September 25^{th} 2025

Restoring causality in far-from-equilibrium fluids

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Funding:

- Vanderbilt Seeding Success Grant
- MERAC Foundation prize grant
- Newton Trust Grant
- Funding from the Cambridge Centre for Theoretical Cosmology

Talk summary

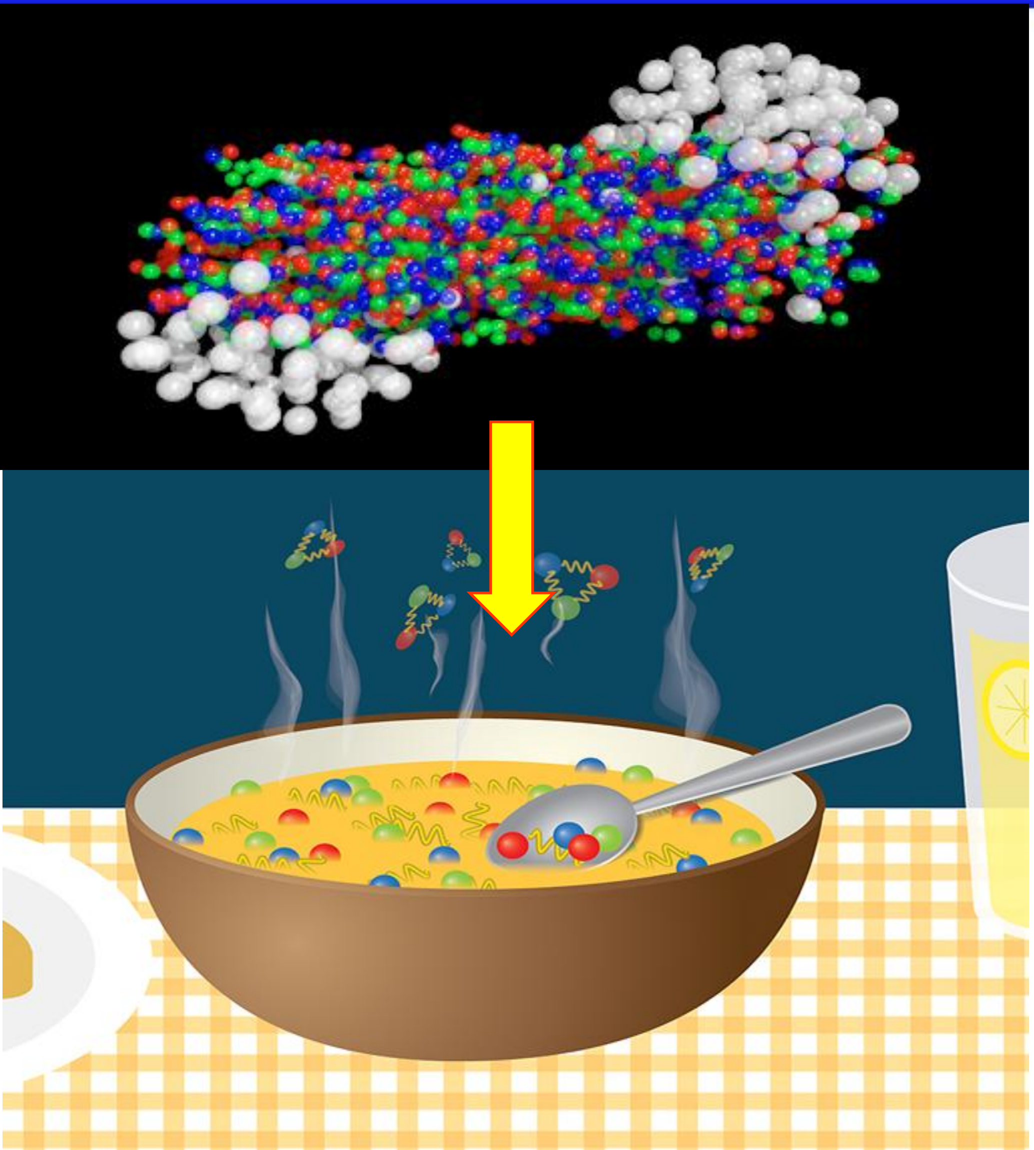
1. I will present a problem.

2. And then I will propose a quick fix.

[Note: Our metric signature is **(-,+,+,+)**]

A bit of context

Fluid description of the quark-gluon plasma



Israel-Stewart framework:

$$u^{\alpha}\nabla_{\alpha}\varepsilon + (\varepsilon + P)\nabla_{\alpha}u^{\alpha} + \pi^{\alpha\beta}\sigma_{\alpha\beta} = 0$$

$$(\varepsilon + P)u^{\beta}\nabla_{\beta}u_{\alpha} + c_s^2\Delta_{\alpha}^{\beta}\nabla_{\beta}\varepsilon + \Delta_{\alpha}^{\beta}\nabla_{\mu}\pi_{\beta}^{\mu} = 0$$

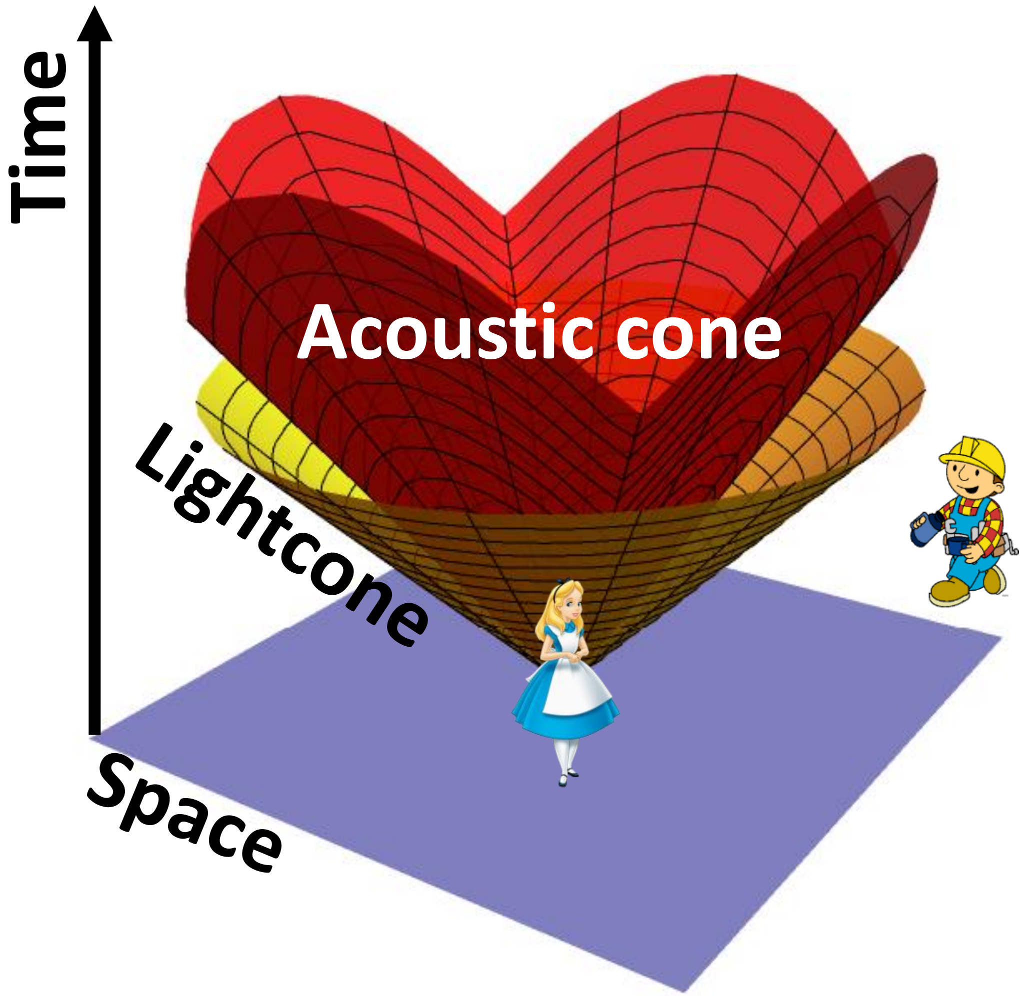
$$\tau_{\pi}\Delta_{\mu\nu}^{\alpha\beta}u^{\lambda}\nabla_{\lambda}\pi_{\alpha\beta} + \pi_{\mu\nu} = -2\eta\sigma_{\mu\nu}$$

(we ignore the other DNMR/BRSSS stuff 😊)

Is this theory causal?

Definition of causality:
Alice cannot use the fluid to send a message to Bob faster than light.

In practice:
- Alice perturbs the fluid at her location.
- The induced changes travel inside an "acoustic cone".
- This cone should be contained inside the Lightcone.



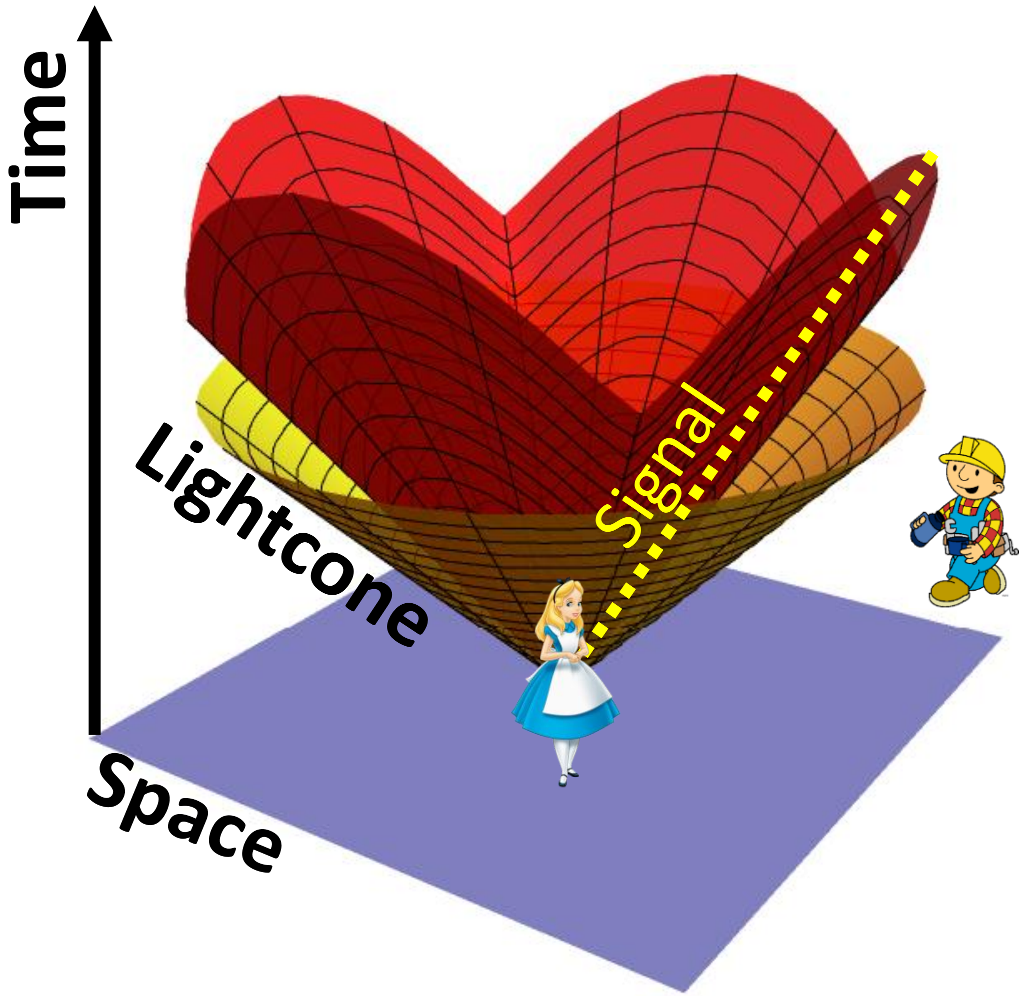
Do some math, and here is the result

Go to a local rest frame and diagonalize the stress tensor:

$$u^\alpha = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \pi_{\alpha\beta} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \pi_{11} & 0 & 0 \\ 0 & 0 & \pi_{22} & 0 \\ 0 & 0 & 0 & \pi_{33} \end{bmatrix}$$

Assume propagation along the x^1 axis.
Information speed:

$$w^2 = c_s^2 + \frac{4\eta}{3\tau_\pi(\varepsilon + P + \pi_{11})}$$



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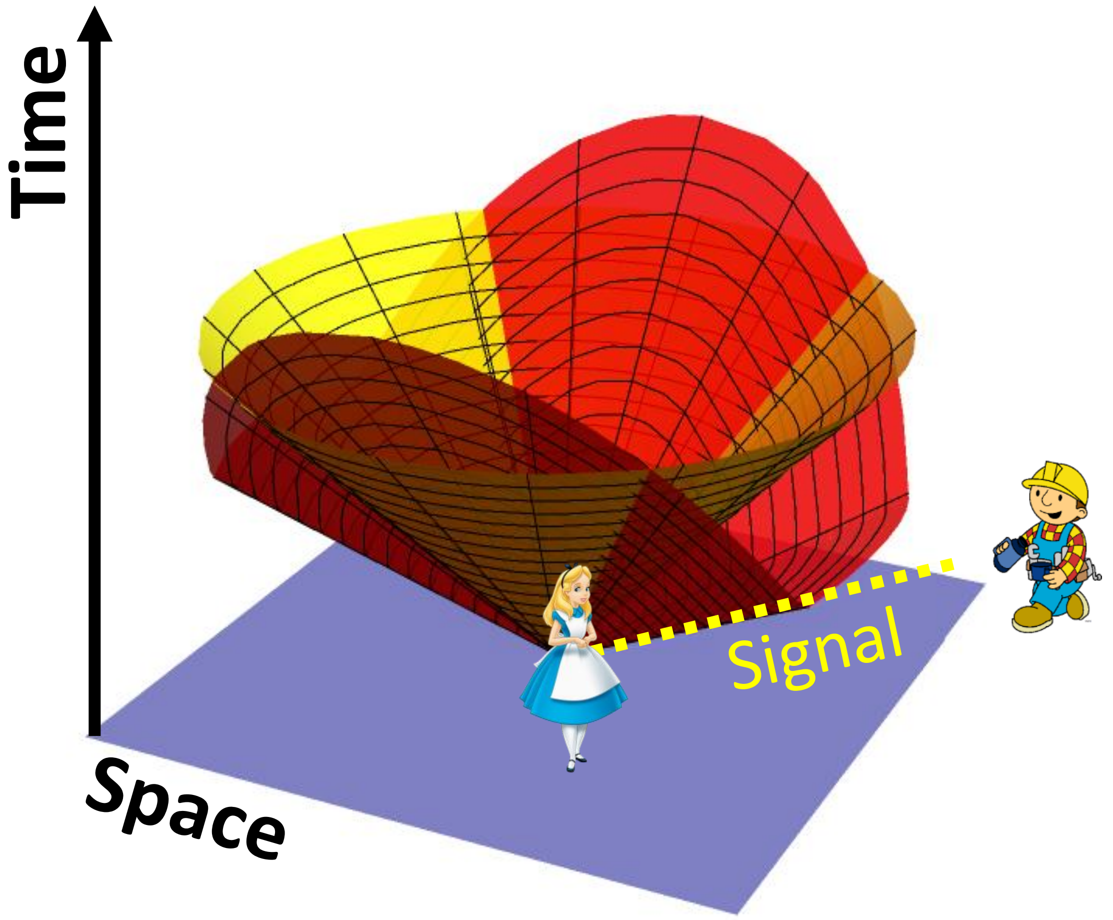
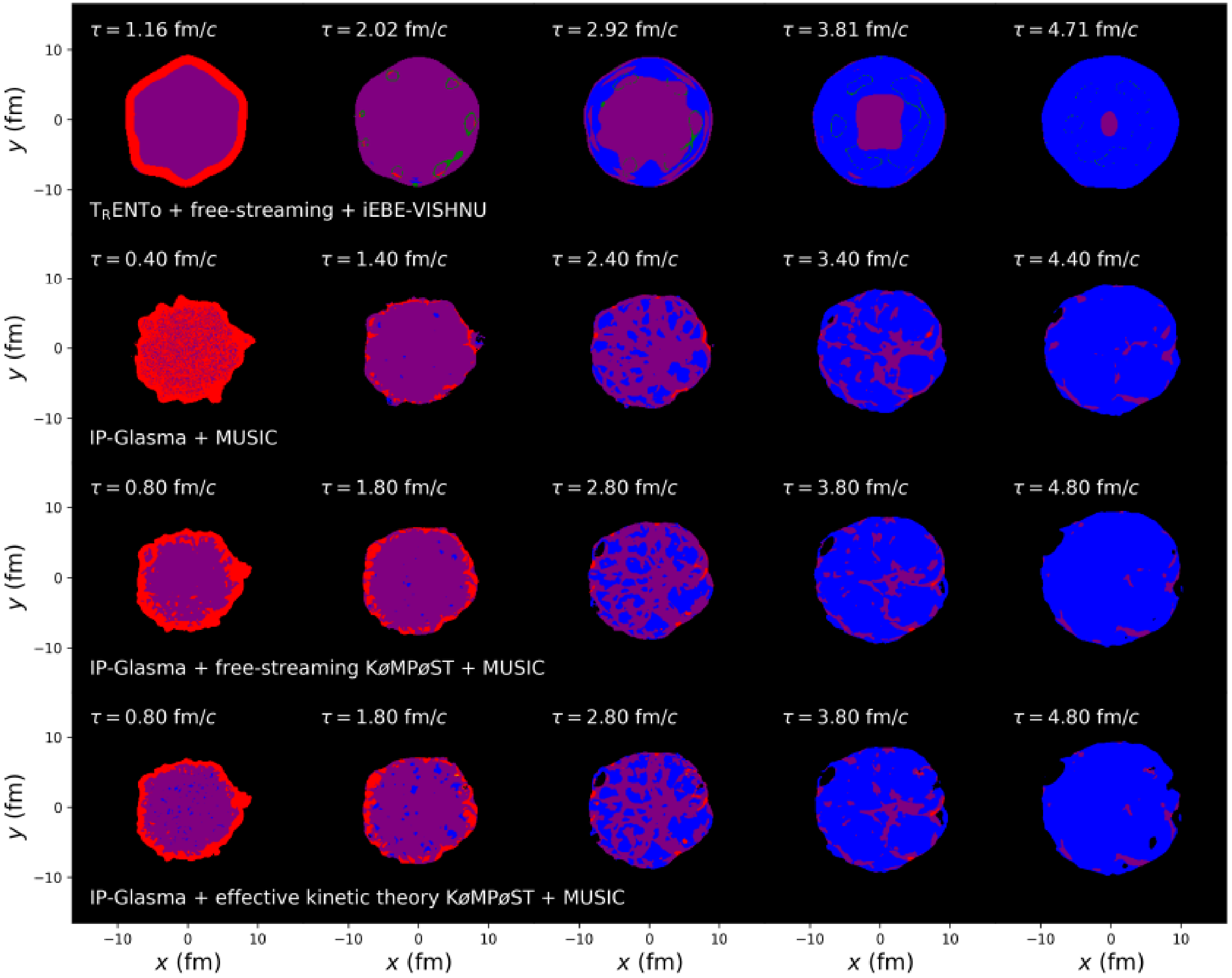
Assume propagation along the x^1 axis.
Information speed:

$$w^2 = c_s^2 + \frac{4\eta}{3\tau_\pi (small)} > c^2$$



Causality violations are not rare



Simulations of the QGP in a heavy-ion collision:

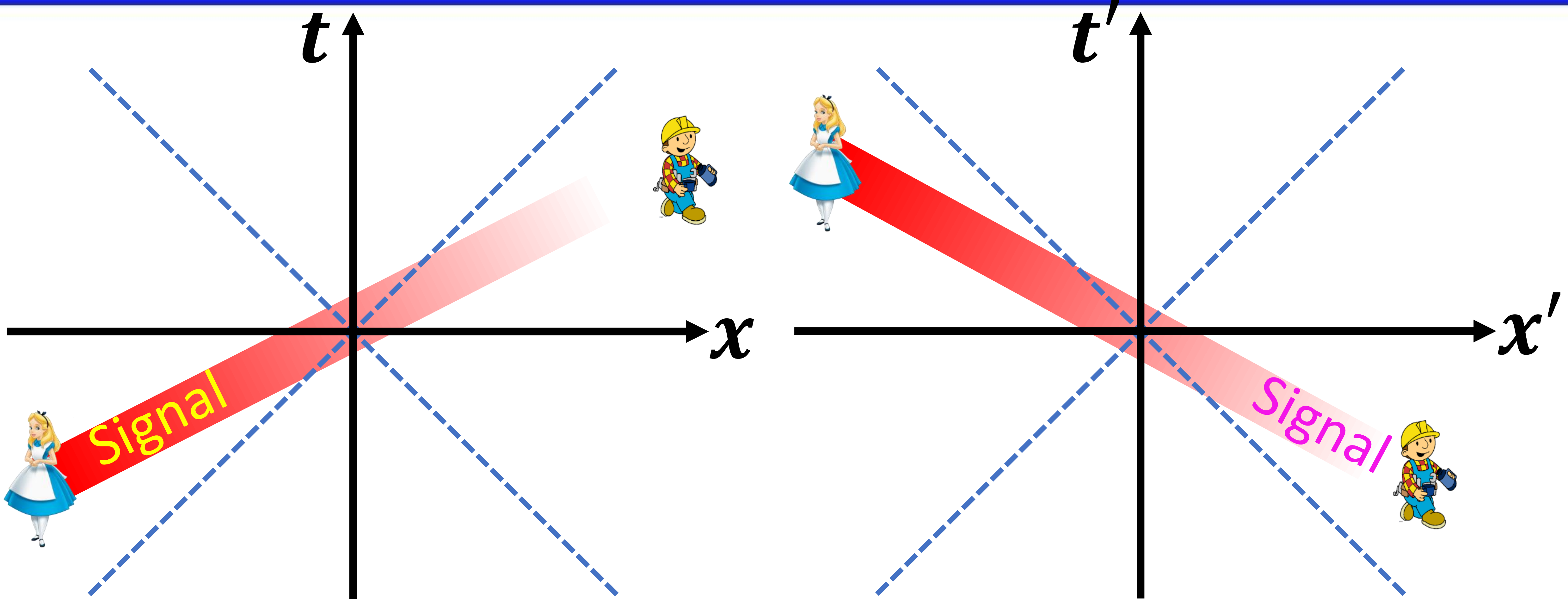
- Red = acausal;
- Purple = unknown;
- Blue = causal.

The best simulations currently available propagate information faster than light.

(Plumberg et al. PRC, 2022)

The problem

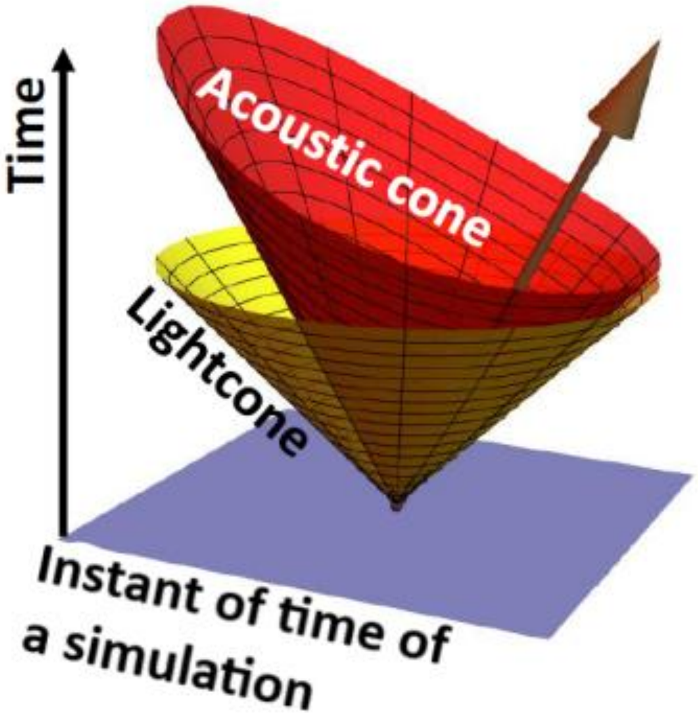
What happens if we break causality?

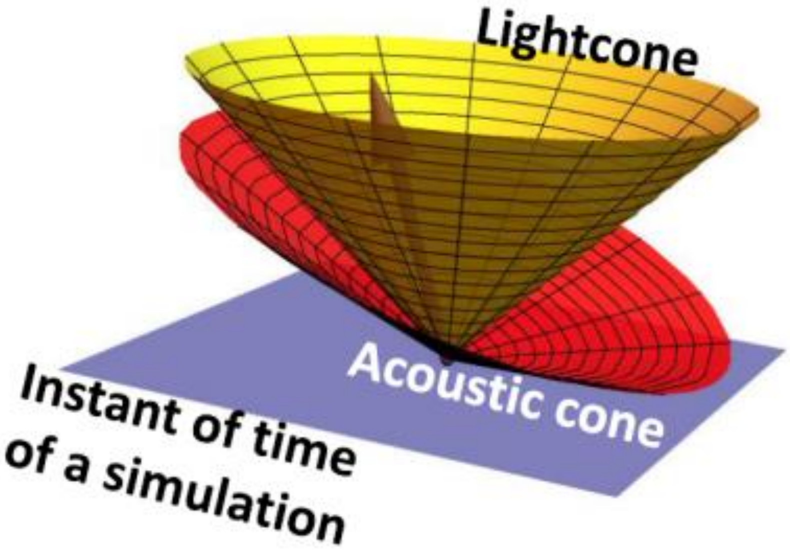


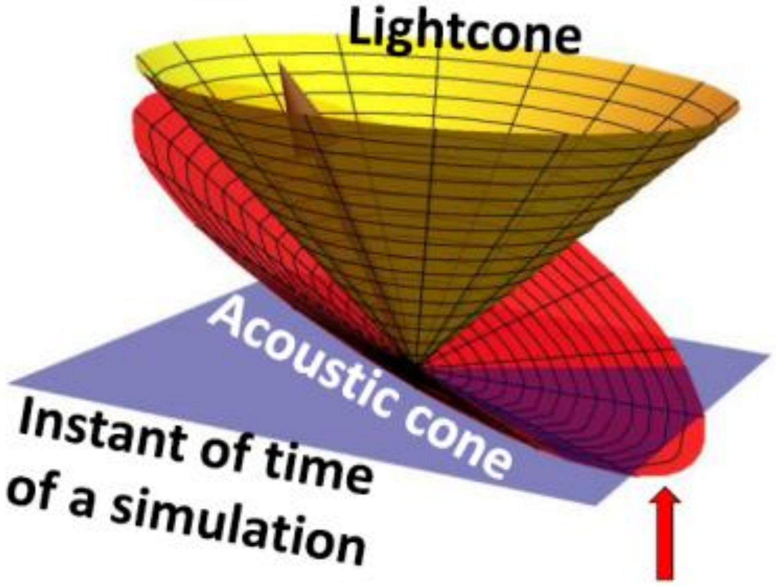
In a reference frame where waves propagate backward in time, the fluid description is unstable (LG PRX, 2022).

Three possible situations

Good: Causal **Bad: Acausal but Stable** **Ugly: Unstable**

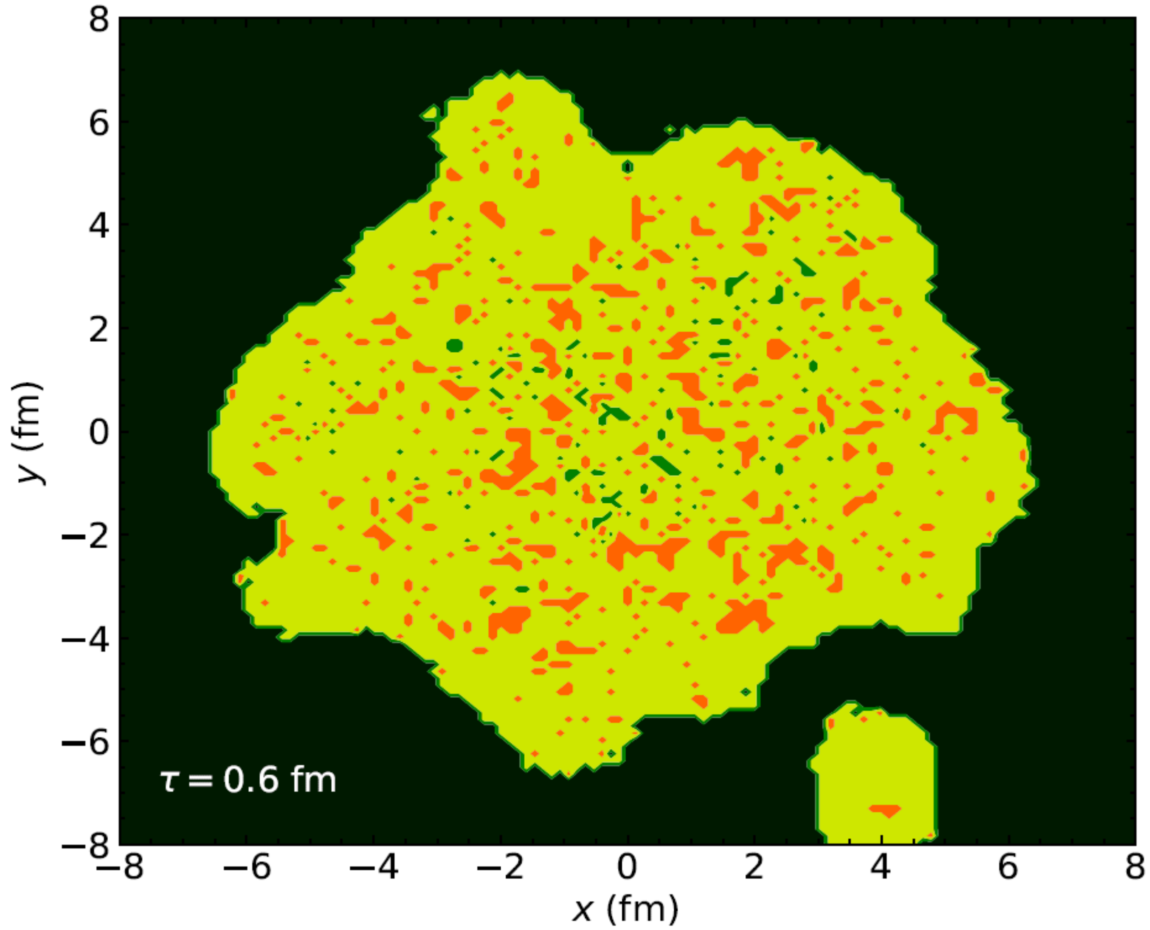






The Good, the Bad, and the Ugly

Good: Causal **Bad: Acausal but Stable** **Ugly: Unstable**



A quick resolution
(work in progress!)

Three possible philosophies

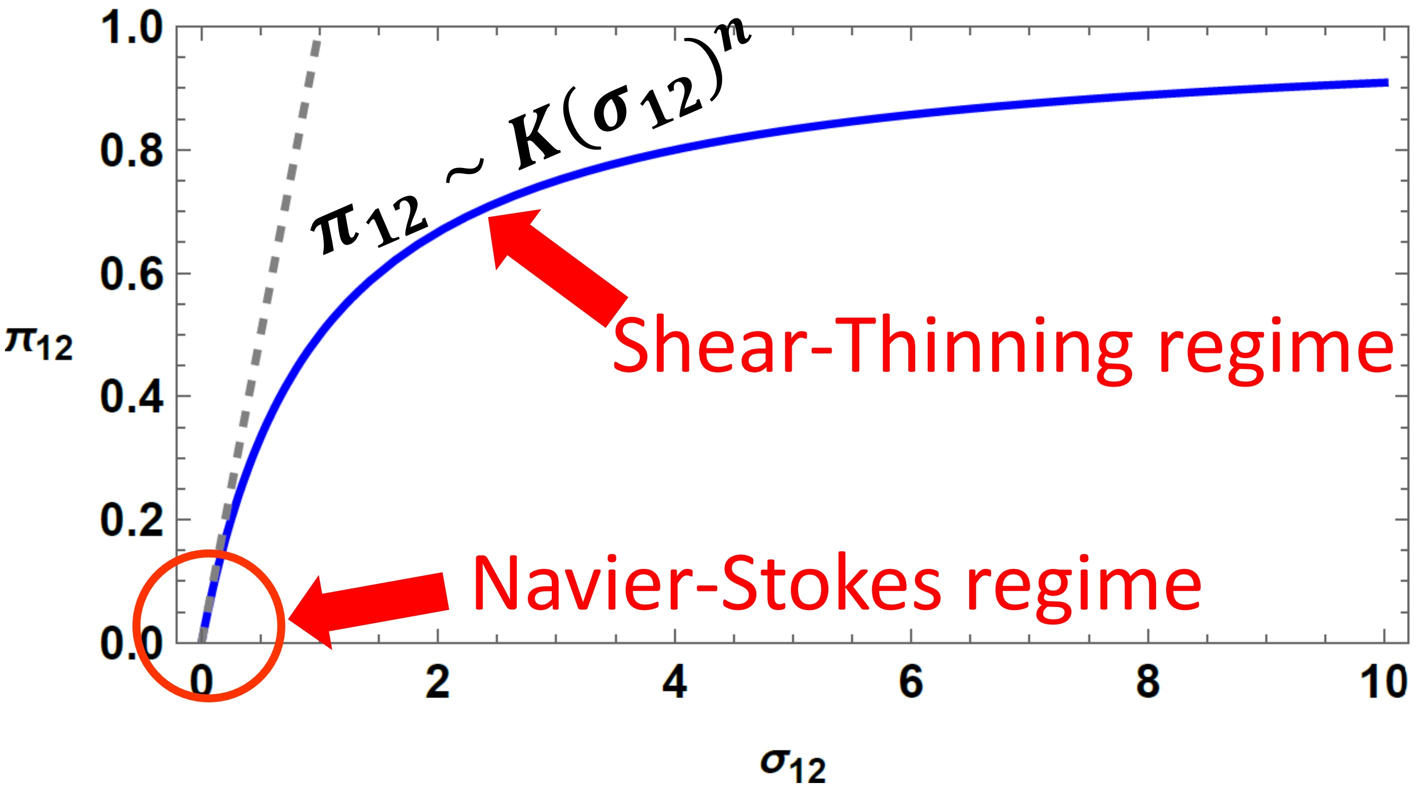
1. "Hydro does not apply far from equilibrium…"

2. Still use hydro, but replace the Israel-Stewart framework with something else entirely (e.g. BDNK, density-frame,…)

3. Try to fix Israel-Stewart

Lessons from Newtonian hydrodynamics

Many everyday fluids evolve outside of the Navier-Stokes regime (e.g., ketchup sprayed out of the bottle). Rheology is the study of the consequent resummations!



Product*	K ($Pa\ s^n$)	n (-)
Apple butter	222.90	0.145
Canned frosting	355.84	0.117
Honey	15.39	0.989
Ketchup	29.10	0.136
Marshmallow cream	563.10	0.379
Mayonnaise	100.13	0.131
Mustard	35.05	0.196
Peanut butter	501.13	0.065
Stick butter	199.29	0.085
Stick margarine	297.58	0.074
Squeeze margarine	8.68	0.124
Tub margarine	106.68	0.077
Whipped butter	312.30	0.057
Whipped cream cheese	422.30	0.058
Whipped desert topping	35.98	0.120

Relativistic rheology: microscopic picture

Setup:

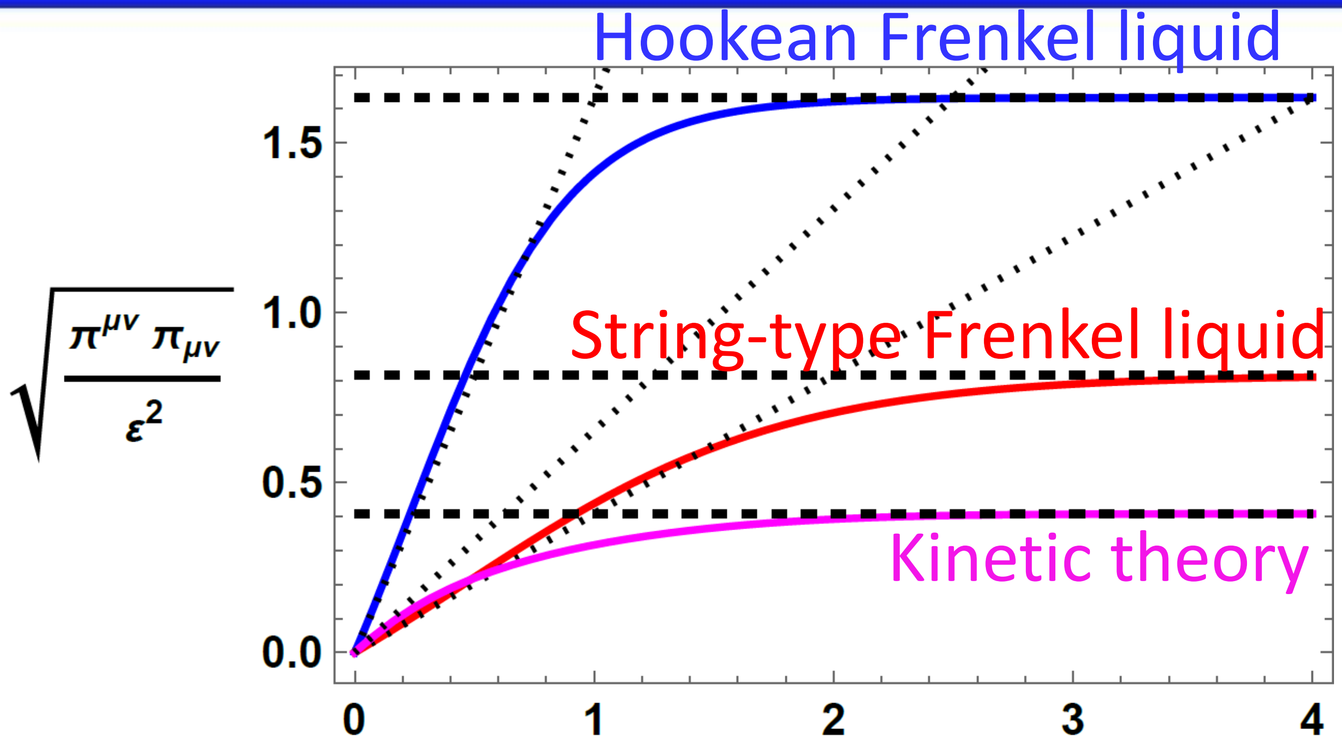
$$ds^2 = -dt^2 + \sum_{j=1}^{3} e^{2a_j t}\left(dx^j\right)^2,$$

$$\mathcal{L}_{\partial_j}\text{“Fluid”} = 0\,, \quad u = \partial_t\,,$$

$$\sigma_{\hat{j}\hat{k}} = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}$$

Find attractor of microscopic theories in this setup.

Relativistic rheology: microscopic picture



$$(a_1, a_2, a_3) = \left(-\frac{a}{2}, -\frac{a}{2}, a\right)$$

Setup:

$$ds^2 = -dt^2 + \sum_{j=1}^{3} e^{2a_j t}\left(dx^j\right)^2,$$

$$\mathcal{L}_{\partial_j}\text{“Fluid”} = 0\,, \quad u = \partial_t\,,$$

$$\sigma_{\hat{j}\hat{k}} = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}$$

Find attractor of microscopic theories in this setup.

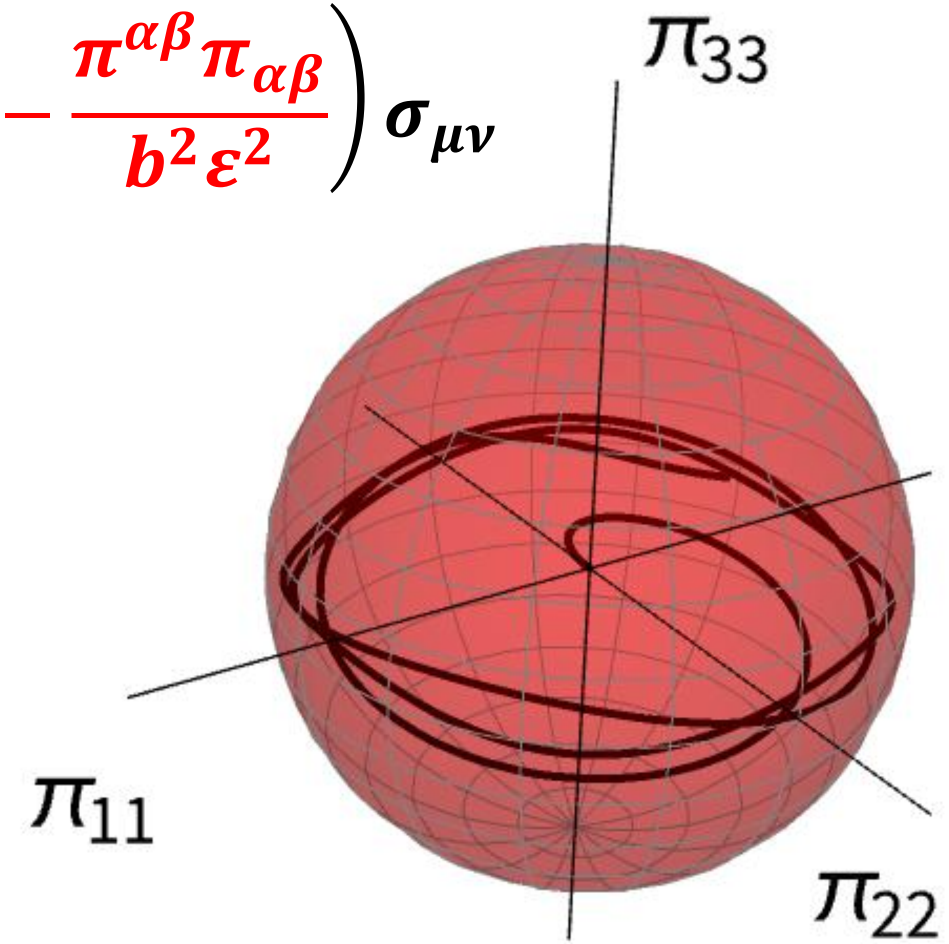
Israel-Stewart theory with maximal deformation

Replace $$\tau_\pi \Delta^{\alpha\beta}_{\mu\nu} u^\lambda \nabla_\lambda \pi_{\alpha\beta} + \pi_{\mu\nu} = -2\eta\sigma_{\mu\nu}$$

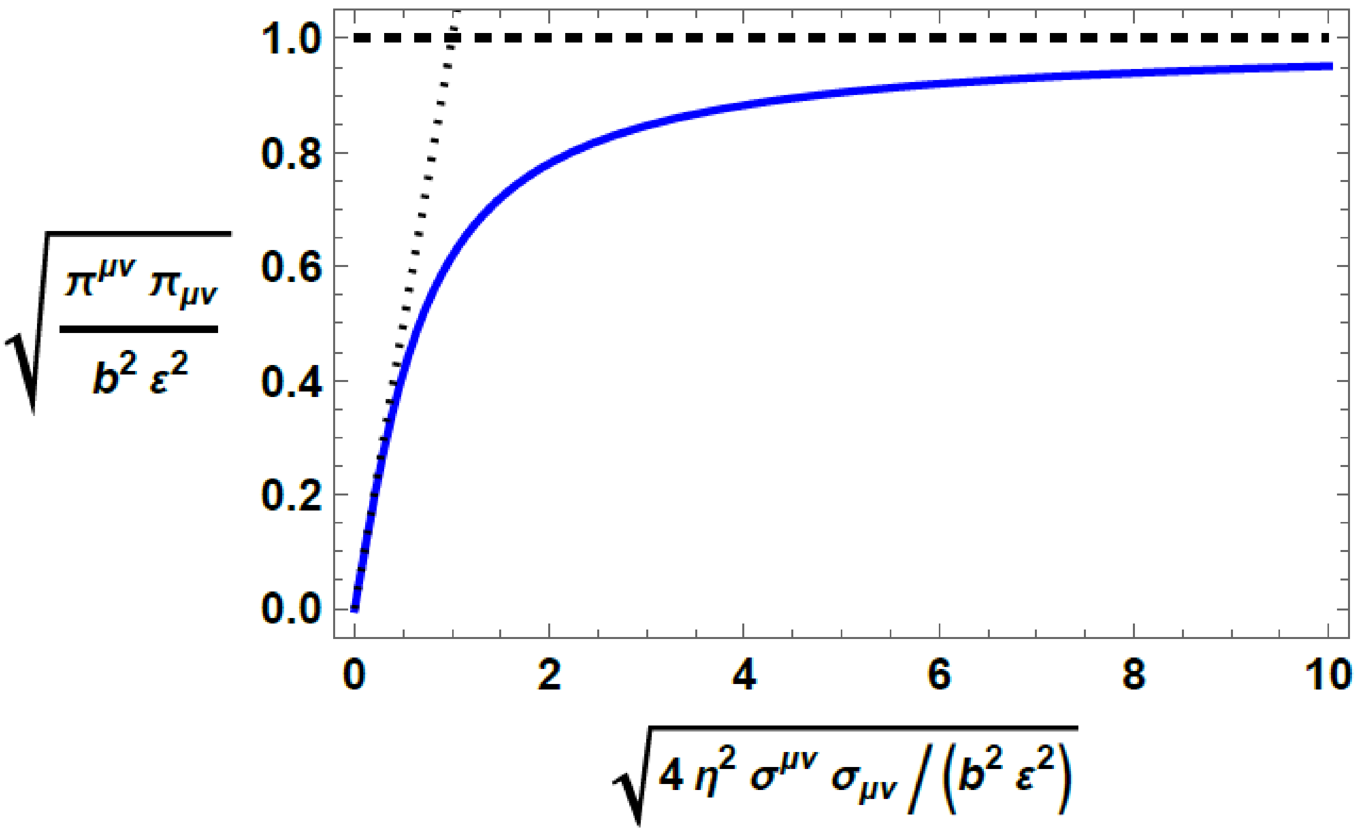
With $$\tau_\pi \varepsilon \Delta^{\alpha\beta}_{\mu\nu} u^\lambda \nabla_\lambda \left(\frac{\pi_{\alpha\beta}}{\varepsilon}\right) + \pi_{\mu\nu} = -2\eta\left(1 - \frac{\pi^{\alpha\beta}\pi_{\alpha\beta}}{b^2\varepsilon^2}\right)\sigma_{\mu\nu}$$

The quantity $\sqrt{\frac{\pi^{\alpha\beta}\pi_{\alpha\beta}}{\varepsilon^2}}$ cannot grow above b.

(see also Chiu et al., arXiv:2508.05292)



In our rheology setup



Recall:

$$ds^2 = -dt^2 + \sum_{j=1}^{3} e^{2a_j t}\left(dx^j\right)^2,$$

$$\mathcal{L}_{\partial_j}\text{“Fluid”} = 0\,,\quad u = \partial_t\,,$$

$$\sigma_{\hat{j}\hat{k}} = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}$$

Find attractor of the theory in this setup.

Causality is back!

Go to a local rest frame and diagonalize the stress tensor:

$$u^\alpha = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \pi_{\alpha\beta} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \pi_{11} & 0 & 0 \\ 0 & 0 & \pi_{22} & 0 \\ 0 & 0 & 0 & \pi_{33} \end{bmatrix}$$

Assume propagation along the x^1 axis. Information speeds:

$$w^2 = c_s^2 + \frac{\pi_{11}}{\varepsilon} + \frac{4\eta\left(1 - \pi^{ab}\pi_{ab}/b^2\varepsilon^2\right)}{3\tau_\pi(\varepsilon + P + \pi_{11})}$$

Conclusions

1) At early times, our simulations should blow up/crash.
They don't.

2) There are some easy fixes that do not involve fully reinventing hydro.

3) Such fixes have physical justifications within rheology.

Appendix

A simple instability mechanism

Quick preliminary.

In non-relativistic fluids, the momentum density is

$$\boldsymbol{\wp} = \boldsymbol{\rho v}$$

In relativistic bulk-viscous fluids, it becomes

$$\boldsymbol{\wp} = \boldsymbol{T^{01}} = (\boldsymbol{\rho} + \boldsymbol{P} + \boldsymbol{\Pi})\boldsymbol{\gamma}^2\boldsymbol{v}$$

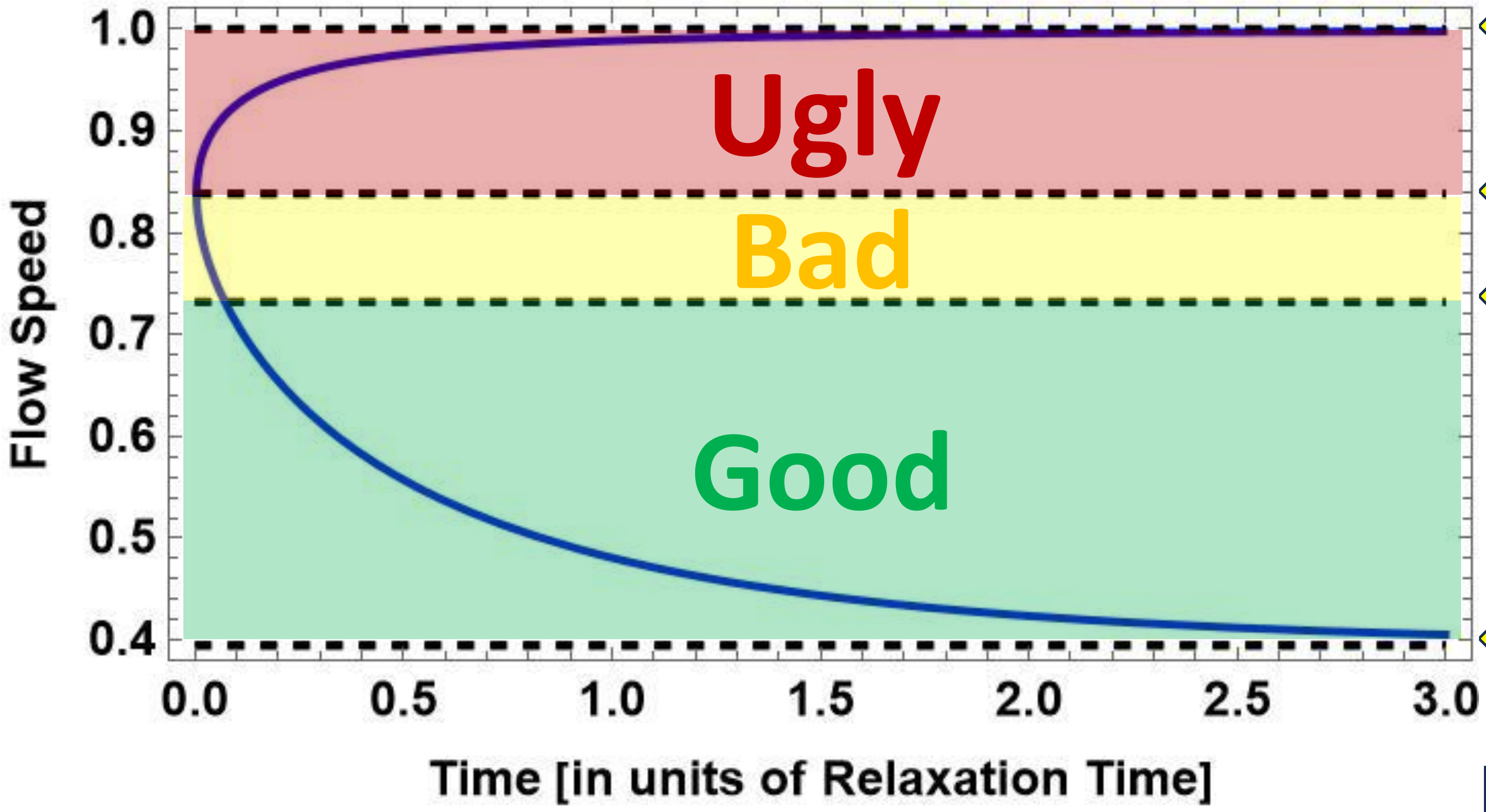
The fluid can accelerate with no external force by varying the value of $\boldsymbol{\Pi}$.

If $\boldsymbol{\Pi} \to -(\boldsymbol{\rho} + \boldsymbol{P})$, the fluid can move at the speed of light, while still having finite momentum (and energy).

A simple instability mechanism

Analytical solution of Israel-Stewart with bulk alone ($\tau_\Pi \dot{\Pi} + \Pi = -\zeta \partial_\mu u^\mu$).

A uniform moving fluid uses Π to "fuel" its own acceleration.



v = Speed of light

$vw = 1$ (stability ends)

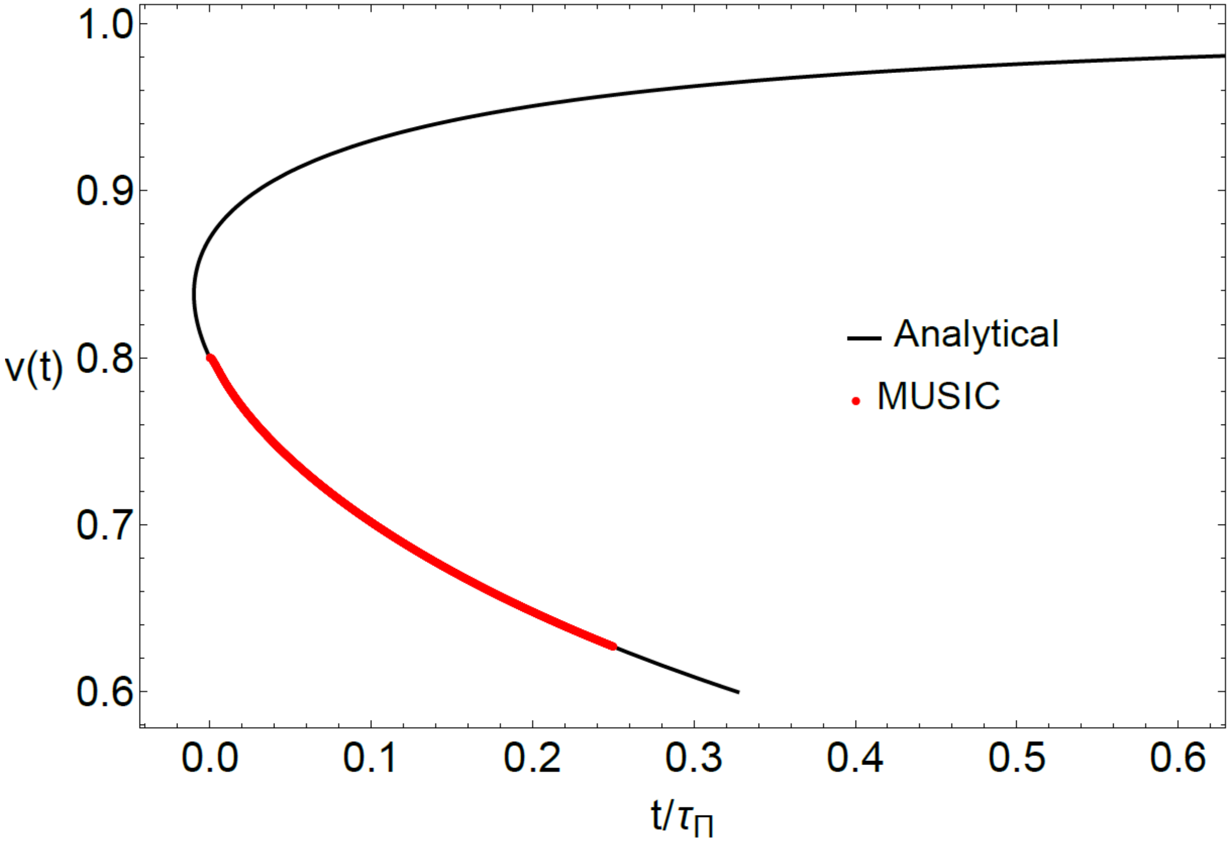
$w = 1$ (causality ends)

v = Speed at equilibrium

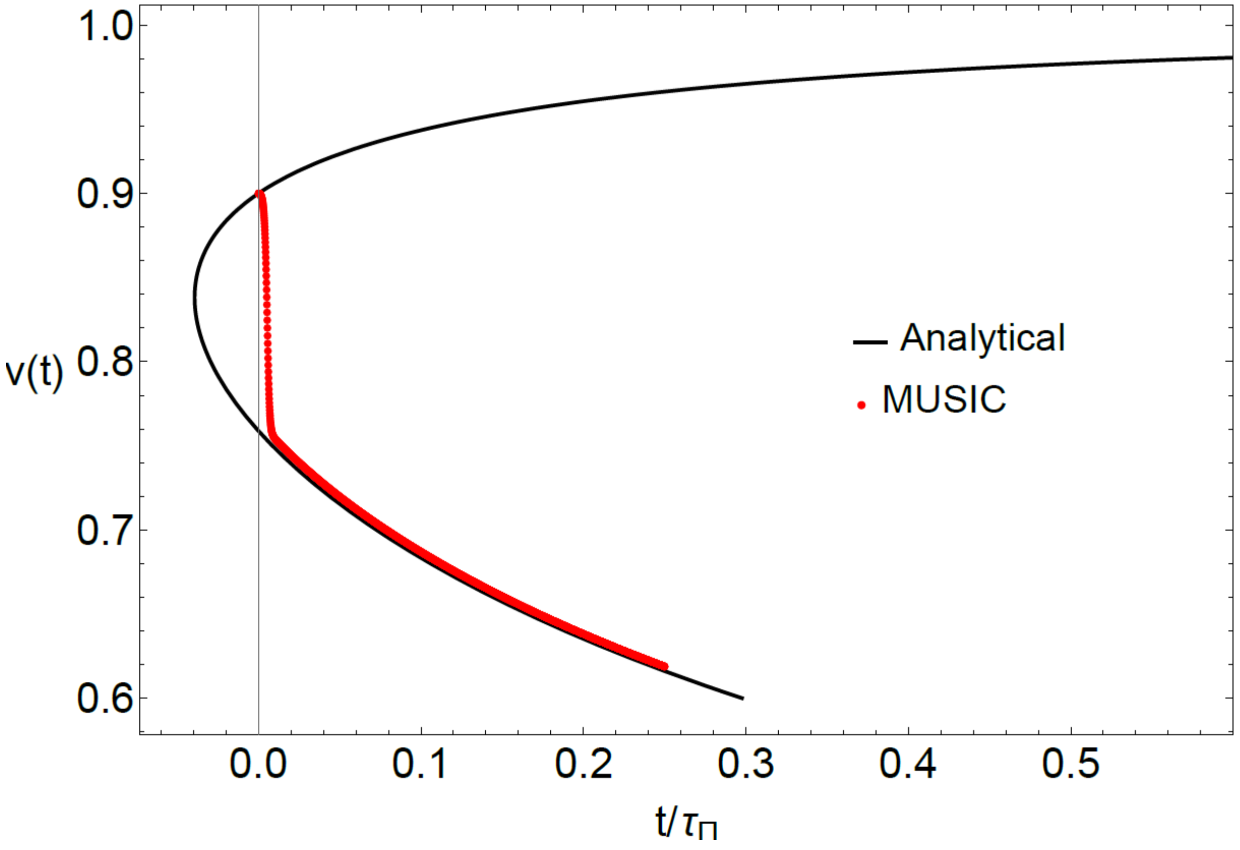
If $vw>1$, the fluid "runs away"!

The simulation does not see the instability!

Start Good or Bad…
Nothing to declare.



Start Ugly…
The numerical solution does not explode.



Resummed hydrodynamics

Proposal by Chiu, Denicol, Luzum, and Shen (arXiv:2508.05292).

Keep

$$u^\alpha \nabla_\alpha \varepsilon + (\varepsilon + P)\nabla_\alpha u^\alpha + \pi^{\alpha\beta}\sigma_{\alpha\beta} = 0$$

$$(\varepsilon + P)u^\beta \nabla_\beta u_\alpha + c_s^2 \Delta_\alpha^\beta \nabla_\beta \varepsilon + \Delta_\alpha^\beta \nabla_\mu \pi_\beta^\mu = 0$$

$$\tau_\pi \Delta_{\mu\nu}^{\alpha\beta} u^\lambda \nabla_\lambda \pi_{\alpha\beta} + \pi_{\mu\nu} = -2\eta\sigma_{\mu\nu}$$

But

$$\eta \to \eta_{res} = \frac{\eta}{1 + \mathrm{arctanh}^2(\text{Reynolds numbers})}$$