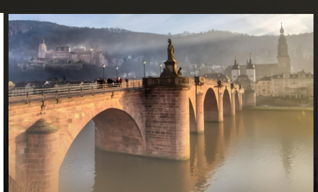
Topology
in far-from-equilibrium dynamics: Sine-Gordon-class universality

CLUSTER OF EXCELLENCE STRUCTURES



Thomas Gasenzer

Synthetic Quantum Systems

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Kirchhoff-Institut für Physik

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Center for Quantum Dynamics

Abundance of non-linear phenomena in...

Topology & NTFPs in 2D superfluids

Video: Approach of a non-thermal fixed point in a 1-component 2D gas

https://www.kip.uni-heidelberg.de/gasenzer/projects/anomalousntfp

...vs. decaying (superfluid) turbulence:

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...in a dipolar superfluid:

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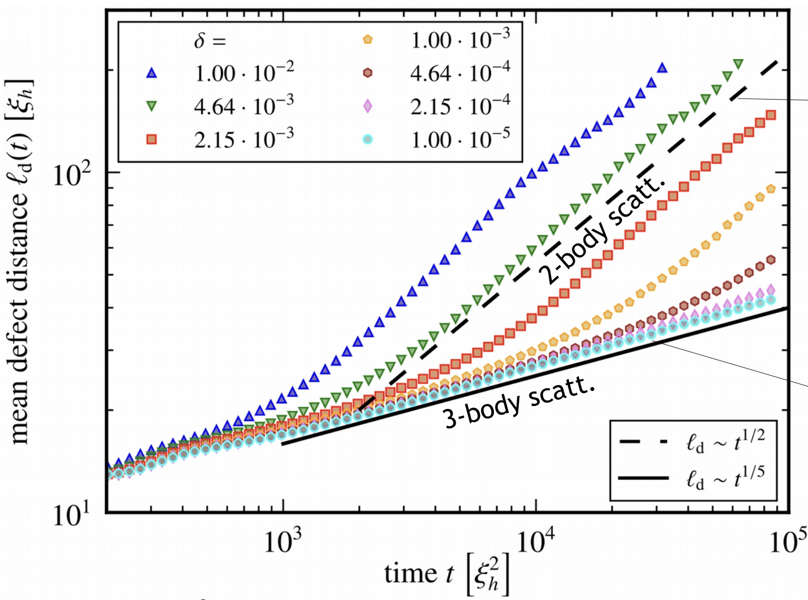
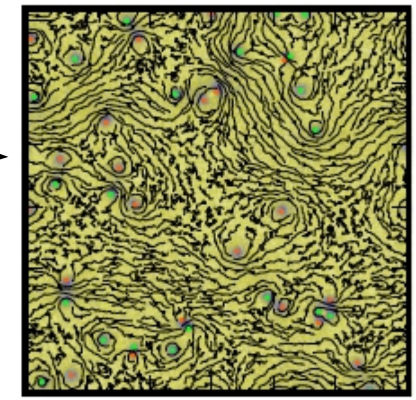
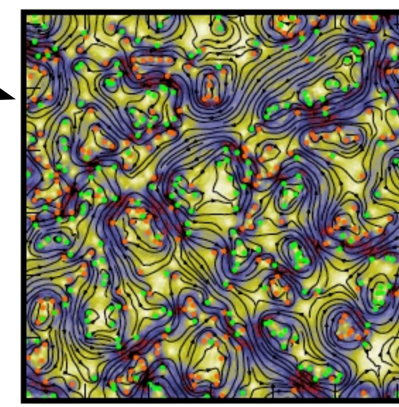
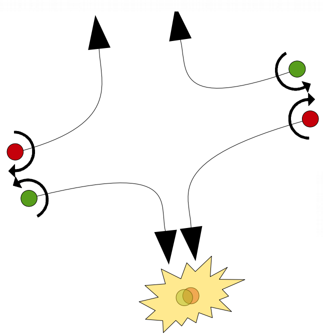
Anomalous NTFP

Vortex coarsening in a 2D Bose gas: mutual annihilation









$$\partial_t N_d \sim -\Gamma_3 N_d \sim -\text{ const.} \times N_d^{7/2}$$

$$\Rightarrow \quad N_d(t) \sim t^{-2/5}$$

M. Karl, TG, NJP 19, 093014 (2017) — also: J. Deng, S. Schlichting, R. Venugopalan, Q. Wang, PRA 97, 053606 (18); D. Spitz, J. Berges, M. Oberthaler, A. Wienhard, SciPost Phys. 11, 060 (21); V. Noel, TG, K. Boguslavski, PRR 7, 033220 (25); N. Rasch, L. Chaumaz, TG, 2506.01653

Anomalous NTFP

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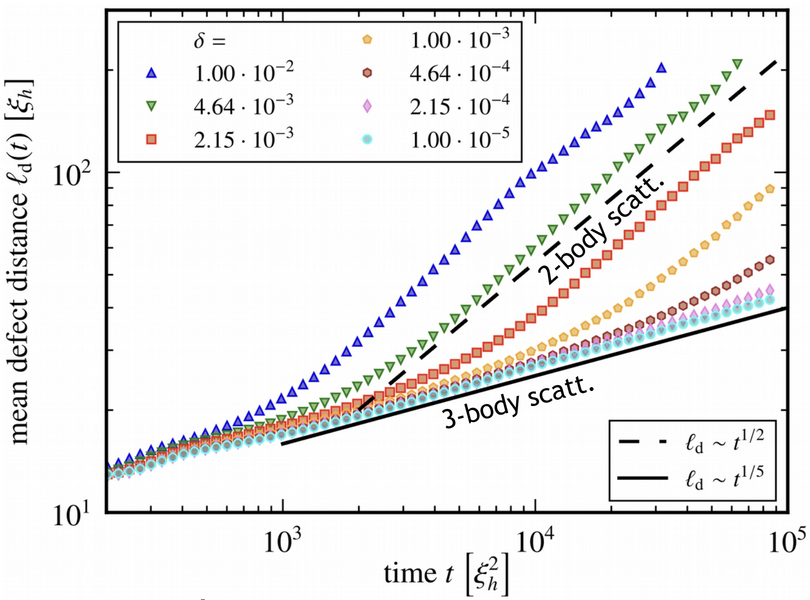
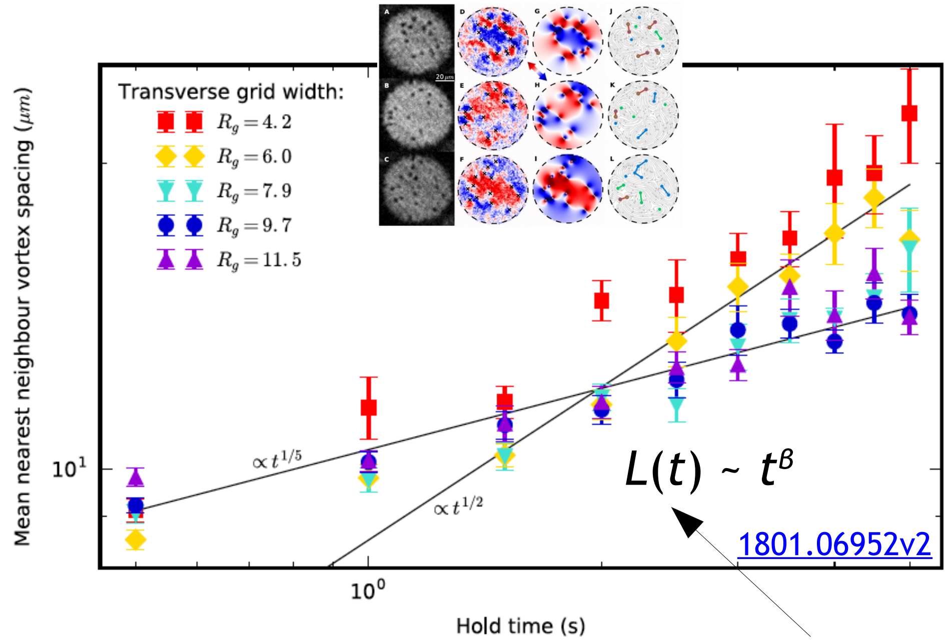
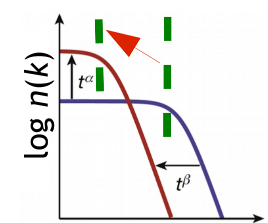
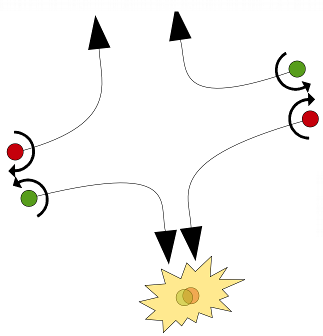




$k_L(t) \sim L(t)^{-1}$





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M. Karl, TG, NJP 19, 093014 (2017)

S.P. Johnstone, A.J. Groszek, P.T. Starkey, C.J. Billington, T.P. Simula, K. Helmerson, Science **364**, 1267 (2019); 1801.06952v2

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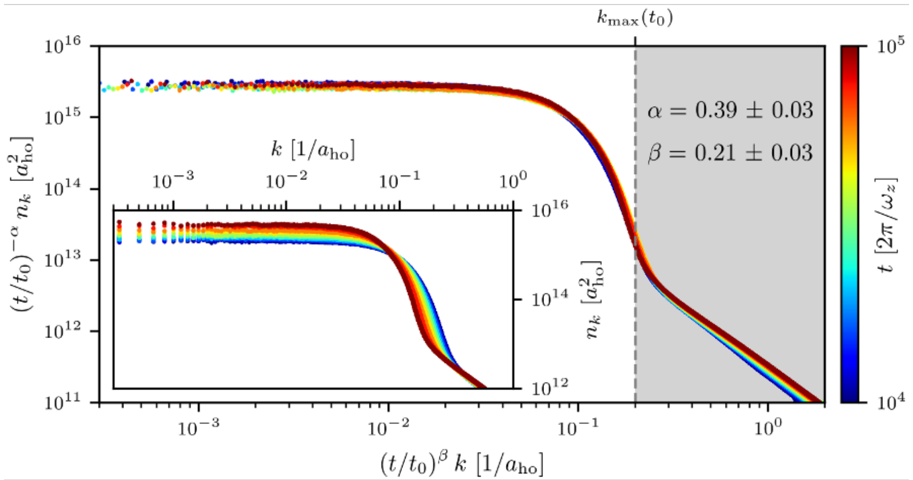
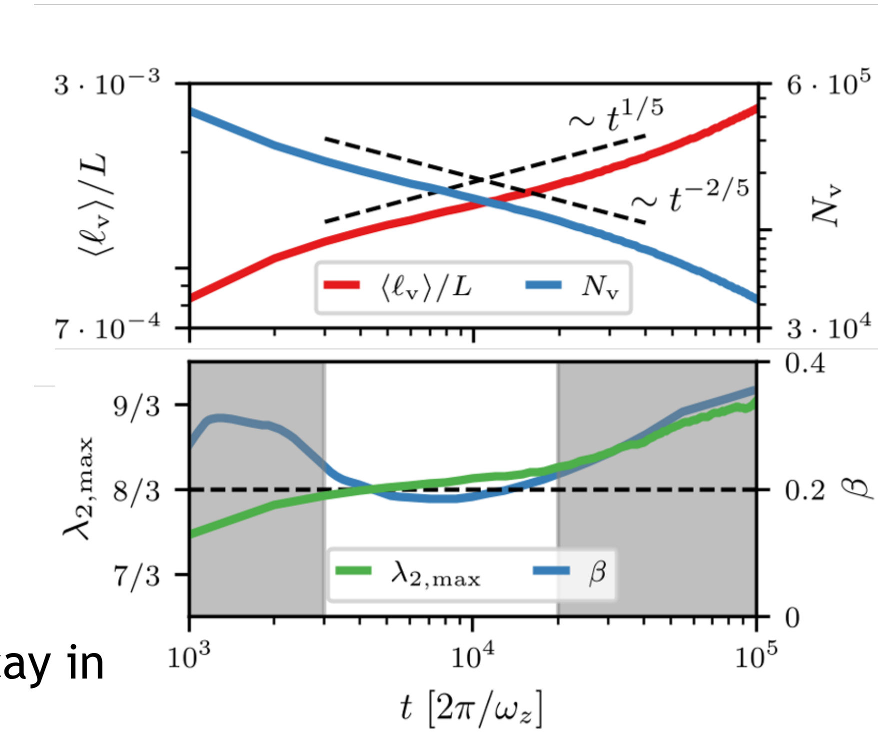
Decaying 2D turbulence: quantum vs. classical





Niklas Rasch, TG, arXiv:2509.21285 [cond-mat.quant-gas].

Power law exponent of vortex density decay in classical decaying 2D Turbulence:

Simulations:

Reference	$\alpha = 2\beta$
Carnevale *et al.*[7]	0.75
Weiss and McWilliams[2]	0.72±0.03
Dritschel[5]	0.29±0.03
Clercx and Nielsen[21]	1.03±0.10 [c]
Bracco *et al.*[8]	0.76±0.03

Experiments:

Reference	$\alpha = 2\beta$
Carnevale *et al.*[7]	0.75
Tabeling *et al.*[9]	0.70±0.1
Cardoso *et al.*[28]	0.44±0.1
Hansen *et al.*[10]	0.70±0.1
Clercx *et al.*[29]	0.70±0.1

For these tables & Refs., see van Bokhoven et al., Phys. Fl. **19**, 046601 (07)

Intermittency in higher-order moments of circulation

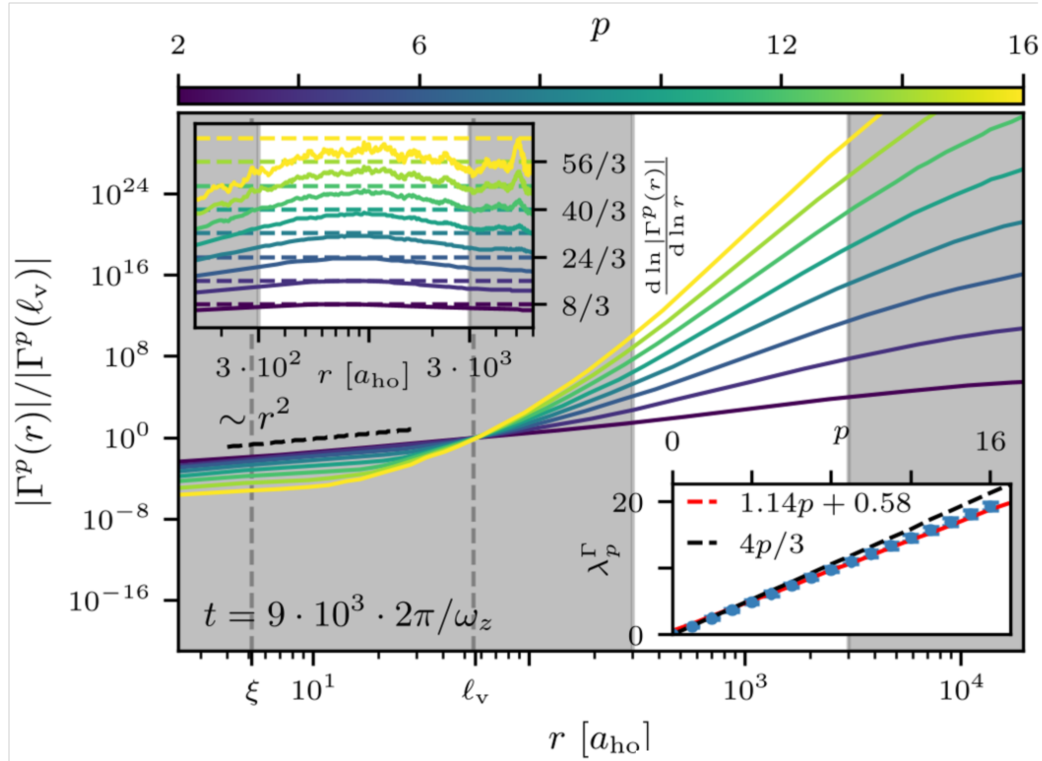
- Intermittency = deviations from

$$\lambda_p = 4p/3$$

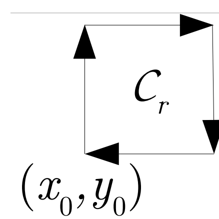
- Logarithmic slopes flatten in inertial range

Bifractal intermittency model:

$$\lambda_p \approx 1.14\,p + 0.58$$



Circulation: $\Gamma(r;x_0,y_0) = \oint_{\mathcal{C}_r} \boldsymbol{v}\,\mathrm{d}\mathbf{l}$



Measured in thin fluid layers:
H.-Y. Zhu et al., PRL **130**, 214001 (2023)
Simulated in nearly incompressible QT:
N. P. Müller and G. Krstulovic, PRL **132**, 094002 (2024)

Niklas Rasch, TG, arXiv:2509.21285 [cond-mat.quant-gas].

Coulomb gas (of vortices) vs. Sine-Gordon (BKT)

2+0D euclidean Sine-Gordon partition sum:

$$\left\langle \mathrm{e}^{2\lambda \int_{\mathbf{x}} \cos(\theta_{\mathbf{x}})} \right\rangle_{\mathcal{L}_0} \sim \left\langle 1 + \sum_{k=1}^{\infty} \frac{\lambda^k}{k!} \int_{\mathbf{x}_1,\ldots,\mathbf{x}_k} \prod_{j=1}^{k} \left(\mathrm{e}^{\mathrm{i}\theta_{\mathbf{x}_j}} + \mathrm{e}^{-\mathrm{i}\theta_{\mathbf{x}_j}} \right) \right\rangle_{\mathcal{L}_0}$$

$$\mathcal{L}_0 \sim (\partial_{\mathbf{x}} \theta)^2$$

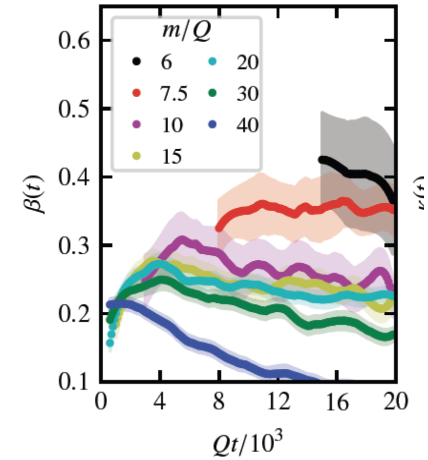
2+0D euclidean 2-point correlations:

$$\left\langle \mathrm{e}^{\mathrm{i}q_i \theta_{\mathbf{x}_i}} \mathrm{e}^{\mathrm{i}q_j \theta_{\mathbf{x}_j}} \right\rangle_{\mathcal{L}_0} \sim |\mathbf{x}_i - \mathbf{x}_j|^{-\eta q_i q_j} = \mathrm{e}^{-\eta q_i q_j \ln |\mathbf{x}_i - \mathbf{x}_j|}$$

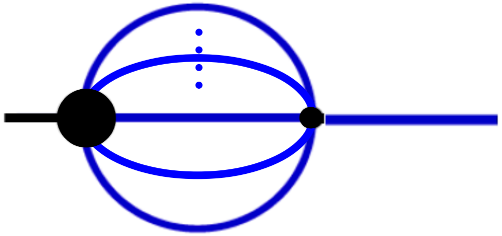
Coulomb potential

Berezinskii, Sov. Phys. JETP 32, 493 (71); 34, 610 (72); Kosterlitz & Thouless, J. Phys. C 6, 1181 (73)

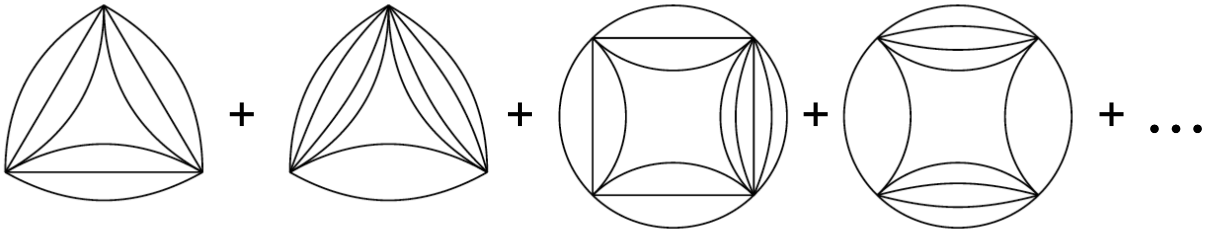
Non-equilibrium Greens Functions

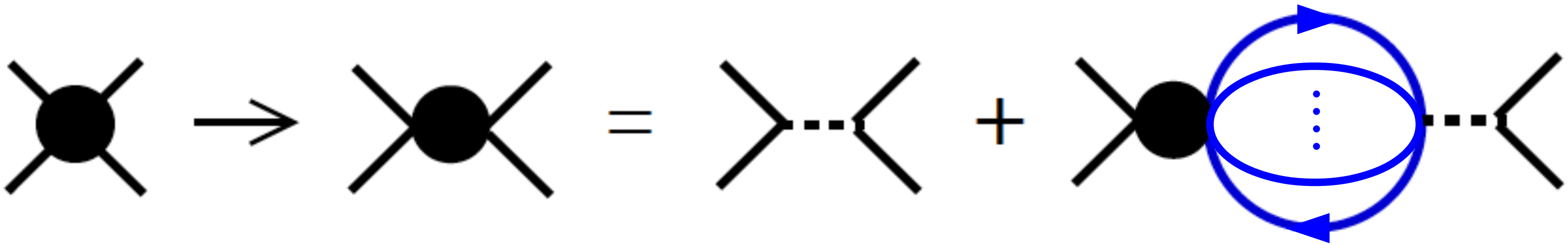


Kadanoff-Baym equation for sine-Gordon (-type)

$\partial_t n(p) =$ 

from **2PI (two-particle-irreducible) effective action**, resumed as

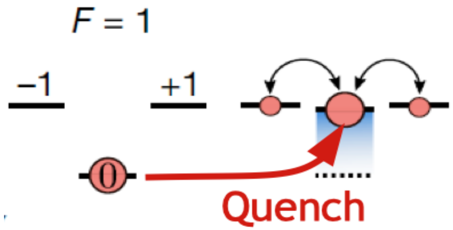
$\Gamma_2 \sim$ 

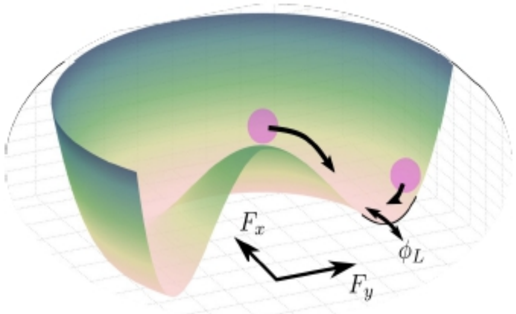


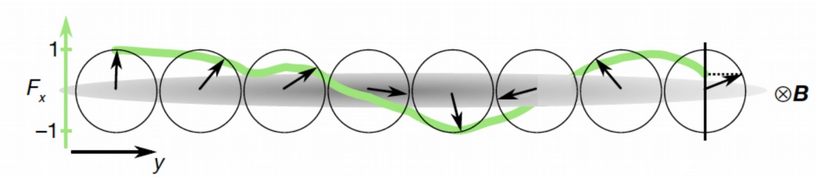
Scaling analysis: $\beta = 1/2$ or $\beta = 1/(2+d)$ [if beyond φ^4 matters]

[P. Heinen, A. Mikheev, TG, Phys. Rev. A **107**, 043303 (23) & arXiv:2212.01162; perturbative calc.: Tavora & A. Mitra (2013)]

1D Spin-1 gas: Pattern formation in F_x-F_y–plane:







$$F_\perp = F_x + iF_y = |F_\perp| e^{i\varphi_L}$$

C.M. Schmied, M. Prüfer, M.K. Oberthaler, TG, PRA **99**, 033611 (2019); arXiv:1812.08571 [cond-mat.quant-gas]
I. Siovitz, S. Lannig, Y. Deller, H. Strobel, M.K. Oberthaler, TG, PRL **131**, 183402 (23); arXiv:2304.09293 [cond-mat.quant-gas]

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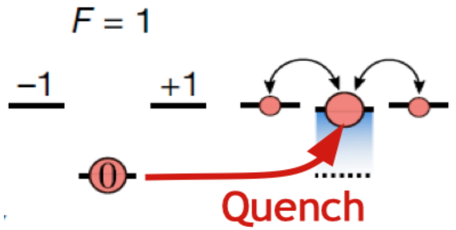

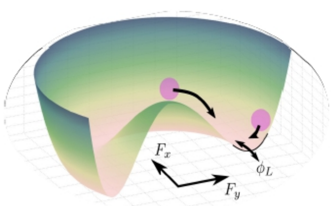

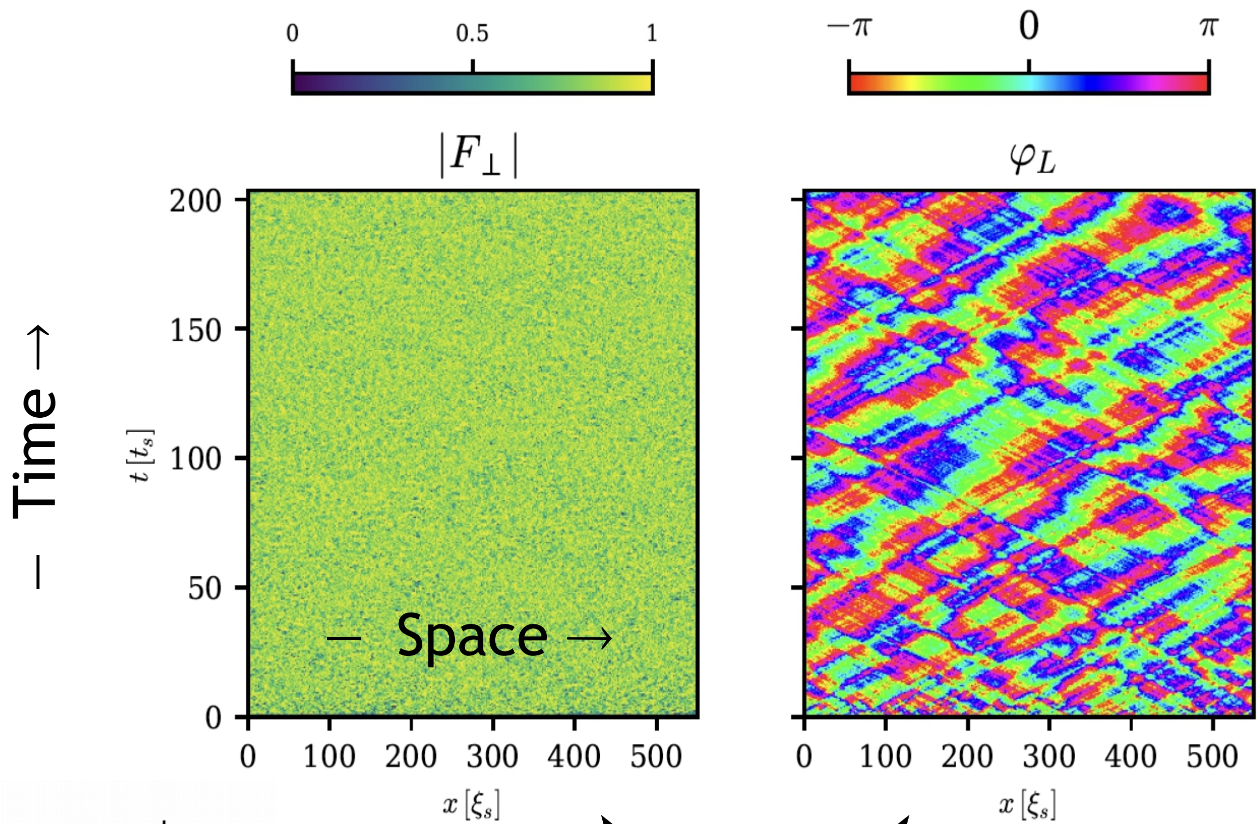

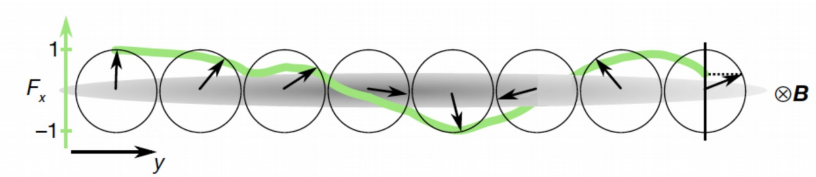

Amplitude & Larmor phase

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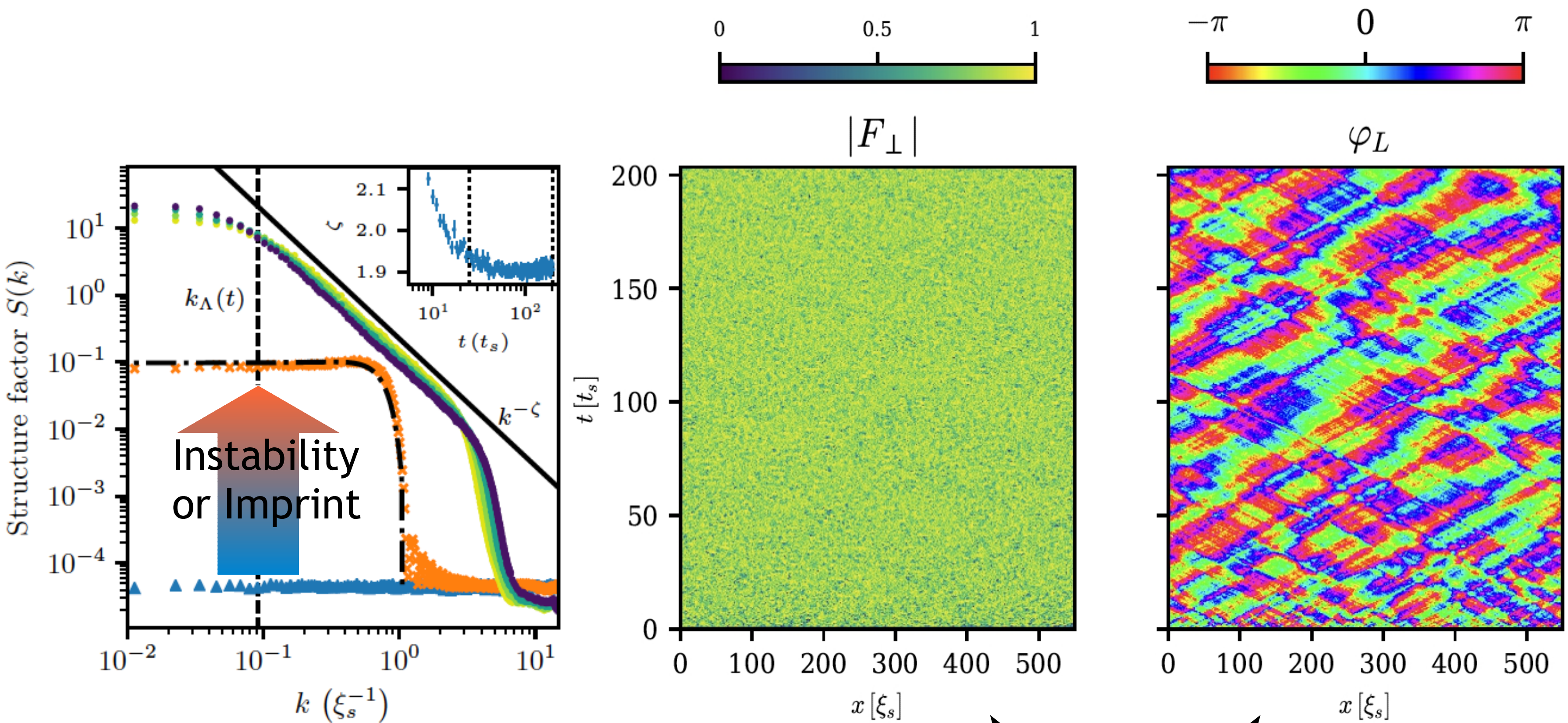
Space-time scaling & Coarsening



Structure factor … of … Amplitude & Larmor phase

$$S(k,t) = \langle |F_\perp(k,t)|^2 \rangle \qquad F_\perp = F_x + iF_y = |F_\perp| e^{i\varphi_L}$$

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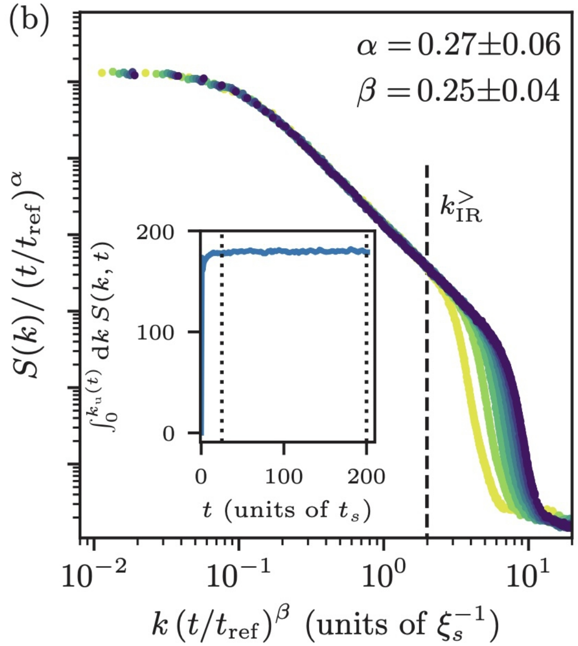
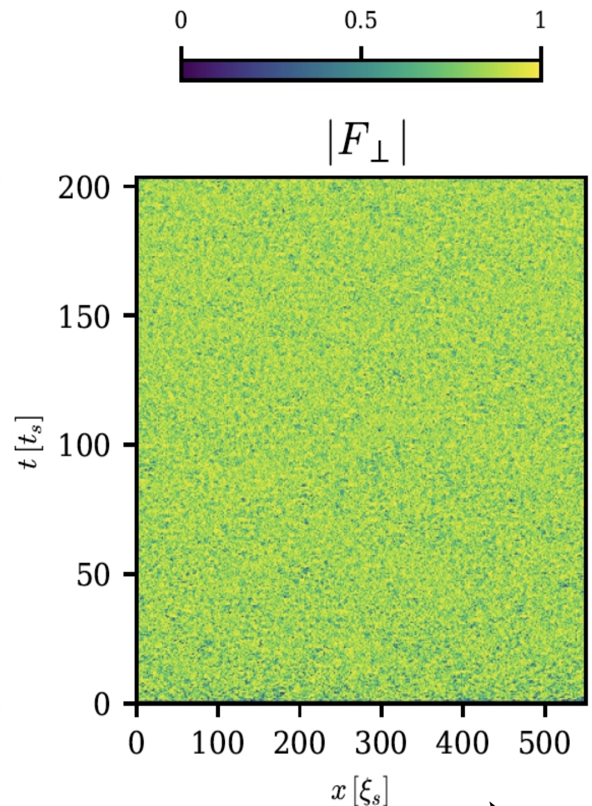
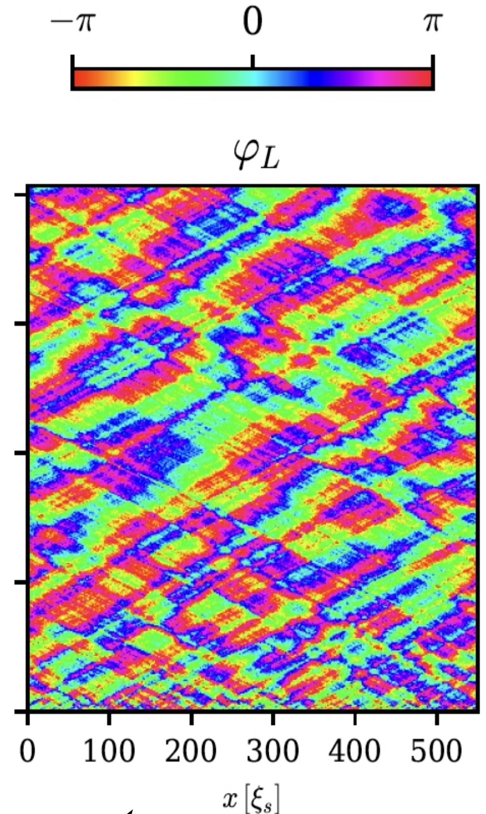
Pattern formation & Coarsening

not ½ !

$\beta \approx 1/4$
$\alpha = d\,\beta$







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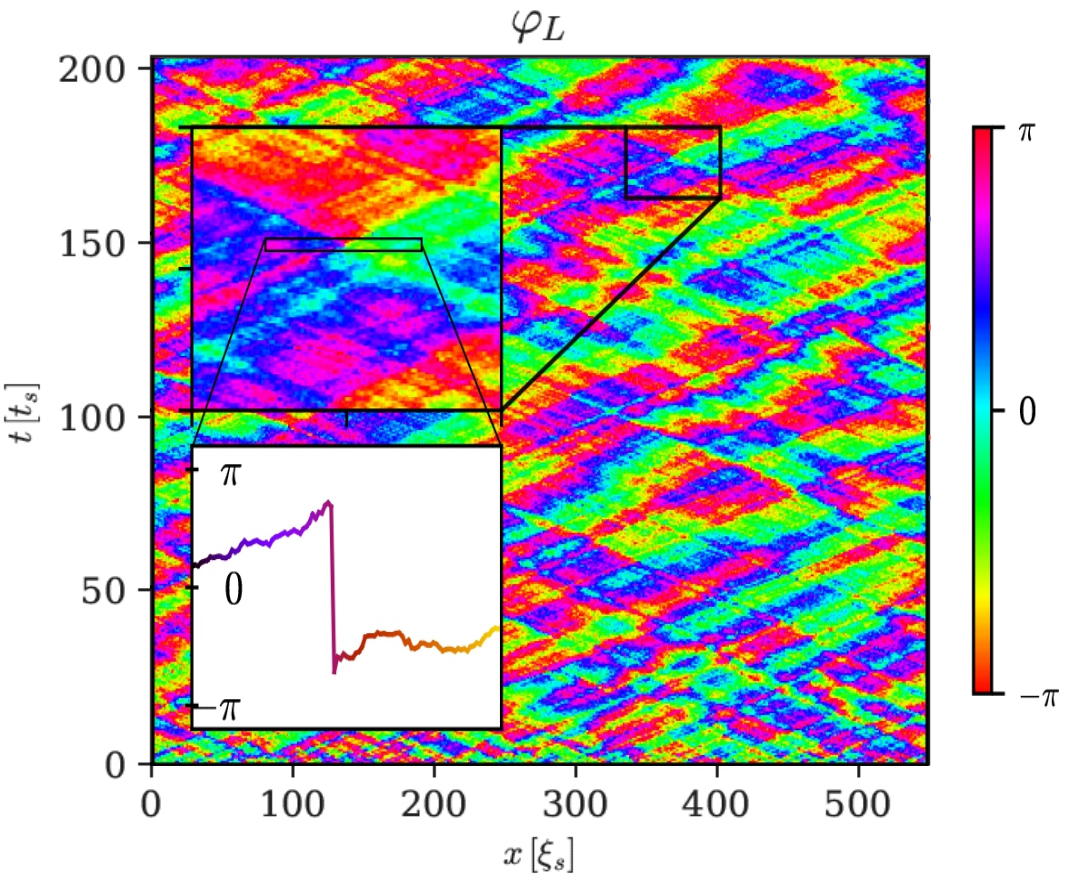
… back to the Larmor phase pattern



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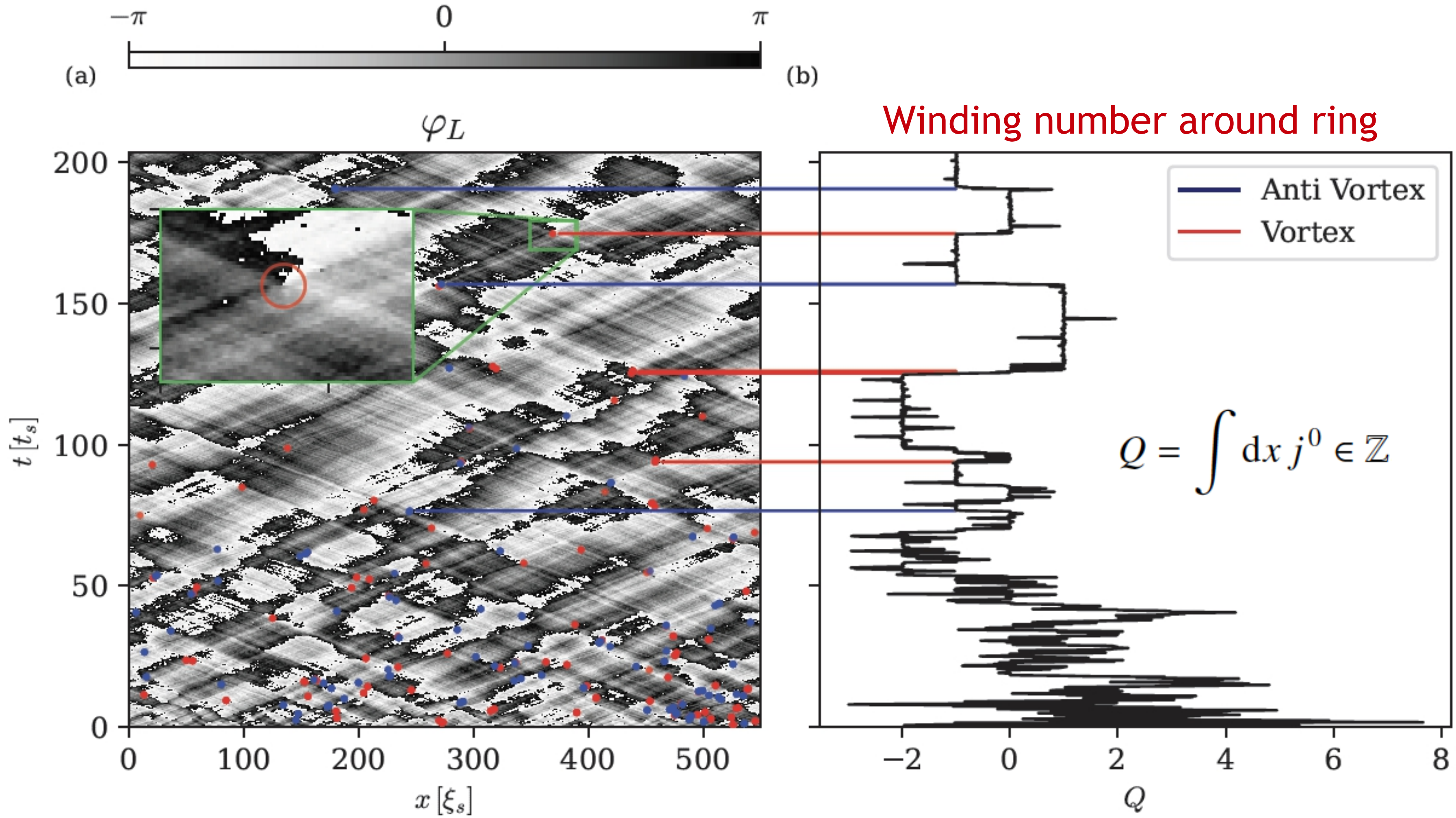
Instantons! (= phase slips)



$$F_\perp = F_x + iF_y = |F_\perp| e^{i\varphi_L}$$

$$j^0 = \partial^x \varphi_L, \quad j^1 = \partial^t \varphi_L$$

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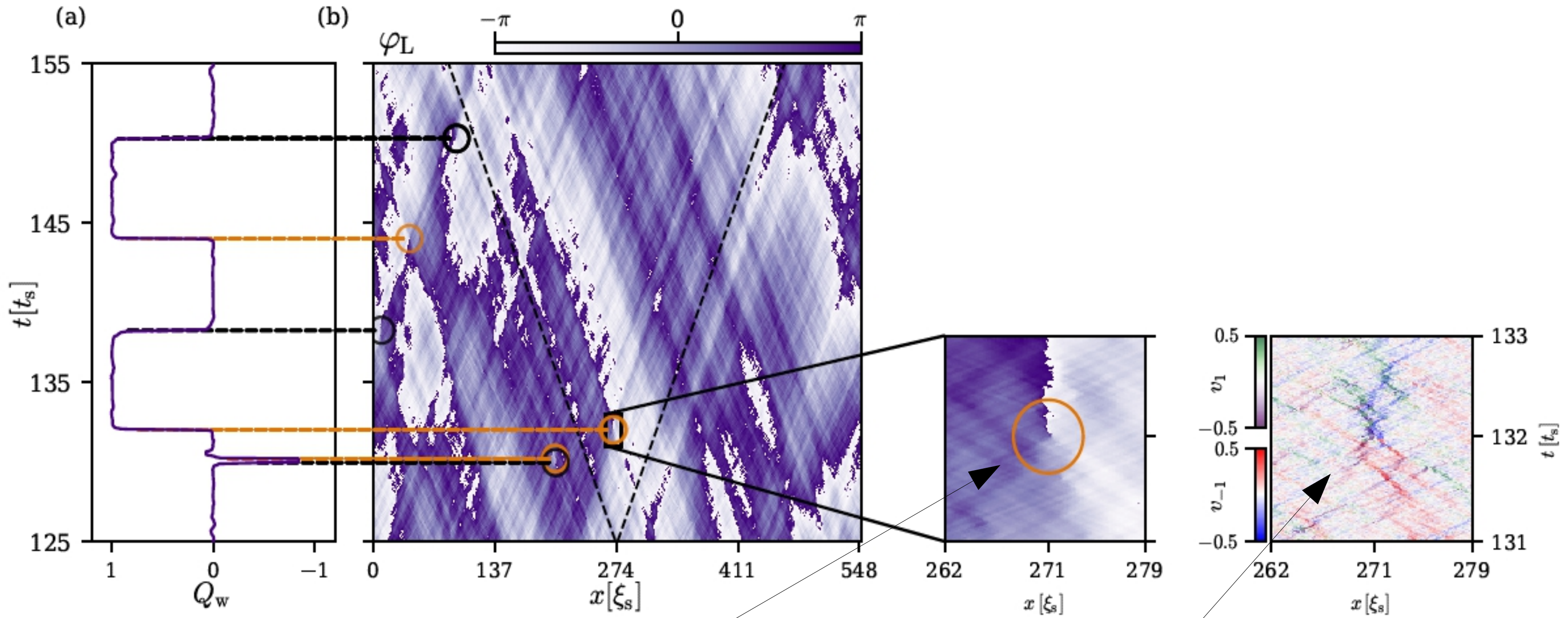
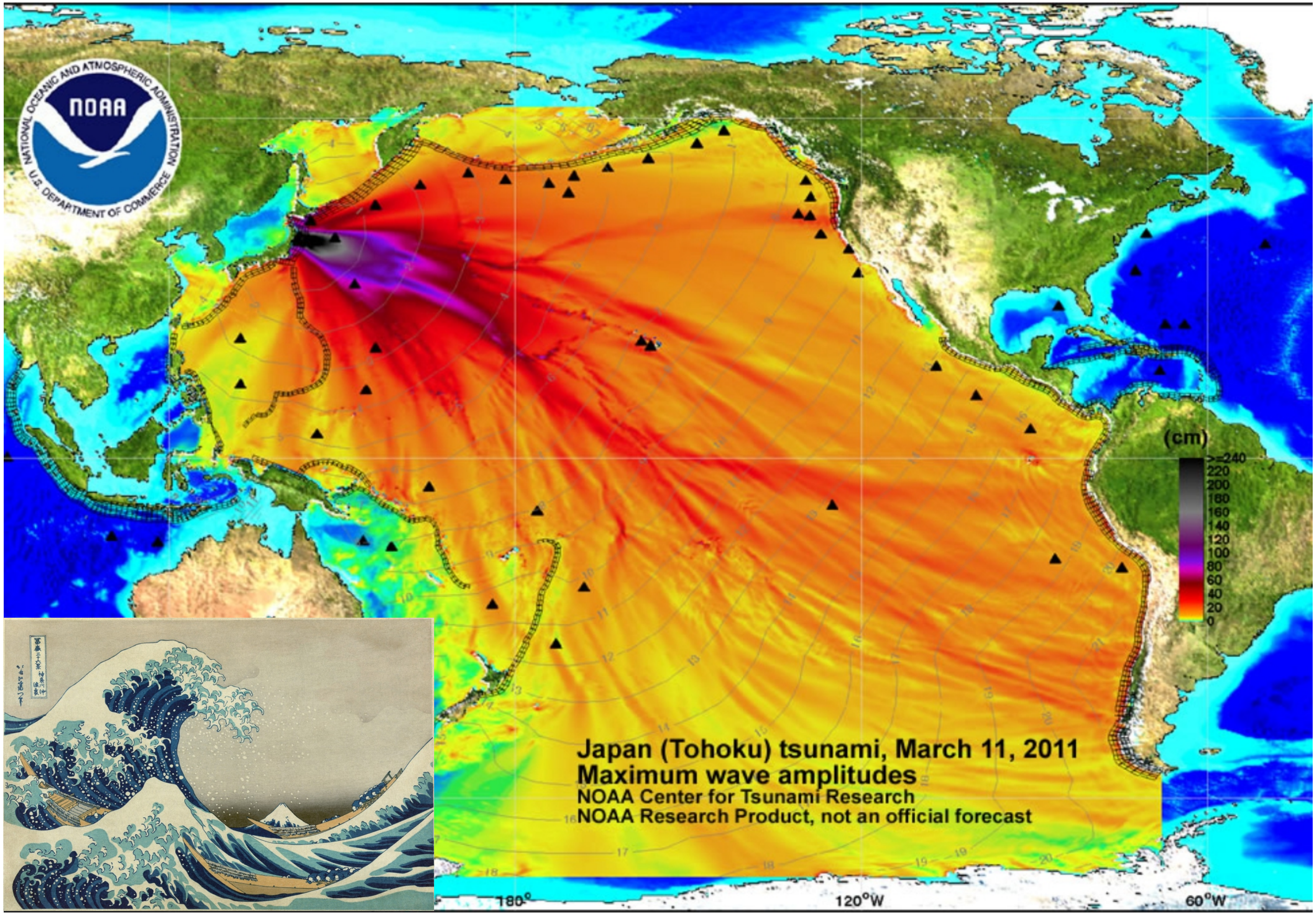
Instantons induced by rogue waves



Instanton = phase slip, induced by **rogue wave** in spinor phase

I. Siovitz, S. Lannig, Y. Deller, H. Strobel, M.K. Oberthaler, TG, PRL **131**, 183402 (23); arXiv:2304.09293 [cond-mat.quant-gas]

Caustics lead to Rogue Waves



Source: NOAA https://nctr.pmel.noaa.gov/honshu20110311/

Sine²-Gordon Model: low-energy effective theory

$$\mathcal{L} = \frac{\mathrm{i}}{2}\left(\psi_a^*\partial_t\psi_a - \psi_a\partial_t\psi_a^*\right) - \frac{1}{2M}\nabla\psi_a^*\nabla\psi_a - q(f^z)^2_{ab}\psi_a^*\psi_b - \frac{c_0}{2}(\psi_a^*\psi_a)^2 - \frac{c_1}{2}\sum_{i\in\{x,y,z\}}(\psi_a^* f^i_{ab}\psi_b)^2,$$

Spin-1 Bose gas

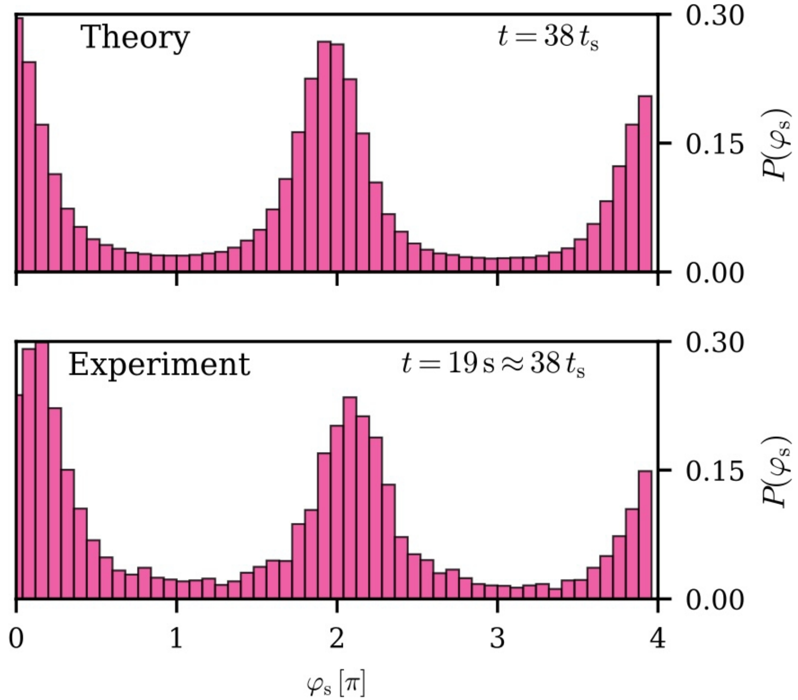
$$\psi_{\pm 1} = \sqrt{\rho \pm \epsilon}\,\mathrm{e}^{\mathrm{i}(\theta \pm \varphi_{\mathrm{L}})/2}$$

$$\psi_0 = \sqrt{\tilde{\rho} - 2\rho}\,\mathrm{e}^{\mathrm{i}(\theta + \varphi_{\mathrm{s}})/2}$$

$$\rho = n + \delta\rho$$
$$\epsilon = \bar{\epsilon} + \delta\epsilon$$

integrate out
@ 2nd order
in flucts

$$\mathcal{L}_{\mathrm{eff}}(\varphi_{\mathrm{s}}) = -\frac{1}{32c_1}\dot{\varphi}_{\mathrm{s}}^2 - \frac{\tilde{\rho} - 2n}{8m}(\nabla\varphi_{\mathrm{s}})^2 - \left[2c_1 n(\tilde{\rho} - 2n) - \frac{q^2}{16c_1}\right]\cos\varphi_{\mathrm{s}} - \frac{q^2}{32c_1}\sin^2\varphi_{\mathrm{s}}$$



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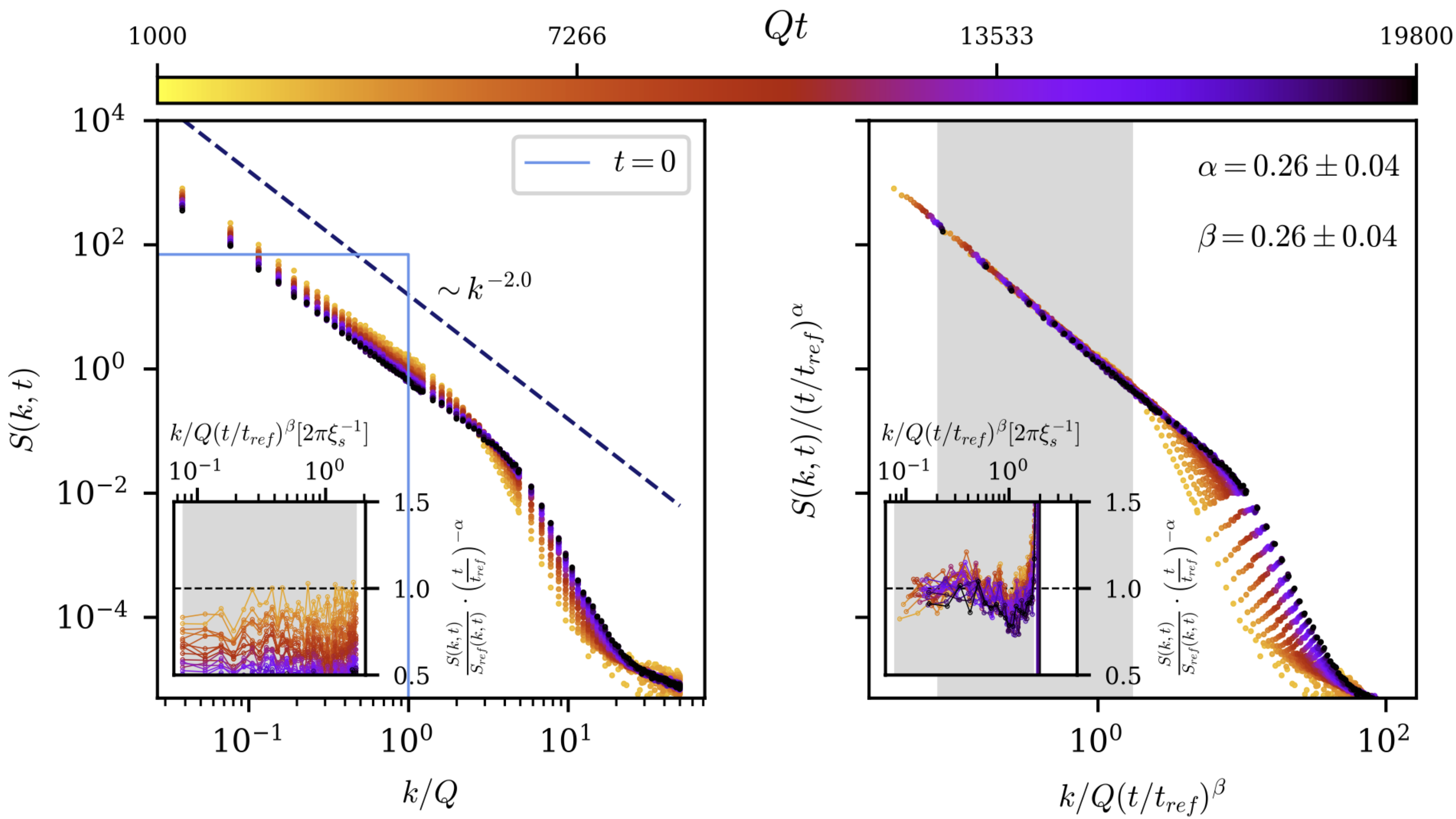
Universal scaling from Sine^2-Gordon



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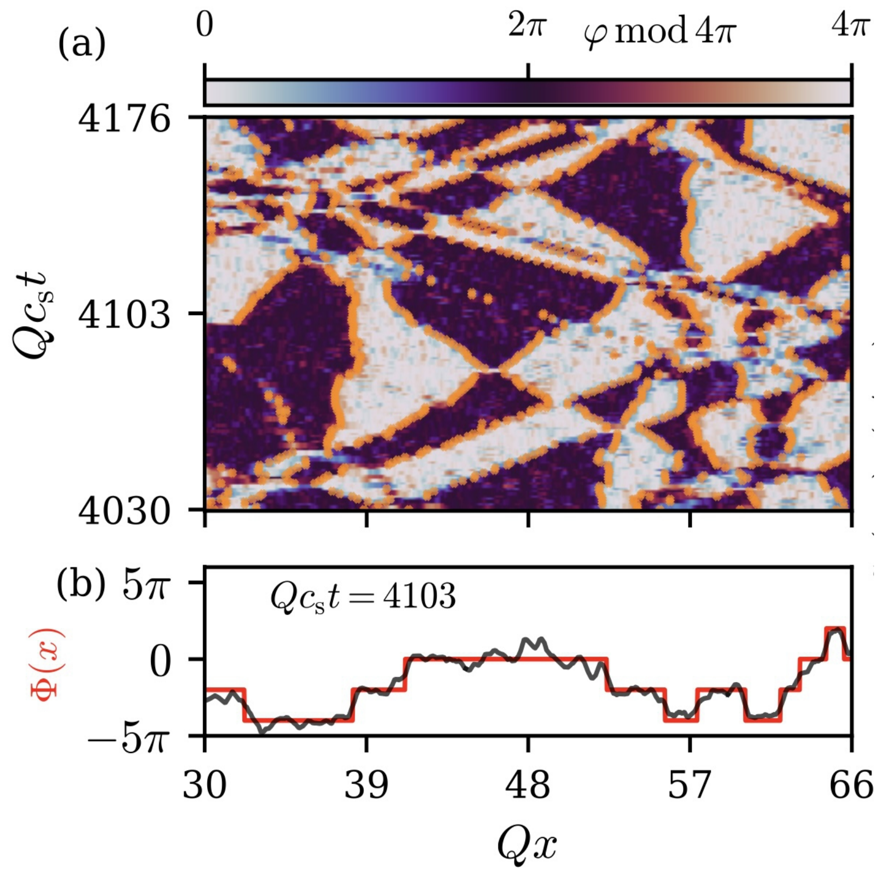
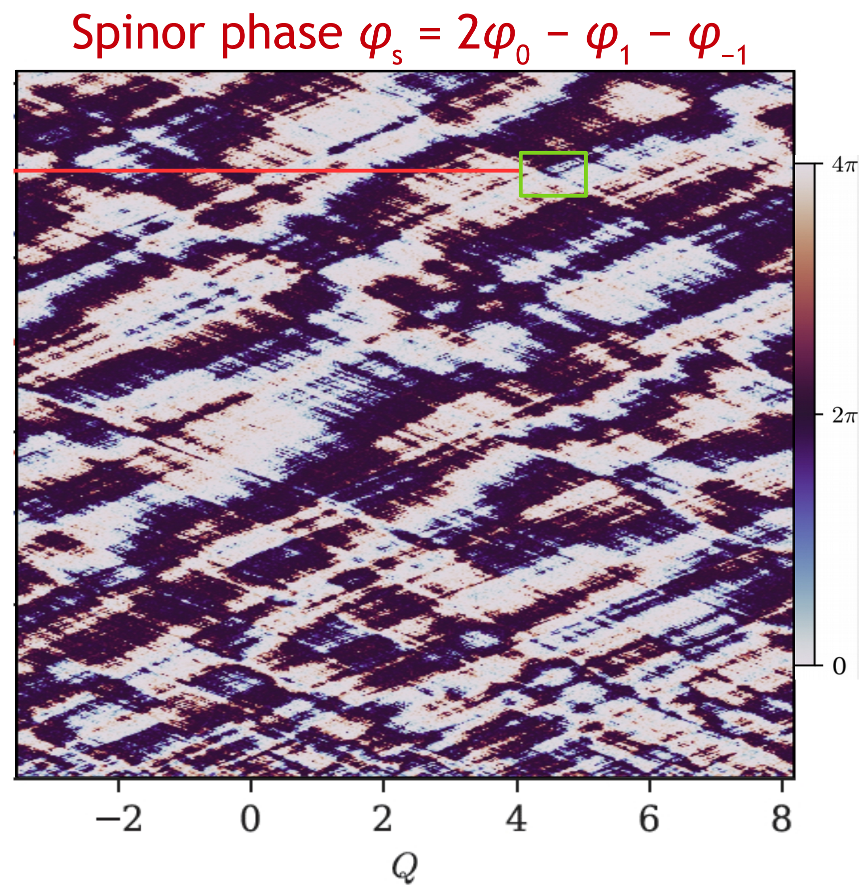
Spinor phase pattern





Sine²-Gordon model vs. full spin-1 gas

I. Siovitz, A.-M. E. Glück, Y. Deller, A. Schmutz, F. Klein, H. Strobel, M.K. Oberthaler, TG, PRA **112,** 023304 (25); 2412.13986 [cond-mat.quant-gas]

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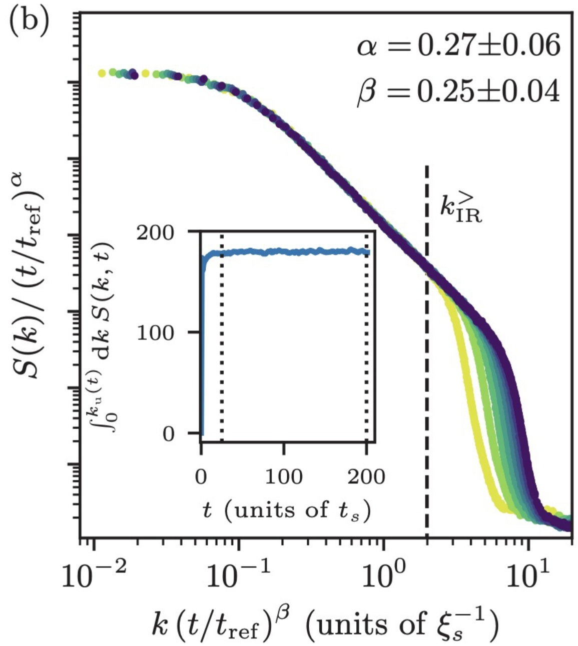

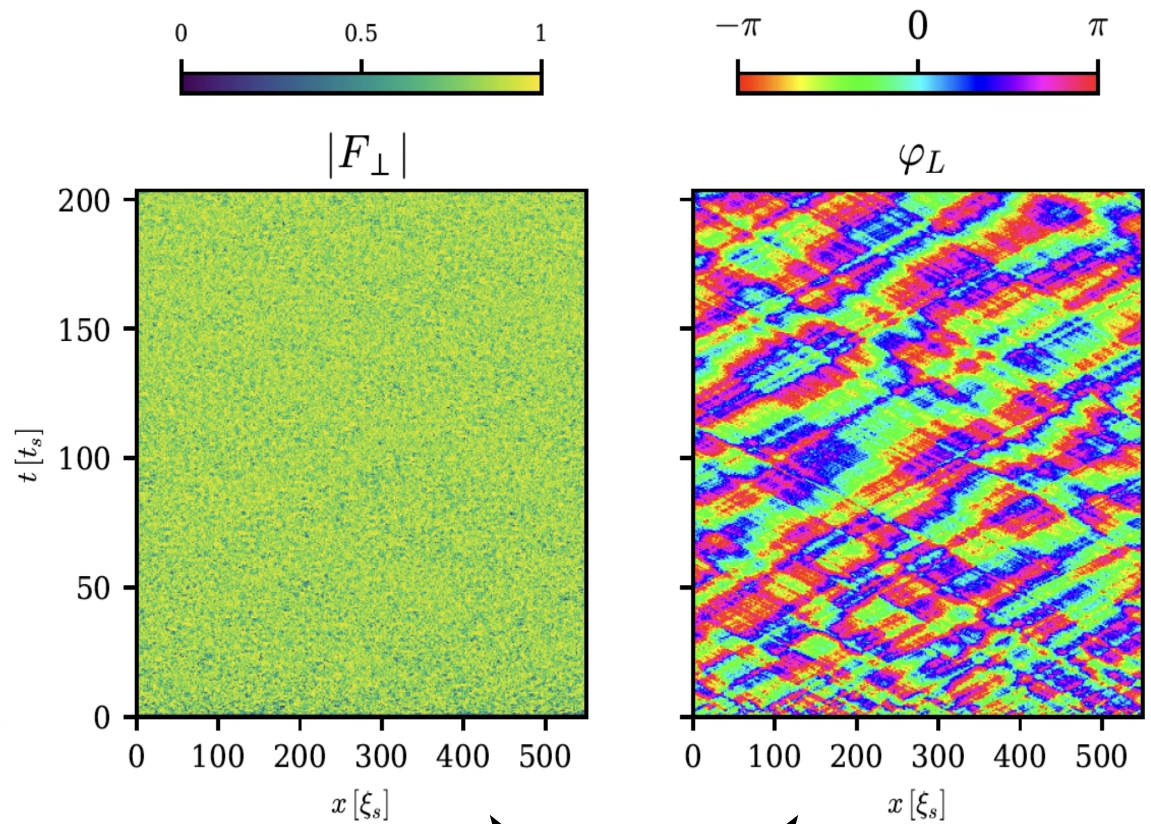

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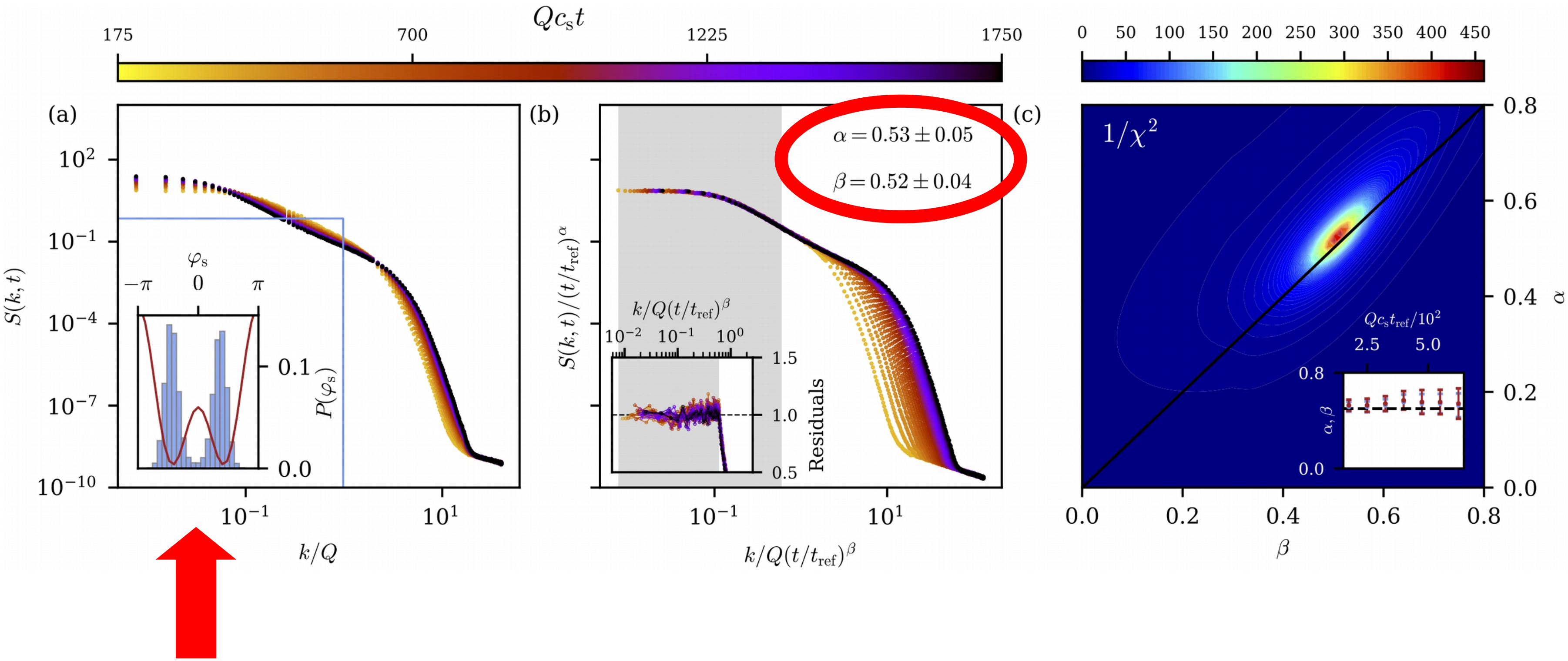
Universal scaling from Sine^2-Gordon: 1D



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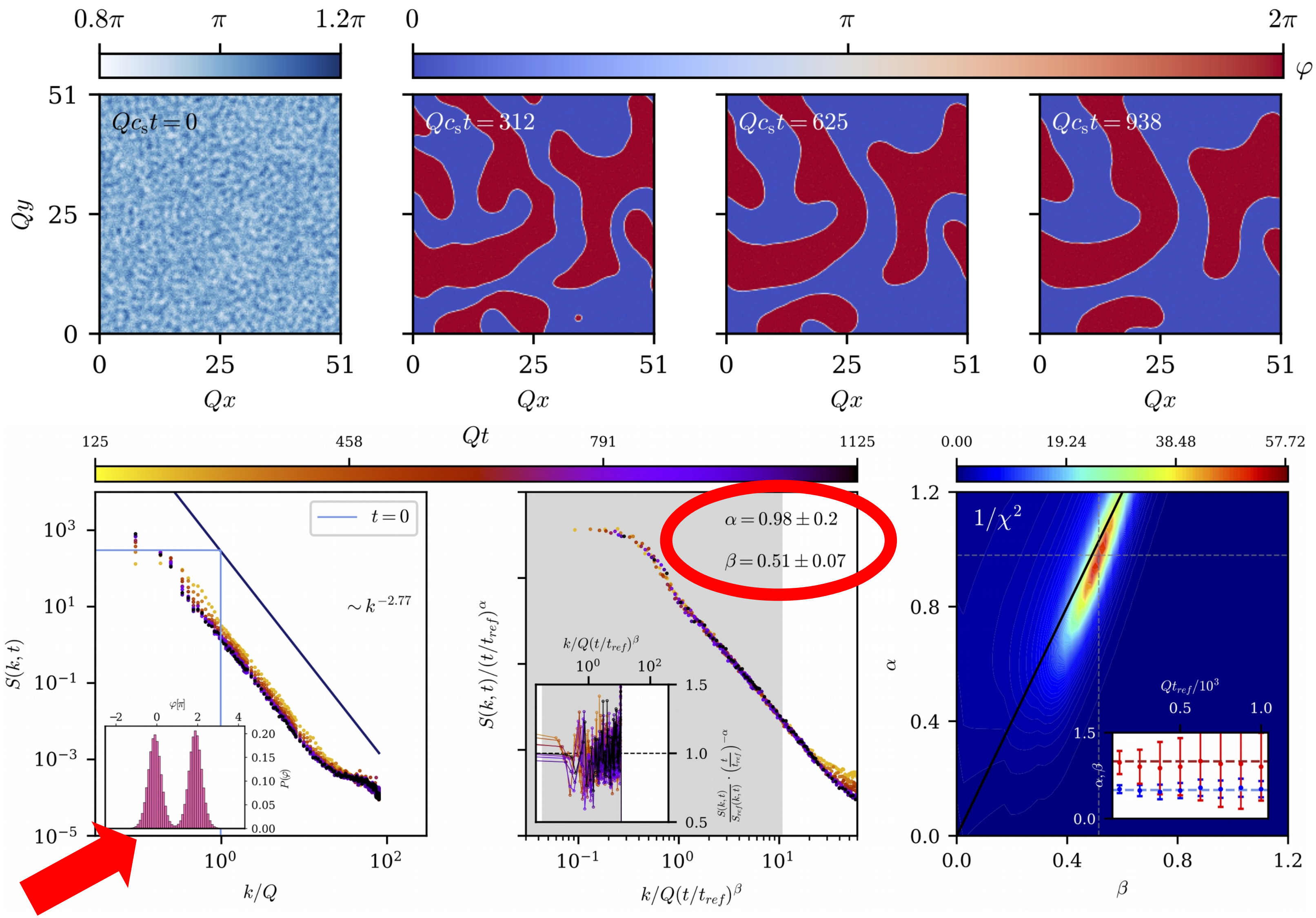
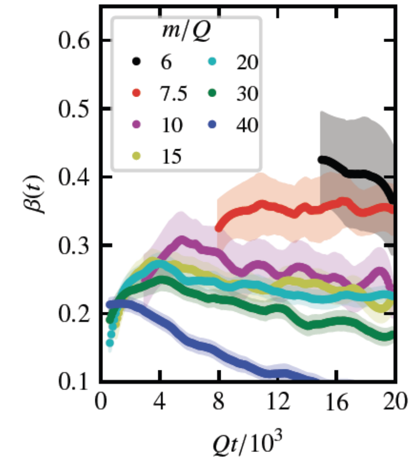
Universal scaling from Sine²-Gordon: 2D

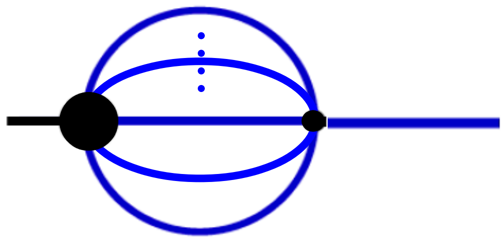


I. Siovitz, A.-M. E. Glück, Y. Deller, A. Schmutz, F. Klein, H. Strobel, M.K. Oberthaler, TG, PRA **112,** 023304 (25); 2412.13986 [cond-mat.quant-gas]

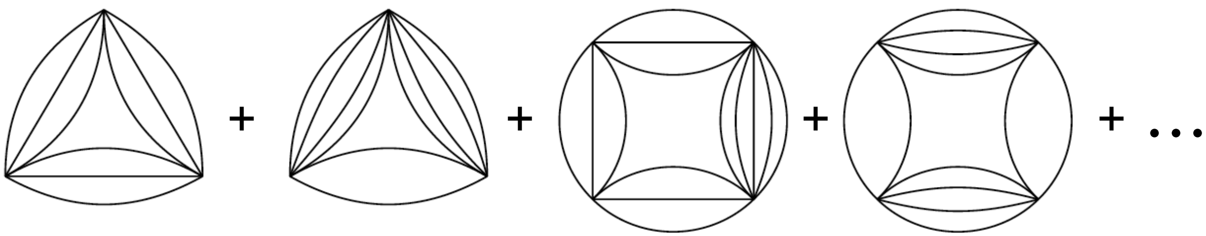
Non-equilibrium Greens Functions

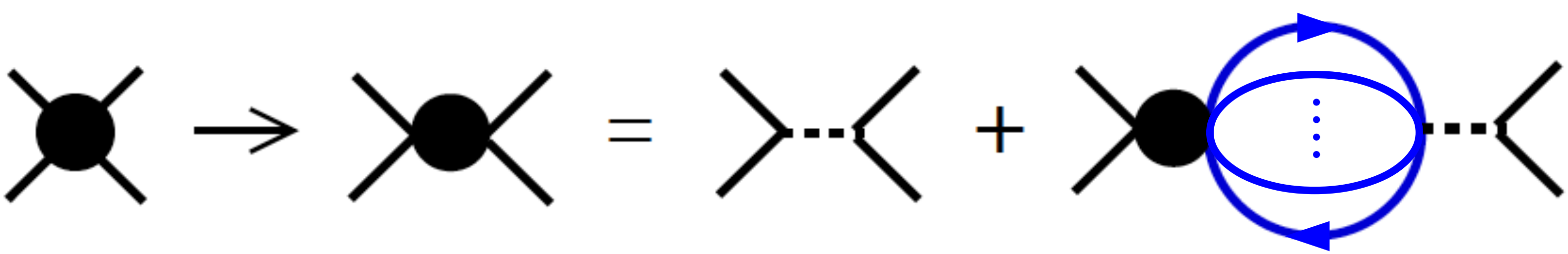


Kadanoff-Baym equation for sine-Gordon (-type)

$\partial_t n(p) =$ 

from **2PI (two-particle-irreducible) effective action**, resumed as

$\Gamma_2 \sim$  + …



Scaling analysis: $\beta = 1/2$ or $\beta = 1/(2+d)$ [if beyond φ^4 matters]

[P. Heinen, A. Mikheev, TG, Phys. Rev. A **107**, 043303 (23) & arXiv:2212.01162; perturbative calc.: Tavora & A. Mitra (2013)]

[& Alexander Flamm + Qinya Li]

Alex Schmutz | Raphael Schäfer | Luise Heiland | Yannik Deller | Helmut Strobel | Markus Oberthaler

Alexander Baum | Martin Zboron | Nils Becker | Julian Mayr | Florian Schmitt | Niklas Rasch | Wyatt Kirkby | Hannes Köper | TG

Elena Garcia Garcia | Ido Siovitz | Andreea Oros | Anna-Maria Glück | Mai Le | Philipp Heinen

Synthetic Quantum Systems

Kirchhoff-Institut @ Uni Heidelberg

SynQS

www.synqs.org

The End

Reviews, Lecture notes, Summary articles, & Recent progress:

Universal dynamics and non-thermal fixed points in quantum fluids far from equilibrium
A. N. Mikheev, I. Siovitz, TG,
EPJST **232**, 3393 (2023), arXiv:2304.12464 [cond-mat.quant-gas]

Universal dynamics of rogue waves in a quenched spinor Bose condensate
I. Siovitz, S. Lannig, Y. Deller, H. Strobel, M. K. Oberthaler, TG,
Phys. Rev. Lett. **131**, 183402; arXiv:2304.09293 [cond-mat.quant-gas]

Observation of two non-thermal fixed points for the same microscopic symmetry
S. Lannig, M. Prüfer, Y. Deller, I. Siovitz, J. Dreher, TG, H. Strobel, M. K. Oberthaler,
arXiv:2306.16497 [cond-mat.quant-gas]

Anomalous scaling at non-thermal fixed points of the sine-Gordon model
P. Heinen, A.N. Mikheev, TG
Phys. Rev. A **107**, 043303 (2019); arXiv:2212.01163 [cond-mat.quant-gas], 2212.01162 [cond-mat.quant-gas]

Low-energy effective theory of non-thermal fixed points in a multicomponent Bose gas
A. N. Mikheev, C.-M. Schmied, TG,
Phys. Rev. A **99**, 063622 (2019); arXiv:1807.10228 [cond-mat.quant-gas]

Prescaling in a far-from-equilibrium Bose gas
C.-M. Schmied, A. N. Mikheev, TG,
Phys. Rev. Lett. **122**: 170404 (2019); arXiv:1807.07514 [cond-mat.quant-gas]

Kinetic theory of non-thermal fixed points in a Bose gas
I. Chantesana, A. Piñeiro Orioli, TG,
Phys. Rev. A **99**, 043620 (2019); arXiv:1801.09490 [cond-mat.quant-gas]

Prethermalization and universal dynamics in near-integrable quantum systems
T. Langen, TG, J. Schmiedmayer,
JSTAT **064009**, 2016; arXiv:1603.09385 [cond-mat.quant-gas]

Inverse Cascade

of energy in 2D classical turbulence

— together with direct k^{-3} *enstrophy* ($\sim$ *vorticity density*) cascade —

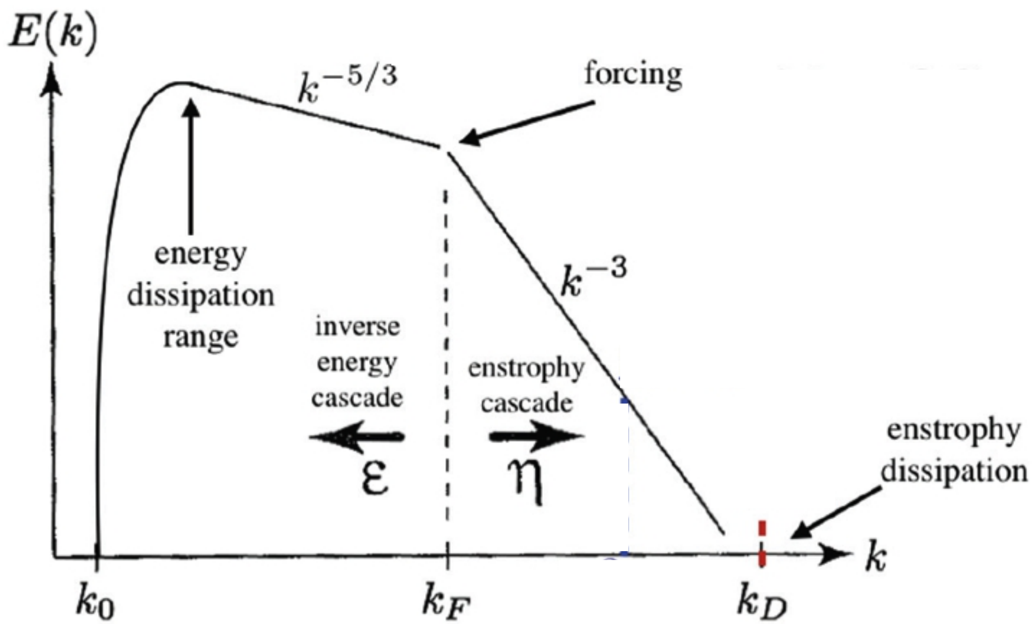


Fig. adapted from Fox-Kemper et al., CLIVAR Exchanges 65 (14) 42.

Robert H. Kraichnan, *Inertial Ranges in Two-Dimensional Turbulence*. Phys. Fl. **10**, 1417 (1967)

Equivalent in nearly incompressible flow in 2D quantum turbulence (GPE):

N. P. Müller, G. Krstulovic, PRL **132**, 094002 (2024)
M. Reeves et al., PRL **110**, 104501 (2013)

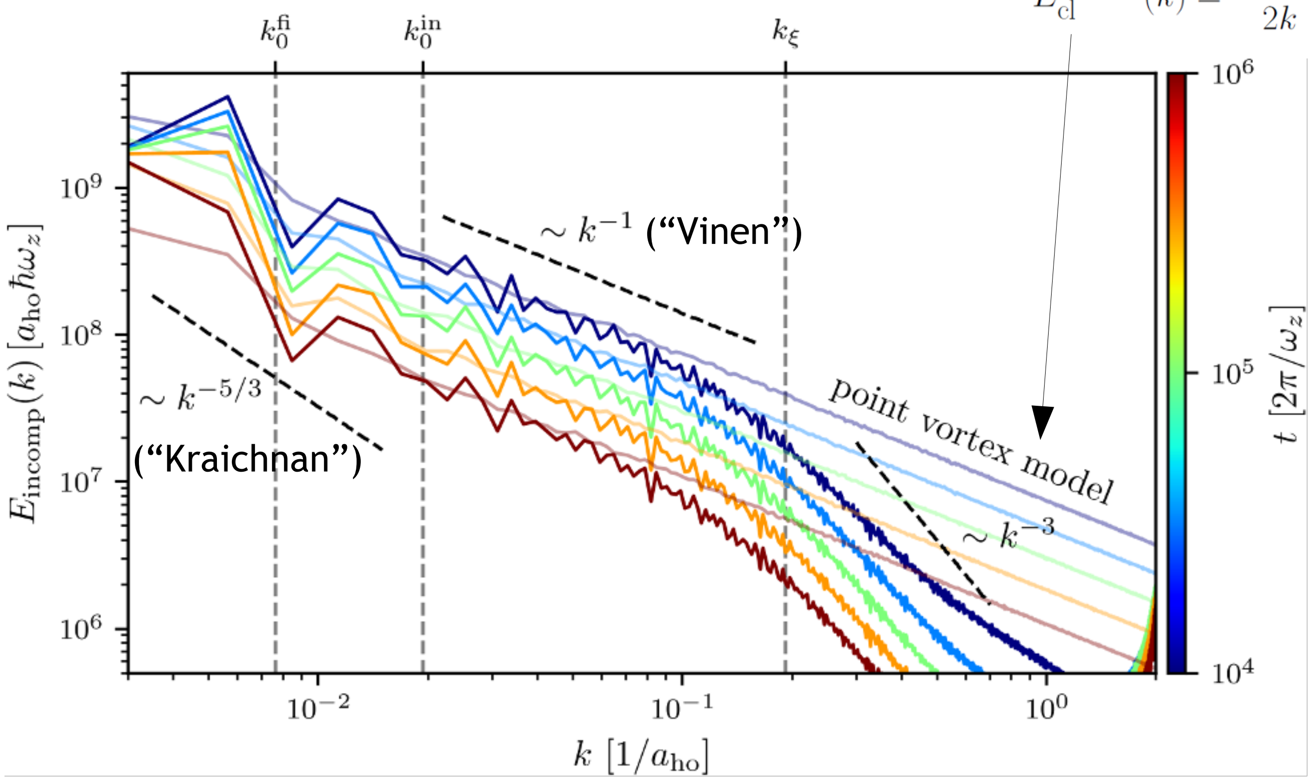
Spectrum of incompressible kinetic energy

Split kinetic energy:

$$E_{\text{kin}} = \frac{1}{2}\int \mathrm{d}^2\boldsymbol{x}\,\Big(\underbrace{|\nabla\sqrt{n}|^2}_{E_\text{q}} + \underbrace{|\sqrt{n}\nabla\varphi|^2}_{E_\text{cl}}\Big) = \frac{1}{2}\int \mathrm{d}^2\boldsymbol{x}\,\Big(|\boldsymbol{v}_\text{q}|^2 + |\boldsymbol{v}_\text{cl}|^2\Big)$$

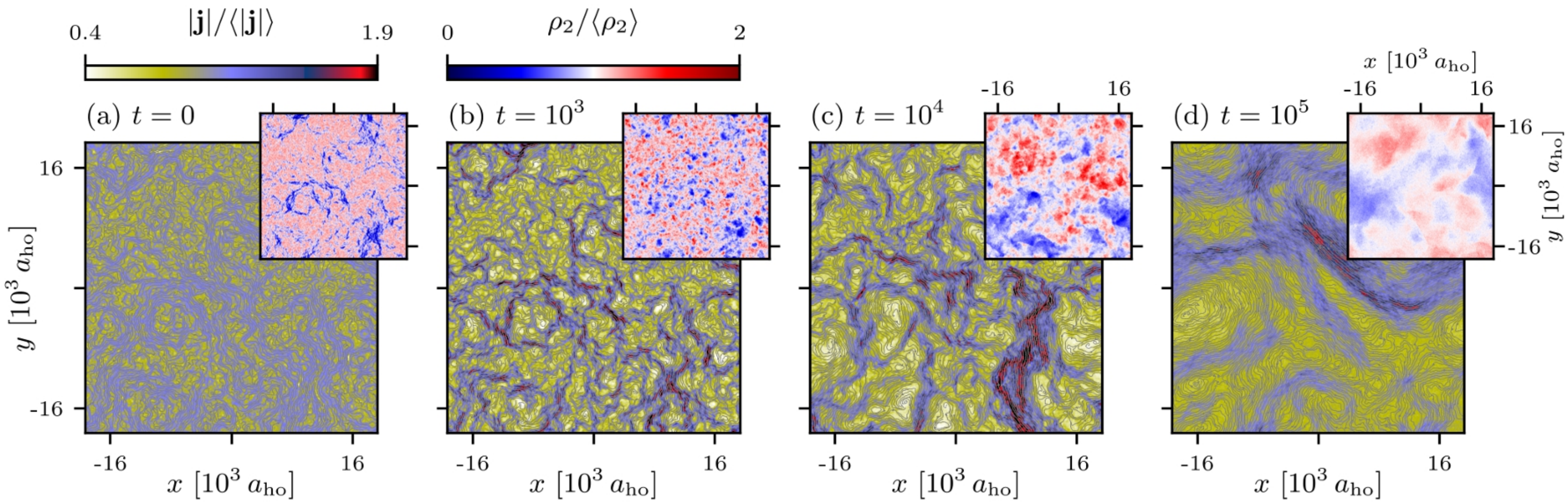
Incompressible energy in point-vortex model

$$E_\text{cl}^\text{incomp}(k) = \frac{(2\pi)^2 n}{2k}\sum_{ij} q_i q_j J_0(k l_{ij})$$



Niklas Rasch, TG, arXiv:2509.21285 [cond-mat.quant-gas].

Pattern in position space



Decreased UV resolution

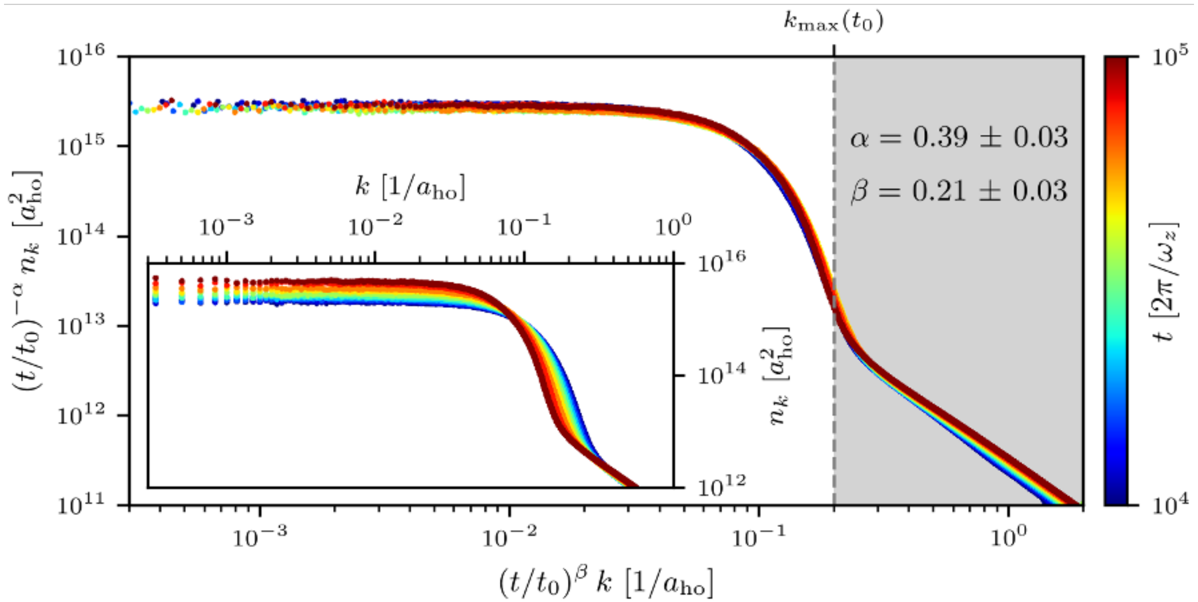
$\xi \approx 2\ dx$

Increased IR resolution

- Larger grid 16384^2
- Higher initial vortex density → increase k_v ($1.4 \cdot 10^6$ vortices & antivortices)

Niklas Rasch, TG, arXiv:2509.21285 [cond-mat.quant-gas].

Momentum spectra



Decreased UV resolution

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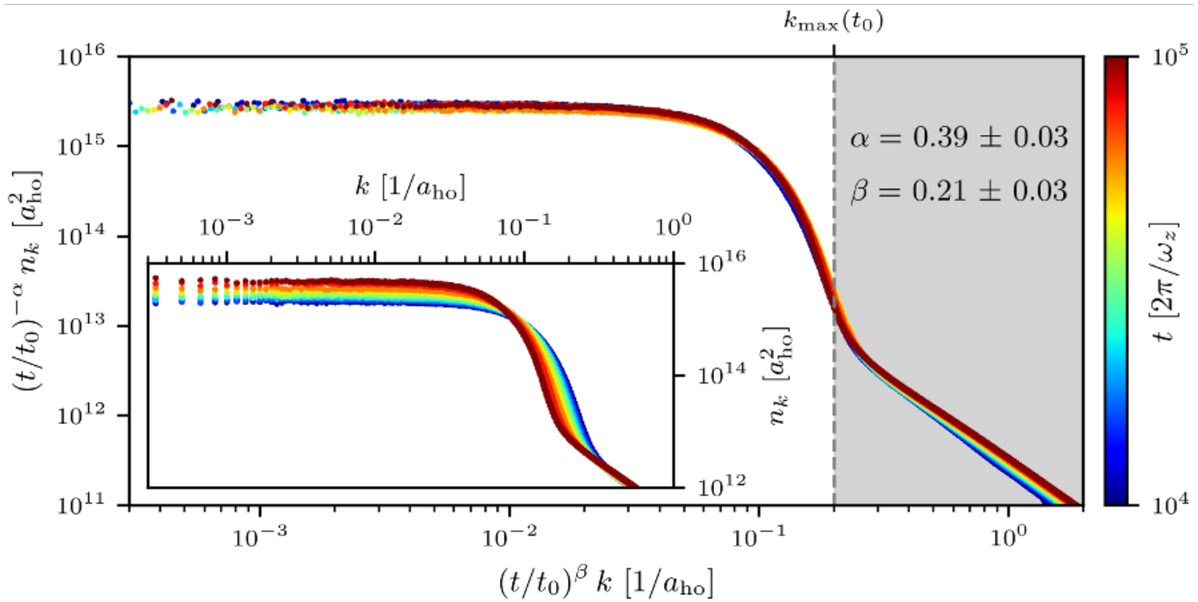
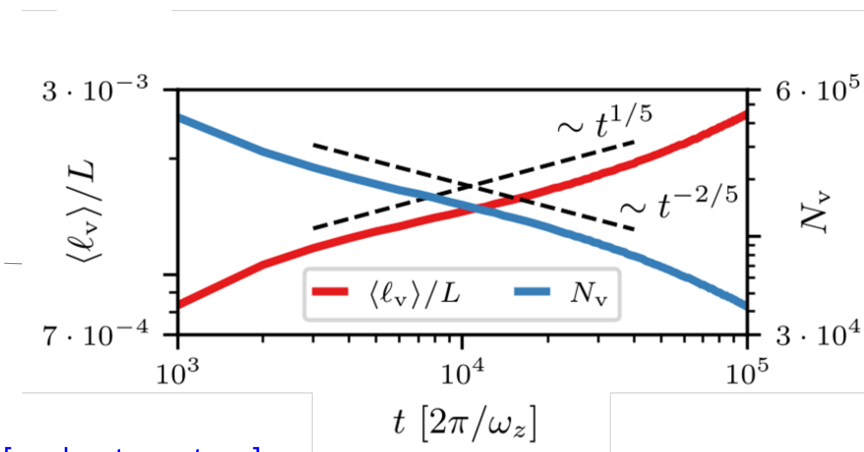
- Higher initial vortex density → increase k_v ($1.4 \cdot 10^6$ vortices & antivortices)

Niklas Rasch, TG, arXiv:2509.21285 [cond-mat.quant-gas].

Momentum spectra & Vortex number/distance



inter-vortex distance

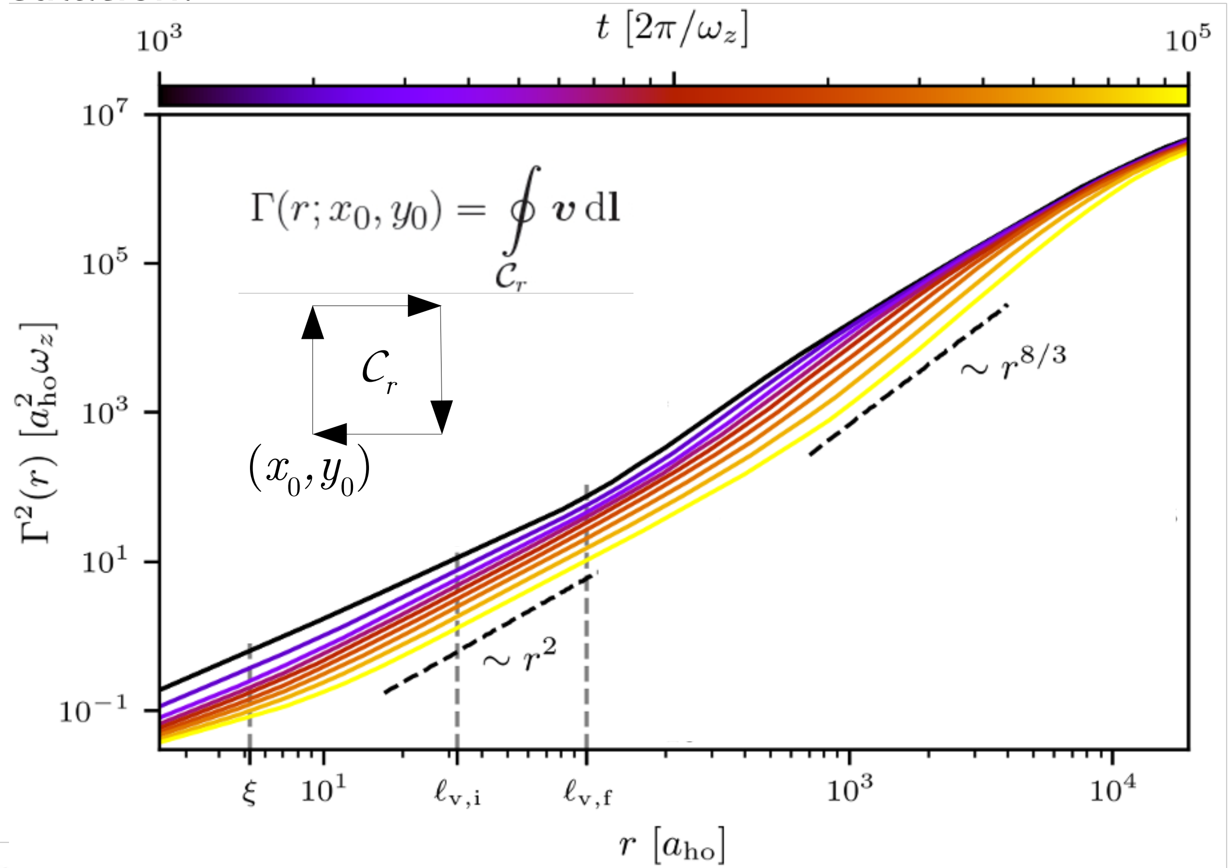


total vortex number

Niklas Rasch, TG, arXiv:2509.21285 [cond-mat.quant-gas].

Analysis in position space

Velocity circulation:

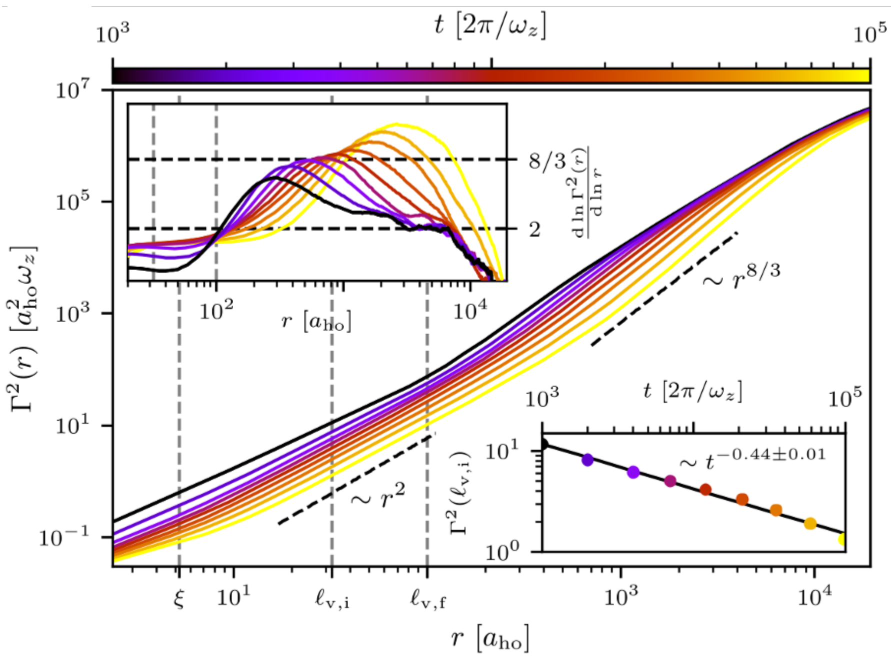


$$\langle |\Gamma(r)|^p \rangle = \frac{1}{V} \int |\Gamma(r; x_0, y_0)|^p \, \mathrm{d}x_0 \, \mathrm{d}y_0$$

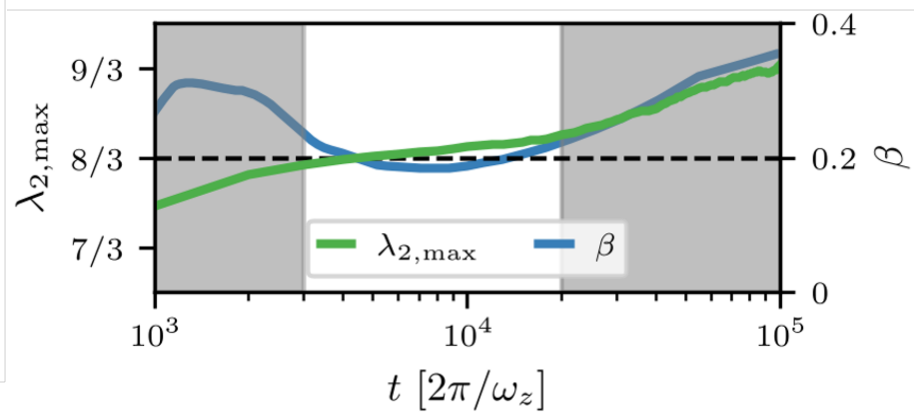
Niklas Rasch, TG, arXiv:2509.21285 [cond-mat.quant-gas].

See also: forced/quenched 2D superfluid turbulence (IEC):
Experiment: H.-Y. Zhu, J.-H. Xie, and K.-Q. Xia, PRL **130**, 214001 (2023)
Simulation: N. P. Müller, G. Krstulovic, PRL **132**, 094002 (2024)

Analysis in position space



Compare maximal local slope $\lambda_{(2,\max)}(t)$ with time-local coarsening $\beta(t)$:



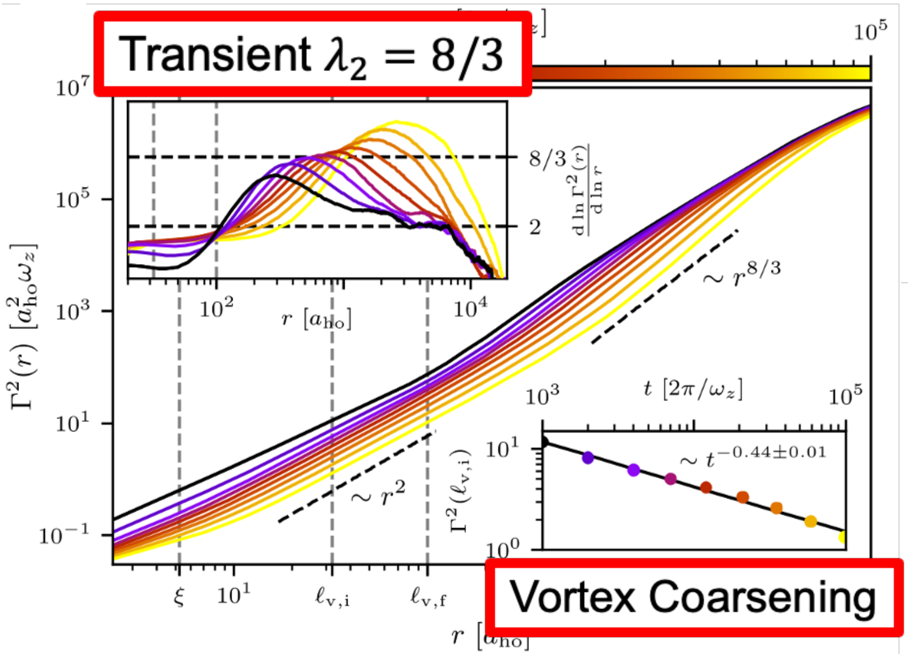
Single vortex scaling: $\langle|\Gamma(r)|^p\rangle \sim r^2$ for $r < \ell_{\rm v}$

Inverse energy cascade: $\langle|\Gamma(r)|^p\rangle \sim r^{\lambda_p}$ for $r > \ell_{\rm v}$

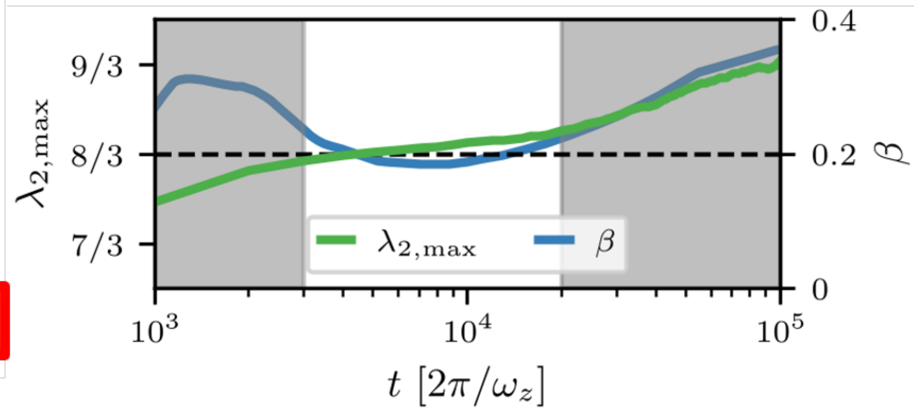
Kraichnan Batchelor: $\lambda_p = 4p/3$ (non-intermittent)

Niklas Rasch, TG, arXiv:2509.21285 [cond-mat.quant-gas].

Analysis in position space



Compare maximal local slope $\lambda_{(2,\max)}(t)$ with time-local coarsening $\beta(t)$:



Indications of simultaneity of transient
inverse energy cascade and
anomalous NTFP in universal interval

Niklas Rasch, TG, arXiv:2509.21285 [cond-mat.quant-gas].

Intermittency in higher-order moments of circulation

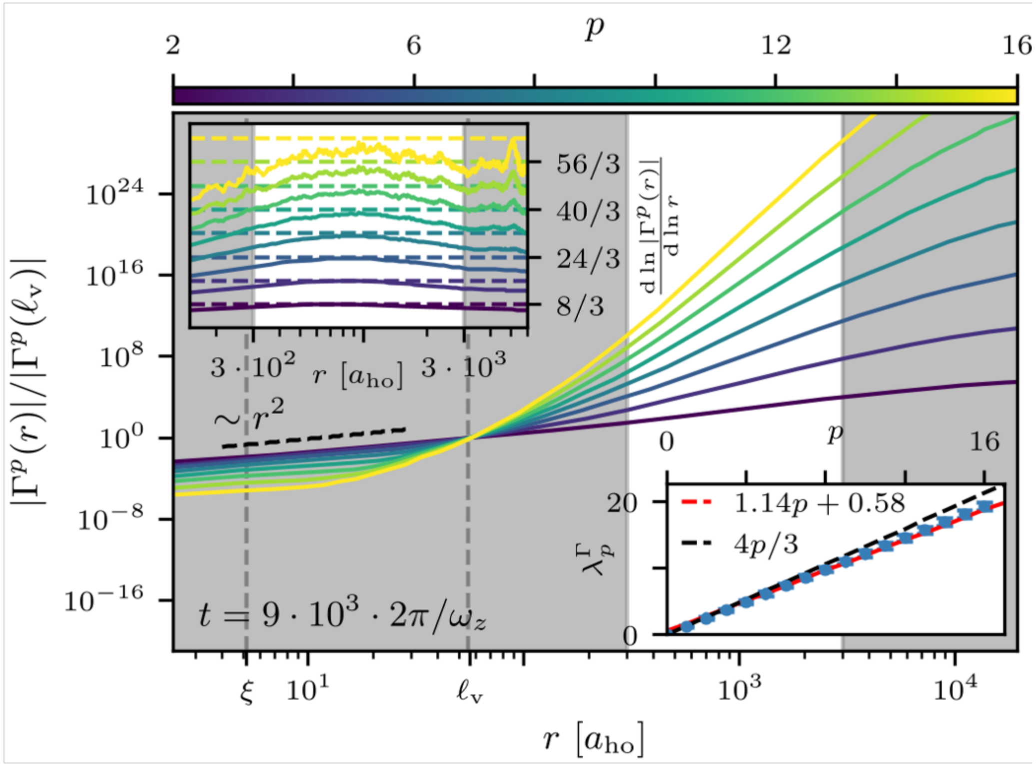
- Intermittency reflected in deviations from

$$\lambda_p = 4p/3$$

- Logarithmic slopes flatten in inertial range

Extracted scaling exponents can be fit with
bifractal intermittency model

$$\lambda_p \approx 1.14\,p + 0.58$$



Measured in thin fluid layers:
H.-Y. Zhu et al., PRL **130**, 214001 (2023)
Simulated in nearly incompressible QT:
N. P. Müller and G. Krstulovic, PRL **132**, 094002 (2024)

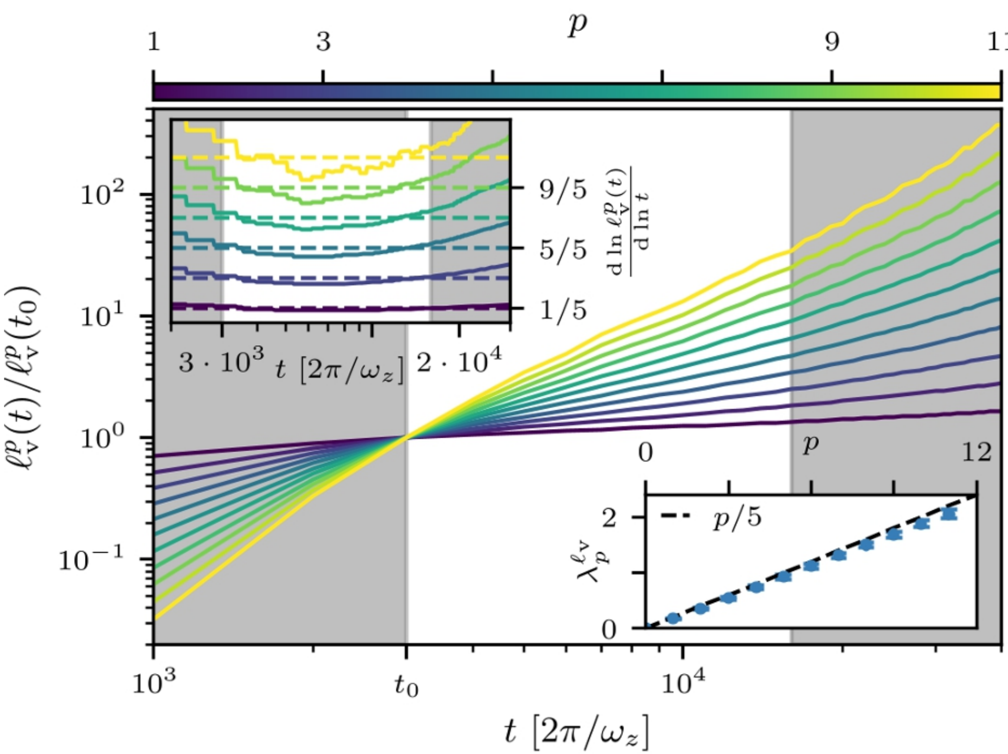
Niklas Rasch, TG, arXiv:2509.21285 [cond-mat.quant-gas].

Higher-order moments of intervortex distance distr.

- Logarithmic slopes flatten in universal interval
- Extracted scaling exponents follow

$$\lambda_p \approx p/5$$

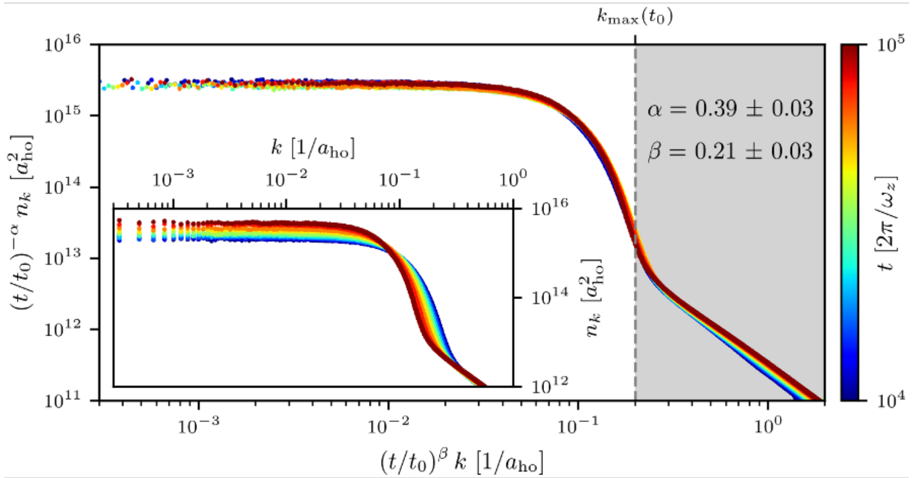
- Inter-defect distribution does not show deviations from Gaussianity



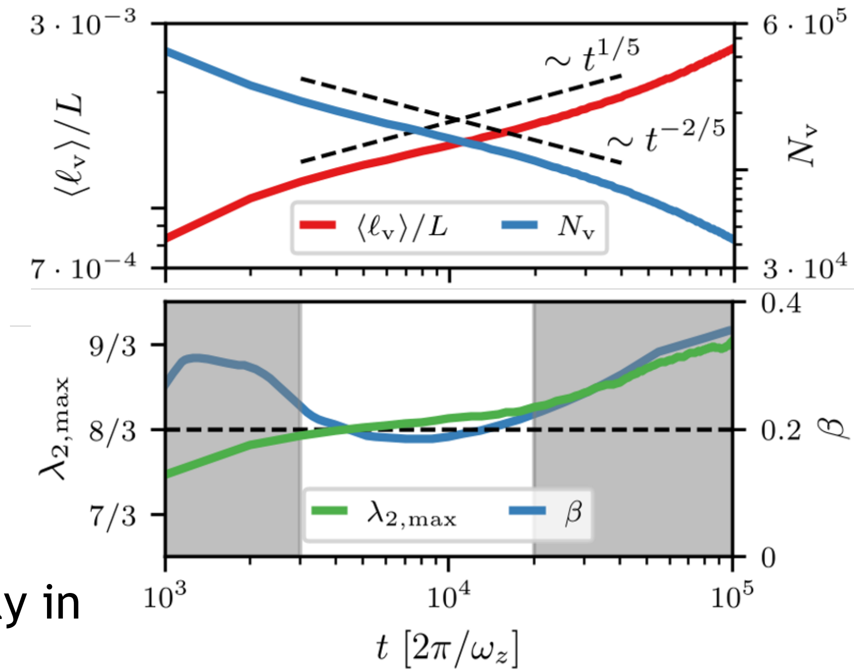
Measured in thin fluid layers:
H.-Y. Zhu et al., PRL **130**, 214001 (2023)
Simulated in nearly incompressible QT:
N. P. Müller and G. Krstulovic, PRL **132**, 094002 (2024)

Niklas Rasch, TG, arXiv:2509.21285 [cond-mat.quant-gas].

Decaying 2D turbulence: quantum vs. classical



Niklas Rasch, TG, arXiv:2509.21285 [cond-mat.quant-gas].



Power law exponent of vortex density decay in classical decaying 2D Turbulence:

Simulations:

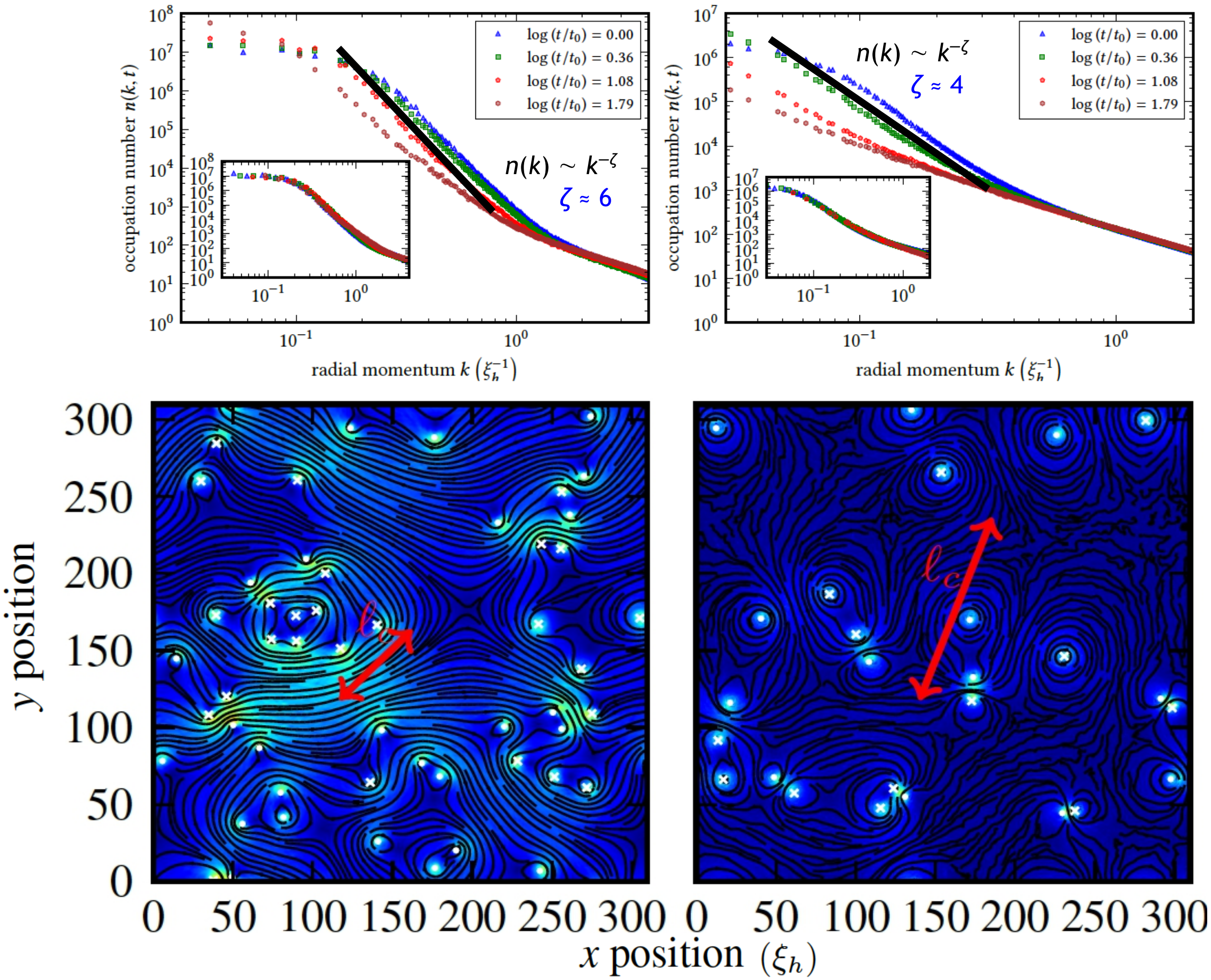
Reference	$\alpha = 2\beta$
Carnevale *et al.*[7]	0.75
Weiss and McWilliams[2]	0.72±0.03
Dritschel[5]	0.29±0.03
Clercx and Nielsen[21]	1.03±0.10 [c]
Bracco *et al.*[8]	0.76±0.03

Experiments:

Reference	$\alpha = 2\beta$
Carnevale *et al.*[7]	0.75
Tabeling *et al.*[9]	0.70±0.1
Cardoso *et al.*[28]	0.44±0.1
Hansen *et al.*[10]	0.70±0.1
Clercx *et al.*[29]	0.70±0.1

For these tables & Refs., see van Bokhoven et al., Phys. Fl. **19**, 046601 (07)

Anomalous vs. Gaussian NTFP



$n(k) \sim k^{-\zeta}$

Conjecture:

$\zeta = d + N - \eta$

ℓ_c = mean vortex distance

[cf. also Billam, Bradley, et al. (14, 15)]

M. Karl, TG, NJP 2017, 1611.01163 [cond-mat.quant-gas]

Universal dynamics in **experiment**

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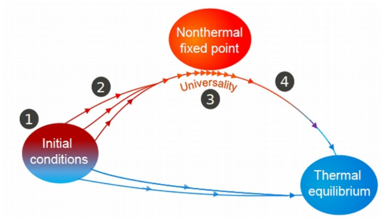
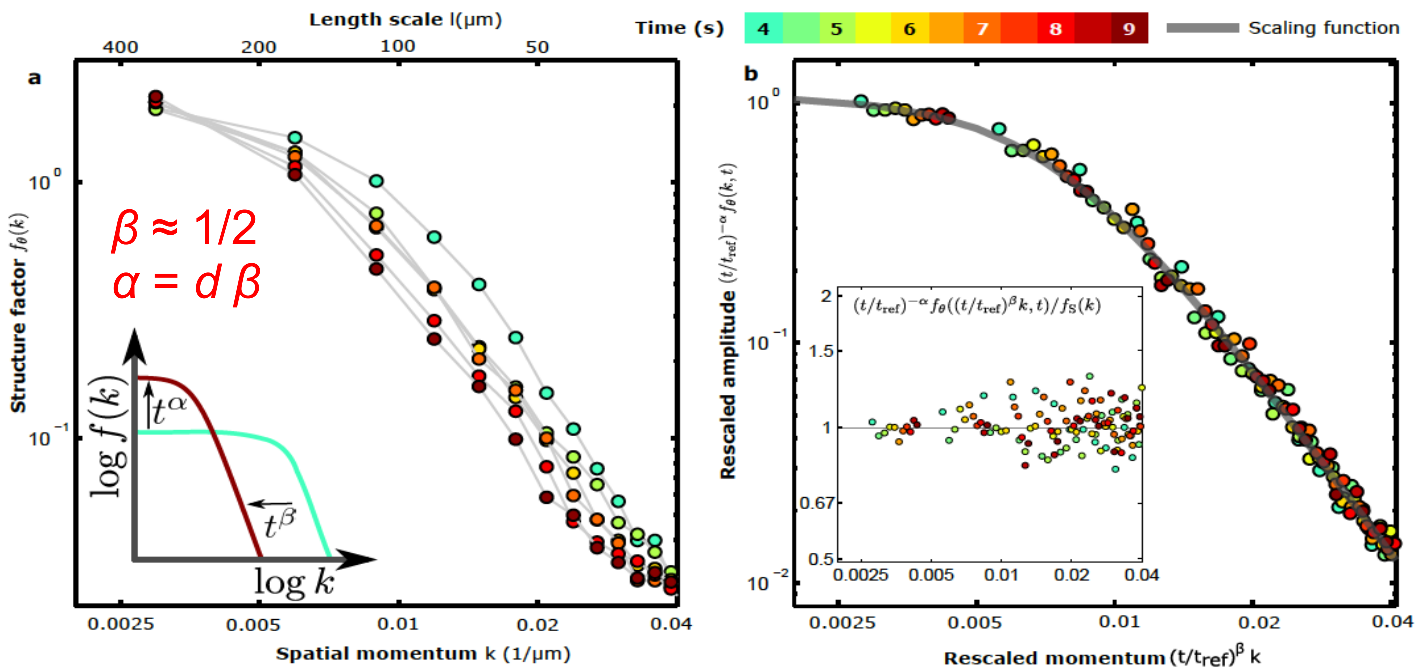
Oberthaler labs

Experimental identification of a non-thermal fixed point in a Spin-1 Bose gas

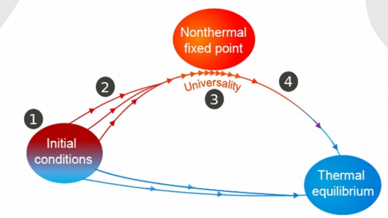




M. Prüfer, et al., Nature **563,** 217 (18); arXiv:[1805.11881 [cond-mat.quant-gas]]

Universal dynamics in **experiment**

Experimental identification of a non-thermal fixed point **in Ultracold Gases**



M. Prüfer, P. Kunkel, H. Strobel, S. Lannig, D. Linnemann, C.-M. Schmied, J. Berges, TG, M. K. Oberthaler, Nature **563**, 217 (18); arXiv:1805.11881 [cond-mat.quant-gas]

C. Eigen, J. A. P. Glidden, R. Lopes, E. A. Cornell, R. P. Smith, and Z. Hadzibabic, Nature **563**, 221 (18), arXiv:1805.09892 [cond-mat.quant-gas]

S. Erne, R. Bücker, TG, J. Berges, and J. Schmiedmayer, Nature **563**, 225 (18); arXiv:1805.12310 [cond-mat.quant-gas]

N. Navon, C. Eigen, J. Zhang, R. Lopes, A. L. Gaunt, K. Fujimoto, M. Tsubota, R. P. Smith, and Z. Hadzibabic, Science **366**, 382 (19), arXiv:1807.07564 [cond-mat.quant-gas]

J. A. P. Glidden, C. Eigen, L. H. Dogra, T. A. Hilker, R. P. Smith, and Z. Hadzibabic, Nature Phys. **17**, 457 (21), arXiv:2006.01118 [cond-mat.quant-gas]

A. D. García-Orozco, L. Madeira, M. A. Moreno-Armijos, A. R. Fritsch, P. E. S. Tavares, P. C. M. Castilho, A. Cidrim, G. Roati, and V. S. Bagnato, Phys. Rev. A **106**, 023314 (22), arXiv:2107.07421 [cond-mat.quant-gas];

G. Martirosyan, C. J. Ho, J. Etrych, Y. Zhang, A. Cao, Z. Hadzibabic, C. Eigen, Phys. Rev. Lett. 132, 113401 (24); arXiv:2304.06697 [cond-mat.quant-gas];

S. Lannig, M. Prüfer, Y. Deller, I. Siovitz, J. Dreher, TG, H. Strobel, M. K. Oberthaler, arXiv:2306.16497 [cond-mat.quant-gas];

M. Gazo, A. Karailiev, T. Satoor, C. Eigen, M. Gałka, Z. Hadzibabic, arXiv:2312.09248 [cond-mat.quant-gas];

M. A. Moreno-Armijos, A. R. Fritsch, A. D. García-Orozco, S. Sab, G. Telles, Y. Zhu, L. Madeira, S. Nazarenko, V. I. Yukalov, V. S. Bagnato, arXiv:2407.11237 [cond-mat.quant-gas];

G. Martirosyan, M. Gazo, J. Etrych, S. M. Fischer, S. J. Morris, C. J. Ho, C. Eigen, Z. Hadzibabic, arXiv:2410.08204 [cond-mat.quant-gas].

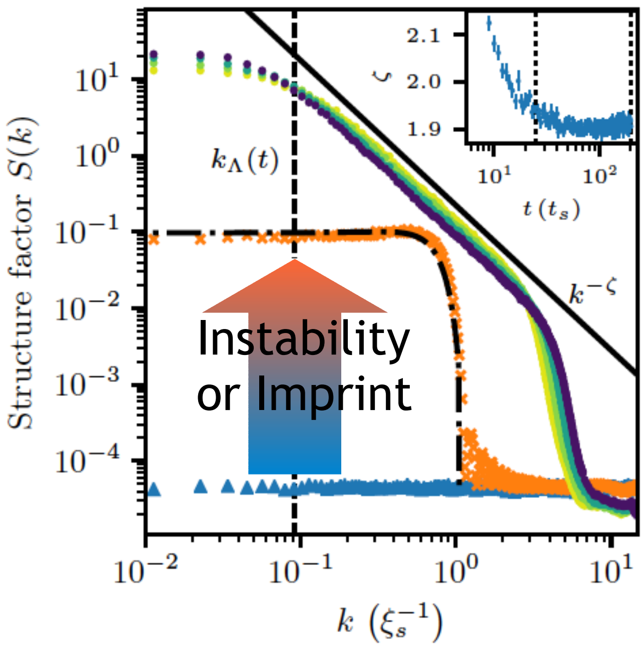
⋮

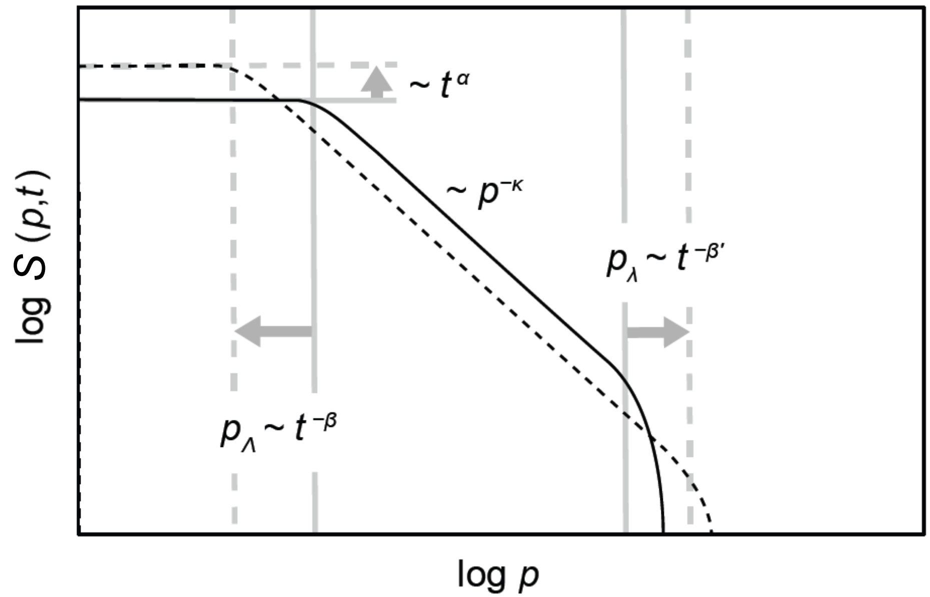
much activity at present

M. Prüfer, et al., Nature **563**, 217 (18); arXiv:1805.11881 [cond-mat.quant-gas]

Universal rescaling dynamics

$\beta \approx 1/4, \quad \alpha \approx d\,\beta \approx 1/4$





Structure factor

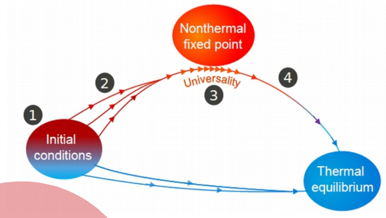
$$S(k,t) = \langle |F_\perp(k,t)|^2 \rangle \qquad F_\perp = F_x + iF_y = |F_\perp| e^{i\varphi_L}$$

C.M. Schmied, M. Prüfer, M.K. Oberthaler, TG, PRA **99**, 033611 (2019); arXiv:1812.08571 [cond-mat.quant-gas]
I. Siovitz, S. Lannig, Y. Deller, H. Strobel, M.K. Oberthaler, TG, PRL **131**, 183402 (23); arXiv:2304.09293 [cond-mat.quant-gas]

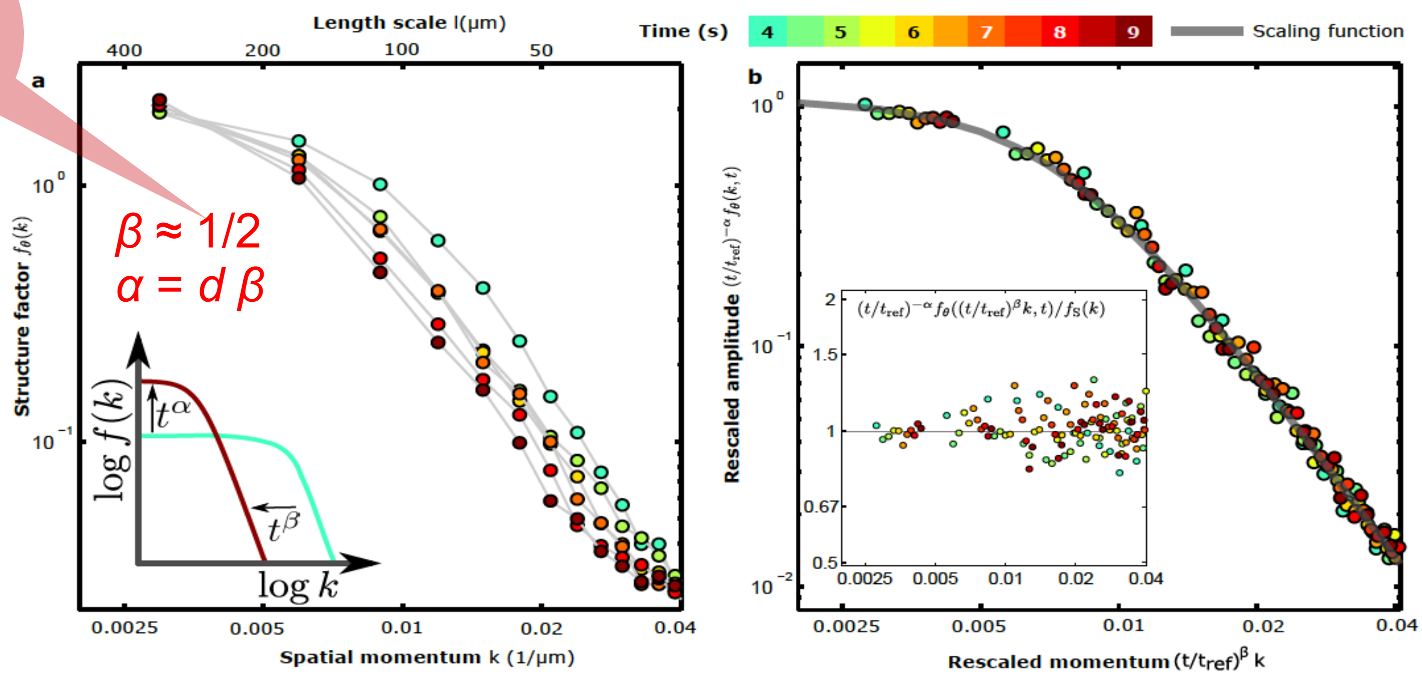
Universal dynamics in **experiment**

SynQS
Oberthaler labs

Experimental identification of a non-thermal fixed point in a Spin-1 Bose gas



why not ¼ ?!



M. Prüfer, et al., Nature **563**, 217 (18); arXiv:1805.11881 [cond-mat.quant-gas]

Experiment

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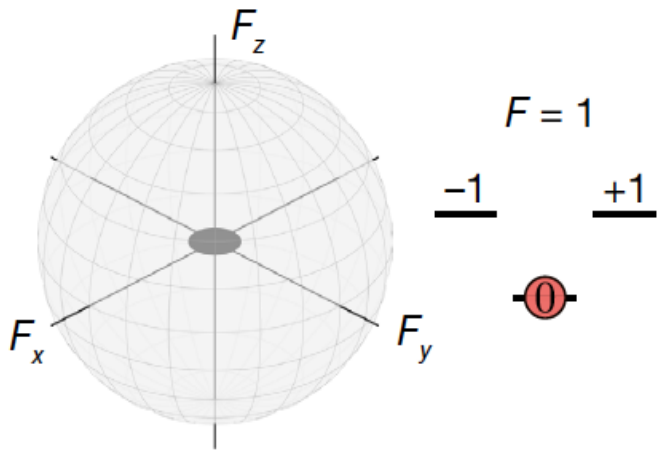
^{87}Rb BEC in a quasi 1D trap:

Spin-1 BEC (F = 1 hyperfine state with magnetic sublevels m_F = 0, ±1)



$$\hat{H} = \int \mathrm{d}\boldsymbol{x} \left\{ \hat{\boldsymbol{\Psi}}^\dagger(\boldsymbol{x},t) \left[-\frac{\hbar^2}{2M}\nabla^2 + q f_z^2 \right] \hat{\boldsymbol{\Psi}}(\boldsymbol{x},t) + \frac{1}{2}c_0 : \hat{n}^2(\boldsymbol{x},t) : + \frac{1}{2}c_1 : \hat{\boldsymbol{F}}^2(\boldsymbol{x},t) : \right\}$$

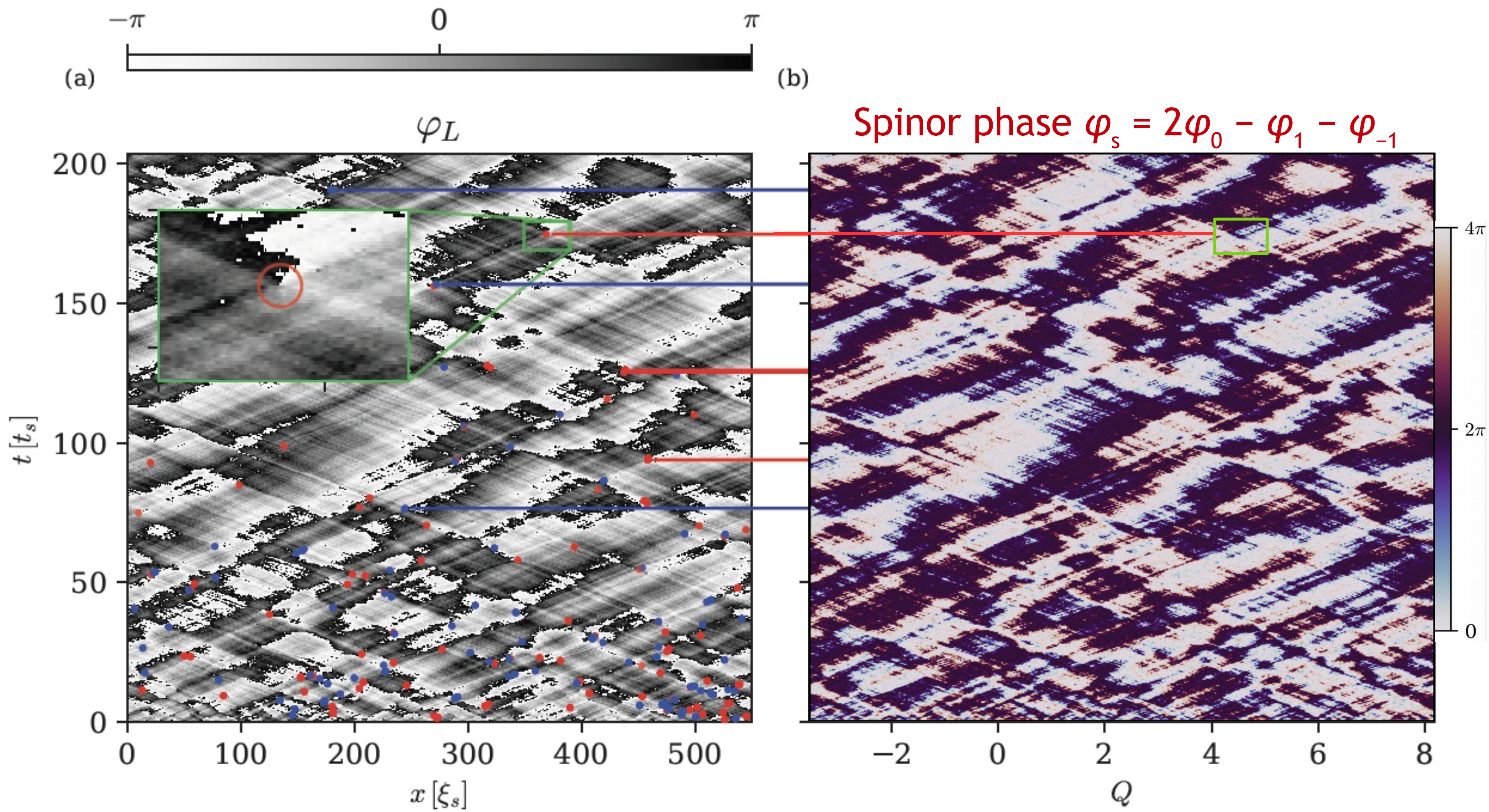
$$\hat{\boldsymbol{\Psi}} = (\Psi_{-1}, \Psi_0, \Psi_1)$$

$$\hat{n}(\boldsymbol{x},t) = \hat{\boldsymbol{\Psi}}^\dagger(\boldsymbol{x},t)\hat{\boldsymbol{\Psi}}(\boldsymbol{x},t)$$

$$\hat{\boldsymbol{F}} = \hat{\boldsymbol{\Psi}}^\dagger(\boldsymbol{x},t) \cdot \boldsymbol{f} \cdot \hat{\boldsymbol{\Psi}}(\boldsymbol{x},t)$$

M. Prüfer, et al., Nature 563, 217 (18); S. Lannig et al., arXiv:2306.16497 [cond-mat.quant-gas];

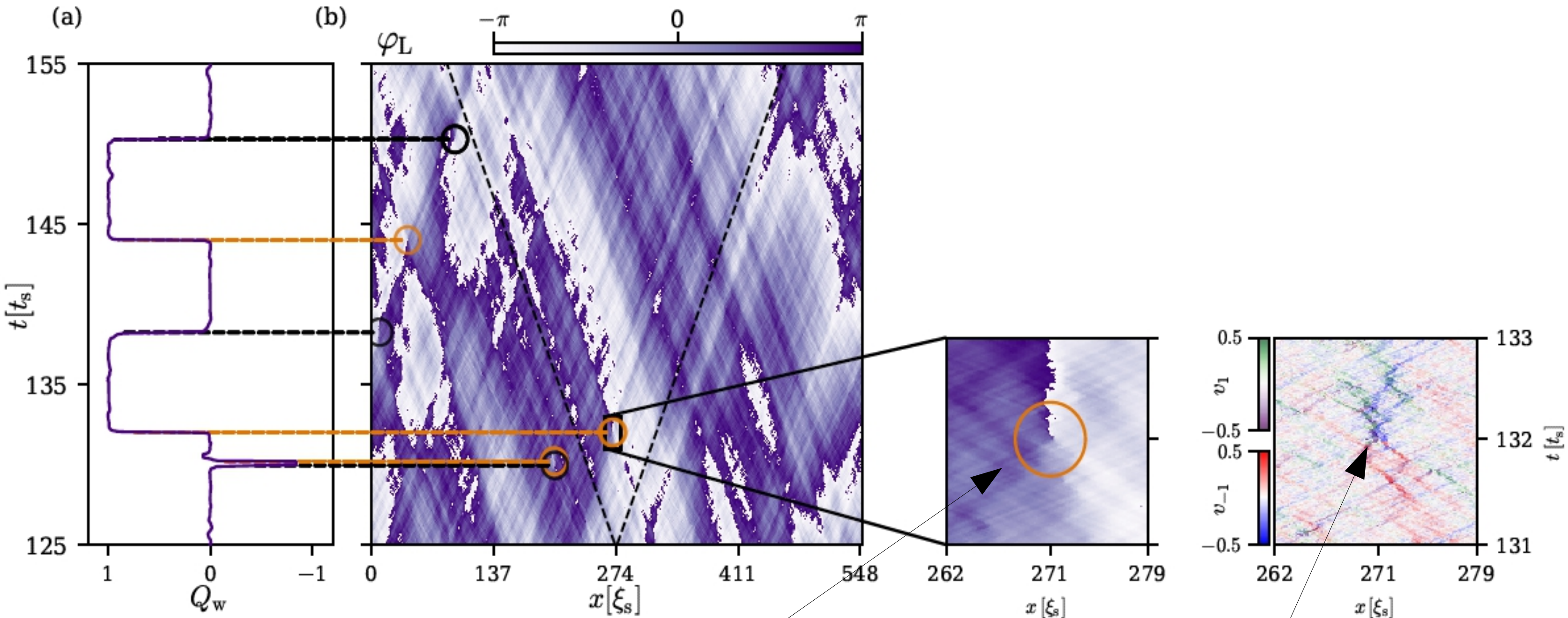
Larmor vs. Spinor phase



$$F_\perp = F_x + iF_y = |F_\perp| e^{i\varphi_L}$$

I. Siovitz, A.-M. E. Glück, Y. Deller, A. Schmutz, F. Klein, H. Strobel, M.K. Oberthaler, TG, arXiv:2412.13986 [cond-mat.quant-gas]

...briefly back to the spinor phase pattern:



Our interpretation: Instantons = phase slips, induced by **rogue waves** in the spinor phase, cause the Larmor phase to wind several times around the potential maximum. Also the spinor phase thereby develops fluctuations with higher winding number...

I. Siovitz, S. Lannig, Y. Deller, H. Strobel, M.K. Oberthaler, TG, PRL **131**, 183402 (23); arXiv:2304.09293 [cond-mat.quant-gas]

Sine^2-Gordon Model: low-energy effective theory??

$$\mathcal{L} = \frac{\mathrm{i}}{2}\left(\psi_a^*\partial_t\psi_a - \psi_a\partial_t\psi_a^*\right) - \frac{1}{2M}\nabla\psi_a^*\nabla\psi_a - q(f^z)^2_{ab}\psi_a^*\psi_b - \frac{c_0}{2}(\psi_a^*\psi_a)^2 - \frac{c_1}{2}\sum_{i\in\{x,y,z\}}(\psi_a^* f^i_{ab}\psi_b)^2,$$

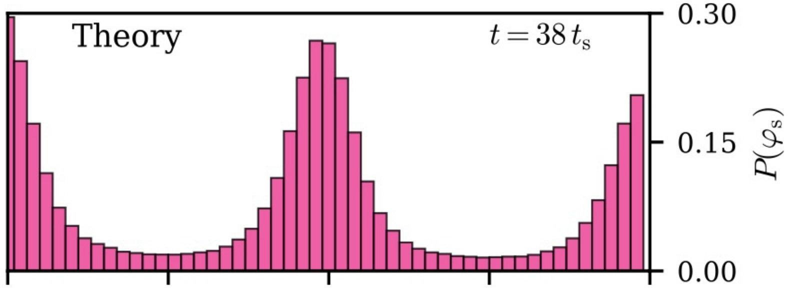
Spin-1 Bose gas

$$\psi_{\pm 1} = \sqrt{\rho \pm \epsilon}\, \mathrm{e}^{\mathrm{i}(\theta \pm \varphi_\mathrm{L})/2}$$

$$\psi_0 = \sqrt{\tilde{\rho} - 2\rho}\, \mathrm{e}^{\mathrm{i}(\theta + \varphi_\mathrm{s})/2}$$

$$\rho = n + \delta\rho$$
$$\epsilon = \bar{\epsilon} + \delta\epsilon$$

integrate out
@ 2nd order
in flucts



$$\mathcal{L}_\mathrm{eff}(\varphi_\mathrm{s}) = -\frac{1}{32c_1}\dot{\varphi}_\mathrm{s}^2 - \frac{\tilde{\rho}-2n}{8m}(\nabla\varphi_\mathrm{s})^2 - \left[2c_1 n(\tilde{\rho}-2n) - \frac{q^2}{16c_1}\right]\cos\varphi_\mathrm{s} - \frac{q^2}{32c_1}\sin^2\varphi_\mathrm{s}$$

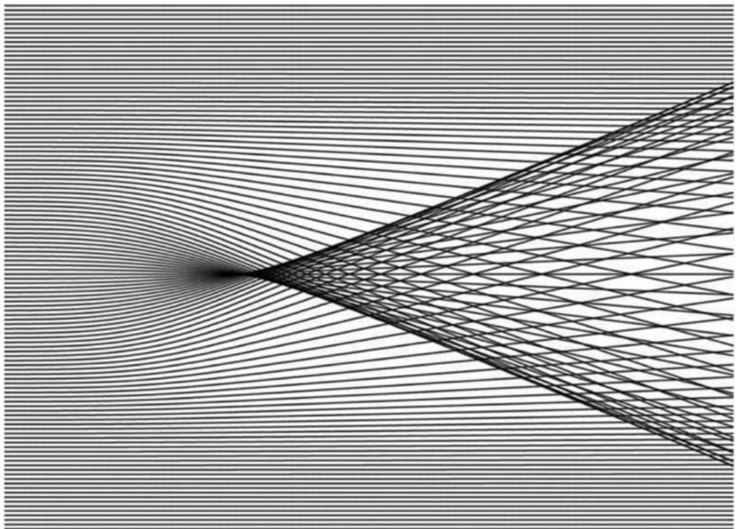
actually from

!

$$\sim \frac{2q^2(\tilde{\rho}-2n)}{k_I^2\tilde{\rho} + 8c_1(\tilde{\rho}-2n)(1+\cos\varphi_\mathrm{s})}$$

I. Siovitz, A.-M. E. Glück, Y. Deller, A. Schmutz, F. Klein, H. Strobel, M.K. Oberthaler, TG, PRA **112,** 023304 (25); 2412.13986 [cond-mat.quant-gas]

Caustics …

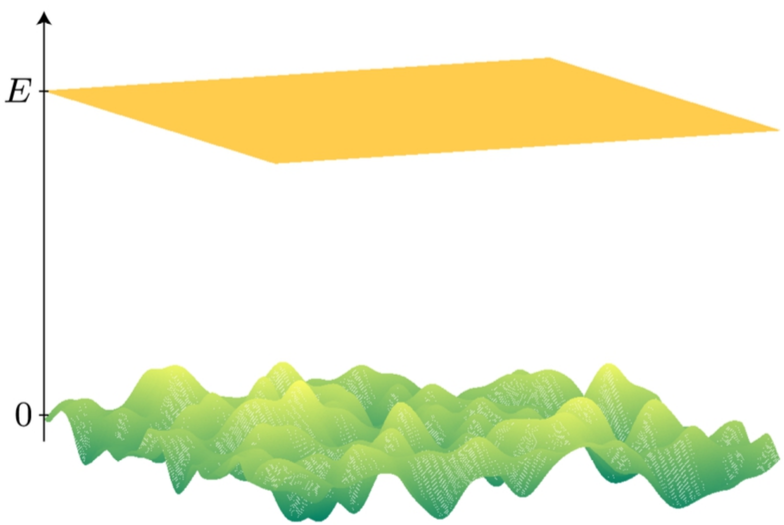


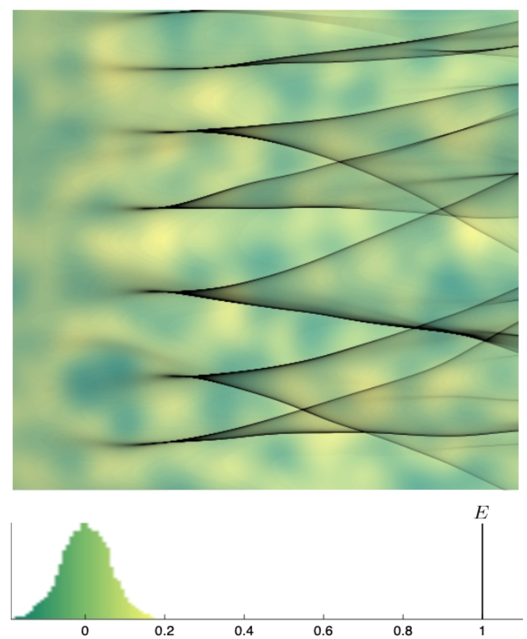
What if many such “bad lenses”?

↔ Classical trajectories in a (correlated) random potential

$$\langle V(x)V(x+r)\rangle \sim e^{-r^2/2l_c^2}$$

Trajectories focus in certain areas
➔ Caustics ➔ Rogue waves



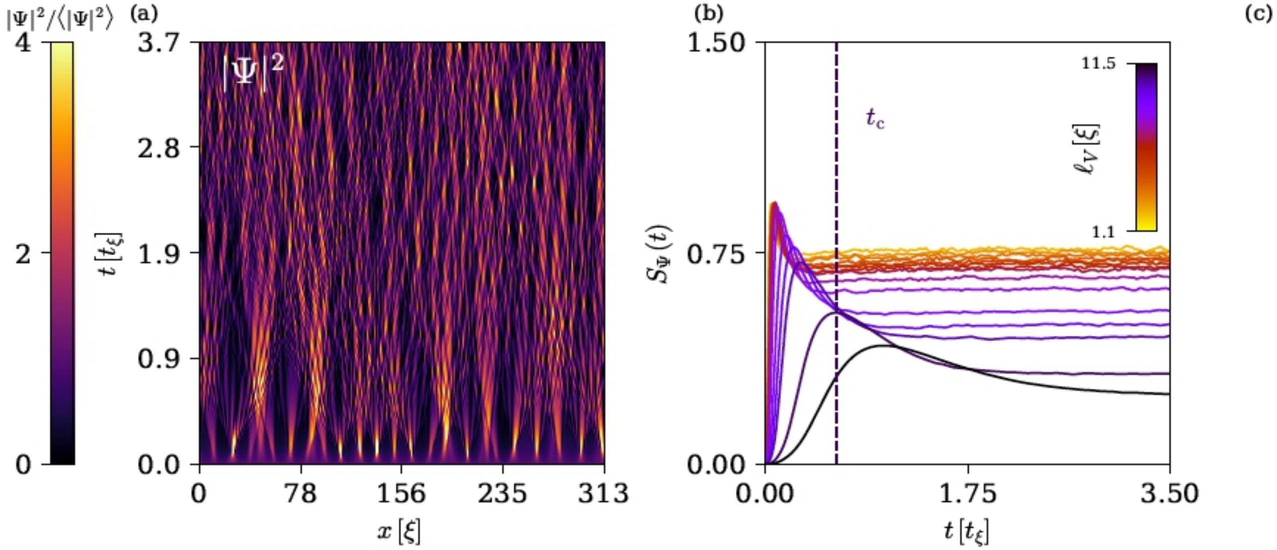


J.J. Metzger, PhD thesis, Univ. Göttingen, 2010.

Caustics as rare & extreme patterns

… here in a single-component 1D GPE dynamics in random noise pot.:

scintillation index $S_\psi(t) = \langle|\psi|^4\rangle_x / \langle|\psi|^2\rangle_x^2 - 1$

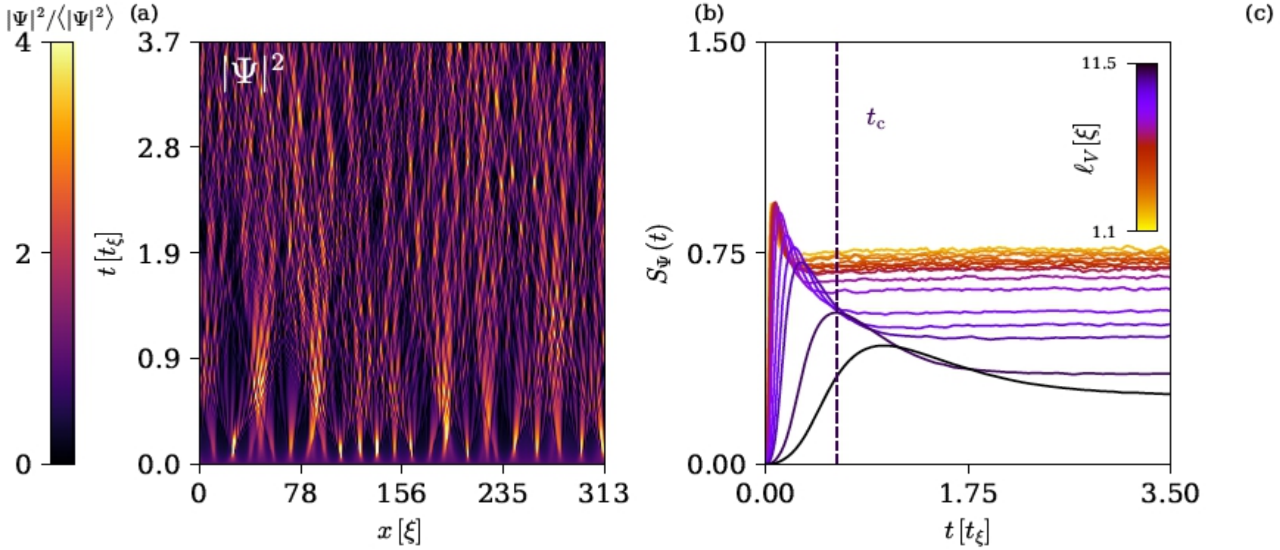


I. Siovitz, S. Lannig, Y. Deller, H. Strobel, M.K. Oberthaler, TG, PRL **131**, 183402 (23); arXiv:2304.09293 [cond-mat.quant-gas]

Caustics as rare & extreme patterns

… "time to 1st caustics" t_c depends on the spatial correlation length:

scintillation index $S_\psi(t) = \langle|\psi|^4\rangle_x / \langle|\psi|^2\rangle_x^2 - 1$



$$\langle V(x,t)V(x+r,t)\rangle \sim \exp(-r/\ell_V) \qquad \left\langle [x - c_s(t-t_0) - x_0]^2 \right\rangle_{m,t} \simeq \xi_s^2 \left\{ 1 + \frac{\pi}{3} V_0^2 \; \frac{\ell_\tau c_s t^3}{\ell_V^2} \right\}$$

$$\langle V(x,t)V(x,t+\tau)\rangle \sim \frac{V_0^2}{1 + [c_s\tau/\ell_\tau(t)]^2}$$

$$\Rightarrow \quad t_c \sim \left(V_0^2 \;\ell_\tau c_s\right)^{-1/3} \ell_V^{4/3}$$

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Caustics as rare & extreme patterns

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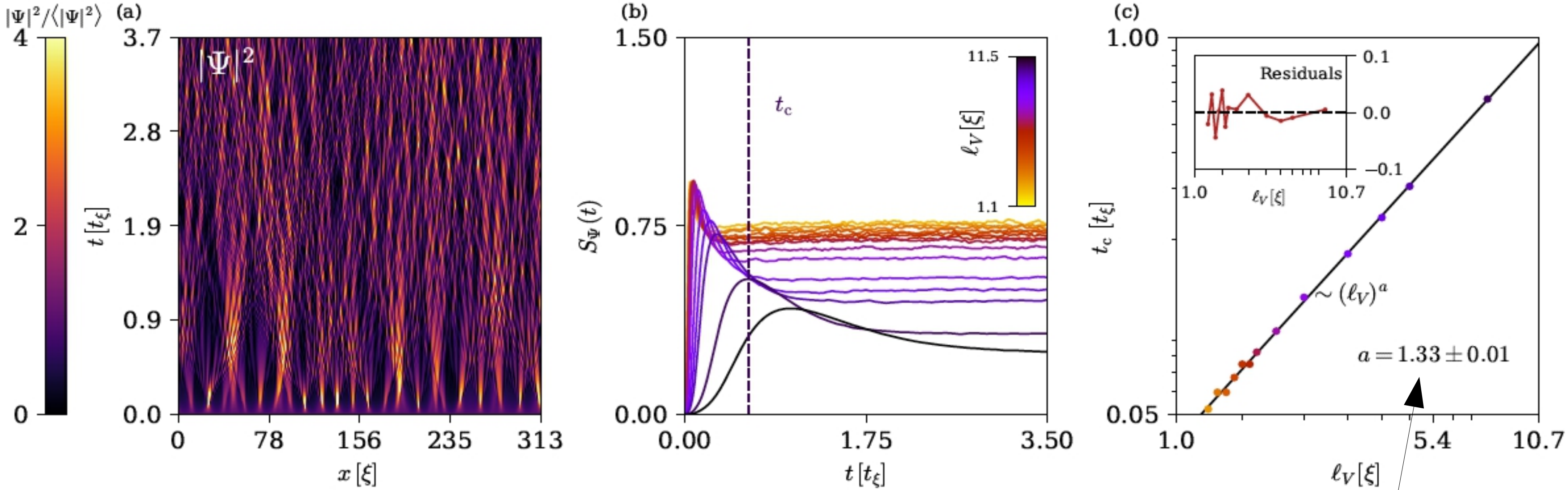

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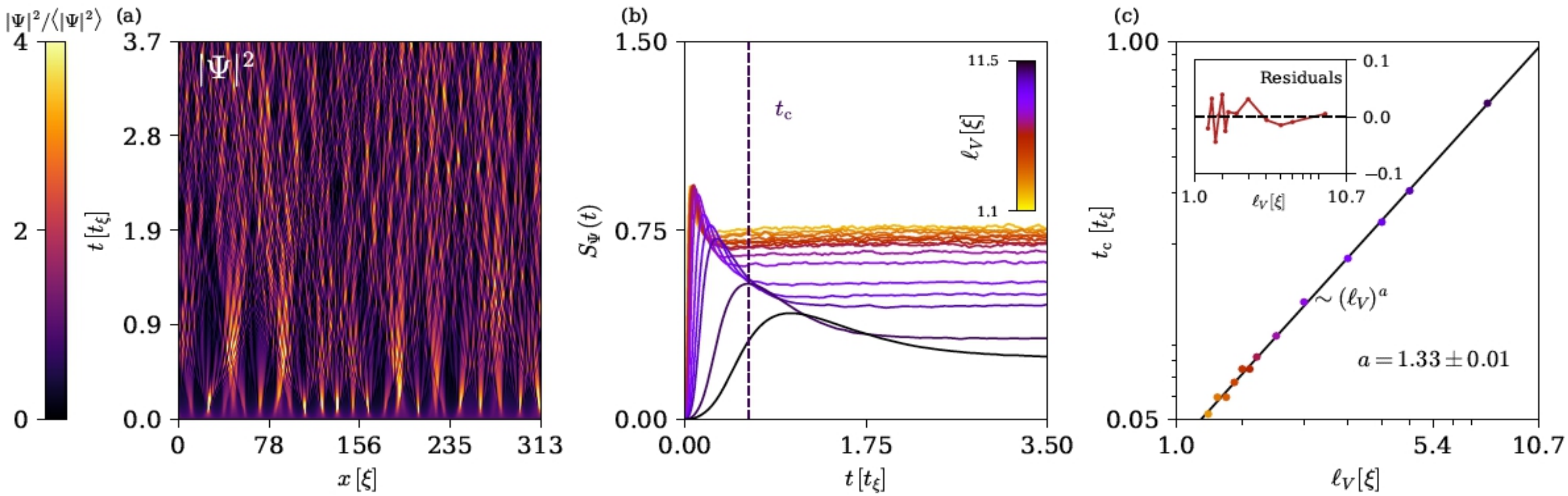
I. Siovitz, S. Lannig, Y. Deller, H. Strobel, M.K. Oberthaler, TG, PRL **131**, 183402 (23); arXiv:2304.09293 [cond-mat.quant-gas]

Universal dynamics of rare events

Caustics in single-component 1D GPE dynamics in random noise pot.

scintillation index $S_{\psi}(t) = \langle|\psi|^4\rangle_x / \langle|\psi|^2\rangle_x^2 - 1$



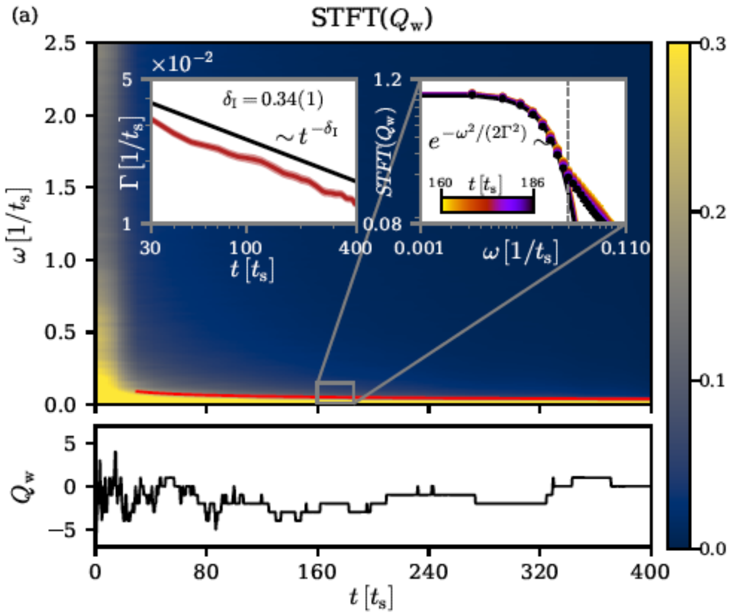
scaling relation

$$\delta = 4\beta/3$$

I. Siovitz, S. Lannig, Y. Deller, H. Strobel, M.K. Oberthaler, TG, arXiv:2304.09293 [cond-mat.quant-gas]

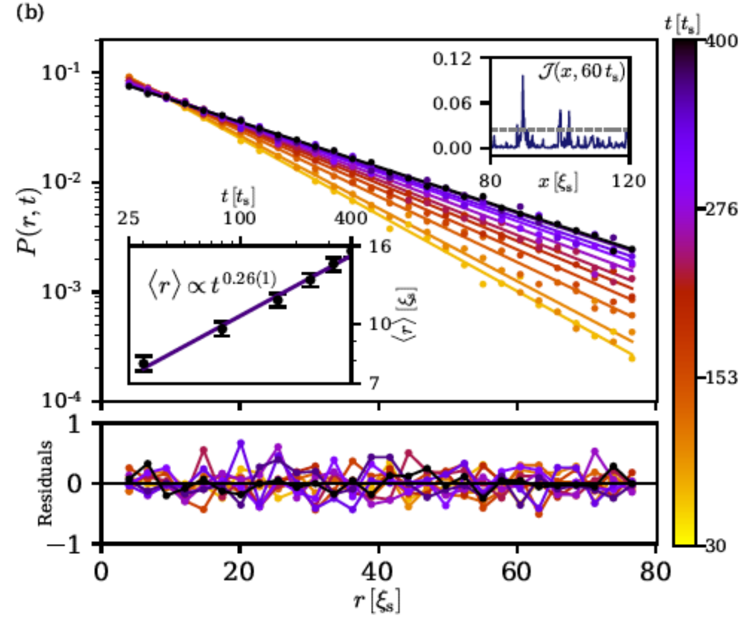
Instanton distribution & dilution

Short-time Fourier Transform (STFT) of winding number $Q_w(t)$



Scaling with $\delta \approx 1/3$

Spatial PDF of instantons as function of time $P(r,t)$



Scaling with $\beta \approx 1/4$

Relation known from caustics: $\delta = 4\beta/3$!

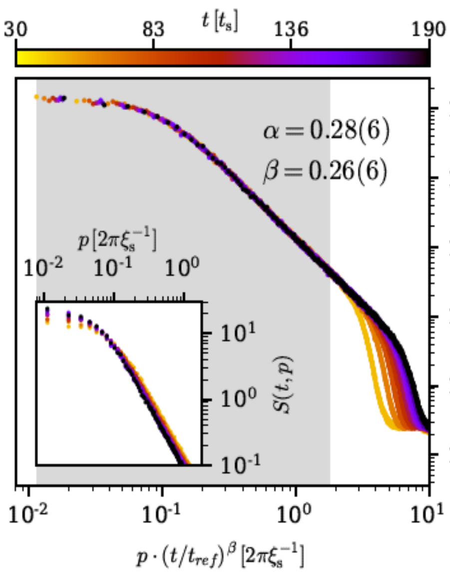
I. Siovitz, S. Lannig, Y. Deller, H. Strobel, M.K. Oberthaler, TG, PRL **131**, 183402 (23); arXiv:2304.09293 [cond-mat.quant-gas]

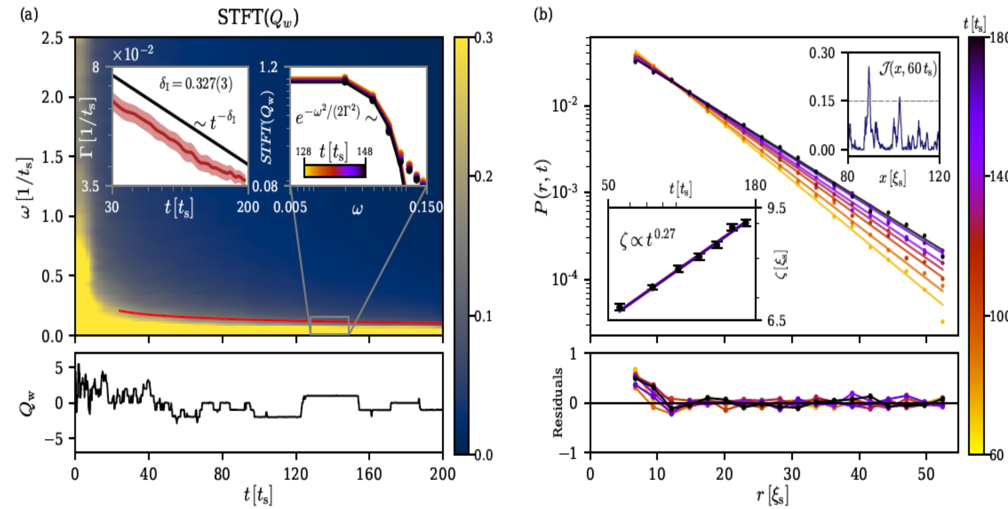
Universal dynamics of rare events

Rogue waves, caustics and instantons in a Spinor gas

New scaling exponent

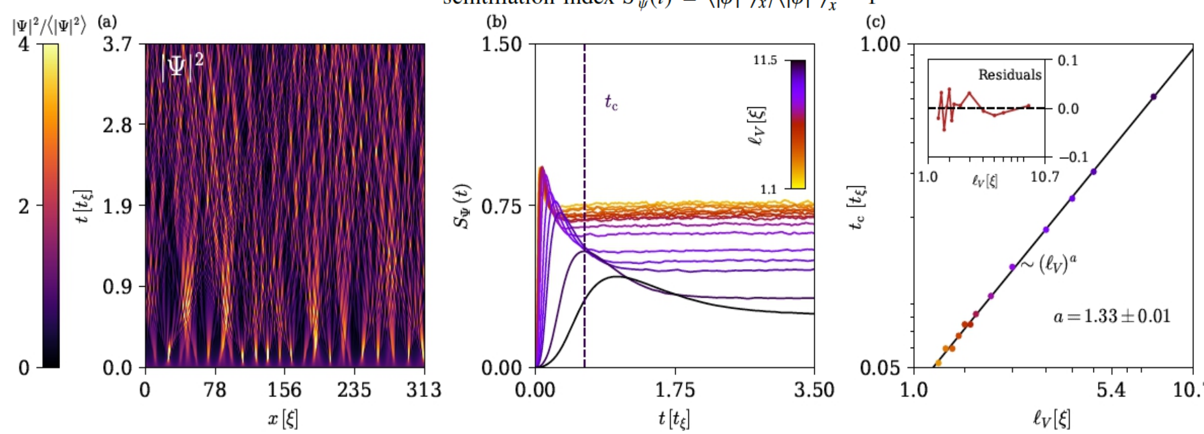
$$\delta = 4\beta/3$$





Caustics in GPE dynamics

scintillation index $S_\psi(t) = \langle|\psi|^4\rangle_x / \langle|\psi|^2\rangle_x^2 - 1$



I. Siovitz, S. Lannig, Y. Deller, H. Strobel, M.K. Oberthaler, TG, PRL **131**, 183402 (23); arXiv:2304.09293 [cond-mat.quant-gas]