Thermalization and entropy production in heavy-ion collisions

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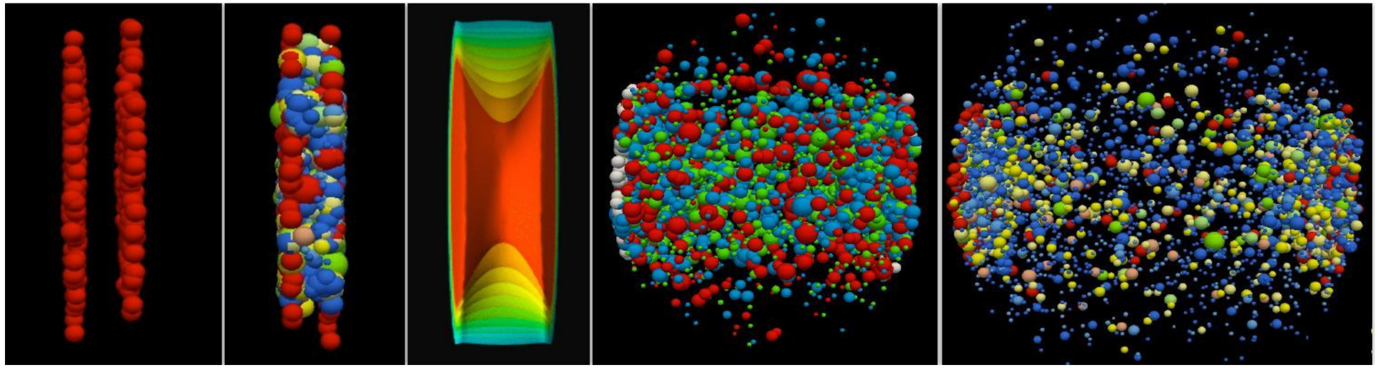
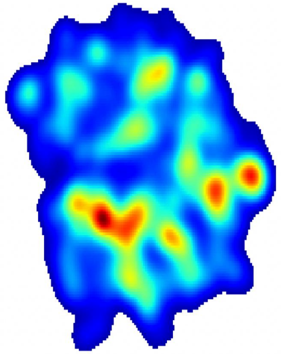
Attractors and thermalization in nuclear collisions and cold quantum gases

Sep. 25, 2025, Trento, Italy

With Chenxi Liang

Hybrid Hydro modeling of heavy-ion collisions





local thermalization

$$
\begin{cases}
\partial_\mu T^{\mu\nu} = 0 \\
T^{\mu\nu} = e u^\mu u^\nu + (P - \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} \\
(\tau_\pi u \cdot \partial + 1)\pi^{\mu\nu} = \eta \sigma^{\mu\nu} + \ldots \\
(\tau_\Pi u \cdot \partial + 1)\Pi = -\zeta \nabla \cdot u + \ldots
\end{cases}
$$

$+$ LEOS: $e(P)\,, e(T)\,, P(T)\,, \ldots$

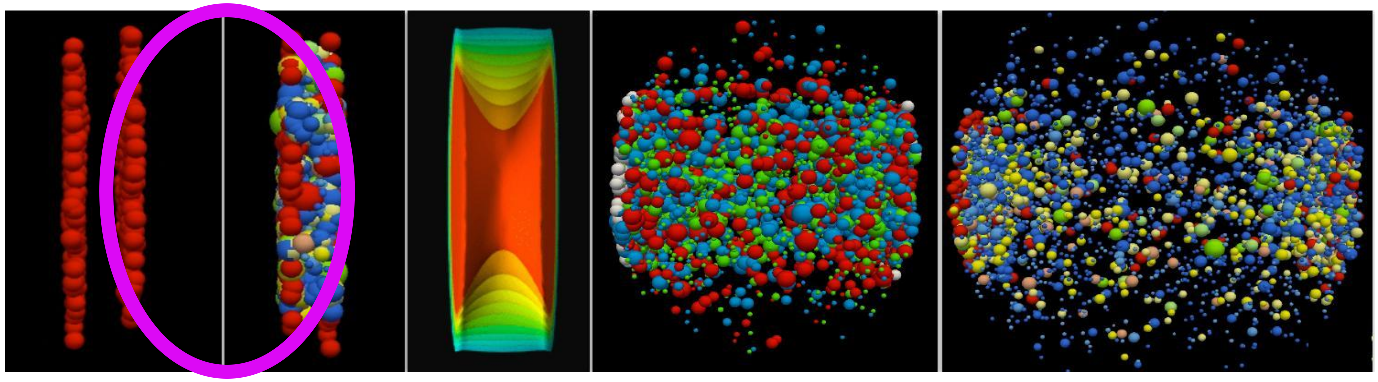
freeze-out

hadron transport

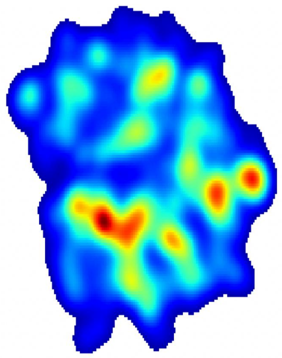
- Relativistic hydrodynamics solves conservation of energy and momentum (baryon charge, etc.)

Hybrid Hydro modeling of heavy-ion collisions

hydro attractor



How and why?



local thermalization →

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$+$ LEOS: $e(P)\,, e(T)\,, P(T)\,, \dots$

freeze-out → hadron transport

- Relativistic hydrodynamics solves conservation of energy and momentum (baryon charge, etc.)

Flow-related observables

[J. Grosse-Oetringhaus and U. Wiedemann, 2407.07484]

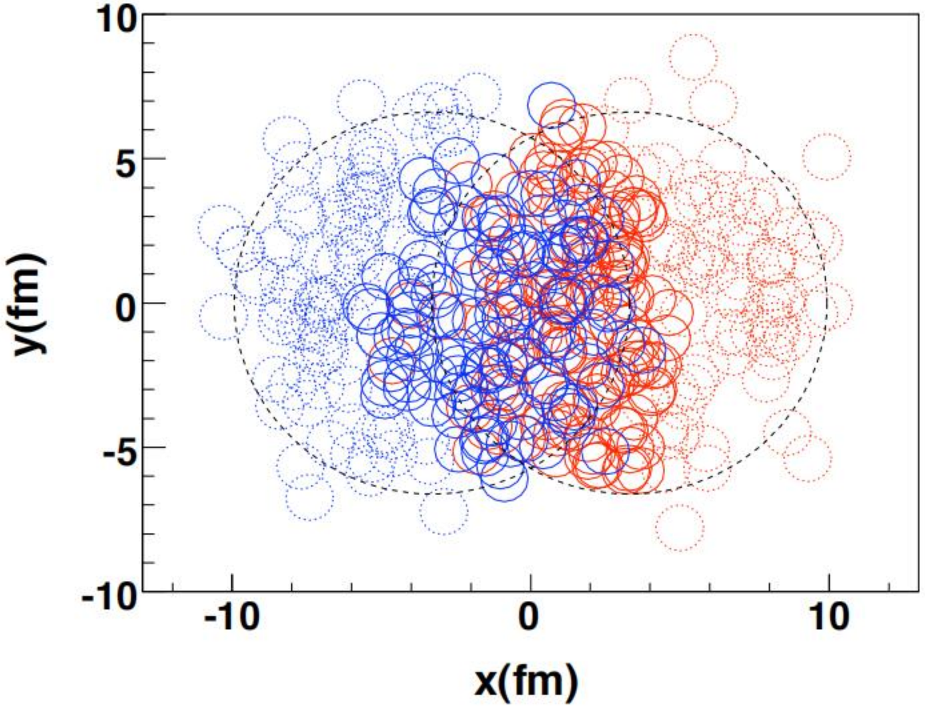
Observable or effect	Pb–Pb, Xe–Xe, Au–Au	p–Pb, a–A (high N)	pp (high N)	Refs.
Near-side ridge yields	yes	yes	yes	33–36,74,76,77,79,79,100
Azimuthal anisotropy	v_1–v_9	v_1–v_5	v_2–v_4	34–36,46,73–89,101,102
Weak η dependence	yes	yes	yes	82,90–98
Characteristic mass dependence	v_2–v_5	v_2, v_3	v_2	78,81,83,87,103–110
Higher-order cumulants (mainly $v_2\{n\}$, $n \geq 4$)	"$4 \approx 6 \approx 8 \approx$ LYZ" +higher harmonics	"$4 \approx 6 \approx 8 \approx$ LYZ" +higher harmonics	"$4 \approx 6$"	83,84,88,96,109,111–123
Symmetric cumulants (SC)	up to $(5,3)$	only $(4,2)$, $(3,2)$	only $(4,2)$, $(3,2)$	86,88,124–130
Non-linear flow modes	up to v_7	not measured	not measured	89,131,132
Factorization breaking	$n = 2$–4, $\{2\}$, $\{4\}$	$n = 2, 3$, $\{2\}$	not measured	77,85,133–137
Event-by-event v_n distributions	v_2–v_4	not measured	not measured	138–140
Flow–p_T correlation	up to v_4	v_2	not measured	141,142
Directed flow (from spectators)	yes	no	no	143
Charge-dependent correlations	yes	yes	yes	144–150
Low p_T spectra ("radial flow")	yes	yes	yes	52,151–161
Intermediate p_T ("recombination")	yes	yes	yes	153,156,160,162–166
Particle ratios	GC level	GC level	GC level	153,154,157,158,167,168
Statistical model	$\gamma_s^{GC} = 1$	$\gamma_s^{GC} \approx 1$	$\gamma_s^{C} < 1$	52,161,169–171
HBT radii ($R(k_T)$, $R(\sqrt[3]{N})$)	$R_{out}/R_{side} \approx 1$	$R_{out}/R_{side} \lesssim 1$	$R_{out}/R_{side} \lesssim 1$	172–180
Direct photons at low p_T	yes	not measured	not observed	181–183

To some extent, hydro works for all these observables! (unreasonable succsess of hydro.)

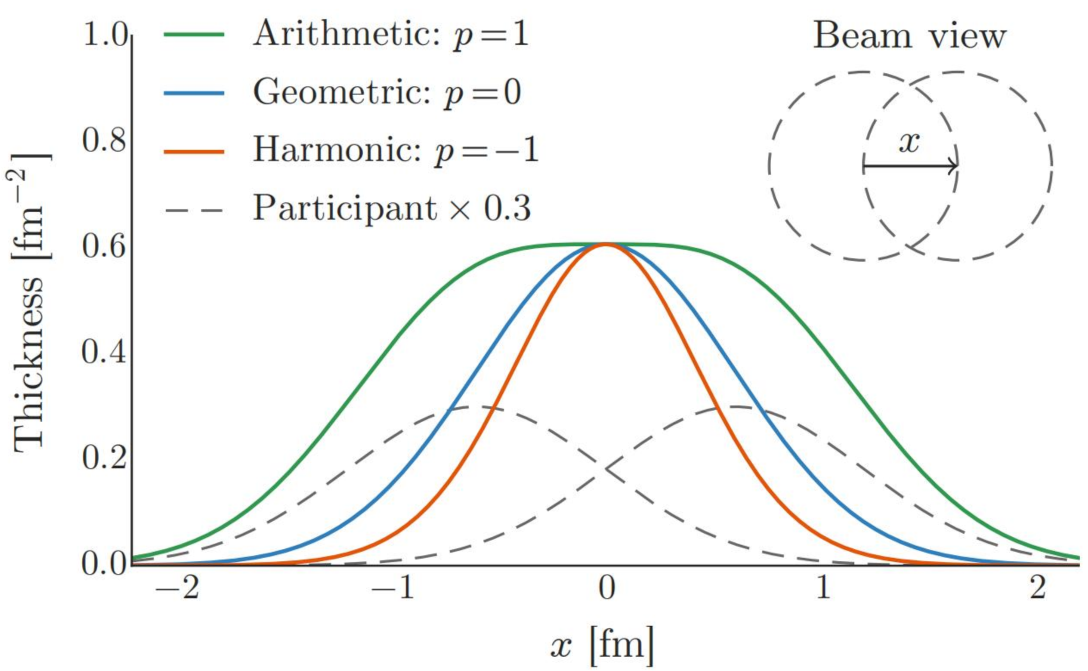
Entropy production $\leftrightarrow$ Thermalization

Thermalization assumption and entropy production in models

[MC-Glauber, 0805.4411]



[Trento Model, J. Moreland, J. Bernhard, and S.Bass, 1412.4708]

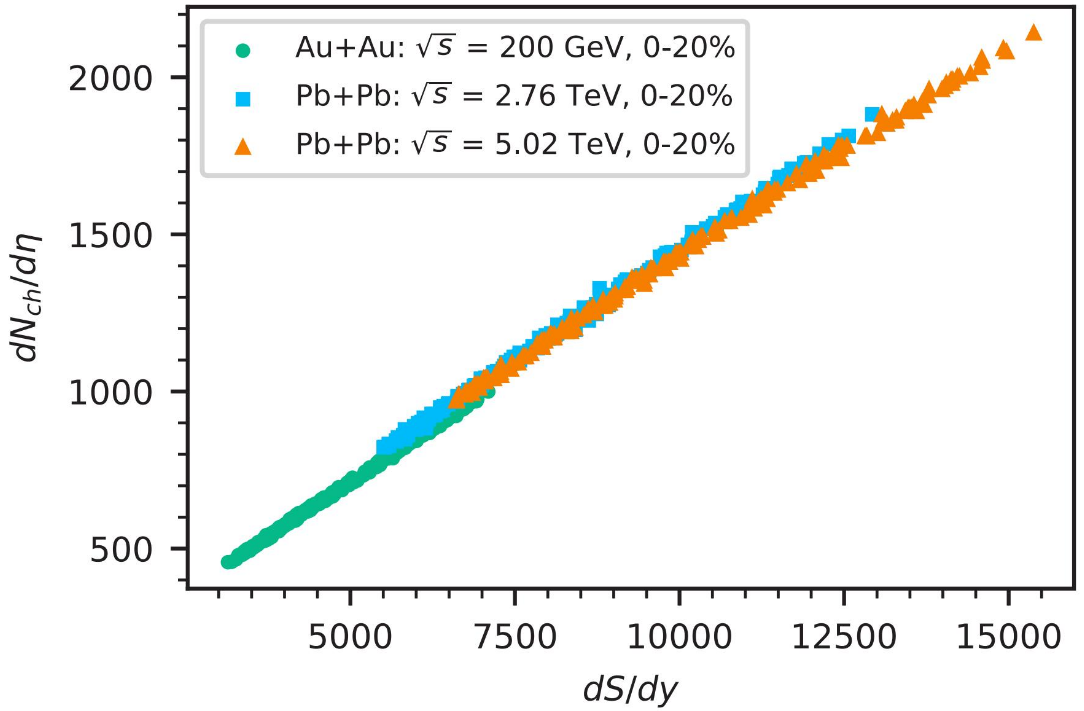


- Determine where entropy is produced.
- Determine whether entropy is produced.
- How much entropy is produced?

$$\frac{dS}{dy}\Big|_{\tau_0} = N_{norm} \underbrace{T_R(T_A, T_B)}_{\text{nucleon-nucleon collision}} \propto \left(\frac{T_A^p + T_B^p}{2}\right)^{1/p}$$

Estimate entropy production from $dN_c\,/d\eta$

- Linearity between $dN_c\,/d\eta$ and dS/dy



$$\frac{dN_c}{d\eta} \approx \frac{1}{7.14}\frac{dS}{dy}\Big|_{hyd}$$

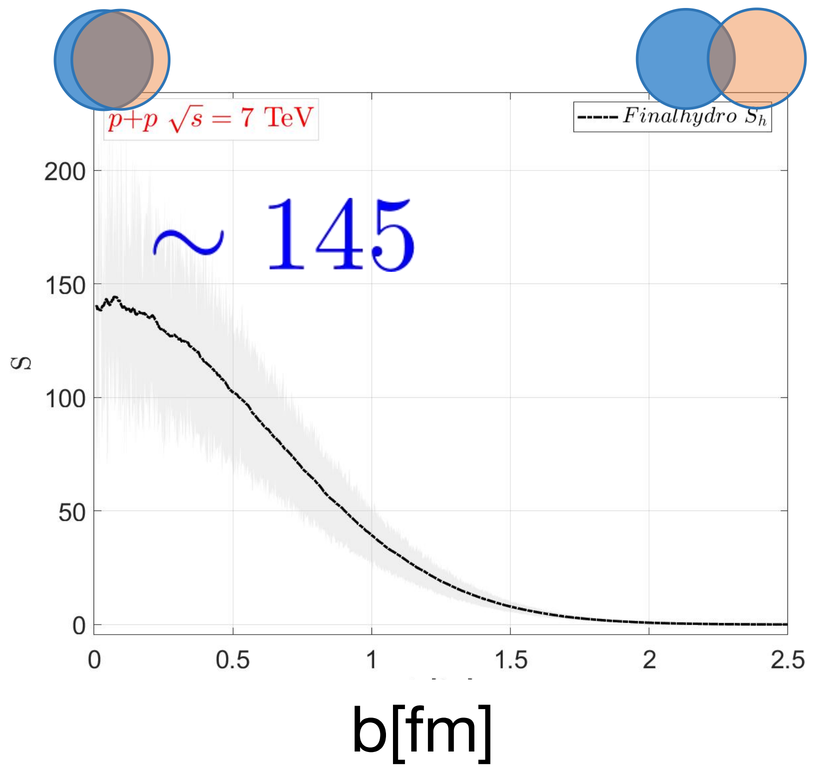
[J. Churchill, LY, S. Jeon and C. Gale, PRC 21]

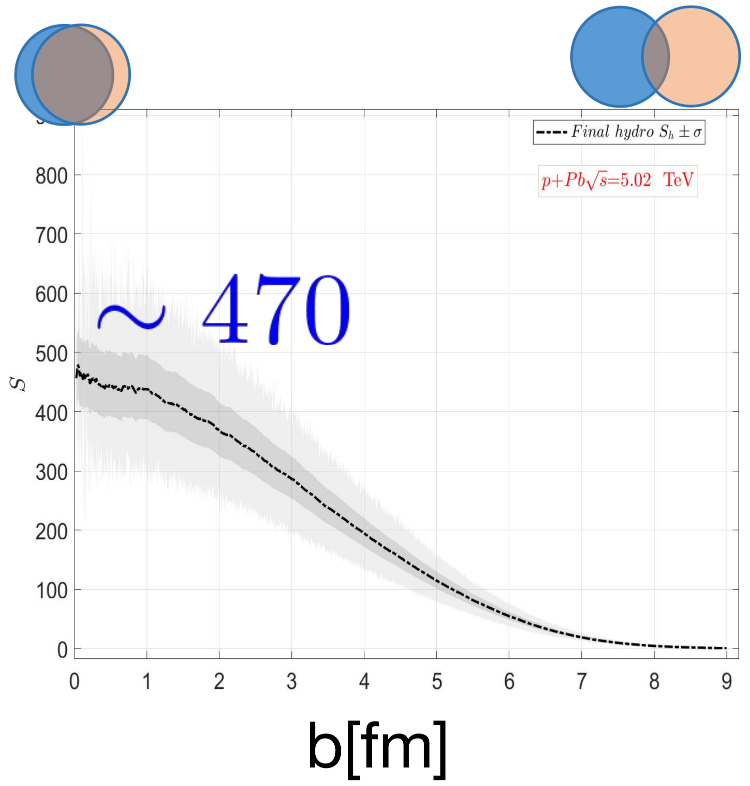
[P. Hanus, K. Reygers and A. Mazeliauskas, 1908.02792]

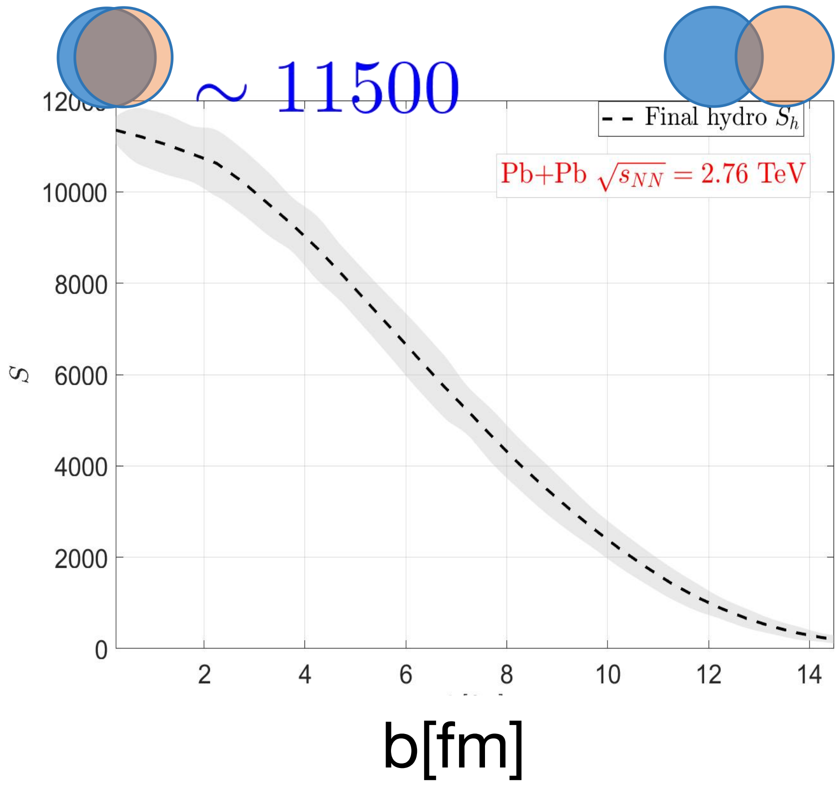
system	dS/dy	(dS/dy)/(dN_{ch}/dy)
Pb–Pb, 0–10%	$11\,335 \pm 1188$	6.7 ± 0.8
pp minimum bias	37.8 ± 3.7	5.2 ± 0.5
pp high mult.	135.7 ± 17.9	5.4 ± 0.7

$$\frac{dN_c}{d\eta} \propto \frac{dS}{dy}\Big|_{total} \gtrsim \frac{dS}{dy}\Big|_{hyd}$$

Empirical estimate of the total entropy S_{total} from Trento



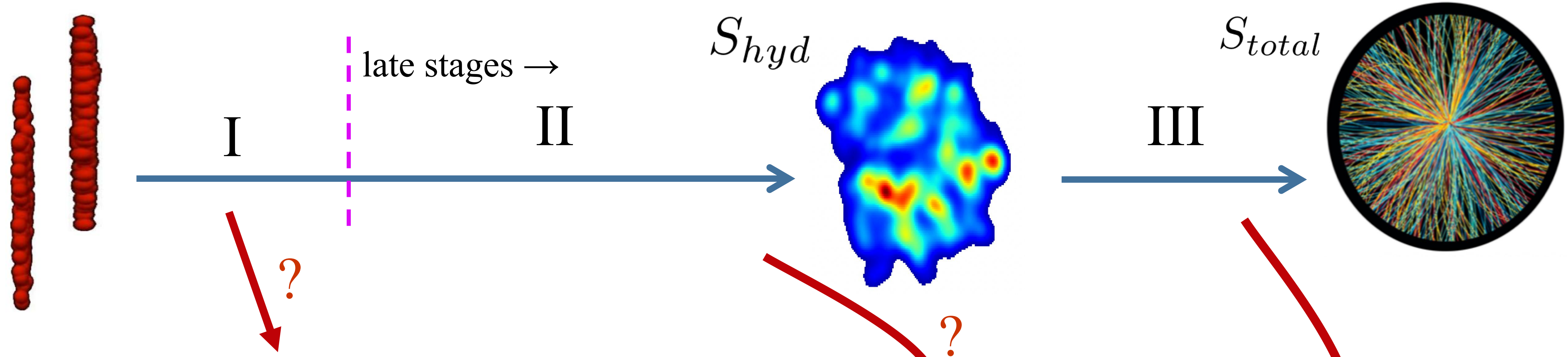




- Calibrate with respect to $N_c \propto S_{total}$ in ultra-central events.
- Determine collision centrality using EbE Trento, estimate S_{total} for all centralities.
- Empirical based on hydro, less reliable at peripheral collisions.

Entropy production in different stages

- Evolution stages and effective model characterization



I. Entanglement Entropy and Quantum Thermalization.

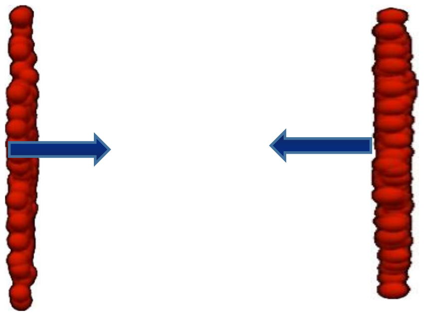
II. Pre-equilbrium Entropy: e.g., attractor, KOMPOST

III. Fluid Entropy production: dissipative fluid:

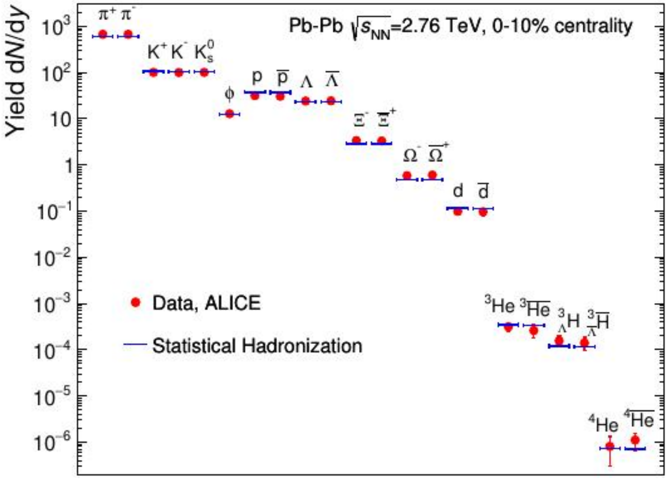
$$\partial_\mu s^\mu = \pi^{\alpha\beta}\langle\nabla_\alpha u_\beta\rangle/T + \dots$$

Thermalization of a QCD matter: quantum thermalization

[Nature 561, 321(2018)]



$$\lim_{t\to\infty}\langle\Psi(t)|\hat{O}|\Psi(t)\rangle \propto \mathrm{Tr}[e^{-\hat{H}/T}\hat{O}]$$

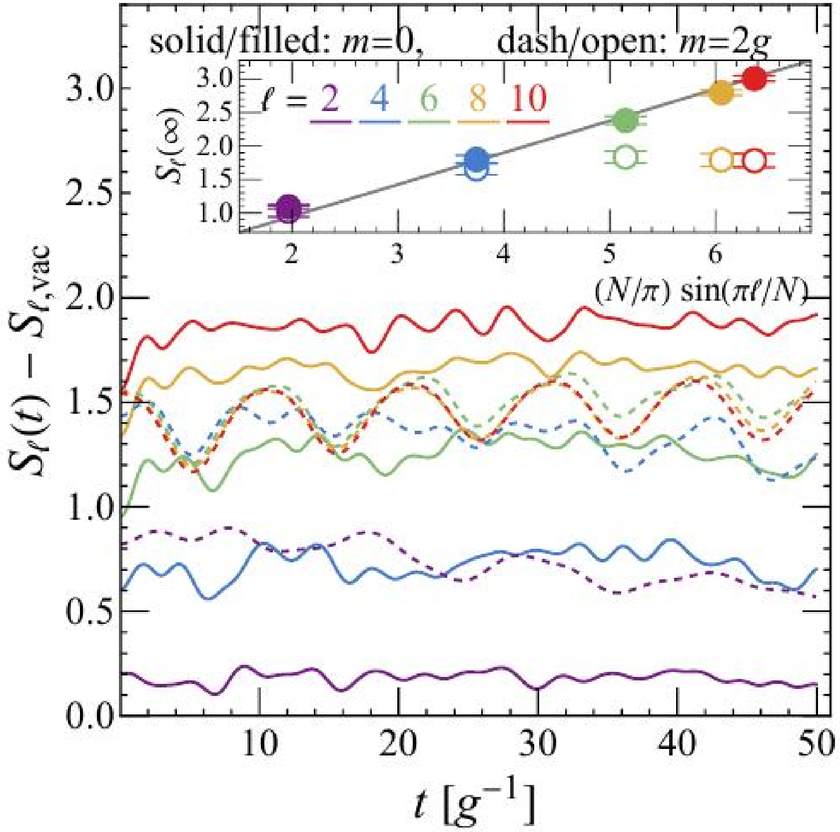


similarly for pp, electron-positron

How thermal ensemble emerges from pure Quantum states, under unitarty interaction?

- QGP is a high-energy (closed) quantum system obeying non-Abelian gauge theory.
- QGP thermalization is beyond perturbative QCD characterization.
- Beyond current lattice QCD simulation: time evolution requires diagonalization of Hamiltonian.

Quantum thermalization and entanglement entropy in QED_2



Entanglement Entropy

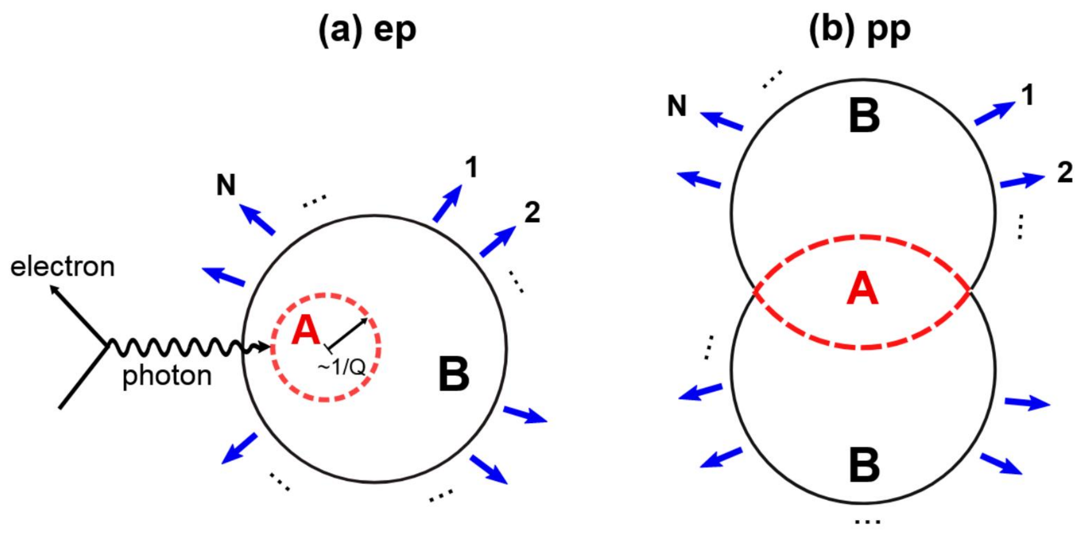
$$\hat{H} = \int \Big(\bar{\psi}(\gamma^1(-i\partial_z - g\,A_1) + m\,e^{i\theta\gamma^5})\psi + \frac{1}{2}\mathcal{E}^2 \Big) \mathrm{d}z$$

- How a system resembles QCD thermalizes quatum mechanically? (with confinement, vacuum topoloty, ...)
- Realization on 1D lattice and quantum computation (classical/quantum device): 1D spin model
- Quantum thermalization realized in “massless”/strongly-coupled system: $\delta m \sim a g^2$

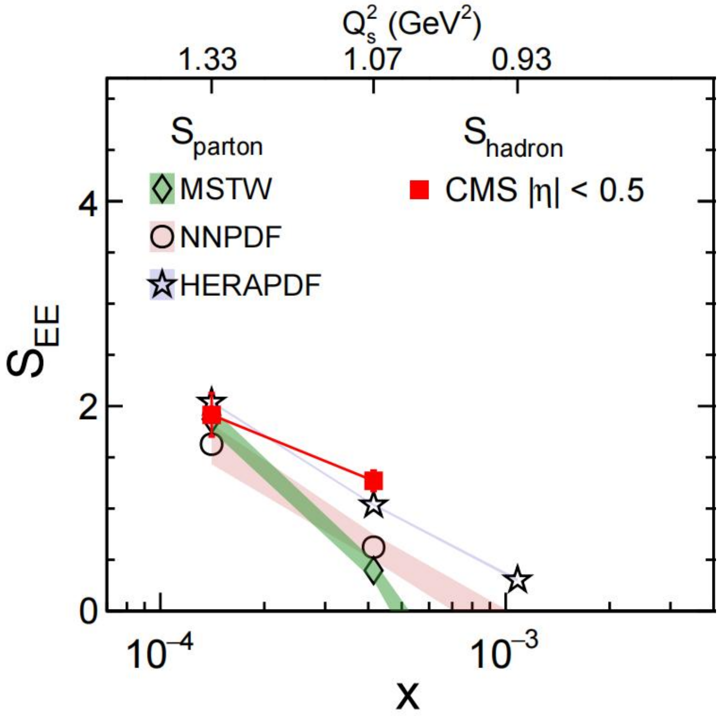
[Shile Chen, Shuzhe Shi, LY, 2412.00662]

Entanglement entropy production in high-Q^2 inclusive scatterings

[D. Kharzeev and E. Levin, PRD 95, 114008 (2017),
Z. Tu, D. Kharzeev and T. Ulirich, PRL19]



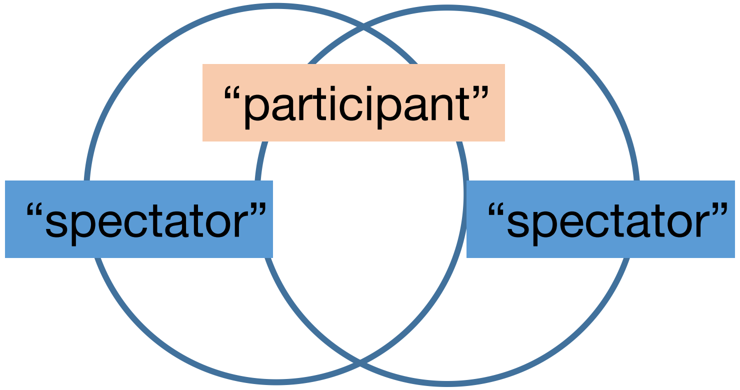
$$S_A = -tr[\hat{\rho}_A \ln \hat{\rho}_A] = S_B = -tr[\hat{\rho}_B \ln \hat{\rho}_B] \overset{?}{=} S_{hadron} = -\sum P(N) \ln P(N)$$



- In high-Q^2 inclusive scatterings no interference among states of different n_{parton} , in small x limit.
- Region involved in collision approaches maximally entanglement.
- Entanglement entropy is calculable and related to the parton distribution function (pdf).

$$S_{En} = \log(xG(x, Q^2))$$

Quantum thermalization in inclusive scatterings



- "Spectators" are traced away → reduced density matrix

$$|\psi\rangle_p = \sum_n \frac{1}{\sqrt{\mathcal{D}}}|n, x, Q^2\rangle \,, \quad \mathcal{D}\text{: dimension of Fock space}$$

- Maximally entangled in a bipartite system: "participant" + "spectator"

[PRA43, 2046 (1991); PRE50.888 (1994)]

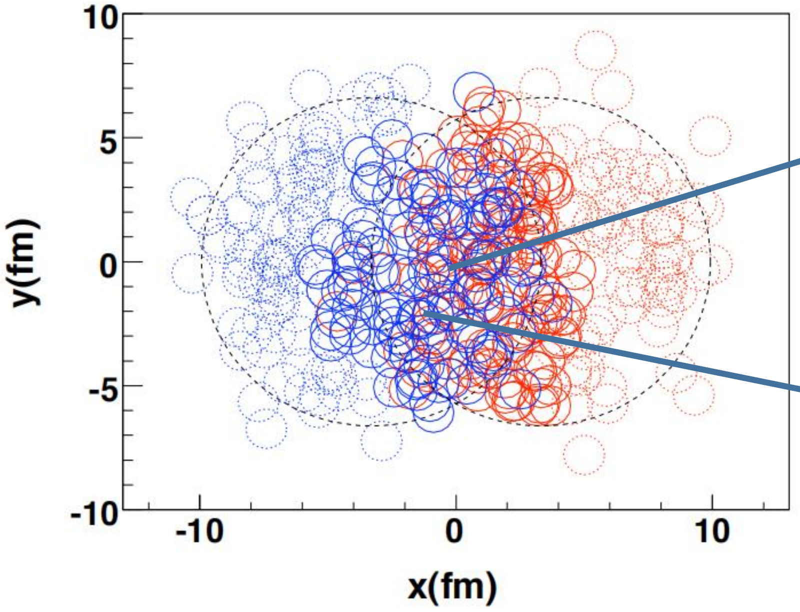
$$xG(x,Q^2) \equiv \langle\psi|\hat{N}|\psi\rangle = \frac{\mathcal{D}}{2} \xleftarrow{\text{ETH}} N_{mn} = N(E)\delta_{mn} + e^{-S} r_{mn}$$

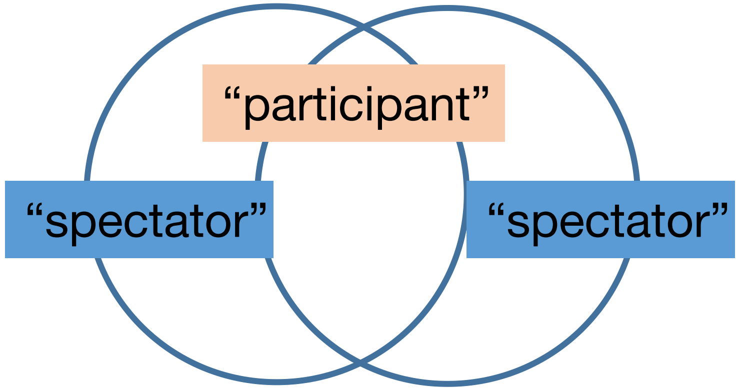
$$\rho_p = |\psi\rangle_{pp}\langle\psi| \to S_{En} = -\text{tr}[\rho_p \log \rho_p] = \log \mathcal{D} \approx \log(xG(x,Q^2))$$

Quantum thermalization in high-Q[2] and small-x inclusive scatterings is generic.

$$Q \gg \Lambda_{QCD} \,, \quad x \ll 1$$

Generalization to heavy-ion collisions





$$S_{En} = N_{part} \times \log[xG_N(x, Q^2)]$$

- Generalization: (pdf → npdf) + nuclear geometry from Trento model [2112.12462]
- Consider central rapidity and Q depends on collision energy and collision system, e.g.,

$$Q \sim A^{1/3}, \qquad Q \sim \sqrt{s}^{\lambda}, \qquad x \sim Q/\sqrt{s}$$

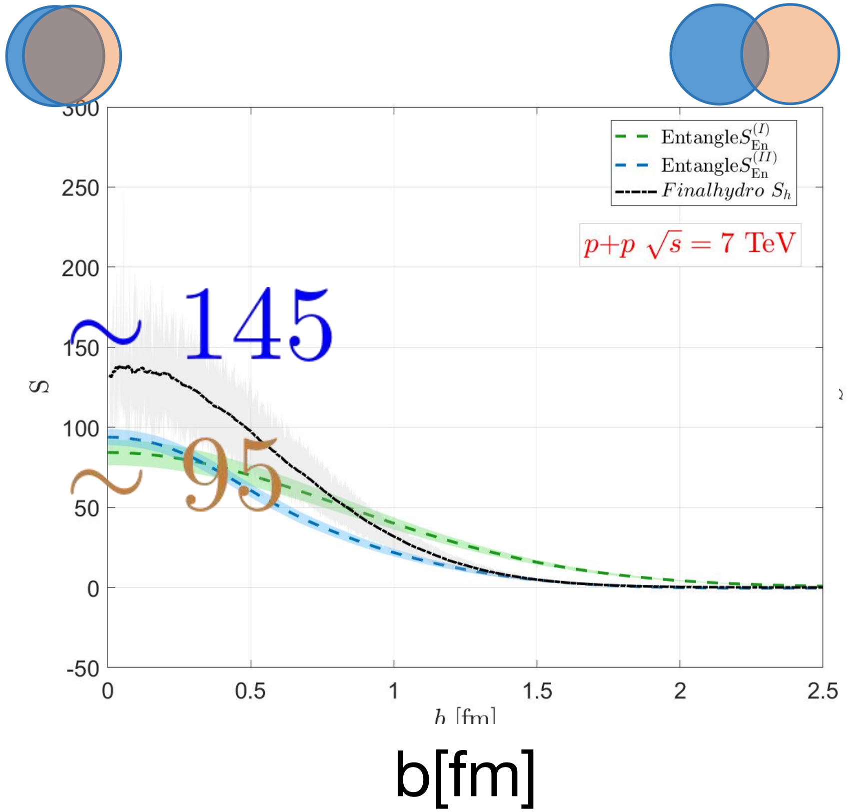
[L.McLerran and R. Venugopalan, PRD 1994]

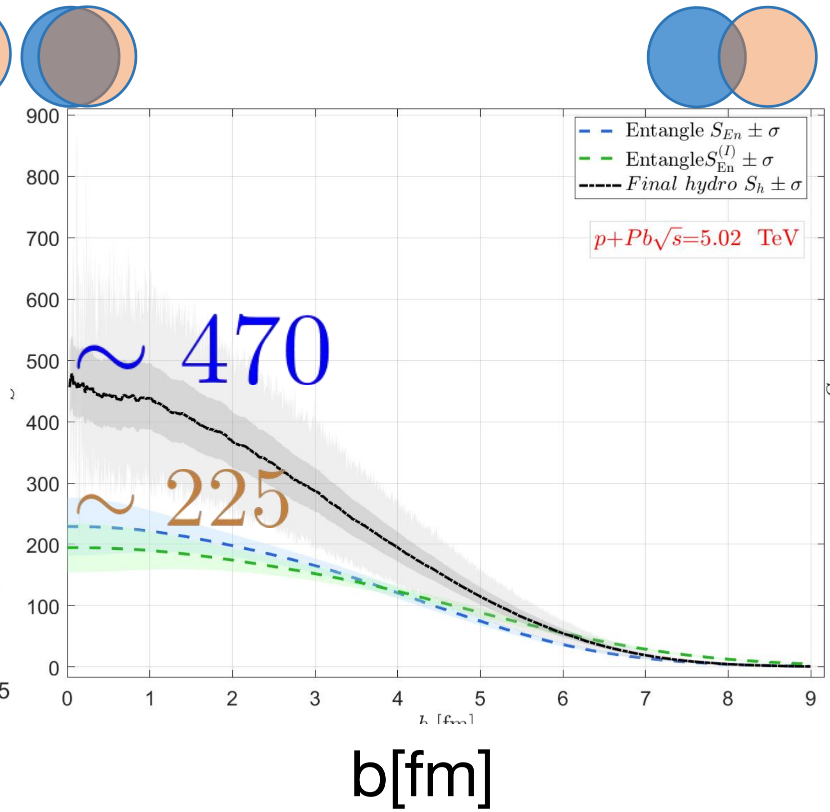
- A more sophisticated/realistic formulation:

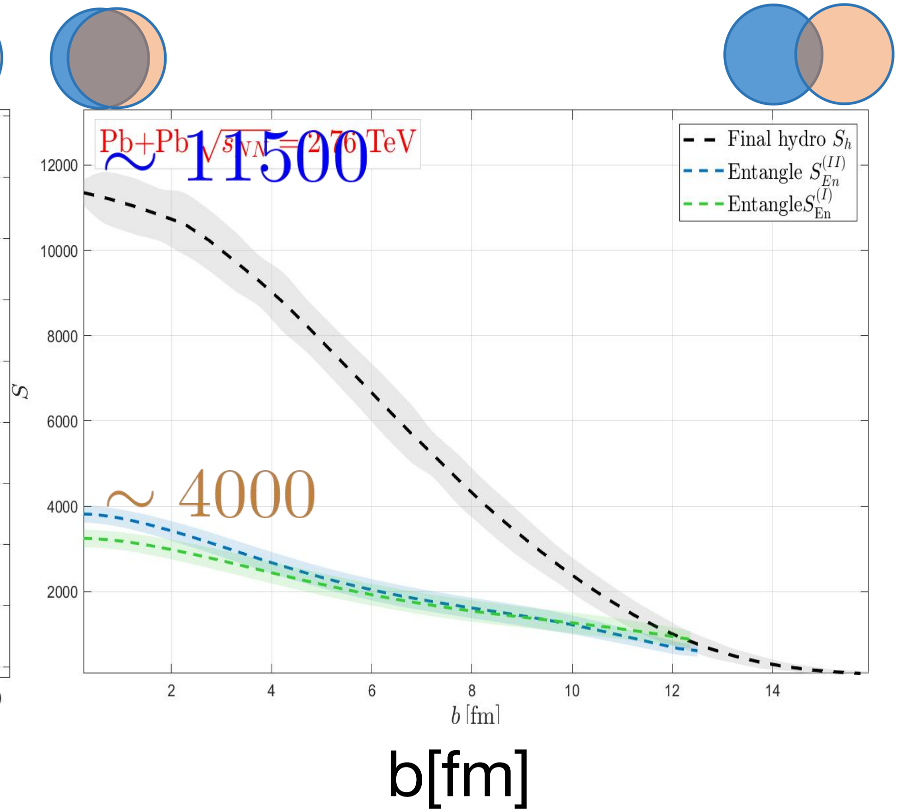
$$S_{En} = \sum_{i}^{N_{part}} \log[xG_N(x, Q^2(\vec{x}_\perp^i, b))]$$

[H. Kowalski and D. Teaney, PRD 2003]

Entanglement entropy production

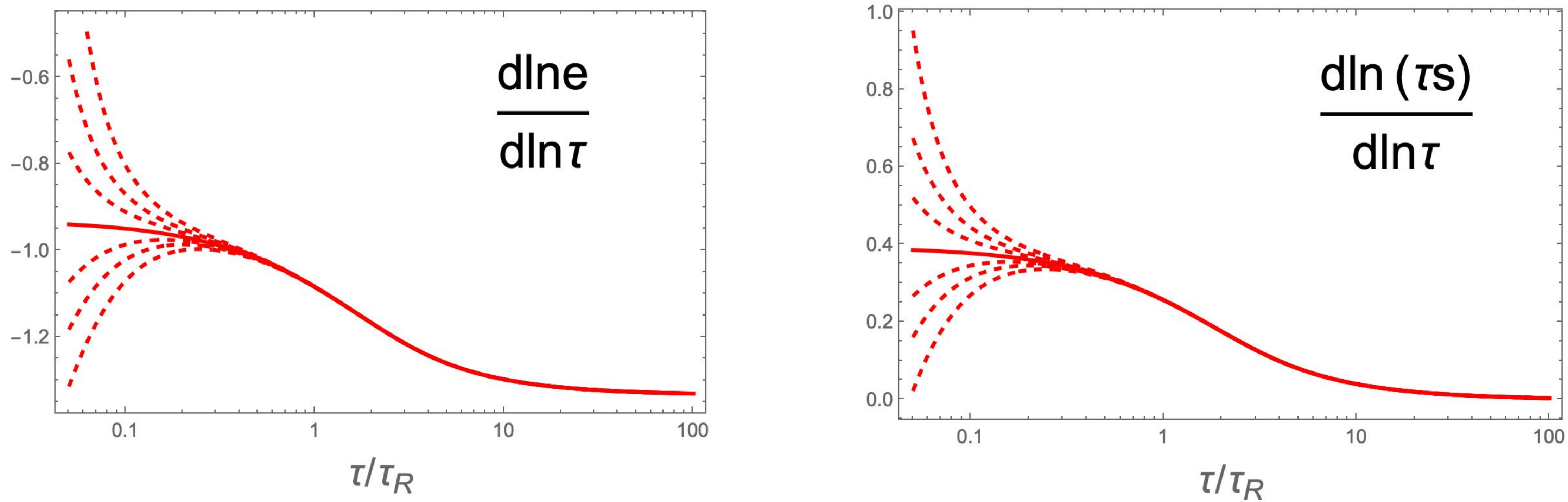






Exclude S_{En}, one does notice an increase of entropy production from late stages, from pp to PbPb.

Pre-equilibrium entropy production wrt. hydro attractor



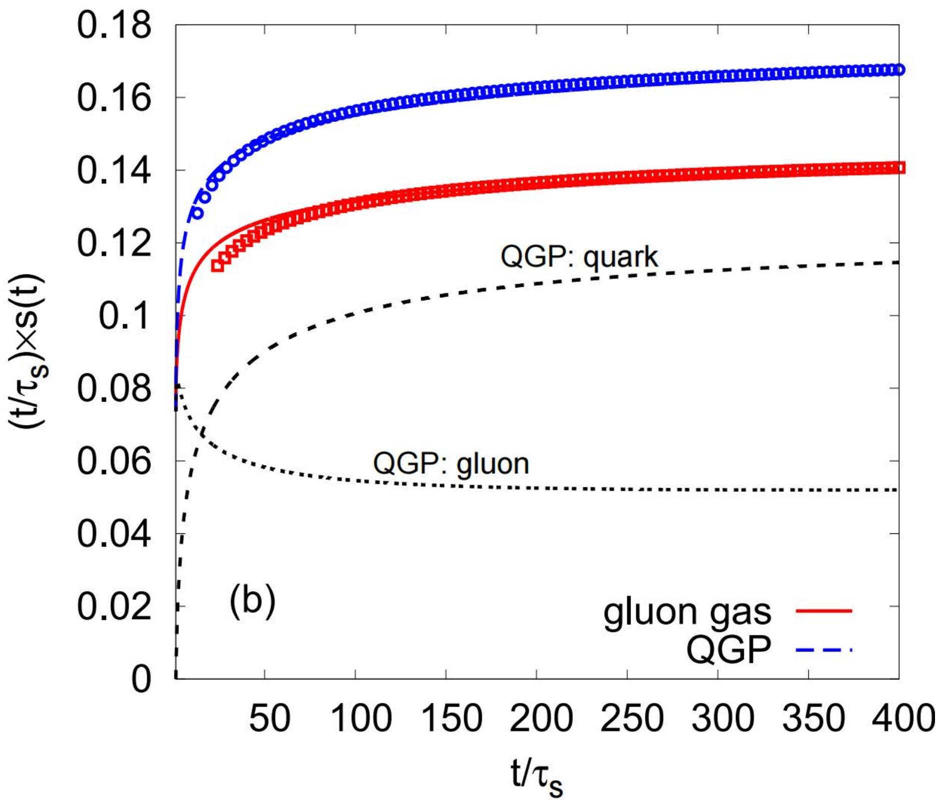
$$\partial_\tau e + \frac{4}{3}\frac{e}{\tau} = \frac{\pi}{\tau}$$

$$\tau_\pi \partial_\tau \pi + \pi = \frac{4}{3}\frac{\eta}{\tau} - a_1 \frac{\tau_\pi}{\tau}\pi \rightarrow g(\tau/\tau_R) \equiv \frac{d\ln e}{d\ln \tau} \xrightarrow{de=Tds} \frac{d\ln(s\tau)}{d\ln\tau} = 1 + \frac{3}{4} g(\tau/\tau_R)$$

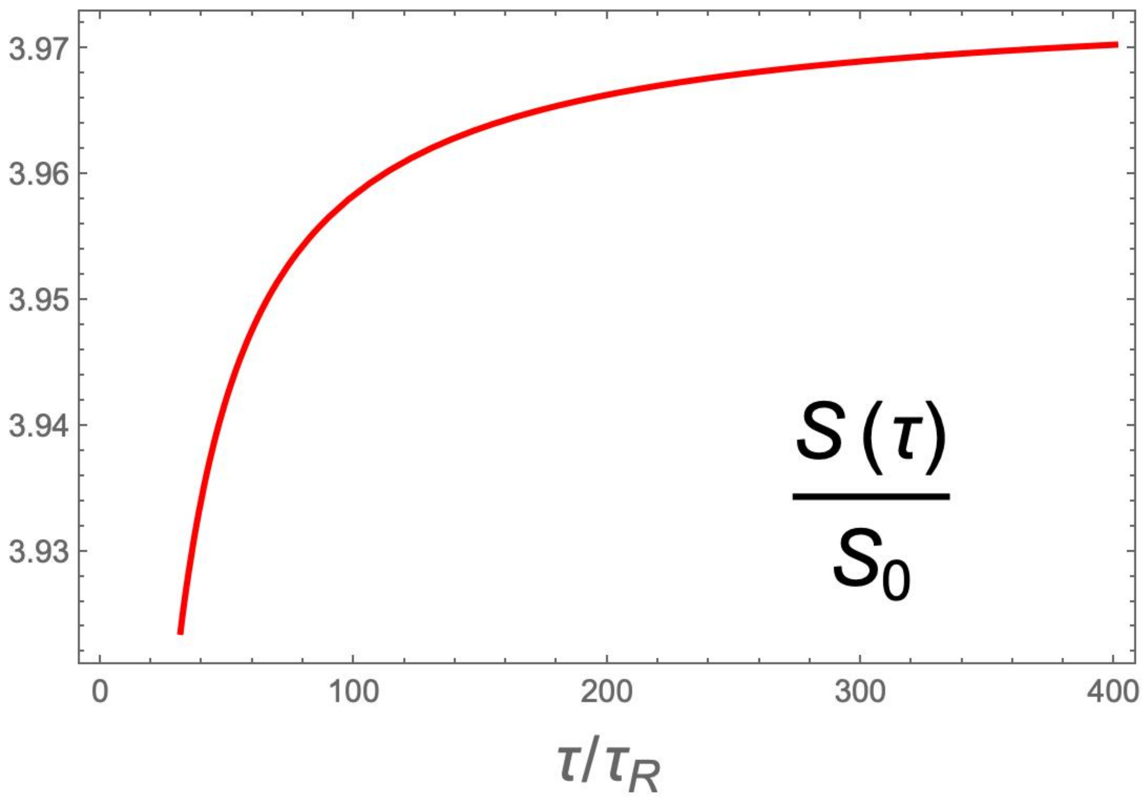
[Blaizot, LY, 2106.10508]

Non-equilibrium entropy production

Boltzmann equ. w/ 2-2 scatterings

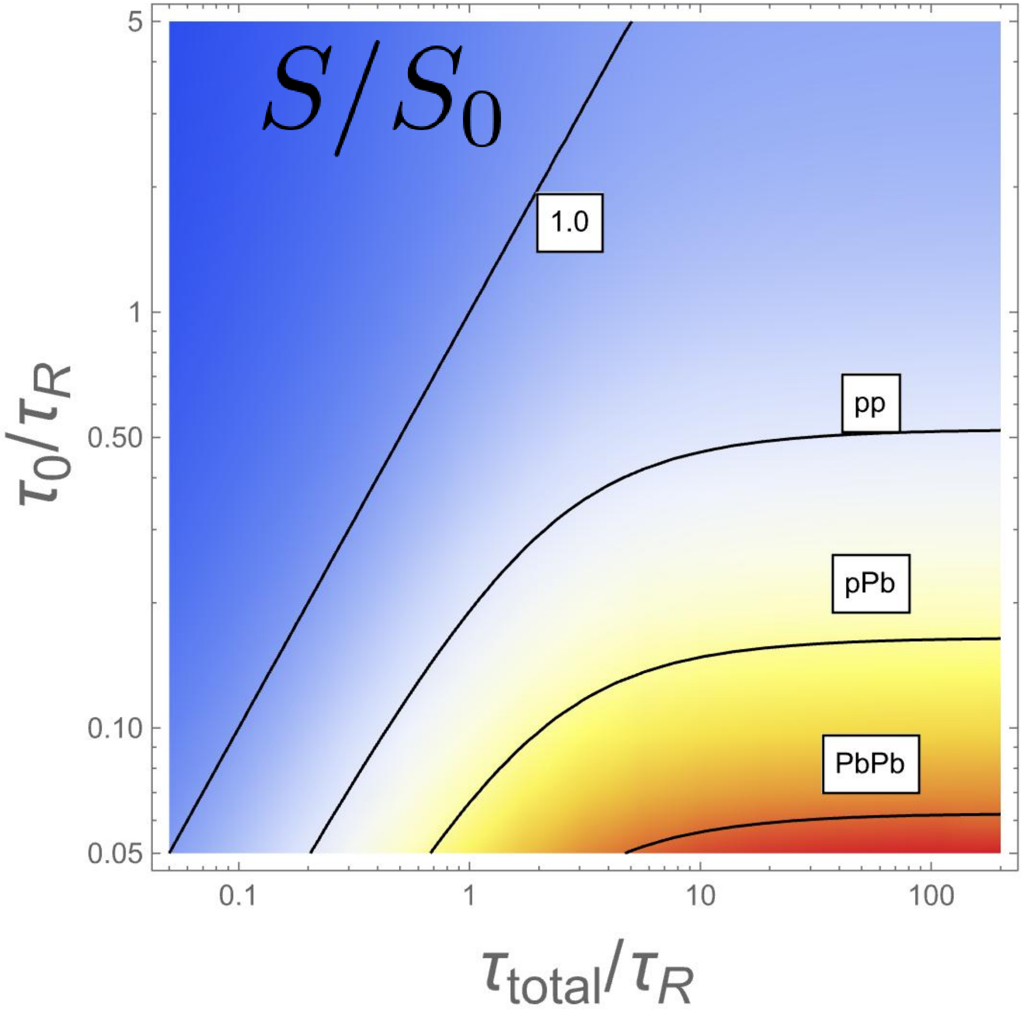


Hydro attractor



- 2nd law: Thermal entropy increases as system approaching equilibrium.

Entropy production and thermalization



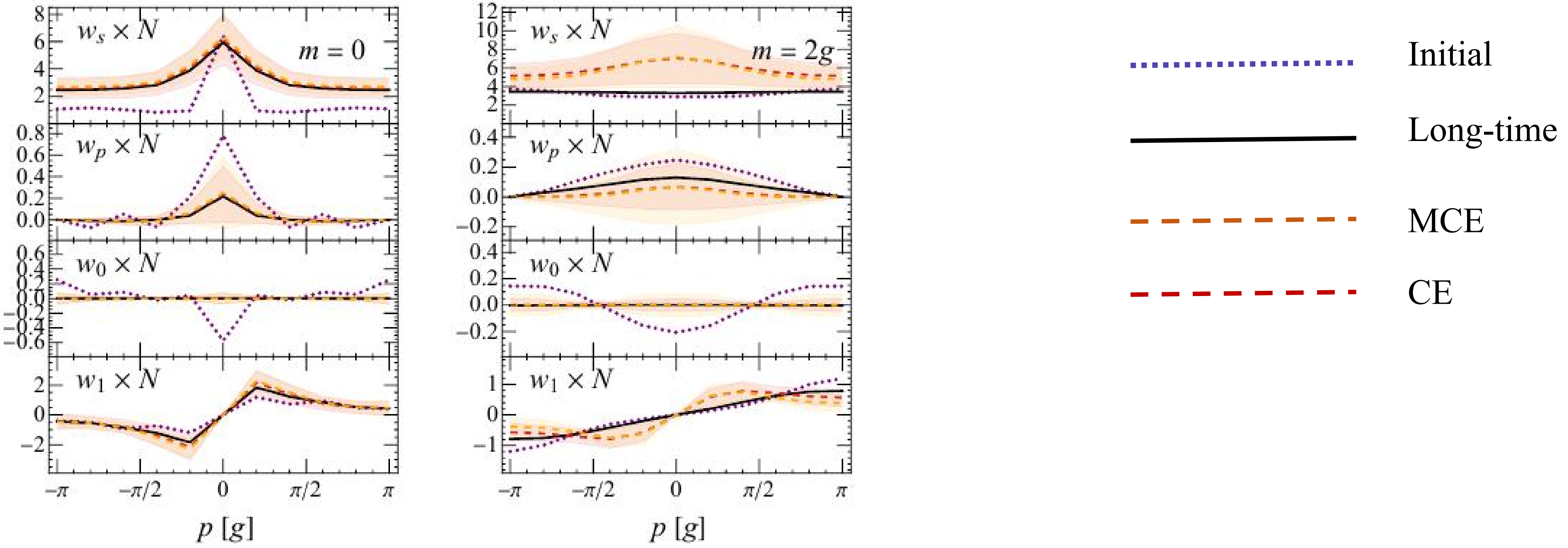
- Structure of thermalization plateau.
- Initial time (~1/Q) is bounded from above in different systems: earlier in larger systems.
- pp is unlikely fully thermalized ...

Summary

- Late-stage entropy production can be estimated in heavy-ion collisions, by experiment.
- Thermalization can be quantified in terms of late-stage entropy production.

Wigner function thermalization in QED_2

$\theta = 0$

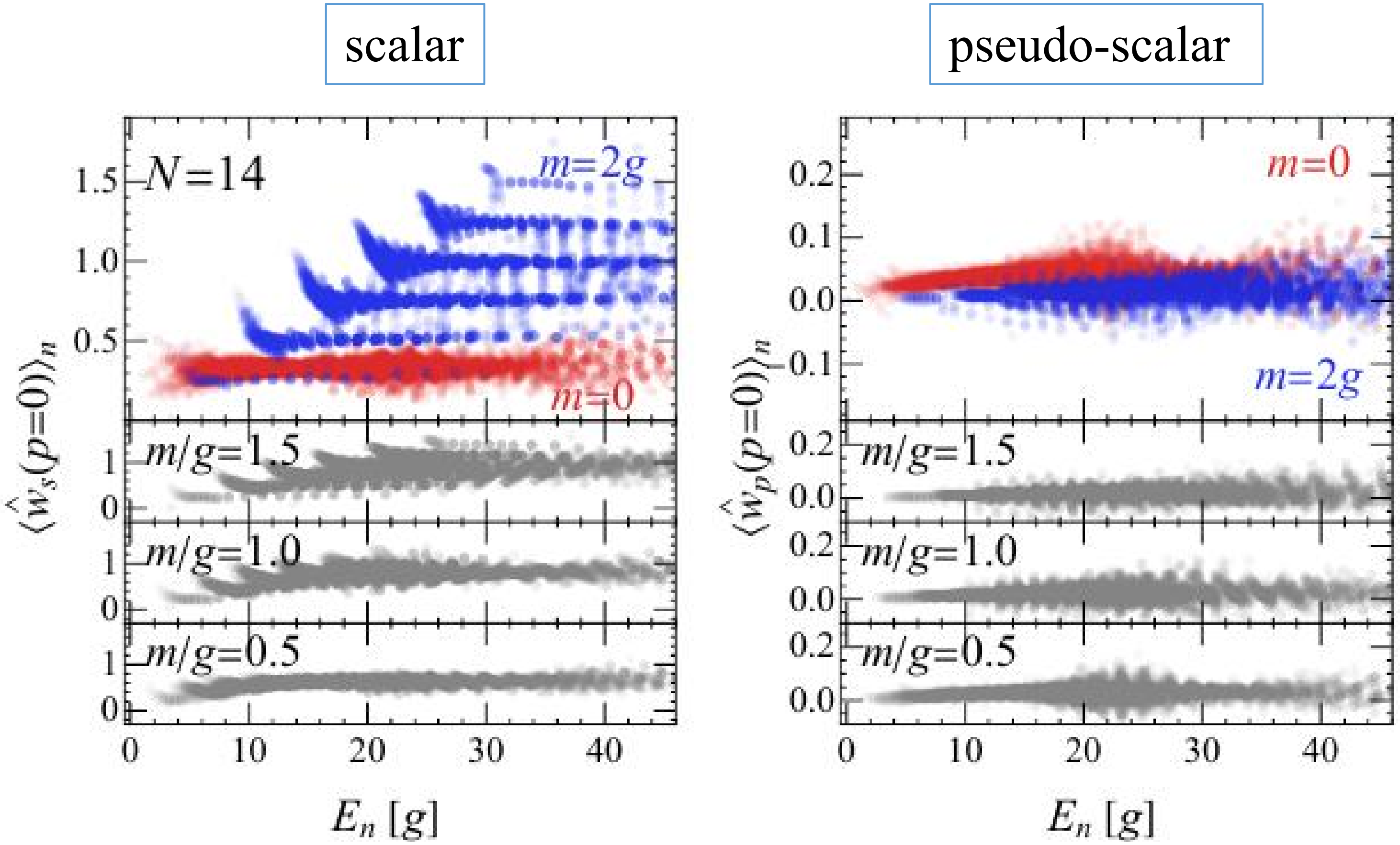


- Quantum thermalization realized in "massless"/strongly-coupled system: $\delta m \sim ag^2$
- Quantum thermalization partly realized in massive/weakly-coupled system: parity dependent!

$$\begin{cases} \text{thermalized: pseudo-scalar and vector charge} \\ \text{not thermlized: scalar and axial vector charge} \end{cases}$$

Distribution of diagonal elements of Wigner function

$\theta = 0$



- Strong coupling: single and narrow band ==> smooth function ==> ETH condition ==> thermalization
- Scalar: The distribution gradually splits as coupling gets weaker: break down of ETH
- Pseudo-scalar: always thermalize