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Attractors in hydrodynamic simulations of heavy ion collisions

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Attractors and thermalization in nuclear collisions and cold quantum gases

What you will see today

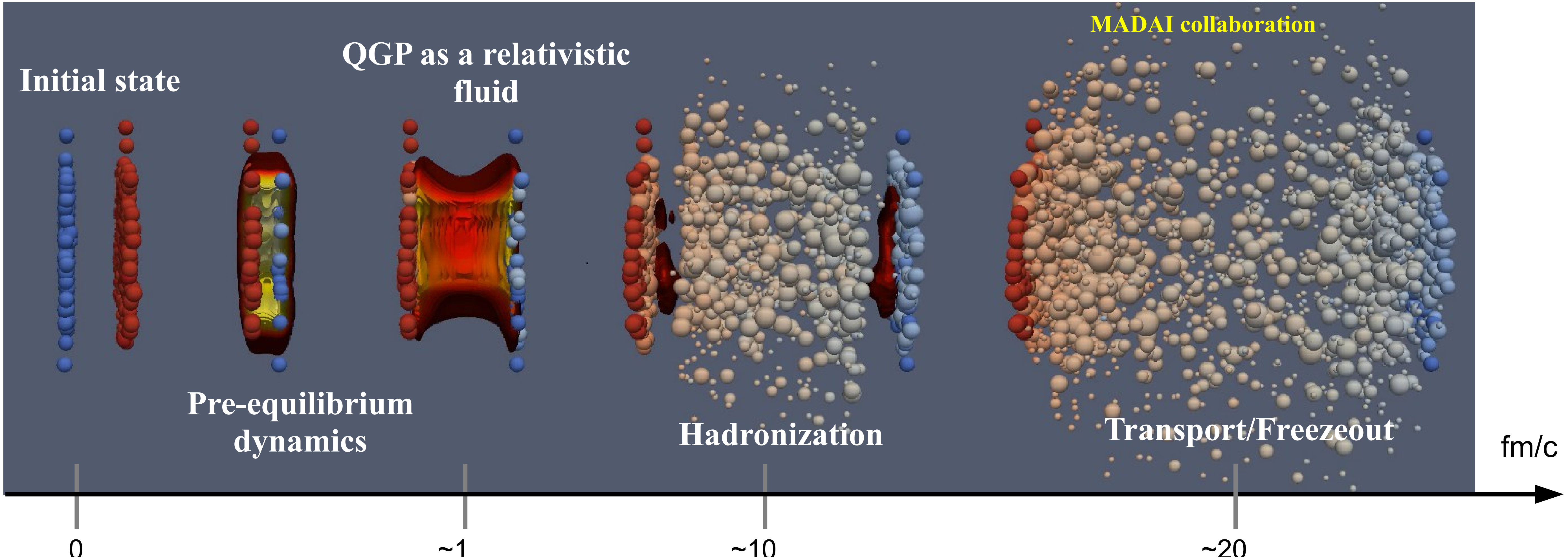
We look for hints of hydrodynamic attractors in simulations of heavy ion collisions

Outline:

- Motivation
- Attractors in (0+1)D Bjorken flow
- (2+1)D Hydrodynamic simulations and attractors
- Discussion and conclusions

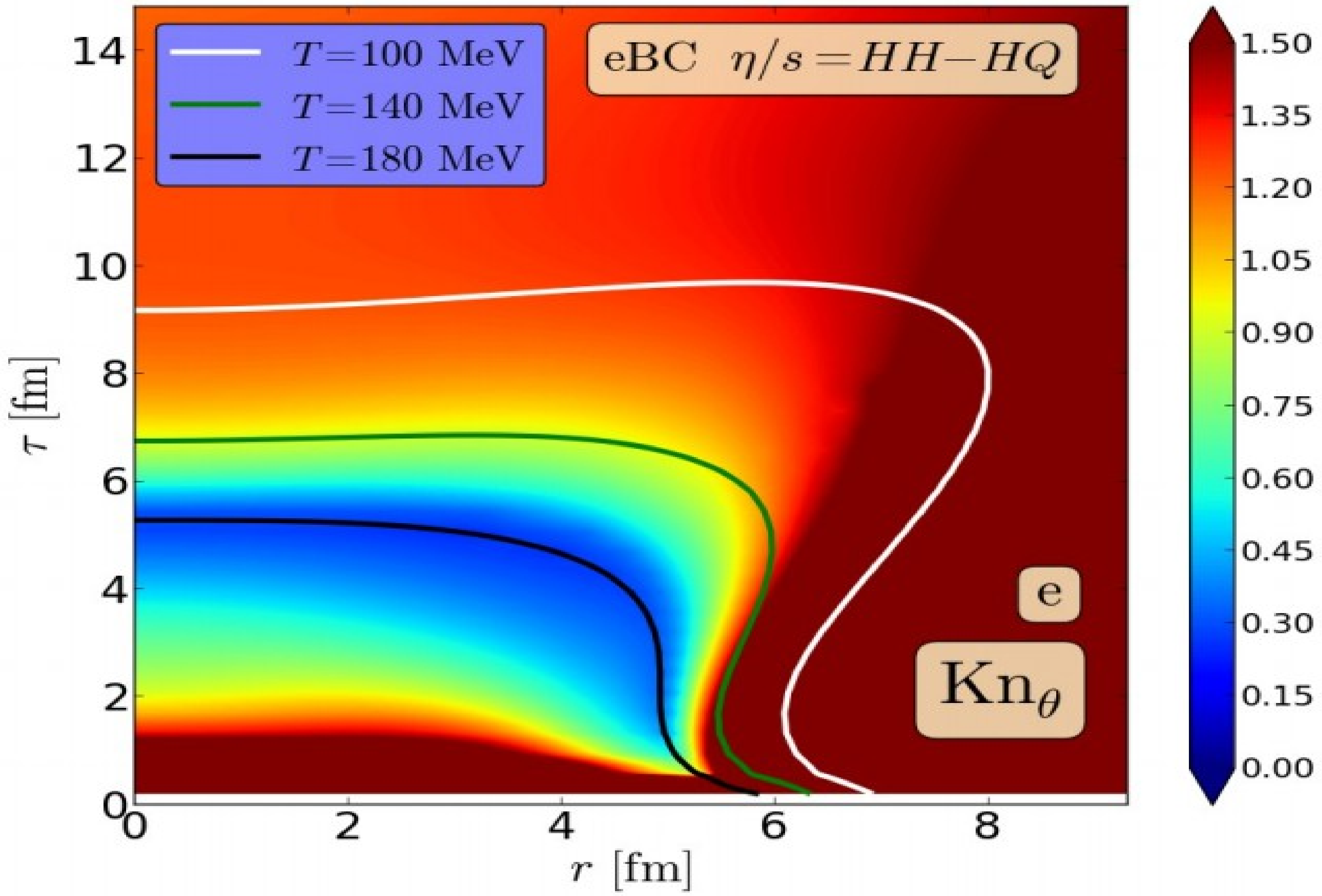
Current picture of a heavy ion collision

Empirical: Fluid-dynamical models of heavy ion collisions work well at RHIC and LHC energies



Main assumption: fluid dynamics can be applied at early times ~ 0.1--1 fm

Extreme Conditions



Knudsen number

$$\mathrm{Kn} = \tau_\pi \nabla_\mu u^\mu$$

is not small at early times

Can this system be close to equilibrium?

Or domain of applicability of hydrodynamic theories better than expected?

Relativistic fluid dynamics

Effective theory describing the dynamics of a system over long-times and long-distances

Conservation laws
+
Equation of state
+
simple constitutive relations

separation of scales: $K_N \sim \frac{\ell}{L} \ll 1$

macroscopic: L

microscopic: ℓ

Relativistic Fluid dynamics

Conservation laws

$$\partial_\mu T^{\mu\nu} = 0$$

EoS: $P_0 = P_0(\varepsilon)$

Tensor decomposition

$$T^{\mu\nu} = \varepsilon\, u^\mu u^\nu - \Delta^{\mu\nu}\left(P_0 + \Pi\right) + \pi^{\mu\nu}$$

Projection operator:

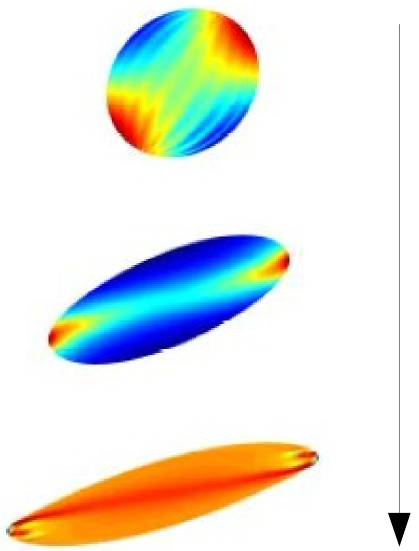
$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

Bulk viscous pressure

Shear stress tensor

Navier-Stokes theory:
constitutive relations

$$\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle}$$



$$\Pi = -\zeta\nabla_\mu u^\mu$$

What we solve is not "traditional" fluid dynamics

Causality: constitutive relations for the dissipative currents *cannot* be imposed

Dynamical equations, e.g. Israel-Stewart theory Annals Phys. 118 (1979) 341-372

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \quad \dots$$

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \quad \dots$$

relaxation times

higher-order terms

expansion rate

$$\theta = \nabla_\mu u^\mu$$

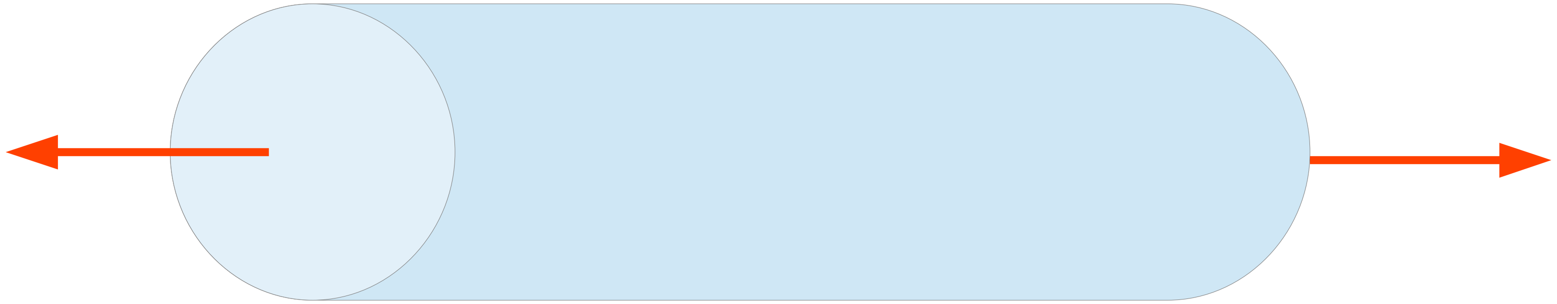
shear tensor

$$\sigma^{\mu\nu} = \nabla^{\langle\mu} u^{\nu\rangle}$$

non-perturbative theory in gradients! Heller&Spalinski, PRL 115 (2015) 7, 072501

Bjorken flow (toy model of heavy ion collisions) J. D. Bjorken, Phys. Rev. D27, 140 (1983)

Simple model for *boost invariant* longitudinal expansion



$v_z = z/t$

Homogeneous fluid in hyperbolic coordinates (τ, x, y, ς)

$\tau = \sqrt{t^2 - z^2} \qquad \varsigma = \tanh^{-1}(z/t)$

Static velocity

$u^\mu = (1, 0, 0, 0)$

Gradients ~ 1/τ

$\sigma^\mu_\nu = \mathrm{diag}\left(0, -\frac{1}{3\tau}, -\frac{1}{3\tau}, \frac{2}{3\tau}\right)$

energy-momentum tensor

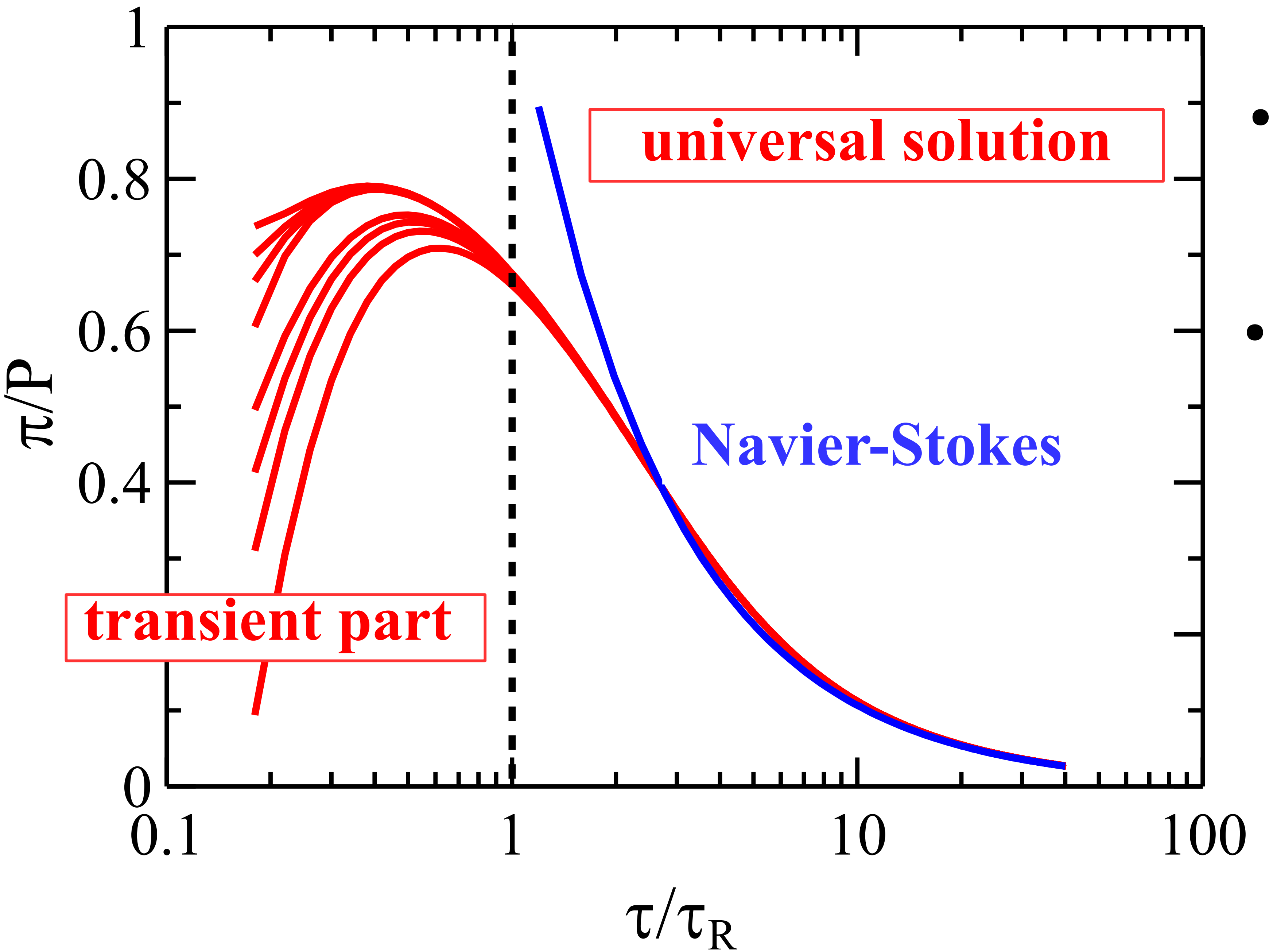
$T^\mu_\nu = \mathrm{diag}\,(\varepsilon, P - \pi/2, P - \pi/2, P + \pi)$

dissipative correction

Knudsen number: $K_N \sim \hat{\tau}^{-1} \equiv \tau_R/\tau$

Fluid-dynamical regime

Late-time solution of the conformal (RTA) Boltzmann equation appears to be universal



- System forgets about initial state very fast
- This occurs far from equilibrium, before NS limit is achieved

Fluid-dynamical regime

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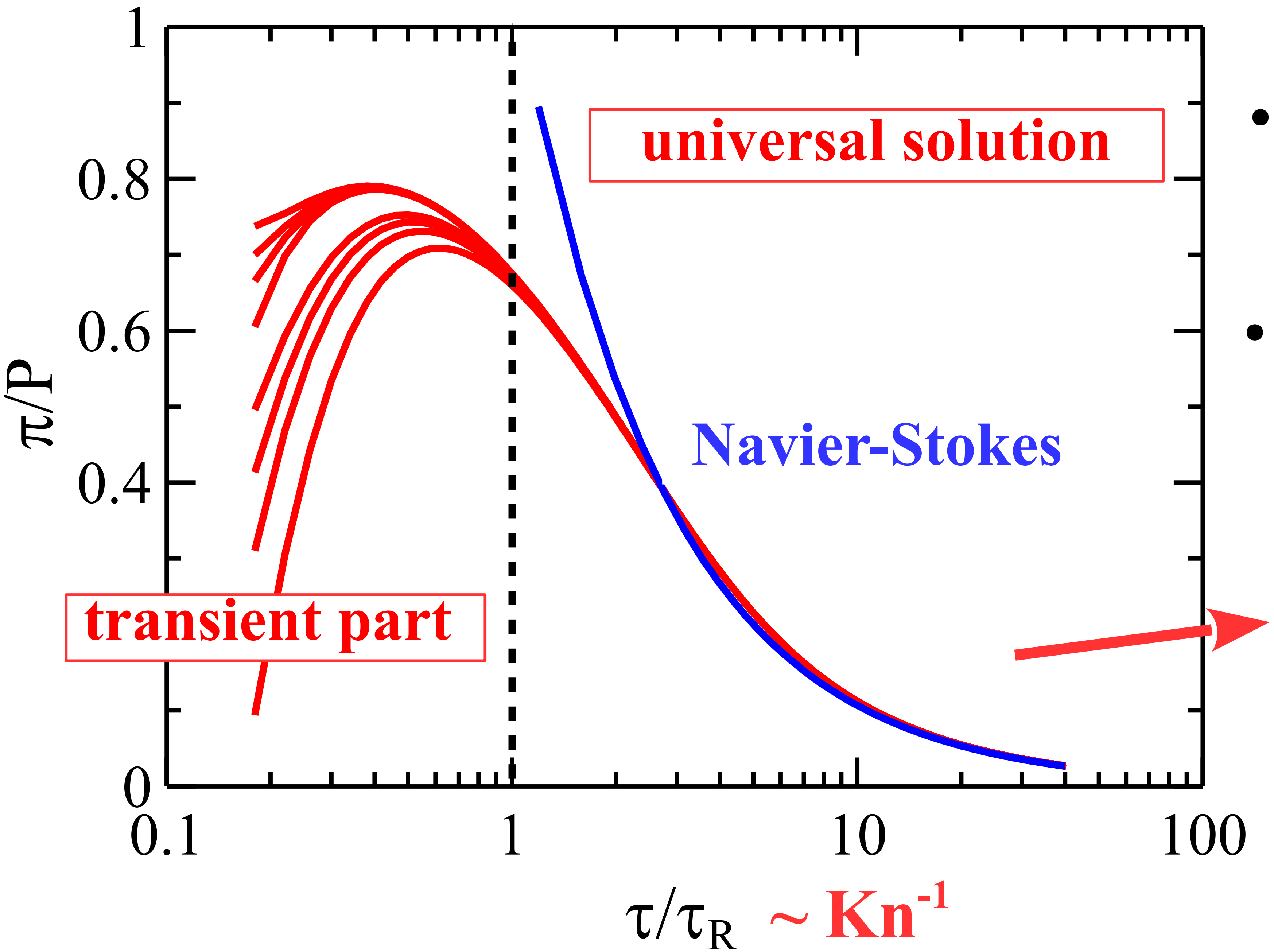

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Emergence of constitutive relations far-from-equilibrium?

Fluid-dynamical regime

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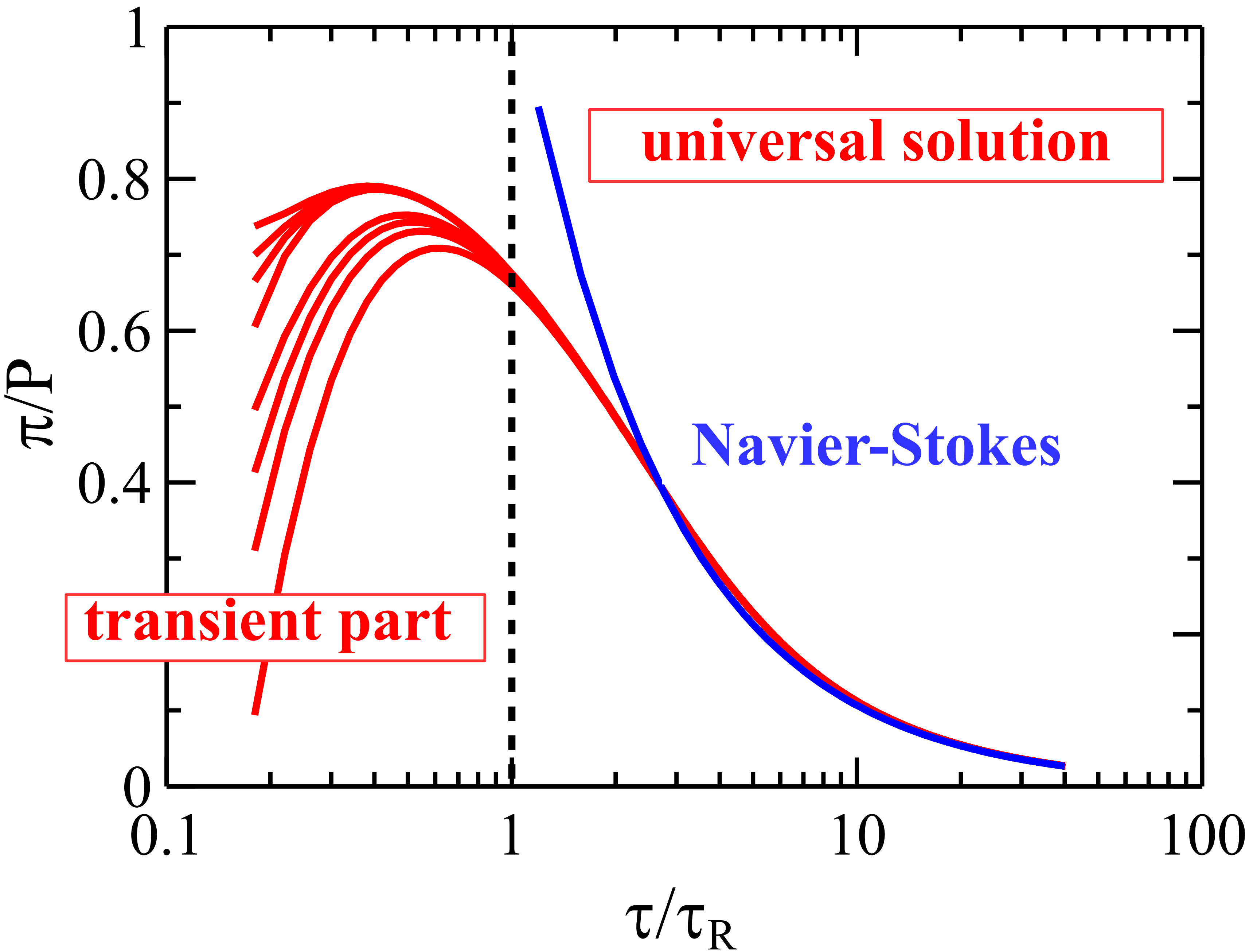
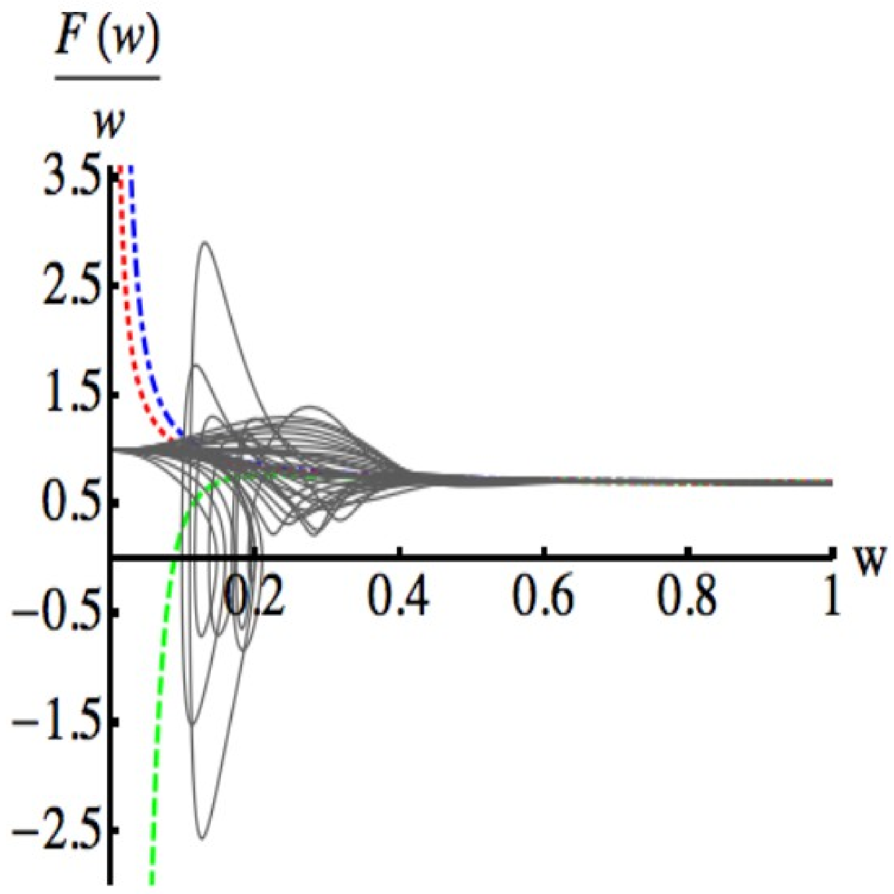


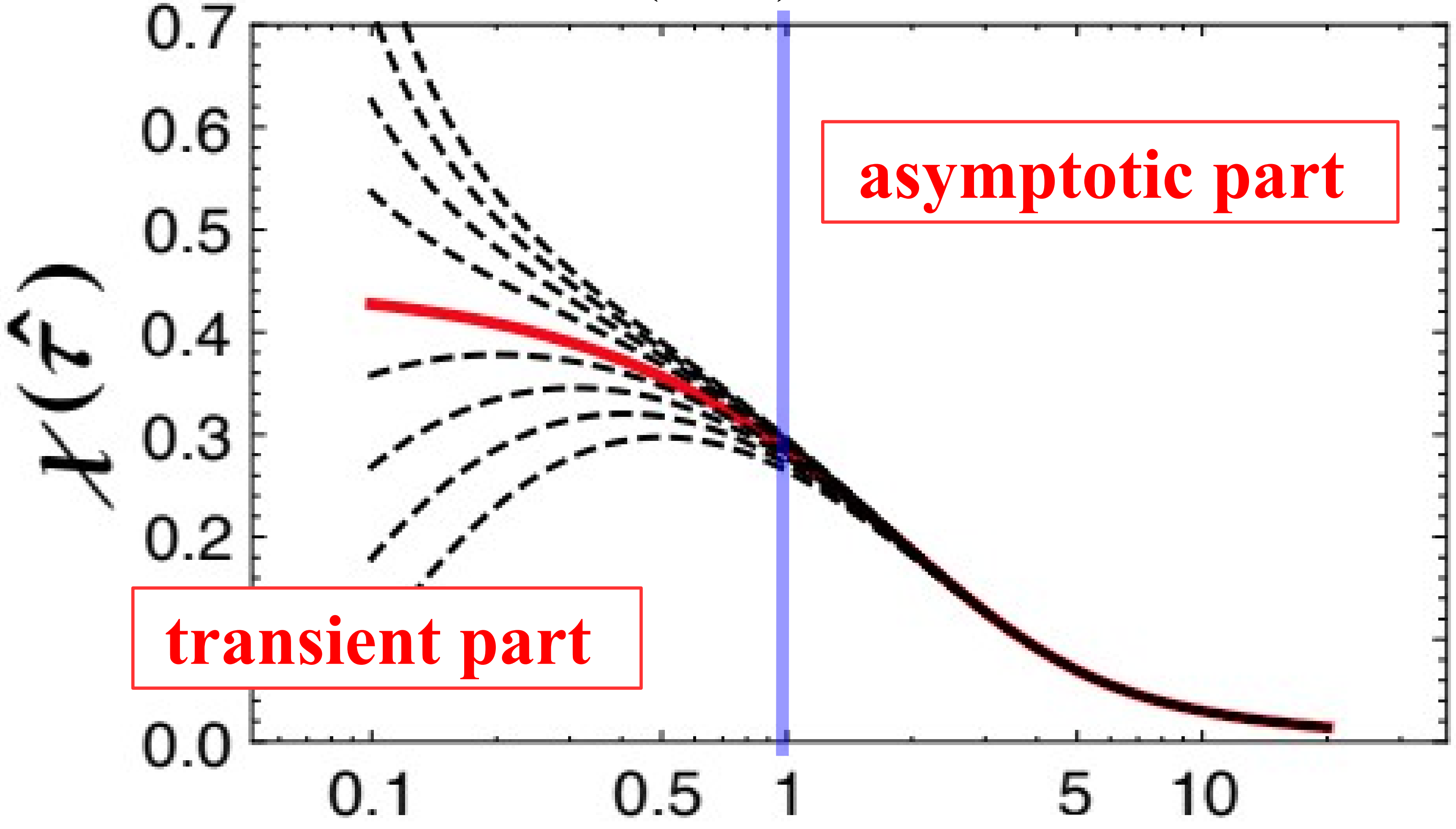
First proposed by Heller&Janik& Witaszczyk in holography, PRL 108 (2012) 201602



Fluid-dynamical regime

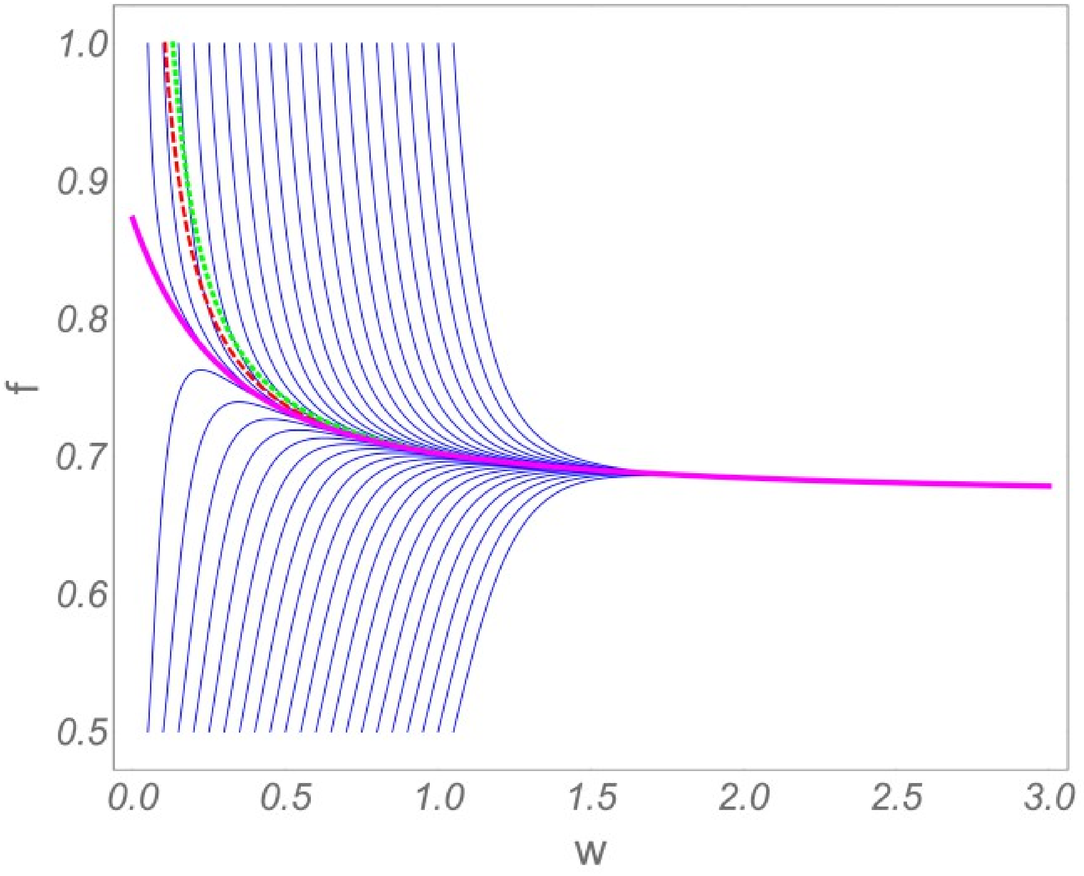
Late-time solution of Israel-Stewart theory displays the same feature

PRD 97 (2018) 5, 056021



$$\chi(\hat{\tau}) = \frac{\pi}{\varepsilon + P} \qquad \hat{\tau} = \frac{\tau}{\tau_R}$$

First proposed by Heller&Spalinski, PRL 115 (2015) 7, 072501

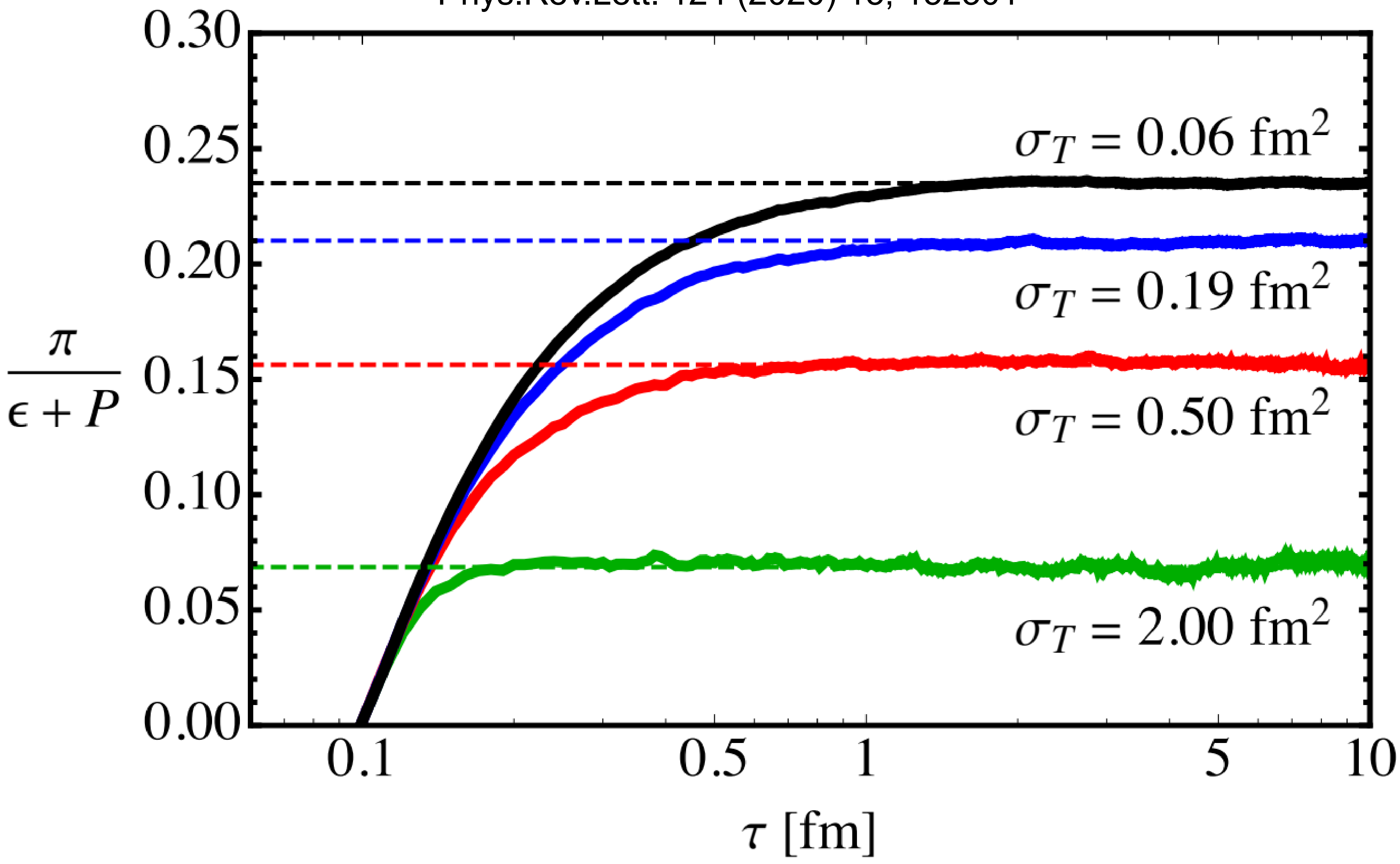


Relativistic gas of hard spheres

System with a contant Knudsen number

Phys.Rev.Lett. 124 (2020) 15, 152301

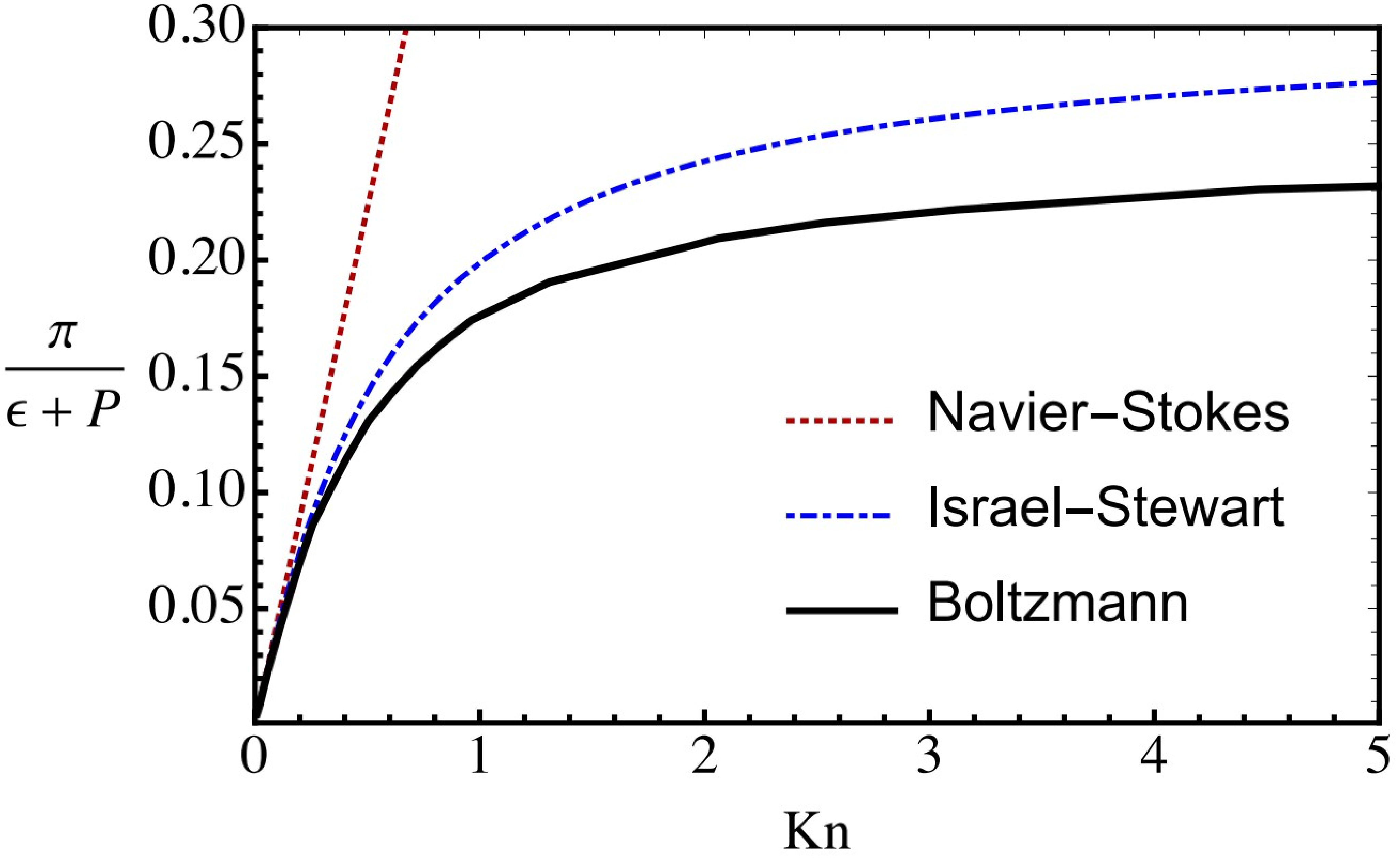
$$\mathrm{Kn} = \frac{\ell_{\mathrm{mfp}}}{\tau} = \frac{1}{n_0 \tau_0 \sigma_T}$$



- Solution for the normalized shear-stress tensor relaxes to a constant (attractor!)

Relativistic gas of hard spheres

Attractor solution of the Boltzmann equation and DNMR theory can be calculated



- DNMR attractor is qualitatively and even quantitatively similar to the attractor of the Boltzmann equation
- Navier-Stokes only works for small Knudsen number

Can we study attractors in more general flow configurations?

In Bjorken flow (boost invariant), every fluid element is identical → same attractor for all.

In more realistic flow profiles, different fluid elements (in a Lagrangian view) may each approach their own local attractor ????

Here we investigate this using Lagrangian solvers: attractor behavior is easier to identify for each moving fluid element

Simulation

✔ We solve the fluid-dynamical equations using a relativistic version of the SPH algorithm

Phys.Rev.C 80 (2009) 064901

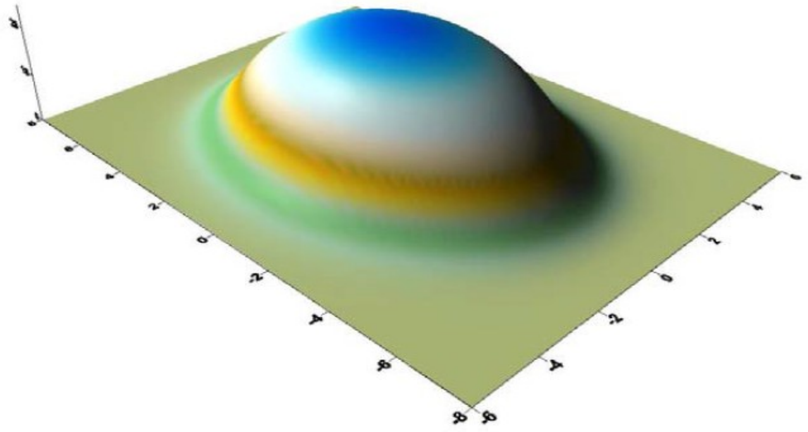
✔ Ideal equation of state: $\varepsilon = 3P$

✔ Transport coefficients: $\eta/s = \text{cte}$ $\qquad \tau_\pi = 5\frac{\eta}{\varepsilon + P}$

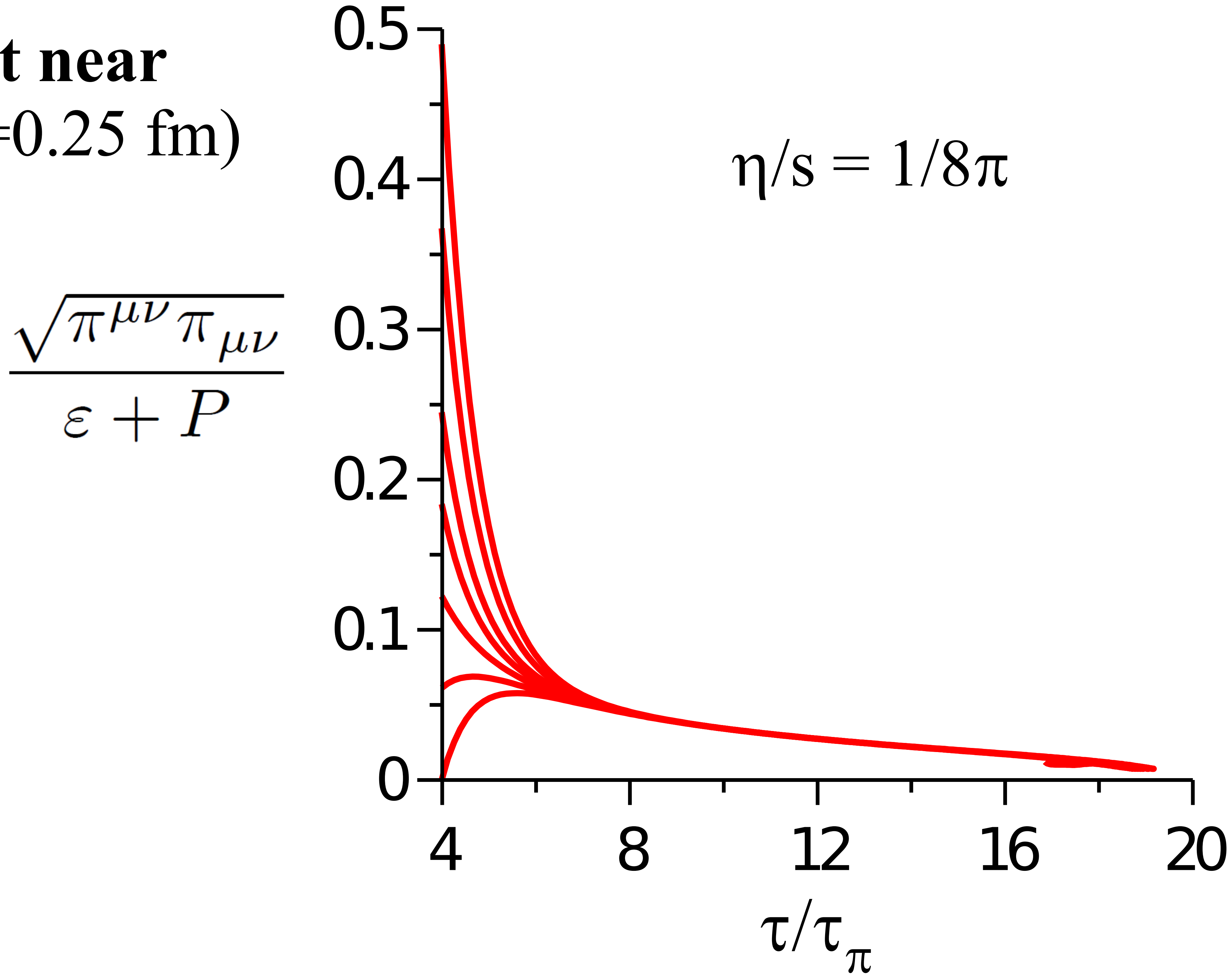
✔ Initial condition: Optical Glauber, central collision

$\varepsilon_0 \sim 40\ \text{GeV/fm}^3$

initial shear stress $\sim \varepsilon + P$



Fluid element near the center (r=0.25 fm)



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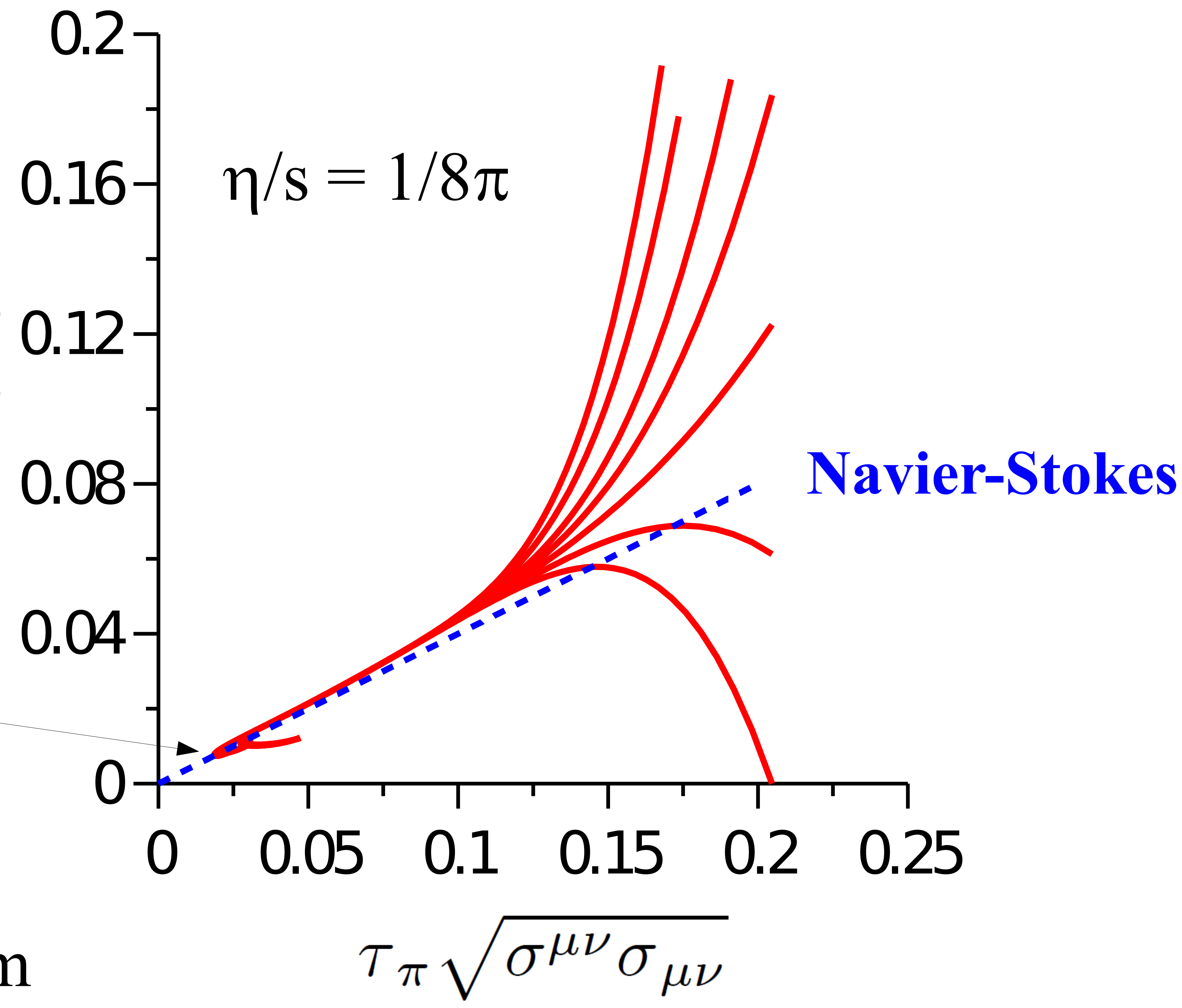
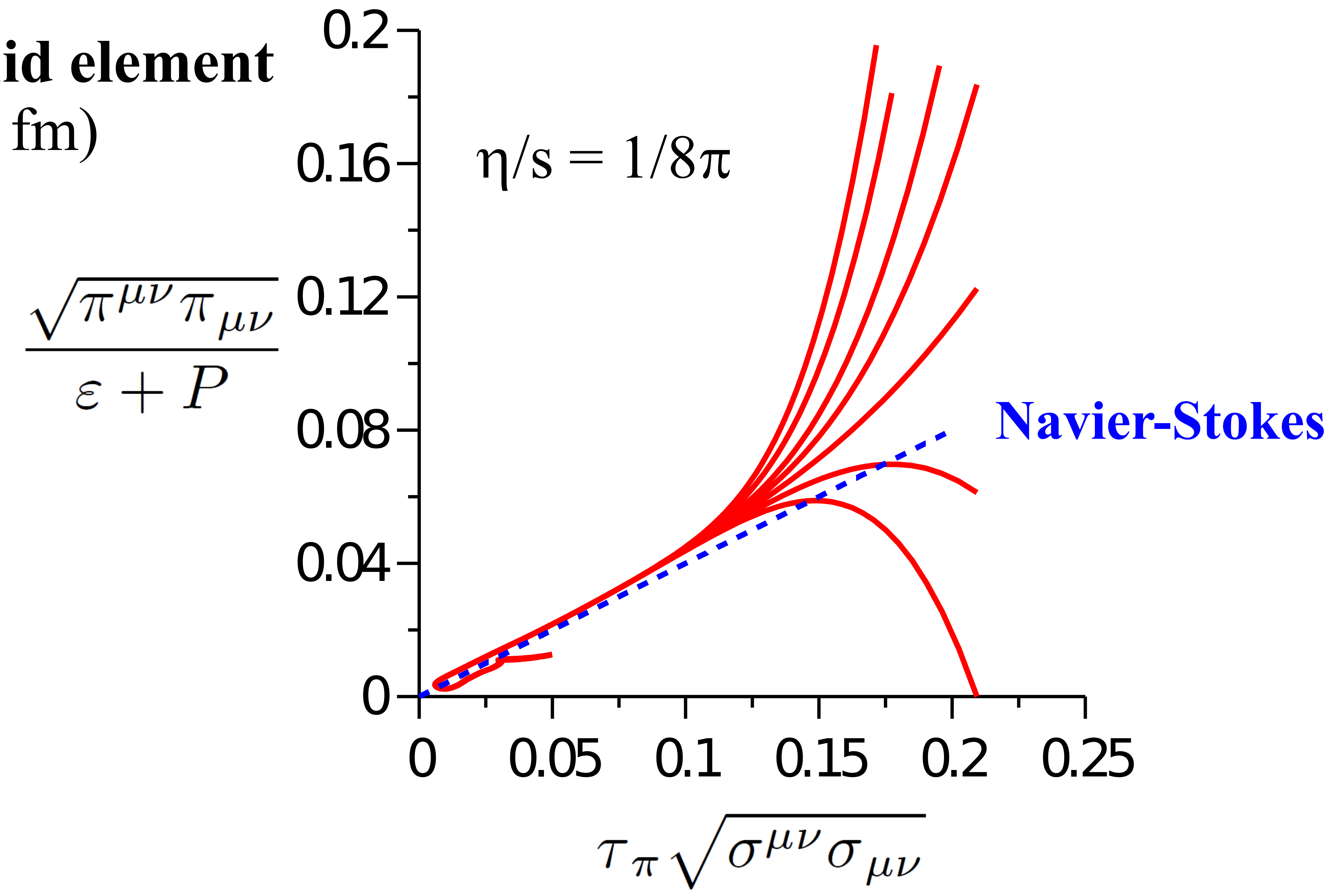


Knudsen number starts to increase again

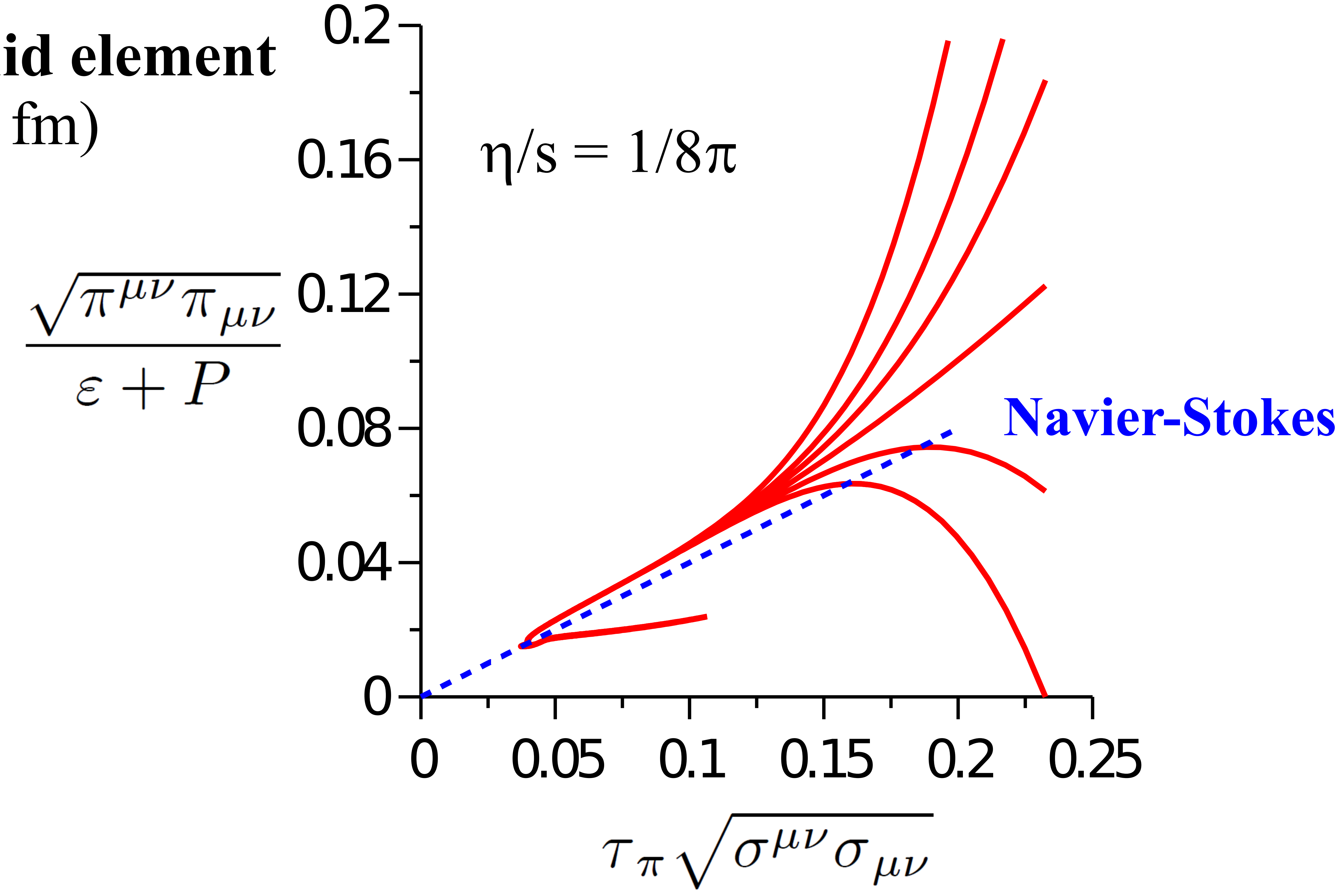
Fluid element starts to deviate from equilibrium

Another fluid element

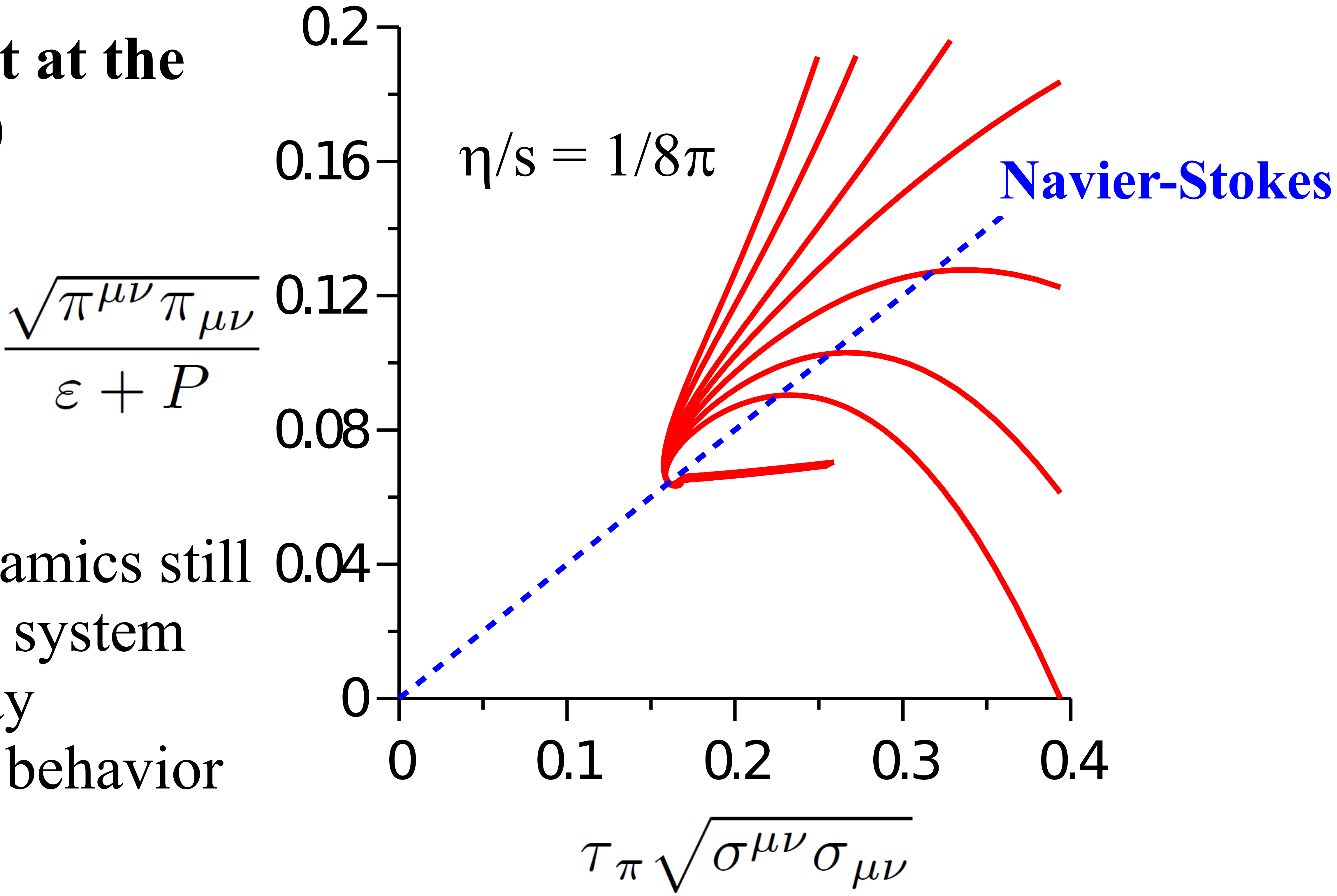
(r=2 fm)



Another fluid element
(r=4 fm)

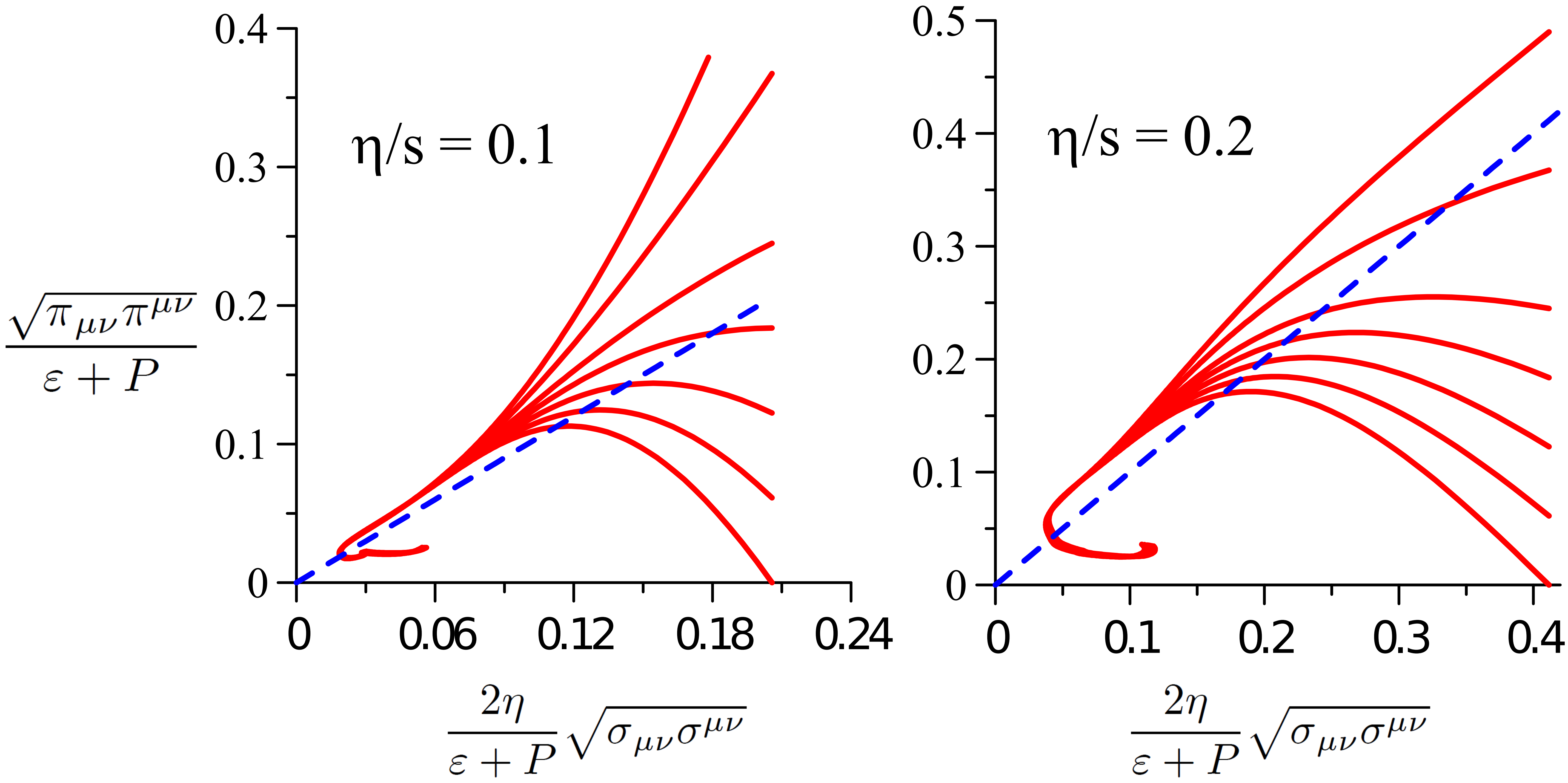


Fluid element at the edge (r=6 fm)



- Transient dynamics still goes away, but system does not display hydrodynamic behavior

Different viscosities



Far-from-equilibrium hydrodynamics for general flows

Consider the usual gradient expansion (Kn <<1)

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \eta_1\sigma_\lambda^{\langle\mu}\sigma^{\nu\rangle\lambda} + \eta_2\sigma_\lambda^{\langle\mu}\omega^{\nu\rangle\lambda} + \eta_3\omega_\lambda^{\langle\mu}\omega^{\nu\rangle\lambda} + \eta_4\theta\sigma^{\mu\nu} + \eta_5\nabla_\perp^{\langle\mu}P\nabla_\perp^{\nu\rangle}P + \eta_6\nabla_\perp^{\langle\mu}\nabla_\perp^{\nu\rangle}P + \mathcal{O}\left[K_N^3\right],$$

When Kn ~1:
- Large rearrangement of the series
- 3rd-order terms like $\sim\sigma^{\alpha\beta}\sigma_{\alpha\beta}\sigma^{\mu\nu}$ may be grouped with $2\eta\sigma^{\mu\nu}$

Resummation of transport coefficients $\eta \rightarrow \eta_R(\mathrm{Kn})$

$$\pi^{\mu\nu} = 2\eta^R\sigma^{\mu\nu} + \eta_1^R\sigma_\lambda^{\langle\mu}\sigma^{\nu\rangle\lambda} + \eta_2^R\sigma_\lambda^{\langle\mu}\omega^{\nu\rangle\lambda} + \eta_3^R\omega_\lambda^{\langle\mu}\omega^{\nu\rangle\lambda} + \eta_5^R\nabla_\perp^{\langle\mu}P\nabla_\perp^{\nu\rangle}P + \eta_6^R\nabla_\perp^{\langle\mu}\nabla_\perp^{\nu\rangle}P + \mathcal{O}\left[(K_N^R)^3\right],$$

Example: slow-roll solution of Israel-Stewart theory

Simplified Israel-Stewart theory:

$$\chi^{\mu\nu} = \frac{\pi^{\mu\nu}}{\varepsilon + P} \qquad \tau_\pi D\chi^{\langle\mu\nu\rangle} + \chi^{\mu\nu} = \frac{2}{5}\tau_\pi\sigma^{\mu\nu} - \frac{4}{3}\tau_\pi\chi^{\mu\nu}\chi^{\alpha\beta}\sigma_{\alpha\beta}$$

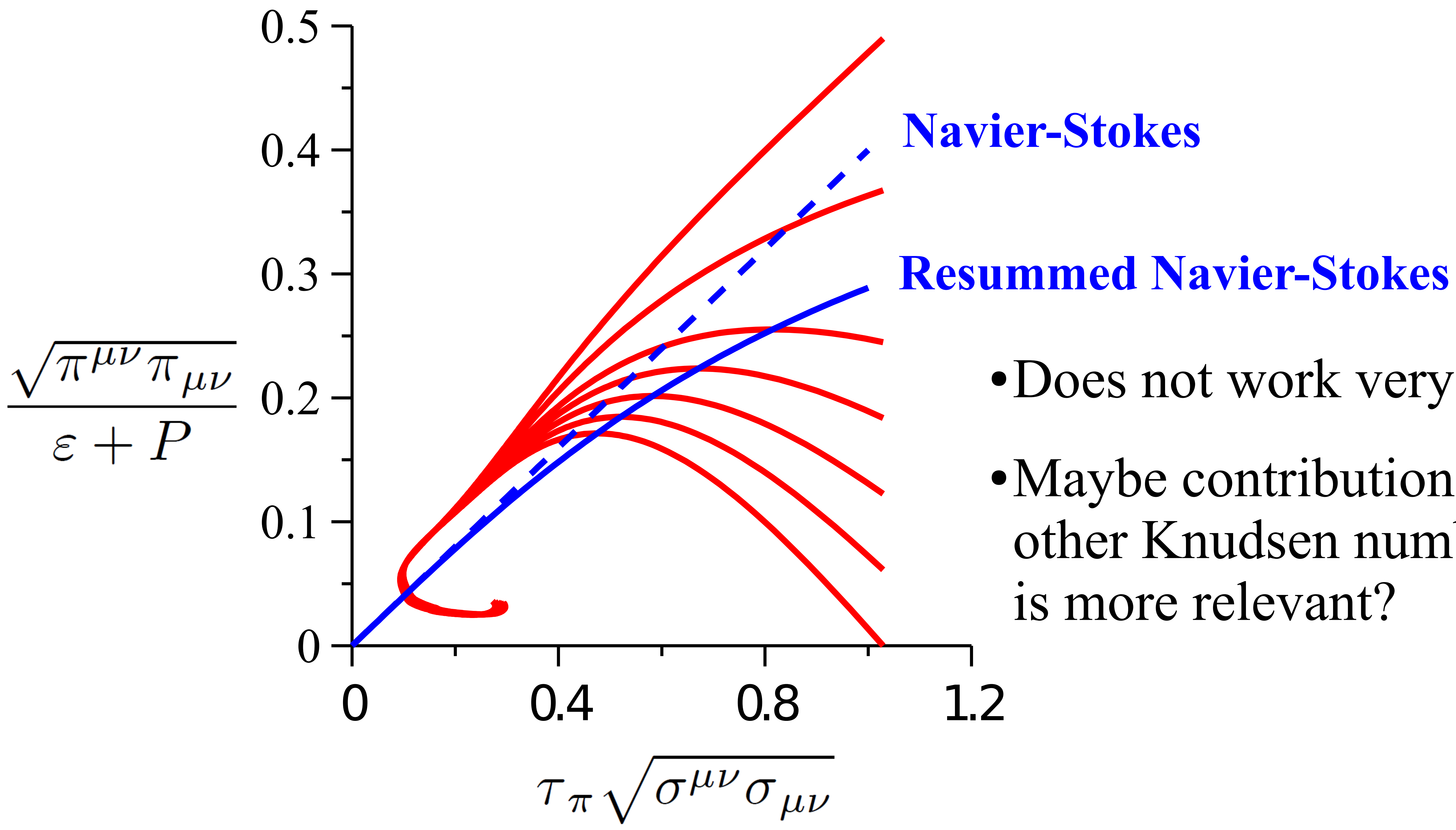
Slow-roll expansion: $\epsilon\,\tau_\pi D\mathcal{X}^{\langle\mu\nu\rangle} = -\mathcal{X}^{\mu\nu} + \frac{2}{5}\tau_\pi\sigma^{\mu\nu} - \frac{4}{3}\tau_\pi\mathcal{X}^{\mu\nu}\mathcal{X}^{\alpha\beta}\sigma_{\alpha\beta}$

$$\mathcal{X}^{\mu\nu}(x;\epsilon) = \sum_{n=0}^{\infty}\epsilon^n\mathcal{X}_n^{\mu\nu}(x)$$

Leading solution: $\mathcal{X}_0^{\mu\nu} = \frac{2}{5}\tau_\pi\sigma^{\mu\nu}\mathcal{S}$

$$\mathcal{S}(K_N) = \frac{15}{16K_N^2}\left(\sqrt{1+\frac{32}{15}K_N^2} - 1\right)$$

$$K_N = \tau_\pi\sqrt{\sigma_{\mu\nu}\sigma^{\mu\nu}}$$



- Does not work very well
- Maybe contribution from other Knudsen numbers is more relevant?

Conclusions

Attractor behavior, previously seen in symmetric (0+1)D Bjorken flow, also manifests locally in complex (2+1)D heavy-ion collisions

- Each fluid element can approach its own far-from-equilibrium attractor, provided it evolves long enough (i.e., not at the very edge of the fireball)

What is the correct functional form of the attractor for a general flow? Is it a function of one Knudsen number, or multiple independent invariants?