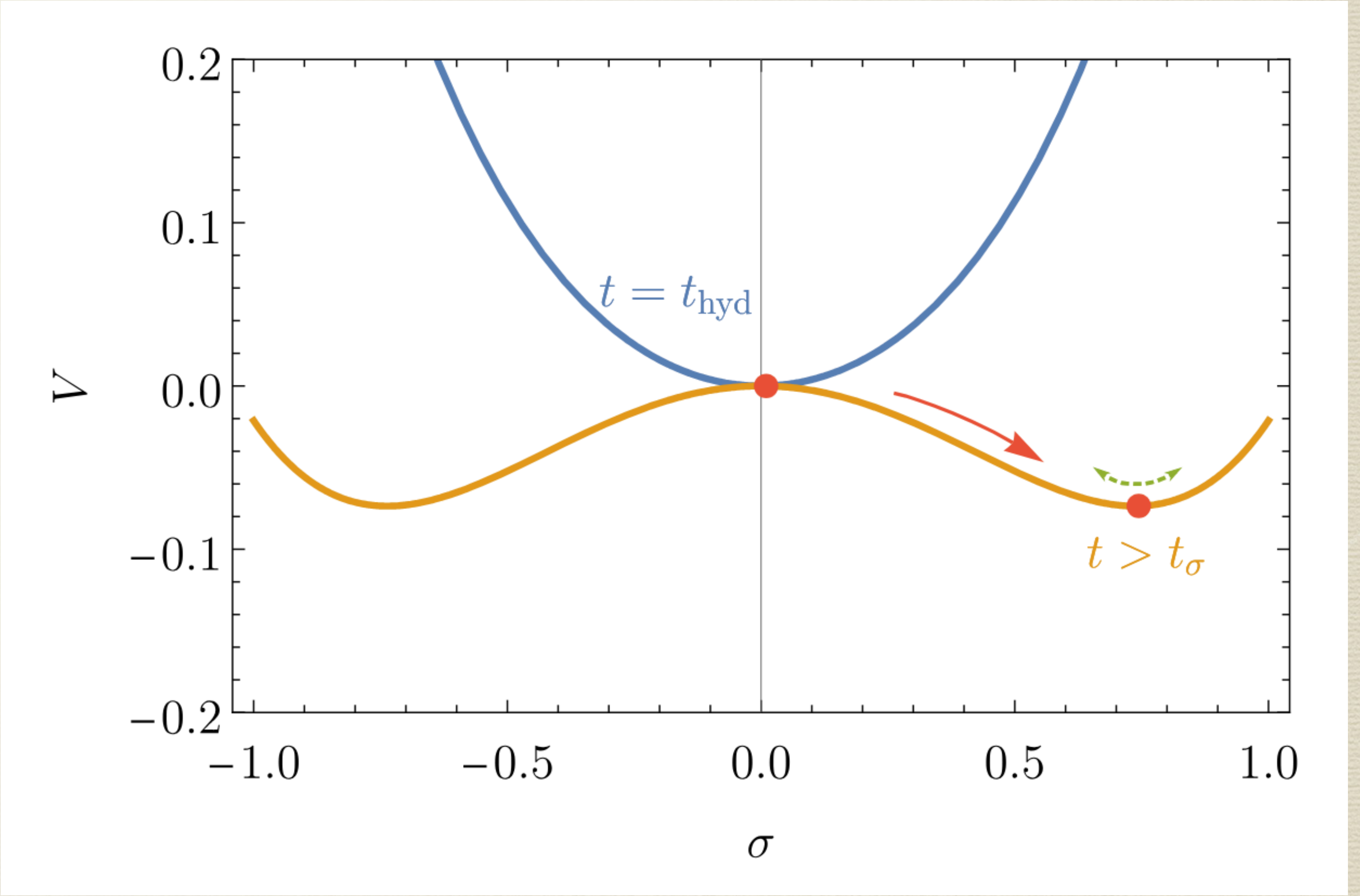
$$S = \int d^4x \sqrt{-g}\; P(T, \mu, \Sigma)$$

$$P(T, \mu, \Sigma) \equiv p(T, \mu) - \frac{1}{2}(D_\mu \Sigma)(D^\mu \Sigma)^\dagger - V(\Sigma, T)$$

Fluid pressure

$$\frac{m_0(T - T_c)\sigma^2}{2} + \frac{1}{4}\lambda\sigma^4$$

σ : order parameter and ψ : $U(1)$ phase



Potential as a function of temperature and condensate.
arXiv:2410.01892

Equations

- ∗ $\mu = 0$ and $\psi = const$

- Conservation equation:

$$\boxed{\nabla_\mu T^{\mu\nu} = 0}$$

- Equation of motion:

$$\boxed{\begin{gathered}\frac{\delta S}{\delta \sigma} = 0 \\ \frac{\delta S}{\delta \sigma} = -\kappa_1 (u \cdot \nabla)\sigma \qquad \kappa_1 = C_{\kappa_1} T\end{gathered}}$$

$$(T^\mu_\nu)_n + (T^\mu_\nu)_{scalar} \qquad \pi^\mu_\nu + \Pi\ \Delta^\mu_\nu$$

- ∗ Dissipation:

$$\boxed{T^\mu_\nu = T^\mu_{\nu\ ideal} + \underbrace{\Pi^\mu_\nu}_{fluid-dissipative}}$$

$$diag(0,\pi/2,\pi/2,-\pi)$$

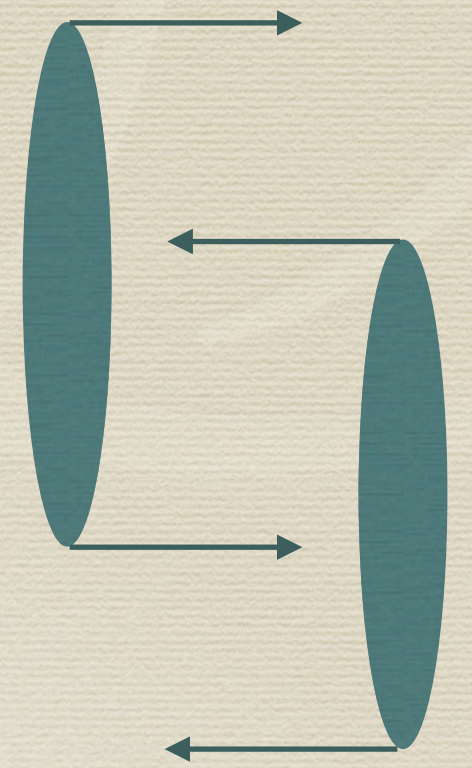
- ∗ MIS formalism:

$$\boxed{\begin{gathered}\tau_\pi \left(u \cdot \nabla + \frac{4}{3} \nabla \cdot u \right) \pi^\mu_\nu + \pi^\mu_\nu = -2\eta\sigma^\mu_\nu, \\ \tau_\Pi (u \cdot \nabla)\ \Pi + \Pi = -\zeta \nabla \cdot u,\end{gathered}}$$

$$\eta = C_\eta\ s \qquad \tau_\pi = C_{\tau_\pi} T^{-1}$$

$$\zeta = C_\zeta\ T^3 \qquad \tau_\Pi = C_{\tau_\Pi} T^{-1}$$

Expanding background



Anisotropic expansion

$\eta \neq 0$

$\zeta = 0$

Bjorken flow

Longitudinal expansion

$$ds_B^2 = -\,d\tau^2 + dx_\perp^2 + \tau^2 d\eta^2$$

$$ds_G^2 = \tau^{-2} ds_B^2$$

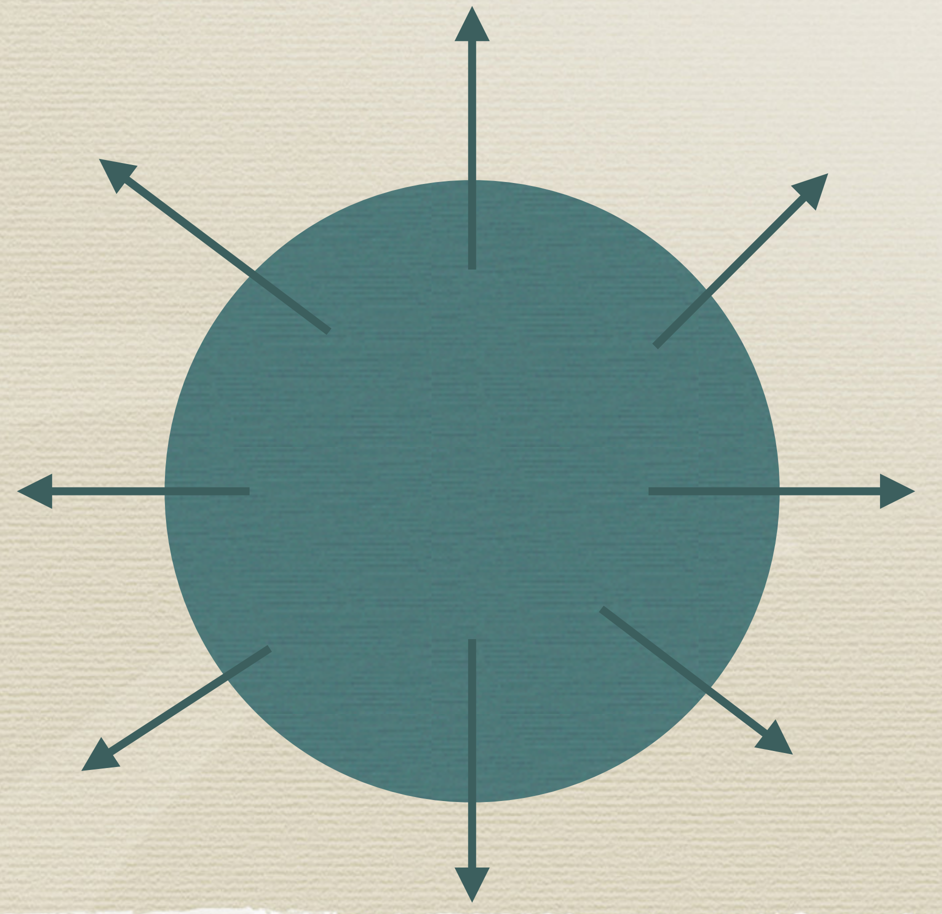
Gubser flow

Transverse + longitudinal expansion

$$ds_G^2 = -\,d\tilde{t}^2 + \cosh^2 \tilde{t}\ d\Omega^2 + d\eta^2$$

$$d\theta^2 + sin^2\theta d\phi^2$$

$$\tilde{t} \equiv \tilde{t}(\tau, x_\perp),\ \ \theta \equiv \theta(\tau, x_\perp)$$

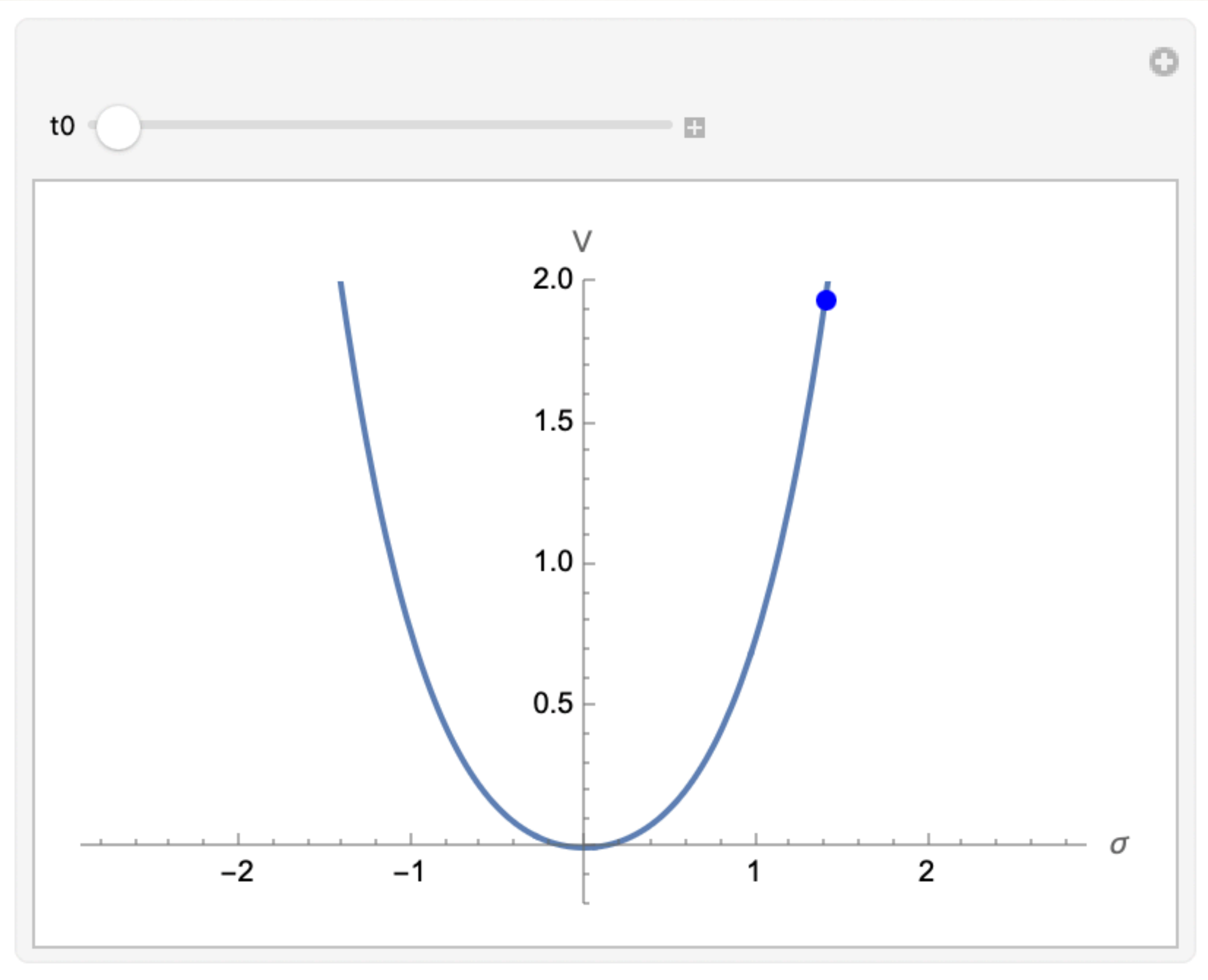


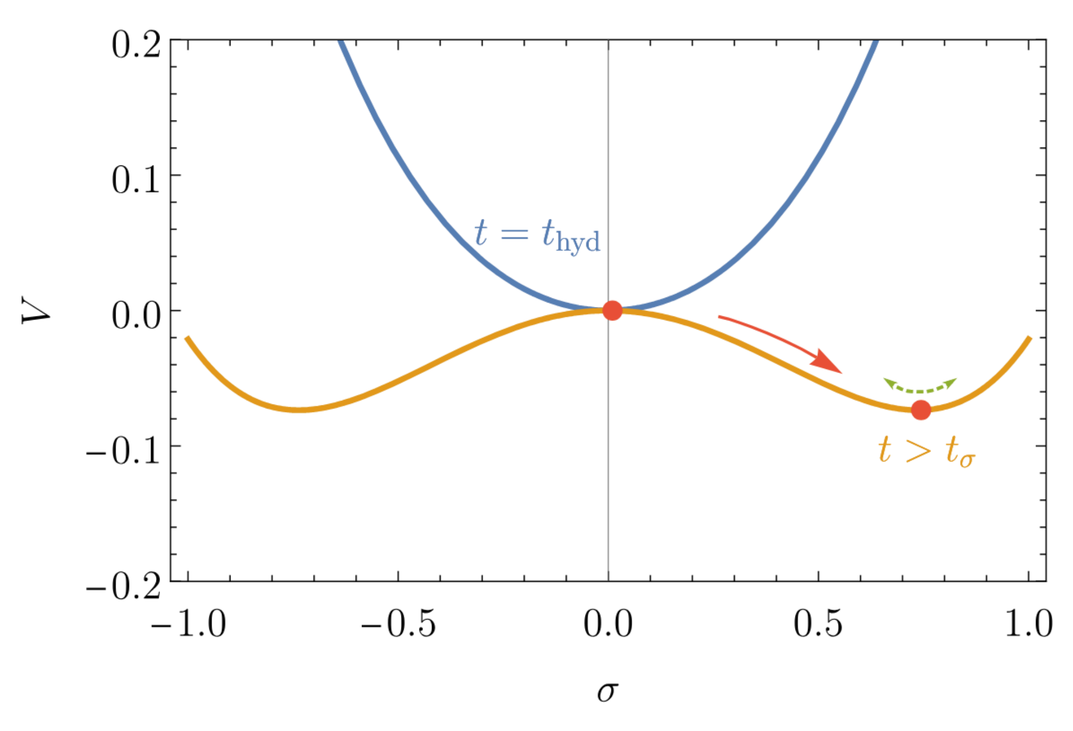
Isotropic expansion

FLRW $\qquad \zeta \neq 0$

$$ds_F^2 = -\,dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$$

$$V = \frac{m_0(T(\tau) - T_c)\sigma^2}{2} + \frac{1}{4}\lambda\sigma^4$$





Potential as a function of temperature and condensate. arXiv:2410.01892

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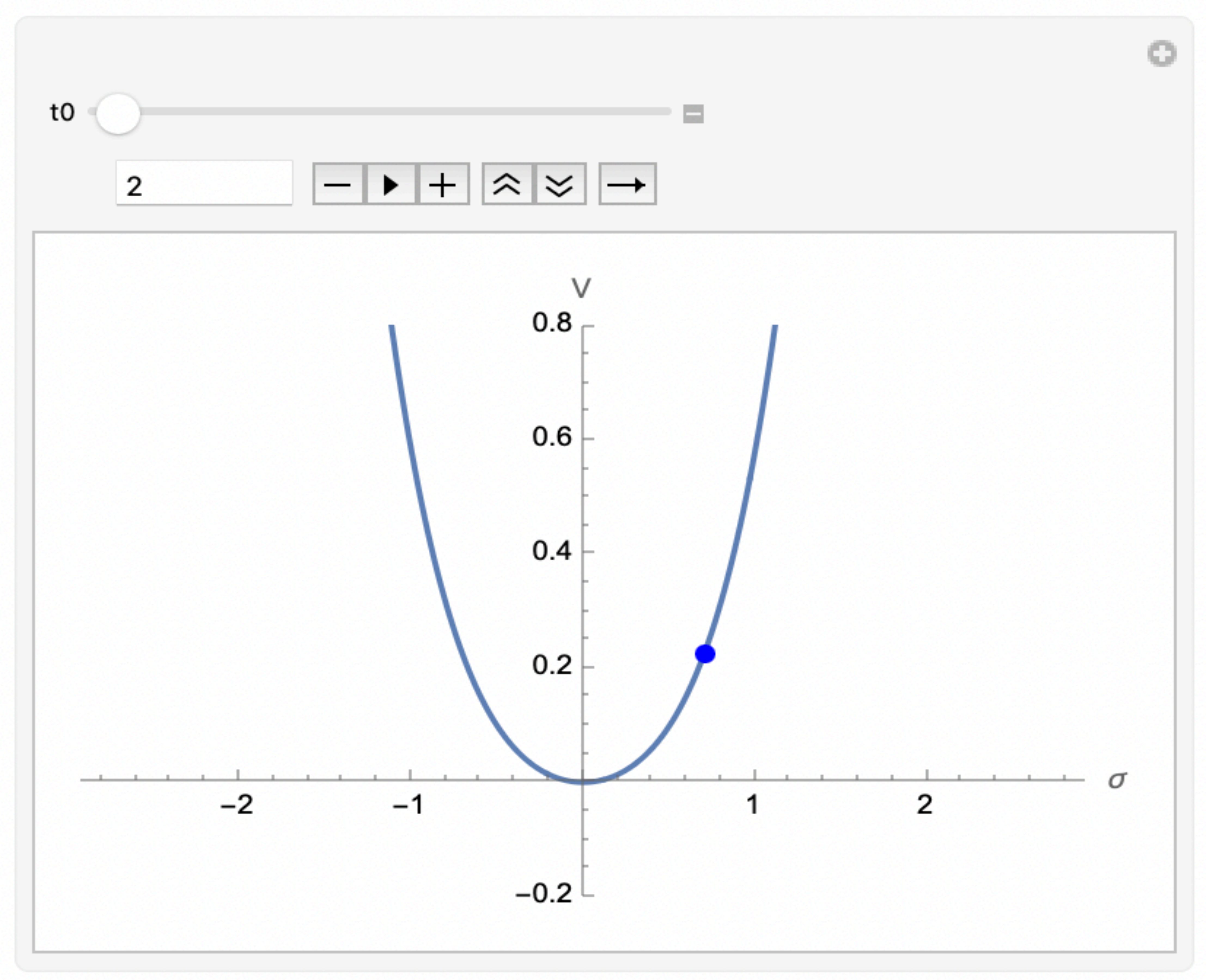

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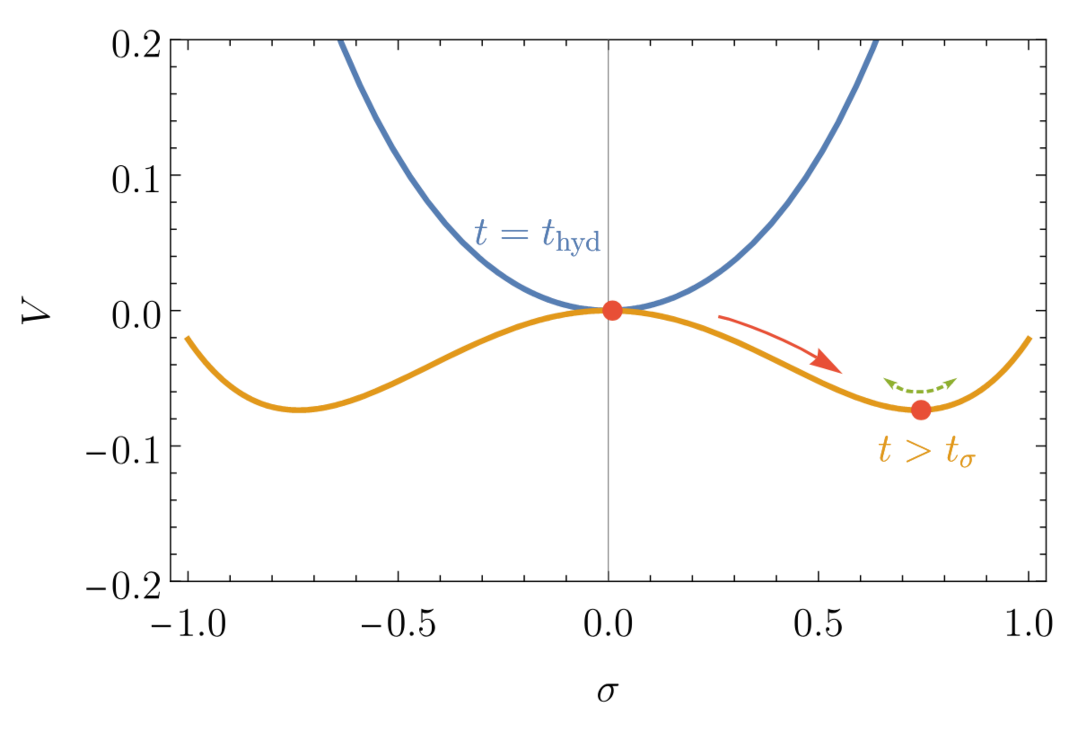

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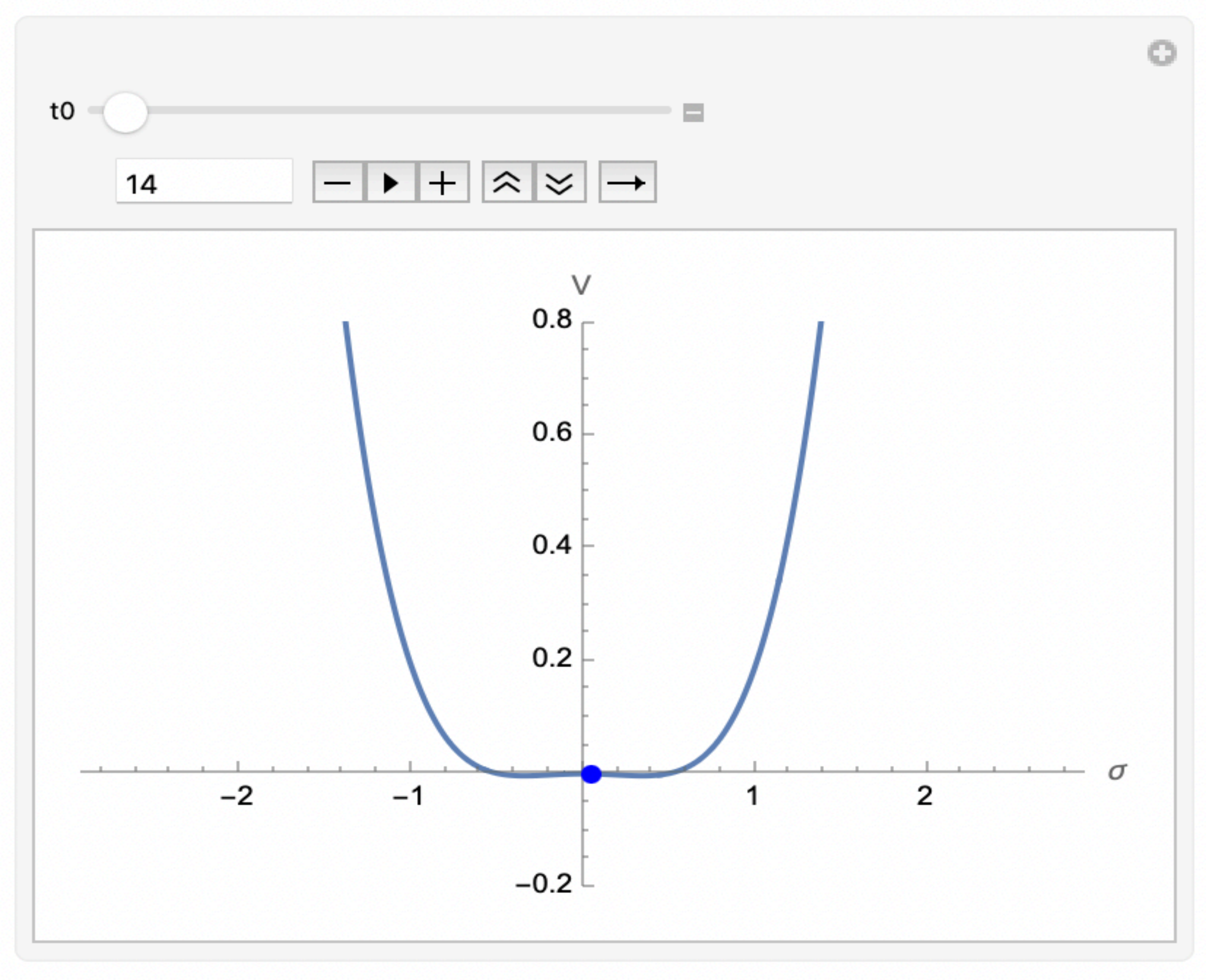

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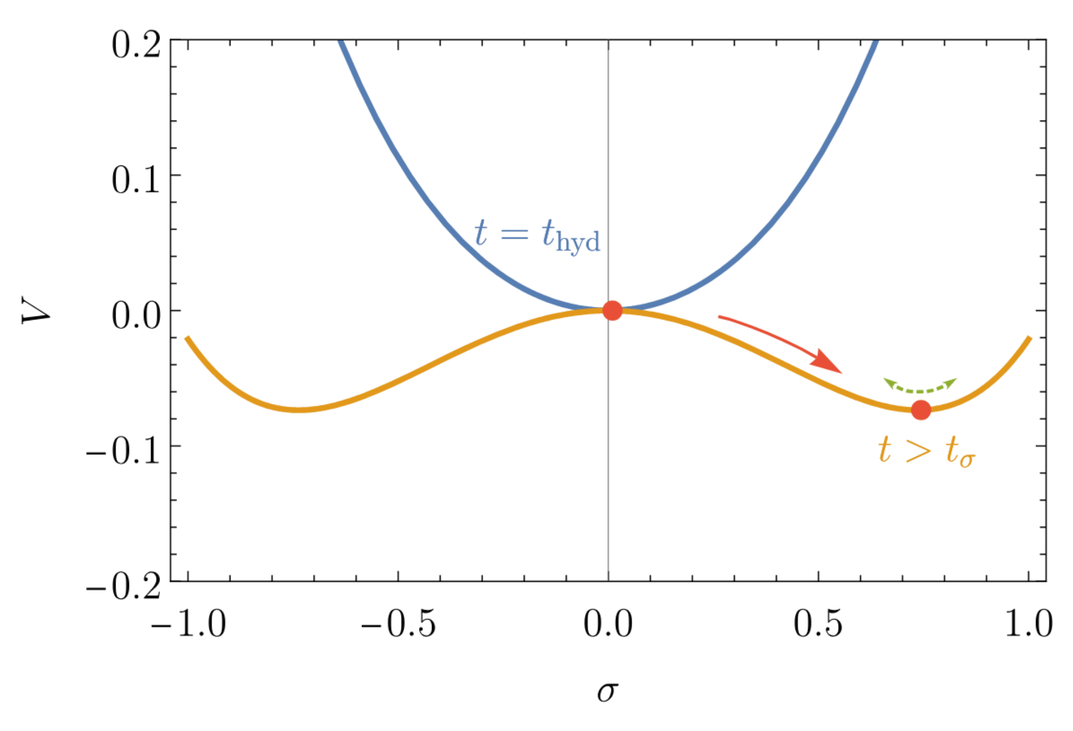

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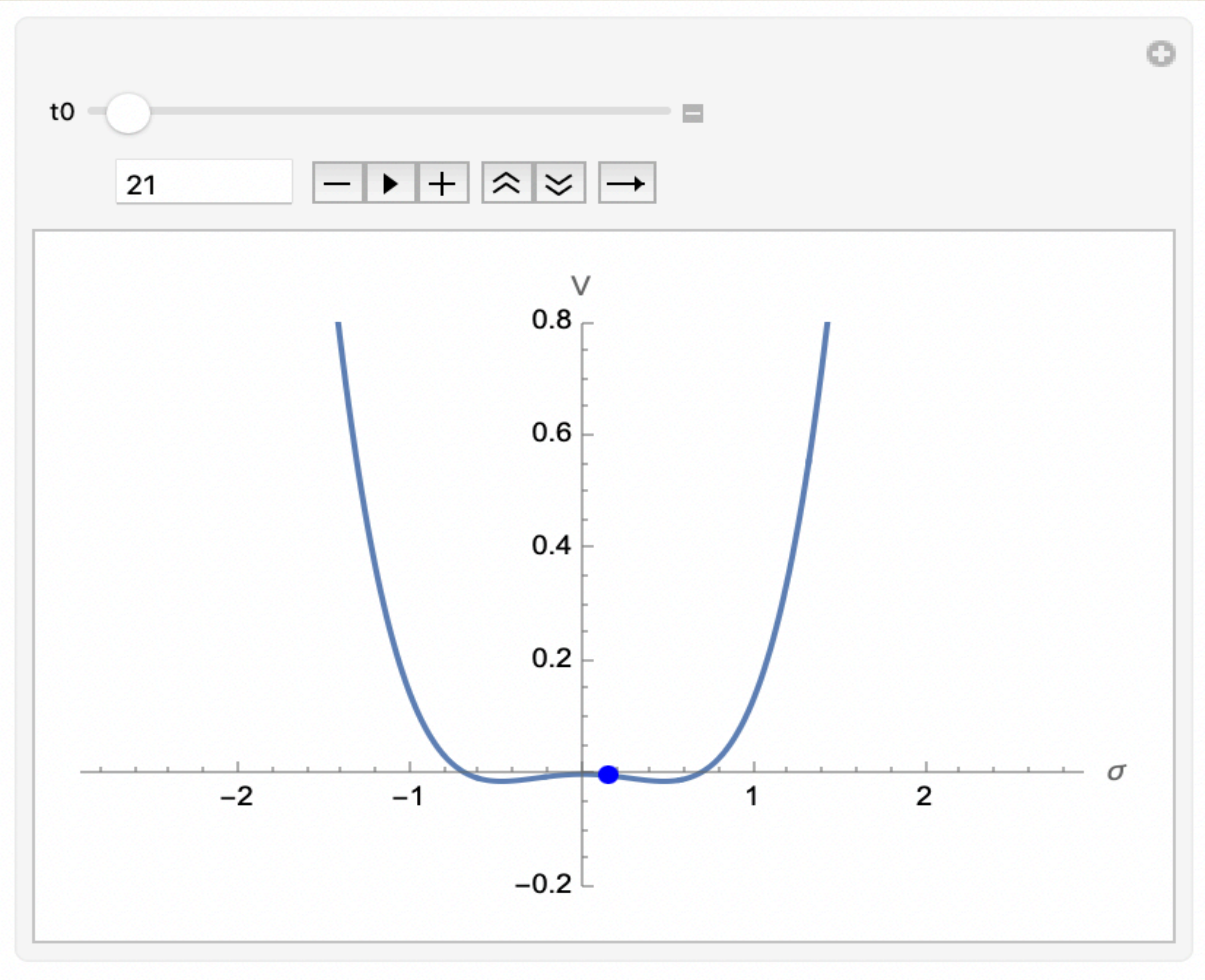

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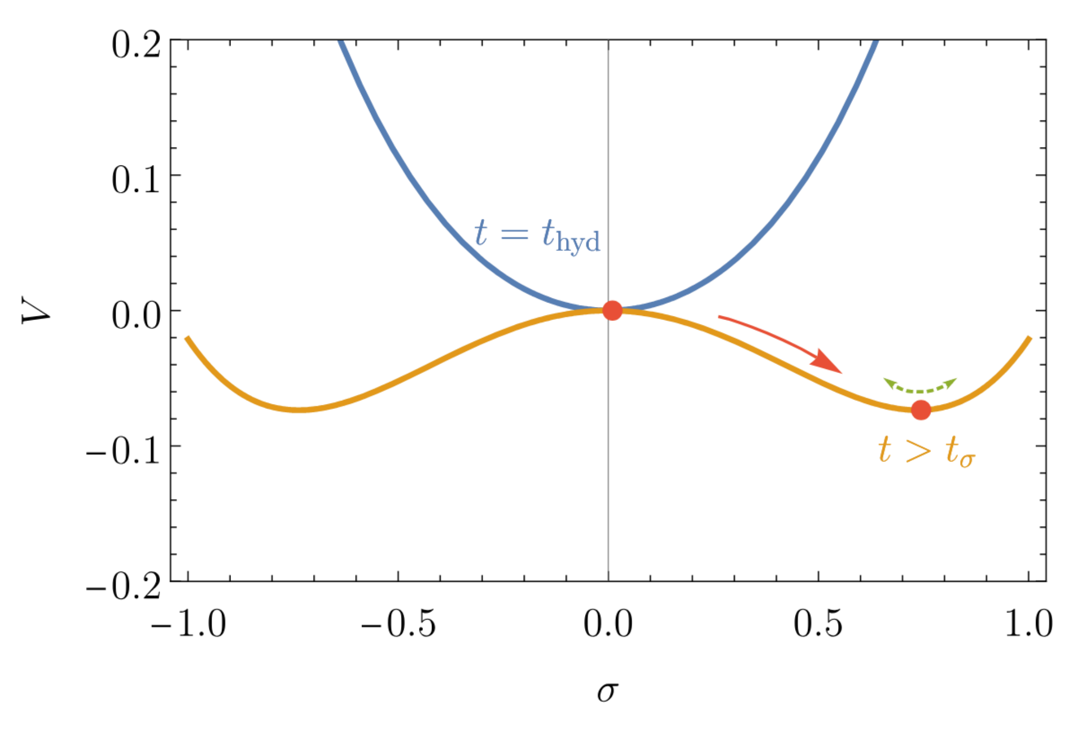

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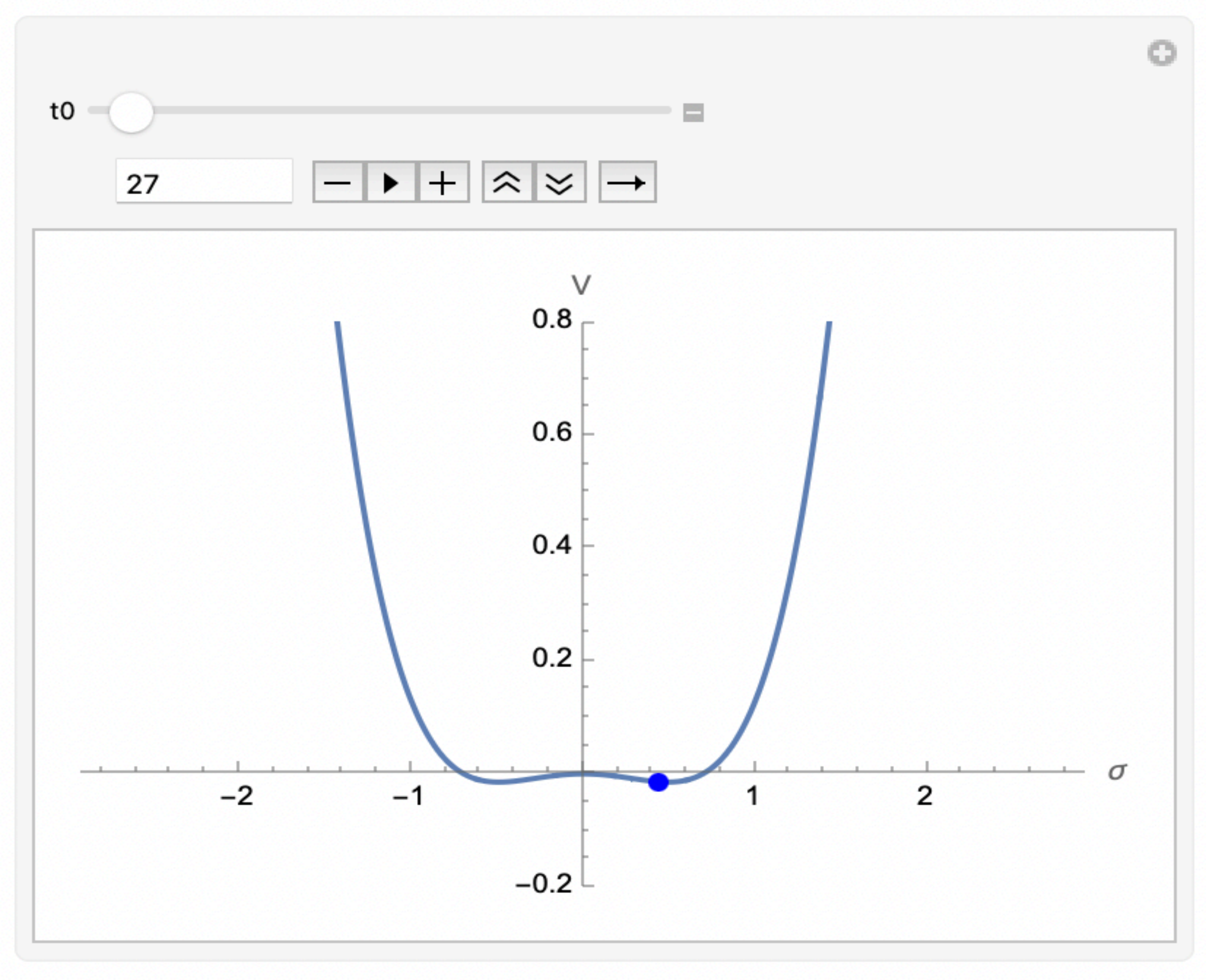

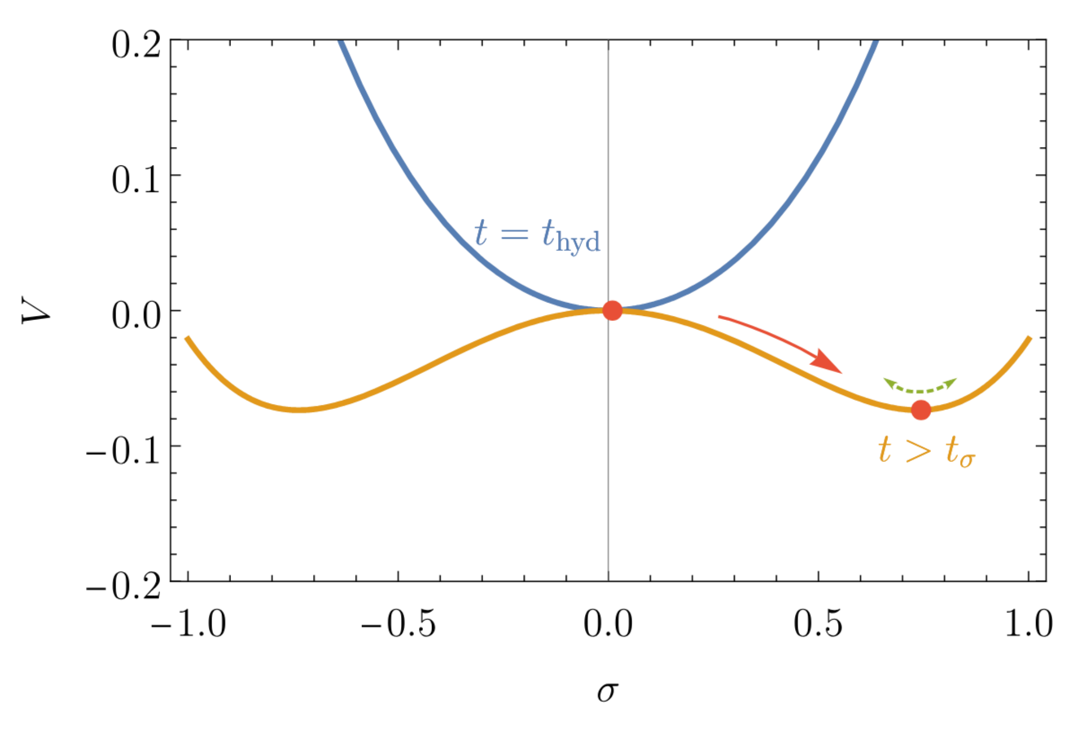

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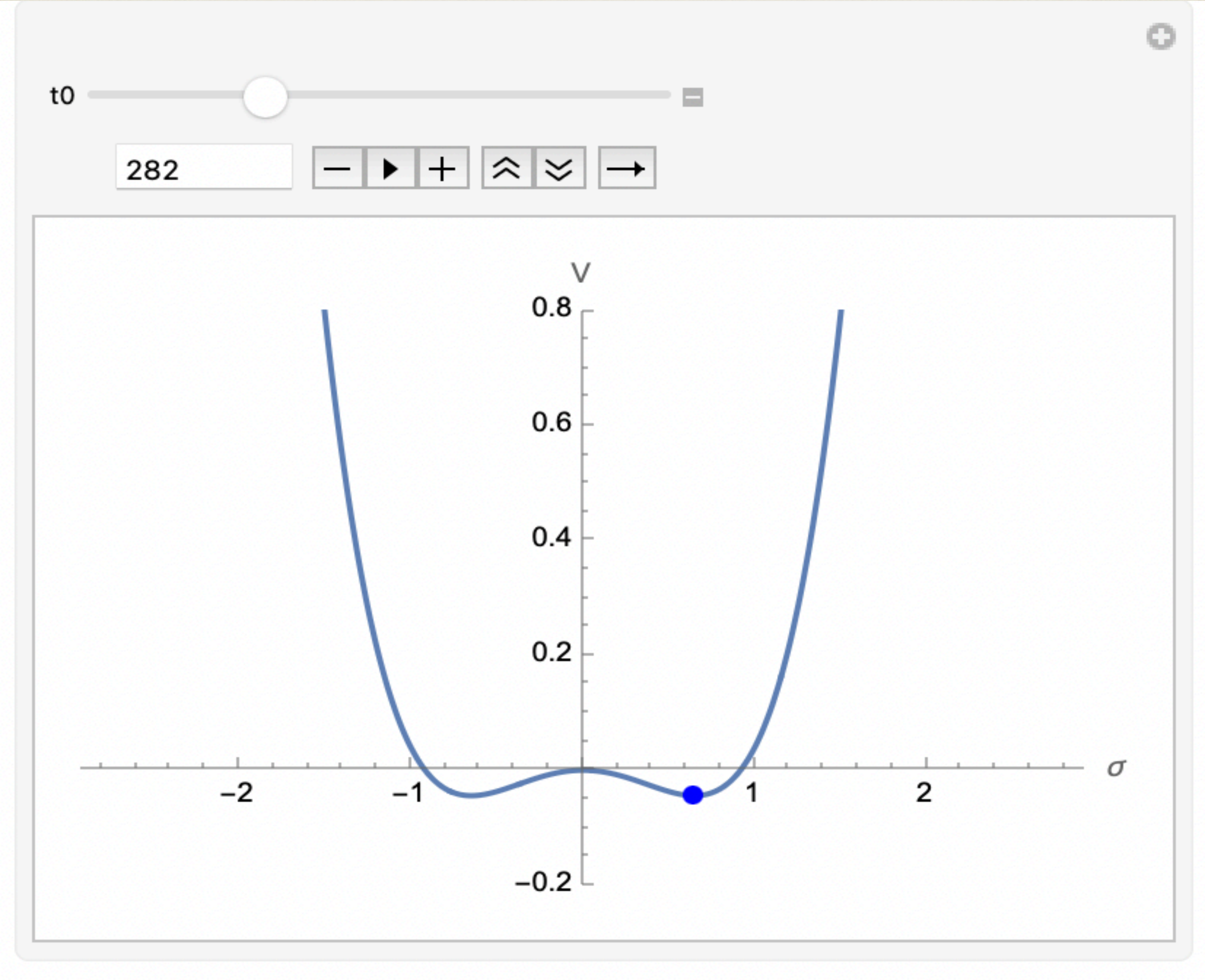

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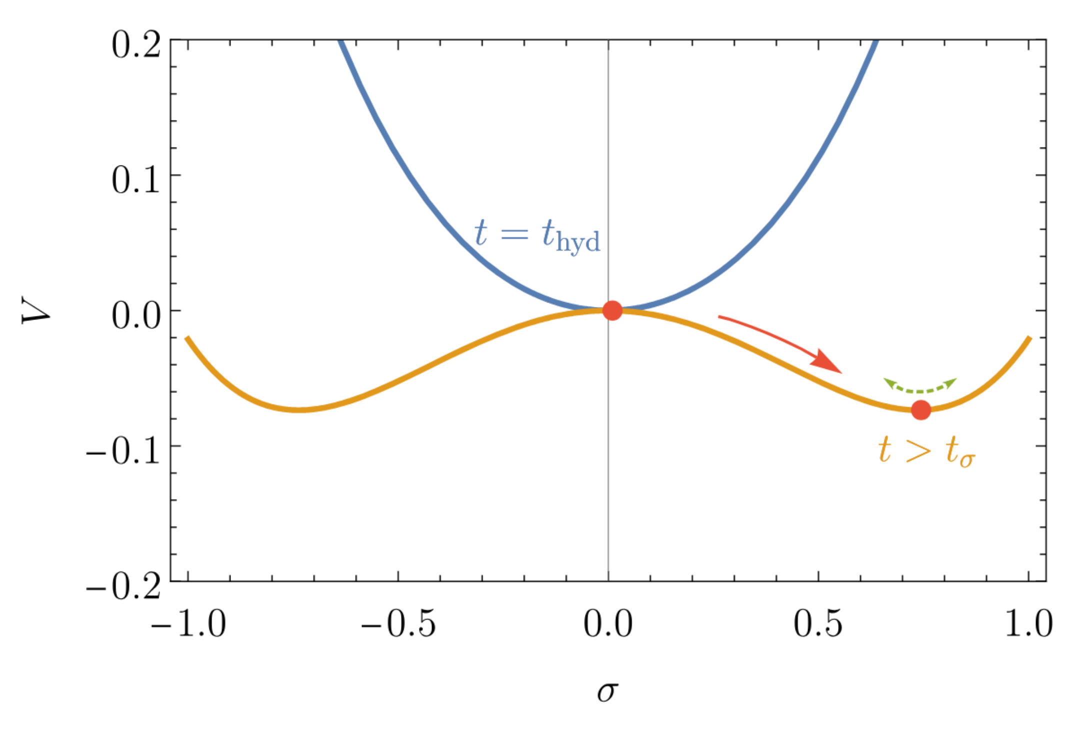

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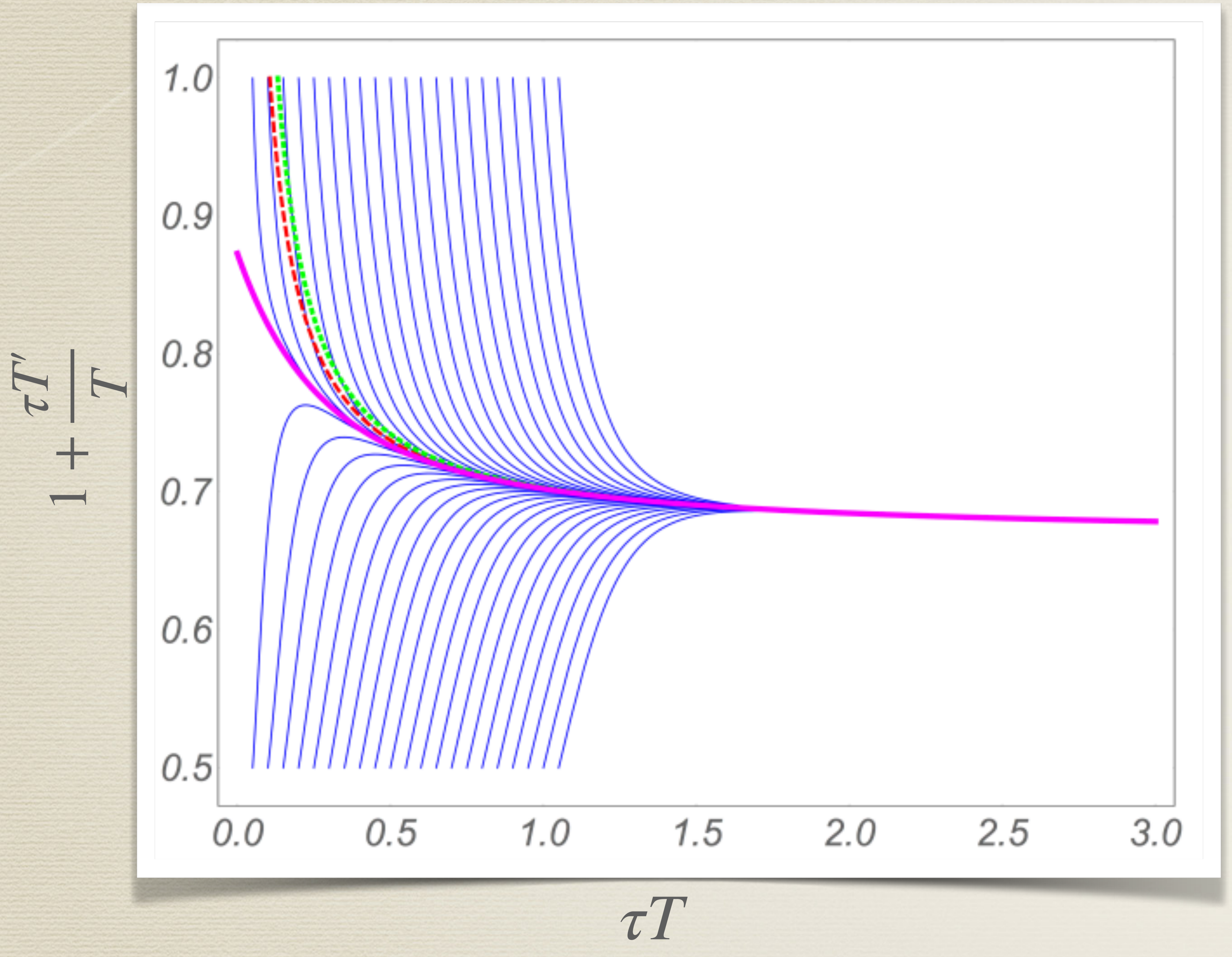
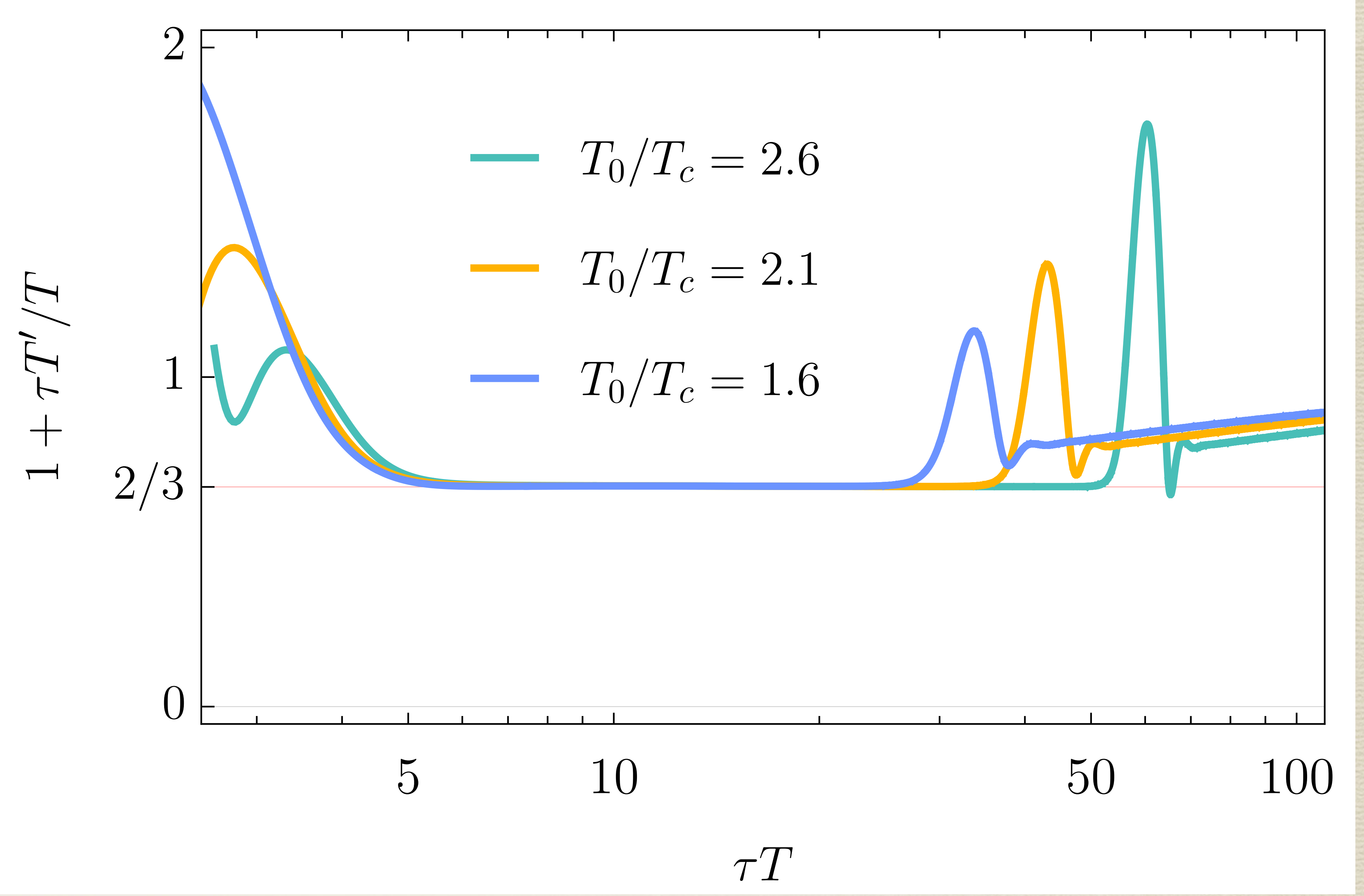
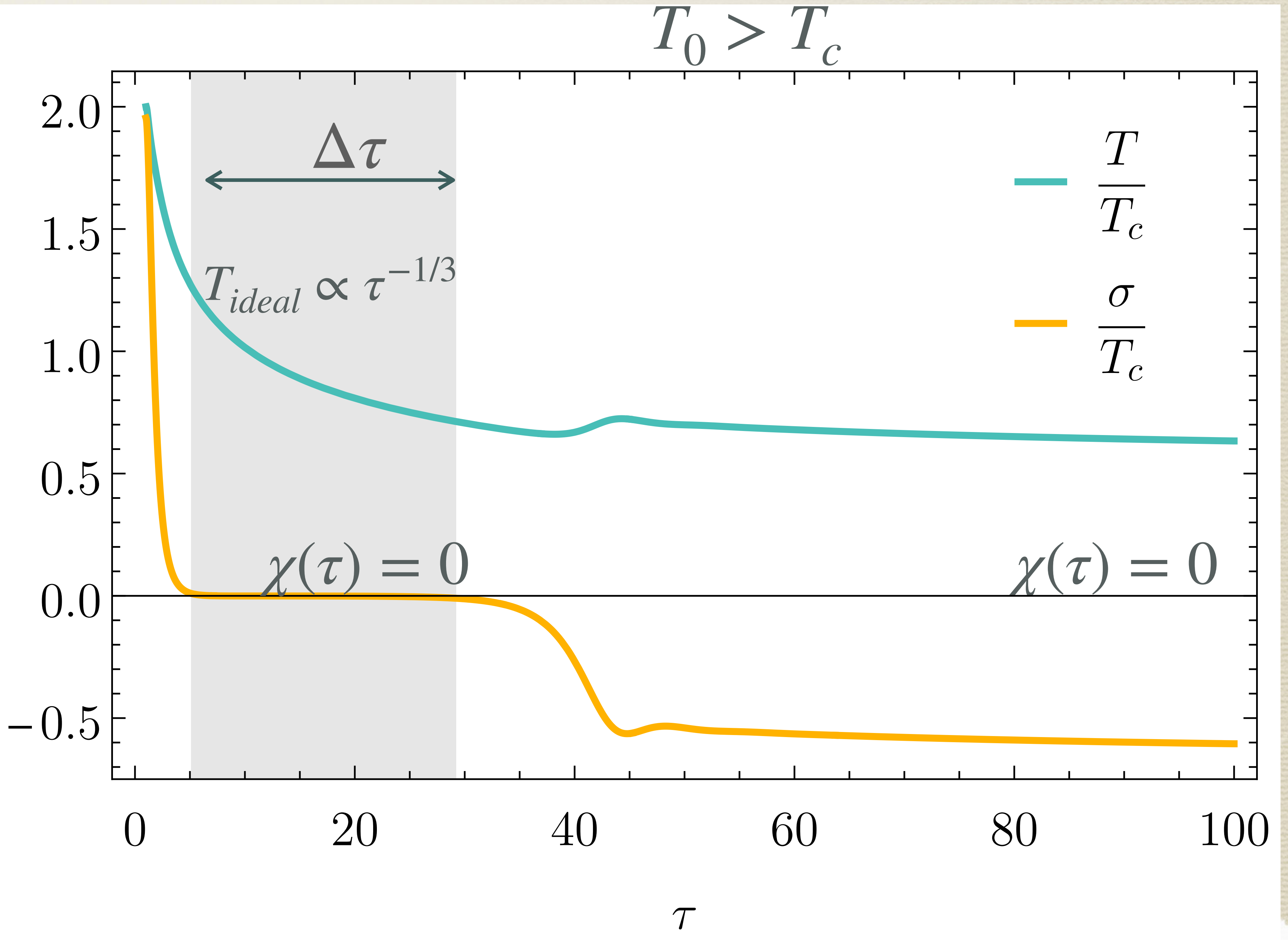
Bjorken flow   D.o.f: $T, u^\mu, \sigma, \chi \propto \pi/T^4$



Heller and Spaliński. 2015





I - Initial condition.

II - Hydro.

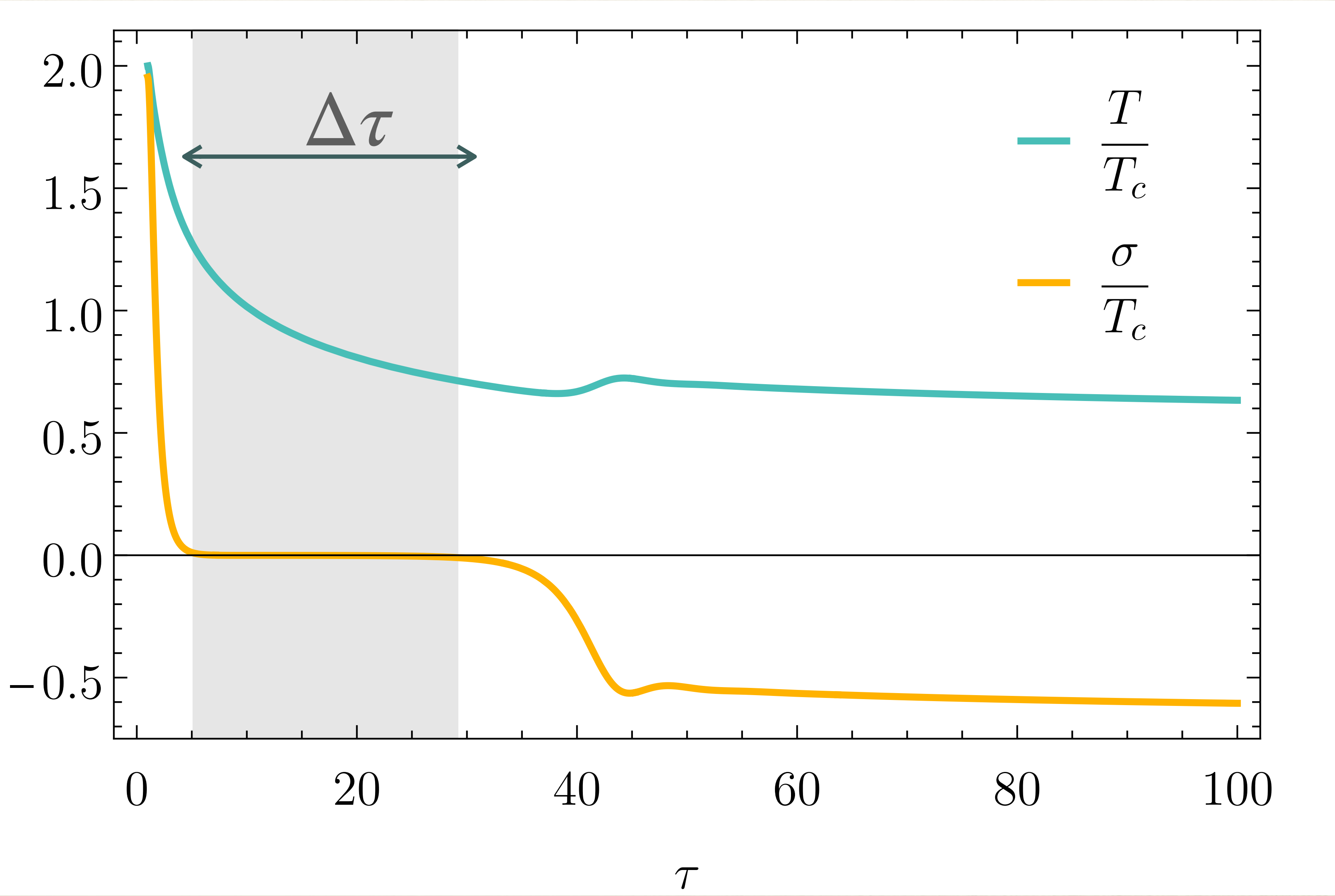
III- Symmetry breaking regime.

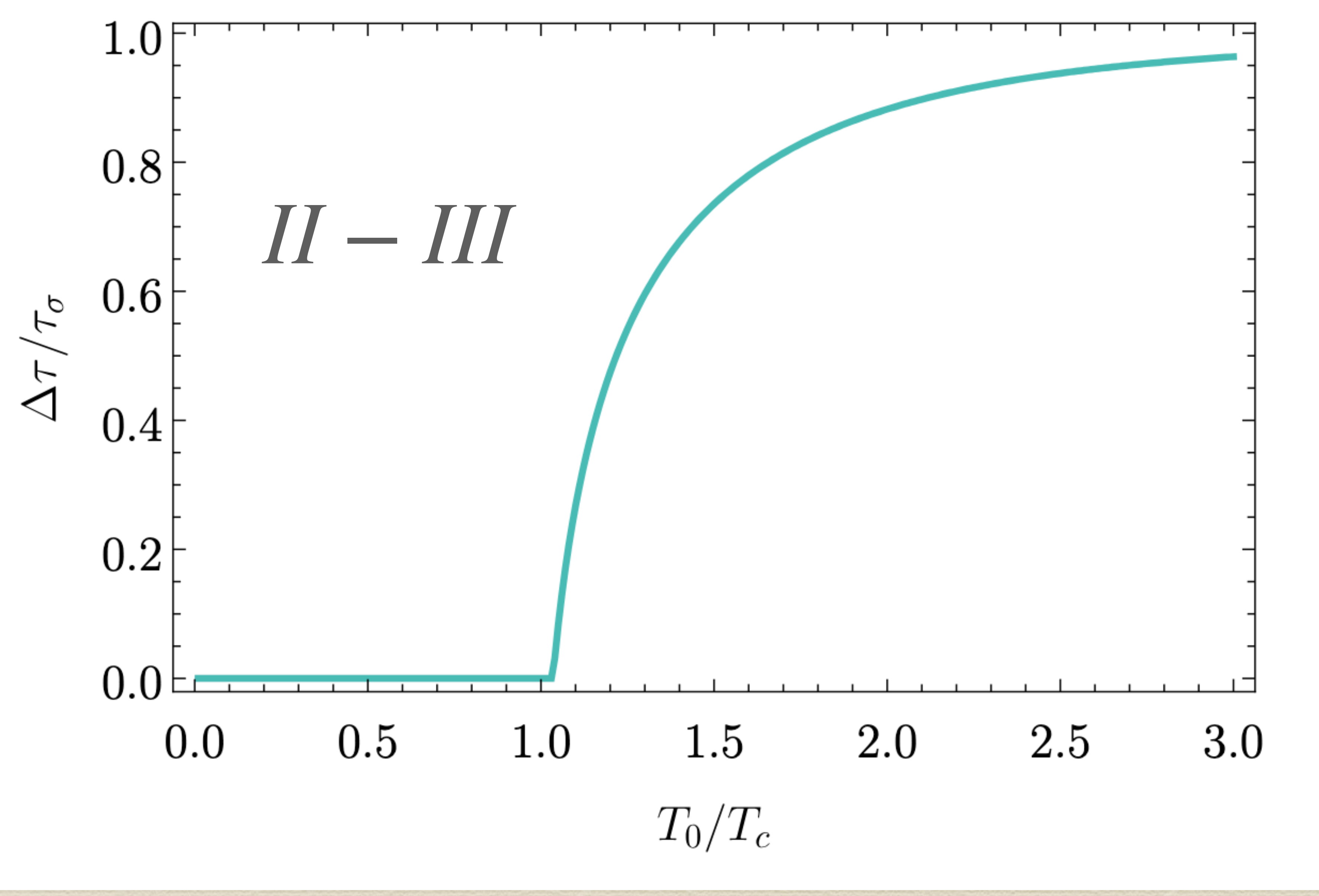
Attractor time

- Transition from hydro-like behaviour to the symmetry breaking fixed points.

- Attractor time: Onset of hydrodynamic behaviour $\tau_{hyd}(\sigma = 0, \chi = 0)$ to the onset of condensate regime $\tau_\sigma(\chi = 0, \sigma \neq 0)$.

- Bjorken: $\Delta\tau = \tau_\sigma - \tau_{hyd} \equiv |\sigma(\tau)| \leq 10^{-3}$





The figure shows the attractor time in Bjorken flow

Gubser flow

I - Initial condition.

II - Inviscid hydro.

III- Viscous hydro.

Denicol, Noronha 2018

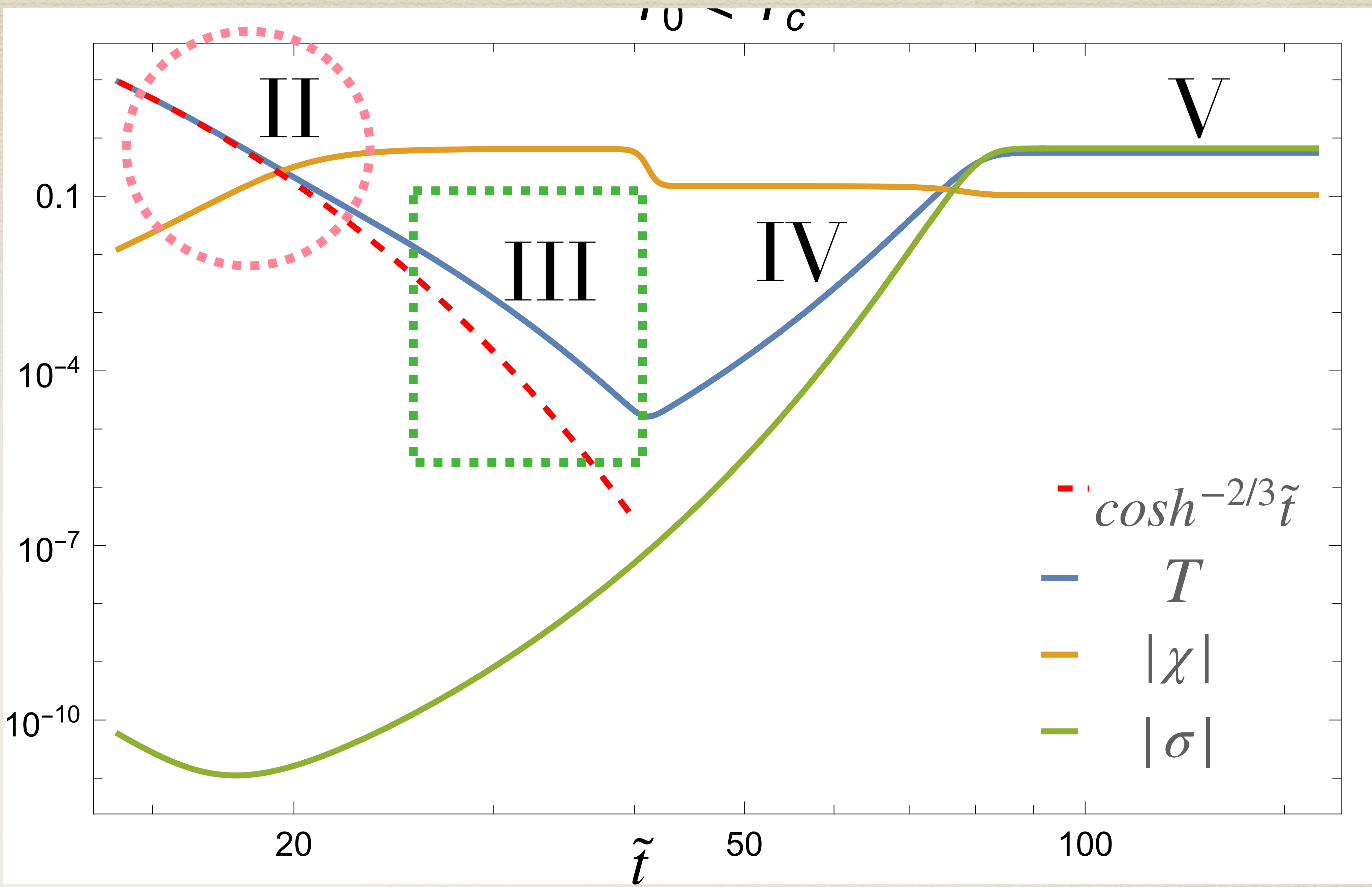
IV- Non-linear regime.

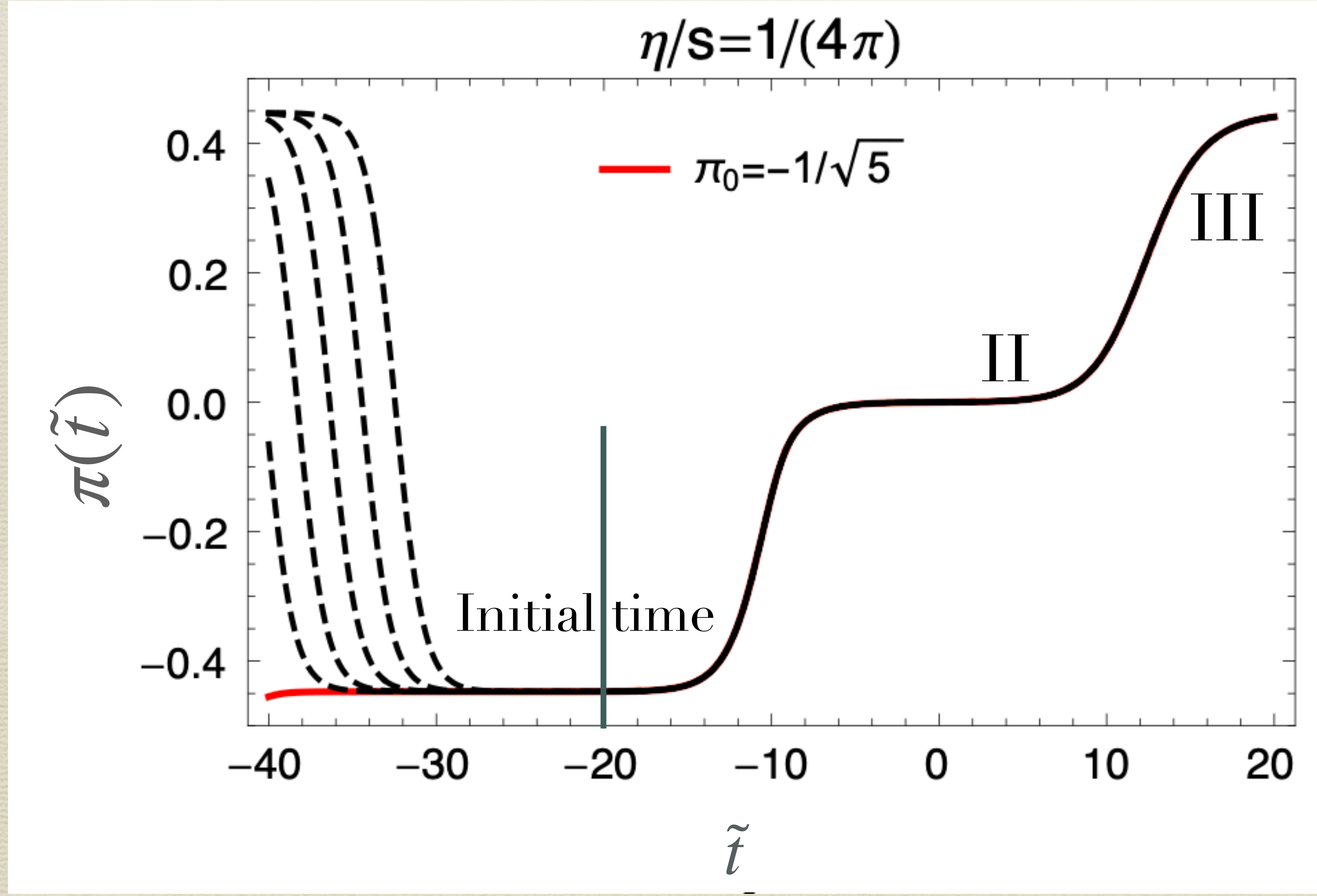
V- Symmetry breaking regime.

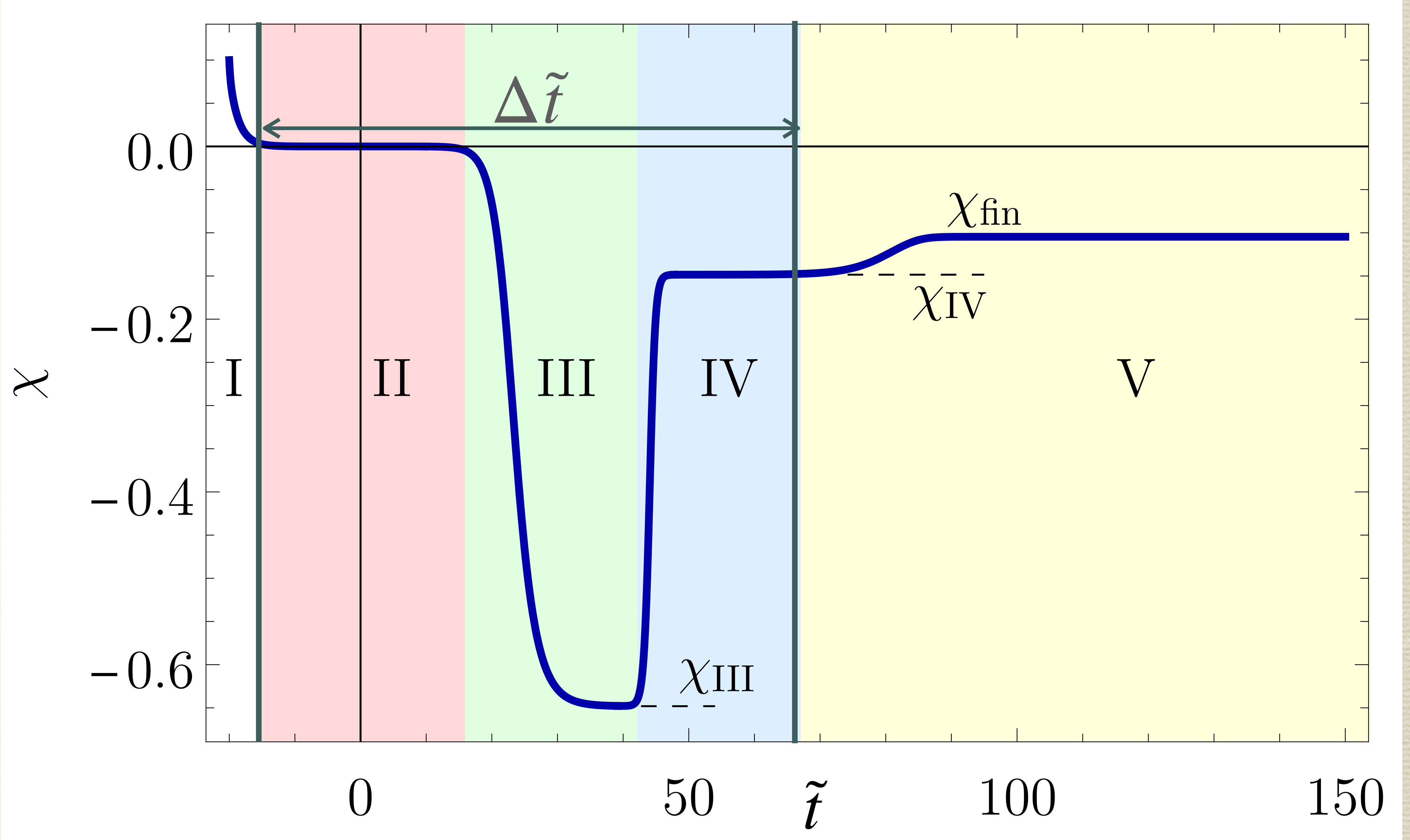
$$\chi_{\rm II} = 0$$

$$\chi_{\rm III} \propto -\left(\frac{C_\eta}{C_{\tau_\pi}}\right)^{1/2}$$

$$\chi_{\rm IV} \propto -\left(\frac{C_\eta}{C_{\tau_\pi}}\right)$$



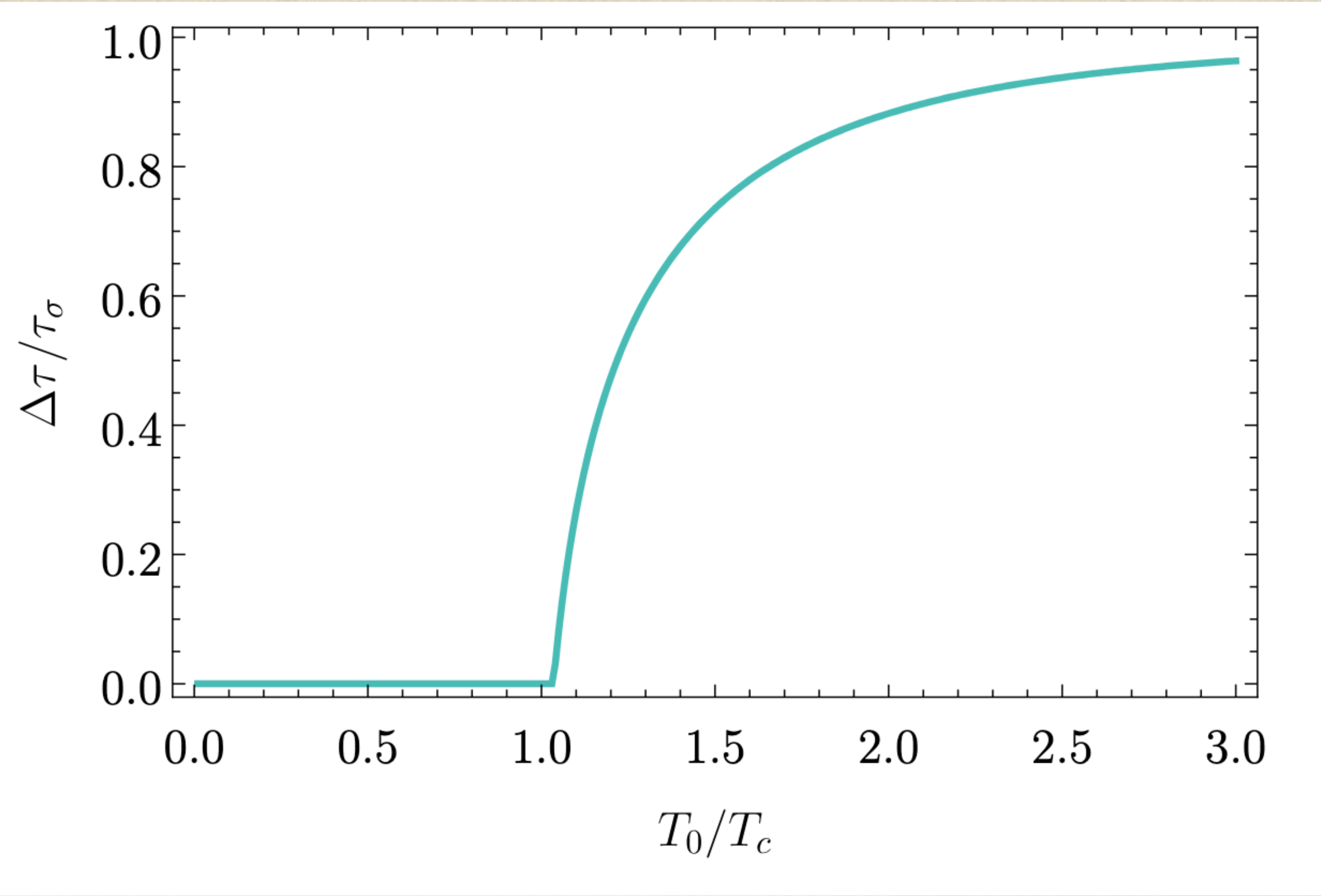




Denicol, Noronha 2018

Attractor time in Gubser:

$$\Delta\tilde{t} := |\sigma(\Delta\tilde{t})| \leq 10^{-3}$$



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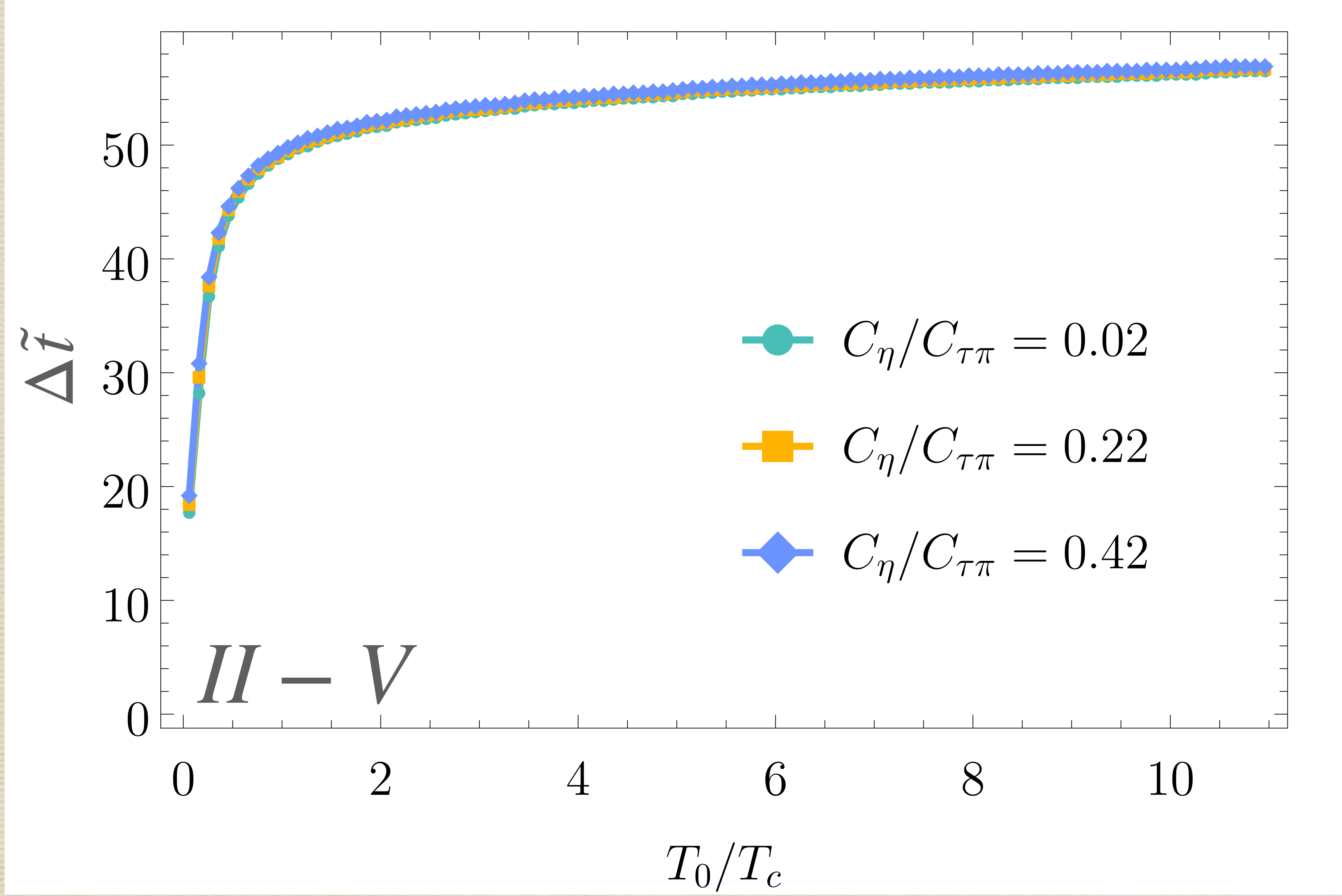
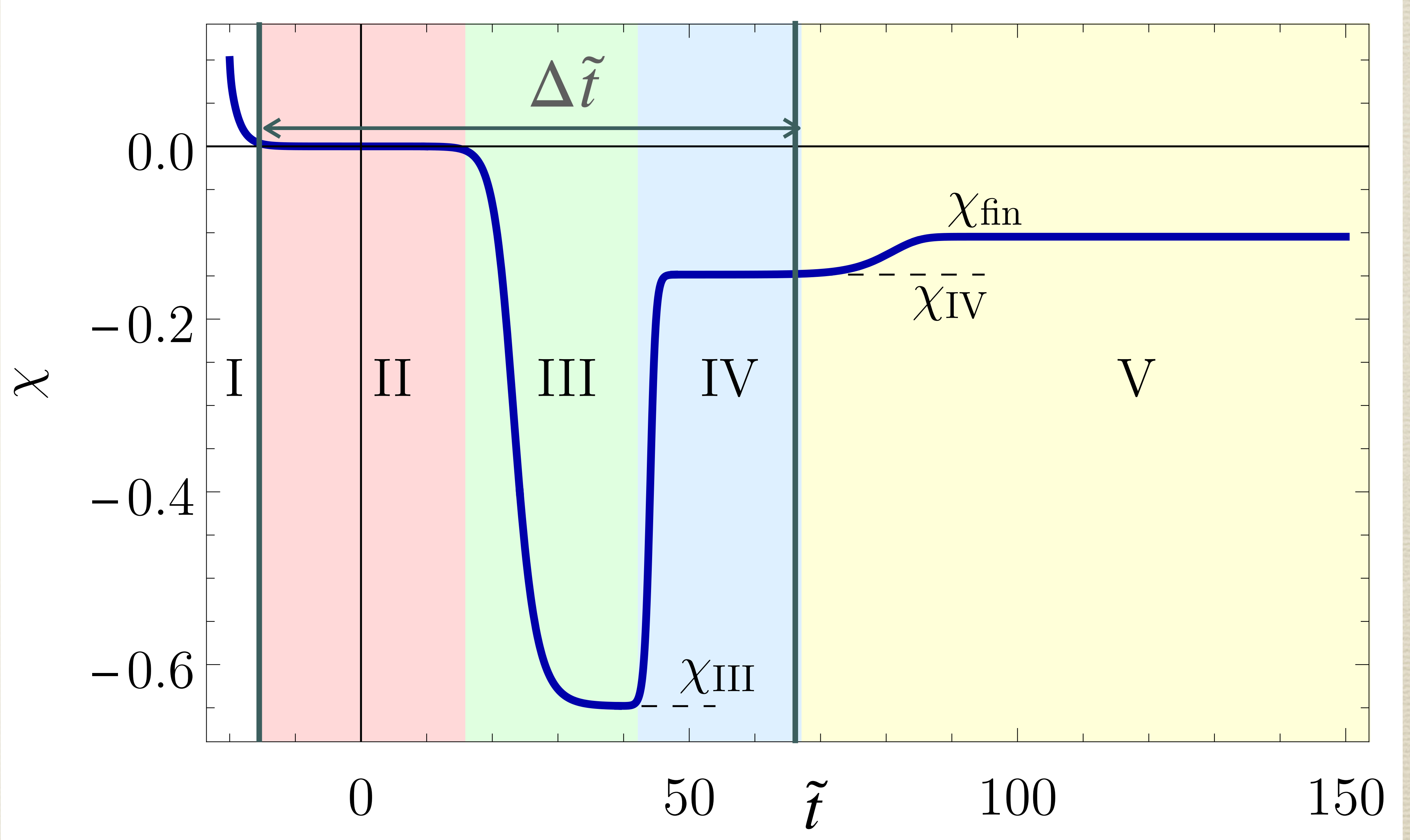




Summary

- Expanding superfluid with dissipative correction features intermediate hydrodynamic behaviour followed by fixed point in broken phase.

- This formalism gives a notion of attractor time.

- Evolving out of hydro-regime depends on the initial conditions.

FLRW - isotropic expansion

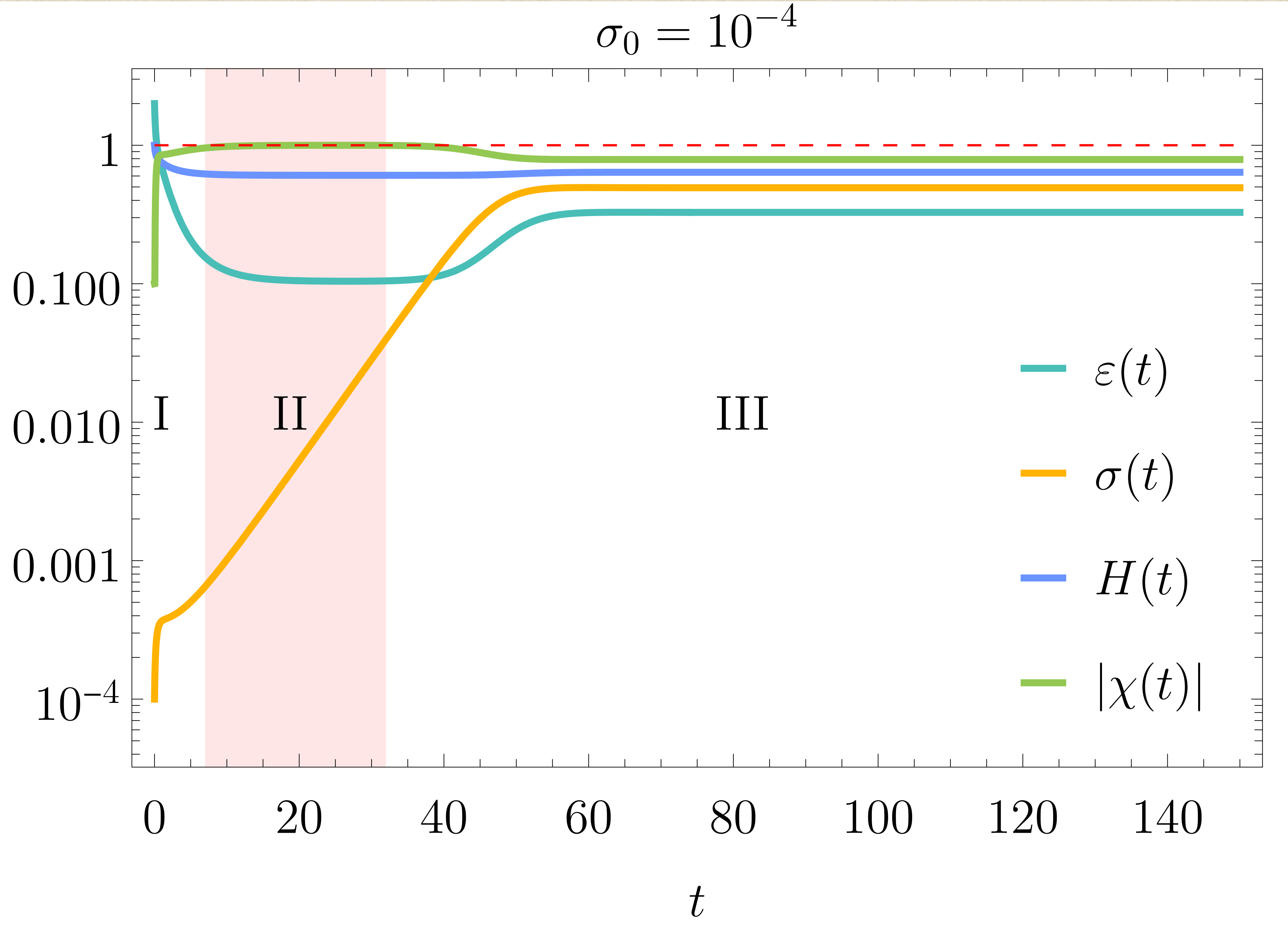
$\zeta \neq 0,\ \sigma^{\mu}_{\nu} = 0$

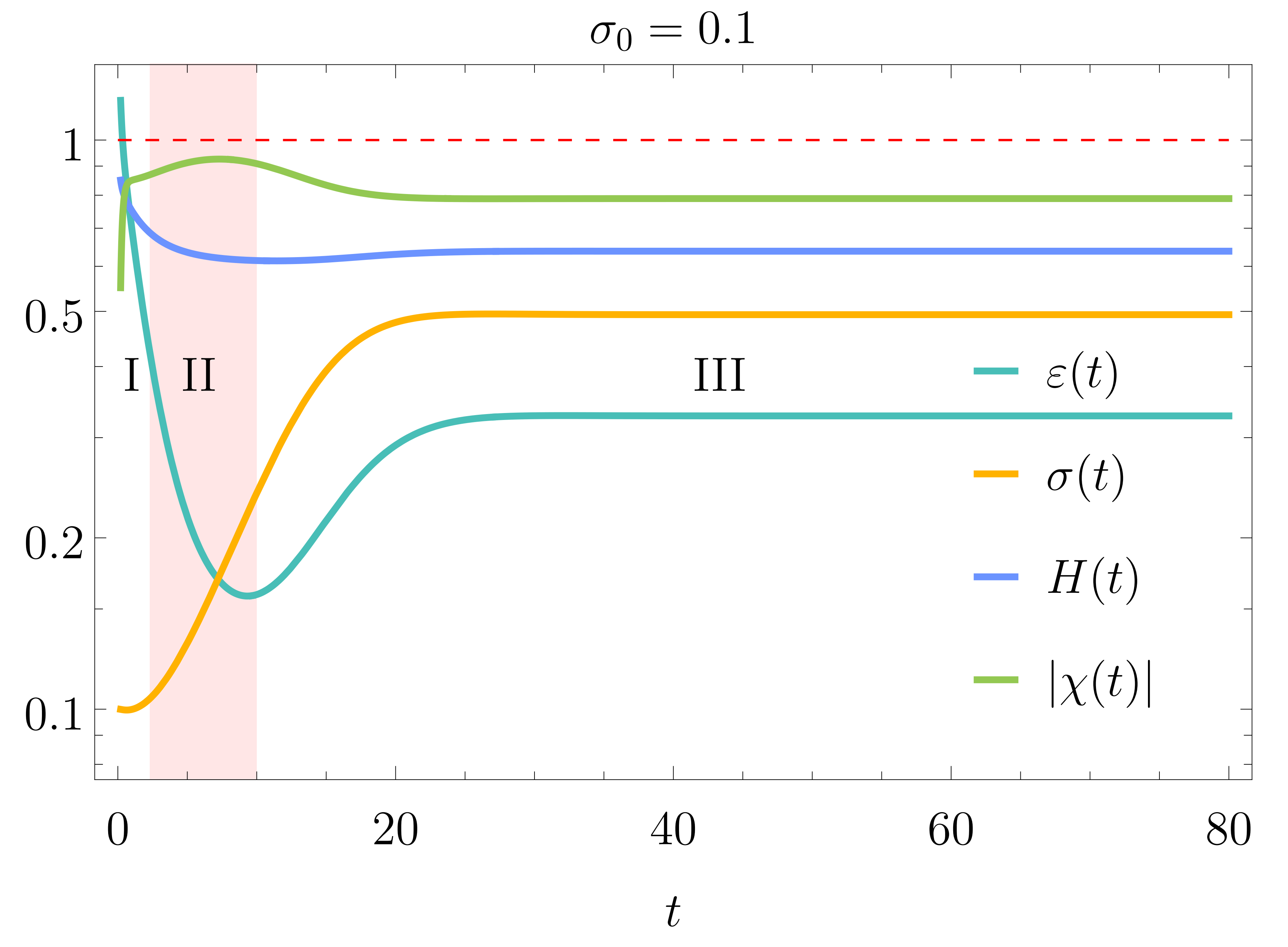
- Dynamical background: $ds_F^2 = -\,dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$

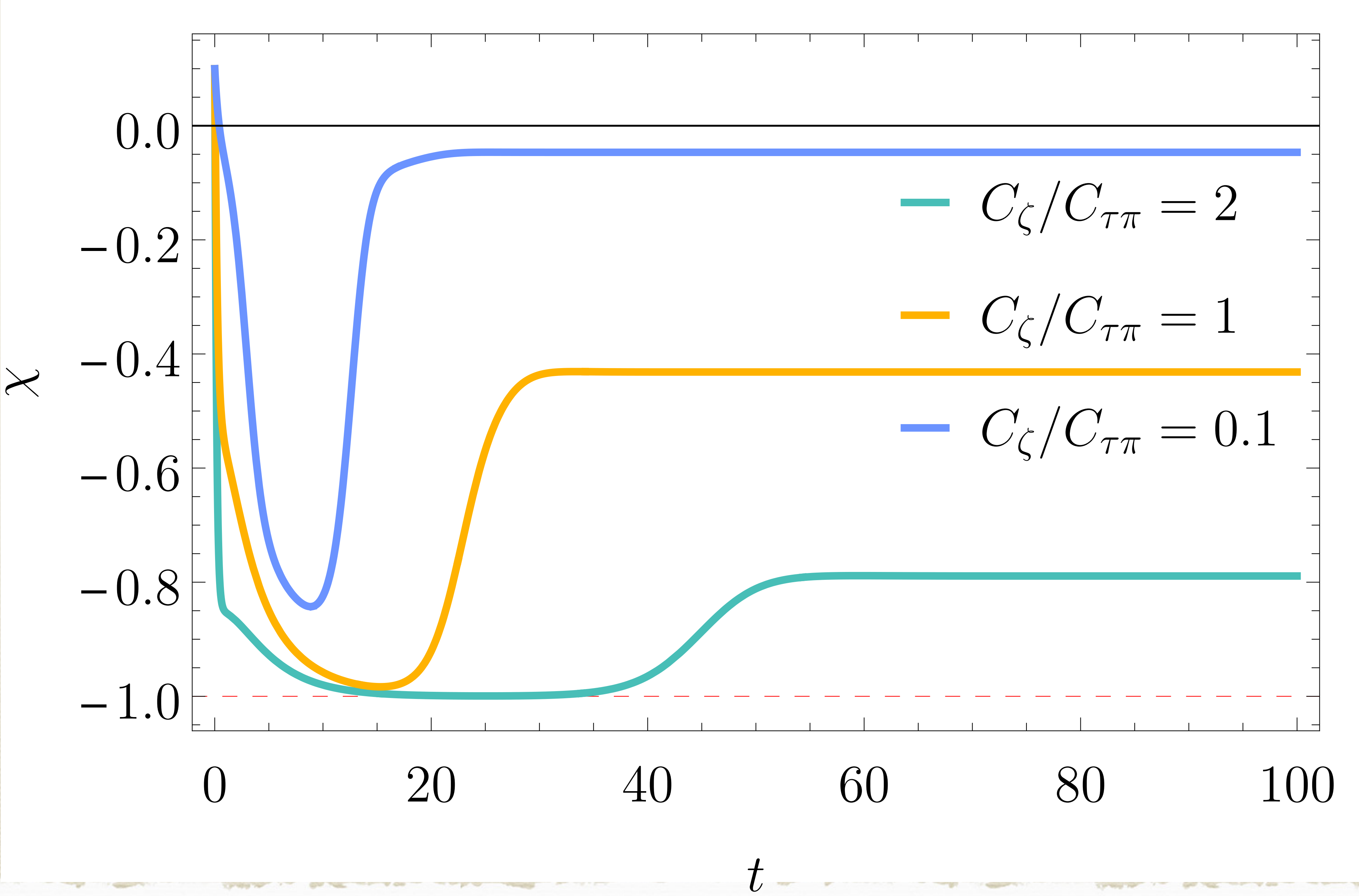
- Einstein -Hilbert action $\boxed{S_{\mathrm{EH}} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,(R - 2\Lambda) + S_m}$

- Equations: $\nabla_\mu T^{\mu\nu} = 0$ $\quad \frac{\delta S}{\delta\sigma} = -\,\kappa_1(u\,.\,\nabla)\sigma$ $\quad \tau_\Pi\,(u\,.\,\nabla)\,\Pi + \Pi = -\,\zeta\,\nabla\,.\,u$

$$R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa\,T_{\mu\nu}, \qquad H = \dot{a}/a$$


$\sigma_0 = 10^{-4}$

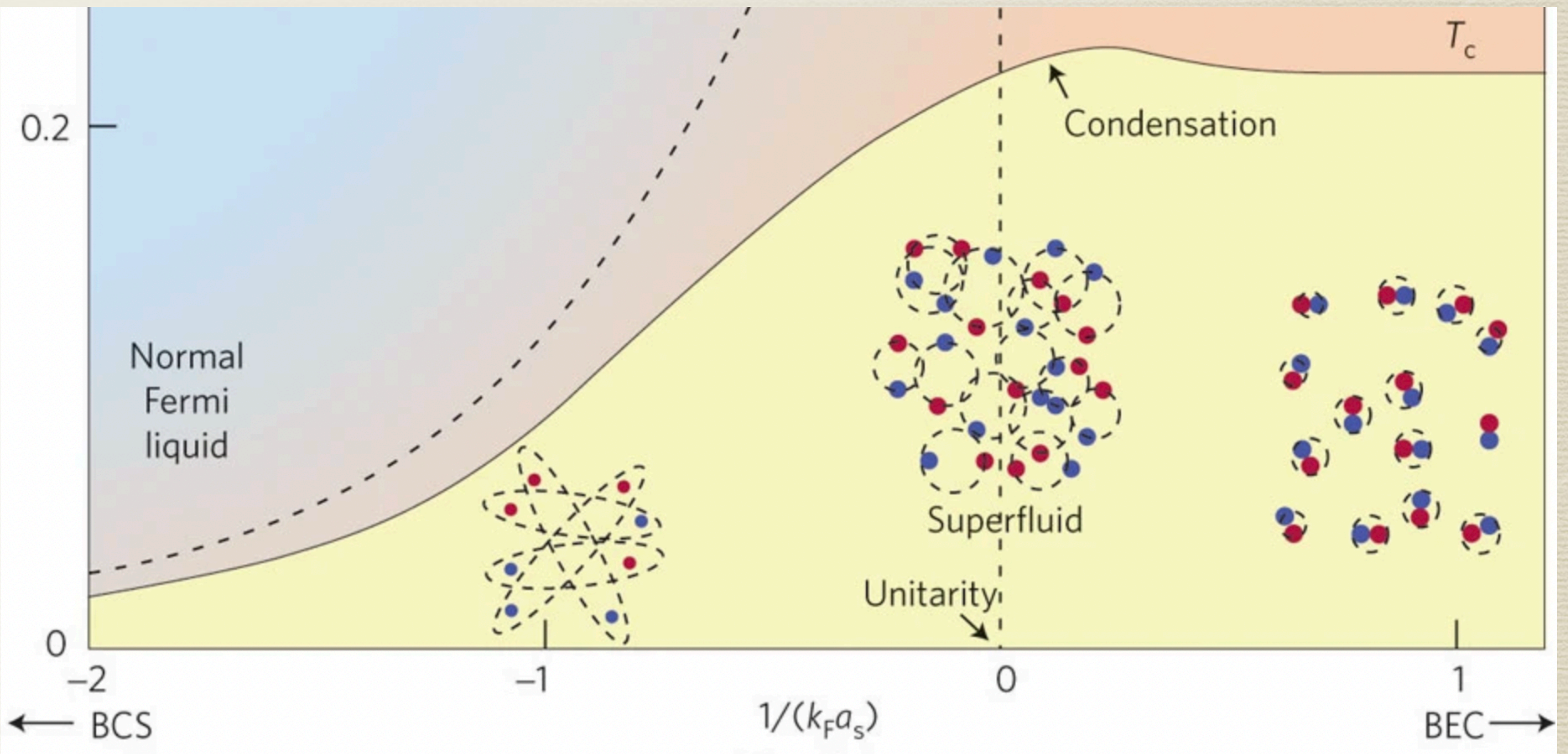

$\sigma_0 = 0.1$



Outlook

- Mathematical understanding of the expanding superfluid.
- Including non-zero chemical potential in the formalism.
- Implementing the formalism to cold atom?

$$S = \int d^4x \;\; p(T, \mu, k_F a_s) + S_{micro}[\psi, \bar{\psi}, k_F a_s]$$



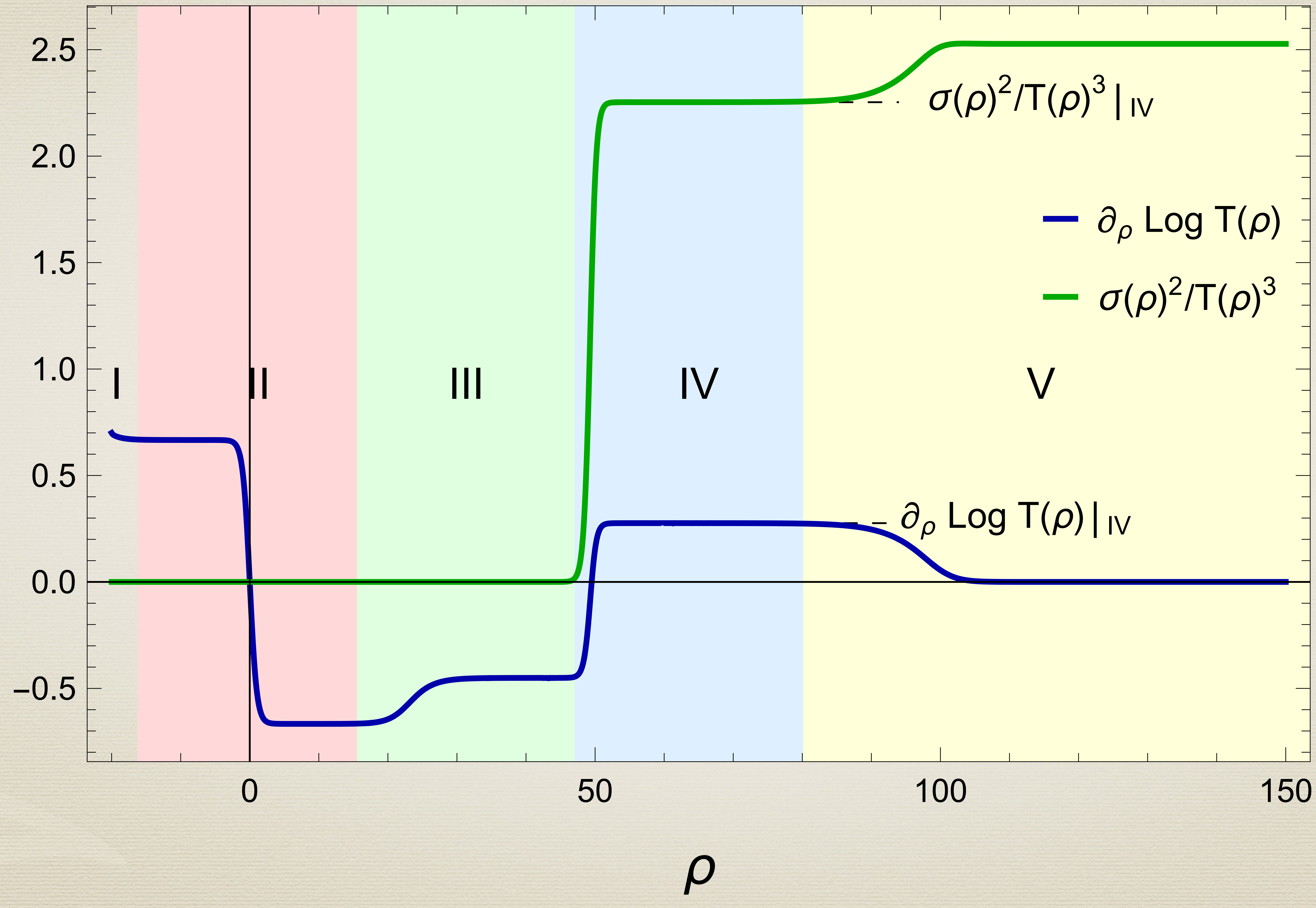
Thank you

Dynamics of non-linear regime - Gubser

$$\left(\frac{\sigma'}{\sigma}\right)' + \frac{\sigma'^2}{\sigma^2} + 2\frac{\sigma'}{\sigma} + L^2\ m_0(T - T_c) + C_{\kappa_1}T\ L\ \frac{\sigma'}{\sigma} = 0$$

$$\frac{T'}{T} + \frac{1}{3}(\chi + 2) - \frac{\sigma^2}{4T^3}\left(m_0 + m_0\frac{\sigma'}{\sigma} + C_{\kappa_1}\frac{\sigma'^2}{\sigma^2}\right) = 0$$

$$\frac{\chi'}{\chi} + \frac{4}{3}\left(2 + \frac{C_\eta}{C_{\tau_\pi}\chi}\right) + 4\frac{T'}{T} + \frac{LT}{C_{\tau_\pi}} = 0.$$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2.0
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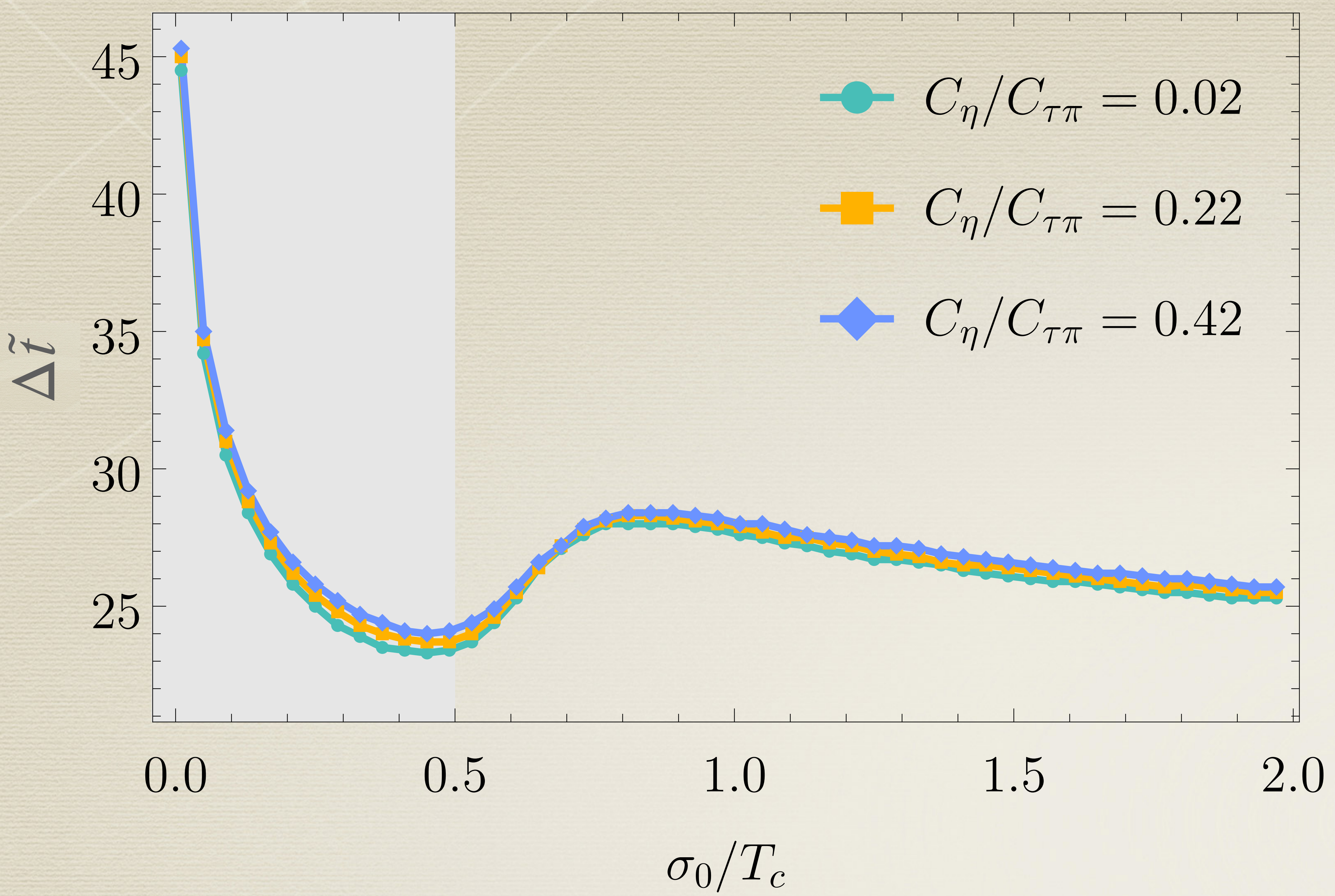
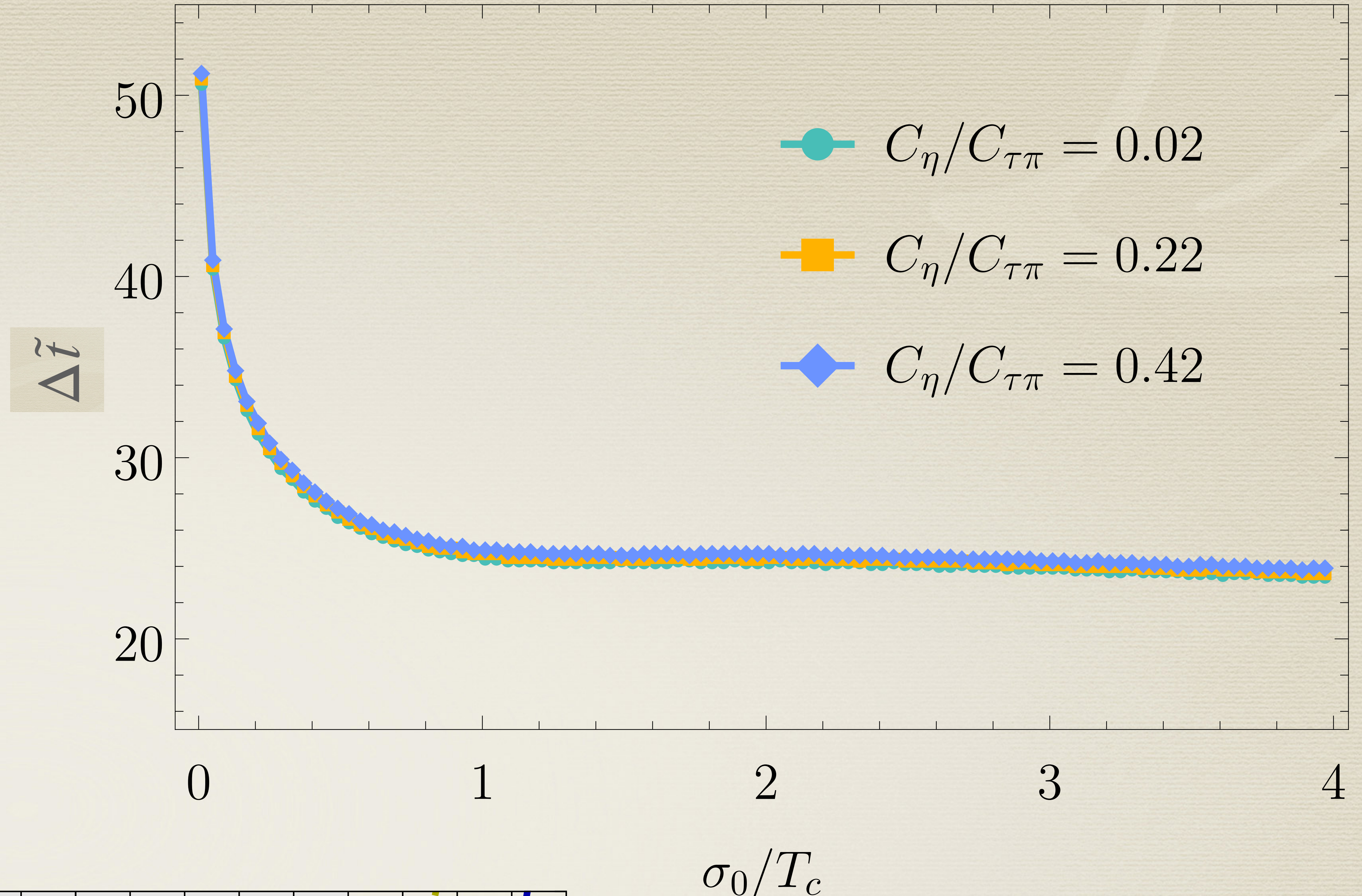
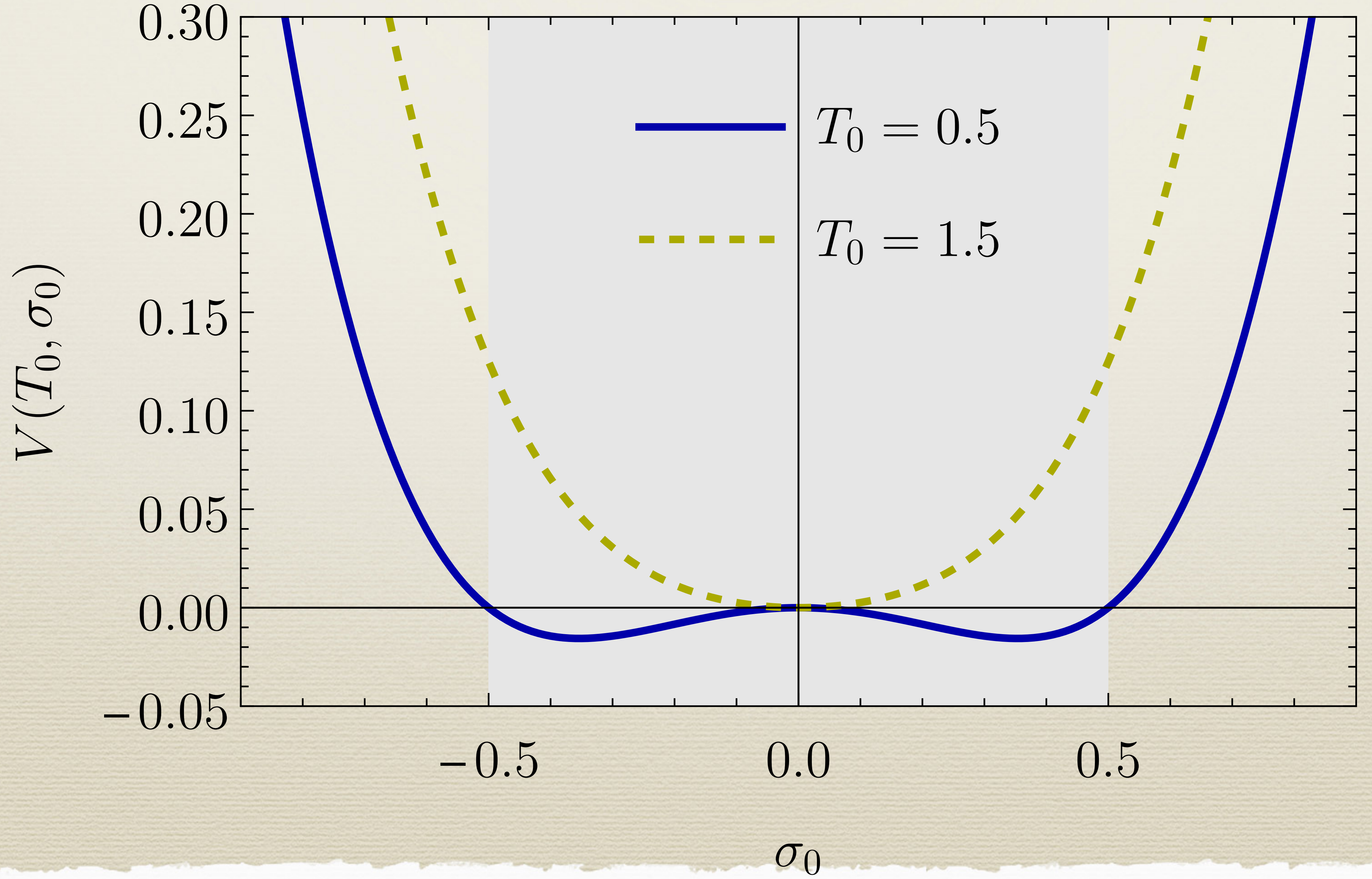
$\sigma(\rho)^2/T(\rho)^3|_{IV}$
∂_ρ Log T(ρ)$|_{IV}$







$$V(\sigma_0, T_0) = 0$$

$$|\sigma_0| \leq \sqrt{2\frac{m_0(T_c - T_0)}{\lambda}}$$